Climate Change, Directed Innovation, and Energy Transition: The Long-run Consequences of the Shale Gas Revolution*

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Abstract

We investigate the short- and long-term effects of a natural gas boom in an economy where energy can be produced with coal, natural gas, or clean sources and the direction of technology is endogenous. In the short run, a natural gas boom reduces carbon emissions by inducing substitution away from coal. Yet, the natural gas boom discourages innovation directed at clean energy, which delays and can even permanently prevent the energy transition to zero carbon. We formalize and quantitatively evaluate these forces using a benchmark model of directed technical change for the energy sector. Quantitatively, the technology response to the shale gas boom results in a significant increase in emissions as the US economy is pushed into a "fossil-fuel trap" where long-run innovations shift away from renewables. Overall, the shale gas boom reduces social welfare under laissez-faire, whereas, combined with the appropriate policy responses, it could have increased welfare substantially.

Keywords: climate change, director technological change, energy, environment, natural gas, shale gas.

JEL Classification: O30, O41, O44, Q33, Q43, Q54, Q55

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1 Introduction

There is growing recognition that transitioning to cleaner, non-fossil sources of energy is an imperative for humanity to reduce and reverse damages from global temperature rises, which are now set to exceed the target of 1.5°C above preindustrial times established at the Paris Agreement. Because renewable energy sources such as wind and solar still face major intermittency challenges and lack sufficient infrastructure, one alternative is to work towards this transition by initially relying on "transition fuels" such as natural gas that generate fewer emissions (see e.g., IEA 2019).

The "US shale gas revolution"—arguably the most notable change in the US energy sector over the last several decades—could thus be seen as an enabler of the much-needed energy transition. Thanks to advances in horizontal drilling and hydraulic fracturing methods, US production of natural gas from shale deposits increased more than twelvefold and overall natural gas production rose by 50% between 2007 and 2018, as depicted in Figure 1 Panel A. The macroeconomic and full environmental consequences of this shale gas revolution have not been systematically studied.

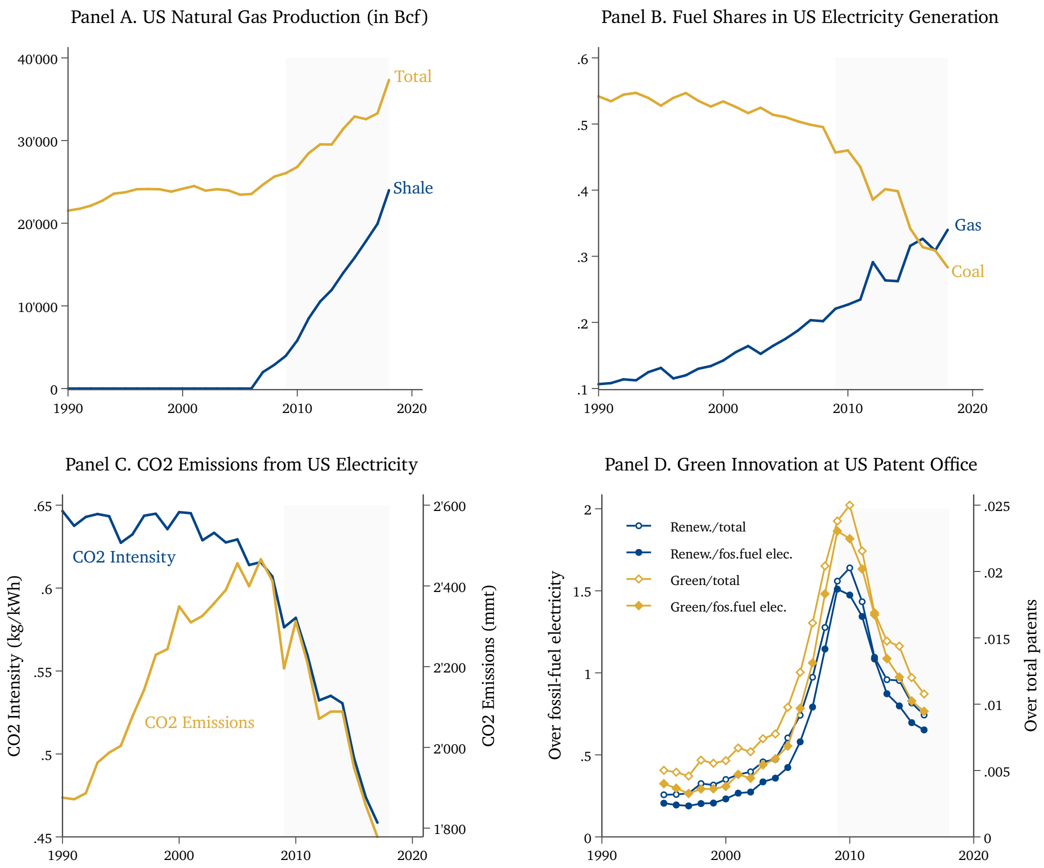
As shale gas production rose rapidly from 2009 onwards, natural gas displaced coal as the major source of fuel for the US electricity sector, as documented in Figure 1.B. Because natural gas emits significantly less carbon than coal per unit of energy, US CO_2 emissions from the electricity sector peaked in 2007 and have been on a downward trend since (see Figure 1.C).

Yet, increasing usage of cleaner fossil fuels like shale gas has a darker side as well. Greater efficiency may increase emissions both statically and dynamically. Statically, a version of the Jevons's paradox applies as growing usage of cheaper shale gas corresponds to an increase in the energy efficiency of fossil fuels overall. The resulting increase in energy consumption can then lead to more, rather than less, emissions. Evidence in Figure 1.C, however, sheds doubt on the relevance of this static channel. In this paper, we focus instead on the dynamic effects, which, to the best of our knowledge, are novel: greater energy efficiency of fossil fuels discourages innovation targeting cleaner (green) energy sources such as renewables and boosts long-run emissions.

Interestingly, a significant slowdown in innovation in renewable has taken place concurrently with the shale gas revolution, as we show in Figure 1.D: renewable patents in the US have declined from about 1.9% of total patents in 2009 to only 0.8% in 2016. One contribution of our paper is to document this new stylized fact (see Popp, Pless, Hascic, and Johnstone 2022, for additional evidence).

If the shale gas boom reduces emissions in the short-run but simultaneously displaces green innovation, then its overall impact on climate change mitigation and welfare are ambiguous and depend on the strength of the two opposing forces. Our major objective in this paper is to model, elucidate, and quantitatively evaluate these forces.

Figure 1—Natural Gas Production, Fuel Use, Emissions and Innovation in the US Electricity Sector



Note: This figure reports trends relevant to the US electricity sector. Panel A plots the total production of natural gas and the production of natural gas from shale (data source: EIA). Shale gas production takes off from 2007 onward. Panel B reports the share of electricity from coal and from gas (data source: EIA). Gas overtakes coal after the shale boom. Panel C shows the CO_2 intensity of electricity production (left-axis) and the total amount of CO_2 emissions (right axis), both drop following the shale boom (data source: US Environmental Protection Agency). Panel D reports ratios of either renewables or green (=renewables + nuclear + biofuel) patents over either fossil-fuel electricity or all patents (data source: PATSTAT). "Patents" here refer to USPTO patent applications. Innovation trends reverse after the boom.

With this purpose in mind, we build a stylized model of energy substitution and innovation. Energy can be produced with coal, natural gas, or a fully clean source, such as renewables. Natural gas has intermediate carbon emissions, and emissions create negative externalities both on domestic consumers and the rest of the world. The unique final good of the economy is produced by combining energy with other intermediates. Innovation is directed towards either fossil fuels or renewables, and can sustain long-run economic growth.

The model delivers the following insights. In the short run, a natural gas boom creates two opposing implications. First, there is a substitution effect, as natural gas is used increasingly in place of both dirtier coal-based energy and cleaner renewables. Under the plausible assumption that renewables are a small part of energy production initially, this substitution effect reduces carbon emissions. Second, and in opposition to this substitution mechanism, there is a scale effect. Namely, the shale gas boom reduces the overall price of energy which encourages energy consumption and thereby increases aggregate CO_2 emissions. The substitution effect dominates the scale effect and short-run emissions decline as long as natural gas is sufficiently clean compared to coal.

Long-run implications are more complex. The natural gas boom always reallocates scarce research inputs away from renewables towards fossil fuels, and consequently delays the energy transition. This effect may go beyond simple delay. We provide sufficient conditions for a "fossil-fuel trap" where the natural gas boom prevents the energy transition, while emissions would have converged to zero without the boom. With or without such a trap, the boom can reduce welfare. Overall, our theoretical analysis establishes how an unmanaged natural gas boom, such as the one unleashed by the recent shale gas revolution, can increase long-run carbon emissions. This conclusion stands in contrast to what the economy could have achieved with optimal policy responses, which we also characterize.

Our theoretical results raise the possibility of paradoxical welfare effects from technological improvements in natural gas extraction. Are these mere theoretical possibilities or actually relevant for the current energy transition challenge? We explore this question in the last part of the paper. We undertake a quantitative analysis of both short-run and long-run implications of natural gas booms. We start by calibrating the model to the electricity sector in the United States. To do this, we collect generator-level micro data on power plants to quantify different components of generation costs, such as fuel resource costs, production input costs, and local pollution abatement expenditures. We combine these estimates with data on electricity production, fossil-fuel extraction productivity, aggregate data on output and profit margins, and estimates of the elasticity of substitution across fuels.

Our benchmark results suggest that, in the short-run, the shale gas revolution led to an 11.2% decline in the CO_2 intensity of US electricity production and reduced emissions levels by about 4.2%. This underscores the possible environmental benefits from natural gas in general and shale gas in particular.

We then move to our main focus—the long-run implications from short-run substitution of natural gas for other types of energy. We estimate that an unmanaged shale gas boom leads to a fossil-fuel trap, with significant declines in green innovations. Our quantitative model matches well several targeted moments, as well as important untargeted moments, such as the pre- and post-boom levels of green relative to fossil-fuel innovations. In our benchmark quantification, electricity sector emissions start rising from 2028 and are about 25% higher by 2100 as a result of the boom. Our calibrated model predicts an overall intertemporal welfare loss from an unmanaged shale gas boom, equivalent to a 1.6% fall in yearly consumption.

We also demonstrate that the shale gas boom could have increased welfare considerably with optimal policies, which should have imposed greater subsidies to green technologies and higher carbon taxes. In fact, our benchmark estimates suggest that the shale gas boom has approximately *doubled* the potential welfare gains from adopting optimal climate policy.

Our paper contributes to a growing literature on the macroeconomics of climate change. A first strand develops "Integrated assessment models" (IAMs) for evaluating the macroeconomic and welfare impacts of climate change and various policies. This literature, pioneered by Nordhaus (e.g., 1994), has since grown considerably, including several recent macroeconomic works building on Golosov, Hassler, Krusell, and Tsyvinski (2014), and is reviewed by, e.g., Hassler, Krusell, and Smith Jr (2016). This literature neither focuses on endogenous and directed technology nor investigates the long-run implications of natural resource booms.

Most closely related to our analysis is the literature on directed technical change (DTC, e.g., Acemoglu 1998, 2002) applied in the context of climate change and the energy sector. Several papers have incorporated induced innovation into models of climate change (see Gillingham, Newell, and Pizer 2008, for a review of the early literature). Hassler, Krusell, and Olovsson (2021) and Smulders and de Nooij (2003) and Casey (2023) explore the consequences of DTC between energy-saving and energy-using technologies. We focus instead on DTC between clean and dirty technologies in energy production, building on and extending Acemoglu, Aghion, Bursztyn, and Hémous (2012), henceforth AABH, who characterize the optimal climate policy in the presence of DTC.[1] A number of papers have since extended AABH: Acemoglu, Akcigit, Hanley, and Kerr (2016) build a firm-level, dynamic model of energy transition; Hémous (2016) embeds AABH in a trade framework to study the effects of unilateral policies; Fried (2018) looks at the implications of an exogenous oil price shock; and Lemoine (forthcoming) analyzes endogenous energy transitions when there are separate resource and non-resource inputs in energy production; Aghion, Bénabou, Martin, and Roulet (2023) investigate the joint impact of consumers' environmental concerns and market competition on firms' incentives to innovate in clean technologies (see Hémous and Olsen (2021) for a literature review). Our contribution relative to this literature stems from the new question we are investigating—the long-run innovation and climate consequences of a natural resource boom—and our detailed quantitative exercise, making use of data on clean and dirty patents and micro estimates of productivity and elasticities from the electricity sector, which is generally recognized as the most important part of the economy for decarbonization (Barron, Fawcett, Hafstead, McFarland, and Morris 2018).

[1] Aghion, Dechezleprêtre, Hémous, Martin, and Reenen (2016) provide empirical evidence for both DTC and path-dependence in the choice between clean and dirty technologies in the car industry. See also Newell, Jaffe, and Stavins (1999), Popp (2002) and Calel and Dechezleprêtre (2016) for further evidence for DTC in the environmental context.

We also build on existing computational energy models and empirical electricity sector analyses to study the shale gas boom. Quantitative models have found mixed net impacts owing to substitution and scale effects (see, e.g., the multi-model study by McJeon, Edmonds, Bauer, Clarke, Fisher, Flannery, Hilaire, Krey, Marangoni, Mi, et al. 2014, and Gillingham and Huang 2019; Burtraw, Palmer, Paul, and Woerman 2012; Venkatesh, Jaramillo, Griffin, and Matthews 2012, and Brown and Krupnick 2010), while empirical studies have estimated significant short-run declines in the CO_2 emissions of electricity production as a result of the boom (e.g., Cullen and Mansur 2017; Fell and Kaffine 2018; Holladay and LaRiviere 2017; Linn and Muehlenbachs 2018). Loosely speaking, these papers correspond to the static component of our model and do not consider long-run technology implications, which are our main focus and contribution (though some works in this literature, such as Bauer, Mouratiadou, Luderer, Baumstark, Brecha, Edenhofer, and Kriegler 2016 or Bosetti, Carraro, Galeotti, Massetti, and Tavoni 2006, feature learning-by-doing in the energy sector). It is also worth noting that the empirical estimates in this literature are broadly in line with the static emission effects of the decline in natural gas prices in our model.

There is also an emerging literature investigating various broader consequences of the shale gas revolution. These include the impacts of hydraulic fracking on a range of outcomes including house prices (Muehlenbachs, Spiller, and Timmins 2015), local economies (Allcott and Keniston 2018; Feyrer, Mansur, and Sacerdote 2017), and local/sectoral welfare (Bartik, Currie, Greenstone, and Knittel 2019; Hausman and Kellogg 2015). More closely related are Knittel, Metaxoglou, and Trindade (2016) and Daubanes, Henriet, and Schubert (2021), who model the possibility of carbon leakage through increased exports of coal and oil following the expansion of shale gas, and EIA (2015), Gillingham and Huang (2019) and Henriet and Schubert (2019), who point out the possibility that shale gas may delay the deployment of renewables. These works do not study the effects of the shale gas boom on the direction of future technology, which is our main contribution.

The rest of the paper is organized as follows. Section 2 presents evidence on the decline in green innovation and the role that natural gas prices have played in this redirection of technology. Section 3 develops our theoretical framework, and provides conditions under which a natural gas boom reduces emissions in the short run but increases them in the long-run because of the induced redirection of innovation. This section also characterizes optimal policy in the presence of a natural gas boom. Using data from the US electricity sector, Section 4 provides a quantitative analysis of the implications of our model, focusing especially on long-run consequences. Section 5 presents some extensions, and Section 6 concludes. Appendix A contains the proofs of our main results, additional empirical analyses, and various robustness exercises, and is included with the paper. Proofs of our secondary results can be found in the Supplementary Material available on our website at https://www.econ.uzh.ch/en/people/faculty/hemous/research.html.

2 The Shale Gas Revolution and Green Innovations

The key building block of our study is the negative impact of natural gas (and specifically the shale gas revolution) on green innovations. A first contribution of our paper is to document a large decline in green patents taking place concurrently with the shale gas boom.

We rely on the World Patent Statistical Database (PATSTAT), which contains detailed information on patents from most patent offices in the world. We use the International Patent Classification (IPC) and the extended Cooperative Patent Classification (CPC) to identify green and fossil-fuel patents.[2] We refer to these patents as fossil-fuel patents (without emphasizing that they are for the electricity sector). Importantly, these *do not* include patents on extraction technologies such as hydraulic fracking and horizontal drilling that have been foundational to the shale gas boom.

Green innovations are identified as a subset of those in the technological subclass Y02 of the CPC, which include technologies that reduce greenhouse gases; those in the group Y02E10, which correspond to renewable electricity (geothermal, hydro, tidal, solar thermal, photovoltaic and wind); those in the group Y02E30 for nuclear energy; and those in Y02E50, which include biofuels and fuel from waste.[3] We assign patents to countries according to the location of the patent office at which they were filed. Because more recent patent data are incomplete, our sample stops in 2016.

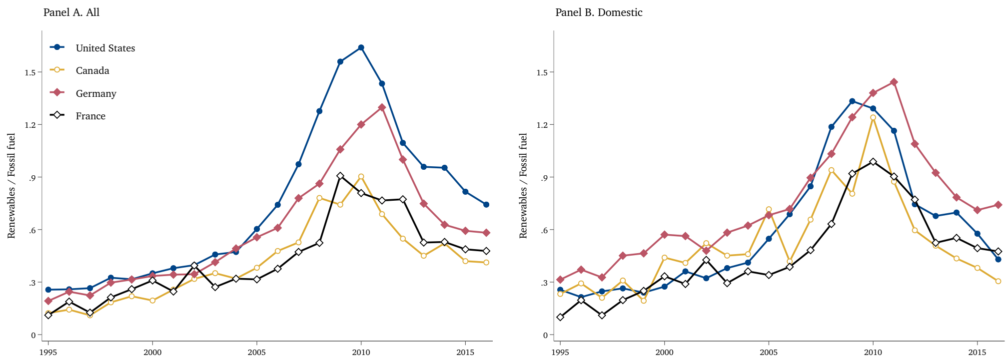
Figure 1.D in the Introduction plots patent applications at the USPTO, with year corresponding to the date of first filing. A sharp decline in the ratio of renewable to fossil-fuel patent applications after 2009-2010 is clearly visible. Figure 2 plots the same ratio for Canada, France and Germany, with Panel A for all patents, and Panel B for patents by domestic inventors (counting patents fractionally when inventors from multiple locations are listed). The same sharp decline from 2009-2010 is again visible. Note additionally that the reversal appears to have occurred a bit earlier for the United States and Canada, the two countries which first exploited shale gas, but is also quite sharp for France and Germany, even though these countries do not exploit shale gas. Nevertheless, innovation trends in France and Germany are relevant for our inquiry, for

[2]Specifically, we build on Lanzi, Verdolini, and Hascic (2011) who identified IPC codes corresponding to fossil-fuel technologies for electricity generation. We count as fossil-fuel patents those with an IPC or CPC code in their list. The full list of codes is given in their Appendix A.1. We exclude the small fraction of patents without CPC codes from our analysis.

[3]Nuclear energy poses environmental and safety hazards, but does not generate greenhouse gases. Biofuels are used for transportation but also for electricity generations. Crucially, our green innovations exclude those aimed at reducing pollution from fossil-fuel electricity generation (Y02E20). They also do not include innovations aimed at improving the efficiency of the grid (Y02E40) and storage (Y02E60), since those are not technologies that compete directly with fossil-fuel technologies.

at least two reasons. First, European inventors sell globally and are therefore affected by North American shocks.[4] Second, natural gas prices have also fallen in these countries, both because of the US shale gas revolution and because of the expansion of Russian natural gas. Appendix A.1 shows similar patterns for the ratio of green to fossil-fuel patents or for renewable patents relative to total patents.

Figure 2—Natural Gas Production, Fuel Use, Emissions and Innovation in the US Electricity Sector



Note: This figure reports the ratio of renewables to fossil-fuel patents in the US, Canada, France and Germany (data source: PATSTAT). Patents are allocated to countries according to their patent office. In Panel A, we count all patents, while in Panel B, we only count patents by domestic inventors (allocating patents fractionally if inventors from multiple countries are listed). The reversal in innovation occurs in all four countries.

While our theory focuses on the effects of the shale gas revolution for innovation incentives, these incentives are also impacted by various other factors. To confirm the central role of natural gas prices and obtain estimates that correspond to the elasticity of green innovation to natural gas prices, we next turn to a regression analysis.

We build an unbalanced panel spanning the time period 1978-2016 for 32 countries using data on indexed real industry natural gas prices from the International Energy Agency (IEA). We then estimate the following empirical relationship between the direction of innovation and natural gas prices:

$$\sinh^{-1}\left(y^{g}_{c,t}\right) - \sinh^{-1}\left(y^{f}_{c,t}\right) = \beta_p \ln p_{c,t-2} + \beta_X X_{c,t-2} + \delta_c + \delta_t + \varepsilon_{c,t}.$$

Here $y^{f}_{c,t}$ is the number of fossil-fuel patents, and $y^{g}_{c,t}$ denotes the number of green patents in the patent office of country c in year t (Appendix A.1 repeats this exercise focusing just on renewable patents). We use the inverse hyperbolic sine transformation, $\sinh^{-1}$ to accommodate zeros in patent counts, so our left-hand side is approximately equal to the log ratio of green to fossil-fuel patents $\ln\left(y^{g}_{c,t}/y^{f}_{c,t}\right)$ when patents counts are positive. The variable $p_{c,t}$ is the real indexed industrial natural gas price, and we use the two-year lag of this price on the right-hand side to accommodate delay in the impact of

[4]Prior empirical work also confirms the role of global incentives in renewable innovation (e.g., Dechezleprêtre and Glachant 2014; Peters, Schneider, Griesshaber, and Hoffmann 2012).

natural gas prices on innovation incentives. In addition, $X_{c,t}$ is a vector of controls which includes GDP per capita (from the OECD), public R&D expenditures in fossil-fuel energy or green energy (from the IEA) and log energy consumption (from the World Bank), δ_c and δ_t are country- and year- fixed effects, and $\varepsilon_{c,t}$ is an error term, meant to capture omitted factors.

Table 1 confirms that there is a positive and significant correlation between the ratio of green to fossil-fuel patents and natural gas prices. Columns (1)-(3) consider all patents, while columns (4)-(6) focus on patents by domestic inventors. The coefficients yield elasticities (except for approximation due to the use of $\sinh^{-1}$ instead of log on the left-hand side). Hence, column (6) indicates that a 1% increase in natural gas prices is associated with a relative increase in domestic green patents compared to fossil-fuel patents of 0.246%. This elasticity is close to Popp's (2002) estimate of the effect of energy prices on energy-saving innovations (which is an elasticity of around 0.3).[5]

Two additional issues are worth discussing. First, the results reported in Table 1 should be interpreted as correlations, which could be impacted by various omitted variables, and other factors may have driven the decline in green innovation documented in Figure 2. Changes in public R&D spending do not appear to have contributed to the decline in green innovation as they do not display a similar reversal. The rise of Chinese solar panel production may have also contributed to this reversal, but is unlikely to be its main driver in the United States, because US innovation in wind technologies show the same decline.[6] Nevertheless, other omitted factors, such as the exhaustion of low-hanging innovation opportunities in green technologies, may have played a role (see Popp et al. 2022, for further discussion).

Second, green innovation matters. While learning-by-doing and scale economies in solar panel production have been important, existing evidence points to a central role for green innovations in the large declines in the costs of renewable energy production. For example, in solar panels, it was new technologies that enabled the deployment of larger crystals for ingots and allowed the cutting of ingots into thinner wafers (Carvalho, Dechezleprêtre, and Glachant 2017). In fact, around half of patented innovations in solar photovoltaic cells in the United States concern the currently dominant technology and are therefore relevant for these cost reductions. In a recent study, Kavlak, McNerney, and Trancik (2018) estimate that around 60% of the global cost decline in solar panels between 1980 and 2012 can be attributed to public and private R&D. Interestingly, even in the Chinese case, innovations play a major role. For example, about half of the decline in wind turbine prices in China between 1998 and 2012 appears driven by new innovations (Yu, Li, Che, and Zheng 2017). New technological breakthroughs may be even more

[5]In these regressions, global shocks to natural gas prices are captured by the time fixed effects, and so is the global response of inventors. We show in the Appendix that if time effects are omitted, the relationship between natural gas prices and green patents we estimate in Table 1 becomes stronger.

[6]See Figure A.3 in the Appendix. Moreover, we see a similar reversal in China as well.

Table 1—Innovation and Gas prices

Dependent Variable:	Green - Fossil-Fuel Electricticity Patents					
Inventors:	All			Domestic		
	(1)	(2)	(3)	(4)	(5)	(6)
ln(Gas Price Index)	0.21	0.25	0.22	0.23	0.23	0.25
	(0.10)	(0.13)	(0.11)	(0.15)	(0.09)	(0.10)
ln(GDP/capita)	1.16	2.08	2.22	−0.99	2.56	2.89
	(0.37)	(0.54)	(0.58)	(0.21)	(0.76)	(0.87)
ln(Public R&D Fossil)		−0.01	0.00		−0.07	−0.06
		(0.03)	(0.02)		(0.03)	(0.04)
ln(Public R&D Green)		0.06	0.08		−0.00	0.01
		(0.04)	(0.06)		(0.09)	(0.09)
ln(Energy consumption)			−0.41			−0.43
			(0.43)			(0.75)
Year fixed effects	✓	✓	✓	✓	✓	✓
Country fixed effects	✓	✓	✓	✓	✓	✓
R-squared	0.91	0.93	0.94	0.83	0.88	0.88
Observations	923	636	636	887	608	608
Countries	32	29	29	32	29	29

Note: This table presents the results of panel regressions of the direction of innovation on gas prices. The direction of innovation is measured as the $\sinh^{-1}$ difference between the number of green patents in a country and the number of fossil-fuel patents. Patents are allocated to a country according to the location of the patent office and dated from the year of first filing. The independent variables are lagged by two periods. Columns (1) to (3) include all patents, Columns (4) to (6) only include patents by domestic inventors (patents are counted fractionally if there are multiple inventors' nationalities). Gas prices are measured as the log of a real gas price index from the IEA. All regressions control for log GDP per capita, country and year fixed effects. Columns (2), (3), (5) and (6) add controls for log public R&D expenditures in green and fossil-fuel technologies. Columns (3) and (6) also control for log energy consumption. Each country is weighted by the sum of its green and fossil-fuel patent counts over the years. The list of countries is: AT, AU, BE, CA, CH, CZ, DE, DK, EE, ES, FI, FR, GB, GR, HU, IE, IT, JP, KR, LT, LU, LV, MX, NL, NZ, PL, PT, SE, SI, SK, TR, US. Standard errors are clustered at the country-level.

important for future advances.

Overall, this section shows that innovation in the electricity sector has been sharply redirected away from renewable and green technologies concurrently with the shale gas revolution in the United States. We next develop our conceptual framework, which will enable us to model the short-run and long-run implications of this technology redirection.

3 Theory

In this section, we present our conceptual framework, which models the static and dynamic substitution between three different types of energy—coal, natural gas and green. Dynamic substitution results from directed innovation. After describing the basic outlines of the model, we solve for the static equilibrium and explore the short-term impact of a natural gas boom. We then turn to the dynamic equilibrium, where the direction of innovation responds to the natural gas price. We finally characterize optimal policy in this framework.

3.1 Preferences, Production Technology and the Environment

Time is discrete and the economy is populated by a mass 1 of identical households who live for one period and do not make any intertemporal decisions. We define social welfare as

$$U_t = \sum_{\tau=t}^{\infty} \frac{1}{(1+\rho)^{\tau-t}} \frac{C_\tau^{1-\vartheta}}{1-\vartheta}, \tag{1}$$

where C_τ is consumption, ρ is the social planner's rate of time preference, and ϑ is the inverse elasticity of intertemporal substitution. Households inelastically supply L units of production labor and one unit of scientist labor used in innovation.

There is a unique final good, produced with the technology

$$Y_t = (1 - D(S_t)) \left((1-\nu) Y_{Pt}^{\frac{\lambda-1}{\lambda}} + \nu \left(\widetilde{A}_E E_t \right)^{\frac{\lambda-1}{\lambda}} \right)^{\frac{\lambda}{\lambda-1}}, \tag{2}$$

where $\nu \in (0,1)$, E_t is an energy composite, Y_{Pt} is a production input, $\widetilde{A}_E$ represents energy efficiency and λ is the elasticity of substitution between energy and the production input. We assume $\lambda \in (0,1)$ so that energy and other inputs are gross complements. There are no savings and the final good is used only for consumption, so that $Y_t = C_t$. The variable S_t is the carbon concentration in the atmosphere and the function $D(S_t)$ represents the environmental damage on production. We adopt Golosov, Hassler, Krusell, and Tsyvinski's (2014) representation and assume that $D(S_t) = 1 - e^{-\zeta(S_t - S_0)}$, where S_0 is the pre-industrial carbon concentration and $\zeta > 0$. The production input is produced according to

$$Y_{Pt} = A_{Pt} L_{Pt},$$

where A_{Pt} is a productivity parameter and L_{Pt} is labor used in the production sector.

The energy composite is generated according to the function

$$E_t = \left(\kappa_c E_{ct}^{\frac{\varepsilon-1}{\varepsilon}} + \kappa_s E_{st}^{\frac{\varepsilon-1}{\varepsilon}} + \kappa_g E_{gt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{3}$$

where E_{ct}, E_{st}, and E_{gt} respectively denote coal, natural gas, and green (renewable) energy. In addition, the κ's are share parameters. This specification implies that the three types of energy are substitutes with an elasticity of substitution $\varepsilon > 1$. In Section 4, we allow for different elasticities within fossil fuels and between fossil fuels and green technologies.

Energy production of each type $i \in \{c, s, g\}$ combines an extracted resource R_{it} (such as coal or gas) together with fuel-specific energy ("power plant") inputs Q_{it}, with the

Leontief production function

$$E_{it} = \min\{Q_{it}, R_{it}\}. \tag{4}$$

The Leontief technology implies that, in equilibrium, $E_{it} = Q_{it} = R_{it}$. The power plant input for each $i \in \{c, s, g\}$ is produced according to a Cobb-Douglas production function,

$$Q_{it} = \exp\left(\int_0^1 \ln q_{ijt} dj\right) \tag{5}$$

where q_{ijt} is an intermediate supplied by technology monopolist j for energy type i. We assume that all intermediates are produced linearly using labor:

$$q_{ijt} = A_{ijt} l_{ijt}^q, \tag{6}$$

where l_{ijt}^q denotes the amount of labor hired and A_{ijt} denotes the productivity of intermediate j for energy type i at time t. Average productivity for energy type i at time t is

$$\ln A_{it} = \int_0^1 \ln A_{ijt} dj, \tag{7}$$

and summarizes one dimension of energy technology.

The other dimension pertains to resource extraction. Extraction for green technology is assumed to be free (e.g., from wind or the sun), while extraction is costly for coal and natural gas. We also allow technological change in resource extraction as we explain below. Specifically, extracting one unit of coal or natural gas requires one unit of an extraction input. With a slight abuse of notation, we denote the extraction input for energy type $i \in \{c, s\}$ by R_{it} (since the amount of resource extracted is equal to this input). We model the production of the extraction input analogously, with a Cobb-Douglas aggregator of intermediates,

$$R_{it} = \exp\left(\int_0^1 \ln r_{ijt} dj\right),$$

where each extraction intermediate r_{ijt} is produced linearly with labor l_{ijt}^r and productivity B_{ijt}: $r_{ijt} = B_{ijt} l_{ijt}^r$. We also define average productivity in extraction for energy type $i \in \{c, s\}$ as

$$\ln B_{it} = \int_0^1 \ln B_{ijt} dj.$$

Finally, we assume that, like renewables, coal and natural gas are in infinite supply, so there is no possibility of resource exhaustion.[7]

[7]Coal reserves that can be recovered with the current technology in the United States are 470 times the current level of consumption, while the "demonstrated reserve base" that can be extracted in the future is

We capture environmental damages by tracking the behavior of the stock of carbon in the atmosphere, S_t. Fossil fuels generate greenhouse gas emissions and increase this carbon stock. We denote the carbon intensity of electricity production from coal and gas by, respectively, ξ_c and $\xi_s < \xi_c$. This inequality implies that natural gas is cleaner than coal. Green energy generates zero greenhouse gas emissions, $\xi_g = 0$. We denote emissions from energy type i at date t by $P_{it} = \xi_i R_{it}$. Aggregate emissions are then given by $P_t = \xi_c R_{ct} + \xi_s R_{st}$. The behavior of the carbon stock S_t depends on these aggregate emissions and on the absorption of the existing carbon stock by oceans and other means. These exact dynamics are not central to our theoretical results, but for completeness and for our quantitative analysis, we adopt the carbon cycle specification of Golosov et al. (2014), so that:

$$S_t = \overline{S} + \sum_{s=0}^{t+T} (\varphi_L + (1-\varphi_L)\varphi_0(1-\varphi_d)^s) P_{t-s}, \tag{8}$$

where $\overline{S}$ is pre-industrial carbon concentration. This formulation reflects that a share φ_L of emissions stay in the atmosphere forever, while out of the remaining emissions, a share $1-\varphi_0$ is immediately absorbed and the rest decays geometrically at the rate φ_d. In the quantitative section, we also incorporate emissions from the rest of the world.

3.2 Innovation and the Direction of Technology

Intermediate productivities, the A_{ijt}'s, increase over time due to innovation, building on the previous best vintage. We assume that innovation uses only scientist labor as input. Scientists that innovate successfully over an intermediate raise that intermediate's productivity by a factor $\gamma > 1$, so that $A_{ijt} = \gamma A_{ij(t-1)}$ when there is innovation at date t for intermediate j for energy type i. Following such an innovation, the scientist becomes the monopolist supplier of the intermediate. We assume that this monopolist is constrained by the next-best (previous) technology and, in order to exclude entrants, sets a limit price with a gross markup of γ.

Innovation is directed, and in particular, scientists decide to allocate their research efforts between the fossil-fuel energy inputs or the green energy input. This formulation is motivated by the fact that, in practice, many inputs in coal and natural gas power plants are similar and keeping track of only two technologies simplifies the analysis (see Section 5.2 for an extension where innovation is directed between the three sectors). There is potentially congestion in research effort, with different scientists chasing similar new ideas when working in the same field. Consequently, the probability of success of innovation directed at energy type i at time t is $\eta s_{it}^{-\psi}$ per scientist, where s_{it} is the

twice this amount (EIA 2021). For natural gas, the amount of recoverable resources are about 98 times the current level of consumption (EIA 2022), and reserves of methane hydrates, which could be commercially viable with future technologies, are estimated to be much larger.

total number of scientists exerting effort towards innovations for this energy type, ψ parameterizes the extent of the congestion effects (diminishing returns), and η represents research productivity. For simplicity, and without any major loss of insight, we assume the same research productivity in both sectors. As a result, the evolution of the average productivity is in the production of the three types of energy can be written as:[8]

$$A_{ct} = \gamma^{\eta s_{ft}^{1-\psi}} A_{c(t-1)}, A_{st} = \gamma^{\eta s_{ft}^{1-\psi}} A_{s(t-1)} \text{ and } A_{gt} = \gamma^{\eta s_{gt}^{1-\psi}} A_{g(t-1)}.$$

As in AABH, we simplify the analysis of the direction of technology by assuming that patents only last one period so that scientists maximize profits in the current period (rather than the discounted sum of future profits). This simplification is immaterial given our focus, and Acemoglu et al. (2016) incorporate forward-looking innovation behavior in a similar setup.

Finally, we assume that the productivities in extraction, B_{ct} and B_{st}, and in input production, A_{Pt}, evolve exogenously. This assumption is adopted for simplicity, and we outline in Section 5.1 how extraction technologies can be endogenized. For added simplicity and to amplify the parallel between the energy inputs and the other inputs, we assume that extraction intermediates and the production input are supplied with the same gross markup as the energy intermediates, $\gamma > 1$. Our main focus will be to study how an exogenous improvement in the extraction of natural gas, B_{st}, affects emissions and the direction of innovation.

3.3 Short-run Effects of a Natural Gas Boom

We first take the productivity of different intermediates, the A_{ijt}'s, and the extraction sector, B_{ct} and B_{st}, as given and focus on the static equilibrium. A static equilibrium is defined as an allocation in which all energy types and the final good production sector minimize costs, the intermediate monopolists maximize profits, and all markets clear. It is straightforward to verify that a static equilibrium always exists and is unique, and we now proceed to characterize it.

For notational simplicity, we drop the subscript t in this subsection. We take the final good to be the numeraire throughout, and let p_i^q denote the price of the energy input and p_i^r the price of the resource extraction input ($p_g^r = 0$ since extraction is free in green technologies). With Cobb-Douglas production and Bertrand competition from the next-best technology, the equilibrium price of the energy intermediate ij is equal to

[8] This follows because $\ln A_t - \ln A_{t-1} = \int_0^1 \ln A_{jt} dj - \int_0^1 \ln A_{j(t-1)} dj = \int_0^1 \varepsilon_{jt} dj$, where ε_{jt} is an iid random variable that takes the value zero with probability $1 - \eta s^{1-\psi}$ (no innovation) and the value $\ln \gamma$ with probability $\eta s_t^{1-\psi}$ (innovation). Appealing to the law of large numbers (and ignoring technical details to do with continuums), this gives $\ln A_t - \ln A_{t-1} = \mathbb{E}[\varepsilon_{jt}] = \eta s_t^{1-\psi} \ln \gamma$. The expression in the text follows by taking exponents.

$p_{ij}^q = \gamma w / A_{ij}$. Aggregating across intermediates, we obtain the price of the energy input i as

$$p_i^q = \frac{\gamma w}{A_i}. \tag{9}$$

The resulting profits for intermediate ij are

$$\pi_{ij}^q \equiv \left(1 - \frac{1}{\gamma}\right) p_i^q Q_i. \tag{10}$$

Under our assumption that there is also a gross markup equal to γ for extraction intermediates, we obtain the price of extracted resource input as

$$p_i^r = \frac{\gamma w}{B_i}.$$

Next, the Leontief technology imposes that the equilibrium price of electricity of type i will be equal to the cost of the power plant and extraction inputs, and thus

$$p_i = p_i^q + p_i^r = \frac{\gamma w}{C_i} \text{ with } \frac{1}{C_i} \equiv \frac{1}{A_i} + \frac{1}{B_i}, \tag{11}$$

where C_i, the harmonic mean of A_i and B_i, gives the overall productivity in the production of electricity of type $i \in \{c, s, g\}$.

For each energy type i, cost-minimization implies

$$E_i = \kappa_i^\varepsilon \left(\frac{C_i}{C_E}\right)^\varepsilon E \text{ with } C_E \equiv \left(\kappa_c^\varepsilon C_c^{\varepsilon-1} + \kappa_s^\varepsilon C_s^{\varepsilon-1} + \kappa_g^\varepsilon C_g^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}. \tag{12}$$

C_E is the overall productivity of the energy sector. The equilibrium price of the energy composite and the equilibrium level of production are then

$$p_E = \frac{\gamma w}{C_E} \text{ and } E = C_E L_E, \tag{13}$$

where L_E is total labor hired by the energy sector.[9]

The relative sizes of the energy subsectors depend on their relative productivities and are given by

$$\Theta_i = \frac{p_i E_i}{p_E E} = \kappa_i^\varepsilon \left(\frac{C_i}{C_E}\right)^{\varepsilon-1}.$$

[9]The allocation of labor follows from cost-minimization in the final good sector. Taking the ratio of the first-order conditions with respect to E and L_P, and using labor market clearing, we get

$$L_E = \frac{\nu^\lambda \widetilde{A}_E^{\lambda-1} C_E^{\lambda-1}}{\nu^\lambda \widetilde{A}_E^{\lambda-1} C_E^{\lambda-1} + (1-\nu)^\lambda A_P^{\lambda-1}} L. \tag{14}$$

Labor in the energy sector decreases with average productivity C_E, because energy and production inputs are gross complements ($\lambda < 1$).

