Can Deficits Finance Themselves?*

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Abstract

We ask how fiscal deficits are financed in environments with two key features: (i) nominal rigidity, and (ii) a violation of Ricardian equivalence due to finite lives or liquidity constraints. In such environments, deficits can contribute to their own financing through two channels: a boom in real economic activity, which expands the tax base; and a surge in inflation, which erodes the real value of nominal government debt. Our main theoretical result establishes that this mechanism becomes more potent as fiscal adjustment is delayed, leading to full self-financing in the limit. In this scenario, the government can run a deficit today, refrain from tax hikes or spending cuts in the future, and still see its debt converge back to its initial level. We further demonstrate that a significant degree of self-financing is achievable when the theory is disciplined by empirical evidence on marginal propensities to consume, nominal rigidities, and the speed of fiscal adjustment.

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1 Introduction

Suppose the government runs a deficit today in order to stimulate aggregate demand. Suppose further that there is no "free lunch" of the type considered in the recent "$r < g$" literature (Blanchard, 2019; Reis, 2022); i.e., the government's net cost of borrowing is positive. What does the government have to do in order to make sure that public debt eventually returns back to its initial level?

The most conventional answer is *fiscal adjustment*: sooner or later, the government must adopt a package of tax hikes and/or spending cuts in order to pay down the accumulated debt. In this paper, we investigate a different margin—what we refer to as the *self-financing* of fiscal deficits.

The basic idea is simple. Insofar as a deficit triggers a boom, it can contribute to its own financing via two complementary channels: by expanding the tax base, which helps generate additional tax revenue without any adjustment in tax rates; and by triggering inflation, which helps reduce the real value of the government's nominal liabilities (i.e., debt erosion). Our contribution is to shed light on the theoretical properties and the quantitative potency of such self-financing .

We first verify that some self-financing obtains naturally in environments that combine two key features: a failure of Ricardian equivalence, so that deficits can stimulate aggregate demand; and nominal rigidity, so that aggregate demand can drive real economic activity and thereby also inflation. We next show that, in such environments, the degree of self-financing increases as the fiscal adjustment is delayed. Intuitively, the further the adjustment is delayed, the larger the boom induced by the initial deficit, thus raising the potency of both of our two self-financing channels. Pushing this logic to its limit, we obtain our headline result: as the fiscal adjustment is delayed more and more, the tax hike needed to bring debt back to trend vanishes. In other words, the initial deficit finances itself through a combination higher output and higher inflation. Our contribution is then completed by evaluating the practical relevance of this lesson.

Environment. Our baseline model is kept purposefully close to the New Keynesian textbook (e.g., see Galí, 2008; Woodford, 2003b). The supply block is exactly the same, and boils down to the usual New Keynesian Phillips curve (NKPC). What changes is the demand block, which now consists of overlapping generations of perpetual-youth consumers (à la Blanchard, 1985), with survival probability $\omega \in (0,1]$. When $\omega = 1$, the model reduces to the standard permanent-income representative-agent (PIH-RANK) benchmark. When instead $\omega < 1$, the model shares two key properties with quantitative heterogeneous-agent (HANK) models: (i) consumers discount future disposable income more heavily than in the PIH benchmark; and (ii) they have a larger short-run propensity to consume (MPC). As will become clear, our lessons derive not from the OLG structure per se, but rather from these two more general and empirically relevant properties of consumer demand.

Fiscal policy is represented by a rule for how tax revenue (and thereby the primary surplus) responds to changes in aggregate income and in public debt. The dependence on aggregate income captures the tax base channel: as output goes up, so automatically does tax revenue. The dependence on public debt, on the other hand, captures the speed of fiscal adjustment, or equivalently the horizon at which the government commits to hike taxes as needed to bring debt back to trend. Our analysis will reveal how this policy parameter is a key determinant of the degree of self-financing.

Finally, monetary policy is parameterized by the cyclicality of (expected) real interest rate rates. For the bulk of our analysis, we concentrate on the case in which the monetary authority keeps (expected) real rates constant. This can be interpreted as a "neutral" monetary policy that neither offsets the fiscal stimulus by discouraging private spending, nor increases fiscal space by allowing the government's real cost of borrowing to fall. This special case is useful for two reasons. First, it sharpens the theoretical analysis by isolating the effects of the fiscal stimulus on aggregate demand. Second, as we will discuss later, it is also arguably empirically relevant.

The self-financing result. Starting from steady state, we shock the economy with a one-off, deficit-financed lump-sum transfer to households. We consider this experiment as a proxy for the "stimulus check" policies recently seen in the U.S. We then ask: How much of this deficit shock will be self-financed in equilibrium via the tax base and inflation channels? Or, conversely, how large are the tax hikes needed, sooner or later, to bring public debt back to its trend?