The equilibrium level of pollution can be computed as

$$P = \xi_E E \text{ with } \xi_E \equiv \xi_c \kappa_c^{\varepsilon} \left(\frac{C_c}{C_E}\right)^{\varepsilon} + \xi_s \kappa_s^{\varepsilon} \left(\frac{C_s}{C_E}\right)^{\varepsilon}, \tag{15}$$

where ξ_E measures the average emission intensity of energy production.

We now consider the implications of a natural gas boom. Motivated by the shale gas revolution in the United States, we take the driver of this natural gas boom to be an increase in the productivity of extraction for gas, B_s. Our main focus is on total emissions, P.

The static impact of the natural gas boom on emissions can be decomposed into a substitution and a scale effect:

$$\frac{\partial \ln P}{\partial \ln B_s} = \underbrace{\frac{\partial \ln \xi_E}{\partial \ln B_s}}_{\text{substitution effect}} + \underbrace{\frac{\partial \ln E}{\partial \ln B_s}}_{\text{scale effect}}. \tag{16}$$

The substitution effect is rooted in the changes in the average pollution intensity of energy resulting from the natural gas boom, while the scale effect is driven by the expansion of energy due to the sector's higher average productivity. The scale effect is closely related to, but different from the Jevons's paradox, which results when the efficiency of a resource increases, raising its overall use. Here, the natural gas boom does not directly increase resource efficiency, though it improves the average efficiency of the energy sector (or the fossil-fuel component of that sector).

Due to the intermediate emission intensity of natural gas, the substitution effect has an ambiguous sign. It will be negative when natural gas mostly replaces coal, but positive when it mostly replaces green energy. Mathematically, we can express this substitution effect as

$$\frac{\partial \ln \xi_E}{\partial \ln B_s} = \varepsilon \frac{\partial \ln C_s}{\partial \ln B_s}\left(\frac{P_s}{P} - \Theta_s\right), \tag{17}$$

where recall that Θ_s is the revenue share of natural gas in the energy sector, while $P_s/P = \xi_c \kappa_c^{\varepsilon} C_c^{\varepsilon} / \left(\xi_c \kappa_c^{\varepsilon} C_c^{\varepsilon} + \xi_s \kappa_s^{\varepsilon} C_s^{\varepsilon}\right)$ is its emissions share. In addition, $\partial \ln C_s / \partial \ln B_s = C_s / B_s > 0$ represents the effect of an increase of the extraction technology on the average productivity of natural gas energy. This expression clarifies that the substitution effect will be negative, and natural gas will reduce emissions at given scale, when $P_s/P - \Theta_s < 0$. This condition is automatically satisfied when the revenue share of green energy, Θ_g, is small, since in that case $\Theta_s = \kappa_s^{\varepsilon} C_s^{\varepsilon-1} / \left[\kappa_c^{\varepsilon} C_c^{\varepsilon-1} + \kappa_s^{\varepsilon} C_s^{\varepsilon-1}\right]$ is always greater than P_s/P since $\xi_s < \xi_c$. It is also more likely to be negative when the emission intensity of natural gas, ξ_s, is relatively low.

The scale effect term, on the other hand, is always positive and equal to

$$\frac{\partial \ln E}{\partial \ln B_s} = \frac{C_s}{B_s}\Theta_s\left(\lambda + (1-\lambda)\Omega_E\right),$$

where $\Omega_E \equiv p_E E/Y$ is the revenue share of energy in the economy. The scale effect is larger when less labor gets reallocated from the energy sector to the production sector, which occurs when the elasticity λ is larger and when the energy input is more important (Ω_E is larger).

Thus the overall impact of a natural gas boom on pollution is given by

$$\frac{\partial \ln P}{\partial \ln B_s} = \frac{C_s}{B_s}\left(\varepsilon\left(\frac{P_s}{P} - \Theta_s\right) + \Theta_s\left(\lambda + (1-\lambda)\Omega_E\right)\right).$$

Since $\varepsilon > 1$ and $\lambda < 1$, a negative substitution effect may dominate the scale effect and in fact does so provided that natural gas is sufficiently clean relative to coal.[10] This establishes the main result of the static analysis:

Proposition 1 *A natural gas boom (a one time increase in B_s) leads to a decrease in emissions in the short-run provided that natural gas is sufficiently clean compared to coal (that is, provided that ξ_s/ξ_c is sufficiently small).*

3.4 Directed Innovation and the Dynamic Equilibrium

A dynamic equilibrium is a sequence of static equilibria with the vector of productivities for power plant inputs in the energy sector, the A_{ijt}'s, evolving according to the equilibrium allocation of scientists and the productivities for extraction inputs, B_{ct} and B_{st}, and the production input A_{Pt} evolving exogenously. The allocation of scientists is determined by an innovation equilibrium condition, requiring that they expect the same returns from devoting effort to fossil-fuel and green innovations.[11] These returns are the static profits, (10), multiplied by the probability of success. Thus, the expected returns from innovation in green energy are

$$\Pi_{gt} = \eta s_{gt}^{-\psi}\left(1 - \frac{1}{\gamma}\right)p_{gt}E_{gt}. \tag{18}$$

[10] Substituting for P_s/P, Θ_s and Ω_E, we have that $\partial \ln P/\partial \ln B_s < 0$ if and only if

$$\frac{\xi_s}{\xi_c} < \frac{\kappa_c^{\varepsilon} C_c^{\varepsilon}\left[\varepsilon - \left(\lambda + (1-\lambda)\frac{\nu^{\lambda}\widetilde{A}_{Et}^{\lambda-1}C_E^{\lambda-1}}{\nu^{\lambda}\widetilde{A}_{Et}^{\lambda-1}C_E^{\lambda-1}+(1-\nu)^{\lambda-1}A_P^{\lambda-1}}\right)\right]}{\left[\kappa_s^{\varepsilon} C_s^{\varepsilon}\left(\lambda + (1-\lambda)\frac{\nu^{\lambda}\widetilde{A}_{Et}^{\lambda-1}C_E^{\lambda-1}}{\nu^{\lambda}\widetilde{A}_{Et}^{\lambda-1}C_E^{\lambda-1}+(1-\nu)^{\lambda-1}A_P^{\lambda-1}}\right) + \varepsilon C_s\left(\kappa_c^{\varepsilon} C_c^{\varepsilon-1} + \kappa_g^{\varepsilon} C_g^{\varepsilon-1}\right)\right]}.$$

[11] Since $\psi > 0$, for any finite t, there cannot be a corner equilibrium in which all scientists work on one type of technology. But asymptotically, the economy can converge to an equilibrium in which all innovation is in one of the two technologies.

Similarly, the expected profits of devoting innovation efforts to fossil fuel are

$$\Pi_{ft} = \eta s_{ft}^{-\psi}\left(1-\frac{1}{\gamma}\right)\left(p_{ct}^{q}Q_{ct}+p_{st}^{q}Q_{st}\right) = \eta s_{ft}^{-\psi}\left(1-\frac{1}{\gamma}\right)\left(\frac{C_{ct}}{A_{ct}}p_{ct}E_{ct}+\frac{C_{st}}{A_{st}}p_{st}E_{st}\right). \quad (19)$$

This last expression incorporates the fact that fossil-fuel innovations are used both by coal and natural gas inputs. Notice also that power plant inputs for energy type i only receive a share C_i/A_i of the revenues generated by this type of energy, with the remainder accruing to the extraction input because of the Leontief technology. Since innovation only responds to current profits, the discount rate, ρ, does not matter for the dynamic equilibrium allocation.

Hence, the innovation equilibrium condition can be written as

$$\frac{\Pi_{gt}}{\Pi_{ft}} = \frac{s_{gt}^{-\psi}\kappa_g^{\varepsilon}C_{gt}^{\varepsilon-1}}{s_{ft}^{-\psi}\left(\kappa_c^{\varepsilon}\frac{C_{ct}^{\varepsilon}}{A_{ct}}+\kappa_s^{\varepsilon}\frac{C_{st}^{\varepsilon}}{A_{st}}\right)} = 1. \quad (20)$$

We show in Appendix A that this condition uniquely determines the allocation of innovation effort in equilibrium provided that the following assumption is satisfied:

Assumption 1 $\eta \ln\gamma < \psi/((\varepsilon-1)(1-\psi))$.

We thus have:

Proposition 2 *Under Assumption 1, a dynamic equilibrium exists and is unique.*

Moreover, we can also derive an approximate explicit expression for relative research effort devoted to green innovations. Specifically, when either γ is sufficiently close to 1 or η is sufficiently small, we have:

$$\left(\frac{s_{gt}}{s_{ft}}\right)^{\psi} \approx \frac{\kappa_g^{\varepsilon}C_{g(t-1)}^{\varepsilon-1}}{\frac{1}{A_{c(t-1)}}\kappa_c^{\varepsilon}\left(\frac{1}{A_{c(t-1)}}+\frac{1}{B_{ct}}\right)^{-\varepsilon}+\frac{1}{A_{s(t-1)}}\kappa_d^{\varepsilon}\left(\frac{1}{A_{s(t-1)}}+\frac{1}{B_{st}}\right)^{-\varepsilon}}. \quad (21)$$

This expression highlights that, as in AABH, the direction of technology in the energy sector features path dependence: higher green productivity at time $t-1$, $A_{g(t-1)}(=C_{g(t-1)})$ increases the relative size of the green energy sector, which then favors further green innovations at time t. Similarly, higher productivity levels, $A_{c(t-1)}$ and $A_{s(t-1)}$, increase the relative size of the fossil-fuel sector, which encourages further fossil-fuel innovations.

The new element in (21) is the role of productivity in the extraction sector. When productivity in fossil-fuel power plant technologies, $A_{c(t-1)}$ and $A_{s(t-1)}$, are high relative to productivity in extraction, B_{ct} and B_{st}, fossil-fuel innovations are discouraged, because a higher share of revenues from fossil-fuel energy goes to extraction, leaving less incentives for further innovations for power plant inputs. As a result, an increase in $A_{c(t-1)}$ or $A_{s(t-1)}$

has generally an ambiguous effect on the direction of innovation. This effect highlights the important role that the evolution of extraction productivity plays in the direction of innovation.

This discussion also starts building an intuition about how a natural gas boom will impact the direction of technology in the energy sector. Because the right-hand side of (21) is decreasing in B_{st}, a higher B_{st} encourages further fossil-fuel innovations and discourages green innovations. Intuitively, cheaper natural gas increases the size of the fossil-fuel sector and, for a given size of this sector, it raises the demand for the complementary power plant inputs.

In sum, a natural gas boom at time 1 (an increase in B_{st} for $t \geq 1$) reduces current innovation in green technologies (i.e., s_{g1} decreases). This leads to higher levels of A_{c1} and A_{s1} and a lower level for the green technology C_{g1}.

The full effects of the natural gas boom over time are more complex, however. On the one hand, an increase in the productivity of power plant inputs further encourages fossil-fuel innovations via path dependence, so that the negative effect of the boom on green innovation builds on itself over time. On the other hand, the same impulse also creates counteracting effects if extraction technologies are too far behind. In what follows, we simplify the discussion by imposing the assumption that $\min\left\{B_{ct}/A_{c(t-1)}, B_{st}/A_{s(t-1)}\right\} > \gamma^{\eta}/(\varepsilon - 1)$, which ensures that this counteracting effect is dominated. This is a sufficient, but not necessary condition, that enables us to provide the following simple characterization of the dynamic implications of a natural gas boom.

Proposition 3 *Under Assumption 1, a natural gas boom (an increase in B_{st} for all $t \geq 1$) reduces s_{g1} and depresses innovation in green technologies. Moreover, if $\min\left\{B_{ct}/A_{c(t-1)}, B_{st}/A_{s(t-1)}\right\} > \gamma^{\eta}/(\varepsilon - 1)$ for all $t > 1$, then green innovation declines for all $t \geq 1$.*

This proposition provides sufficient conditions under which a natural gas boom leads to a permanent reallocation of innovation effort away from green technologies. The overall climate impact of a natural gas boom will be determined by a balance between its short-run effects (which are beneficial under the conditions of Proposition 1) and its potential negative long-run effects via reduced green innovations, as we study next.

3.5 Long-run Emission Consequences of a Natural Gas Boom

To fully characterize the effect of the natural gas boom on emissions, consumption and welfare, we need to specify the growth processes for the extraction and the production input technologies. With this aim, we suppose that A_{Pt} grows at the rate $\gamma^{\eta} - 1$ and that the extraction technologies B_{ct} and B_{st} grow at the rate $\gamma^{\eta_B} - 1$, with $\eta_B \in [0, \eta]$. In what follows, we say that the economy is on a *green path*, if, asymptotically, innovation only occurs in green technologies. Conversely, we say that the economy is on a *fossil-fuel* *path*, if, asymptotically, innovation only occurs in fossil-fuel technologies. Notice that output gross of climate damages (without the $D(S_t)$ term) grows at the rate $\gamma^{\eta}-1$ if the economy is on a green path, and at the rate $\gamma^{\eta_B}-1$ if it is on a fossil-fuel path.

In this section, we simplify the discussion by focusing on the case where extraction technologies grow at a sufficiently fast rate, that is η_B is above some threshold $\overline{\eta}$.[12] This assumption has two important consequences. First, because in this case extraction technologies are not a limiting factor, the allocation of innovation is asymptotically "bang-bang" as in AABH, with either all scientists working on green innovation or on fossil fuels (except for a knife-edge case). More specifically, there exists a threshold value $\overline{A}_{g0}(A_{s0}, A_{c0}, B_{s1}, B_{c1})$, which depends on the initial productivities in fossil-fuel technologies, such that if initially, productivity in the green technology lies below this threshold (that is, if $A_{g0} < \overline{A}_{g0}$), then the economy is on a fossil-fuel path. The opposite occurs and the economy is on a green path if the initial green technology is above this threshold, that is, if $A_{g0} > \overline{A}_{g0}$. Second, we can characterize conditions under which if the inequality $\min\left\{B_{ct}/A_{c(t-1)}, B_{st}/A_{s(t-1)}\right\} > \gamma^{\eta_f}/(\varepsilon-1)$ holds for $t=1$, then it holds for all t. In that case, Proposition 3 implies that the natural gas boom permanently reallocates research inputs away from green technologies.[13]

The next proposition provides a characterization of the conditions under which a natural gas boom can shift the economy from a green path to a fossil-fuel path (proof in Appendix A.4).

Proposition 4 *Suppose Assumption 1 holds,* $\min\{B_{c1}/A_{c0}, B_{s1}/A_{s0}\} > \gamma^{\eta}/(\varepsilon-1)$ *and* B_{ct} *and* B_{st} *grow exogenously at the rate* $\gamma^{\eta_B}-1$ *with* $\eta_B > \overline{\eta}$*. Then, there exist thresholds for initial green energy productivity,* $\underline{A_{g0}}$ *and* $\overline{A_{g0}} > \underline{A_{g0}}$*, such that:*

1. *When* $A_{g0} \in \left(\underline{A_{g0}}, \overline{A_{g0}}\right)$*, the shale gas boom decreases green innovation permanently. Asymptotically, all innovation takes place in fossil-fuel technologies following a natural gas boom at time* $t=1$*, but all innovation would have been in green technologies without the boom. Long-run emissions grow asymptotically at the rate* $\gamma^{\eta_B}-1$ *with the boom but converge to zero without the boom.*

2. *When* $A_{g0} < \underline{A_{g0}}$*, asymptotically all innovation is in fossil-fuel technologies with or without the boom. Emissions grow asymptotically at the rate* $\gamma^{\eta_B}-1$ *with or without the boom.*

[12] Notice that $\overline{\eta} < \eta$. In particular, in Appendix A.4, we show that $\overline{\eta} = \frac{\eta}{2^{1-\psi}}$ if $\varepsilon \geq 2$ and $\overline{\eta} = \eta \max\left(\frac{1}{2^{1-\psi}}, \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)\right)$ if $\varepsilon < 2$.

[13] For η_B sufficiently small, the condition $\min\left\{B_{ct}/A_{c(t-1)}, B_{st}/A_{s(t-1)}\right\} > \gamma^{\eta_f}/(\varepsilon-1)$ cannot be satisfied at all times and the economy cannot converge toward a fossil-fuel path in the long-run. We study this case in Section 5.1. We derive conditions under which the natural gas boom still delays the transition toward green innovation. Appendix A.3 characterizes the long-run behavior of the economy for any value of η_B.

3. *When $A_{g0} > \overline{A_{g0}}$, asymptotically all innovation is in green technologies with or without the boom but the boom permanently decreases green innovation. Long-run emissions converge to zero with or without the boom, but there exists a $\bar{t}$ such that for $t > \bar{t}$, emissions are larger with the boom than without.*

This Proposition 4 contains two of the most important results of our analysis. First, the natural gas boom generally leads to a permanent decline in green innovation and greater long-run emissions (provided that we are not already on a fossil-fuel path).[14] Second, the natural gas boom increases the threshold value $\overline{A}_{g0}$, such that, for intermediate values of the initial green productivity A_{g0}, we can have the following "fossil-fuel trap" configuration (part 1): without the natural gas boom, the economy was on a green innovation path, but after the natural gas boom it is pushed into the fossil-fuel path. Implications for long-run emissions and output are striking. While on a green innovation path emissions asymptotically converge to zero, they keep growing along the fossil-fuel path. As a result, output grows at a positive rate in the long-run on the green path, but it converges to zero on the fossil-fuel path as the term $D(S_t)$ in (2) converges to one.

We next discuss the welfare effects of the natural gas boom and optimal policy, and then in the next section turn to a quantitative analysis of these effects, where one of our key questions will be whether the US economy is near the intermediate values for the productivity of the green technology that leads to a fossil-fuel trap.

3.6 Welfare and Optimal Policy

Proposition 4 shows how a natural gas boom increases long-run emissions. But counterbalancing this, such a boom reduces short-run emissions (provided that the conditions in Proposition 1 are satisfied) and short-run output always increases. The next proposition explores the implications of these two opposing forces on welfare (proof in Appendix A.5).

Proposition 5 *Suppose Assumption 1 holds,* $\min\{B_{c1}/A_{c0}, B_{s1}/A_{s0}\} > \gamma^{\eta}/(\varepsilon - 1)$*,* B_{ct} *and* B_{st} *grow exogenously at the rate* $\gamma^{\eta_B} - 1$ *with* $\eta_B > \overline{\eta}$ *and* $A_{g0} \in \left(\underline{A_{g0}}, \overline{A_{g0}}\right)$*. Then the natural gas boom reduces social welfare if the discount rate* ρ *is less than some threshold* $\overline{\rho}$ *(where* $\overline{\rho} > 0$*) or if the inverse elasticity of intertemporal substitution* ϑ *is greater than* 1.

To understand this result, first note that, in our model, a natural gas boom always creates short-run benefits and long-run costs. Hence, the finding that the costs will exceed the benefits for sufficiently small discount rates is intuitive.

To gain additional intuition, let us consider the three cases in Proposition 4 separately. When $A_{g0} \in \left(\underline{A_{g0}}, \overline{A_{g0}}\right)$, the natural gas boom shifts the economy from a green path to a

[14] Technically, we can prove that the boom decreases green innovation when $A_{g0} > \underline{A}_{g0}$. When $A_{g0} < \underline{A}_{g0}$, we can also prove this as long as long as fossil-fuel innovations are not too high to start with, that is for $s_{ft} \leq \left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}$, but even this condition is not necessary.

fossil-fuel path, with dramatic effects on long-run emissions and thus on output (inclusive of environmental damages captured by the term $D(S_t)$ in (2)). In particular, output net of climate damages grows at the rate $\gamma^{\eta}-1$ without the boom but converges to 0 with the boom (which is in line with the exponential net-of-damage function adopted in Golosov et al., 2014). The resulting very low levels of utility in the future matter more when ρ is low. Moreover, when ϑ is above 1, the flow utility tends to $-\infty$.[15]

When $A_{g0} > \overline{A_{g0}}$, the natural gas boom raises emissions in the long-run, but the economy still remains on the green path and long-run emissions still converge to zero. Nevertheless, even in this case, such a boom can reduce welfare, because by reducing green innovation, it depresses long-run output (since long-run energy is entirely met by clean technologies in the green path). This reduces welfare provided that the future matters sufficiently—meaning that the interest rate minus the growth rate is sufficiently low. Recall that $r-g$ being small is equivalent to $\frac{\gamma^{\eta(1-\vartheta)}}{1+\rho}$ being large, since $r \approx \rho + \vartheta g$ and $g \approx \eta \ln \gamma$. Hence, low levels of ρ again make the negative welfare effects more likely.[16]

Finally, when $A_{g0} < \underline{A_{g0}}$, long-run net output becomes very low in the long-run since emissions grow exponentially. This leads to a very low welfare, with or without the boom, and even more so when ρ is small and ϑ is large.

We next determine how optimal policy should respond to a natural gas boom. As in AABH, there are two inefficiencies in this economy: the environmental externality (due to the fact that fossil-fuel technologies lead to carbon emissions) and innovation distortions (because scientists do not fully appropriate the returns from the technologies they invent).[17] Optimal policy has to deal with both margins of inefficiency leading to the next proposition (the proof is straightforward and is presented in Supplementary Appendix B.1.3):

Proposition 6 1. *Optimal policy can be implemented by a carbon tax and a subsidy to green innovation.*

2. *Under the optimal policy, a natural gas boom always increases welfare.*

As in AABH, the optimal carbon tax is given by the standard Pigovian formula and corrects for the environmental externality.[18] The research subsidy, on the other hand, is intended to correct the distorted allocation of scientists between fossil-fuel and green

[15] More specifically, we can show that when $\vartheta > 1$, flow utility limits to $-\infty$ sufficiently fast, as carbon concentration in the atmosphere increases. In this case, therefore, the welfare effects of the natural gas boom are negative for any discount rate.

[16] In this case, negative welfare effects are also more likely when carbon concentrations depend more on current emissions than the existing stock of carbon—i.e., when φ_L is small and φ_D is large.

[17] Since all sectors share the same monopolistic structure and the final good is not used for production, there is no monopoly distortion in the final good production.

[18] If we assume log preferences ($\vartheta = 1$) as in Golosov et al. (2014), then we obtain the same closed-form solution for the carbon tax, $\tau_t = Y_t \zeta (1+\rho)\left(\frac{\varphi_L}{\rho} + \frac{(1-\varphi_L)\varphi_0}{\rho+\varphi_d}\right)$. In addition, it is straightforward to establish that if $\vartheta > 1$ or if $\rho < \bar{\rho}$, then optimal policy always induces a green path.

innovations. The laissez-faire allocation of research effort is distorted because scientists do not capture the full social value of their innovation. The optimal allocation of scientists can be computed as

$$\left(\frac{s_{ft}}{s_{gt}}\right)^{\psi} = \frac{\sum_{u=0}^{\infty} \frac{1}{1+r_{t,t+u}} \left(\frac{C_{c(t+u)}}{A_{c(t+u)}} p_{c(t+u)} E_{c(t+u)} + \frac{C_{s(t+u)}}{A_{s(t+u)}} p_{s(t+u)} E_{s(t+u)}\right)}{\sum_{u=0}^{\infty} \frac{1}{1+r_{t,t+u}} p_{g(t+u)} E_{g(t+u)}},$$

where $r_{t,t+u}$ is the (shadow) interest rate between t and $t+u$, given by $1 + r_{t,t+u} = (1+\rho)^u C_{t+u}^{\vartheta}/C_t^{\vartheta}$. The right-hand side of this expression corresponds to the ratio between the discounted sum of benefits from innovations in fossil-fuel and green technologies. Notice that the (social) benefits from innovation are proportional to the revenues of the sectors and, in the case of fossil-fuel technologies, they are also adjusted for the share of revenues going to extraction rather than power plants (which is what the the ratio of $C_{i(t+u)}/A_{i(t+u)}$ achieves). Compared to this, the laissez-faire equilibrium only features expected profits in the current period on the right-hand side, accounting for the divergence between the optimum and the equilibrium, which optimal policy corrects for. Intuitively, when an economy is transitioning to a green path, a greater share of long-run innovations will be for green technologies and the spillover of current research on these innovations is uninternalized.

This formula also provides an intuition for why a natural gas boom generally necessitates higher subsidies to green innovation. While contemporaneous private returns from innovation shift in favor of fossil-fuel technologies after a natural gas boom, long-run relative social values of fossil-fuel and green innovations do not change as much (provided that the social planner still prefers a green path). Consequently, more aggressive research subsidies to green innovation are needed to align social and private returns.

The second part of the proposition is intuitive as well. A natural gas boom improves the production possibilities frontier of the economy. If the social planner can induce the optimal allocation, then she will always improve welfare.

Finally, we note that the results in Proposition 6 do not depend on the simplifying assumption that innovators only capture current profits. With long-lasting patents, similar results apply because innovators remain unable to capture the full social returns from new technologies, notably originating from the building on the shoulder of giants externality, as shown in Acemoglu et al. (2016), Greaker, Heggedal, and Rosendahl (2018), and Hémous and Olsen (2021).

4 Quantitative Model

We now use our model as the basis for a quantitative evaluation of the implications of the US shale gas boom. The details of parameter choices are presented in Section 4.1. We then present estimates of the short-run implications of the boom in Section 4.2 and its long-run implications in Section 4.3. Section 4.4 presents our results for optimal climate policy. Finally, Section 4.5 discusses welfare effects.

4.1 Calibration and Parameter Choices

A model period corresponds to five years. The pre-boom base period, to which we calibrate the model, covers the years 2006-10. As is common in the macro-climate literature, we consider an economy with a 400-year horizon.[19]

We first describe the calibration of energy and final goods production to the pre-shale period, which proceeds in three steps. First, we construct measures of electricity generation costs and other key moments from the data. Second, we select a number of parameters directly based on prior literature. Third, we solve for the remaining parameters and initial equilibrium outcomes to match the data and other moments given the parameters from the first two steps. We now describe each of these steps in more detail.

First, some of the costs of coal and gas generation are due to mandated expenditures on local pollution abatement (e.g., sulfur dioxide), and we model these abatement expenditures explicitly. Letting $\overline{\Lambda_i}$ denote the fraction of the intermediate inputs devoted to local pollution abatement, the equilibrium price of energy type j now satisfies (see Appendix A.7.1):

$$p_i = p_i^q \left(1 + \overline{\Lambda_i}\right) + p_i^r, \tag{22}$$

where p_i^q denotes the price of the energy input ($p_i^q = \gamma w / A_i$) and p_i^r is again the resource price. Naturally, with this modification, all of our previous results apply replacing A_i by $A_i / \left(1 + \overline{\Lambda_i}\right)$.

To quantify electricity generation costs (p_i) and their components (p_i^q, $p_i^q \overline{\Lambda_i}$ and p_i^r) by energy type, we collect plant- and generator-level data on electricity generation, fuel inputs and costs, operation and management (O&M) expenditures, plant capital, and abatement expenditures as outlined in Table 2.

Appendix Section A.7.2 presents further details on how we use these data. Before proceeding, we note that the FERC data only covers investor-owned utilities meeting certain generation thresholds. Consequently, the "green" energy generators represented in FERC tilt towards existing nuclear power plants. In order to improve our measure of

[19] Cai and Lontzek (2019) consider 600 years, while Barrage and Nordhaus (2023) focus on a 500–year horizon.

Table 2—Data Sources for Costs of Electricity Generation

Item	Data Source(s)
Intermediate costs/MWh $p^q_{it}(1+\overline{\Lambda_i})$ (Plant O&M expenditures, capital, output)	Federal Energy Regulatory Commission (FERC) Form 1
Abatement costs/MWh $p^q_{it}\overline{\Lambda_i}$ (Local abatement investment, O&M, output)	Energy Information Administration (EIA) Form 767, Form 923
Fuel resource costs/MWh p^r_{it}	Federal Energy Regulatory Commission (FERC); EIA Form 423, EIA Form 923

green generation costs, we also consult levelized cost estimates (LCOE) from Lazard to compute the generation-weighted average capital-labor cost for green technologies in our base period.[20]

The second step of the calibration uses direct information from the literature and matches selected moments in the data, as summarized in Table 3. The benchmark substitution elasticities (ε, λ) are calibrated externally based on other studies. Specifically, we set the elasticity of substitution between fuels, ε, to 1.8561 based on recent empirical estimates for green and fossil electricity from Papageorgiou, Saam, and Schulte (2017), and the elasticity of substitution between electricity and the production input, λ, to 0.4 in line with estimates of both energy-capital labor elasticities (e.g., Van der Werf 2008) and electricity-other energy elasticities (e.g., Chen, Paltsev, Reilly, Morris, Karplus, Gurgel, Winchester, Kishimoto, Blanc, and Babiker (2017); Bosetti, Massetti, and Tavoni (2007), see Appendix A.7.3 for further discussion).[21] Next, we set $\nu = 0.5$ without loss of generality since different values of ν can be accommodated by adjusting the level of $\widetilde{A_{E,0}}$.

Finally, we solve for the κ's to match profit-maximizing fossil and green electricity input demands according to the equations:

$$\frac{E_{ct}}{E_{st}} = \left(\frac{\kappa_c}{\kappa_s}\frac{p_{st}}{p_{ct}}\right)^{\varepsilon} \text{ and } \frac{E_{gt}}{E_{ct}} = \left(\frac{\kappa_g}{\kappa_c}\frac{p_{ct}}{p_{gt}}\right)^{\varepsilon}, \tag{23}$$

using data on electricity consumption in Table 4 for the base period (2006-10), and imposing $1 = \kappa_c + \kappa_s + \kappa_g$. These estimates then yield the initial electricity composite quantity E_0, price p_{E0} and energy efficiency parameter $\widetilde{A_{E0}}$ (see Appendix A.7.3).

Beyond the base period, we assume that energy composite efficiency $\widetilde{A}_{Et}$ is constant, and that the productivity of the general production input A_{Pt} grows at 2% per year. These assumptions, together with our quantification of the innovation process and rest of

[20]We compute the generation-weighted average LCOE (without subsidies) for green energy for all available years in the base period (2008, 2009, and 2010). We then average FERC and Lazard estimates for green generation costs. Hydroelectricity generation is excluded from these calculations in light of limited projected expansion potential (see e.g., EIA 2019).

[21]We consider different substitution elasticities between gas and renewables and between gas and coal in the extended quantitative model in Section 5.2.

Table 3—Base Year Model Calibration Summary

Parameter	Value(s)	Sources and notes
ε	1.8561	Papageorgiou et al. (2017) avg. estimate of elasticity of subs. btw. clean, dirty inputs in electricity production
λ	0.4	Literature (e.g., Van der Werf, 2008)
ν	0.5	Normalized (without loss of generality)
κ_c, κ_s	0.2779, 0.3644	Rationalize electricity demands
κ_g	0.3577	Data (from EIA) and costs (estimated from FERC, EIA data)
γ	1.07	Match profits data (2004-2014, US Census)
ξ_c, ξ_s	1.001, 0.429	Billion metric tons of CO_2 / trillion kWh (EIA, 2016)
$\widetilde{A}_{E,0}$		Rationalize final goods producer's electricity demand (in trillion kWhs) in base period (2006-10) at GDP Y_0 (in bil. \$2010, BEA)
$A_{g,0}, A_{c,0}, A_{s,0}$, $B_{c,0}, B_{s,0}, C_{f,0}$ $C_{E,0}, A_{P,0}$		Match equilibrium conditions at observed GDP, energy production, and cost estimates $(p^y_{i,t}, p^r_{i,t})$
η_B	1.1585	Match fossil-fuel extraction productivity growth data 1.58%/yr (1987-2010, BLS)
η	1.4634	Match assumed growth rate of 2%/yr
ψ	0.5	Blundell, Griffith and Windmeijer (2002)
ζ	$5.3 \cdot 10^{-5}$	Golosov et al. (2014)
ρ	0.01 / year	DICE-2023 (Barrage and Nordhaus, 2023)
ϑ	1.5	DICE-2023 (Barrage and Nordhaus, 2023)

Note: This table reports how we choose parameters and initial conditions based on either values from the literature or data moments we try to match.

Table 4—Base Year Energy Production and Prices

	Production $E_{i,0}$ (tril. kWh)	Total price $p_{i,0}$ (\$/MWh)	Resource price $p^r_{i,0}$ (\$/MWh)	Local pollutant abatement cost $\overline{\Lambda_i}$ (avg., %)
Coal	9.5	37.7	21.8	9.6%
Gas	4.1	77.9	61.5	0.5%
Green	4.4	73.3	-	-

Note: This table reports total electricity production decomposed by source for the period 2006-2010, which we compute using micro-data. The table also reports the average cost of production for each source decomposed between resource costs, local pollution abatement costs and other costs. Data source: FERC, Lazard and authors' computation.

the world carbon emissions described below, guarantee that, along the green path, the long-run growth rate of the economy is 2% per year.

To quantify the future productivity of coal and gas extraction (B_{st} and B_{ct}), we obtain Bureau of Labor Statistics estimates of labor productivity in coal mining (NAICS 2121) and oil and gas extraction (NAICS 2111) for all available years until the shale gas boom (1987-2010). The base period generation-weighted average annual extraction productivity growth rate was 1.58%. Since productivity growth is slower in extraction than in the rest of the economy, the price of fossil-fuel resources increases over time. We use this quantification for η_B as a benchmark, but consider an alternative scenario with slower fossil extraction productivity growth as well.

Next, we calibrate the innovation step size $\gamma = 1.07$ based on profit data from the US Census Bureau (*Quarterly Financial Reports*) to match that profits are a share $1-1/\gamma$ of sectoral income (see Appendix A.7.3 for details).

Given these values, we set the remaining 10 initial equilibrium parameters and unknown variables ($A_{g0}, A_{c0}, A_{s0}, B_{c0}, B_{s0}, C_{E0}, A_{P0}, L_{E0}, L_{P0}, w_0$) by solving the system of equations implied by the equilibrium conditions of the model (given in Appendix A.7.3). We then set pollution intensities ξ_c and ξ_s based on the benchmark pollution intensity of each type of electricity generation (EIA, 2016).[22]

Our quantification of the innovation process assumes equal research productivities in fossil-fuel and green energy, $\eta_f = \eta_g \equiv \eta$. We choose η such that, along the asymptotic green path (where all energy innovation is in green technology) A_{gt} grows at 2% per year ($\eta = 5 \ln 1.02 / \ln \gamma = 1.4634$). We also set the exponent parameter $\psi = 0.5$ in line with other models (e.g., Acemoglu, Akcigit, Alp, Bloom, and Kerr 2018) as motivated by empirical evidence of an elasticity of R&D expenditures with respect to R&D costs close to one (e.g., Bloom, Van Reenen, and Williams 2019; Bloom, Griffith, and Van Reenen 2002; Blundell, Griffith, and Windmeijer 2002, etc.).

On the climate side, we adopt the carbon cycle specification of Golosov et al. (2014) with appropriate modifications for our five-year time periods (see Appendix A.7.3). We also adopt their damage function specification $(1-D(S_t)) = e^{-\zeta(S_t - S_0)}$ and consider two potential values for the damage parameter ζ. The first is Golosov, Hassler, Krusell, and Tsyvinski's (2014) benchmark value for deterministic models ($\zeta = 5.3 \cdot 10^{-5}$). The second is a "high" damage specification that doubles the projected effects of unmitigated end-of century warming, yielding $\zeta = 1.1 \cdot 10^{-4}$.[23] This specification is motivated both by recent

[22]One may be concerned about the implications of methane leaks and other life-cycle emissions (e.g., coal mine methane). A comprehensive Department of Energy analysis of greenhouse gas emissions in the US energy sector suggests slightly higher CO_2-equivalent emissions coefficients ($\xi_c = 1.124$, $\xi_s = 0.489$) but a similar ratio of coal-to-gas emissions per kWh once life-cycle emissions of both fuels are taken into account (Skone, Littlefield, Marriott, Cooney, Jamieson, Jones, Demetrion, Mutchek, Shih, Curtright, et al. 2016). To the extent that our calibration underestimates natural gas-related warming differentially, our estimates of the shale boom's negative impacts on emissions may be a lower bound.

[23]We double damages to $S_t = 1{,}761$ GtC or approximately 826.8 ppm CO_2, which is the projected

syntheses of global damage estimates, which point to higher benchmark damages than the earlier literature (e.g., Barrage and Nordhaus 2023; Howard and Sterner 2017), and by recent studies emphasizing several additional damages from climate change. For example, Dietz, Rising, Stoerk, and Wagner (2021) estimate that the combined effects of eight climate tipping points increase the social cost of carbon by about 25%.

Since the benchmark model only endogenizes greenhouse gas emissions from the US electricity sector, we additionally specify, but take as exogenous, a path for emissions from other countries and sources, P_t^{ROW}. We use the business-as-usual (BAU) emissions projections from the 2010 RICE model for all but one-third of US emissions for this purpose (Nordhaus 2010).[24]

Finally, following Barrage and Nordhaus (2023), we consider the benchmark values for the pure rate of social time preference ($\rho = 1\%/yr$) and the intertemporal elasticity of substitution (implying $\vartheta = 1.5$), but we additionally present results for lower discount rates as well. We further assume that consumers may experience disutility from climate change impacts on the rest of the world (ROW), thus replacing (1) with

$$U_t = \sum_{\tau=t}^{\infty} \frac{1}{(1+\rho)^{\tau-t}} \left(\frac{C_\tau^{1-\vartheta}}{1-\vartheta} + \upsilon(S_\tau) \right) \text{ where } \upsilon(S_\tau) \equiv \iota_{ROW} \cdot \varsigma_\tau \cdot (1 - e^{-\zeta(S_\tau - S_0)}), \quad (24)$$

where $\upsilon'(S) \leq 0$, and $\iota_{ROW} \in [0,1]$ is interpreted as an altruism parameter that captures how much US consumers care about global damages. In addition, $(1 - e^{-\zeta(S_t - S_0)})$ is the fraction of global output lost due to carbon concentrations S_t, and ς_t is a time-varying preference parameter set so that, with full altruism ($\iota_{ROW} = 1$), the US utility loss is approximately equivalent to the value of ROW output losses due to climate change (this implies $\varsigma_t \approx (-1) \cdot Y_t^{ROW} \cdot C_t^{-\vartheta}$, as detailed in Appendix A.7.3). This specification implies that with full altruism, the US social planner would set a carbon tax equal to the global social cost of carbon. In our benchmark, we followed the most common approach in policy work and set $\iota_{ROW} = 1$ (e.g., Greenstone, Kopits, and Wolverton 2020; Interagency Working Group, White House 2021). Recall finally that the term $\upsilon(S_t)$ has no impact on the equilibrium analysis and only affects optimal policy. As a result, none of our analysis in the previous section needs to be modified.

business-as-usual concentration in 2100 in the 2016 DICE Model (Nordhaus 2017).

[24] More precisely, we take all but 31.5% of US emissions—corresponding to the average US electricity greenhouse gas emissions share between 1990-2008—as exogenous. Therefore, we replace the law of motion for carbon concentration (8) with $S_t = \overline{S} + \sum_{s=0}^{t+T} (\varphi_L + (1-\varphi_L)\varphi_0(1-\varphi_d)^s)(P_{t-s} + P_{t-s}^{ROW})$. In Appendix A.8.1, we also allow for emissions spillovers from the US electricity sector to other sources of emissions.

4.2 Short-Run Impacts

This subsection presents quantitative estimates of the static effects of a natural gas boom driven by advances in gas extraction technology. We consider a doubling of B_{s0}. This is motivated by the relative price changes of coal and gas observed after the US shale gas boom, which show a decline in average gas fuel cost relative to coal from 2.8 in the 2006-10 to 1.4 in 2011-15.

Table 5 presents both benchmark results and their sensitivity to a number of variations. As expected, the net effect of an improvement in gas extraction technology on contemporaneous carbon emissions is consistently negative, with a 4.2% decline in emissions in the benchmark calibration. A higher (lower) elasticity of substitution ε between energy types is associated with slightly higher (lower) declines in CO_2 emissions. This is because a higher ε implies stronger substitution from both coal and clean technologies towards natural gas, but the former shift is more powerful and thus yields lower emissions. We also find that a higher (lower) value for the elasticity of substitution λ between the production and energy inputs is associated with a smaller (larger) decline in CO_2 emissions since this increases (decreases) the scale effect—as C_{Et} raises, workers get reallocated toward the production input but less so for a high λ.

Table 5—Short-run Effects of the Shale Gas Boom

	$\%\Delta\xi_E$	$\%\Delta E$	$\%\Delta CO_2$
Benchmark	−11.2%	7.9%	−4.2%
Higher $\varepsilon = 2.2$	−13.4%	8.4%	−6.1%
Lower $\varepsilon = 1.5$	−8.9%	7.4%	−2.2%
Higher $\lambda = 0.5$	−11.2%	9.9%	−2.4%
Lower $\lambda = 0.3$	−11.2%	5.9%	−5.9%

Note: This table shows predicted short-run change in emissions intensity (ξ_E), electricity aggregate (E), and CO_2 emissions following a 100% increase in B_{s0} in the benchmark case and for alternative values of the elasticities of substitution across electricity types and between electricity and production. In all cases, the substitution effect is negative and dominates the scale effect.

It is useful to compare these results to the data and empirical studies of the shale gas boom's impacts. Aggregate data suggest that CO_2 emissions from US electricity generation declined 11.4% between 2006-10 and 2011-15, an almost identical magnitude to our benchmark estimate of -11.2%. Microeconometric studies quantifying short-run effects of natural gas price changes on electricity producers yield similar estimates.[25]

[25] Cullen and Mansur (2017) estimate that the decline in natural gas prices from 2008 to 2012 led to a 10% reduction in the CO_2 emissions intensity of electricity generation. Linearly extrapolating Linn and Muehlenbachs's (2018) estimate to the observed price reduction suggests an emission intensity decline of 4%. These estimates, which hold factors such as generating capacity constant, are naturally smaller than our five-year aggregate impacts. We also note that several other estimates in the literature are harder to compare to our results, because they focus on different metrics or outcomes. For example, Knittel,

4.3 Dynamic Impacts

We now examine the dynamic effects of a natural gas boom in our model. We assume that the economy is in laissez-faire in 2006-2010 and is then hit by the shale gas boom from the next period (in Section 5.2 we assume calibrated 2006-2010 BAU policies and keep those constant). We then contrast the evolution of the economy with a counterfactual world where there is no shale gas boom.[26]

Figure 3 presents the effects of a doubling of B_{s0}. Panel A shows that the resulting natural gas boom not only decreases the share of scientists in green innovations, but in fact permanently delays a green transition that would have otherwise occurred. This is of course the quantitative equivalent of part 1 of Proposition 4, where the natural gas boom shifts the economy from a green path to a fossil-fuel path, permanently increasing emissions. Indeed, we see in Panel B that emissions start raising as early as 2028, and by 2100 they are about 25% higher than in the counterfactual world without the shale gas boom. Panel C plots impacts on output net of climate damages, which are initially positive but turn substantially negative over time.

These benchmark results take emissions outside of the US electricity sector as given. In reality, US energy technologies may impact technology and emissions in the rest of the world. We explore this question in Appendix A.8.1 and show that this response can magnify the negative long-run consequences of a natural gas boom.

Finally, it is useful to compare the model's predictions to two untargeted moments in the data. Table 6 confirms that the model matches both the initial and post-boom levels of the green innovation intensity remarkably well, increasing our confidence in the calibration of the model and its counterfactual implications.

4.4 Policy Implications

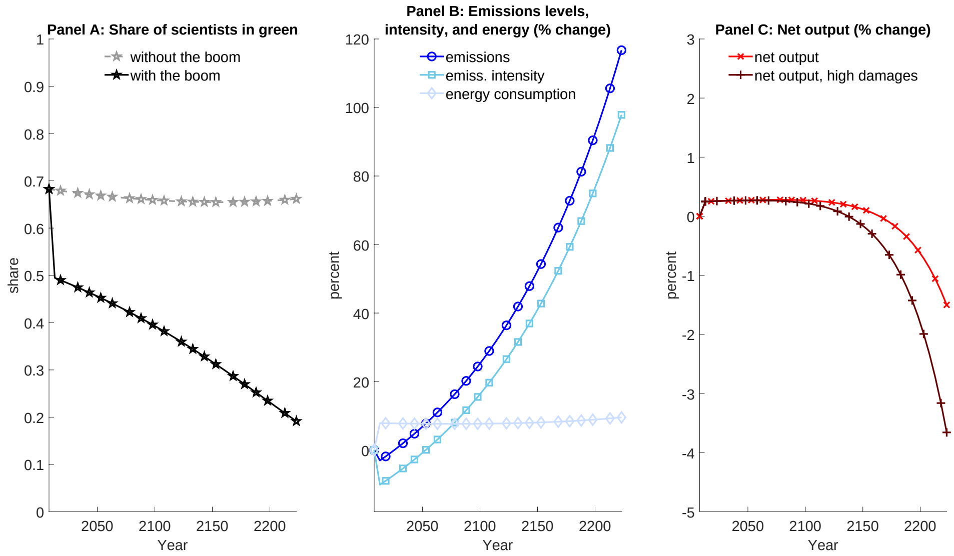
We next turn to the optimal policy responses to a natural gas boom mimicking the shale gas revolution in the United States. We focus on the choices of a social planner maximizing discounted US welfare. Recall from Section 3.6 that the optimal allocation can be decentralized using a carbon tax and a green research subsidy, and we focus on these two instruments. The planner takes the path of carbon emissions in the rest of the world and outside of the US electricity sector as given.

We start by characterizing the optimal allocation of researchers, after the natural

Metaxoglou, and Trindade (2015) compare shale gas share and CO_2 emissions responses to natural gas price variation across different types of power plants focusing on entities with both coal- and gas-fired capacity.

[26]In reality, there were other relevant shocks, such as policy changes and increased production of renewable inputs in China. For this reason, our results should not be viewed as predictive about future trajectories, but as informative about the effects of the shale gas boom relative to a counterfactual without such a boom.

Figure 3—Shale Boom Impact on Laissez-faire Outcomes



Note: This figure shows the dynamic effects of the shale gas boom in laissez-faire in our baseline calibration. Panel A depicts the allocation of scientists with and without the shale boom. While innovation is increasingly directed toward green technology without the boom, it moves toward fossil-fuel technologies with the boom. Panel B shows the changes (in %) in emission intensity, energy consumption and emissions that result from the boom. The boom is associated with an initial decline in emission intensity that is reversed over time. As a result emissions eventually rise following the boom. Panel C shows the effect on net output of the boom for two calibrations of the damage function. The boom eventually decreases net output.

gas boom and focusing on the GHKT formulation of damages. Panel A of Figure 4 compares the share of researchers in green technologies in laissez-faire against the optimal allocation. It shows that optimal policy should strongly prioritize research effort in green technologies (compared to the laissez-faire). Panels B and C explore how the natural gas boom impacts optimal policy. We see that, consistent with AABH, the optimal clean innovation subsidy is quite high, around 70%, even in the absence of the natural gas boom. Furthermore, the subsidy should increase further, by another 25 percentage points in the early decades of the boom. It is also worth noting that, as Panel C highlights,

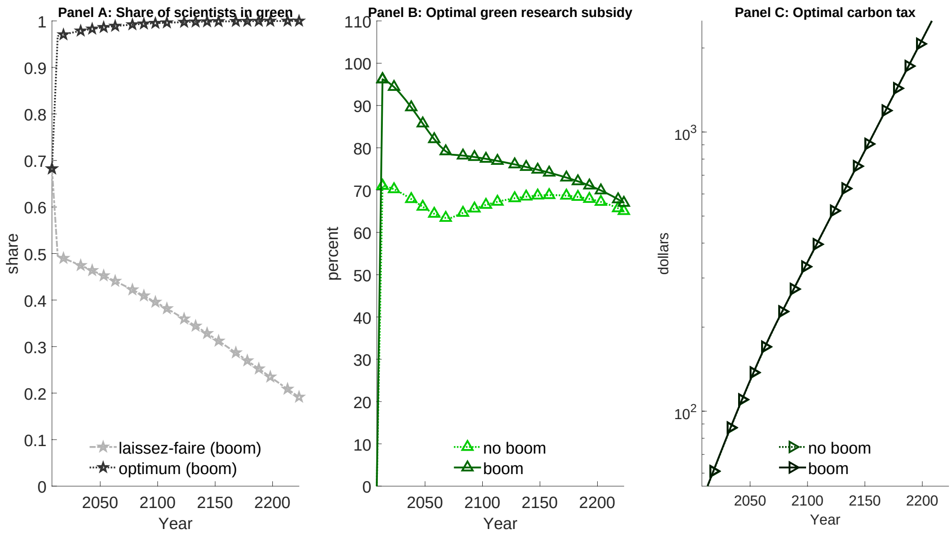
Table 6—Model vs. Data for Untargeted Innovation Moments

	Ratio of Green to Fossil-Fuel Patents	
	Data	Model
2006-2010	1.47	1.47
2011-2015	1.02	0.99

Note: This table depicts the ratios of the number of green to fossil-fuel electricity patents filed in the US by domestic inventors in the data and according to the model predictions for the base and the subsequent periods. The model's implications match the data well.

optimal policy involves a sizable carbon tax that increases over time, though this tax is not very sensitive to the boom. This latter result is because, as in GHKT, the optimal carbon price (as a fraction of GDP) depends mainly on damages and the rate of time preference.[27]

Figure 4—Optimal Green Innovation Subsidies and Carbon Prices Over Time



Note: This figure shows the optimal policy. Panel A shows the allocation of scientists in laissez-faire and in the optimum (with the boom). The optimal policy redirects innovation toward green technologies. Panel B shows the optimal clean research subsidy with and without the boom. The subsidy is higher with the boom. Panel C shows the optimal carbon tax with and without the boom, the tax remains similar in both cases.

4.5 Welfare Effects of (Unmanaged) Natural Gas Booms

Finally, we explore whether an unmanaged natural gas boom—meaning without the appropriate policy responses—improves or damages welfare. The results presented in Section 4.3 indicate that the fossil-fuel trap configuration in part 1 of Proposition 5 applies in our benchmark calibration, and thus an unmanaged natural gas boom, approximating the shale gas boom in the United States which did not lead to significant changes in climate policy, will have unambiguously negative welfare effects. Recall, in particular, that the inverse intertemporal elasticity of substitution parameter is $\vartheta = 1.5$ and, given our parameter estimates, the US economy is in the range of fossil-fuel trap, as in part 1 of Proposition 5, so that the natural gas boom shifts the economy from a green path to a fossil-fuel path.

[27]We note two additional points. First, the benchmark GHKT result can be extended to a setting with non-logarithmic CRRA preferences as in our model, in which case consumption growth also affects the optimal carbon tax-GDP ratio (Barrage 2013). Second, the results are similar if we use higher damages than GHKT, except that the initial green subsidy and carbon tax levels are higher in this case.

A shift to a fossil-fuel path has highly damaging effects in the long-run, as explained in the context of Proposition 5. In Table 7 we focus on welfare effects for a standard climate-economy model time horizon of 400 years. We focus on a benchmark rate of social time preference of $\rho^{yr} = 1\%$. From the first row of the table we see that an unmanaged natural gas boom (similar to the US shale gas boom) is expected to reduce welfare by 1.6% in consumption equivalent terms with the GHKT damages and by 2.8% in the high damages case. Columns (3) and (4) give the threshold values of time preference below which welfare effects are negative—2.1% and 2.5% in the two cases, respectively. Figure 5 plots the welfare effects of the shale gas boom for discount rates between 0.1% and 1%

The next six rows of Table 7 demonstrate that these results are robust to varying the elasticities of substitution ε and λ and the extent of congestion (diminishing returns) effects ψ. While the negative welfare effects fluctuate—from a low of -0.6% to a high of -4.5%—the general pattern is very similar to the benchmark in the first row.

The next row of the table shows that if we focus on the effects that completely ignore the rest of the world, then the shale gas boom is neutral with the GHKT damages and reduces US welfare by about 0.4% with the high damages.

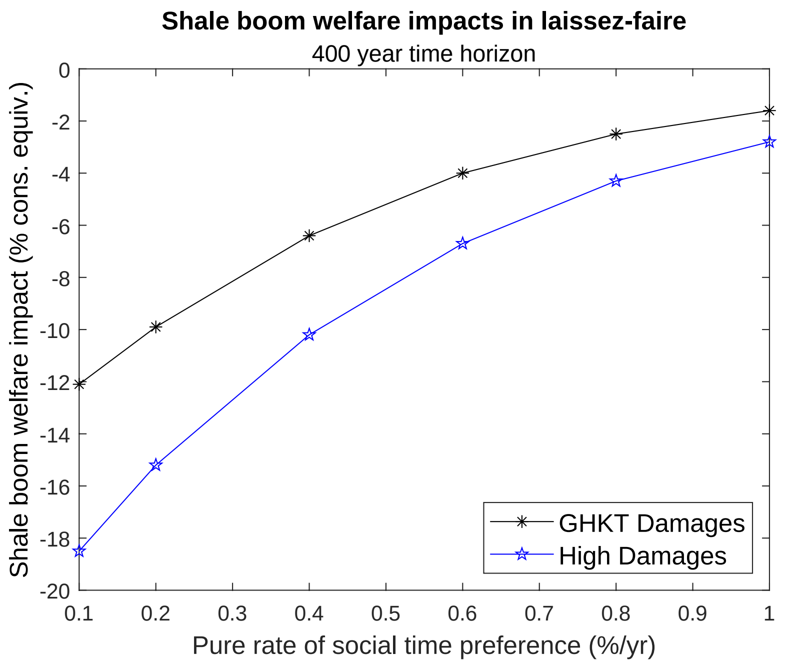
The remaining two rows confirm that there are significant gains from switching to optimal policy and that these gains increase substantially following the shale boom. For example, from the last row, we see that, under GHKT damages, optimal policy would have increased welfare by about 2.4% without the boom but 4.2% with the boom (in consumption equivalent terms).

Table 7—Welfare Effects of the Shale Gas Boom

	Welfare Impacts $\rho^{yr} = 1\%$		Threshold ρ^{yr}	
Damages:	GHKT	High	GHKT	High
Effect of boom in laissez-faire				
Benchmark	-1.6%	-2.8%	2.1%	2.5%
Higher $\varepsilon = 2.2$	-2.6%	-4.5%	2.4%	2.9%
Lower $\varepsilon = 1.5$	-0.6%	-1.2%	1.7%	2.1%
Higher $\lambda = 0.5$	-1.6%	-2.9%	2.0%	2.4%
Lower $\lambda = 0.3$	-1.5%	-2.7%	2.1%	2.6%
Higher $\psi = 0.55$	-1.2%	-2.2%	1.9%	2.3%
Lower $\psi = 0.45$	-1.9%	-3.4%	2.2%	2.7%
No altruism toward ROW	0.0%	-0.4%	1.0%	1.4%
Effect of optimal climate policy (no boom)	2.4%	4.8%		
Effect of optimal policy (with boom)	4.2%	8.1%		

Note: This table reports, across a range of scenarios, the welfare impacts of the shale gas boom (in consumption equivalent terms) ("Welfare Impacts"), and the threshold on the annual pure rate of social time preference below which these welfare impacts are negative ("Threshold ρ_{yr}"). In both cases the economy is in laissez-faire. In all but one cases, the welfare impacts of the shale gas boom are negative for a 1% discount rate. Welfare is computed for a 400-year time horizon.

Figure 5—Welfare Impacts of the Shale Boom in Laissez-faire Across Utility Discount Rates



Note: This figure shows the welfare impacts of the shale gas boom (in consumption equivalent terms) for different values of the pure rate of social time preference, both for the GHKT and high damages scenarios. Welfare is computed for a 400-year time horizon. In all cases, the shale gas boom is associated with welfare losses which increase in absolute value when the pure rate of social time preference is lower and damages are higher.

5 Extensions

In this section, we briefly discuss some extensions to our theoretical and quantitative analysis.

5.1 Alternative Growth Processes in Extraction

Our analysis so far has focused on the case where extraction technologies grow exogenously at a sufficiently fast rate, ensuring that they do not become a bottleneck on the energy sector. In this subsection, we discuss two alternative scenarios, one in which extraction technologies grow slowly and another one where there is endogenous innovation in extraction.

Slow Progress in Extraction Technologies. We now consider the case where the growth rate of B_{st} and B_{ct} is small. In this scenario, fossil-fuel prices increase rapidly over time so that, eventually, it becomes unprofitable for firms to innovate in power plant technologies for coal and natural gas, and innovation is always redirected to clean energy. Intuitively, since extraction technologies do not improve much and since extraction and power plant inputs are complements, the share of revenues within the fossil-fuel sector accruing to power plant inputs goes to zero. This discourages innovation in fossil-fuel power plant technologies. Emissions in this scenario decrease toward zero. Nevertheless, a natural gas boom can still impact emissions and welfare because it encourages innovation in fossil-fuel technologies in the short run. Formally, we establish (proof in Supplementary Material Appendix B.1.4):

Proposition 7 *Assume that Assumption 1 holds, $\varepsilon \geq 2$ and $\eta_B < \eta/\varepsilon$. Then:*

1. *There exists a time t_{switch} such that for all $t > t_{switch}$, $s_{gt} > 1/2$ and eventually all innovation takes place in green technologies.*

2. *A natural gas boom at $t = 1$ delays the time t_{switch} and reduces green innovation until then.*

3. *For $\ln \gamma$ small, the natural gas boom increases emissions in the long-run and decreases output.*

Overall, this case is similar to the third part of Proposition 4 where the economy converges to the green path with or without the boom. More specifically, part 1 of Proposition 7 establishes that the economy always transitions to a green path, but part 2 clarifies that this switch is delayed by the natural gas boom. Finally, part 3 shows that emissions increase in the long-run. In addition, since the economy eventually relies on green technologies, the reduction in green innovation along the transition path reduces output. Welfare decreases under the same conditions as in the case where $A_{g0} > \overline{A_{g0}}$ in Section 3.6.[28] In Appendix A.8.2, we present quantitative results for this case, focusing on the simple limiting scenario with zero progress in extraction technologies.

Endogenous Innovation in Extraction Technology. Appendix A.6 considers the case of endogenous innovation in extraction technologies. This economy behaves similarly to the one with exogenous fast growth in extraction technologies as it also exhibits path dependence in green versus fossil-fuel innovations. We prove the equivalent of Proposition 3 for this economy, establishing that a natural gas boom (an exogenous increase in B_{s0}) decreases innovation in the green technology A_{g1} relative to fossil-fuel innovation, and that when $\varepsilon C_{s0} \geq B_{s0}$, it also reduces green innovation in absolute terms.

[28] Recall that these conditions are: φ_L is small and φ_D is large, so that carbon concentration depends more on current emissions, and $\frac{\gamma^{\eta(1-\vartheta)}}{1+\rho}$ sufficiently large. Note also that the assumption $\varepsilon \geq 2$ is a sufficient but not necessary condition for this proposition. It plays a role similar to the assumption that $\min\left\{B_{ct}/A_{c(t-1)}, B_{st}/A_{s(t-1)}\right\} > \gamma^{\eta_f}/(\varepsilon - 1)$ used in Proposition 3. We also make the technical assumption that $\ln \gamma$ is small for part 3, which is useful in proving that following the boom, green innovation decreases at all future dates.

5.2 An Extended Quantitative Model

This subsection considers an extended version of our quantitative model. We present a brief overview of the three changes we implement here and refer the reader to Appendix A.9 for details. First, we allow natural gas and coal to be more substitutable with each other than with renewables, for example, because of the intermittency of renewables. Namely, we now assume that the energy composite E_t is produced as

$$E_t = \left(\left(\kappa_c E_{ct}^{\frac{\sigma-1}{\sigma}} + \kappa_s E_{st}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}\frac{\varepsilon-1}{\varepsilon}} + \kappa_g E_{gt}^{\frac{\varepsilon-1}{\varepsilon}} \right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{25}$$

where $\sigma \geq \varepsilon$ is the elasticity of substitution between the two types of fossil fuels. In the quantitative implementation, we maintain the benchmark elasticity between clean and dirty fuels ($\varepsilon = 1.8561$) and set $\sigma = 2$ in line with the coal-gas electricity elasticity of substitution in empirical studies and other quantitative models (e.g., Bosetti et al. 2007; Ko and Dahl 2001; Söderholm 1998)

Second, we assume that before the arrival of the natural gas boom there were BAU climate policies—specifically, separate ad-valorem taxes on the generation of coal-based, gas-based, or green electricity, and R&D subsidies for green or fossil-fuel innovations. We quantify baseline tax values based on levelized cost estimates with and without subsidies and National Science Foundation survey data (see Appendix A.9 for details).

Third, we relax the assumption that all fossil-fuel innovations apply equally to coal and gas power plants. Instead, each innovation in coal-based power plants is coal-specific with probability $1-\chi$ but can also be used in natural gas power plants with probability χ, and vice-versa. We simplify the analysis by assuming that in any period, there is always at most one innovation for a given intermediate. As a result, the law of motions for the power plant technologies are now given by

$$A_{ct} = \gamma^{\eta_f \left(s_{ct}^{1-\psi} + \chi s_{st}^{1-\psi}\right)} A_{c(t-1)}, \, A_{st} = \gamma^{\eta_f \left(\chi s_{ct}^{1-\psi} + s_{st}^{1-\psi}\right)} A_{s(t-1)} \text{ and } A_{gt} = \gamma^{\eta_g s_g^{1-\psi}} A_{g(t-1)}, \tag{26}$$

where s_{ct} denotes the share of scientists in coal research and s_{st} is the share of scientists in natural gas research. To maintain the assumption that fossil-fuel and green technologies can grow at the same rate, we impose that the research productivity parameters in green and fossil-fuel innovations are now related by $\eta_g = \eta_f \left(1 + \chi^{\frac{1}{\psi}}\right)^{\psi}$. We choose the parameter χ so that the extended model matches the observed ratio of green to fossil-fuel patents in the pre-boom period (2006-10), which yields $\chi = 0.945$. This estimate is consistent with the fact that many intermediates are shared between gas and coal generation (e.g., boilers, steam engines, super-heaters, etc., see, e.g., discussion in Lanzi, Verdolini, and Hascic 2011). We further verify the robustness of our quantitative results to different values of χ.

Clearly, our baseline model is a special case of this extended model with $\varepsilon = \sigma$, BAU policies set at zero and $\chi = 1$ (provided that η_f is properly adjusted).

Proposition 1 on the short-run effects of the shale gas boom can be extended to this setup with minor modifications (see Proposition A.5 in Appendix A.9). Most notably, the short-run impact of the natural gas boom on the emission rate can now be written as

$$\frac{\partial \ln \xi_{Et}}{\partial \ln B_{st}} = \frac{C_{st}}{B_{st}} \left[\sigma \frac{P_{st}}{P_t} - (\sigma - \varepsilon)\, \theta_{sft} - \varepsilon \Theta_{st} \right]. \tag{27}$$

Here θ_{sft} is the revenue share of the gas industry within the fossil-fuel energy subsector. Expression (27) reveals that a natural gas boom is more likely to lead to a short-run reduction in emissions when the elasticity of substitution between fossil fuels is large relative to the elasticity of substitution between fossil fuels and green electricity ($\sigma > \varepsilon$).

In Appendix A.9, we derive explicit conditions under which the natural gas boom reduces green innovation (see Proposition A.6). We further show that with $\chi < 1$, a shale gas boom favors natural gas-based over coal-based innovations, and this tends to reduce emissions in the medium-run relative to our benchmark economy (since natural gas is cleaner than coal).

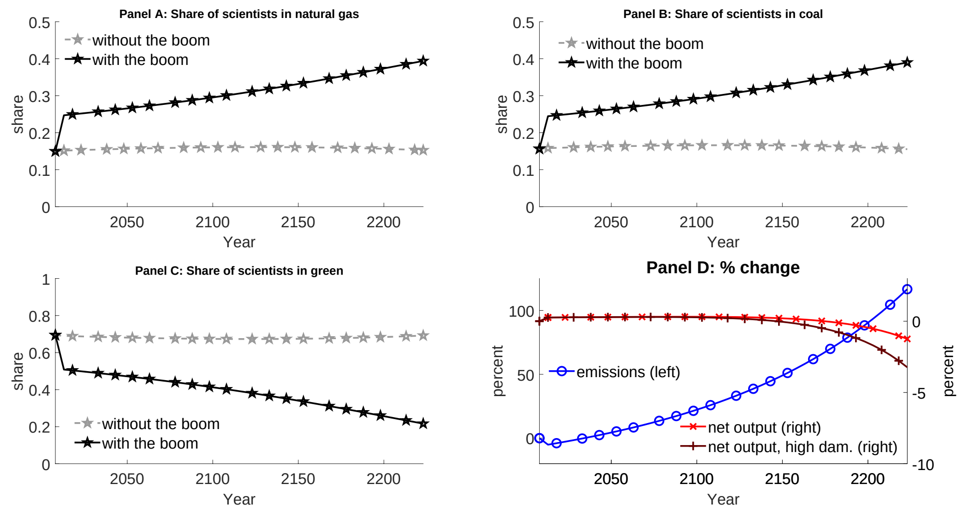
Figure 6 presents the impacts of the natural gas boom in our extended quantitative model. The boom reduces carbon emissions and increases output in the short run, but leads to a permanent delay in the green transition, raising emissions and reducing net output in the long run. Interestingly, the welfare impacts of the natural gas boom are similar to our benchmark calibration and they remain robust when we consider a lower value of coal-gas innovation spillovers ($\chi = 0.5$) and alternative assumptions about BAU policies (see Appendix A.9).

5.3 Complementarity between Natural Gas and Renewable

Renewables are intermittent energy sources, and this can introduce some complementarity between them and natural gas, the production of which can be ramped up and down easily.[29] Our model in Section 5.2 already captures this complementarity to some degree, since it allows for $\sigma > \varepsilon$—which implies greater complementarity between renewables and natural gas than between the two fossil fuels. In Supplementary Material Appendix B.4, we present an alternative model with a hybrid energy source, combining renewables and natural gas. We show with this extended model that a natural gas boom now leads to a greater reduction of emissions in the short-run, but continues to reduce green innovations for reasonable parameter values (which we confirm with a brief quantitative exercise).

[29]Once better storage technologies are developed, this source of complementarity may be weakened.

Figure 6—Shale Boom Impacts in Laissez-faire in the Extended Model



Note: This figure shows the dynamic effects of the shale gas boom in laissez-faire in the extended model. Panel A depicts the share of scientists allocated to natural gas power plant technologies with and without the shale boom. Panel B and C do same for scientists allocated to coal power plant technologies and green technologies, respectively. While innovation is increasingly directed toward green technologies without the boom, it moves toward fossil-fuel technologies with the boom. Panel D shows the changes (in %) in emissions and in net output that result from the boom. The boom increases emissions in the long-run and decreases net output.

6 Conclusion

Engineering a transition from fossil fuels to renewables and other cleaner sources of energy is one of the major challenges of the current generation. One question is how energy sources with intermediate CO_2 emissions, such as natural gas, should be used in this process. These sources can reduce emissions in the short run, but it remains an open question whether they would help or hinder the longer-run transition.

This paper investigates the short- and long-term effects of a natural gas boom in an economy where energy can be produced with coal, natural gas, or a clean energy source, and innovation can be directed either toward fossil-fuel or clean energy. In the short run, a natural gas boom reduces CO_2 emissions under plausible conditions—in particular, provided that renewables are a small fraction of energy consumption to start with and natural gas is sufficiently clean relative to coal. However, such a boom also discourages clean innovations.

We characterize conditions under which this negative effect on innovation has drastic consequences: the natural gas boom can create a "fossil-fuel trap", permanently shifting the economy from a "green path" (where in the long run all innovation is in green technologies) to a "fossil-fuel path" (where in the long run all innovation is in fossil-fuel technologies). Even when it has less extreme effects, a natural gas boom can raise long-run CO_2 emissions and reduce welfare.

We document that the US shale gas boom, which massively expanded natural gas extraction from shale reserves, reduced short-run emissions, but there was also a concurrent and notable decline in the share of green and renewable electricity patents relative to fossil-fuel patents. We then investigate the emission, output and welfare consequences of this boom using a quantitative version of our model.

We calibrate our model parameters to the US electricity generation sector and then quantitatively assess how a natural gas boom affects the direction of innovation in the energy sector and how it impacts long-run emissions and output. Our quantitative results confirm the short-run negative effect of the shale gas boom on CO_2 emissions, but also show that it raises emissions significantly in the longer run. In fact, according to our estimates, the US economy is in the range of parameters and initial conditions for a fossil-fuel trap. Accordingly, for reasonable values of the social rate of time preference, a natural gas boom reduces long-run welfare (output inclusive of environmental damages). In contrast, both our theory and quantitative results confirm that with the optimal policies, the shale gas boom could have massively improved welfare and output. Our findings thus suggest that the need for appropriate policy responses—which in general take the form of a carbon tax and a subsidy to clean research—is amplified by the US shale gas boom.

There are several research directions suggested by our study. First, further empirical and quantitative analysis of the emission and innovation implications of the shale gas boom would be highly valuable. Second, our analysis assumed that there was no similar natural gas boom in the rest of the world. In practice, natural gas production increased in other countries as well and shale gas is likely to spread to other parts of the world. Incorporating these into a more detailed model with cross-country trade and innovation linkages would be another area for future study. Third, we abstracted from the possibility of future policy responses to rising emissions. Welfare implications could be significantly different if a natural gas boom triggers future policy reactions. An exploration of these topics requires a detailed model of dynamic policy-making, which is another worthwhile topic for the future.

Finally, we note that the lessons of our model may be relevant to other "intermediate solutions" to the energy transition problem. Several of the proposed solutions, including biofuels, fission nuclear energy or geoengineering, also raise the possibility of other types of environmental damages, and a more general model incorporating different types of environmental externalities may be necessary to study their long-run implications. More generally, our analysis suggests that the use of natural gas as a solution to the climate change challenge may have much in common with other historical episodes that accidentally but permanently directed innovation toward potentially inefficient solutions. Examples may include the use of a uranium cycle instead of a thorium cycle in nuclear fission, or Henry Ford's technology choices for mass production which enabled internal combustion engines to replace early electric vehicles. Developing models for the study of the short-run vs. long-run trade-offs when technology can be directed to different technology classes or platforms is another major area of research.

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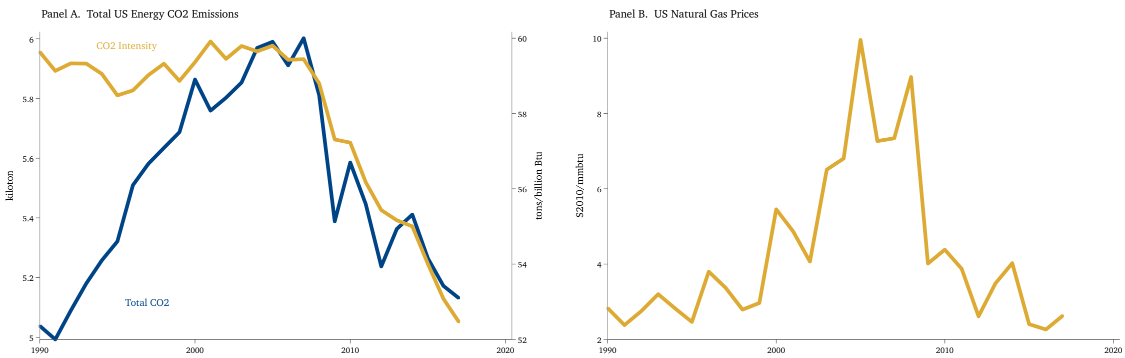
A. Online Appendix for "Climate Change, Directed Innovation, and Energy Transition: The Long-run Consequences of the Shale Gas Revolution"

A.1 Additional Empirical Results

This section provides additional empirical results, which complement those presented in the Introduction and in Section 2.

Further Results on Emission and Patenting Trends. We first note that total primary energy consumption and total energy CO_2 emissions behave very similarly to the trends shown in Figure 1 Panel C for the electricity sector. This is depicted in Figure A.1 Panel A (data are from the US Energy Information Administration). Next, Figure A.1 Panel B verifies the sharp decline in US natural gas prices during the shale gas boom period (data are from the World Bank and the Bureau of Labor Statistics).

Figure A.1—Emissions for the Whole US Economy and Natural Gas Prices



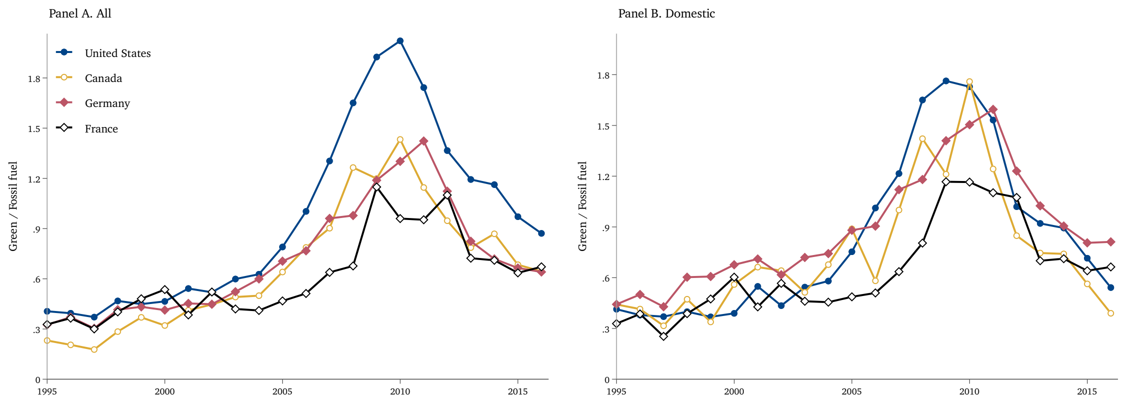
Note: Panel A reports the emission intensity (emissions divided by total energy consumption) and the total emissions of the entire US economy (data source: EIA). Both decrease sharply after the shale gas boom. Panel B reports the US natural gas price, which also collapses after the boom.

Next, Figure A.2 reproduces Figure 2 but for the ratio of green over fossil-fuel patents, leading to similar patterns. In unreported results, we verified that the patterns are similar when (renewable or green) patents are weighted by citations.[1]

Figure A.3 unpacks renewable technologies and separately shows the ratio of wind power over fossil-fuel patents (panel A) and the ratio of solar photovoltaic over fossil-fuel patents (panel B) for domestic inventions. The pattern for wind power is less pronounced

[1] We can also look at clean and fossil-fuel electricity patents separately by taking the ratio over total patents. The ratios of clean (renewable or green) patents over total patents display a hump-shape pattern with a peak around 2010 (see also Figure 1.D for the US). The trends are less clear-cut for fossil-fuel electricity patents over total patents. Since these trends may be dominated by variations in other sectors with fast patenting growth (such as IT) and since our interest is in the direction of innovation within the electricity sector, we focus on the ratios of clean patents relative to fossil-fuel electricity patents.

Figure A.2—Ratio of Green to Fossil-Fuel Patents Across Countries



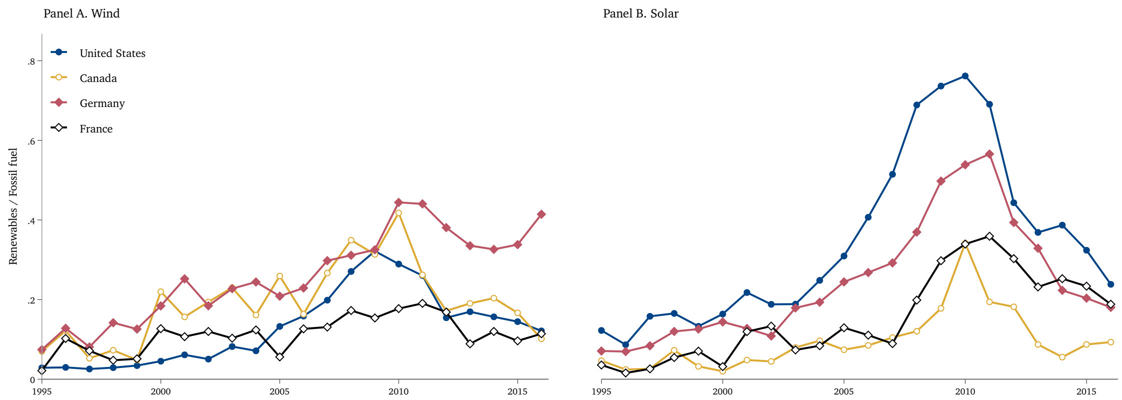
Note: This figure reports the ratio of green (= renewables + nuclear + biofuel) to fossil-fuel patents in the US, Canada, France and Germany (data source: PATSTAT). Patents are allocated to countries according to their patent office. In Panel A, we count all patents, while in Panel B, we only count patents by domestic inventors (allocating patents fractionally if inventors from multiple countries are listed). The reversal in innovation occurs in all four countries.

than for solar and there is no consistent decline in France and Germany after 2010. This suggests that at least for wind power, the factors behind the recent decline in renewable innovations are stronger in North America than in the United States. We also looked at the ratio of storage patents (Y02E70/30) over fossil-fuel patents. We found a relative decline in storage patents, though with a slight delay (from 2013 instead of 2011 for renewables). This is consistent with the decline in green innovations spilling over to storage technologies, which is a complementary input. Yet, these series are noisier due to the relatively low number of storage patents.[2]

Further Results on Panel Regressions. Table A.1 presents robustness checks for Table 1. We start from the specification of column 6 in the baseline table where the dependent variable is the $\sinh^{-1}$ difference between green and fossil-fuel patents. In these specifications, we count domestic patents only and we include all of our controls. Column (1) removes the year fixed effects, which leads to a somewhat larger coefficient. Column (2) does not weigh observations by country size. The coefficient remains of a similar magnitude but becomes less precise. Column (3) uses the log difference instead of the $\sinh^{-1}$ difference, leading to similar estimates (though we lose a few observations with zero green or fossil-fuel patents). Column (4) focuses on granted patents for which the results are slightly stronger. Column (5) weighs patents by the number of citations received. Column (6) uses only renewable patents instead of all green patents. Column (7) replaces the real gas price index from the IEA with the IEA wholesale price index deflated by the GDP deflator (from OECD data), which is available for a smaller set of

[2]Within fossil-fuel electricity patents, one can also distinguish between energy saving patents (which can be considered "grey" innovations since they allow to reduce the use of fossil fuel to produce fossil-fuel electricity) and others. We did not find a clear trend-break around 2010 in the direction of innovation within fossil-fuel electricity patents.

Figure A.3—Ratio of Wind and Solar Patents to Fossil-Fuel Patents across Countries



Note: This figure reports the ratio of wind (Panel A) or solar pv (Panel B) to fossil-fuel patents in the US, Canada, France and Germany (data source: PATSTAT). Patents are allocated to countries according to their patent office. We only count patents by domestic inventors (allocating patents fractionally if inventors from multiple countries are listed). While solar innovations decrease markedly for all countries following the boom, wind innovations only do so in the US and Canada.

countries. In all three cases, the coefficient on gas prices remains very similar.