Our first observation is that some self-financing is possible if and only if $\omega < 1$. When $\omega = 1$ (i.e., PIH-RANK), Ricardian equivalence holds, and thus aggregate demand is invariant to the initial deficit shock and the timing of the subsequent fiscal adjustment. It follows that both self-financing channels are idle—the date-0 tax cut is fully financed with later tax hikes. When instead $\omega < 1$, Ricardian equivalence fails, and the deficit—which now represents a net transfer from future generations to current generations or, less literally, helps some consumers overcome liquidity constraints—stimulates aggregate demand. With nominal rigidities, this increase in aggregate demand translates to an increase in real income and, via the NKPC, to an increase in the nominal price level. The two self-financing channels are thus operative: the real boom leads to higher tax revenues even without changes in tax rates; and the accompanying inflation lowers the real government debt burden.

The main question of the paper is how large such self-financing can be. We let ν denote the overall degree of self-financing; i.e., the fraction of the initial deficit that is financed via the tax base and inflation channels. The residual, $1-\nu$, then equals the discounted present value of the future tax hikes needed for each dollar of current deficit. Our headline result is that, if $\omega < 1$, then ν converges monotonically towards 1 as fiscal adjustment is pushed further into the future. In other words, delaying the fiscal adjustment reduces its magnitude, achieving *full* self-financing in the limit. Furthermore, the stickier the prices, the larger the share of self-financing through tax base expansion, and *vice-versa*.

The economics of self-financing. Why does the strength of self-financing increase with the delay in fiscal adjustment, and why is the limit one of *full* self-financing?

To develop intuition, we consider an analogy with a simple two-period economy ($t \in \{0,1\}$). Here, the first period represents the "short run", when the effects of the fiscal stimulus play out; and second period represents the "long run," or the return to steady state. The government pays out transfers of size $\varepsilon > 0$ at $t = 0$, and at $t = 1$ hikes taxes as much as necessary to bring total government debt down to its initial level. Aggregate consumer demand at $t = 0$ is proportional to concurrent disposable income; in particular, consumers are myopic and do not respond to any possible $t = 1$ tax hike. Prices at $t = 0$ are rigid, so output is fully demand-determined. The date-0 boom contributes to financing of the initial deficit via our tax base channel: for any dollar of additional aggregate income, tax revenue is assumed to increase automatically by $\tau_y \in (0,1)$ dollars.

Let y denote aggregate income at $t = 0$, normalized so that $y = 0$ when $\varepsilon = 0$. By applying simple, static, Keynesian cross logic, we infer that

$$y = \frac{\text{MPC}}{1 - \text{MPC}(1-\tau_y)} \times \varepsilon,$$

where MPC $\in (0,1)$ is the household's marginal propensity to consume out of income. The output boom generates additional tax revenue of $\tau_y \times y$, so the degree of self-financing is given by

$$\nu \equiv \frac{\tau_y \times y}{\varepsilon} = \frac{\tau_y \times \text{MPC}}{1 - \text{MPC} \times (1-\tau_y)}.$$

We note two key properties of this expression. First, self-financing ν is strictly increasing in the MPC. Intuitively, a larger MPC maps to both (i) a larger partial equilibrium effect of the transfer on consumer spending (numerator) and (ii) stronger general equilibrium amplification (denominator). Second, $\nu \to 1$ if and only if MPC $\to 1$. In this limit, the partial equilibrium response is 1 and the Keynesian multiplier is $\frac{1}{1-(1-\tau_y)} = \frac{1}{\tau_y}$, so every dollar of initial transfer ends up generating exactly one dollar of additional tax revenue—i.e., full self-financing obtains.

Our main insight is that, if $\omega < 1$, then our fully-fledged, micro-founded, infinite-horizon economy closely echoes these intuitions. To see the connection, translate "$t = 0$" to an initial time interval in our model that is both sufficiently long and sufficiently distant from the eventual tax hike (i.e., the "short run"); "$t = 1$" to the time of the delayed tax hike (i.e., the "long run"); and "MPC" as the *cumulative* MPC over the short run. Two properties of our economy then complete the connection. The first property is *discounting*: consumers discount any future income, and hence also any future taxes, at a higher rate than the interest rate faced by the government. This implies that tax hikes in the far-ahead future have little effect on consumer demand in the short run, echoing the lack of feedback from date-1 taxes to date-0 demand in the two-period economy. The second property is *front-loading*: consumers spend both the initial fiscal transfer as well as any additional income generated in general equilibrium relatively quickly. This implies that, when fiscal adjustment is sufficiently delayed, the cumulative MPC over the short run approaches 1. By the same token, the cumulative fiscal multiplier approaches $\frac{1}{\tau_y}$, and so the degree of self-financing ν approaches 1, just as in the above analogy.