Table A.1—Robustness Checks

	No Y FE	Unweighted	Log	Granted	Citation weighted	Renewable over FF	Wholesale
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
ln(Gas Price Index)	0.52	0.31	0.23	0.50	0.21	0.25	0.24
	(0.07)	(0.24)	(0.12)	(0.15)	(0.11)	(0.12)	(0.05)
ln(GDP/capita)	1.95	1.05	3.04	2.64	3.10	2.25	2.05
	(0.16)	(1.06)	(0.99)	(1.30)	(0.98)	(1.14)	(1.03)
ln(Public R&D Fossil)	−0.05	−0.01	−0.06	−0.18	−0.07	−0.06	−0.14
	(0.04)	(0.05)	(0.04)	(0.07)	(0.04)	(0.07)	(0.05)
ln(Public R&D Green)	0.26	−0.10	0.01	0.04	0.03	−0.03	−0.01
	(0.10)	(0.08)	(0.10)	(0.11)	(0.10)	(0.11)	(0.06)
ln(Energy consumption)	0.03	−0.65	−0.42	−0.51	−0.35	−0.09	0.95
	(0.55)	(0.60)	(0.84)	(0.79)	(0.75)	(1.00)	(0.59)
Year fixed effects		✓	✓	✓	✓	✓	✓
Country fixed effects	✓	✓	✓	✓	✓	✓	✓
R-squared	0.76	0.66	0.90	0.84	0.90	0.89	0.94
Observations	608	618	479	618	479	618	226
Countries	29	29	27	29	27	29	13

Note: This table considers deviations from our baseline specification (column (6) of Table 1, see Table 1 notes for further details): a panel regression of the $\sinh^{-1}$ difference between the number of green patents in a country and the number of fossil-fuel patents on the log gas price and controls. Only domestic patents are included and the independent variables are lagged by 2 periods. Column (1) removes the year fixed effects. Column (2) runs an unweighted regression. Column (3) replaces the $\sinh^{-1}$ difference with the log difference, dropping the zeros in this case. Column (4) restricts attention to granted patents, rather than patent applications. Column (5) weighs patent applications by citations. Column (6) looks at renewable patents only (and adjusts weights accordingly). Column (7) uses a wholesale price index to measure gas prices. All regressions include country fixed effects, Columns (2)-(7) also include year fixed effects. Standard errors are clustered at the country-level.

A.2 Uniqueness of Equilibrium and Proof of Proposition 3

We can rewrite (20) as:

$$f\left(s_{gt}, A_{c(t-1)}, B_{ct}, A_{s(t-1)}, B_{st}, C_{g(t-1)}\right) = 1 \tag{A-1}$$

where the function f is defined as

$$f \equiv \frac{\left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{c(t-1)}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{c(t-1)}}+\frac{1}{B_{ct}}\right)^{-\varepsilon}+\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{s(t-1)}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{-\eta_f s_{ft}^{1-\psi}}}{A_{s(t-1)}}+\frac{1}{B_{st}}\right)^{-\varepsilon}\right)s_{gt}^{\psi}}{\kappa_g^{\varepsilon}C_{g(t-1)}^{\varepsilon-1}s_{ft}^{\psi}\gamma^{\eta_g s_{gt}^{1-\psi}(\varepsilon-1)}}. \tag{A-2}$$

This implies

$$\begin{aligned}
\frac{\partial \ln f}{\partial \ln s_{gt}} &= \psi-\eta(\varepsilon-1)(1-\psi)(\ln\gamma)s_{gt}^{1-\psi}+\psi\frac{s_{gt}}{s_{ft}} \\
&+\frac{\eta(1-\psi)\ln(\gamma)\frac{s_{gt}}{s_{ft}^{\psi}}\left(\kappa_c^{\varepsilon}\frac{C_{ct}^{\varepsilon}}{A_{ct}}\left(1-\varepsilon\frac{B_{ct}}{B_{ct}+A_{ct}}\right)+\kappa_s^{\varepsilon}\frac{C_{st}^{\varepsilon}}{A_{st}}\left(1-\varepsilon\frac{B_{st}}{B_{st}+A_{st}}\right)\right)}{\kappa_c^{\varepsilon}\frac{C_t^{\varepsilon}}{A_{ct}}+\kappa_s^{\varepsilon}\frac{C_{st}^{\varepsilon}}{A_{st}}} \\
&\geq \psi-\eta(\varepsilon-1)(1-\psi)(\ln\gamma)s_{gt}^{1-\psi}+\left(\psi-\eta(\varepsilon-1)(1-\psi)(\ln\gamma)s_{ft}^{1-\psi}\right)\frac{s_{gt}}{s_{ft}}.
\end{aligned}$$

Therefore, we get that $\frac{\partial \ln f}{\partial \ln s_{gt}} > 0$ if Assumption 1 holds. In that case, since $f(0,.) = 0$ and $\lim_{s_g\to 1} f\left(s_g,.\right) = \infty$, (20) defines a unique equilibrium innovation allocation.

We have $\frac{\partial f}{\partial B_{st}} > 0$ so that an increase in B_{s1} leads to a lower value for s_{g1}.

Further, we have $\frac{\partial f}{\partial C_{g(t-1)}} < 0$, so that a higher value for $C_{g(t-1)}$ leads to more clean innovation. Then, we get

$$\frac{\partial \ln f}{\partial \ln A_{c(t-1)}} = \frac{\frac{1}{A_{ct}}\kappa_c^{\varepsilon}C_{ct}^{\varepsilon}}{\frac{1}{A_{ct}}\kappa_c^{\varepsilon}C_{ct}^{\varepsilon}+\frac{1}{A_{st}}\kappa_s^{\varepsilon}C_{st}^{\varepsilon}}\left(\varepsilon\frac{B_{ct}}{B_{ct}+\gamma^{\eta s_{ft}^{1-\psi}}A_{c(t-1)}}-1\right).$$

Therefore $\frac{\partial \ln f}{\partial \ln A_{c(t-1)}} \geq 0$ for all values of s_{ft} provided that $\frac{B_{ct}}{A_{c(t-1)}} > \frac{\gamma^{\eta}}{\varepsilon-1}$. Similarly, $\frac{\partial \ln f}{\partial \ln A_{s(t-1)}} \geq 0$ for all values of s_{ft} provided that $\frac{B_{st}}{A_{s(t-1)}} > \frac{\gamma^{\eta}}{\varepsilon-1}$. If these conditions are satisfied, then an increase in B_{s1} leads to higher values of A_{s1}, A_{c1} and a lower value of C_{g1}, which imply a lower value of s_{g2}. This in turns leads to even higher values of A_{s2}, A_{c2} and a lower value for C_{g2}. By iteration, all s_{gt} decrease for $t \geq 1$.

A.3 Long-run Dynamics for General η_B

With exogenous growth in extraction technologies and for A_{Pt} growing at the rate $\gamma^{\eta_P} - 1$, the long-run behavior of the economy is characterized by the following two propositions which, respectively, deal with the case where $\varepsilon \geq 2^{1-\psi}$ and the case where $1 < \varepsilon < 2^{1-\psi}$.

Proposition A.1 *Assume that $\varepsilon \geq 2^{1-\psi}$ and that Assumption 1 holds.*

1. *If $\frac{\eta_B}{\eta} < \frac{1}{\varepsilon}$, then the economy always converges to a green path where asymptotically all innovation occurs in green technologies.*

2. *If $\frac{\eta_B}{\eta} > \frac{1}{\varepsilon}$ and i) $\varepsilon \geq 2$ or ii) $\frac{\eta_B}{\eta} \notin \left(\frac{1}{2^{1-\psi}}, \frac{\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}}{\varepsilon} \right)$, then, depending on initial technology levels, the economy converges either to a path where all innovation asymptotically occurs in fossil-fuel technologies, or to a path where all innovation occurs in green technologies (except for an unstable knife-edge case with an interior allocation of innovation in the limit).*

3. *If $\varepsilon < 2$ and $\frac{\eta_B}{\eta} \in \left(\frac{1}{2^{1-\psi}}, \frac{\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}}{\varepsilon} \right)$, then, depending on initial technology levels, the economy converges either (i) to a path where all innovation asymptotically occurs in fossil-fuel technologies, or (ii) to a path where fossil-fuel technologies develop faster than clean technologies and both exhibit positive growth rates in the long-run, or ()iii) to a path where all innovation occurs in green technologies (except for two unstable knife-edge cases with interior allocations of innovation in the limit).*

The first case is characterized by slow growth in extraction technologies (including no growth, $\eta_B = 0$), as in Section 5.1. The second case displays bang-bang long-run behavior. This occurs if growth in extraction technologies is sufficiently fast, as in Section 3.5. The third case, obtained for intermediate values of η_B/η when $\varepsilon < 2$, features an interior and stable asymptotic steady state on top of the fossil-fuel and green paths.

Proposition A.2 *Assume that $\varepsilon < 2^{1-\psi}$ and that Assumption 1 holds. Then:*

1. *If $\frac{\eta_B}{\eta} < \frac{1}{2^{1-\psi}}$, then the economy always converges to a green path where, asymptotically, all innovation occurs in green technologies.*

2. *If $\frac{1}{2^{1-\psi}} < \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}$, then, depending on initial technology levels, the economy converges either to a path where all innovation asymptotically occurs in green technologies, or to a path where fossil-fuel technologies develop faster than clean technologies and both exhibit positive growth rates in the long-run (except for an unstable knife-edge case with an interior allocation of innovation in the limit).*

3. *If $\frac{1}{\varepsilon} < \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$, then, depending on initial technology levels, the economy converges either (i) to a path where all innovation asymptotically occurs in*

fossil-fuel technologies, or (ii) to a path where fossil-fuel technologies develop faster than clean technologies and both exhibit positive growth rates in the long-run, or (iii) to a path where all innovation occurs in green technologies (except for two unstable knife-edge cases with interior allocations of innovation in the limit).

4. *If* $\frac{\eta_B}{\eta} > \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$*, then, depending on initial technology levels, the economy converges either to a path where all innovation asymptotically occurs in fossil-fuel technologies, or to a path where all innovation occurs in green technologies (except for an unstable knife-edge case with an interior allocation of innovation in the limit).*

In the first case, growth in extraction technologies is slow and all innovation is allocated to green technologies asymptotically. In the second case, which only occurs for $\varepsilon < 2^{1-\psi}$, the asymptotic fossil-fuel steady state is interior. The third case is analogous to case 3 in Proposition A.1. The fourth case features bang-bang long-run behavior, and occurs under sufficiently fast growth in extraction technologies, as in Section 3.5.

The proofs of these two Propositions are in our Supplementary Material Appendix B.1.1.

A.4 Proof of Proposition 4

The proof of Proposition 4 proceeds in five steps. First, we establish two lemmas, which are then used in the rest of the proof. Then, we show the existence of thresholds on A_{g0} that determine the long-run behavior of the economy. We then look at the effect of the boom on innovation. Finally, we derive the consequences of the boom for emissions. We also establish that $\overline{\eta} = \frac{1}{2^{1-\psi}}$ for $\varepsilon \geq 2$ and $\overline{\eta} = \max\left(\frac{1}{2^{1-\psi}}, \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)\right)$ if $\varepsilon < 2$; and that the shale gas boom decreases green innovation when $A_{g0} < \underline{A}_{g0}$ as long as $s_{ft} \leq \left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}$. Both statements are mentioned in the text.

Lemma A.1 *Assume that Assumption 1 holds, that* $\min\{B_{c1}/A_{c0}, B_{s1}/A_{s0}\} > \gamma^{\eta}/(\varepsilon-1)$ *and that* B_{ct} *and* B_{st} *grow exogenously at the rate* $\gamma^{\eta_B}-1$*. Then an increase in* B_{s1} *or a decrease in* A_{g0} *is associated with a decline in* s_{gt} *as long as* $s_{f\tau} \leq \left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}$ *for all* $\tau \in [1, t-1]$.

Proof. Assume that $s_{f\tau} \leq \left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}$ for all $\tau \in [1, t-1]$. Then given that inequality $\min\left\{B_{c\tilde{\tau}}/A_{c(\tilde{\tau}-1)}, B_{s\tilde{\tau}}/A_{s(\tilde{\tau}-1)}\right\} > \gamma^{\eta_f}/(\varepsilon-1)$ holds for $\tilde{\tau}=1$, it must also hold for all $\tau \in [1, t]$. Proposition 3 establishes that an increase in B_{s1} decreases green innovation, and the same logic applies following a decrease in A_{g0}. ■

Lemma A.2 *Assume that* $s_{ft} > \left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}$ *and* $\frac{\eta_B}{\eta} \geq 2^{\psi-1}$, $s_{f\tau} > \left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}$ *for all* $\tau \geq t$.

Proof. Consider the equilibrium allocation of scientists first. Let us denote $f_\tau(s_g) \equiv f\left(s_g, A_{c(\tau-1)}, B_{c\tau}, A_{s(\tau-1)}, B_{s\tau}, C_{g(\tau-1)}\right)$, where f is defined in (A-2), so that the equilibrium allocation obeys $f_\tau(s_{g\tau}) = 1$. We then obtain:

$$f_{\tau+1}\left(1-\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) = \frac{\frac{\gamma^{-\eta_B}}{A_{c\tau}}\kappa_c^\varepsilon\left(\frac{\gamma^{-\eta_B}}{A_{c\tau}}+\frac{\gamma^{-\eta_B}}{B_{c\tau}}\right)^{-\varepsilon}+\frac{\gamma^{-\eta_B}}{A_{s\tau}}\kappa_s^\varepsilon\left(\frac{\gamma^{-\eta_B}}{A_{s\tau}}+\frac{\gamma^{-\eta_B}}{B_{s\tau}}\right)^{-\varepsilon}}{\kappa_g^\varepsilon A_{g\tau}^{\varepsilon-1}\gamma^{(\varepsilon-1)\left(\eta^{\frac{1}{1-\psi}}-\eta_B^{\frac{1}{1-\psi}}\right)^{1-\psi}}}\left(\frac{1-\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}}{\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}}\right)^{\psi}$$

$$= \frac{\frac{\kappa_c^\varepsilon}{A_{c\tau}}\left(\frac{1}{A_{c\tau}}+\frac{1}{B_{c\tau}}\right)^{-\varepsilon}+\frac{\kappa_s^\varepsilon}{A_{s\tau}}\left(\frac{1}{A_{s\tau}}+\frac{1}{B_{s\tau}}\right)^{-\varepsilon}}{\gamma^{(\varepsilon-1)\eta_B\left[\left(\left(\frac{\eta}{\eta_B}\right)^{\frac{1}{1-\psi}}-1\right)^{1-\psi}-1\right]}\kappa_g^\varepsilon A_{g\tau}^{\varepsilon-1}}\left(\frac{1-\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}}{\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}}\right)^{\psi}$$

With $\frac{\eta}{\eta_B} \geq 2^{\psi-1}$, we get that $\left(\left(\frac{\eta}{\eta_B}\right)^{\frac{1}{1-\psi}}-1\right)^{1-\psi} < 1$, so that $\gamma^{(\varepsilon-1)\eta_B\left[\left(\left(\frac{\eta}{\eta_B}\right)^{\frac{1}{1-\psi}}-1\right)^{1-\psi}-1\right]} < 1$. This implies

$$f_{\tau+1}\left(1-\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) \geq \frac{\frac{\kappa_c^\varepsilon}{A_{c\tau}}\left(\frac{1}{A_{c\tau}}+\frac{1}{B_{c\tau}}\right)^{-\varepsilon}+\frac{\kappa_s^\varepsilon}{A_{s\tau}}\left(\frac{1}{A_{s\tau}}+\frac{1}{B_{s\tau}}\right)^{-\varepsilon}}{\kappa_g^\varepsilon A_{g\tau}^{\varepsilon-1}}\left(\frac{1-\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}}{\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}}\right)^{\psi} = f_\tau\left(1-\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right).$$

If $s_{f\tau} > \left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}$, then we get that $f_\tau\left(1-\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) > f(s_{g\tau}) = 1$ (since f_τ is increasing in g). This immediately implies that $f_{\tau+1}\left(1-\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) > 1$ so that $s_{f(\tau+1)} > \left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}$. Therefore if $s_{ft} > \left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}$, then $s_{f\tau} > \left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}$ for all $\tau \geq t$. ■

Thresholds. Assume that either (i) $\varepsilon \geq 2$ and $\frac{\eta_B}{\eta} > 2$, which also implies that $\frac{\eta_B}{\eta} > \frac{1}{\varepsilon}$; or (ii) that $\varepsilon < 2$ and $\frac{\eta_B}{\eta} > \max\left\{\frac{1}{2^{1-\psi}}, \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)\right\}$. Using Propositions A.1 and A.2, we know that, except for a knife edge case, the asymptotic allocation of scientists is either all in green or all in fossil-fuel innovations. Using Lemma A.2, we then get that if at any point in time $s_{ft} > \left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}$, then s_{ft} must converge to 1.

Consider an equilibrium path where innovation is asymptotically allocated all in green technologies. On that equilibrium path, it must be that $s_{ft} \leq \left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}$. Using Lemma A.1, we then get that had the initial clean technology A_{g0} been higher, green innovation on that alternative path should be higher as well. Therefore, innovation is also asymptotically allocated entirely to the green technology on this alternative path.

Consider now an equilibrium path where asymptotically all innovation is in fossil-fuel technologies together with an alternative path characterized by a lower green technology A_{g0}. Using Lemma A.1, fossil-fuel innovation is higher on the alternative path as long as s_{ft} remains below $\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}$, but if s_{ft} crosses $\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}$, then innovation is eventually all allocated in fossil-fuel technologies. Therefore, it must be the case that asymptotically all innovation is on fossil-fuel technologies on the alternative path.

This establishes the existence of the threshold $\overline{A_{g0}}$: without the boom, the economy converges to the green path for $A_{g0} > \overline{A_{g0}}$, and to the fossil-fuel path for $A_{g0} < \underline{A_{g0}}$.

Effect of the Shale Gas Boom on Innovation. Assume that $A_{g0} < \underline{A_{g0}}$. Then using Lemma A.1, the shale gas boom reduces green innovation until s_{ft} crosses $(\eta_B/\eta)^{\frac{1}{1-\psi}}$, and the economy converges to the fossil-fuel asymptotic steady state.

Assume that $A_{g0} > \underline{A_{g0}}$, then there are two options: either the economy still converges to the green asymptotic steady state or it converges toward the fossil-fuel asymptotic steady state. This defines a threshold $\overline{A_{g0}}$. If $A_{g0} > \overline{A_{g0}}$, innovation asymptotes the green steady state with or without the boom. In that case, it must be that $s_{ft} < (\eta_B/\eta)^{\frac{1}{1-\psi}}$ at all future dates, and hence the boom reduces green innovation. If $A_{g0} \in \left(\underline{A_{g0}}, \overline{A_{g0}}\right)$, the boom decreases green innovation until s_{ft} becomes higher than $(\eta_B/\eta)^{\frac{1}{1-\psi}}$ on the post-boom path. Meanwhile, we always have $s_{ft} < (\eta_B/\eta)^{\frac{1}{1-\psi}}$ on the pre-boom path. Therefore, the boom must always reduce green innovation.

Effect of the Shale Gas Boom on Emissions. Using (13), (14), and (15), we get.

$$P = \left(\xi_c \kappa_c^{\varepsilon} \left(\frac{C_{ct}}{C_{Et}}\right)^{\varepsilon} + \xi_s \kappa_s^{\varepsilon} \left(\frac{C_{st}}{C_{Et}}\right)^{\varepsilon}\right) C_{Et} \frac{\nu^{\lambda} \widetilde{A}_E^{\lambda-1} C_{Et}^{\lambda-1}}{\nu^{\lambda} \widetilde{A}_E^{\lambda-1} C_{Et}^{\lambda-1} + (1-\nu)^{\lambda} A_{Pt}^{\lambda-1}} L. \tag{A-3}$$

If $s_{gt} \to 1$, we get that $C_{ct} \to A_{ct}$, $C_{st} \to A_{st}$ and $C_{Et} \to \kappa_g^{\frac{\varepsilon}{\varepsilon-1}} A_{gt}$ so that:

$$P_t \sim \left(\xi_c \kappa_{ct}^{\varepsilon} A_{ct}^{\varepsilon} + \xi_s \kappa_s^{\varepsilon} A_{st}^{\varepsilon}\right) \kappa_g^{-\varepsilon} A_{gt}^{1-\varepsilon} \frac{\nu^{\lambda} \widetilde{A}_E^{\lambda-1} \kappa_g^{\frac{\varepsilon}{\varepsilon-1}(\lambda-1)} A_{gt}^{\lambda-1}}{\nu^{\lambda} \widetilde{A}_E^{\lambda-1} \kappa_g^{\frac{\varepsilon}{\varepsilon-1}(\lambda-1)} A_{gt}^{\lambda-1} + (1-\nu)^{\lambda} A_{Pt}^{\lambda-1}} L$$

which tends to zero since L_{Et} is bounded, $\xi_c \kappa_c^{\varepsilon} C_{ct}^{\varepsilon} + \xi_s \kappa_s^{\varepsilon} C_{st}^{\varepsilon}$ does not grow exponentially and $A_{gt}^{1-\varepsilon}$ decreases exponentially. If $A_{g0} > \overline{A_{g0}}$, then the boom reduces green innovation, which increases C_{ct} and C_{st} and decreases A_{gt}. The expression on the right-hand side is decreasing in A_{gt}, so emissions increase for t large enough following the boom.

Alternatively if $s_{gt} \to 0$, then C_{ct} and C_{st} grow at the rate $\gamma^{\eta_B} - 1$ and A_{gt} does not grow asymptotically. Therefore C_{Et} also grows at the rate $\gamma^{\eta_B} - 1$. This ensures that ξ_{Et} tends toward a constant. Using that A_{Pt} grows at the rate $\gamma^{\eta} - 1$, we get from (A-3) that P_t grows at the rate $\gamma^{\eta_B} - 1$. Output gross of climate damages also grows at the rate $\gamma^{\eta_B} - 1$. Note finally that we immediately conclude that the shale gas boom increases emissions in the long-run if it switches the economy from a green path to a fossil-fuel path, i.e., when we have $A_{g0} \in \left(\underline{A_{g0}}, \overline{A_{g0}}\right)$. This completes the proof of Proposition 4.

A.5 Proof of Proposition 5

We first derive the asymptotic behavior of output on the green and the fossil-fuel paths. We then establish Proposition 5. In Supplementary Material Appendix B.1.2, we look at welfare effects in the case where $A_{g0} > \overline{A_{g0}}$.

Output. Using (2) and (14), we get that output can be written as:

$$Y_t = e^{-\zeta S_t}\left(\nu^\lambda \widetilde{A}_E^{\lambda-1} C_{Et}^{\lambda-1} + (1-\nu)^\lambda A_{Pt}^{\lambda-1}\right)^{\frac{1}{\lambda-1}} L. \tag{A-4}$$

On a green path where $s_{gt} \to 1$, S_t asymptotes to a constant since emissions decrease exponentially. In addition C_{Et} asymptotically grows like A_{gt} at the rate $\gamma^\eta - 1$, since A_{Pt} also grows at the rate $\gamma^\eta - 1$, then Y_t asymptotically grows at the rate $\gamma^\eta - 1$.

On a fossil-fuel path where $s_{gt} \to 0$, the growth rate of overall energy productivity C_{Et} is constrained by the growth rate of the extraction technologies, so that C_{Et} asymptotically grows at the rate $\gamma^{\eta_B} - 1$. Then, output gross of climate damages $\left(\nu^\lambda \widetilde{A}_E^{\lambda-1} C_{Et}^{\lambda-1} + (1-\nu)^\lambda A_{Pt}^{\lambda-1}\right)^{\frac{1}{\lambda-1}}$ also grows asymptotically at the rate $\gamma^{\eta_B} - 1$, but so do emissions. Therefore, given the exponential net-of-damages function $e^{-\zeta S_t}$, output net of climate damages Y_t converges to 0.

Welfare with $A_{g0} \in \left(\underline{A_{g0}}, \overline{A_{g0}}\right)$. We first consider the case where $A_{g0} \in \left(\underline{A_{g0}}, \overline{A_{g0}}\right)$. If $\vartheta < 1$, then without the boom, the economy is on a green path and the utility flow $\frac{C_\tau^{1-\vartheta}}{1-\vartheta}$ is positive and asymptotically grows at the rate $\gamma^{\eta(1-\vartheta)} - 1$. With the boom, the economy is on a dirty path and the utility flow tends to zero. For a sufficiently small discount rate, the utility (1) is then larger without the boom than with the boom.

If $\vartheta > 1$, the utility flow converges to zero without the boom. With the boom,

$$\frac{1}{(1+\rho)^\tau} \frac{C_\tau^{1-\vartheta}}{1-\vartheta} \sim -K_1 e^{K_2 \zeta(\vartheta-1)\gamma^{\eta_B \tau}} \left(\frac{\gamma^{\eta_B}}{1+\rho}\right)^\tau$$

where K_1 and K_2 are positive constant. Therefore, $\frac{1}{(1+\rho)^\tau} \frac{C_\tau^{1-\vartheta}}{1-\vartheta}$ tends to $-\infty$, so that $U = -\infty$ regardless of the discount rate ρ. Therefore, the boom reduces welfare.

If $\vartheta = 1$, we have

$$\frac{1}{(1+\rho)^\tau} \ln C_\tau = \frac{1}{(1+\rho)^\tau}\left[-\zeta S_\tau + \ln\left(\left(\nu^\lambda \widetilde{A}_E^{\lambda-1} C_{E\tau}^{\lambda-1} + (1-\nu)^\lambda A_{P\tau}^{\lambda-1}\right)^{\frac{1}{\lambda-1}} L\right)\right].$$

Without the boom, the utility flow $\ln C_\tau$ tends toward a term growing linearly, so that $\frac{1}{(1+\rho)^\tau} \ln C_\tau$ tends to zero. With the boom, the utility flow $\ln C_\tau$ is asymptotically proportional to $-\zeta S_\tau$ and tends to $-\infty$ at the rate $\gamma^{\eta_B} - 1$. Therefore we get that $U = -\infty$ if $\rho < \gamma^{\eta_B} - 1$, and more generally, welfare is reduced for sufficiently small ρ.

A.6 Endogenous Innovation in Extraction

We now consider the case where productivities of the extraction technologies, B_{ct} and B_{st}, are endogenous and determined by the allocation of scientists. We denote by s_{ft} the mass of innovators in the fossil-fuel sectors, which can now be separated into s_{Aft} innovators in the fossil-fuel power plant technologies A_{ct} and A_{st} (these innovations still apply to both technologies), s_{Bst} innovators in natural gas extraction technologies B_{st} and s_{Bct} innovators in coal extraction technologies B_{ct}. For all innovations in the fossil-fuel sector we impose for simplicity (and without loss of generality) the same probability of success η_{ft}. Innovations in extraction technologies features the same congestion externality, so that the probability of success is $\eta_f s_{it}^{-\psi}$ for $i \in \{Af, Bc, Bs\}$. Since advancing the average fossil-fuel technology now requires endogenous innovation in both power plant and extraction technologies, we let the productivity of research in green technology, η_g, be potentially different from η_f. This is necessary to ensure that long-run growth of gross output can in principle be the same with both technologies.

Expected profits in green innovations and fossil-fuel power plant technologies are still respectively given by (18) (with η_g instead of η) and by (19) (with s_{Aft} instead of s_{ft} and η_f instead of η). Expected profits in extraction technologies are given by:

$$\Pi_{Bct} = \eta_f s_{Bct}^{-\psi}\left(1-\frac{1}{\gamma}\right)\frac{C_{ct}}{B_{ct}}p_{ct}E_{ct} \text{ and } \Pi_{Bst} = \eta_f s_{Bst}^{-\psi}\left(1-\frac{1}{\gamma}\right)\frac{C_{st}}{B_{st}}p_{st}E_{st}.$$

In equilibrium, expected profits are equalized for the 4 innovation activities. This leads to equations determining the allocation of innovation within fossil-fuel technologies:

$$\left(\frac{s_{Bct}}{s_{Aft}}\right)^{\psi} = \frac{\frac{C_{ct}}{B_{ct}}\kappa_c^{\varepsilon}C_{ct}^{\varepsilon-1}}{\frac{C_{ct}}{A_{ct}}\kappa_c^{\varepsilon}C_{ct}^{\varepsilon-1}+\kappa_s^{\varepsilon}\frac{C_{st}}{A_{st}}C_{st}^{\varepsilon-1}} \text{ and } \left(\frac{s_{Bst}}{s_{Aft}}\right)^{\psi} = \frac{\frac{C_{st}}{B_{st}}\kappa_s^{\varepsilon}C_{st}^{\varepsilon-1}}{\frac{C_{ct}}{A_{ct}}\kappa_c^{\varepsilon}C_{ct}^{\varepsilon-1}+\frac{C_{st}}{A_{st}}\kappa_s^{\varepsilon}C_{st}^{\varepsilon-1}}, \tag{A-5}$$

and the allocation of innovation between green and fossil-fuel technologies:

$$\left(\frac{s_{Aft}}{s_{gt}}\right)^{\psi} = \frac{\eta_f\left(\kappa_c^{\varepsilon}\frac{C_{ct}}{A_{ct}}C_{ct}^{\varepsilon-1}+\frac{C_{st}}{A_{st}}\kappa_s^{\varepsilon}C_{st}^{\varepsilon-1}\right)}{\eta_g\kappa_g^{\varepsilon}A_{gt}^{\varepsilon-1}}. \tag{A-6}$$

Since it is possible to improve the extraction technology, this case is similar in spirit to that of high growth in extraction technologies, and in Supplementary Material Appendix B.2, we establish the following proposition.

Proposition A.3 *There is path dependence in fossil-fuel versus green innovation.*

We now look at the short-run effects of a natural gas boom on innovation. We assume that $\ln\gamma$ is small so that the three equations in (A-5) and (A-6) define a unique equilibrium and that one can ignore the dependence of the right-hand sides of (A-5) and (A-6) on the innovation allocation when taking comparative statics with respect to changes in technology levels. In Supplementary Material Appendix B.2, we establish:

Proposition A.4 *Suppose that* $\ln\gamma$ *is small. Then an exogenous increase in* B_{s0} *reduces green innovation relative to fossil-fuel power plant innovation. If* $\varepsilon C_{s0} \geq B_{s0}$*, it also reduces green innovation absolutely.*

A.7 Calibration and Electricity Producers Data

In this section, we provide further information on the calibration of parameters.

A.7.1 Accounting for Local Pollution Abatement

Due to regulations such as the Clean Air Act and the Clean Water Act, US power plants are already subject to a range of command-and-control regulations that enforce expenditures to control emissions of local pollutants such as sulfur dioxides, nitrogen oxides, and fly ash (for coal plants).

Formally, we denote by P_i^l local pollution of energy resource i, by ξ_i^l the baseline local pollution intensity, and by μ_i the share of local emissions abated, so that $P_i^l = (1-\mu_i)\xi_i^l R_i$. We assume that to abate a share μ_i of its local emissions, the producer of energy resource i needs to use an additional $\Lambda(\mu_i)$ units of power plant inputs. We denote by $\underline{\mu_i}$ the mandated minimum level of pollution abatement and assume that it is binding.

Then, the profit-maximizing input choices of energy producer of type i satisfy $R_i = E_i$ and $Q_i = \left(1 + \Lambda\left(\underline{\mu_i}\right)\right) R_i$. The equilibrium price of energy type i is then given by (22) where we define $\overline{\Lambda_i} \equiv \Lambda\left(\underline{\mu_i}\right)$. Our previous results naturally extend to this case, with A_i replaced by $\frac{A_i}{1+\overline{\Lambda_i}}$—so that we now have $C_i = \left(\frac{1+\overline{\Lambda_i}}{A_i} + \frac{1}{B_i}\right)^{-1}$ for $i \in \{s, c\}$.

A.7.2 Electricity Generation Cost Data Processing Notes

Production Input Costs. We first obtain estimates of plants' non-fuel generation costs using plant-level micro data from annual FERC Form 1 filings for our base period (2006-2010). The data provide information on plants' capital costs, annual generation, generation costs, fuel input usage, fuel heat content, etc. The data are provided as filed by utilities and can thus contain some errors and pathological observations, such as plants that are not engaged in regular operations during a given year. We exclude plant-years that are inactive, report negative operation and maintenance costs or generally negative generation costs per KWh, and those with missing information on fuel inputs. We also exclude plant-years with reported generation costs in excess of $300/MWh as this level of costs is above even the upper bounds of ranges of typical generation cost estimates and these operations generally appear unusual.[3]

For each plant-year we directly observe operational costs ("OM" for which we consider all non-capital and non-fuel production expenditures, such as on maintenance, engineering, etc.) and the plant's total capital costs (including for land, structures, and equipment). We infer annualized capital expenditures ("CAPEX") assuming a 7% interest rate in line with the literature.[4] We then compute each plant's OM and CAPEX expenditures per KWh of electricity generated by each fuel.[5] Prior to aggregation, we winsorize both OM and CAPEX per KWh at the 1% level and convert all costs into $2010.

Finally, for each year and fuel (e.g., coal in 2006), we compute the generation share-weighted average across plants of OM and CAPEX per MWh for each fuel type in each year, add them, and compute the 5-year average over our base period.

Fuel Resource Costs. Next we quantify plants' fuel resource costs using data from FERC/EIA Forms 423 (2006-07) and EIA Form 923 (2006-10).[6] The data provide fuel costs at the generator-fuel-month level, from which we compute the average cost per British thermal unit (Btu) at the plant-year-fuel level. We merge these data with plant-fuel-year level electricity generation and fuel consumption data from Form 923 to calculate fuel costs per MWh of electricity generation. Prior to aggregation, we drop observations with reported negative net electricity generation, winsorize fuel costs per MWh for each fuel type-year (e.g., coal in 2006) at the 1% level, and convert all costs in $2010. Across all plants, we then calculate the generation-weighted average fuel cost per MWh in 2006-10 for coal and natural gas generation, respectively. Note that EIA Form 923 fuel cost estimates are only available for regulated plants.

Abatement Costs. We quantify local pollution abatement expenditures based on EIA Form 767 (1985-2005) and Form 923 (2008-2010). These are mandatory surveys of both regulated and unregulated power plants. For each plant-year we observe electricity output and fuel inputs at the generator level. We drop plant-years with zero or negative net electricity generation. We assign electricity to fuels based on their heat input shares. For abatement, we attribute both OM and investment outlays for flue gas desulfurization and ash disposal to coal exclusively, and split other costs (e.g., on water abatement) between coal and gas based on their generation shares in each plant-year. We convert all

[3]For example, in 2006, the median number of hours of load operations reported among the (dropped) excessive average cost plants was only 63.5 hours *per year*, suggesting that most of these plants were not engaged in regular operations.

[4]For example, the EIA NEMS model assumes an average interest rate of 6.2% for the electricity sector (EIA 2022), whereas Lazard's assumptions imply a baseline rate of 9.2% based on a 60/40 split of debt/equity (Lazard, e.g., 2015).

[5]In the data, power plants frequently use multiple fuels. We attribute both electricity generation and costs to the fuels in question (e.g., coal vs. gas) based on their shares in the total heat content of the fuel inputs reported in a given plant-year.

[6]For 2006-07, the data also include EIA Form 906 information. Both these and Form 423 (fuel delivery information) data were consolidated into Form 923 beginning in 2008.

costs into \$2010 and use the perpetual inventory method to construct abatement capital stock estimates (assuming an annual depreciation rate of 10%), which we annualize into CAPEX again assuming a 7% interest rate. We then compute each power plant's annual abatement spending per MWh and compute the generation-share weighted average across plants for coal and gas, respectively.[7] Finally, we combine these estimates with those on general production input costs to compute the share of generation costs due to mandated local pollution abatement ($\overline{\Lambda_i}$) at the fuel-year level. We ultimately wish to quantify this abatement cost share for our model base period of 2006-2010. Unfortunately the EIA did not collect abatement expenditure data in 2006 and 2007. We thus take the average of the estimated year 2005 & 2008-10 data instead.

A.7.3 Calibration of the Parameters and Initial Technologies

Electricity Substitution Parameter λ. In the literature, the elasticity of substitution between electricity and other inputs is commonly modeled as part of a nested production function with both electricity and non-electricity energy. That is, in the background of our framework one may imagine a production function:

$$Y = \left\{ \gamma_Y (A_P Y_P)^{\frac{\sigma_1 - 1}{\sigma_1}} + (1-\gamma_Y) \left[\gamma_{Elec} E_{Elec}^{\frac{\sigma_2 - 1}{\sigma_2}} + (1-\gamma_{Elec}) E_{NonElec}^{\frac{\sigma_2 - 1}{\sigma_2}} \right]^{\frac{\sigma_2(\sigma_1 - 1)}{(\sigma_2 - 1)\sigma_1}} \right\}^{\frac{\sigma_1}{\sigma_1 - 1}}. \quad \text{(A-7)}$$

We are interested in the elasticities of substitution between the production input and electricity $\sigma_{Y_P,Elec}$ and σ_{Elec,Y_P}. The Morishima elasticities are:[8]

$$\sigma_{Elec,Y_P} = \gamma_{Elec} \cdot \sigma_1 + (1-\gamma_{Elec}) \cdot \sigma_2 \text{ and } \sigma_{Y_P,Elec} = \sigma_1.$$

The literature provides examples or estimates of the parameters in (A-7). Common values for $\sigma_1 \sim \sigma_{KL,E}$ are $0.4 - 0.5$ (e.g., Chen et al., 2017; Van der Werf, 2008; Böhringer and Rutherford 2008; Bosetti et al., 2007). As various modelers also assume $\sigma_2 = 0.5$ (e.g., Chen et al., 2017; Bosetti et al., 2007), we would have $\sigma_{Elec,Y_P} = \sigma_{Y_P,Elec} = 0.5$ for any value of γ_{Elec}. We ultimately use a slightly lower value of 0.4 in recognition of recent empirical evidence of a near-zero capital-labor and energy substitution elasticities (albeit at the yearly level) presented by Hassler et al. (2021).

Calibration of the Energy Composite. Given our estimates of the κ's and initial electricity prices, we can back out the initial electricity composite quantity and price as follows, where we note that electricity generation is measured in trillions of kWhs and

[7]The raw data contain some extreme outliers in implied abatement OM costs per MWh for some gas operators. We winsorize the right tail (top 1 percentile) of these observations.

[8]Intuitively, they are not symmetric because a change in the price of electricity also changes the relative prices of electricity and non-electricity energy, whereas a change in the price of Y_P does not.

costs are measured in \$2010:

$$p_{E,0}(\$2010\ bil./tril.kWh-eq) = \left(\kappa_g^{\varepsilon} p_{g,0}^{1-\varepsilon} + \kappa_s^{\varepsilon} p_{s,0}^{1-\varepsilon} + \kappa_c^{\varepsilon} p_{c,0}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}} = 182.41, \quad \text{(A-8)}$$

$$E_0(tril.kWh-eq) = \left(\kappa_g E_{g,0}^{\frac{\varepsilon-1}{\varepsilon}} + \kappa_s E_{s,0}^{\frac{\varepsilon-1}{\varepsilon}} + \kappa_c E_{c,0}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}} = 5.49. \quad \text{(A-9)}$$

We then solve for $\widetilde{A}_{E,0}$ based on the final goods producer's electricity first order condition:

$$p_{E,0} = \frac{\partial Y_0}{\partial E_0} = [Y_0]^{\frac{1}{\lambda}} \nu \widetilde{A_{E,0}}^{\frac{\lambda-1}{\lambda}} E_0^{\frac{-1}{\lambda}} \rightarrow \widetilde{A}_{E,0} = 1.4791+05. \quad \text{(A-10)}$$

Profit Margins and γ Calibration. We calibrate γ based on profits data, specifically to match that profits are a share $1-1/\gamma$ of sectoral income in laissez-faire. We collect data on after-tax profits per dollar of sales for corporations in three relevant industries ("Petroleum and coal products", "All Durable Manufacturing", and "All Wholesale Trade") from the US Census Bureau's *Quarterly Financial Report* for 2004-2014. With an average weighted profit share of 6.53%, we obtain $\gamma = 1.07$.

Calibration of Remaining Base Year Technologies. To calibrate the initial technology levels, we normalize $L = 10$ and we solve for the remaining 10 unknowns to satisfy the following set of 10 equations at the initial observed GDP Y_0, energy production, and energy prices (we reproduce the equations derived earlier here for clarity):

$$\begin{aligned}
\text{Unknowns} &: A_{g0}, A_{c0}, A_{s0}, B_{c0}, B_{s0}, C_{E0}, A_{P0}, w_0, L_{E0}, L_{P0} \qquad \text{(A-11)}\\
Y_0 &= (1-D(S_0))\left((1-\nu)(A_{P0}L_{P0})^{\frac{\lambda-1}{\lambda}} + \nu\left(\widetilde{A}_{E0}E_0\right)^{\frac{\lambda-1}{\lambda}}\right)^{\frac{\lambda}{\lambda-1}}\\
E_0 &= C_{E0}L_{E0} \text{ with } C_{E0} = \left(\kappa_g^{\varepsilon} A_{g0}^{\varepsilon-1} + \kappa_c^{\varepsilon}\left(\frac{1+\overline{\Lambda}_c}{A_{c0}} + \frac{1}{B_{c0}}\right)^{1-\varepsilon} + \kappa_s^{\varepsilon}\left(\frac{1+\overline{\Lambda}_s}{A_{s0}} + \frac{1}{B_{s0}}\right)^{1-\varepsilon}\right)^{\frac{1}{\varepsilon-1}}\\
A_{g0} &= \frac{\gamma w_0}{p_{g0}^q}, A_{c0} = \frac{\gamma w_0}{p_{c0}^q}, A_{s,0} = \frac{\gamma w_0}{p_{s0}^q}, B_{c,0} = \frac{\gamma w_0}{p_{c0}^r} \text{ and } B_{s,0} = \frac{\gamma w_0}{p_{s0}^r}\\
\frac{\gamma w_0}{A_{P0}} &= (1-\nu) Y_0^{\frac{1}{\lambda}} (A_{P,0}L_{P,0})^{\frac{-1}{\lambda}} \text{ with } L_{P0} = 10 - L_{E0}
\end{aligned}$$

The resulting parameter values are as follows:

$A_{g,0}$	$A_{c,0}$	$A_{s,0}$	$B_{c,0}$	$B_{s,0}$	$C_{E,0}$	$A_{P,0}$	w_0	L_{E0}
98.35	452.96	441.16	330.77	117.18	38.96	4.8047e+03	6,738	1.409%

Calibration of the Carbon Cycle. We adopt the carbon cycle of Golosov et al. (2014), with the following modifications to match our time period and base period. First, a fraction of carbon emissions remains permanently in the atmosphere, set at $\varphi_L = 0.2$.

Another fraction φ_0 exits the atmosphere within a decade, and the remainder decays at rate φ. For the latter, GHKT match an atmospheric half life of 300 years, implying, in our setting, that decay parameter φ should solve $(1-\varphi)^{60} = 0.5$ and hence $\varphi = 0.0115$. For the former, GHKT match the moment that about half of a CO_2 impulse is removed after 30 years. In our setting, this implies $1-\frac{1}{2} = 0.2 + 0.8\varphi_0(1-0.0115)^5$ yielding $\varphi_0 = .3973$. Finally, we update initial carbon stocks from the year-2000 levels in GHKT to our base period (2006-2010) levels. Total CO_2 concentrations are set to $S_0 = 830$ Gtc based on the 2010 average from the Mauna Loa observatory.[9]

Calibration of Disutility over ROW Climate Damages. The main parameter needed to include rest of the world climate damages in US utility in (24) is ς_t. Conceptually, we would like ς_t to approximate the product of rest of the world output Y_t^{ROW} and the US marginal utility of consumption, which in our model is equivalent to:

$$-\varsigma_t = Y_t^{ROW} \cdot (Y_t^{US})^{-\vartheta}. \tag{A-12}$$

Figure A.4 displays different estimates for the US share of world GDP over time based on data and forecasts by the International Monetary Fund (IMF), from the RICE Model (Nordhaus, 2011), and from the Shared Socioeconomic Pathway (SSP) 2 Scenario based on the SSP database hosted by the International Institute for Applied Systems Analysis Energy Program.[11]

We then infer values of ς_t by (i) assuming that US GDP grows 2% per year from its initial value (taken from the data in the first model period of 2011+), (ii) inferring rest-of-the-world GDP based on our predicted US share of World GDP, and (iii) evaluating (A-12) at these values. We note that ς_t is sensitive to the units in which GDP is reported. For our calibration, which reports GDP in 5-year flows of billions of \$2010, its starting value is $\varsigma_{2011-15} = -0.0223$.

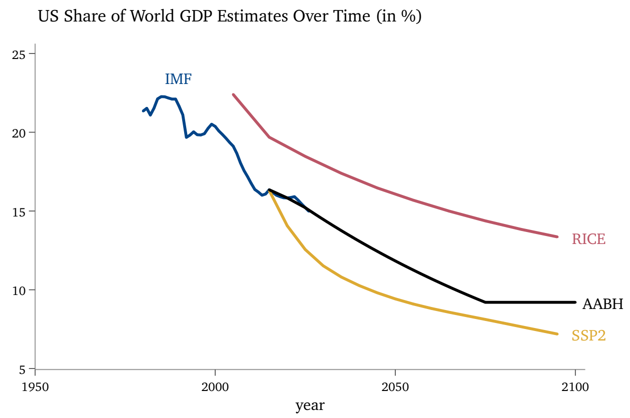
A.8 Further Quantitative Results

We now present a few additional quantitative results

[9]Data are from the National Oceanic and Atmospheric Administration's Global Monitoring Laboratory website with URL (accessed December 2021): https://gml.noaa.gov/ccgg/trends/data.html The permanent reservoir is set at GHKT's initial value plus 20% of 2005-2010 global emissions (CDIAC, 2020), yielding $S_{1,0} = 684 + 10 = 694$.[10] The remaining increase in total concentrations is assigned to the second reservoir, $S_{2,0} = 136$ (up from 118 in GHKT).

[11]We specifically consult the IMF's World Economic Outlook October 2021 projections of "GDP based on PPP, share of the world", available at URL (accessed October 2021): https://www.imf.org/external/datamapper/PPPSH@WEO/OEMDC/ADVEC/WEOWORLD. The SSP database is available at (accessed December 2021): https://tntcat.iiasa.ac.at/SspDb. All projections are in PPP-adjusted dollars. We adopt an intermediate approach which uses the available IMF forecasts through 2025, assumes that the US share of world GDP declines at 1% per year thereafter until 2075, and then stabilizes at 9.2%.

Figure A.4—Projections of US Share of World GDP



Note: This figure plots the projected US share of World GDP. The historical data and medium-run projections are from the IMF, and projections include those from the RICE model (data source: Nordhaus, 2011), the Shared Socioeconomic Pathway 2 from the IPCC (SSP2, data source: IIASA), and the one we use in our paper (AABH).

A.8.1 Spillover Effects

This section presents results for a variant of our benchmark model which allows for spillover effects of the shale gas boom to non-electricity emissions in the United States and to the rest of the world (ROW). Intuitively, we might expect electrification of other sectors (e.g., transportation) to be affected by both changes in electricity prices and in terms of emissions. Moreover, shale extraction technology may also spill over to other countries and perhaps more importantly, even if the ROW does not use shale gas, than there may be spillovers from the redirection of US innovation in the electricity sector toward fossil fuels (i.e. the increase in A_{ct} and A_{st} with a decrease in A_{gt} in the US).

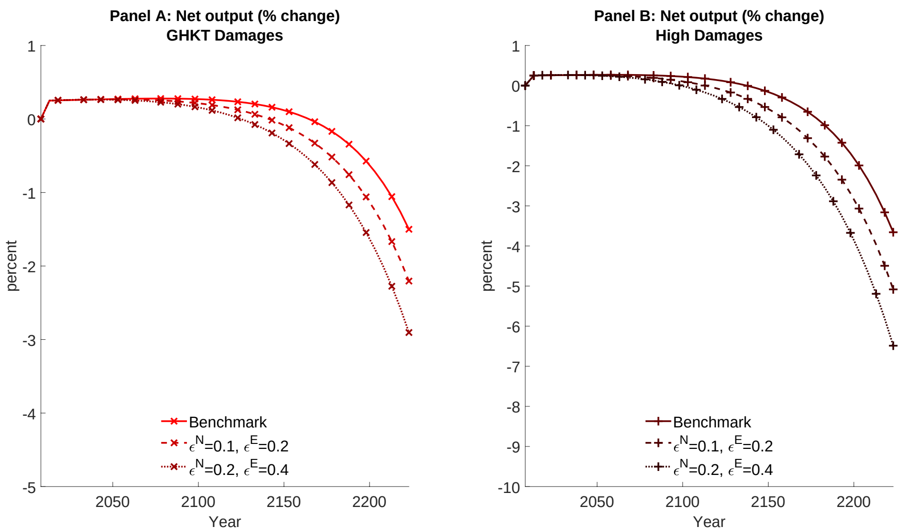
We capture these effects in a stylized manner by assuming that both US and ROW non-electricity pollution flows ($\overline{P_t^{N.Elec}}$) and ROW electricity emissions ($\overline{P_t^{ROW,Elec}}$) respond to changes in US electricity emissions ($\%\Delta P_t^{US,Elec}$) based on response elasticities ϵ^N and ϵ^E, respectively. That is, global emissions at time t are given by:

$$P_t^{Global} = P_t^{US,Elec} + \overline{P_t^{ROW,Elec}} \cdot (1 + \%\Delta P_t^{US,Elec} \cdot \epsilon^E) + \overline{P_t^{N.Elec}} \cdot (1 + \%\Delta P_t^{US,Elec} \cdot \epsilon^N)$$

where upper bars denote business-as-usual emissions levels, $P_t^{US,Elec}$ denotes endogenous US emissions as defined in (15), and where $\%\Delta P_t^{US,Elec}$ is formally defined as the percent change in US electricity emissions at time t due to the shale gas boom.

Figure A.5 compares the benchmark model results (where $\epsilon^N = \epsilon^E = 0$) with two alternate specifications, focusing on US GDP impacts of the shale gas boom in laissez-faire. Allowing for spillover effects increases the projected effects of the shale gas boom. While both the initial output benefit of the boom and the longer-term negative effects are strengthened by spillovers, the impact on the latter is larger. Intuitively, this asymmetry is driven by the facts that (i) initial output benefits are largely due to cheaper energy prices, which are invariant to emissions spillovers, and (ii) the relative importance of ROW emissions for climate change increases significantly over time. That is, due to the projected future rise of emissions from countries such as China and India, the climate benefits of reducing ROW emissions by 1% today are much smaller than the climate damages of increasing ROW damages by 1% in the year 2100. Overall, the results thus suggest that abstracting from spillovers in the benchmark is conservative in that it will lead us to understate the overall effects of the shale gas boom.

Figure A.5—Effect of the Shale Boom on GDP in the Presence of Emissions Spillovers



Note: This figure shows the effect of the shale boom on net output in laissez-faire when there are spillovers from technological development in the US electricity sector to non-electricity US and global emissions. The boom has a more detrimental long-run effect to net output in the presence of spillovers.

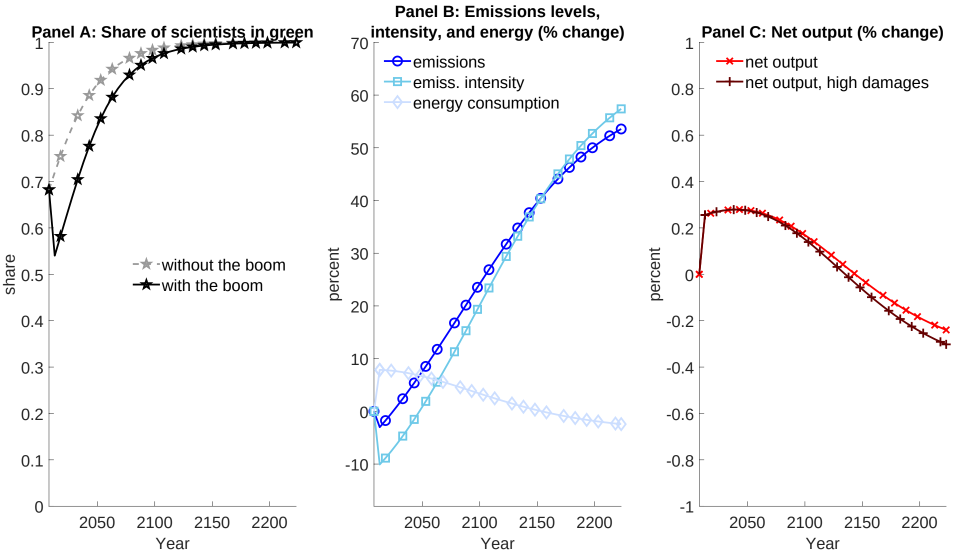
A.8.2 Slow Progress in Extraction Technologies

Here we present results from a revised version of the benchmark model with slow progress in extraction technologies (B_{ct} and B_{st}). We specifically consider the limiting case with zero progress after the shale gas boom. Figure A.6 shows the results, mirroring Figure 3 for the baseline case. In line with Proposition 7, the shale gas boom (i) delays the transition to a green economy (Panel A), (ii) increases CO_2 emissions in the long run (Panel B), and (iii) decreases output in the long run (Panel C).

A.9 Extended Model

In this part of the Appendix, we analyze the extended model presented in Section 5.2. First, we present the static equilibrium conditions. We then solve for the dynamic equilibrium and subsequently discuss the calibration of the model and present quantitative results.

Figure A.6—Shale Boom Impact on Laissez-faire Outcomes in the Absence of Technological Progress in Extraction Technologies



Note: This figure shows the dynamic effects of the shale gas boom in laissez-faire when there is no progress in extraction technologies (except for the boom). Panel A shows the allocation of scientists with and without the shale boom. While a green transition occurs in both cases, the boom significantly slows it down. Panel B shows the changes (in %) in emission intensity, energy consumption and emissions that result from the boom. As in the baseline case, the boom is associated with an initial decline in emission intensity that is reversed over time, and an increase in long-run emissions. Panel C shows the effects on net output of the boom for two calibrations of the damage function. The boom eventually decreases net output.

A.9.1 Static Equilibrium and Short-Run Effect in the Extended Model

We now derive the static equilibrium conditions of the extended model. We define the fossil-fuel energy composite as

$$E_{f,t} \equiv \left(\kappa_c E_{c,t}^{\frac{\sigma-1}{\sigma}} + \kappa_s E_{s,t}^{\frac{\sigma-1}{\sigma}} \right)^{\frac{\sigma}{\sigma-1}}. \tag{A-13}$$

To find the demand for coal and natural gas energy, we solve for the maximization problem of a fossil-fuel energy composite producer:

$$\max_{E_{st}, E_{ct}} p_{ft} E_{ft} - (1+\widetilde{\tau}_s) p_{st} E_{st} - (1+\widetilde{\tau}_c) p_{ct} E_{ct},$$

where $\widetilde{\tau}_i$ denotes the add-valorem tax on energy i. Using that the energy prices are still given by $p_{it} = \gamma w_t / C_{it}$, for $i = c, g, s$, we get:

$$\begin{aligned} E_{c,t} &= \kappa_c^{\sigma} \left(\frac{C_{ct}}{(1+\widetilde{\tau}_c) C_{ft}} \right)^{\sigma} E_{ft} \text{ and } E_{s,t} = \kappa_s^{\sigma} \left(\frac{C_{st}}{(1+\widetilde{\tau}_s) C_{ft}} \right)^{\sigma} E_{ft}, \quad \text{(A-14)} \\ \text{with } C_{ft} &\equiv \left(\kappa_c^{\sigma} \left(\frac{C_{ct}}{1+\widetilde{\tau}_c} \right)^{\sigma-1} + \kappa_s^{\sigma} \left(\frac{C_{st}}{1+\widetilde{\tau}_s} \right)^{\sigma-1} \right)^{\frac{1}{\sigma-1}}. \end{aligned}$$

C_{ft} is the fossil-fuel aggregate productivity and the fossil-fuel aggregate price is:

$$p_{ft} = \gamma w_t / C_{ft}. \tag{A-15}$$

Profit maximization by the electricity producer then leads to

$$E_{f,t} = \left(\frac{C_{ft}}{C_{Et}}\right)^{\varepsilon} E_t \text{ and } E_{g,t} = \kappa_g^{\varepsilon}\left(\frac{C_{gt}}{\left(1+\widetilde{\tau}_g\right) C_{Et}}\right)^{\varepsilon} E_t \tag{A-16}$$

$$\text{with } C_{Et} \equiv \left(C_{ft}^{\varepsilon-1} + \kappa_g^{\varepsilon}\left(\frac{A_{gt}}{1+\widetilde{\tau}_g}\right)^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}. \tag{A-17}$$

C_{Et} is still the aggregate productivity in energy and the energy price obeys (13). For given E_t, the production of the different energy inputs is given by (A-16) and (A-14).

To solve for E_t, we follow steps similar to those in the baseline model and detailed in Supplementary Material B.3.1. We find that

$$E_t = \widetilde{C}_{Et} L_{Et} \text{ with } \widetilde{C}_{Et} \equiv C_{Et}^{\varepsilon}\left[C_{ft}^{\varepsilon}\widetilde{C}_{ft}^{-1} + \kappa_g^{\varepsilon}\left(1+\widetilde{\tau}_g\right)^{-\varepsilon} A_{gt}^{\varepsilon-1}\right]^{-1}, \tag{A-18}$$

$$\widetilde{C}_{ft} \equiv C_{ft}^{\sigma}\left[\kappa_c^{\sigma}\left(1+\widetilde{\tau}_c\right)^{-\sigma} C_{ct}^{\sigma-1} + \kappa_s^{\sigma}\left(1+\widetilde{\tau}_s\right)^{-\sigma} C_{st}^{\sigma-1}\right]^{-1}, \tag{A-19}$$

$$\text{and } L_{Et} = \frac{\nu^{\lambda}\left(\widetilde{A}_E\right)^{\lambda-1} C_{Et}^{\lambda}\widetilde{C}_{Et}^{-1}}{\nu^{\lambda}\left(\widetilde{A}_E\right)^{\lambda-1} C_{Et}^{\lambda}\widetilde{C}_{Et}^{-1} + (1-\nu)^{\lambda} A_{Pt}^{\lambda-1}}. \tag{A-20}$$

To derive emissions, we use (A-14) and (A-16), and we obtain $P_t = \xi_{Et} E_t$ with the emission rate ξ_{Et} now given by:

$$\xi_{Et} = \left(\xi_c \kappa_c^{\sigma}\left(\frac{C_{ct}}{\left(1+\widetilde{\tau}_c\right) C_{ft}}\right)^{\sigma} + \xi_s \kappa_s^{\sigma}\left(\frac{C_{st}}{\left(1+\widetilde{\tau}_s\right) C_{ft}}\right)^{\sigma}\right)\left(\frac{C_{ft}}{C_{Et}}\right)^{\varepsilon}.$$

The short-run effect of the natural gas boom on emissions can again be decomposed into a substitution effect which affects the emission rate ξ_{Et} and a scale effect which affects energy demand E_t. We get the modified Proposition 1 (proof in Supplementary Material Appendix B.3.2):

Proposition A.5 *A natural gas boom (that is a one time increase in B_s at time $t = 0$) leads to a change in the emission rate given by (27). Emissions decrease in the short-run provided that natural gas is sufficiently clean compared to coal (for ξ_s/ξ_c small enough) and the ad-valorem taxes ($\widetilde{\tau}_c, \widetilde{\tau}_s$ and $\widetilde{\tau}_g$) are small.*

A.9.2 Dynamic Equilibrium and Innovation Effect in the Extended Model

We now derive the innovation allocation. Expected profits from clean research still obey (18) but with η_g instead of η. Instead of (19), expected profits from an innovation in fossil-fuel technologies are now given by

$$\Pi_{ct} = \eta_f s_{ct}^{-\psi}\left(1-\frac{1}{\gamma}\right)\left(\frac{C_{ct}\left(1+\overline{\Lambda}_c\right)}{A_{ct}}p_{ct}E_{ct} + \chi\frac{C_{st}\left(1+\overline{\Lambda}_s\right)}{A_{st}}p_{st}E_{st}\right), \tag{A-21}$$

for an innovation directed at the coal technologies and by

$$\Pi_{st} = \eta_f s_{st}^{-\psi}\left(1-\frac{1}{\gamma}\right)\left(\chi\frac{C_{ct}\left(1+\overline{\Lambda}_c\right)}{A_{ct}}p_{ct}E_{ct} + \frac{C_{st}\left(1+\overline{\Lambda}_s\right)}{A_{st}}p_{st}E_{st}\right), \tag{A-22}$$

for an innovation directed at natural gas technologies.

In equilibrium, scientists are indifferent between innovating in the three sectors; therefore, denoting by q_j a R&D subsidy for sector j, we get

$$\frac{\Pi_{ct}}{1-q_c} = \frac{\Pi_{st}}{1-q_s} \text{ and } \frac{\Pi_{ct}}{1-q_c} + \frac{\Pi_{st}}{1-q_s} = 2\frac{\Pi_{gt}}{1-q_g}. \tag{A-23}$$

Using (A-14) and (A-16), we get the revenue shares within the energy sector:

$$\Theta_{gt} = \frac{p_{gt}E_{gt}}{p_{Et}E_t} = \frac{\kappa_g^{\varepsilon}}{\left(1+\tilde{\tau}_g\right)^{\varepsilon}}\left(\frac{C_{gt}}{C_{Et}}\right)^{\varepsilon-1} \text{ and } \Theta_{it} = \frac{p_{it}E_{it}}{p_{Et}E_t} = \frac{\kappa_i^{\sigma}}{(1+\tilde{\tau}_i)^{\sigma}}\left(\frac{C_{it}}{C_{ft}}\right)^{\sigma-1}\left(\frac{C_{ft}}{C_{Et}}\right)^{\varepsilon-1} \text{ for } i \in \{c,s\}. \tag{A-24}$$

Using (A-24), (A-21), and (A-22) allows us to rewrite the indifference condition for innovation within the fossil-fuel sector (in A-23) as:

$$\left(\frac{s_{ct}}{s_{st}}\right)^{\psi} = \frac{(1-q_s)\left(\frac{\kappa_c^{\sigma}}{(1+\tilde{\tau}_c)^{\sigma}}\frac{(1+\overline{\Lambda}_c)C_{ct}^{\sigma}}{A_{ct}} + \chi\frac{\kappa_s^{\sigma}}{(1+\tilde{\tau}_s)^{\sigma}}\frac{(1+\overline{\Lambda}_s)C_{st}^{\sigma}}{A_{st}}\right)}{(1-q_c)\left(\chi\frac{\kappa_c^{\sigma}}{(1+\tilde{\tau}_c)^{\sigma}}\frac{(1+\overline{\Lambda}_c)C_{ct}^{\sigma}}{A_{ct}} + \frac{\kappa_s^{\sigma}}{(1+\tilde{\tau}_s)^{\sigma}}\frac{(1+\overline{\Lambda}_s)C_{st}^{\sigma}}{A_{st}}\right)}. \tag{A-25}$$

Using the same equations together with (18) and (A-24), we can rewrite the indifference condition for innovation between the green and the fossil-fuel sector (in A-23) as:

$$\frac{\eta_f C_{ft}^{\varepsilon-\sigma}\left[\left(\frac{s_{ct}^{-\psi}}{1-q_c} + \frac{\chi s_{st}^{-\psi}}{1-q_s}\right)\frac{\kappa_c^{\sigma}}{(1+\tilde{\tau}_c)^{\sigma}}\frac{(1+\overline{\Lambda}_c)C_{ct}^{\sigma}}{A_{ct}} + \left(\frac{\chi s_{ct}^{-\psi}}{1-q_c} + \frac{s_{st}^{-\psi}}{1-q_s}\right)\frac{\kappa_s^{\sigma}}{(1+\tilde{\tau}_s)^{\sigma}}\frac{(1+\overline{\Lambda}_s)C_{st}^{\sigma}}{A_{st}}\right]}{\eta_g \frac{s_{gt}^{-\psi}}{1-q_g}\frac{\kappa_g^{\varepsilon}}{\left(1+\tilde{\tau}_g\right)^{\varepsilon}}C_{gt}^{\varepsilon-1}} = 2. \tag{A-26}$$

Finally, the scientists market clearing equilibrium condition is now given by

$$s_{ct} + s_{st} + s_{gt} = 1. \tag{A-27}$$

We then define a dynamic equilibrium of this economy.

Definition A-1 *The dynamic equilibrium is defined by the indifference conditions (A-25) and (A-26), the scientist market clearing condition (A-27), the laws of motion for A_j and the definitions of C_{ft}, C_{st} and C_{ct} and the laws of motion for A_{jt} (26).*

We note that the equilibrium is unique for sufficiently small innovation size $\ln \gamma$ (proof in Supplementary Material Appendix B.3.3).

As in the baseline model, without enough technological progress in extraction technology, innovation must occur in clean technologies in the long-run; whereas there is path dependence in innovation if there is sufficiently fast progress in extraction technology. When the extraction technologies grow at the rate $\gamma^{\eta_f\left(1+\chi^{\frac{1}{\psi}}\right)^{\psi}}-1$, and innovation occurs only in the fossil-fuel sector in the long-run, then the energy productivity variables C_{Et} and $\widetilde{C}_{Et}$ grow asymptotically at most at the same rate $\gamma^{\eta_f\left(1+\chi^{\frac{1}{\psi}}\right)^{\psi}}-1$, which is achieved if $q_c = q_s$ — see proof in Supplemental Material Appendix B.3.3. Similarly, if innovation occurs only in the green sector in the long-run, then energy productivity asymptotically grows at the rate $\gamma^{\eta_g}-1$. In the calibration, we impose $\eta_g = \eta_f\left(1+\chi^{\frac{1}{\psi}}\right)^{\psi}$, which ensures that the long-run growth potential for output gross of climate damages is the same on a clean path and on a fossil-fuel path.

We now look at the effect of the natural gas boom on innovation allocation at $t=1$. We assume that $\ln \gamma$ is low, so that we ignore the dependence of A_{ct}, A_{st} and A_{gt} on the innovation allocation at time t. For simplicity, we focus on two cases where we can derive analytical results: (1) when the elasticity of substitution between green and fossil fuels is not much lower than that between fossil fuels, $\sigma \approx \varepsilon$, which corresponds to our calibration; and (2) when most fossil-fuel innovations are common to coal and natural gas ($\chi \approx 1$). We show in Supplementary Material Appendix B.3.4, the following proposition.

Proposition A.6 *Suppose $\ln \gamma$ is small (which ensures that the equilibrium is unique).*

1. *Assume that $\sigma \approx \varepsilon$. Then a natural gas boom increases innovation in natural gas technology and decreases innovation in green technology. The effect on coal technology is ambiguous: it is positive if χ is sufficiently close to 1, and negative if χ is sufficiently close to zero.*

2. *Assume that $\chi \approx 1$ and $\frac{\left(1+\overline{\Lambda}_s\right)C_{st}}{(1+\widetilde{\tau}_s)A_{st}} \geq \frac{\left(1+\overline{\Lambda}_c\right)C_{ct}}{(1+\widetilde{\tau}_c)A_{ct}}$. Then a natural gas boom leads to a decrease in green innovation and an increase in both types of fossil-fuel innovations.*

In this extension, a shale gas boom need not necessarily be associated with a decline in green innovation. The reason is that when $\sigma > \varepsilon$, green technologies are more complementary to natural gas technologies than coal technologies are. As a result, a natural gas boom could potentially encourage green innovation. Clearly, this channel is dominated if σ is sufficiently close to ε. Even if σ is large relative to ε, this channel could be dominated when most fossil-fuel innovations are mostly common to coal and natural gas (χ is large) and $\frac{(1+\overline{\Lambda}_s)C_{st}}{(1+\tilde{\tau}_s)A_{st}} \geq \frac{(1+\overline{\Lambda}_c)C_{ct}}{(1+\tilde{\tau}_c)A_{ct}}$, which means that the "adjusted" productivity of the power plant technology relative to the extraction technology is not too large in the natural gas sector relative to coal sector. In the calibration, σ turns out to be close to ε and the natural gas boom leads to a reduction in green innovation, as in our main analysis. Finally, we remark that when $\chi \neq 1$, the effect of the natural boom on coal-based innovation is ambiguous: for χ small, the natural gas boom may relocate innovation away from coal.

A.9.3 Calibration of the Extended Model

The calibration of the extended model follows similar steps as the benchmark, with appropriate modifications and additions. First, we retain the same parameters from the literature as in Table 3 (i.e., ε, λ, ν, γ, $\xi_{c,}$ and ξ_s), and set $\sigma = 2$ as described in Section 5.2. Second, we solve for the energy share parameters $\kappa_{c,}$ $\kappa_{s,}$ and κ_g in (25) jointly with the initial fossil price index p_{ft} and quantity E_{ft} via (A-13), $1 = \kappa_c + \kappa + \kappa_g$ and the modified set of equations:

$$\frac{E_{c,t}}{E_{s.t}} = \left(\frac{\kappa_c}{\kappa_s}\frac{(1+\tau_{st})p_{st}}{(1+\tau_{ct})p_{ct}}\right)^{\sigma} \text{ and } \frac{E_{g,t}}{E_{f.t}} = \left(\kappa_g \frac{p_{ft}}{(1+\tau_g)p_{gt}}\right)^{\varepsilon}, \tag{A-28}$$

$$p_{ft} = (\kappa_c^{\sigma}(p_{ct}(1+\tau_{ct}))^{(1-\sigma)} + \kappa_s^{\sigma}(p_{st}(1+\tau_{st}))^{(1-\sigma)})^{\frac{1}{1-\sigma}}.$$

This quantification requires estimates of BAU taxes on coal, gas, and green electricity production (above and beyond the effective tax resulting from local pollution abatement regulations, which is already captured in the $\overline{\Lambda}_c$ and $\overline{\Lambda}_s$ terms). For green electricity, we compare Lazard estimates of the levelized costs of each type of green electricity with vs. without subsidies (e.g., investment and production tax credits) to infer the effective overall subsidy for each energy type in the available years of our base period (2008-10).[12] For nuclear power, there are no subsidy-inclusive estimates. We assume a zero value as there is only limited tax support for nuclear power generation.[13] These calculations yield a generation-weighted average green tax in our base period of $\tau_g = -3.68\%$. For fossil generation, only a small fraction of US electricity generation has been subject to a carbon price (OECD 2018), but renewable portfolio standards had been enacted by more than half of US states by our base period, with measurable impacts on prices (Greenstone and

[12] Lazard generally presents ranges of LCOE estimates. We use averages between the bounds.

[13] Nuclear generators enjoy a special tax rate on reserve funds set aside for decommissioning, however the effects of this policy on generation have been argued to be minimal (Nordhaus, Merrill, and Beaton 2013). The Energy Policy Act of 2005 introduced a temporary generation tax credit of 1.8 cents per kWh for the first eight years of operation of *new* nuclear capacity beginning operation by 2020.

Nath, 2019). As a benchmark, we thus assume low but positive taxes on fossil generation in line with relative carbon contents ($\tau_s = 2.5\%$ and $\tau_c = 5\%$), but consider robustness to other values below. The benchmark calibration implies $\kappa_c = 0.2806$, $\kappa_s = 0.3711$, $\kappa_g = 0.3484$, $p_{f0} = 269.26$, and $E_{f0} = 2.6114$.

Next, we compute the initial energy price index by extending (A-8) to

$$p_{E0} = \left(\kappa_g^{\varepsilon}\left[p_{g0}(1+\tau_g)\right]^{1-\varepsilon} + p_{f0}^{1-\varepsilon}\right)^{\frac{1}{1-\varepsilon}},$$

the initial energy aggregate E_0 via (A-9), and the productivity parameter $\widetilde{A_E}$ via (A-10). Analogous to the benchmark, we then solve for the remaining unknown variables in initial equilibrium ($A_{g0}, A_{c0}, A_{s0}, B_{c0}, B_{s0}, C_{c0}, C_{s0}, C_{f0}, C_{E0}, \widetilde{C}_{f0}, \widetilde{C}_{E0}, A_{P0}, L_{E0}, L_{P0}, w_0$) in an extended version of (A-11) with 15 equations:[14]

$$\begin{aligned}
Y_0 &= (1-D(S_0))\left((1-\nu)(A_{P0}L_{P0})^{\frac{\lambda-1}{\lambda}} + \nu\left(\widetilde{A}_{E0}E_0\right)^{\frac{\lambda-1}{\lambda}}\right)^{\frac{\lambda}{\lambda-1}} \\
A_{g0} &= \frac{\gamma w_0}{p^q_{g0}}, A_{c0} = \frac{\gamma w_0}{p^q_{c0}}, A_{s0} = \frac{\gamma w_0}{p^q_{s0}}, B_{c0} = \frac{\gamma w_0}{p^r_{c0}} and B_{s0} = \frac{\gamma w_0}{p^r_{s0}} \\
C_{c0} &= \left(\frac{1+\overline{\Lambda}_c}{A_{c0}} + \frac{1}{B_{c0}}\right)^{-1} \text{ and } C_{s0} = \left(\frac{1+\overline{\Lambda}_s}{A_{s0}} + \frac{1}{B_{s0}}\right)^{-1} \\
C_{f0} &= \left(\kappa_c^{\sigma}\left(\frac{C_{c0}}{1+\tau_c}\right)^{\sigma-1} + \kappa_s^{\sigma}\left(\frac{C_{s0}}{1+\tau_s}\right)^{\sigma-1}\right)^{\frac{1}{\sigma-1}} \text{ and } \widetilde{C}_{f0} = \frac{C^{\sigma}_{f,0}}{\frac{\kappa_c^{\sigma}C_{ct}^{\sigma-1}}{(1+\tau_{ct})^{\sigma}} + \frac{\kappa_s^{\sigma}C_{st}^{\sigma-1}}{(1+\tau_{st})^{\sigma}}} \\
E_0 &= \widetilde{C}_{E0}L_{E0} \text{ with } C_{E0} = \left(\frac{\kappa_g^{\varepsilon}A_{g0}^{\varepsilon-1}}{\left(1+\tau_g\right)^{\varepsilon-1}} + C_{f0}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}} \text{ and } \widetilde{C}_{Et} = \frac{C^{\varepsilon}_{E,0}}{\frac{C^{\varepsilon}_{ft}}{\widetilde{C}_{ft}} + \frac{\kappa_g^{\varepsilon}A_{gt}^{\varepsilon-1}}{\left(1+\tau_{gt}\right)^{\varepsilon}}} \\
\gamma w_0 &= A_{P0}(1-\nu)Y_0^{\frac{1}{\lambda}}(A_{P0}L_{P0})^{\frac{-1}{\lambda}} \text{ and } L_0 = L_{E0} + L_{P0}.
\end{aligned}$$

Finally, we quantify the innovation-related parameters as follows. First, we compute BAU subsidies based on the National Science Foundation's "Industrial Research and Development" Survey, which breaks down both private and government-supported R&D spending by energy technology category. We found this to be the most comprehensive database of both public and private R&D spending by technology type.[15] On average during the model base years available in the data (2006-07), 3.85% of R&D spending at firms was funded by the government in the fossil-fuel sector, leading us to set

[14]In the benchmark calibration, the results imply that $A_{g,0} = 98.34$, $A_{c,0} = 452.88$, $A_{s,0} = 441.08$, $B_{c,0} = 330.71$, $B_{s,0} = 117.17$, $C_{c,0} = 183.69$, $C_{s,0} = 92.47$, $C_{f,0} = 26.20$, $C_{E,0} = 40.51$, $\widetilde{C}_{f,0} = 27.19$, $\widetilde{C}_{E,0} = 41.06$, $A_{P,0} = 4.8054e+03$, $L_{E,0} = 0.1411$, $L_{P,0} = 9.8589$, $w_0 = 6.7365e+03$.

[15]U.S. Energy Information Administration financial reporting data provided R&D expenditures by major energy producers through 2009, but observations on renewable technologies are withheld in many years for privacy reasons. Conversely, while the International Energy Agency collects robust data on public R&D support, some of their estimates of private R&D expenditures appear implausibly noisy, with zero private expenditure shares reported in many years.

$q_c = q_s = 0.0385$. For green technology, we take the total R&D spending-weighted average subsidy rate across renewables (13.9%) and nuclear (0%) in the most recent year with disclosed data (2004), yielding $q_g = 0.125$.

Then, we set the probability that gas patents are also relevant for coal, χ, so that the model matches the ratio of green to fossil patents observed in the data in the pre-boom period ($\eta_g s_g^{1-\psi}/[\eta_f(s_s^{1-\psi}+s_c^{1-\psi})]$ =1.47). This yields a benchmark value of $\chi = 0.945$. This high figure is in line with the empirical fact that many technologies are shared : improved boilers, steam engines, super-heaters, combined-cycle generation, technologies such as gas turbines due to coal gasification, etc. (see discussion in Lanzi et al., 2011). Yet, for robustness, we also present results for a much lower value of $\chi = 0.5$.

As in the baseline model, we set research productivities, the η's, to permit balanced long-run growth at 2% per year. This now implies: $\eta_g = 5\ln 1.02/\ln\gamma = 1.4634$ for green and an adjusted value of $\eta_f = \eta_g\left(1+\chi^{\frac{1}{\psi}}\right)^{-\psi} = 1.0636$ for fossil innovation. The remainder of the parameters are as in the baseline model.

A.9.4 Quantitative Results

First, Table A.2 shows that the predicted short-run effects of the shale gas boom in the extended model are very similar to our baseline framework results in Table 5.

Table A.2—Short-run Effects of the Shale Gas Boom in the Extended Model

Extended Model Version	$\%\Delta\xi_E$	$\%\Delta E$	$\%\Delta CO_2$
Benchmark	-13.3%	8.2%	-6.2%
More pro-fossil BAU ($\tau_j = 0, 2\times q_j, j = c, s$)	-13.3%	8.2%	-6.3%
More pro-green BAU ($2\times\tau_g, q_g$)	-13.6%	8.2%	-6.5%
No BAU policies	-13.0%	8.1%	-6.0%

Note: This table shows the projected short-run impacts of a 100% increase in B_s0 on emissions intensity (ξ_E), electricity consumption (E), and electricity-based CO_2 emissions, respectively, in the benchmark case, a more pro-fossil fuel scenario (without taxes on fossil generation and with double fossil innovation subsidies), a more pro-green scenario (with double the green generation and innovation subsidies), and a scenario with no policies. In all cases, the results are similar to those of the baseline model: the substitution effect is negative and dominates the scale effect.

Figure 6 in the text shows the predicted long-run impacts of the shale gas boom in the extended model. We conclude by also depicting predicted welfare impacts of the boom in the extended model, again over a 400 year time horizon, including for several sensitivity checks (Table A.3). As in the main model, the projected welfare effects of the boom with market-based discounting are around -1.5% to -2.5% in the GHKT and high damages cases, respectively. The results change only minimally across different assumptions about BAU policies. Even lowering the assumed coal-gas technology overlap parameter χ to 50% mitigates the negative welfare impacts of an unmanaged shale gas boom only slightly (by dampening the boom's positive impacts on innovation in coal), and continues to imply a "fossil-fuel trap" impact of the boom.