While the above discussion has assumed rigid prices, the result extends to any degree of nominal rigidity. With an upward-sloping aggregate Phillips curve, the stimulus-led boom is now accompanied by a surge in inflation and an erosion in the real debt burden. This allows both channels of self-financing to operate, but does not alter the monotonicity and limit properties of ν. The only material change, naturally, is the relative contribution of the two channels: as prices become more flexible, more self-financing occurs through debt erosion.

Theoretical generality. We illustrate the robustness of the logic behind our self-financing result with a number of model extensions.

Our first two extensions generalize the demand block. The first one accommodates a more flexible specification for consumer demand. Echoing the intuition above, the key conditions for self-financing to occur are discounting and front-loading: consumers need to respond relatively little to expectations of future taxes, so that demand is indeed stimulated; and they need to spend their transfers relatively quickly, so that the resulting boom plays out fast. The second extension accommodates investment. Under standard assumptions about firm behavior, the Keynesian cross logic behind our result continues to apply, and our self-financing result goes through almost unchanged.

The remaining extensions generalize our assumptions about fiscal and monetary policy. We first ask what happens when the monetary authority moves away from our "neutral" benchmark of fixed real rates. If nominal rates rise less than inflation, then real rates fall, so households front-load spending further and the deficit-driven boom plays out even faster. Conversely, if real rates are increased, then the deficit-driven boom is delayed, and so is convergence to the self-financing limit. If the monetary response is too aggressive, full self-financing ceases to be possible. On the fiscal side, we show that our results generalize with relatively little change to government purchases (rather than transfers) and to distortionary (rather than lump-sum) future fiscal adjustment.

Practical relevance. Is self-financing merely a theoretical property, or is it possible in practice? And if so, where will self-financing come from—inflation or the tax base?

To address these questions, we study the propagation of deficit shocks in a quantitative version of our model, disciplined by evidence on its crucial ingredients: (i) the deviation from the permanent-income benchmark; (ii) the speed of fiscal adjustment; (iii) the degree of nominal rigidity; and (iv) the monetary policy reaction. For (iv), we look at time-series evidence on the response of interest rates to identified fiscal shocks (e.g., from Ramey, 2011). For (iii), we draw from ample empirical evidence on the slope of the Phillips curve. For (ii), we turn to recent work that measures the speed of fiscal adjustment in practice (Galí et al., 2007; Bianchi and Melosi, 2017; Auclert and Rognlie, 2020). Finally, for (i), we review the available microeconomic evidence on the response of household consumption to income shocks (Fagereng et al., 2021), including at relatively far-out horizons. We then calibrate an extended version of our baseline model—one that adds a margin of hand-to-mouth consumers—to match this evidence. This extended, yet still tractable and parsimonious model, is not only capable of matching the evidence, but also serves as a close proxy for consumer behavior in fully-fledged HANK model environments (Auclert et al., 2023; Wolf, 2021).[1]

In this empirically disciplined model, we find that one-off deficit shocks—i.e., "stimulus checks"—are indeed largely self-financed. Intuitively, household MPCs are sufficiently front-loaded, relative to fiscal adjustment, to ensure that the self-financing boom can play out fast. Furthermore, given the flatness of our NKPC, this self-financing predominantly occurs through tax base expansion, with only limited inflationary pressure. We then conclude by clarifying how these conclusions depend on our assumptions about the policy response and the supply side. If there is less slack in the economy, in the sense of a steeper NKPC, then the fiscal stimulus necessarily generates a bigger surge in inflation. Holding real rates constant, this changes the split between the two forms of self-financing, but without reducing its overall potency. However, if this surge in inflation instead translates—via a hawkish monetary policy response—into a hike in real interest rates, then there will be less scope for a Keynesian boom and, consequently, less room for self-financing overall.

Literature. Our analysis relates and contributes to several strands of literature. First, we offer a different perspective on fiscal space than the "$r < g$" literature (Blanchard, 2019; Mehrotra and Sergeyev, 2021; Reis, 2022). Similar to this literature, our results suggest that deficits can be financed without future tax hikes. However, unlike that literature, we do not require the real interest rate on government debt to be lower than the economy's real growth rate, nor do we necessitate that the government collects seigniorage in the form of a convenience yield on its debt (Angeletos et al., 2023). Instead, we emphasize how fiscal deficits can contribute to their own financing by triggering a Keynesian boom. Mian et al. (2022) touch upon both these issues ("$r < g$" and deficit-driven Keynesian booms), but emphasize a self-financing mechanism different from ours: they focus on how additional debt issuance can reduce the real interest rate on government debt by triggering inflation along the ZLB.