Table A.3—Welfare effects of the Shale Gas Boom in the Extended Model

Extended Model Version	Welfare Impacts $\rho^{yr} = 1\%$		Threshold ρ^{yr}	
Damages:	GHKT	High	GHKT	High
Benchmark	-1.4%	-2.3%	1.9%	2.3%
More pro-fossil BAU ($\tau_j = 0, 2 \times q_j, j = c, s$)	-1.5%	-2.7%	2.0%	2.4%
More pro-green BAU ($2 \times \tau_g, q_g$)	-1.4%	-2.5%	1.9%	2.3%
No BAU policies	-1.5%	-2.7%	2.0%	2.4%
Lower $\chi = 0.5$	-1.3%	-2.2%	1.9%	2.2%

Note: This table reports, across a number of scenarios, the welfare impacts of the shale gas boom (in consumption equivalent terms), "Welfare Impacts", and the threshold on the annual pure rate of social time preference below which these welfare impacts are negative ("Threshold ρ_{yr}"). In all cases, the welfare effects of the shale gas boom are negative for a 1% annual utility discount rate. Welfare is computed over 400 years.

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B Supplementary Material for "Climate Change, Directed Innovation, and Energy Transition: The Long-run Consequences of the Shale Gas Revolution"

B.1 Additional Proofs for the Baseline Model

B.1.1 Proofs of Propositions A.1 and A.2

To prove these results, we start by defining the function $I(s) \equiv (\varepsilon-1)(1-s)^{1-\psi}+s^{1-\psi}-\varepsilon\frac{\eta_B}{\eta}$ and characterize its zeros in the following two Lemmas.

Lemma B.1 *Assume that $\varepsilon \geq 2^{1-\psi}$. Over the interval $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}},1\right)$, the function $I(s)$ has:*

1. *no zero if $\frac{\eta_B}{\eta}<\frac{1}{\varepsilon}$;*
2. *one zero with $\frac{1}{\varepsilon}<\frac{\eta_B}{\eta}<\frac{1}{2^{1-\psi}}$ and this zero satisfies $I'(s^*)<0$;*
3. *no zero if $\frac{\eta_B}{\eta}>\frac{1}{2^{1-\psi}}$ and i) $\varepsilon\geq 2$ or ii) $\frac{\eta_B}{\eta}>\frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$;*
4. *two zeros if $\frac{1}{2^{1-\psi}}<\frac{\eta_B}{\eta}<\frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$ and $\varepsilon<2$, the first zero satisfies $I'\left(s_1^*\right)>0$ and the second zero satisfies $I'\left(s_2^*\right)<0$.*

Proof. Differentiating $I(s)$, we obtain

$$I'(s)=\left(s^{-\psi}-(\varepsilon-1)(1-s)^{-\psi}\right)(1-\psi), \tag{B-1}$$

$$I''(s)=-\psi\left(s^{-\psi-1}+(\varepsilon-1)(1-s)^{-\psi-1}\right)(1-\psi)<0.$$

Therefore the function I is concave in s and always decreasing in s for s large enough (since $I'(1)=-\infty$).

Further, at the boundaries of the interval, one gets:

$$I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right)=(\varepsilon-1)\left[\left(1-\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right)^{1-\psi}-\frac{\eta_B}{\eta}\right],$$

and we get that $I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right)>0$ if and only if $\frac{\eta_B}{\eta}<\frac{1}{2^{1-\psi}}$. In addition $I(1)=1-\varepsilon\frac{\eta_B}{\eta}$, and we obtain that $I(1)>0$ if and only if $\frac{\eta_B}{\eta}<\frac{1}{\varepsilon}$. Since $\varepsilon>2^{1-\psi}$, we get that $I(1)>0 \Rightarrow I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right)>0$. As I is concave, it has no zeros for $\varepsilon<1/2^{1-\psi}$. This establishes part 1 of Lemma B.1.

Assume now that $\frac{1}{\varepsilon} < \frac{\eta_B}{\eta} < \frac{1}{2^{1-\psi}}$, then $I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) > 0$ but $I(1) < 0$, since I is concave, then I has only 1 zero over the interval $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$ and this zero features $I'(s^*) < 0$. This establishes part 2 of Lemma B.1.

Consider now the case where $\frac{\eta_B}{\eta} > \frac{1}{2^{1-\psi}}$, so that $I(1) < 0$ and $I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) < 0$. Then either I has 2 zeros (one for I increasing and one for I decreasing) or I has no zero. First, note that I is decreasing on $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$ if $I'\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) < 0$. In that case, we have

$$\begin{aligned} I'\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) &< 0 \iff \left(\frac{\eta_B}{\eta}\right)^{\frac{-\psi}{1-\psi}} - (\varepsilon-1)\left(1-\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right)^{-\psi} < 0 \\ &\iff \frac{\eta_B}{\eta} > \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}. \end{aligned}$$

If $\varepsilon \geq 2$, then $\frac{1}{2^{1-\psi}} \geq \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}$ so that $\frac{\eta_B}{\eta} > \frac{1}{2^{1-\psi}} \Rightarrow \frac{\eta_B}{\eta} > \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}$. Then, I has no zero over the interval $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$. This establishes part 3i) of Lemma B.1.

We now consider the case where $\varepsilon < 2$, and $\frac{\eta_B}{\eta} < \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}$, then I has a maximum, which is reached at $s = \widetilde{s}$, where $\widetilde{s}$ solves $I'(\widetilde{s}) = 0$. Using (B-1), we get $\widetilde{s} = \left[1+(\varepsilon-1)^{\frac{1}{\psi}}\right]^{-1}$ and

$$I(\widetilde{s}) = \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi} - \varepsilon\frac{\eta_B}{\eta}.$$

Therefore,

$$I(\widetilde{s}) > 0 \iff \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}.$$

We note that when $\varepsilon < 2$, $\frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi} < \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}$, so that $\frac{\eta_B}{\eta} < \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$ immediately implies $\frac{\eta_B}{\eta} < \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}$. Therefore, if $I(\widetilde{s}) > 0$, then I will have two zeros, the first one when I is increasing and the second one when I is decreasing. This establishes part 4) of Lemma B.1. Finally, if instead, $I(\widetilde{s}) < 0$, then $I(s)$ will have no zeros, establishing part 3ii) of Lemma B.1. Note that $\frac{1}{2^{1-\psi}} \leq \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$ for all ε with strict inequality unless $\varepsilon = 2$, therefore the interval $\left(\frac{1}{2^{1-\psi}}, \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}\right)$ is non-empty for $\varepsilon \neq 2$. ■

We establish a similar Lemma for the case $\varepsilon < 2^{1-\psi}$.

Lemma B.2 *Assume that $\varepsilon < 2^{1-\psi}$. Over the interval $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$, the function $I(s)$ has:*

1. *no zeros if $\frac{\eta_B}{\eta} < \frac{1}{2^{1-\psi}}$;*
2. *one zero with $\frac{1}{2^{1-\psi}} < \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}$ and this zero satisfies $I'(s^*) > 0$;*

3. *two zeros if* $\frac{1}{\varepsilon} < \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$, *the first zero satisfies* $I'\left(s_1^*\right) > 0$ *and the second zero satisfies* $I'\left(s_2^*\right) < 0$.

4. *no zero if* $\frac{\eta_B}{\eta} > \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$*;*

Proof. The proof is similar to the previous case. With $\varepsilon < 2^{1-\psi}$, $I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) > 0 \Rightarrow I(1) > 0$. Since I is concave, it has no zeros for $\frac{\eta_B}{\eta} < \frac{1}{2^{1-\psi}}$, which establishes part 1.

Assume now that $\frac{1}{2^{1-\psi}} < \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}$, then $I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) < 0$ but $I(1) > 0$, since I is concave, then I has only 1 zero over the interval $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$ and this zero features $I'(s^*) > 0$. This establishes part 2.

Consider now the case where $\frac{\eta_B}{\eta} > \frac{1}{\varepsilon}$. As in the previous proof, $I(1) < 0$ and $I\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) < 0$, so I has either 2 zeros (one for I increasing and one for I decreasing) or I no zero. I is decreasing on $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$ if $I'\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}\right) < 0$, which is equivalent to $\frac{\eta_B}{\eta} > \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}$. Otherwise, I has a maximum $\widetilde{s} = \left[1+(\varepsilon-1)^{\frac{1}{\psi}}\right]^{-1}$ and we still get that $I(\widetilde{s}) > 0 \iff \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$. With $\varepsilon < 2^{1-\psi}$, then we always have that $\frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi} < \left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi-1}$. We can then consider two cases: $\frac{1}{\varepsilon} < \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$ and $\frac{\eta_B}{\eta} > \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$. In the former case, I has 2 zeros, in the latter I has no zero (since either I decreases or its maximum is negative). This establishes parts 3) and 4). ■

We now establish Propositions A.1 and A.2. To do that, we derive the respective conditions under which each type of asymptotic equilibrium exists. Using (20), the allocation of innovation follows:

$$\left(\frac{s_{gt}}{s_{ft}}\right)^{\psi} = \frac{\kappa_g^{\varepsilon} A_{gt}^{\varepsilon-1}}{\frac{1}{A_{ct}}\kappa_c^{\varepsilon}\left(\frac{1}{A_{ct}}+\frac{1}{B_{ct}}\right)^{-\varepsilon} + \frac{1}{A_{st}}\kappa_d^{\varepsilon}\left(\frac{1}{A_{st}}+\frac{1}{B_{st}}\right)^{-\varepsilon}}. \tag{B-2}$$

Corner Asymptotic Steady State with Clean Innovation. In an asymptotic steady state where $s_{gt} \to 1$, the (B-2) grows without bonds, which in turn confirms the corner allocation for innovation. Therefore such a steady state is always possible and occurs whenever A_{g0} is sufficiently large relative to the fossil-fuel technologies.

Corner Asymptotic Steady State with Fossil-Fuel Innovation. Alternatively, consider a steady state where $s_{ft} \to 1$. Then (B-2) implies that:

$$\left(\frac{s_{gt}}{s_{ft}}\right)^{\psi} = O\left(\frac{A_{gt}^{\varepsilon-1}}{\frac{1}{A_{ct}}\kappa_c^{\varepsilon}B_{ct}^{\varepsilon} + \frac{1}{A_{st}}\kappa_d^{\varepsilon}B_{st}^{\varepsilon}}\right).$$

The LHS tends toward 0 and the RHS tends toward 0 only if $B_{ct}^{\varepsilon}/A_{ct}$ grows without bound (knowing that $B_{st}^{\varepsilon}/A_{st}$ behaves similarly). This occurs if $\varepsilon\eta_B > \eta$. Therefore, we get that for $\eta_B/\eta < 1/\varepsilon$, an asymptotic steady state where all innovation occurs in the fossil-fuel technologies cannot exist. In contrast, such an asymptotic steady state occurs for $\eta_B/\eta > 1/\varepsilon$ provided that A_{g0} is sufficiently small.

Interior Asymptotic Steady State. We now analyze whether an interior asymptotic steady state is possible. There are three possible cases: A_{ct} grows faster, at the same rate or less fast than B_{ct}.

Assume first that A_{ct} grows less fast than B_{ct} (that is, $\eta\left(s_{ft}^*\right)^{1-\psi} < \eta_B$ where s_{ft}^* is the limit of s_{ft}). Then (B-2) implies that

$$\left(\frac{s_g^*}{s_f^*}\right)^{\psi} \sim \frac{\kappa_g^{\varepsilon}A_{gt}^{\varepsilon-1}}{A_{ct}^{\varepsilon-1}\kappa_c^{\varepsilon}+A_{st}^{\varepsilon-1}\kappa_d^{\varepsilon}}. \tag{B-3}$$

The RHS can only converge asymptotically to a constant if A_{gt} and A_{ct} grow at the same rate in the long-run. This is possible only if $s_f^* = s_g^* = 1/2$, which combined with condition $\eta\left(s_{ft}^*\right)^{1-\psi} < \eta_B$, requires that $\eta_B/\eta > 2^{\psi-1}$. In addition, if $A_{g(t-1)}$ is shocked in such a way that the RHS in (B-3) increases, then s_{gt} should increase as well: so that the interior asymptotic state can only exist in a knife-edge case and it is unstable.

The case where A_{ct} and B_{ct} grow at the same rate follows the same logic since in that case (B-3) still holds up to a constant. We must then have $s_f^* = 1/2$ and $\eta\left(s_{ft}^*\right)^{1-\psi} = \eta_B$, which can only occur for $\eta_B = \eta 2^{\psi-1}$. Again this interior steady state will always be unstable.

Consider now the case where A_{ct} grows faster than B_{ct} (that is $\eta\left(s_{ft}^*\right)^{1-\psi} > \eta_B$). Then (B-2) implies that

$$\left(\frac{s_{gt}^*}{s_{ft}^*}\right)^{\psi} \sim \frac{\kappa_g^{\varepsilon}A_{gt}^{\varepsilon-1}}{\frac{1}{A_{ct}}\kappa_c^{\varepsilon}B_{ct}^{\varepsilon}+\frac{1}{A_{st}}\kappa_d^{\varepsilon}B_{st}^{\varepsilon}},$$

which is possible only if the RHS tends toward a constant. This implies that s_{ft}^* must also satisfy $I\left(s_{ft}^*\right) = \varepsilon\eta_B/\eta$. An interior steady state will therefore exist if $I\left(s_{ft}^*\right) = 0$ has a solution in the interval $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$. That steady state will be unstable if $I'\left(s_{ft}^*\right) < 0$ since then a shock leading to a temporarily higher s_{ft} is associated with permanently higher s_{ft}. The steady state will be stable if $I'\left(s_{ft}^*\right) > 0$.

Lemma B.1 immediately characterizes the conditions under which this case occurs for $\varepsilon \geq 2^{1-\psi}$ and we get that:

1) There is no interior asymptotic steady state if $\frac{\eta_B}{\eta} < \frac{1}{\varepsilon}$;

2) There is one unstable interior asymptotic steady state if $\frac{1}{\varepsilon} < \frac{\eta_B}{\eta}$ and i) $\varepsilon \geq 2$ or ii) $\varepsilon < 2$ and $\frac{\eta_B}{\eta} \notin \left(\frac{1}{2^{1-\psi}}, \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)\right)$;

3) There are two unstable interior asymptotic steady states and one stable interior

asymptotic steady state if $\frac{1}{\varepsilon} < \frac{\eta_B}{\eta}$, $\varepsilon < 2$ and $\frac{\eta_B}{\eta} \in \left(\frac{1}{2^{1-\psi}}, \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)\right)$.

Similarly, Lemma B.2 characterizes the conditions under which $I\left(s^*_{ft}\right) = 0$ has a solution in $\left(\left(\frac{\eta_B}{\eta}\right)^{\frac{1}{1-\psi}}, 1\right)$ for $\varepsilon < 2^{1-\psi}$ and we get that:

1) There is no interior asymptotic steady state if $\frac{\eta_B}{\eta} < \frac{1}{2^{1-\psi}}$;

2) There is one unstable interior asymptotic steady state if $\frac{1}{2^{1-\psi}} < \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}$ and a stable interior asymptotic steady state;

3) There are two unstable interior asymptotic steady states if $\frac{1}{\varepsilon} < \frac{\eta_B}{\eta} < \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$ and a stable asymptotic steady state;

4) There is one unstable interior asymptotic steady state if $\frac{\eta_B}{\eta} > \frac{1}{\varepsilon}\left(1+(\varepsilon-1)^{\frac{1}{\psi}}\right)^{\psi}$.

Conclusion. Bringing together the three cases establishes Propositions A.1 and A.2.

B.1.2 Complement to Proposition 5

In this Appendix, we complement Proposition 5 by showing that when $A_{g0} > \overline{A_{g0}}$, the natural gas boom decreases welfare provided that $\frac{\gamma^{\eta(1-\vartheta)}}{1+\rho}$ is sufficiently large, that φ_L is small and that φ_D is large as mentioned in the text.

Proof. In that case, the economy is on a green path whether the boom occurred or not. From Proposition 4, however, we get that emissions are lower without the boom for t large enough. Therefore, if the stock of carbon depends mostly on current emissions (which is the case when φ_L is sufficiently small and φ is sufficiently large enough), then S_t is lower without the boom for t large enough (though in both cases, S_t tends toward a constant). In addition, since innovation is reallocated away from clean technologies, A_{gt} is lower with the boom than without. Therefore, for t sufficiently large, we obtain that C_{Et} is also lower with the boom than without. As a result, for t large enough output is lower with the boom than without.

For T large but finite, Y_t grows approximately at the rate $\gamma^\eta - 1$. Using (A-4), we can then write the change in welfare following (a small) boom as:

$$dU \approx \sum_{\tau=0}^{T-1} \frac{1}{(1+\rho)^\tau} \frac{d\left(Y_\tau\right)^{1-\vartheta}}{1-\vartheta} + \sum_{\tau=T}^{\infty} \frac{Y_T^{1-\vartheta}}{(1+\rho)^T}\left(\frac{\gamma^{\eta(1-\vartheta)}}{1+\rho}\right)^{(\tau-T)}\left(\frac{\nu^\lambda \widetilde{A}_{E\tau}^{\lambda-1} C_{E\tau}^{\lambda-1} d\ln C_{E\tau}}{(1-\nu)^\lambda A_{P\tau}^{\lambda-1} + \nu^\lambda \widetilde{A}_{E\tau}^{\lambda-1} C_{E\tau}^{\lambda-1}} - \zeta dS_\tau\right).$$

As argued above, for T large $dS_\tau > 0$. Furthermore, $d\ln C_{E\tau} \approx d\ln A_{g\tau} = \sum_{s=u}^{\tau} \eta(1-\psi) s_{gu}^{-\psi} ds_{gu}$ where all $ds_{gu} < 0$, so that $d\ln C_{E\tau}$ is negative and bounded away from 0. Therefore, the second sum becomes arbitrarily large if $\frac{\gamma^{\eta(1-\vartheta)}}{1+\rho}$ is sufficiently close to 1. The latter condition is met when ρ is sufficiently small and $\vartheta \leq 1$ for instance. ■

B.1.3 Proof of Proposition 6

Anticipating that the social planner allocates labor symmetrically within intermediates and that she maintains the equality $E_{it} = Q_{it}$, we can write the social planner problem as maximizing

$$U_0 = \sum_{t=0}^{\infty} \frac{1}{(1+\rho)^t} \frac{Y_t^{1-\vartheta}}{1-\vartheta}, \tag{B-4}$$

subject to the final good equation (2) with Lagrange parameter λ_t, the energy equation (3) with Lagrange parameter λ_{Et},

$$\lambda_{it} : E_{it} = C_{it} L_{it},$$

where for this equation and the following ones the term before the : is the associated Lagrange parameter,

$$\lambda_{Pt} : Y_{Pt} = A_{Pt} L_{Pt}$$

$$\lambda_{Lt} : L_{ct} + L_{st} + L_{gt} + L_{Pt} = L$$

$$\mu_{ct} : A_{ct} = \gamma^{\eta s_{ft}^{1-\psi}} A_{c(t-1)},\ \mu_{st} : A_{st} = \gamma^{\eta s_{ft}^{1-\psi}} A_{s(t-1)} \text{ and } \mu_{gt} : A_{gt} = \gamma^{\eta s_{gt}^{1-\psi}} A_{g(t-1)},$$

$$\chi_t : s_{ft} + s_{gt} = 1,$$

$$\omega_{Pt} : \xi_c E_{ct} + \xi_s E_{st} = P_t,$$

$$\omega_{St} : S_t = \overline{S} + \sum_{s=0}^{t+T} (\varphi_L + (1-\varphi_L)\varphi_0 (1-\varphi_d)^s) P_{t-s}.$$

The first order condition with respect to Y_t imposes that λ_t be equal to the marginal value of consumption at time t,

$$\lambda_t = \frac{Y_t^{-\vartheta}}{(1+\rho)^t}.$$

The first order condition with respect to Y_{Pt} ensures that $\frac{\partial Y_t}{\partial Y_{Pt}} = \frac{\lambda_{Pt}}{\lambda_t} \equiv p_{Pt}$, where the ratio λ_{Pt}/λ_t is the shadow price of the production input. Similarly, the first order condition with respect to E_t implies $\frac{\partial Y_t}{\partial Y_{Et}} = \frac{\lambda_{Et}}{\lambda_t} \equiv p_{Et}$. The first order condition with respect to E_{gt} implies $\lambda_{Et} \frac{\partial E_t}{\partial E_{gt}} = \lambda_{gt}$, so that $\frac{\partial E_t}{\partial E_{gt}} = \frac{\lambda_{gt}}{\lambda_t} = p_{gt}$. The first order condition with respect to Y_{ct} gives

$$\lambda_{Et} \frac{\partial E_t}{\partial E_{ct}} = \lambda_{ct} + \xi_c \omega_{Pt} \Longrightarrow \frac{\partial E_t}{\partial E_{ct}} = p_{ct} + \xi_c \tau_t,$$

with $p_{ct} \equiv \lambda_{ct}/\lambda_t$ being the shadow producer price of coal-based energy and $\tau_t = \omega_{Pt}/\lambda_t$ being the shadow price of emissions. Similarly, we have

$$\frac{\partial E_t}{\partial E_{st}} = p_{st} + \xi_s \tau_t.$$

First order conditions with respect to L_{it} for $i = c, s, g$ yield $p_{it} \partial E_{it} / \partial L_{it} = \lambda_{Lt} / \lambda_t \equiv w_t$ which is the shadow wage and similarly, $p_{Pt} \partial Y_{Pt} / \partial L_{Pt} = w_t$. Therefore, and unsurprisingly, the static optimal allocation is identical to the decentralized allocation provided that there is a carbon tax given by τ_t. Note that there is no monopoly distortion to be addressed because all sectors are equally affected and there is no roundabout production (yet the shadow wage differs from the decentralized wage by a constant).

The first order condition with respect to S_t yields

$$\omega_{St} = \lambda_t \zeta Y_t, \tag{B-5}$$

whereas the first order condition with respect to P_t implies:

$$\omega_{Pt} = \sum_{s=0}^{\infty} \left(\varphi_L + (1 - \varphi_L) \varphi_0 (1 - \varphi_d)^s\right) \omega_{St+s}.$$

We can rewrite this as

$$\tau_t = Y_t^{\vartheta} \sum_{s=0}^{\infty} \frac{\left(\varphi_L + (1 - \varphi_L) \varphi_0 (1 - \varphi_d)^s\right)}{(1 + \rho)^s} \zeta Y_{t+s}^{1-\vartheta}. \tag{B-6}$$

If $\vartheta = 1$, we obtain the closed form solution of Golosov et al. (2014), namely $\tau_t = Y_t \gamma (1 + \rho) \left(\frac{\varphi_L}{\rho} + \frac{(1 - \varphi_L) \varphi_0}{\rho + \varphi_d} \right)$.

The first order conditions with respect to A_{it} for $i = c, s$ yield

$$\mu_{it} = \lambda_{it} \left(\frac{C_{it}}{A_{it}} \right)^2 L_{it} + \gamma^{\eta_f s_{f(t+1)}^{1-\psi}} \mu_{i(t+1)}.$$

Multiply by A_{it} and iterate forward to get

$$\mu_{it} A_{it} = \lambda_{it} \frac{C_{it}}{A_{it}} E_{it} + A_{it+1} \mu_{i(t+1)} = \sum_{s=0}^{\infty} \lambda_{it+s} \frac{C_{it+s}}{A_{it+s}} E_{it+s}.$$

The first order condition with respect to A_{gt} gives

$$\mu_{gt} = \lambda_{gt} L_{gt} + \gamma^{\eta s_{g(t+1)}^{1-\psi}} \mu_{g(t+1)},$$

which similarly leads to

$$\mu_{gt} A_{gt} = \sum_{s=0}^{\infty} \lambda_{gt+s} E_{gt+s}.$$

The first order conditions with respect to s_{ft} and s_{gt} imply

$$(1 - \psi) \ln(\gamma) s_{ft}^{-\psi} \left(\mu_{ct} A_{ct} + \mu_{st} A_{st}\right) = \chi_t = (1 - \psi) \ln(\gamma) s_{gt}^{-\psi} \mu_{gt} A_{gt}.$$

Therefore the innovation allocation obeys

$$\left(\frac{s_{ft}}{s_{gt}}\right)^{\psi} = \frac{\mu_{ct}A_{ct} + \mu_{st}A_{st}}{\mu_{gt}A_{gt}} = \frac{\sum_{s=0}^{\infty} \frac{1}{1+r_{t,t+s}} \left(\frac{C_{c(t+s)}}{A_{c(t+s)}} p_{c(t+s)} E_{c(t+s)} + \frac{C_{cs(t+s)}}{A_{s(t+s)}} p_{s(t+s)} E_{s(t+s)}\right)}{\sum_{s=0}^{\infty} \frac{1}{1+r_{t,t+s}} p_{g(t+s)} E_{g(t+s)}},$$

where $r_{t,t+s} = \lambda_t/\lambda_{t+s} - 1$ is the shadow interest rate between t and $t+s$. At the optimum, the allocation of innovation depends on the ratio of the social values of innovation in each sector. These social values are equal to the discounted sum of the marginal benefit of innovation in all future periods. This contrasts with the decentralized economy where the allocation of innovation is given by:

$$\left(\frac{s_{ft}}{s_{gt}}\right)^{\psi} = \frac{\frac{C_{ct}}{A_{ct}} p_{ct} E_{ct} + \frac{C_{st}}{A_{st}} p_{st} E_{st}}{p_{gt} E_{gt}},$$

including in the presence of the carbon tax (since p_{ct} and p_{st} are pre-tax producer prices of energy). The optimal scientist allocation can be decentralized through research subsidies.

In the quantitative analysis, we add an exogenous path of emissions from the rest of the world P_t^{ROW} and direct disutility costs from carbon concentration on utility (to capture the effect of climate change on the rest of the world), see (24). The former does not affect our analysis, whereas the latter simply turns (B-5) into $\omega_{St} = \lambda_t \frac{D'(S_t)}{1-D(S_t)} Y_t - \frac{\nu'(S_t)}{(1+\rho)^t}$, so that we get $\tau_t = Y_t^{\vartheta} \sum_{s=0}^{\infty} \frac{(\varphi_L + (1-\varphi_L)\varphi_0(1-\varphi_d)^s)}{(1+\rho)^s} \left(\zeta Y_{t+s}^{1-\vartheta} - \nu'(S_{t+s})\right)$ instead of (B-6).

B.1.4 Proof of Proposition 7

We prove Proposition 7 and also establish that the shale gas boom decreases welfare provided that $\frac{\gamma^{\eta(1-\vartheta)}}{1+\rho}$ is sufficiently large, that φ_L is sufficiently small and that φ_D is sufficiently large.

Proof of Part 1). With $\varepsilon \geq 2$, Proposition A.1 applies and establishes that for $\eta_B < \eta/\varepsilon$, the economy converges toward a green path, so that $s_{gt} \to 1$ and t_{switch} must be finite. We then show that from t_{switch} onward, green innovation increases over time. Using the notation f_t introduced in Appendix A.4, we get:

$$f_{t+1}(s_{gt}) = \frac{\frac{\gamma^{-2\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}} \kappa_c^{\varepsilon} \left(\frac{\gamma^{-2\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}} + \frac{\gamma^{-\eta_B}}{B_{ct}}\right)^{-\varepsilon} + \frac{\gamma^{-2\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}} \kappa_s^{\varepsilon} \left(\frac{\gamma^{-2\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}} + \frac{\gamma^{-\eta_B}}{B_{st}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon} C_{g(t-1)}^{\varepsilon-1} \gamma^{2\eta s_{gt}^{1-\psi}(\varepsilon-1)}} \left(\frac{s_{gt}}{s_{ft}}\right)^{\psi}.$$

Assume that $s_{gt} \geq 1/2$ and that $\eta s_{ft}^{1-\psi} > \eta_B$ then

$$\begin{aligned}
& f_{t+1}\left(s_{gt}\right) \\
= \quad & \gamma^{(\varepsilon-1)\eta\left(s_{ft}^{1-\psi}-s_{gt}^{1-\psi}\right)} \frac{\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}+\frac{\gamma^{\eta s_{ft}^{1-\psi}-\eta_B}}{B_{ct}}\right)^{-\varepsilon}+\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}+\frac{\gamma^{\eta s_{ft}^{1-\psi}-\eta_B}}{B_{st}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon}C_{g(t-1)}^{\varepsilon-1}\gamma^{\eta s_{gt}^{1-\psi}(\varepsilon-1)}}\left(\frac{s_{gt}}{s_{ft}}\right)^{\psi} \\
< \quad & f_t\left(s_{gt}\right)=1,
\end{aligned}$$

therefore $s_{g(t+1)} > s_{gt}$.

Assume now that $s_{gt} \geq 1/2$ but that $\eta s_{ft}^{1-\psi} \leq \eta_B$, then:

$$\begin{aligned}
f_{t+1}\left(s_{gt}\right) \quad = \quad & \gamma^{\varepsilon\eta_B} \frac{\frac{\gamma^{-2\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{\eta_B-2\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}+\frac{1}{B_{ct}}\right)^{-\varepsilon}+\frac{\gamma^{-2\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{\eta_B-2\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}+\frac{1}{B_{st}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon}C_{g(t-1)}^{\varepsilon-1}\gamma^{2\eta s_{gt}^{1-\psi}(\varepsilon-1)}}\left(\frac{s_{gt}}{s_{ft}}\right)^{\psi} \\
\leq \quad & \gamma^{\varepsilon\eta_B-\eta s_{ft}^{1-\psi}-\eta(\varepsilon-1)s_{gt}^{1-\psi}} \frac{\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}\kappa_c^{\varepsilon}\left(\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{c(t-1)}}+\frac{1}{B_{ct}}\right)^{-\varepsilon}+\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}\kappa_s^{\varepsilon}\left(\frac{\gamma^{-\eta s_{ft}^{1-\psi}}}{A_{s(t-1)}}+\frac{1}{B_{st}}\right)^{-\varepsilon}}{\kappa_g^{\varepsilon}C_{g(t-1)}^{\varepsilon-1}\gamma^{\eta s_{gt}^{1-\psi}(\varepsilon-1)}}\left(\frac{s_{gt}}{s_{ft}}\right)^{\psi}.
\end{aligned}$$

We wish to establish that $\varepsilon\eta_B - \eta s_{ft}^{1-\psi} - \eta(\varepsilon-1)s_{gt}^{1-\psi} < 0$. To do so, we define $h(s) \equiv \varepsilon\eta_B - \eta s^{1-\psi} - \eta(\varepsilon-1)(1-s)^{1-\psi}$. Twice differentiating h, one gets $h''(s) > 0$, so that h is convex. Furthermore $h(0) = \varepsilon\eta_B - \eta(\varepsilon-1)$. Since $\varepsilon \geq 2$, $\frac{\varepsilon-1}{\varepsilon} \geq \frac{1}{\varepsilon}$, so that $\frac{\eta_B}{\eta} < \frac{1}{\varepsilon} \leq \frac{\varepsilon-1}{\varepsilon}$, which ensures that $h(0) < 0$. In addition, $h\left(\frac{1}{2}\right) = \varepsilon\left(\eta_B - \eta 2^{\psi-1}\right) < 0$ since $\eta_B/\eta < 1/\varepsilon$ and $\varepsilon \geq 2 > 2^{1-\psi}$. Therefore, $\varepsilon\eta_B - \eta s_{ft}^{1-\psi} - \eta(\varepsilon-1)s_{gt}^{1-\psi} < 0$ when $s_{gt} \geq 1/2$. This ensures that $f_{t+1}\left(s_{gt}\right) < f_t\left(s_{gt}\right)$ so that $s_{g(t+1)} > s_{gt}$. This establishes Part 1).

Proof of Part 2). To prove Part 2, it suffices to show that an increase in B_{s0} leads to an increase in s_{gt} as long as $t \leq t_{switch}$. We define

$$\widehat{f}_t\left(s_{gt}, s_{g(t-1)}, \ldots, s_{g1}, B_{s0}\right) \equiv \frac{s_{gt}^{\psi}}{\kappa_g^{\varepsilon}C_{g0}^{\varepsilon-1}s_{ft}^{\psi}\gamma^{\eta(\varepsilon-1)\sum\limits_{\tau=1}^{t}s_{g\tau}^{1-\psi}}}\left(\begin{array}{c}\frac{\kappa_c^{\varepsilon}\gamma^{-\eta\sum\limits_{\tau=1}^{t}s_{f\tau}^{1-\psi}}}{A_{c0}}\left(\frac{\gamma^{-\eta\sum\limits_{\tau=1}^{t}s_{f\tau}^{1-\psi}}}{A_{c0}}+\frac{1}{B_{ct}}\right)^{-\varepsilon} \\ +\frac{\kappa_s^{\varepsilon}\gamma^{-\eta\sum\limits_{\tau=1}^{t}s_{f\tau}^{1-\psi}}}{A_{s0}}\left(\frac{\gamma^{-\eta\sum\limits_{\tau=1}^{t}s_{f\tau}^{1-\psi}}}{A_{s0}}+\frac{1}{B_{st}}\right)^{-\varepsilon}\end{array}\right),$$

so that the equilibrium innovation allocation is still defined by $\widehat{f}_t\left(s_{gt}, s_{g(t-1)}, ..., s_{g1}, B_{s0}\right) = 1$ with $\widehat{f}_t$ increasing in s_{gt} and in B_{s0}. We obtain for $\widetilde{\tau} \in [1, t-1)$

$$\frac{\partial \ln \widehat{f}_t}{\partial \ln s_{g\widetilde{\tau}}} = \left[\frac{\frac{\kappa_c^\varepsilon}{A_{ct}}\left(\frac{1}{A_{ct}} + \frac{1}{B_{c0}}\right)^{-\varepsilon}\left(1 - \varepsilon \frac{\frac{1}{A_{ct}}}{\frac{1}{A_{ct}} + \frac{1}{B_{ct}}}\right) + \frac{\kappa_s^\varepsilon}{A_{st}}\left(\frac{1}{A_{st}} + \frac{1}{B_{s0}}\right)^{-\varepsilon}\left(1 - \varepsilon \frac{\frac{1}{A_{st}}}{\frac{1}{A_{st}} + \frac{1}{B_{st}}}\right)}{\frac{\kappa_c^\varepsilon}{A_{ct}}\left(\frac{1}{A_{ct}} + \frac{1}{B_{c0}}\right)^{-\varepsilon} + \frac{\kappa_s^\varepsilon}{A_{st}}\left(\frac{1}{A_{st}} + \frac{1}{B_{s0}}\right)^{-\varepsilon}} s_{f\widetilde{\tau}}^{-\psi} - (\varepsilon - 1) s_{g\widetilde{\tau}}^{-\psi} \right] s_{g\widetilde{\tau}} \eta (1-\psi) \ln \gamma.$$

Yet, if $t \leq t_{switch}$, then $s_{f\widetilde{\tau}} \geq s_{g\widetilde{\tau}}$, so that

$$\frac{\partial \ln \widehat{f}_t}{\partial \ln s_{g\widetilde{\tau}}} \leq - \left[\varepsilon - 2 + \varepsilon \frac{\frac{\kappa_c^\varepsilon}{A_{ct}^2}\left(\frac{1}{A_{ct}} + \frac{1}{B_{ct}}\right)^{-\varepsilon-1} + \frac{\kappa_s^\varepsilon}{A_{st}^2}\left(\frac{1}{A_{st}} + \frac{1}{B_{st}}\right)^{-\varepsilon-1}}{\frac{\kappa_c^\varepsilon}{A_{ct}}\left(\frac{1}{A_{ct}} + \frac{1}{B_{ct}}\right)^{-\varepsilon} + \frac{\kappa_s^\varepsilon}{A_{st}}\left(\frac{1}{A_{st}} + \frac{1}{B_{st}}\right)^{-\varepsilon}} \right] s_{f\widetilde{\tau}}^{-\psi} s_{g\widetilde{\tau}} \eta (1-\psi) \ln \gamma.$$

Therefore $\frac{\partial \ln \widehat{f}_t}{\partial \ln s_{g\widetilde{\tau}}} < 0$ if $\varepsilon \geq 2$.

Therefore, the natural gas boom reduces $\widehat{f}_1$ leading to a lower value for s_{g1}. It then reduces $\widehat{f}_2$ both directly and because of its negative effect on s_{g1}, leading to a lower value for s_{g2}. By iteration, the natural gas boom will reduce all s_{gt} at least until the switch toward green innovation occurs.

Three Useful Lemmas. We establish three lemmas which are useful to prove part 3.