Second, we offer a different rationale for why deficits can finance themselves than that found in the Fiscal Theory of the Price Level (FTPL). Like its original, flexible-price version (Leeper, 1991; Sims, 1994; Woodford, 1995; Bassetto, 2002; Cochrane, 2005), the modern, sticky-price version of this theory (Cochrane, 2017, 2018, 2023) assumes a representative, infinitely-lived, fully rational consumer, simi-

[1] Our self-financing result depends on the consumer block of the model only through the matrix of "intertemporal" marginal propensities to consume, or iMPCs (Auclert et al., 2023). Since our model provides a close match to iMPCs in HANK, our conclusions are essentially unchanged in a quantitative HANK model, as shown in Online Supplement E.6.1.

larly to Barro (1974)'s classic on Ricardian equivalence (and thus the $\omega = 1$ case in our model). It then breaks Ricardian equivalence through the force of equilibrium selection: the Taylor principle is abandoned, opening the door to multiple self-fulfilling equilibria in aggregate demand, and selecting an equilibrium in which debt and deficits serve, in effect, as coordination devices. In our environment, this mechanism is never at play: the failure of Ricardian equivalence is grounded in finite horizons or liquidity constraints; the equilibrium studied is the conventional one; and our self-financing result is consistent with an "active" monetary authority and a "passive" fiscal authority.[2] Last but not least, we shift the focus from the inflation/debt erosion channel—the focal point of the FTPL literature—to the tax base channel, which in our quantitative explorations turns out to be dominant.[3]

Third, we add to the large literature on the effects of fiscal policy within the New Keynesian framework. Woodford (2011) and Christiano et al. (2011) are the classic references for fiscal multipliers in the PIH-RANK benchmark, while Galí et al. (2007), Kaplan et al. (2018), Hagedorn et al. (2019), Auclert et al. (2023), and Aguiar et al. (2023) study environments in which Ricardian equivalence fails; applications to the post-covid fiscal stimulus include Diamond et al. (2022) and Elenev et al. (2021). Financing through monetary accommodation is the focus of Galí (2020). Overall, it is well-understood within this literature that fiscal deficits can generate booms, thus contributing to their own financing. From this perspective, our paper's added value lies in (i) characterizing the determinants of self-financing; (ii) highlighting that self-financing is possible without monetary accommodation;[4] and (iii) showing that self-financing is complete as fiscal adjustment gets delayed more and more, with the cumulative fiscal multiplier converging to $\frac{1}{\tau_y}$ in the rigid-price, constant-real-rate case.

Finally, our take-home message echoes that of DeLong and Summers (2012). The paper highlights the tax base channel and develops high-level arithmetic on fiscal multipliers and self-financing. However, lacking modern micro-foundations, the paper does not illuminate how the degree of self-financing depends on primitives of the economic environment. Our paper addresses this gap by connecting the theory to relevant evidence and evaluating the quantitative potential for self-financing.

Outline. Sections 2 and 3 present our baseline model and characterize its equilibrium. Section 4 then develops our self-financing result and discusses the economics behind it. Various model extensions and quantitative explorations follow in Sections 5 and 6, respectively. Section 7 concludes. The Online Appendix and Online Supplement contain proofs as well as various additional results.

[2] By the same token, our self-financing result, unlike that of the FTPL, is also robust the kind of global-game-like perturbations used in Angeletos and Lian (2023) to remove the equilibrium indeterminacy of the New Keynesian model.

[3] This also distinguishes our paper from Kaplan et al. (2023), a recent contribution to the FTPL literature that allows for non-PIH consumers but assumes flexible prices, thus shutting Keynesian propagation and so our tax base channel.

[4] This differs from, for example, the environment studied in Denes et al. (2013), where self-financing occurs largely due to an effective monetary expansion, in the sense of lower real rates driven by higher inflation at the zero lower bound.

2 Model

For our main analysis we consider a perpetual-youth, overlapping-generations (OLG) version of the textbook New Keynesian model. Similarly to Del Negro et al. (2015), Farhi and Werning (2019), and Angeletos and Huo (2021), mortality risk (or finite lives) is a convenient proxy for liquidity frictions.[5] In our context, it breaks Ricardian Equivalence and thus lets fiscal policy—i.e., debt and deficits—affect aggregate demand. As will become clear, this departure from the permanent-income benchmark is central to our results. We will show later how the insights obtained from our baseline model extend to more general aggregate demand structures, including those found in the HANK literature.[6]

Throughout we study log-linearized dynamics in response to a surprise increase in fiscal deficits. We use uppercase variables to indicate levels; unless indicated otherwise, lowercase variables denote log-deviations from the economy's deterministic steady state. Time is discrete, indexed by $t \in \{0, 1, ...\}$.

2.1 Households

We index households by $i = (i_1, i_2)$, where $i_1 \in \{0, 1, \dots\}$ denotes their age and $i_2 \in [0, 1]$ their name. A household survives from one period to the next with probability $\omega \in (0, 1]$, so that $1 - \omega$ is the mortality rate. Whenever a household dies, it is replaced by a new household (with the same name i_2 but age reset to $i_1 = 0$). Households do not altruistically value the utility of the future households that replace them. Taking into account the mortality risk, the expected lifetime utility of any (alive) household i in period $t \in \{0, 1, \dots\}$ is therefore given by

$$\mathbb{E}_t \left[\sum_{k=0}^{\infty} (\beta\omega)^k \left[u(C_{i,t+k}) - v(L_{i,t+k}) \right] \right], \tag{1}$$

where $C_{i,t+k}$ and $L_{i,t+k}$ denote household i's consumption and labor supply in period $t + k$ (conditional on survival), and preferences take the standard form $u(C) \equiv \frac{C^{1-1/\sigma}-1}{1-\frac{1}{\sigma}}$ and $v(L) = \chi \frac{L^{1+\frac{1}{\varphi}}}{1+\frac{1}{\varphi}}$.

Households can save and borrow by trading an actuarially fair, risk-free nominal annuity. Conditional on survival, households enjoy a nominal rate of return equal to I_t/ω, where I_t is the nominal interest rate on government bonds. Households furthermore receive labor income and dividend income, given respectively by $W_t L_{i,t}$ and $Q_{i,t}$ (both in real terms), and pay taxes. The real tax payment $T_{i,t}$ depends on both the individual's income and aggregate fiscal conditions: i.e., $T_{i,t} = \mathscr{T}(Y_{i,t}, Z_t)$, where $Y_{i,t} \equiv W_t L_{i,t} + Q_{i,t}$ is the household's total real income, Z_t captures aggregate conditions (including outstanding government debt), and $\mathscr{T}$ is a function describing tax policy, to be specified later.

[5] Galí (2021) also studies an OLG version of the NK model, but with a different goal: to accommodate asset price bubbles.

[6] Also note that, consistent with Woodford (2003b) and Galí (2008), we will consider a "moneyless" economy (or, equivalently, an economy in which money earns the same interest as Treasury bonds). There is thus no seignorage and the channel in Sargent and Wallace (1981) will not help finance deficits.

Finally, old households are obliged to make contributions to a "social fund" whose proceeds are distributed to the newborn households; the role of this fund will be explained momentarily. All in all, the date-t budget constraint of household i is given as

$$A_{i,t+1} = \underbrace{\frac{I_t}{\omega}}_{\text{annuity}} \left(A_{i,t} + P_t \cdot \big(\underbrace{W_t L_{i,t} + Q_{i,t}}_{Y_{i,t}} - C_{i,t} - T_{i,t} + S_{i,t}\big)\right), \tag{2}$$

where $A_{i,t}$ denotes i's nominal saving at the beginning of date t, P_t is the date-t price level, and $S_{i,t}$ is the transfer from or contribution to the fund, with $S_{i,t} = S^{\text{new}} > 0$ for newborns and $S_{i,t} = S^{\text{old}} < 0$ for old households (and where $(1-\omega)S^{\text{new}} + \omega S^{\text{old}} = 0$, ensuring that the fund is balanced).

Compared to Blanchard (1985), the only novelty in our set-up is the social fund. We set $S^{\text{new}} = D^{ss}$ (and therefore $S^{\text{old}} = -\frac{1-\omega}{\omega} D^{ss}$), where D^{ss} is the real steady-state value of public debt (and so private wealth). The fund thus ensures that all cohorts, regardless of their age, enjoy the same wealth and hence consumption in steady state. This then in turn affords two simplifications. First, it simplifies aggregation when we log-linearize the model around its steady state, with every cohort equally weighted in aggregate demand. Second, it implies that the steady state of our model is invariant to both ω as well as the steady-state level of public debt, and hence is the same as its RANK counterpart. In particular, the fund guarantees—together with the annuities, which offset mortality risk—that the steady-state rate of interest is β^{-1} (thus "$r > g$"). The two models thus differ *only* in terms of how fiscal policy influences output gaps, isolating the mechanism that we are interested in.[7]