Lemma B.3 *Consider a small increase in B_s. Denote by t_A the smallest t such that $d \ln A_{st_A} < 0$ and assume that $t_A < \infty$. Then $d \ln A_{gt_A} > d \ln A_{st_A}$.*

Proof. Noting that

$$\ln A_{ct} = \ln A_{c0} + \eta (\ln \gamma) \sum_{\tau=1}^{t} s_{f\tau}^{1-\psi} \text{ and } \ln A_{st} = \ln A_{s0} + \eta (\ln \gamma) \sum_{\tau=1}^{t} s_{f\tau}^{1-\psi},$$

we obtain

$$d \ln A_{ct} = d \ln A_{st} = \eta (1-\psi)(\ln \gamma) \sum_{\tau=1}^{t} s_{f\tau}^{-\psi} ds_{f\tau}. \tag{B-7}$$

By definition of t_A, $d \ln A_{c(t_A-1)} > 0$ and $d \ln A_{ct_A} < 0$, so that we must have $ds_{ft_A} < 0$. Since $ds_{ft} > 0$ for $t \leq t_{switch}$, it must be that $t_A > t_{switch}$. We can similarly write

$$d \ln A_{gt} = -\eta (1-\psi)(\ln \gamma) \sum_{\tau=1}^{t} s_{g\tau}^{-\psi} ds_{f\tau}. \tag{B-8}$$

Using (B-7) and (B-8), we get

$$d\ln A_{st_A} - d\ln A_{gt_A} = \eta(1-\psi)(\ln\gamma)\left(\sum_{\tau=1}^{t_A}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau}\right).$$

We know that $ds_{ft} > 0$ for $t \leq t_{switch}$ and that $ds_{ft_A} < 0$, therefore ds_{ft} must change sign as t increases at least once. We index the times where ds_{ft} switches signs by t_{2p} and t_{2p+1}, such that ds_{ft} becomes negative at t_{2p+1} and positive at t_{2p} and p is a weakly positive integer in the integer set $\{0, ..., P-1\}$ with $P \geq 1$. We denote by $t_0 = t_{switch}$ and $t_{2P} = t_A + 1$. We get:

$$\begin{aligned}
& d\ln A_{st_A} - d\ln A_{gt_A} \qquad \text{(B-9)}\\
= \;& \eta(1-\psi)(\ln\gamma)\left(\begin{array}{c}\displaystyle\sum_{\tau=1}^{t_{switch}-1}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau} \\ \displaystyle+\sum_{p=0}^{P-1}\left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau} + \sum_{\tau=t_{2p+1}}^{t_{2p+2}-1}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau}\right)\end{array}\right)\\
= \;& \eta(1-\psi)(\ln\gamma)\left(\begin{array}{c}\displaystyle\sum_{\tau=1}^{t_{switch}-2}\left(s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi}\right) ds_{f\tau} \\ \displaystyle+\sum_{p=0}^{P-1}\left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(1-\frac{s_{f\tau}^{\psi}}{s_{g\tau}^{\psi}}\right) s_{f\tau}^{-\psi} ds_{f\tau} + \sum_{\tau=t_{2p+1}}^{t_{2p+2}-1}\left(1-\frac{s_{f\tau}^{\psi}}{s_{g\tau}^{\psi}}\right) s_{f\tau}^{-\psi} ds_{f\tau}\right)\end{array}\right).
\end{aligned}$$

Using that $s_{f\tau}^{-\psi} - s_{g\tau}^{-\psi} < 0$ for $\tau < t_{switch}$, that $\frac{s_{f\tau}^{\psi}}{s_{g\tau}^{\psi}}$ is decreasing for $\tau \geq t_{switch}$ (as established in the Proof of Part 1), that $ds_{f\tau} > 0$ only on intervals $\left[t_{2p}, t_{2p+1} - 1\right]$, we get

$$d\ln A_{st_A} - d\ln A_{gt_A} < \eta(1-\psi)(\ln\gamma)\sum_{p=0}^{P-1}\left(1 - \frac{s_{f\,t_{2p+1}}^{\psi}}{s_{g\,t_{2p+1}}^{\psi}}\right)\sum_{\tau=t_{2p}}^{t_{2p+2}-1} s_{f\tau}^{-\psi} ds_{f\tau}.$$

By definition t_A is the smallest t such that $\sum_{\tau=1}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau} < 0$, therefore for any $t_X < t_A$, we have $\sum_{\tau=1}^{t_X} s_{f\tau}^{-\psi} ds_{f\tau} > 0$ and $\sum_{\tau=t_X+1}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau} < 0$. Therefore, we get that

$$\begin{aligned}
& \sum_{p=P-2}^{P-1}\left(1 - \frac{s_{f\,t_{2p+1}}^{\psi}}{s_{g\,t_{2p+1}}^{\psi}}\right)\sum_{\tau=t_{2p}}^{t_{2p+2}-1} s_{f\tau}^{-\psi} ds_{f\tau}\\
= \;& \left(1 - \frac{s_{f\,t_{2P-3}}^{\psi}}{s_{g\,t_{2P-3}}^{\psi}}\right)\sum_{\tau=t_{2P-4}}^{t_{2P-2}-1} s_{f\tau}^{-\psi} ds_{f\tau} + \left(1 - \frac{s_{f\,t_{2P-1}}^{\psi}}{s_{g\,t_{2P-1}}^{\psi}}\right)\sum_{\tau=t_{2P-2}}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau}\\
< \;& \left(1 - \frac{s_{f\,t_{2P-3}}^{\psi}}{s_{g\,t_{2P-3}}^{\psi}}\right)\sum_{\tau=t_{2P-4}}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau}.
\end{aligned}$$

Iterating, we get

$$d\ln A_{st_A} - d\ln A_{gt_A} < \eta(1-\psi)(\ln\gamma)\left(1-\frac{s_{ft_1}^{\psi}}{s_{gt_1}^{\psi}}\right)\sum_{\tau=t_{switch}}^{t_A} s_{f\tau}^{-\psi} ds_{f\tau} \le 0.$$

Therefore $d\ln A_{gt_A} > d\ln A_{st_A}$. ■

We establish a symmetric lemma:

Lemma B.4 *Consider a small increase in B_s. Denote by t_A the smallest t such that $d\ln A_{gt_A} > 0$ and assume that $t_A < \infty$. Then $d\ln A_{gt_A} > d\ln A_{st_A}$.*

Proof. The proof starts as for the previous lemma: $d\ln A_{gt_A} > 0$ requires that $ds_{ft_A} < 0$, which implies $t_A \ge t_{switch}$ and that ds_{ft} switches sign an odd number of times. We use (B-9) to write:

$$\begin{aligned}
& d\ln A_{st_A} - d\ln A_{gt_A} \\
= \; & \eta(1-\psi)(\ln\gamma)\left(\begin{array}{c} \sum_{\tau=1}^{t_{switch}-1}\left(s_{f\tau}^{-\psi}-s_{g\tau}^{-\psi}\right)ds_{f\tau} \\ +\sum_{p=0}^{P-1}\left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(\frac{s_{g\tau}^{\psi}}{s_{f\tau}^{\psi}}-1\right)s_{g\tau}^{-\psi}ds_{f\tau}+\sum_{\tau=t_{2p+1}}^{t_{2p+2}-1}\left(\frac{s_{g\tau}^{\psi}}{s_{f\tau}^{\psi}}-1\right)s_{g\tau}^{-\psi}ds_{f\tau}\right)\end{array}\right) \\
< \; & \eta(1-\psi)(\ln\gamma)\sum_{p=0}^{P-1}\left(\frac{s_{gt_{2p+1}}^{\psi}}{s_{ft_{2p+1}}^{\psi}}-1\right)\sum_{\tau=t_{2p}}^{t_{2p+2}-1} s_{g\tau}^{-\psi}ds_{f\tau},
\end{aligned}$$

following the same logic as before. By definition t_A is the smallest t such that $\sum_{\tau=1}^{t_A} s_{g\tau}^{-\psi}ds_{g\tau} > 0$, then for any $t_X < t_A$, we have $\sum_{\tau=1}^{t_X} s_{g\tau}^{-\psi}ds_{g\tau} < 0$ and $\sum_{\tau=t_X+1}^{t_A} s_{g\tau}^{-\psi}ds_{g\tau} > 0$. As $ds_{g\tau} = -ds_{f\tau}$, then $\sum_{\tau=t_X+1}^{t_A} s_{g\tau}^{-\psi}ds_{g\tau} < 0$. Using the same reasoning as before, we get $d\ln A_{st_A} - d\ln A_{gt_A} < 0$. ■

We can then derive:

Lemma B.5 *For $\ln\gamma$ small, the shale gas boom increases A_{ct}, A_{st} and decreases A_{gt}.*

Proof. We prove this result by contradiction. Assume that A_{gt} does not decrease for all t following the shale gas boom. Denote by t_A the first time that $d\ln A_{gt} > 0$, then if $\ln\gamma$ is small enough, it must be that $d\ln A_{gt_A} \approx d\ln A_{gt_A-1} \approx 0$. According to Lemma B.4, $d\ln A_{gt_A} > d\ln A_{st_A}$, therefore either $d\ln A_{st_A} \approx 0$ or $d\ln A_{st_A} < 0$. Log differentiating f_{t_A}, one obtains:

$$d\ln f_{t_A} = -(\varepsilon-1)\,d\ln A_{g(t_A-1)} + \frac{\frac{1}{A_{st_A}}\kappa_s^\varepsilon C_{st_A}^\varepsilon}{\frac{1}{A_{ct_A}}\kappa_c^\varepsilon C_{ct_A}^\varepsilon + \frac{1}{A_{st_A}}\kappa_s^\varepsilon C_{st_A}^\varepsilon}\frac{C_{st_A}}{B_{st_A}}\varepsilon d\ln B_{st_A}$$
$$+\frac{\frac{1}{A_{ct_A}}\kappa_c^\varepsilon C_{ct_A}^\varepsilon\left(\varepsilon\frac{C_{ct_A}}{A_{ct_A}}-1\right)+\frac{1}{A_{st_A}}\kappa_s^\varepsilon C_{st_A}^\varepsilon\left(\varepsilon\frac{C_{st_A}}{A_{st_A}}-1\right)}{\frac{1}{A_{ct_A}}\kappa_c^\varepsilon C_{ct_A}^\varepsilon + \frac{1}{A_{st_A}}\kappa_s^\varepsilon C_{st_A}^\varepsilon}d\ln A_{s(t_A-1)}.$$

Assume that $d\ln A_{st_A} \approx 0$, then for $\ln\gamma$ small $d\ln A_{s(t_A-1)} \approx 0$, and we get $d\ln f_{t_A} \approx \frac{\frac{1}{A_{st_A}}\kappa_s^\varepsilon C_{st_A}^\varepsilon}{\frac{1}{A_{ct_A}}\kappa_c^\varepsilon C_{ct_A}^\varepsilon + \frac{1}{A_{st_A}}\kappa_s^\varepsilon C_{st_A}^\varepsilon}\frac{C_{st_A}}{B_{st_A}}\varepsilon d\ln B_{st_A}$. Following the shale gas boom $d\ln B_s > 0$, in order for A_{gt_A} to increase, it must be that $d\ln s_{ft}$ has been negative for a number of periods before t_A, which requires that $\varepsilon\frac{C_{ct}}{A_{ct}} < 1$ for a number of periods. This ensures that $\frac{C_{st_A}}{B_{st_A}}$ is bounded above, so that $\frac{\frac{1}{A_{st_A}}\kappa_s^\varepsilon C_{st_A}^\varepsilon}{\frac{1}{A_{ct_A}}\kappa_c^\varepsilon C_{ct_A}^\varepsilon + \frac{1}{A_{st_A}}\kappa_s^\varepsilon C_{st_A}^\varepsilon}\frac{C_{st_A}}{B_{st_A}}\varepsilon$ is not too small. As a result, $d\ln f_{t_A} > 0$ so that $d\ln s_{ft_A} > 0$ which contradicts the fact that $d\ln A_{gt_A} > 0 > d\ln A_{gt_A-1}$.

Similarly, assume now that A_{st} decreases at some point. We denote by t_B the first time at which $d\ln A_{ct_B} < 0$ (t_B could be equal to t_A). Since $d\ln A_{ct_{B-1}} > 0 > d\ln A_{ct_B}$, then for $\ln\gamma$ small, we have $d\ln A_{ct_{B-1}} \approx d\ln A_{ct_B} \approx 0$. Using Lemma B.3, we get $d\ln A_{g(t_B-1)} < 0$ or $d\ln A_{gt_B} \approx 0$. Following the same reasoning as above, we get that $d\ln s_{ft_B} > 0$, which contradicts $d\ln A_{ct_{B-1}} > 0 > d\ln A_{ct_B}$.

Therefore, it must be that A_{ct}, A_{st} increase for all t and A_{gt} decreases for all t. ∎

Proof that Emissions Increase Asymptotically. We now show that emissions increase asymptotically. Log-differentiating (A-3), we get:

$$\begin{aligned} & d\ln P_t \qquad\qquad \text{(B-10)} \\ = \; & \varepsilon\left(\frac{\xi_c\kappa_c^\varepsilon C_{ct}^\varepsilon}{\xi_c\kappa_c^\varepsilon C_{ct}^\varepsilon+\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon}d\ln C_{ct}+\frac{\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon}{\xi_c\kappa_c^\varepsilon C_{ct}^\varepsilon+\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon}d\ln C_{st}\right) \\ & +\left(1-\varepsilon+\frac{(\lambda-1)(1-\nu)^{\lambda-1}A_P^{\lambda-1}}{\nu^\lambda\widetilde{A}_E^{\lambda-1}C_E^{\lambda-1}+(1-\nu)^{\lambda-1}A_P^{\lambda-1}}\right)\left(\frac{\kappa_c^\varepsilon C_{ct}^{\varepsilon-1}d\ln C_{ct}+\kappa_s^\varepsilon C_{st}^{\varepsilon-1}d\ln C_{st}+\kappa_g^\varepsilon A_{gt}^{\varepsilon-1}d\ln A_{gt}}{\kappa_c^\varepsilon C_{ct}^{\varepsilon-1}+\kappa_s^\varepsilon C_{st}^{\varepsilon-1}+\kappa_g^\varepsilon A_{gt}^{\varepsilon-1}}\right). \end{aligned}$$

As $s_{gt} \to 1$, we get:

$$\begin{aligned} d\ln P_t \sim \; & \varepsilon\left(\frac{\xi_c\kappa_c^\varepsilon C_{ct}^\varepsilon}{\xi_c\kappa_c^\varepsilon C_{ct}^\varepsilon+\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon}d\ln C_{ct}+\frac{\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon}{\xi_c\kappa_c^\varepsilon C_{ct}^\varepsilon+\xi_s\kappa_s^\varepsilon C_{st}^\varepsilon}d\ln C_{st}\right) \qquad \text{(B-11)} \\ & -\left(\varepsilon-1+\frac{(1-\lambda)(1-\nu)^{\lambda-1}A_{Pt}^{\lambda-1}}{\nu^\lambda\widetilde{A}_E^{\lambda-1}C_{Et}^{\lambda-1}+(1-\nu)^{\lambda-1}A_{Pt}^{\lambda-1}}\right)d\ln A_{gt}. \end{aligned}$$

We can rewrite this expression as:

$$\begin{aligned} d\ln P_t \quad &\to \quad -\left(\varepsilon - 1 + \frac{(1-\lambda)(1-\nu)^{\lambda} A_{Pt}^{\lambda-1}}{\nu^{\lambda}\widetilde{A}_E^{\lambda-1}\kappa_g^{\frac{\varepsilon(\lambda-1)}{\varepsilon-1}} A_{gt}^{\lambda-1} + (1-\nu)^{\lambda} A_{Pt}^{\lambda-1}}\right) d\ln A_{gt} \\ &+\varepsilon \frac{\xi_c \kappa_c^{\varepsilon} C_{ct}^{\varepsilon}\frac{C_{ct}}{A_{ct}} + \xi_s \kappa_s^{\varepsilon} C_{st}^{\varepsilon}\frac{C_{st}}{A_{st}}}{\xi_c \kappa_c^{\varepsilon} C_{ct}^{\varepsilon} + \xi_s \kappa_s^{\varepsilon} C_{st}^{\varepsilon}} d\ln A_{ct} + \varepsilon \frac{\xi_s \kappa_s^{\varepsilon} C_{st}^{\varepsilon}}{\xi_c \kappa_c^{\varepsilon} C_{ct}^{\varepsilon} + \xi_s \kappa_s^{\varepsilon} C_{st}^{\varepsilon}} \frac{C_{st}}{B_{st}} d\ln B_{st}. \end{aligned}$$

Since A_{gt} decreases and A_{ct} and A_{st} increase, emissions increase asymptotically following the natural gas boom.

Proof that Gross Output Decreases Asymptotically. Using (A-4), we can write output gross of climate damages $\widetilde{Y}_t \equiv Y_t/(1-D(S_t))$ as:

$$\widetilde{Y}_t = \left((1-\nu)^{\lambda} A_{Pt}^{\lambda-1} + \nu^{\lambda}\widetilde{A}_{Et}^{\lambda-1} C_{Et}^{\lambda-1}\right)^{\frac{1}{\lambda-1}} L.$$

Log-differentiating, one gets

$$d\ln \widetilde{Y}_t = \frac{\nu^{\lambda}\widetilde{A}_{Et}^{\lambda-1} C_{Et}^{\lambda-1}}{(1-\nu)^{\lambda} A_{Pt}^{\lambda-1} + \nu^{\lambda}\widetilde{A}_{Et}^{\lambda-1} C_{Et}^{\lambda-1}} d\ln C_{Et}. \tag{B-12}$$

In return, log-differentiating C_{Et} yields:

$$d\ln C_{Et} = \frac{\kappa_c^{\varepsilon}\frac{C_{ct}^{\varepsilon}}{A_{ct}} d\ln A_{ct} + \kappa_s^{\varepsilon}\frac{C_{ct}^{\varepsilon}}{A_{ct}} d\ln A_{st} + \kappa_s^{\varepsilon}\frac{C_{st}^{\varepsilon}}{B_{st}} d\ln B_{st} + \kappa_g^{\varepsilon} C_{gt}^{\varepsilon-1} d\ln A_{gt}}{C_{Et}^{\varepsilon-1}}. \tag{B-13}$$

Plugging (B-7) and (B-8) in (B-13) and using that A_{gt} grows exponentially but C_{st} and C_{ct} do not, we get for t large enough:

$$\begin{aligned} &d\ln C_{Et} \\ \sim \quad &\eta(1-\psi)(\ln\gamma)\left[\sum_{\tau=1}^{t}\left(\frac{\kappa_c^{\varepsilon}\frac{C_{ct}^{\varepsilon}}{A_{ct}} + \kappa_s^{\varepsilon}\frac{C_{ct}^{\varepsilon}}{A_{ct}}}{\kappa_g^{\varepsilon} C_{gt}^{\varepsilon-1}} s_{g\tau}^{\psi} s_{f\tau}^{-\psi} - 1\right) s_{g\tau}^{-\psi} ds_{f\tau}\right]. \end{aligned}$$

Further, use (20) to get:

$$\begin{aligned} &d\ln C_{Et} \\ \sim \quad &\eta(1-\psi)(\ln\gamma)\left[\sum_{\tau=1}^{t}\left(\frac{C_{g\tau}^{\varepsilon-1}\left(\kappa_c^{\varepsilon}\frac{C_{ct}^{\varepsilon}}{A_{ct}} + \kappa_s^{\varepsilon}\frac{C_{ct}^{\varepsilon}}{A_{ct}}\right)}{C_{gt}^{\varepsilon-1}\left(\kappa_c^{\varepsilon}\frac{C_{c\tau}^{\varepsilon}}{A_{c\tau}} + \kappa_s^{\varepsilon}\frac{C_{c\tau}^{\varepsilon}}{A_{c\tau}}\right)} - 1\right) s_{g\tau}^{-\psi} ds_{f\tau}\right]. \end{aligned} \tag{B-14}$$

We want to establish that $d\ln C_{Et} < 0$, but since $ds_{f\tau}$ may not be positive for all τ, we cannot show that directly. As above, we index the times where $ds_{f\tau}$ switches signs by

t_{2p} and t_{2p+1}, such that $ds_{f\tau}$ becomes negative at t_{2p+1} and positive at t_{2p}. The first sign switch occurs after t_{switch} and we also define $t_0 = t_{switch}$. We assume that at t, ds_{ft} is negative and denote $t = t_{2P} - 1$ (the reasoning extends easily to the case where $ds_{ft} > 0$). We can then decompose:

$$\frac{d\ln C_{Et}}{\eta(1-\psi)(\ln\gamma)} \sim \sum_{\tau=1}^{t_{switch}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}-1\right)\frac{ds_{f\tau}}{s_{g\tau}^{\psi}} + \sum_{p=0}^{P-1}\left(\begin{array}{c}\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}-1\right)\frac{ds_{f\tau}}{s_{g\tau}^{\psi}}\\ +\sum_{\tau=t_{2p+1}}^{t_{2p+2}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}-1\right)\frac{ds_{f\tau}}{s_{g\tau}^{\psi}}\end{array}\right).$$

Using that $ds_{ft} < 0$ on $[t_{2P-1}, t_{2P} - 1]$ and that $\frac{s_{g\tau}}{s_{f\tau}}$ is increasing after t_{switch}, we can write:

$$\begin{aligned}\frac{d\ln C_{Et}}{\eta(1-\psi)(\ln\gamma)} &< \sum_{\tau=1}^{t_{switch}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}-\left(\frac{s_{gt_{2P-1}}}{s_{ft_{2P-1}}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}\right)s_{g\tau}^{-\psi}ds_{f\tau}\\ &+\sum_{p=0}^{P-2}\left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}-\left(\frac{s_{gt_{2P-1}}}{s_{ft_{2P-1}}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}\right)\frac{ds_{f\tau}}{s_{g\tau}^{\psi}}+\sum_{\tau=t_{2p+1}}^{t_{2p+2}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}-\left(\frac{s_{gt_{2P-1}}}{s_{ft_{2P-1}}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}\right)\frac{ds_{f\tau}}{s_{g\tau}^{\psi}}\right)\\ &+\sum_{\tau=t_{2P-2}}^{t_{2P-1}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}-\left(\frac{s_{gt_{2P-1}}}{s_{ft_{2P-1}}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}\right)s_{g\tau}^{-\psi}ds_{f\tau}+\left(1-\left(\frac{s_{gt_{2P-1}}}{s_{ft_{2P-1}}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}\right)\frac{d\ln A_{gt}}{\eta(1-\psi)(\ln\gamma)}.\end{aligned}$$

where we use (B-8). Reiterating the same procedure, one gets:

$$\begin{aligned}\frac{d\ln C_{Et}}{\eta(1-\psi)(\ln\gamma)} &< \left(\frac{s_{ft}}{s_{gt}}\right)^{\psi}\sum_{\tau=1}^{t_{switch}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\right)^{\psi}-\left(\frac{s_{gt_1}}{s_{ft_1}}\right)^{\psi}\right)s_{g\tau}^{-\psi}ds_{f\tau}\\ &+\sum_{p=0}^{P-2}\frac{s_{ft}^{\psi}}{s_{gt}^{\psi}}\left(\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\right)^{\psi}-\left(\frac{s_{gt_{2p+1}}}{s_{ft_{2p+1}}}\right)^{\psi}\right)\frac{ds_{f\tau}}{s_{g\tau}^{\psi}}+\frac{\left(\left(\frac{s_{gt_{2p+3}}}{s_{ft_{2p+3}}}\right)^{\psi}-\left(\frac{s_{gt_{2p+1}}}{s_{ft_{2p+1}}}\right)^{\psi}\right)d\ln A_{gt_{2p+2}-1}}{\eta(1-\psi)(\ln\gamma)}\right)\\ &+\left(\frac{s_{ft}}{s_{gt}}\right)^{\psi}\sum_{\tau=t_{2P-2}}^{t_{2P-1}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\right)^{\psi}-\left(\frac{s_{gt_{2P-1}}}{s_{ft_{2P-1}}}\right)^{\psi}\right)s_{g\tau}^{-\psi}ds_{f\tau}+\left(1-\left(\frac{s_{gt_{2P-1}}}{s_{ft_{2P-1}}}\frac{s_{ft}}{s_{gt}}\right)^{\psi}\right)\frac{d\ln A_{gt}}{\eta(1-\psi)(\ln\gamma)}.\end{aligned}$$

The first term is negative because $t_1 > t_{switch}$, so $s_{gt_1} > s_{ft_1}$ while for $\tau < t_{switch}$, $s_{g\tau} < s_{f\tau}$ and $ds_{f\tau} > 0$. The terms in $\sum_{\tau=t_{2p}}^{t_{2p+1}-1}\left(\left(\frac{s_{g\tau}}{s_{f\tau}}\right)^{\psi}-\left(\frac{s_{gt_{2p+1}}}{s_{ft_{2p+1}}}\right)^{\psi}\right)s_{g\tau}^{-\psi}ds_{f\tau}$ are negative because over such intervals $ds_{f\tau} > 0$ and since $t > t_{switch,}$ s_{gt} is increasing so $\left(\frac{s_{g\tau}}{s_{f\tau}}\right)^{\psi}-\left(\frac{s_{gt_{2p+1}}}{s_{ft_{2p+1}}}\right)^{\psi} < 0$.

In addition, we have established in Lemma B.5 that $d\ln A_{gt} < 0$ for all $t's$. Therefore we get that for t large enough $d\ln C_{Et} < 0$. This ensures that gross output decreases asymptotically.

Proof that Welfare Decreases. For T large but finite, we can write the change in welfare as:

$$
\begin{aligned}
& dU \\
\approx \; & \sum_{\tau=0}^{T-1} \frac{1}{(1+\rho)^{\tau}} \frac{d\left(e^{-\zeta S_\tau}\widetilde{Y}_\tau\right)^{1-\vartheta}}{1-\vartheta} \\
& + \sum_{\tau=T}^{\infty} \frac{\widetilde{Y}_T^{1-\vartheta} e^{-\zeta(1-\vartheta)S_\tau}}{(1+\rho)^T} \left(\frac{\left(1+g_{\widetilde{Y}}\right)^{1-\vartheta}}{1+\rho}\right)^{(\tau-T)} \left(\frac{\nu^\lambda \widetilde{A}_{E\tau}^{\lambda-1} C_{E\tau}^{\lambda-1} d\ln C_{E\tau}}{(1-\nu)^\lambda A_{P\tau}^{\lambda-1} + \nu^\lambda \widetilde{A}_{E\tau}^{\lambda-1} C_{E\tau}^{\lambda-1}} - \zeta dS_\tau\right).
\end{aligned}
$$

As emissions decrease exponentially, S_τ is bounded above, so that $e^{-\zeta(1-\vartheta)S_\tau}$ remains bounded away from 0. For T large and if the stock of carbon depends mostly on current emissions (which is the case when φ_L is small enough, potentially 0, and φ large enough), $dS_\tau > 0$. Furthermore, $d\ln C_{E\tau} < 0$. Therefore, as $\frac{\nu^\lambda \widetilde{A}_{E\tau}^{\lambda-1} C_{E\tau}^{\lambda-1}}{(1-\nu)^\lambda A_{P\tau}^{\lambda-1} + \nu^\lambda \widetilde{A}_{E\tau}^{\lambda-1} C_{E\tau}^{\lambda-1}}$ tends toward a constant, the second sum becomes arbitrarily large if $\frac{\left(1+g_{\widetilde{Y}}\right)^{1-\vartheta}}{1+\rho}$ is sufficiently close to 1. The latter condition is met in particular when ρ is low enough and $\vartheta \le 1$.

B.2 Proofs for the model with endogenous innovation in extraction

Proof of Proposition A.3. Assume first that we have asymptotically positive growth in fossil-fuel power plant technologies A_{st} and A_{ct}. We first establish that there must be growth at the same rate in either B_{st} or B_{ct}. Assume instead that both extraction technologies grow more slowly than A_{st} and A_{ct}. Then using (A-5), we get

$$
\left(\frac{s_{Bct}}{s_{Aft}}\right)^{\psi} \sim \frac{\kappa_c^\varepsilon B_{ct}^{\varepsilon-1}}{\frac{B_{ct}}{A_{ct}}\kappa_c^\varepsilon B_{ct}^{\varepsilon-1} + \kappa_s^\varepsilon \frac{B_{st}}{A_{st}} B_{st}^{\varepsilon-1}} \text{ and } \left(\frac{s_{Bst}}{s_{Aft}}\right)^{\psi} \longrightarrow \frac{\kappa_s^\varepsilon B_{st}^{\varepsilon-1}}{\frac{B_{ct}}{A_{ct}}\kappa_c^\varepsilon B_{ct}^{\varepsilon-1} + \kappa_s^\varepsilon \frac{B_{st}}{A_{st}} B_{st}^{\varepsilon-1}}.
$$

Assume without loss of generality that $\frac{B_{ct}}{A_{ct}} B_{ct}^{\varepsilon-1}$ grows at least as fast as $\frac{B_{st}}{A_{st}} B_{st}^{\varepsilon-1}$, then we get

$$
\left(\frac{s_{Bc}}{s_{Aft}}\right)^{\psi} = O\left(\frac{A_{ct}}{B_{ct}}\right),
$$

so that $s_{Aft} \to 0$. This leads to a contradiction as it implies that B_{ct} cannot grow more slowly than A_{ft}. Hence at least one of the two extraction technologies must grow at least as fast as A_{ct}.

Assume now that B_{ct} grows faster than A_{ct}, then (A-5) implies

$$\left(\frac{s_{Bct}}{s_{Aft}}\right)^{\psi} \sim \frac{A_{ct}}{B_{ct}} \frac{\kappa_c^{\varepsilon} A_{ct}^{\varepsilon-1}}{\kappa_c^{\varepsilon} A_{ct}^{\varepsilon-1} + \kappa_s^{\varepsilon} \frac{C_{st}}{A_{st}} C_{st}^{\varepsilon-1}} \leq \frac{A_{ct}}{B_{ct}}.$$

As a result, s_{Bct} tends to 0, which is, again, a contradiction. Therefore, extraction technologies cannot grow faster than A_{ct} on a fossil-fuel path, and at least one extraction technology must grow at the same rate as A_{ct}.

Without loss of generality, assume that B_{ct} grows at the same rate as A_{ct} (while B_{st} grows weakly less fast), using (A-6) we get:

$$\left(\frac{s_{Aft}}{s_{gt}}\right)^{\psi} = O\left(\frac{A_{ct}}{A_{gt}}\right)^{\varepsilon-1}.$$

Then, if A_{ct} grows faster than A_{gt}, $s_{gt} \to 0$. In contrast, if A_{ct} grows more slowly than A_{gt} then $s_{ft} \to 0$, which contradicts the assumption of positive growth in the fossil-fuel sector. Therefore, there is path dependence in innovation and (except for a knifed-edge case) innovation is asymptotically entirely in the fossil-fuel sector or entirely in the green sector.

Proof of Proposition A.4. Log-differentiate (A-5) for the natural gas sector (assuming that one can ignore the dependence of the right-hand side on the allocation of innovation) to get:

$$\psi d\ln s_{B_{st}} - \psi d\ln s_{Aft} = \left(\varepsilon \frac{C_{st}}{B_{st}} \frac{\frac{C_{ct}}{A_{ct}} \kappa_c^{\varepsilon} C_{ct}^{\varepsilon-1}}{\frac{C_{ct}}{A_{ct}} \kappa_c^{\varepsilon} C_{ct}^{\varepsilon-1} + \frac{C_{st}}{A_{st}} \kappa_s^{\varepsilon} C_{st}^{\varepsilon-1}} - 1\right) d\ln B_{st}. \quad \text{(B-15)}$$

Log-differentiating the ratio of the two equations in (A-5) gives:

$$\psi d\ln s_{B_{st}} - \psi d\ln s_{B_{ct}} = \left(\varepsilon \frac{C_{st}}{B_{st}} - 1\right) d\ln B_{st}. \quad \text{(B-16)}$$

Log-differentiate the ratio of (A-5) for natural gas and (A-6) to get:

$$\psi d\ln s_{B_s} - \psi d\ln s_{gt} = \left(\varepsilon \frac{C_{st}}{B_{st}} - 1\right) d\ln B_{st}. \quad \text{(B-17)}$$

Log-differentiating the scientists market clearing condition gives:

$$s_{Bst} d\ln s_{Bst} + s_{Aft} d\ln s_{Aft} + s_{Bct} d\ln s_{Bct} + s_{gt} d\ln s_{gt} = 0. \quad \text{(B-18)}$$

Take the difference between (B-15) and (B-17) to get:

$$d\ln\left(\frac{s_{gst}^{1-\psi}}{s_{Aft}^{1-\psi}}\right) = -\frac{\varepsilon\,(1-\psi)}{\psi}\frac{C_{st}}{B_{st}}\frac{\frac{C_{st}}{A_{st}}\kappa_s^{\varepsilon}C_{st}^{\varepsilon-1}}{\frac{C_{ct}}{A_{ct}}\kappa_{ct}^{\varepsilon}C_{ct}^{\varepsilon-1}+\frac{C_{st}}{A_{st}}\kappa_s^{\varepsilon}C_{st}^{\varepsilon-1}}d\ln B_{st}, \tag{B-19}$$

which establishes that a natural gas boom redirects innovation away from green technologies relative to fossil-fuel power plant technologies.
Plugging (B-15), (B-16), and (B-17) in (B-18) implies:

$$d\ln s_{Bst} = \frac{1}{\psi}\left[s_{Aft}\left(\varepsilon\frac{C_{st}}{B_{st}}\frac{\frac{C_{ct}}{A_{ct}}\kappa_c^{\varepsilon}C_{ct}^{\varepsilon-1}}{\frac{C_{ct}}{A_{ct}}\kappa_c^{\varepsilon}C_{ct}^{\varepsilon-1}+\frac{C_{st}}{A_{st}}\kappa_s^{\varepsilon}C_{st}^{\varepsilon-1}}-1\right)+\left(s_{Bct}+s_{gt}\right)\left(\varepsilon\frac{C_{st}}{B_{st}}-1\right)\right]d\ln B_{st}.$$

Then (B-17) gives:

$$d\ln s_{gt} = -\frac{1}{\psi}\left[s_{Bst}\left(\varepsilon\frac{C_{st}}{B_{st}}-1\right)+s_{Aft}\varepsilon\frac{C_{st}}{B_{st}}\frac{\frac{C_{st}}{A_{st}}\kappa_s^{\varepsilon}C_{st}^{\varepsilon-1}}{\frac{C_{ct}}{A_{ct}}\kappa_c^{\varepsilon}C_{ct}^{\varepsilon-1}+\frac{C_{st}}{A_{st}}\kappa_s^{\varepsilon}C_{st}^{\varepsilon-1}}\right]d\ln B_{st}.$$

Therefore a natural gas boom decreases green innovation if $\varepsilon\frac{C_{st}}{B_{st}}-1\geq 0$.

B.3 Additional Proofs for the Extended Model

B.3.1 Equilibrium energy production

This section derives the energy production in equilibrium, E_t, which requires solving for the labor allocation. To do that, we first note that the price of the production input is given by $p_{Pt}=\gamma w_t/A_{Pt}$. Therefore, as in the baseline model, relative demand by the final good producer leads to

$$\frac{E_t}{Y_{Pt}} = \left(\widetilde{A}_E\right)^{\lambda-1}\left(\frac{\nu C_{Et}}{(1-\nu)A_{Pt}}\right)^{\lambda}. \tag{B-20}$$

Using that $E_{it}=C_{it}L_{it}$ for $i=c,s$ in (A-14), we get that

$$L_{ct} = \frac{\kappa_c^{\sigma}\left(1+\widetilde{\tau}_c\right)^{-\sigma}C_{ct}^{\sigma-1}L_{ft}}{\kappa_c^{\sigma}\left(1+\widetilde{\tau}_c\right)^{-\sigma}C_{ct}^{\sigma-1}+\kappa_s^{\sigma}\left(1+\widetilde{\tau}_s\right)^{-\sigma}C_{st}^{\sigma-1}} \text{ and } L_{st} = \frac{\kappa_s^{\sigma}\left(1+\widetilde{\tau}_s\right)^{-\sigma}C_{st}^{\sigma-1}L_{ft}}{\kappa_c^{\sigma}\left(1+\widetilde{\tau}_c\right)^{-\sigma}C_{ct}^{\sigma-1}+\kappa_s^{\sigma}\left(1+\widetilde{\tau}_s\right)^{-\sigma}C_{st}^{\sigma-1}}. \tag{B-21}$$

Therefore, from (A-13), we get that $E_{ft}=\widetilde{C}_{ft}L_{ft}$ with $\widetilde{C}_{ft}$ defined in (A-19). Similarly, using this expression with $E_{gt}=C_{gt}L_{gt}$ in (A-16), we get:

$$\frac{L_{ft}}{L_{gt}} = \frac{C_{ft}^{\varepsilon}\widetilde{C}_{ft}^{-1}}{\kappa_g^{\varepsilon}\left(1+\widetilde{\tau}_g\right)^{-\varepsilon}C_{gt}^{\varepsilon-1}},$$

which directly leads to (A-18). We then plug (A-18) and $Y_{Pt} = A_{Pt} L_{Pt}$ in (B-20) and obtain:

$$\frac{L_{Et}}{L_{Pt}} = \frac{\nu^{\lambda} \left(\widetilde{A}_E\right)^{\lambda-1} C_{Et}^{\lambda} \widetilde{C}_{Et}^{-1}}{(1-\nu)^{\lambda} A_{Pt}^{\lambda-1}}. \tag{B-22}$$

Together with labor market clearing equation, this equation implies (A-20). Note that we recover (14) when $\widetilde{\tau}_g = \widetilde{\tau}_c = \widetilde{\tau}_s = 0$.

B.3.2 Proof of Proposition A.5

We can decompose the change in the emission rate as:

$$\frac{\partial \ln \xi_E}{\partial \ln B_{st}} = \underbrace{\varepsilon \frac{\partial \ln \left(C_{ft}/C_{Et}\right)}{\partial \ln B_{st}}}_{Sub_g:\ \text{substitution effect away from green}} + \underbrace{\frac{\partial \ln \left(\xi_c \kappa_c^{\sigma} \left(\frac{C_{ct}}{(1+\widetilde{\tau}_c)C_{ft}}\right)^{\sigma} + \xi_s \kappa_s^{\sigma} \left(\frac{C_{st}}{(1+\widetilde{\tau}_s)C_{ft}}\right)^{\sigma}\right)}{\partial \ln \left(B_{st}\right)}}_{Sub_f:\ \text{substitution within fossil fuels}}.$$

The substitution effect away from green electricity is naturally positive:

$$Sub_g = \varepsilon \frac{\kappa_g^{\varepsilon}}{C_{Et}^{\varepsilon-1}} \left(\frac{A_{gt}}{1+\widetilde{\tau}_g}\right)^{\varepsilon-1} \frac{\kappa_s^{\sigma}}{C_{ft}^{\sigma-1}} \left(\frac{C_{st}}{1+\widetilde{\tau}_s}\right)^{\sigma-1} \frac{C_{st}}{B_{st}}, \tag{B-23}$$

where we use the fact that

$$\frac{\partial \ln C_{ft}}{\partial \ln B_{st}} = \frac{\kappa_s^{\sigma}}{C_{ft}^{\sigma-1}} \left(\frac{C_{st}}{1+\widetilde{\tau}_s}\right)^{\sigma-1} \frac{C_{st}}{B_{st}} \text{ and } \frac{\partial \ln C_{Et}}{\partial \ln B_{st}} = \frac{C_{ft}^{\varepsilon-1}}{C_{Et}^{\varepsilon-1}} \frac{C_{st}}{B_{st}}. \tag{B-24}$$

Combining (A-15) and (A-14), we get that the tax-inclusive expenditure share of gas electricity in fossil-fuel electricity obeys:

$$\theta_{sft} = \frac{(1+\tau_{st}) p_{st} E_{st}}{p_{ft} E_{ft}} = \frac{\kappa_s^{\sigma}}{C_{ft}^{\sigma-1}} \left(\frac{C_{st}}{1+\widetilde{\tau}_s}\right)^{\sigma-1}. \tag{B-25}$$

The tax-inclusive expenditure share on clean energy, using (A-16) is given by:

$$\Theta_{gt} = \frac{\left(1+\tau_{gt}\right) p_{gt} E_{gt}}{p_{Et} E_t} = \frac{\kappa_g^{\varepsilon}}{C_{Et}^{\varepsilon-1}} \left(\frac{A_{gt}}{1+\widetilde{\tau}_g}\right)^{\varepsilon-1}.$$

Then, we can rewrite (B-23) as

$$Sub_g = \varepsilon \Theta_{gt} \theta_{sft} \frac{C_{st}}{B_{st}}. \tag{B-26}$$

Further, we have

$$\begin{aligned} Sub_f &= -\sigma \frac{C_{ft}^{1-\sigma}\kappa_c^\sigma \kappa_s^\sigma \left(\frac{C_{st}}{1+\widetilde{\tau}_s}\right)^{\sigma-1}\left(\frac{C_{ct}}{1+\widetilde{\tau}_c}\right)^{\sigma-1}}{\left(\xi_c \kappa_c^\sigma \left(\frac{C_{ct}}{(1+\widetilde{\tau}_c)}\right)^\sigma + \xi_s \kappa_s^\sigma \left(\frac{C_{st}}{(1+\widetilde{\tau}_s)}\right)^\sigma\right)} \left[\xi_c \frac{C_{ct}}{1+\widetilde{\tau}_c} - \xi_s \frac{C_{st}}{1+\widetilde{\tau}_s}\right] \frac{C_{st}}{B_{st}} \quad \text{(B-27)} \\ &= -\sigma \theta_{sft} \frac{P_{c,t}}{P_t} \left[1 - \frac{\xi_s}{\xi_c} \frac{C_{st}}{1+\widetilde{\tau}_s} \frac{1+\widetilde{\tau}_c}{C_{ct}}\right] \frac{C_{st}}{B_{st}}, \end{aligned}$$

where

$$\frac{P_{ct}}{P_t} = \frac{\xi_c \kappa_c^\sigma \left(\frac{C_{ct}}{(1+\widetilde{\tau}_c)C_{ft}}\right)^\sigma}{\xi_c \kappa_c^\sigma \left(\frac{C_{ct}}{(1+\widetilde{\tau}_c)C_{ft}}\right)^\sigma + \xi_s \kappa_s^\sigma \left(\frac{C_{st}}{(1+\widetilde{\tau}_s)C_{ft}}\right)^\sigma} \tag{B-28}$$

is the pollution share of coal based electricity. Therefore the substitution effect within fossil-fuel is negative as long as $\xi_c \frac{C_{ct}}{1+\widetilde{\tau}_c} > \xi_s \frac{C_{st}}{1+\widetilde{\tau}_s}$ holds. Combining (B-26) and (B-27), and using (B-25) and (B-28), we obtain equation (27).