It remains to specify how household income is determined. First, we assume that all households receive identical shares of dividends. Second, we abstract from heterogeneity in labor supply. Specifically, we assume that labor supply is intermediated by labor unions.[8] Those unions demand identical hours worked from all households and bargain on behalf of those households, equalizing the (post-tax) real wage and the average marginal rate of substitution between consumption and labor supply; i.e., we have that

$$(1-\tau_y)W_t = \frac{\chi L_t^{\frac{1}{\varphi}}}{\int_0^1 C_{i,t}^{-1/\sigma} di}, \tag{3}$$

where τ_y is the (time-invariant) tax on labor income, to be discussed in detail in Section 2.3. Putting all the pieces together, we conclude that $Y_{i,t} = Y_t$ and $T_{i,t} = T_t$—in any given period, all households receive the same income and face the same taxes.

[7]Standard incomplete-market models predict that the steady-state natural rate of interest is depressed below β^{-1}, by an amount that decreases with the steady-state quantity of public debt (Aiyagari and McGrattan, 1998; Angeletos et al., 2023). This allows the government to collect a monopoly rent from the liquidity services of public debt, akin to seignorage in monetary models, but does not affect the logic of our results.

[8]This assumption simplifies the analysis by avoiding deficit-driven heterogeneity in the labor supply and income of different generations, without changing the essence of our self-financing results.

2.2 Firms

The production side of the economy is the same as in the textbook New Keynesian model: there is a unit-mass continuum of monopolistically competitive retailers, who set prices subject to the standard Calvo friction, hire labor on a spot market, produce according to a technology that is linear in labor, and pay out all their profits as dividends back to the households. Combined with our aforementioned assumptions on labor supply, this guarantees that the supply block of our economy reduces exactly to the standard New Keynesian Phillips curve (NKPC).

2.3 Policy

The government consists of two blocks: a fiscal authority issuing one-period riskless nominal debt and setting taxes, and a monetary authority setting nominal interest rates.

Fiscal policy. We (for now) abstract from government spending and let B_t denote the total nominal public debt outstanding at the beginning of period t. We can then write the *nominal* flow budget constraint of the government as follows:

$$\frac{1}{I_t} B_{t+1} = B_t - P_t T_t, \tag{4}$$

where $T_t \equiv \int T_{i,t} di$ is the total real tax revenue at t. Letting $D_t \equiv B_t / P_t$ denote the *real* value of public debt, $\Pi_{t+1} \equiv P_{t+1}/P_t$ the realized inflation between t and $t+1$, and $R_t \equiv I_t / \mathbb{E}_t [\Pi_{t+1}]$ the (expected) real rate at t, we can rewrite the government budget in *real* terms as

$$D_{t+1} = R_t (D_t - T_t) \left(\frac{\mathbb{E}_t [\Pi_{t+1}]}{\Pi_{t+1}} \right).$$

This underscores how an inflation surprise between t and $t+1$ erodes the real value of the outstanding nominal debt, thus reducing the tax revenue needed to balance the government budget.

We log-linearize around a steady state in which inflation is zero ($\Pi_{ss} = 1$), real allocations are given by their flexible-price counterparts, and the real debt burden is constant at some level $D^{ss} \geq 0$. As discussed above, our assumptions on annuities and the social fund ensure that $R^{ss} = \frac{1}{\beta} > 1$; steady-state taxes then satisfy $T^{ss} = (1-\beta) D^{ss}$. While we will throughout focus on on the empirically relevant scenario with $D^{ss} > 0$, we do wish to accommodate $D^{ss} = 0$, and so we let $d_t \equiv (D_t - D^{ss}) / Y^{ss}$, $b_t \equiv (B_t - B^{ss}) / Y^{ss}$, and $t_t \equiv (T_t - T^{ss}) / Y^{\mathrm{ss}}$—i.e., we measure fiscal variables in terms of absolute deviations (rather than log-deviations) from the steady state, scaled by steady-state output. Re-writing (4) in real terms and linearizing, we obtain

$$d_{t+1} = \underbrace{\frac{1}{\beta}(d_t - t_t) + \frac{D^{ss}}{Y^{ss}} r_t}_{\text{expected debt burden tomorrow}} - \underbrace{\frac{D^{ss}}{Y^{ss}} (\pi_{t+1} - \mathbb{E}_t[\pi_{t+1}])}_{\text{debt erosion due to inflation surprise}} \tag{5}$$

where $r_t \equiv \log(R_t/R^{ss})$, $\pi_{t+1} \equiv \log(\Pi_{t+1}/\Pi_{ss})$, and Y^{ss} is the steady-state level of output. The government must satisfy both the above flow constraint (at each t) as well as the familiar no-Ponzi condition (in the limit as $t \to \infty$). We can thus go back and forth between the infinite sequence of flow budget constraints and the corresponding integrated intertemporal budget constraint.[9]