To compute the scale effect, we log differentiate (A-18) and get:

$$d \ln E_t = d \ln \widetilde{C}_{Et} + d \ln L_{Et}. \tag{B-29}$$

Log-differentiating (A-20), we get:

$$d \ln L_{Et} = \frac{L_{Pt}}{L} \left(\lambda d \ln C_{Et} - d \ln \widetilde{C}_{Et}\right). \tag{B-30}$$

As long as $d \ln \widetilde{C}_{Et} \approx d \ln C_{Et}$, then an increase in B_{st} is associated with a decline in labor in the energy sector L_{Et}.

From (B-29), we then obtain the change in total energy production:

$$d \ln E_t = \frac{L_{Pt}}{L} \lambda d \ln C_{Et} + \frac{L_{Et}}{L} d \ln \widetilde{C}_{Et}, \tag{B-31}$$

which is positive (as long as $d \ln \widetilde{C}_{Et}$ is not largely negative).

Using the definition of $\widetilde{C}_{Et}$ in (A-19), we get:

$$d \ln \widetilde{C}_{ft} = \left(\sigma \theta_{sft} - \frac{(\sigma-1)\kappa_s^\sigma (1+\widetilde{\tau}_s)^{-\sigma} C_{st}^{\sigma-1}}{\kappa_c^\sigma (1+\widetilde{\tau}_c)^{-\sigma} C_{ct}^{\sigma-1} + \kappa_s^\sigma (1+\widetilde{\tau}_s)^{-\sigma} C_{st}^{\sigma-1}}\right) d \ln C_{st}.$$

Using the definition of $\widetilde{C}_{Et}$ in (A-18) and plugging in the previous expression, we can express the change in the productivity variable $\widetilde{C}_{Et}$ as:

$$d \ln \widetilde{C}_{Et} = \left[\varepsilon \Theta_{st} + \frac{C_{ft}^\varepsilon \widetilde{C}_{ft}^{-1}\left((\sigma-\varepsilon)\theta_{sft} - \frac{(\sigma-1)\kappa_s^\sigma (1+\widetilde{\tau}_s)^{-\sigma} C_{st}^{\sigma-1}}{\kappa_c^\sigma (1+\widetilde{\tau}_c)^{-\sigma} C_{ct}^{\sigma-1} + \kappa_s^\sigma (1+\widetilde{\tau}_s)^{-\sigma} C_{st}^{\sigma-1}}\right)}{C_{ft}^\varepsilon \widetilde{C}_{ft}^{-1} + \kappa_g^\varepsilon \left(1+\widetilde{\tau}_g\right)^{-\varepsilon} A_{gt}^{\varepsilon-1}}\right] d \ln C_{st}. \tag{B-32}$$

For $\widetilde{\tau}_g, \widetilde{\tau}_c$ and $\widetilde{\tau}_s$ small, we get

$$d \ln \widetilde{C}_{Et}|_{\widetilde{\tau}_g,\widetilde{\tau}_c,\widetilde{\tau}_s\approx 0} \approx d \ln \widetilde{C}_{Et} = \Theta_{st} d \ln C_{st}, \tag{B-33}$$

which, using (B-31), leads to the same scale effect as in the baseline:

$$\frac{\partial \ln E_t}{\partial \ln B_{st}}|_{\widetilde{\tau}_g,\widetilde{\tau}_c,\widetilde{\tau}_s\approx 0} \approx \frac{L_{Et} + \lambda L_{Pt}}{L} \Theta_{st} \frac{C_{st}}{B_{st}}. \tag{B-34}$$

The overall effect on emissions is then given by the sum of (27) and (B-34), which we can rewrite as

$$\frac{\partial \ln P_t}{\partial \ln B_{st}}|_{\widetilde{\tau}_g,\widetilde{\tau}_c,\widetilde{\tau}_s\approx 0} \approx -\frac{C_{st}}{B_{st}} \left[(\sigma-\varepsilon)\,\theta_{sft} + (\varepsilon-1)\,\Theta_{st} + \frac{(1-\lambda)\,L_{Pt}}{L}\Theta_{st} - \sigma \frac{P_{st}}{P_t} \right].$$

For ξ_s/ξ_c small, the term $\frac{P_{st}}{P_t}$ is small, given that $\sigma \geq \varepsilon$ and $\lambda \leq 1$, then emissions decrease following the natural gas boom.

B.3.3 Proof of Uniqueness and Maximal Growth Rate

We show that the equilibrium is unique for $\ln \gamma$ small enough. Using (A-25) and defining $s_{ft} = s_{ct} + s_{st}$, we can write:

$$\begin{aligned} s_{ct} &= \frac{(1-q_s)^{\frac{1}{\psi}} \left(\frac{\kappa_c^\sigma (1+\overline{\Lambda}_c) C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}} + \frac{\chi \kappa_s^\sigma (1+\overline{\Lambda}_s) C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}} \right)^{\frac{1}{\psi}} s_{ft}}{(1-q_c)^{\frac{1}{\psi}} \left(\frac{\chi \kappa_c^\sigma (1+\overline{\Lambda}_c) C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}} + \frac{\kappa_s^\sigma (1+\overline{\Lambda}_s) C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}} \right)^{\frac{1}{\psi}} + (1-q_s)^{\frac{1}{\psi}} \left(\frac{\kappa_c^\sigma (1+\overline{\Lambda}_c) C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}} + \frac{\chi \kappa_s^\sigma (1+\overline{\Lambda}_s) C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}} \right)^{\frac{1}{\psi}}} \\ s_{st} &= \frac{(1-q_c)^{\frac{1}{\psi}} \left(\frac{\chi \kappa_c^\sigma (1+\overline{\Lambda}_c) C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}} + \frac{\kappa_s^\sigma (1+\overline{\Lambda}_s) C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}} \right)^{\frac{1}{\psi}} s_{ft}}{(1-q_c)^{\frac{1}{\psi}} \left(\frac{\chi \kappa_c^\sigma (1+\overline{\Lambda}_c) C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}} + \frac{\kappa_s^\sigma (1+\overline{\Lambda}_s) C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}} \right)^{\frac{1}{\psi}} + (1-q_s)^{\frac{1}{\psi}} \left(\frac{\kappa_c^\sigma (1+\overline{\Lambda}_c) C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}} + \frac{\chi \kappa_s^\sigma (1+\overline{\Lambda}_s) C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}} \right)^{\frac{1}{\psi}}}. \end{aligned} \tag{B-35}$$

For $\ln \gamma$ small enough, we can ignore the dependence of the RHS on s_{ct} and s_{st}, so that the previous equations define s_{ct} and s_{st} as increasing (and nearly linear) functions of s_{ft}. We then get that the numerator in the LHS (A-26) is decreasing in s_{ft} (as for $\ln \gamma$ small, we can ignore the dependence of C_{it} and A_{it} on innovation). The denominator is increasing in s_{ft} as $s_{gt} = 1 - s_{ft}$ (and again ignoring the dependence of C_{gt} on the innovation allocation). Therefore the LHS decreases from infinity to 0 in s_{ft}, and the equation defines a unique solution.

We show that the maximal growth rate that can be achieved on a fossil-fuel path corresponds to the growth rate $\gamma^{\eta_f \left(1+\chi^{\frac{1}{\psi}}\right)^{\psi}} - 1$. The growth rate of C_{Et} is maximized if the growth rate of C_{ft} is maximized which occurs if the growth rates of either C_{st} or C_{ct} are maximized. Without loss of generality, assume that A_{ct} grows faster than A_{st}. Then, the growth rates of C_{ct} and that of A_{ct} are maximized when $s_{ct}^{1-\psi}+\chi s_{st}^{1-\psi}$ is maximized, which occurs if $s_{ct}=s_{ft}/\left(1+\chi^{\frac{1}{\psi}}\right)$. In that case, B_{ct} and B_{st} grow faster than A_{st}, and (B-35) gives $s_{ct}\to s_{ft}/\left(1+\chi^{\frac{1}{\psi}}\right)$ for $q_c=q_s$, so that this optimal growth rate can be achieved. We then get that C_{Et} and $\widetilde{C}_{Et}$ grow asymptotically at the rate $\gamma^{\eta_f\left(1+\chi^{\frac{1}{\psi}}\right)^{\psi}}-1$.

B.3.4 Proof of Proposition A.6

Log differentiating (B-35), and assuming that $\ln\gamma$ is sufficiently small that we can ignore the dependence of A_{it} on s_{it}, we can write:

$$d\ln s_{ct}\approx d\ln s_{ft}-\frac{\sigma}{\psi}\frac{s_{st}}{s_{ft}}\frac{\left(1-\chi^2\right)\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}}\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\frac{C_{st}}{B_{st}}d\ln B_{st}}{\left(\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}}+\chi\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\right)\left(\chi\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}}+\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\right)},\tag{B-36}$$

$$d\ln s_{st}\approx d\ln s_{ft}+\frac{\sigma}{\psi}\frac{s_{ct}}{s_{ft}}\frac{\left(1-\chi^2\right)\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}}\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\frac{C_{st}}{B_{st}}d\ln B_{st}}{\left(\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}}+\chi\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\right)\left(\chi\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}}+\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\right)}.\tag{B-37}$$

This directly implies that the ratio s_{st}/s_{ct} increases with B_{st}. Log-differentiating (A-26) and using (A-25) and (B-24) (and $\ln\gamma$ small) leads to

$$\left[\begin{array}{c}\frac{\left(\left(\chi^2+1\right)\frac{\kappa_c^\sigma}{(1+\widetilde{\tau}_c)^\sigma}\frac{(1+\overline{\Lambda}_c)C_{ct}^\sigma}{A_{ct}}+2\chi\frac{\kappa_s^\sigma}{(1+\widetilde{\tau}_s)^\sigma}\frac{(1+\overline{\Lambda}_s)C_{st}^\sigma}{A_{st}}\right)\frac{\kappa_s^\sigma(1+\overline{\Lambda}_s)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\sigma\frac{C_{st}}{B_{st}}d\ln B_{st}}{2\left(\chi\frac{\kappa_c^\sigma(1+\overline{\Lambda}_c)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}}+\frac{\kappa_s^\sigma(1+\overline{\Lambda}_s)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\right)\left(\frac{\kappa_c^\sigma(1+\overline{\Lambda}_c)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}}+\chi\frac{\kappa_s^\sigma(1+\overline{\Lambda}_s)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\right)}\\+(\varepsilon-\sigma)\frac{\kappa_s^\sigma}{C_{ft}^{\sigma-1}}\left(\frac{C_{st}}{1+\widetilde{\tau}_s}\right)^{\sigma-1}\frac{C_{st}}{B_{st}}d\ln B_{st}-\frac{\psi}{2}d\ln s_{st}-\frac{\psi}{2}d\ln s_{ct}+\psi d\ln s_{gt}\end{array}\right]\approx 0.$$

Noting that $d\ln s_{gt}=-\frac{s_{ft}}{s_{gt}}d\ln s_{ft}$ and plugging in (B-36) and (B-37), we get:

$$\begin{aligned}&d\ln s_{ft}\\&\approx\frac{s_{gt}}{\psi}\left[\frac{(\varepsilon-\sigma)\kappa_s^\sigma C_{st}^{\sigma-1}}{C_{ft}^{\sigma-1}(1+\widetilde{\tau}_s)^{\sigma-1}}+\sigma\frac{\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\left(\left(\frac{s_{st}}{s_{ft}}+\chi^2\frac{s_{ct}}{s_{ft}}\right)\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}}+\chi\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\right)}{\left(\chi\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}}+\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\right)\left(\frac{\kappa_c^\sigma\left(1+\overline{\Lambda}_c\right)C_{ct}^\sigma}{(1+\widetilde{\tau}_c)^\sigma A_{ct}}+\chi\frac{\kappa_s^\sigma\left(1+\overline{\Lambda}_s\right)C_{st}^\sigma}{(1+\widetilde{\tau}_s)^\sigma A_{st}}\right)}\right]\frac{C_{st}d\ln B_{st}}{B_{st}}.\end{aligned}\tag{B-38}$$

The second term in the brackets is positive whereas the first term is weakly negative since $\varepsilon\leq\sigma$. Therefore if $\varepsilon\approx\sigma$, then the first term is small and the shale gas boom increases the mass of scientists in fossil-fuel innovations and decreases green innovation. When $\sigma>\varepsilon$, then green energy is more complementary to natural gas than coal is, this creates a force that pushes toward more green innovation following the shale gas boom.

Combining (B-37) with (B-38), it is also immediate that for $\varepsilon\approx\sigma$, an increase in B_{st}

leads to an increase in natural gas innovation. Combining (B-36) with (B-38), we get:

$$
\begin{aligned}
& d\ln s_{ct} \\
\approx\ & \left[\frac{s_{gt}(\varepsilon-\sigma)\kappa_s^{\sigma} C_{st}^{\sigma-1}}{C_{ft}^{\sigma-1}(1+\widetilde{\tau}_s)^{\sigma-1}} + \frac{\sigma \frac{\kappa_s^{\sigma}\left(1+\overline{\Lambda}_s\right)C_{st}^{\sigma}}{(1+\widetilde{\tau}_s)^{\sigma}A_{st}} \left(\left[-s_{st}+\chi^2\left(s_{gt}+s_{st}\right)\right] \frac{\kappa_c^{\sigma}\left(1+\overline{\Lambda}_c\right)C_{ct}^{\sigma}}{(1+\widetilde{\tau}_c)^{\sigma}A_{ct}} + s_{gt}\chi \frac{\kappa_s^{\sigma}\left(1+\overline{\Lambda}_s\right)C_{st}^{\sigma}}{(1+\widetilde{\tau}_s)^{\sigma}A_{st}} \right)}{\left(\chi \frac{\kappa_c^{\sigma}\left(1+\overline{\Lambda}_c\right)C_{ct}^{\sigma}}{(1+\widetilde{\tau}_c)^{\sigma}A_{ct}} + \frac{\kappa_s^{\sigma}\left(1+\overline{\Lambda}_s\right)C_{st}^{\sigma}}{(1+\widetilde{\tau}_s)^{\sigma}A_{st}}\right)\left(\frac{\kappa_c^{\sigma}\left(1+\overline{\Lambda}_c\right)C_{ct}^{\sigma}}{(1+\widetilde{\tau}_c)^{\sigma}A_{ct}} + \chi \frac{\kappa_s^{\sigma}\left(1+\overline{\Lambda}_s\right)C_{st}^{\sigma}}{(1+\widetilde{\tau}_s)^{\sigma}A_{st}}\right)} \right] \frac{C_{st}\, d\ln B_{st}}{\psi B_{st}}
\end{aligned}
$$

The effect of an increase in B_{st} on s_{ct} is ambiguous even for $\varepsilon=\sigma$: the second term in brackets is positive if χ is close to 1 but negative for χ close to 0. This establishes Part i).

Assume now that $\chi=1$, then (B-38) gives:

$$
\begin{aligned}
d\ln s_{ft}\big|_{\chi=1} \ \approx\ & \frac{s_{gt}}{\psi}\left[(\varepsilon-\sigma)\frac{\kappa_s^{\sigma}\left(\frac{C_{st}}{1+\widetilde{\tau}_s}\right)^{\sigma-1}}{\kappa_c^{\sigma}\left(\frac{C_{ct}}{1+\widetilde{\tau}_c}\right)^{\sigma-1}+\kappa_s^{\sigma}\left(\frac{C_{st}}{1+\widetilde{\tau}_s}\right)^{\sigma-1}} + \sigma\frac{\frac{\kappa_s^{\sigma}\left(1+\overline{\Lambda}_s\right)C_{st}^{\sigma}}{(1+\widetilde{\tau}_s)^{\sigma}A_{st}}}{\frac{\kappa_c^{\sigma}\left(1+\overline{\Lambda}_c\right)C_{ct}^{\sigma}}{(1+\widetilde{\tau}_c)^{\sigma}A_{ct}}+\frac{\kappa_s^{\sigma}\left(1+\overline{\Lambda}_s\right)C_{st}^{\sigma}}{(1+\widetilde{\tau}_s)^{\sigma}A_{st}}}\right]\frac{C_{st}}{B_{st}}d\ln B_{st} \\
\approx\ & \frac{s_{gt}}{\psi}\left[\varepsilon+\frac{(\sigma-\varepsilon)\kappa_c^{\sigma}C_{ct}^{\sigma-1}}{(1+\widetilde{\tau}_c)^{\sigma-1}C_{ft}^{\sigma-1}}\left(1-\frac{(1+\widetilde{\tau}_s)\left(1+\overline{\Lambda}_c\right)A_{st}C_{ct}}{(1+\widetilde{\tau}_c)\left(1+\overline{\Lambda}_s\right)A_{ct}C_{st}}\right)\right]\frac{\frac{\kappa_s^{\sigma}\left(1+\overline{\Lambda}_s\right)C_{st}^{\sigma}}{(1+\widetilde{\tau}_s)^{\sigma}A_{st}}\frac{C_{st}}{B_{st}}d\ln B_{st}}{\frac{\kappa_c^{\sigma}\left(1+\overline{\Lambda}_c\right)C_{ct}^{\sigma}}{(1+\widetilde{\tau}_c)^{\sigma}A_{ct}}+\frac{\kappa_s^{\sigma}\left(1+\overline{\Lambda}_s\right)C_{st}^{\sigma}}{(1+\widetilde{\tau}_s)^{\sigma}A_{st}}}.
\end{aligned}
$$

s_{ft} increases following the shale gas boom when $\chi=1$ provided that $\frac{(1+\overline{\Lambda}_s)C_{st}}{(1+\widetilde{\tau}_s)A_{st}} \geq \frac{(1+\overline{\Lambda}_c)C_{ct}}{(1+\widetilde{\tau}_c)A_{ct}}$ (or more generally as long as $(\sigma-\varepsilon)\left(1-\frac{(1+\widetilde{\tau}_s)(1+\overline{\Lambda}_c)A_{st}C_{ct}}{(1+\widetilde{\tau}_c)(1+\overline{\Lambda}_s)A_{ct}C_{st}}\right)$ is not too negative).

B.4 Complementarity between Natural Gas and Renewables

In this Appendix, we present and solve the model sketched in Section 5.3. To capture the notion of greater complementarity between natural gas and renewables, we now assume that energy is produced according to:

$$
E_t=\left(\kappa_c E_{ct}^{\frac{\varepsilon-1}{\varepsilon}}+\kappa_s E_{sat}^{\frac{\varepsilon-1}{\varepsilon}}+\kappa_g E_{gat}^{\frac{\varepsilon-1}{\varepsilon}}+\kappa_b E_{bt}^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}. \tag{B-39}
$$

E_{bt} is a hybrid energy which uses gas (sb) and green (gb) as inputs according to the Cobb-Douglas technology $E_{bt}=E_{sbt}^{1-\alpha}E_{gbt}^{\alpha}$. E_{sat} and E_{gat} represent natural gas and green technologies which are used "alone" (e.g., nuclear power).

In the following, we solve for the competitive equilibrium and derive the effect of the natural gas boom on emissions. Then, we solve for the dynamic equilibrium and derive the effect of the boom on innovation. The effect is theoretically ambiguous, but we quantify the model and show that for reasonable parameter values, the shale gas boom still decreases green innovation.

B.4.1 Competitive Equilibrium

To solve for the competitive equilibrium, we follow the same strategy as for the baseline model. The Cobb-Douglas structure within the bridge technology implies that the effective productivity of the bridge technology is given by

$$C_{bt} \equiv \frac{C_{st}^{1-\alpha} C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}, \tag{B-40}$$

so that the price of the bridge technology is given by $p_{bt} = \frac{\gamma w}{C_{bt}}$. Total energy production is still given by $E_t = C_{Et} L_{Et}$ and the price of energy is $p_{Et} = \gamma w / C_{Et}$ with C_{Et} now given by

$$C_{Et} \equiv \left(\kappa_c^{\varepsilon} C_{ct}^{\varepsilon-1} + \kappa_s^{\varepsilon} C_{st}^{\varepsilon-1} + \kappa_g^{\varepsilon} C_{gt}^{\varepsilon-1} + \kappa_b^{\varepsilon} C_{bt}^{\varepsilon-1}\right)^{\frac{1}{\varepsilon-1}}. \tag{B-41}$$

Similarly to (12), we get

$$\begin{aligned} E_{c,t} &= \kappa_c^{\varepsilon}\left(\frac{C_{ct}}{C_{Et}}\right)^{\varepsilon} E_t \; ; E_{sa,t} = \kappa_s^{\varepsilon}\left(\frac{C_{st}}{C_{Et}}\right)^{\varepsilon} E_t, \\ E_{ga,t} &= \kappa_g^{\varepsilon}\left(\frac{C_{gt}}{C_{Et}}\right)^{\varepsilon} E_t \; ; E_{b,t} = \kappa_b^{\varepsilon}\left(\frac{C_{bt}}{C_{Et}}\right)^{\varepsilon} E_t. \end{aligned}$$

Using that the bridge technology is produced in a Cobb-Douglas way, we have $p_{st}E_{sbt} = (1-\alpha)\, p_{bt}E_{bt}$ and $p_{gt}E_{gbt} = \alpha p_{bt}E_{bt}$ so that

$$E_{gb,t} = \frac{\alpha C_{gt}}{C_{bt}} E_{bt} \text{ and } E_{sb,t} = \frac{(1-\alpha)\, C_{st}}{C_{bt}} E_{bt}.$$

The aggregate clean and natural gas energy productions are then respectively equal to:

$$E_{gt} = \left(\kappa_g^{\varepsilon}\left(\frac{C_{gt}}{C_{Et}}\right)^{\varepsilon} + \frac{\alpha C_{gt}}{C_{bt}} \kappa_b^{\varepsilon}\left(\frac{C_{bt}}{C_{Et}}\right)^{\varepsilon}\right) E_t$$

$$\text{and } E_{s,t} = \left(\kappa_s^{\varepsilon}\left(\frac{C_{st}}{C_{Et}}\right)^{\varepsilon} + \frac{(1-\alpha)\, C_{st}}{C_{bt}} \kappa_b^{\varepsilon}\left(\frac{C_{bt}}{C_{Et}}\right)^{\varepsilon}\right) E_t.$$

Total emissions are given by $P_t = \xi_{Et} E_t$, where the emission rate is now:

$$\xi_{Et} = \xi_c \kappa_c^{\varepsilon}\left(\frac{C_{ct}}{C_{Et}}\right)^{\varepsilon} + \xi_{st}\left(\kappa_s^{\varepsilon}\left(\frac{C_{st}}{C_{Et}}\right)^{\varepsilon} + \frac{(1-\alpha)\, C_{st}}{C_{bt}} \kappa_b^{\varepsilon}\left(\frac{C_{bt}}{C_{Et}}\right)^{\varepsilon}\right).$$

Labor allocation is still given by (14).

B.4.2 Emission Effects of a Natural Gas Boom

As before we derive the effect of a natural gas boom on emissions (at a constant level of extraction technologies). We get that:

$$\frac{\partial \ln P}{\partial \ln B_s} = \frac{\partial \ln \xi_E}{\partial \ln C_s}\frac{\partial \ln C_s}{\partial \ln B_s} + \frac{\partial \ln E}{\partial \ln C_s}\frac{\partial \ln C_s}{\partial \ln B_s}.$$

$\frac{\partial \ln \xi_E}{\partial \ln C_s}$ represents the substitution effect and is given by:

$$\begin{aligned}\frac{\partial \ln \xi_E}{\partial \ln C_s} &= -\varepsilon \frac{P_c}{P}\frac{\partial \ln C_{Et}}{\partial \ln C_s} + \varepsilon \frac{P_{sa}}{P}\left(1 - \frac{\partial \ln C_E}{\partial \ln C_s}\right) + \frac{P_{sb}}{P}\left(\frac{\partial \ln\left(C_s C_b^{\varepsilon-1}\right)}{\partial \ln C_s} - \varepsilon \frac{\partial \ln C_E}{\partial \ln C_s}\right)\\ &= \varepsilon \frac{P_{sa}}{P} + (1 + (1-\alpha)(\varepsilon - 1))\frac{P_{sb}}{P} - \varepsilon \frac{\partial \ln C_E}{\partial \ln C_s};\end{aligned}$$

where

$$\frac{\partial \ln C_E}{\partial \ln C_s} = \frac{\kappa_s^{\varepsilon} C_s^{\varepsilon-1}}{C_E^{\varepsilon-1}} + (1-\alpha)\frac{\kappa_b^{\varepsilon} C_b^{\varepsilon-1}}{C_E^{\varepsilon-1}} = \frac{p_{st}E_{sat}}{p_{Et}E_t} + \frac{(1-\alpha)\,p_{bt}E_{bt}}{p_{Et}E_t} = \frac{p_{st}E_{st}}{p_{Et}E_t} \equiv \Theta_s,$$

where as before Θ_s denotes the revenue share of natural gas in the energy sector. We then get that the substitution effect is determined by:

$$\frac{\partial \ln \xi_E}{\partial \ln C_s} = \left(\varepsilon \frac{P_{sa}}{P} + (1 + (1-\alpha)(\varepsilon - 1))\frac{P_{sb}}{P} - \varepsilon \Theta_s\right),$$

which, for given revenue share and pollution share of natural gas, is lower than in the no bridge technology case. Since $\frac{\partial \ln C_E}{\partial \ln C_s} = \Theta_s$, the scale effect is still determined by:

$$\frac{\partial \ln E_t}{\partial \ln C_{st}} = \frac{\partial \ln C_{Et} L_{Et}}{\partial \ln C_{st}} = \Theta_s\left(\lambda + (1-\lambda)\Omega_E\right).$$

Therefore, one gets:

$$\frac{\partial \ln P}{\partial \ln B_s} = \frac{C_s}{B_s}\left(\varepsilon\left(\frac{P_{sa} + \frac{(1+(1-\alpha)(\varepsilon-1))}{\varepsilon}P_{sb}}{P} - \Theta_s\right) + \Theta_s\left(\lambda + (1-\lambda)\Omega_E\right)\right),$$

which is lower than in the baseline case for given observables (Θ_s, Ω_E and P_s/P). We get:

Proposition B.1 *When there is some degree of complementarity between natural gas and the green technology, a natural gas boom leads to a larger reduction in emissions.*

Intuitively, an improvement in the natural gas technology improves the bridge technology which is less polluting than natural gas alone, this tends to make the substitution effect more negative than without the bridge technology.

B.4.3 Innovation Effects of a Natural Gas Boom

We keep the same structure for innovation as in the baseline model, so that again the direction of innovation depends on relative profits from innovating in the various technologies. We now have that expected profits from clean innovations obey:

$$\Pi_{gt} = \eta s_{gt}^{-\psi}\left(1-\frac{1}{\gamma}\right)\left(p_{gt}E_{gat} + p_{gt}E_{gbt}\right),$$

and expected profits from fossil-fuel innovations obey:

$$\Pi_{ft} = \eta s_{ft}^{-\psi}\left(1-\frac{1}{\gamma}\right)\left(\frac{C_c}{A_c}p_{ct}E_{ct} + \frac{C_{st}}{A_{st}}\left(p_{st}E_{sat} + p_{st}E_{sbt}\right)\right).$$

The revenue share of green technologies alone is given by:

$$\frac{p_{gt}E_{gat}}{p_{Et}E_t} = \frac{\kappa_g^{\varepsilon}C_{gt}^{\varepsilon-1}}{C_{Et}^{\varepsilon-1}}$$

and the revenue share of green technologies within the bridge technology is given by:

$$\frac{p_{gt}E_{gbt}}{p_{Et}E_t} = \frac{\alpha p_{bt}E_{bt}}{p_{Et}E_t} = \frac{\alpha\kappa_b^{\varepsilon}C_{bt}^{\varepsilon-1}}{C_{Et}^{\varepsilon-1}} = \frac{\alpha\kappa_b^{\varepsilon}\left(\frac{C_{st}^{1-\alpha}C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right)^{\varepsilon-1}}{C_{Et}^{\varepsilon-1}}.$$

With similar expressions for the revenue shares associated with natural gas, and using that $\Pi_{gt} = \Pi_{ft}$ in equilibrium, one gets:

$$\left(\frac{s_{ft}}{s_{gt}}\right)^{\psi} = \frac{\frac{C_c}{A_c}\kappa_c^{\varepsilon}C_{ct}^{\varepsilon-1} + \frac{C_{st}}{A_{st}}\left(\kappa_s^{\varepsilon}C_{st}^{\varepsilon-1} + (1-\alpha)\kappa_b^{\varepsilon}\left(\frac{C_{st}^{1-\alpha}C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right)^{\varepsilon-1}\right)}{\kappa_g^{\varepsilon}C_{gt}^{\varepsilon-1} + \alpha\kappa_b^{\varepsilon}\left(\frac{C_{st}^{1-\alpha}C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right)^{\varepsilon-1}}.$$

To look at the effect of the natural gas boom on the innovation allocation at $t=1$, we log differentiate the right-hand side of this expression with respect to B_s. If that derivative is positive (and $\ln\gamma$ is sufficiently small that the innovation allocation is unique), then a natural gas boom leads to an increase in fossil-fuel innovations and a decline in green innovations. We get:

$$\frac{\partial \ln\left(\frac{s_{ft}}{s_{gt}}\right)^{\psi}}{\partial \ln B_{st}} = \left[\frac{\frac{C_{st}}{A_{st}}\left[\kappa_s^{\varepsilon}C_{st}^{\varepsilon-1}\varepsilon + (1-\alpha)\kappa_b^{\varepsilon}\left(\frac{C_{st}^{1-\alpha}C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right)^{\varepsilon-1}((\varepsilon-1)(1-\alpha)+1)\right]}{\frac{C_c}{A_c}\kappa_c^{\varepsilon}C_{ct}^{\varepsilon-1} + \frac{C_{st}}{A_{st}}\left(\kappa_s^{\varepsilon}C_{st}^{\varepsilon-1} + (1-\alpha)\kappa_b^{\varepsilon}\left(\frac{C_{st}^{1-\alpha}C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right)^{\varepsilon-1}\right)} - \frac{(\varepsilon-1)(1-\alpha)\alpha\kappa_b^{\varepsilon}\left(\frac{C_{st}^{1-\alpha}C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right)^{\varepsilon-1}}{\kappa_g^{\varepsilon}C_{gt}^{\varepsilon-1} + \alpha\kappa_b^{\varepsilon}\left(\frac{C_{st}^{1-\alpha}C_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\alpha^{\alpha}}\right)^{\varepsilon-1}}\right]\frac{\partial \ln C_{st}}{\partial \ln B_{st}}.$$

This expression is not necessarily positive, so that the natural gas boom could lead to an increase in green innovation. Intuitively, the natural gas boom leads to an increase in the hybrid share, which can in return boost innovation. This effect may dominate when the coal technology is very advanced relative to the natural gas and hybrid technologies (C_{ct} is large so that the first term is arbitrarily small): in that case, since most of the revenues of the fossil-fuel power plant sector come from coal, the natural gas boom has a small effect on the incentive to introduce fossil-fuel innovations.

Therefore, one gets

$$\frac{d\ln}{d\ln B_s}\frac{s_{ft}}{s_{gt}} \approx \frac{1}{\psi}\left[\frac{\frac{C_{st}}{A_{st}}\left[\varepsilon\frac{E_{sa}}{E_s}+((\varepsilon-1)(1-\alpha)+1)\frac{E_{sb}}{E_s}\right]\Theta_s}{\frac{C_c}{A_c}\Theta_c+\frac{C_{st}}{A_{st}}\Theta_s}-\frac{(\varepsilon-1)(1-\alpha)E_{ga}}{E_g}\right]\frac{\partial\ln C_{st}}{\partial\ln B_{st}},$$

where the approximation comes from the fact that we ignore the dependence of the A's on the current innovation allocation. In contrast, without the hybrid technology, the corresponding expression is

$$\frac{d\ln}{d\ln B_s}\frac{s_{ft}}{s_{gt}}\bigg|_{\kappa_b=0} \approx \frac{1}{\psi}\frac{\frac{C_{st}}{A_{st}}\Theta_{st}}{\frac{C_c}{A_c}\Theta_{ct}+\frac{C_{st}}{A_{st}}\Theta_{st}}\frac{\partial\ln C_{st}}{\partial\ln B_{st}},$$

which is larger for given observables (the revenue shares). However, rearranging terms, we get that the natural gas boom still increases fossil-fuel innovation provided that:

$$\begin{aligned} & \varepsilon\frac{\kappa_s^\varepsilon\kappa_g^\varepsilon C_{st}^{\varepsilon-1}C_{gt}^{\varepsilon-1}}{\kappa_b^\varepsilon C_{bt}^{\varepsilon-1}}+((\varepsilon-1)(1-\alpha)+1)(1-\alpha)\kappa_g^\varepsilon C_{gt}^{\varepsilon-1} \\ & +[\varepsilon-(\varepsilon-1)(1-\alpha)]\alpha\kappa_s^\varepsilon C_{st}^{\varepsilon-1}+\alpha(1-\alpha)\kappa_b^\varepsilon C_{bt}^{\varepsilon-1} \\ > \quad & (\varepsilon-1)(1-\alpha)\alpha\frac{A_{st}}{C_{st}}\frac{C_c}{A_c}\kappa_c^\varepsilon C_{ct}^{\varepsilon-1}. \end{aligned} \tag{B-42}$$

We then obtain:

Proposition B.2 *When there is a hybrid technology, the increase in fossil-fuel innovation following the natural gas boom is smaller, though it is still positive when (B-42) is satisfied.*

Intuitively, a drop in the price of natural gas may incentivize clean innovation through its effect on the hybrid technology. This counteracting force may dominate if the natural gas and the hybrid shares are small compared to the coal share. In that case, the natural gas boom has little impact on the returns to fossil-fuel innovation (which are dominated by coal), but some positive effect on the returns to clean innovation (through the hybrid technology). For this effect to dominate, however, the coal share needs to be very large (as stipulated in (B-42)) and we now show that for reasonable parameter values, this does not occur so that the natural gas boom still reduces green innovation.

B.4.4 Quantification

This section presents a quantification of the model with complementarity in order to investigate whether condition (B-42) holds in the data. To map (B-39) to the data, we assume that all solar and wind generation is in the hybrid nest E_{gbt}, whereas all other green base period generation (e.g., nuclear, biomass) is in the stand-alone green category E_{gat}. To begin, we solve for the Cobb-Douglas exponent α based on the equilibrium price of the renewable-gas bundle:

$$p_{bt} = \frac{p_{st}^{1-\alpha} p_{gt}^{\alpha}}{(1-\alpha)^{1-\alpha}\, \alpha^{\alpha}}. \tag{B-43}$$

The University of Chicago Energy Policy Institute (EPIC) has produced recent estimates of the levelized costs of renewables *backed up by natural gas* for both (onshore) wind (p_{bt} = \$54/MWh) and solar photovoltaic energy (p_{bt} = \$61/MWh) (Greenstone and Nath 2021). The corresponding EIA's Annual Energy Report posits levelized costs *without backup* for onshore wind (p_{gt} =\$34/MWh), and for solar (p_{gt} =\$33/MWh).[1] Combined with EPIC's estimate for the levelized cost of natural gas generation (p_{st} = \$42/MWh), we can use (B-43) to back out the implied value of α for wind generation ($\widehat{\alpha} = 0.8457$) and solar ($\widehat{\alpha} = 0.7561$). We take the generation-weighted average between wind and solar for 2011, yielding $\alpha = 0.8446$.

Next, in order to calibrate the distribution parameters in (B-39), we must specify the remaining base year quantities. For natural gas, we proxy stand-alone generation E_{sat} through combined-cycle plant output, and treat all combustion or steam engine gas generation as in the nest with renewables (E_{sbt}). This distinction is motivated by the EIA's observation that combined-cycle plants are "often used as baseload generation" whereas combustion and steam turbines are "generally only run during hours when electricity demand is high."[2]Importantly, this approach almost surely overstates the amount of natural gas that is complementary to renewables since many areas may rely on gas peaker plants to deal with demand fluctuations even in the absence of renewable generation. In 2011, combined cycle accounted for 82% of utility scale net generation from natural gas, with combustion and steam turbines accounting for the remaining 18%.[3]

Applying these assumptions to our base period data (2006-10) and using $E_{b0} = E_{gb0}^{\alpha} E_{sb0}^{1-\alpha}$ to compute the initial E_{b0} (equal to 0.3343 tril. KWh) enables us to back out

[1] For consistency we utilize levelized cost estimates based on the same year assumptions to calibrate α.

[2] U.S. Energy Information Administration, "Today in Energy," Dec. 18, 2017. URL (accessed September 2021): https://www.eia.gov/todayinenergy/detail.php?id=34172#tab1.

[3] EIA "Electricity Power Monthly" Table 1.7.C., Utility Scale Facility Net Generation from Natural Gas by Technology: Total (All Sectors), 2011-October 2021. URL (accessed Septembre 2021): https://www.eia.gov/electricity/monthly/epm_table_grapher.php?t=table_1_07_c.

the $\kappa's$ in (B-39) via the standard profit-maximization conditions,

$$\frac{p_{c0}}{p_{s0}}=\frac{\kappa_c E_{c0}^{-\frac{1}{\varepsilon}}}{\kappa_s E_{sa0}^{-\frac{1}{\varepsilon}}},\ \frac{p_{g0}}{p_{b0}}=\frac{\kappa_g E_{ga0}^{-\frac{1}{\varepsilon}}}{\kappa_b E_{b0}^{-\frac{1}{\varepsilon}}},\ \frac{p_{c0}}{p_{b0}}=\frac{\kappa_c E_{c0}^{-\frac{1}{\varepsilon}}}{\kappa_b E_{b0}^{-\frac{1}{\varepsilon}}},$$

and the condition that $1=\kappa_c+\kappa_s+\kappa_b+\kappa_g$. We note that, in order to ensure time period consistency, we back out the price of the hybrid bundle relevant for the base period (2006-10) based on (B-43) instead of using the aforementioned EPIC estimates. We also note that we now assume the within-fossil nest elasticity of substitution value from the extended model $\sigma=2$ as value for ε since intermittency concerns that were motivating driving the lower benchmark value of $\varepsilon=$1.8561 in the benchmark are now explicitly accounted for. However, the results below are completely robust to using $\varepsilon=1.8561$ here as well. Solving these four equations in four unknowns yields $\kappa_c=0.25$, $\kappa_s=0.30$, $\kappa_b=0.14$, and $\kappa_g=0.31$.

In order to evaluate (B-42), it remains to solve for initial technology levels consistent with equilibrium in the modified model. We do so by solving a modified version of benchmark system of equations (A-11), with equation (B-40) for C_{b0} added and with (B-41) replacing the benchmark condition for C_{E0}. As inputs to this computation, we also calculate the modified model's E_0 from (B-39), p_{E0} based on the equilibrium condition that $p_{ct}=\kappa_c E_{ct}^{-\frac{1}{\varepsilon}} p_{Et} E_t^{\frac{1}{\varepsilon}}$, and $\widetilde{A}_{E0}$ from (A-10) which remains valid. The results are very similar to the benchmark:

$A_{g,0}$	$A_{c,0}$	$A_{s,0}$	$B_{c,0}$	$B_{s,0}$	$C_{b,0}$	$C_{E,0}$	$A_{P,0}$	w_0	L_{E0}
100.25	461.69	449.66	337.14	119.44	152.95	32.69	4.7318e+03	6.8676e+03	1.258%

Finally, we evaluate the innovation inequality (B-42), yielding:

$$594.7 >>> 2.9.$$

These results imply that condition (B-42) holds easily, suggesting that the impact of the shale gas boom is to increase incentives for fossil innovation even after accounting for the possibility of complementarity between renewables and natural gas.

References

Greenstone, M. and I. Nath (2021). "U.S. Energy & Climate Roadmap". In: Energy Policy Instiute at the University of Chicago.