It remains to specify a fiscal rule for how the total tax t_t is determined as a function of exogenous shocks and endogenous outcomes. First, there is a proportional tax $\tau_y \in [0,1)$ on household labor and dividend income (and thus, on their total income). This tax is distortionary but time-invariant and captures our tax base channel: when aggregate income increases by one dollar, tax revenue automatically increases by τ_y dollars. Second, there is a time-varying lump-sum component, which includes both any initial fiscal stimulus (i.e., the exogenous deficit shock) as well as any subsequent tax hikes that are used to aid in bringing government debt back to steady state.[10] We will consider two different rules for this time-varying component—a baseline rule and an alternative rule—, each serving a distinct purpose. The two rules differ on whether the tax hikes are spread out gradually over time (baseline rule) or concentrated in one period (alternative rule). Either way, these tax hikes will capture our notion of "fiscal adjustment." The key policy parameter that we will vary across experiments is the *speed* at which this fiscal adjustment takes place.

1. *Baseline fiscal rule.* Our baseline rule sets total taxes as follows:

$$T_{i,t} = \tau_y Y_{i,t} + \bar{T} - \mathscr{E}_t + \tau_d \left(D_t - D^{ss} + \mathscr{E}_t\right), \tag{6}$$

   where $\bar{T} = T^{ss} - \tau_y Y^{ss}$ is set to guarantee budget balance at steady state, $\mathscr{E}_t$ is a mean-zero and i.i.d. deficit shock (e.g., issuance of stimulus checks), and $\tau_d \in (0,1)$ is a scalar that parameterizes the speed of fiscal adjustment. Intuitively, fiscal adjustment is arbitrarily fast for $\tau_d \to 1^-$ and arbitrarily slow for $\tau_d \to 0^+$. Thanks to our simplifying assumption that all households receive the same income, we can drop the i index and rewrite (6), after (log-)linearization, as follows:

$$t_t = \underbrace{\tau_y y_t}_{\text{tax base}} + \underbrace{\tau_d (d_t + \varepsilon_t)}_{\text{fiscal adjustment}} - \underbrace{\varepsilon_t}_{\text{deficit shock}}, \tag{7}$$

   where $\varepsilon_t \equiv \mathscr{E}_t / Y^{ss}$ is the deficit shock (conveniently rescaled), $y_t \equiv \ln(Y_t/Y^{ss})$ is the deviation of output from steady state, and d_t is the corresponding deviation of the real value of public debt.

[9]Note that this does not necessarily rule out explosive debt: debt could still explode, provided that it does so at a rate lower that the steady-state real interest. But such an explosion would be at odds, not only with the spirit of our exercise, but also with the log-linearization of the economy. Throughout the subsequent analysis, we will therefore restrict attention to policies (and equilibria) that guarantee that, following any given shock, public debt converges eventually back to steady state. Formally, we require that $\mathbb{E}_t[\lim_{k\to\infty} d_{t+k}] = 0$ for all t and all realizations of uncertainty.

[10]While our main analysis treats the required tax hikes as non-distortionary, we emphasize that this is only for the sake of simplicity in exposition. As we demonstrate in Section 5.3, allowing the tax hikes to be distortionary has minimal impact on our self-financing results.

2. *Alternative fiscal rule.* Our second rule is a time-dependent variant of (6). This rule sets, after (log-)linearization,

$$t_{i,t} = t_t = \begin{cases} \tau_y y_{i,t} - \varepsilon_0 & t = 0 \\ \tau_y y_{i,t} & t \in \{1, \cdots, H-1\} \\ d_t & t \geq H \end{cases}, \tag{8}$$

   Note that this rule shuts down any deficit shocks at dates $t \geq 1$, thus allowing us to focus cleanly on impulse responses to a date-0 shock.[11] The interpretation of the rule is as follows: following the date-0 shock, the government abstains from any fiscal adjustment for the first H periods, before then adjusting taxes from date H onwards to return government debt to steady state. We can thus identify H as the lag between the initial deficit and the future adjustment.

Intuitively, we can capture a longer delay in fiscal adjustment through either (i) a low τ_d under the first rule or (ii) a high H under the second rule. This interchangeability notwithstanding, we like to study both rules. On the one hand, the baseline rule (7) facilitates a tractable, recursive characterization of the equilibrium (which we use for our main theoretical results), as well as a mapping between the theory and some relevant empirical work (which we use for our quantitative exercises). Furthermore, (7) allows a sharp comparison to prior theoretical work.[12] On the other hand, the alternative rule (8) captures more transparently the timing of fiscal adjustment and thus allows us to develop a sharper intuition for our self-financing result. It also makes clear that our fiscal policy is "passive" or "Ricardian" in the sense of the FTPL literature: (8) embeds a commitment to raise taxes as needed to make sure that debt is back in steady state at $t = H$ (and thereafter), no matter what path the economy has followed up to that point, and no matter what path it is expected to follow thereafter.

Monetary policy. The monetary authority sets I_t, the nominal rate of interest, according to the following policy rule:

$$\frac{I_t}{\Pi^{ss}} = R^{ss} \mathbb{E}_t \left[\frac{\Pi_{t+1}}{\Pi^{ss}} \right] \left(\frac{Y_t}{Y^{ss}} \right)^{\phi}, \tag{9}$$

for some $\phi \in \mathbb{R}$. This is equivalent to saying that the monetary authority implements the following relation between the (expected) real interest rate and real output:

$$r_t = \phi y_t. \tag{10}$$

[11] See Wolf (2021) for a discussion of the—conceptually simple but notationally involved—mapping from policy rules in sequence space to their state-space analogues. Also, the disappearance of the term $\tau_y y_{i,t}$ in (8) for $t \geq H$ may suggest that the tax distortion on labor supply also disappears at $t \geq H$, which would complicate our upcoming NKPC by adding a non-stochastic but time-varying cost-push term. We can abstract from this nuisance by setting individual taxes as $t_{i,t} = d_t - \tau_y y_t + \tau_y y_{i,t}$ for $t \geq H$. This then reduces to (8) above and implies that the NKPC is unchanged.

[12] In particular, (7) nests the fiscal rule found in Leeper (1991). As in that paper, the term $\tau_d (d_t + \varepsilon_t)$ captures fiscal adjustment: the variation in taxes induced by the exogenous deficit shock. The key novelty here is the inclusion of the term $\tau_y y_t$, which captures the emphasized tax base channel.

Monetary policy in our model is thus parameterized by the pro-cyclicality of the *real* rate of interest. Since deficits will be expansionary in equilibrium (provided that $\omega < 1$, i.e., that Ricardian equivalence fails), ϕ also parameterizes the co-movement between real rates and deficits. We can thus interpret $\phi < 0$ as an "accommodative" monetary authority that, in response to a positive deficit shock, lets real interest rates fall so as to reduce the government's cost of borrowing and increase fiscal space. Conversely, we can interpret $\phi > 0$ as a "hawkish" or "fiscally conservative" monetary authority that uses interest rate hikes to lean against any boom (and any inflation) triggered by deficits.

For our main analysis, we will let monetary policy be "neutral" in the specific sense that $\phi = 0$; that is, the real rate is kept fixed throughout. This is the same baseline policy as in Woodford (2011) and allows us to cleanly isolate how the interaction of fiscal policy and private spending shapes the scope for the self-financing of fiscal deficits. We will relax this restriction in Section 5.1. In Section 6.2, we also show how our insights extend to more standard Taylor-rule specifications for monetary policy.

3 Equilibrium

This section lays the groundwork for our self-financing result by characterizing the economy's equilibrium. We start by reducing the economy to a system of three equations: one for aggregate demand, one for aggregate supply, and one for the dynamics of public debt. We then characterize the unique bounded solution to this system. Throughout this section, we employ our baseline fiscal rule (7). Derivations for the alternative rule (8) are slightly different and relegated to Appendix A.3, though the economic essence is identical.

3.1 Aggregate demand

The consumption-savings problem of a household i is to choose sequences of consumption and asset holdings to maximize (1) subject to (2). Using the simplifying property that all households receive the same income (and pay the same taxes), we can express the (log-linearized) consumption function of household i in period t as follows:

$$c_{i,t} = (1-\beta\omega)\left(\tilde{a}_{i,t} + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\left(y_{t+k} - t_{t+k}\right)\right]\right) - \gamma\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k r_{t+k}\right], \tag{11}$$

where $\tilde{a}_{i,t}$ denotes the household's *real* financial wealth (inclusive of social fund payments) and $\gamma \equiv \sigma\beta\omega - (1-\beta\omega)\beta\frac{\tilde{A}^{ss}}{P^{ss}Y^{ss}}$ combines the intertemporal substitution and wealth effects of real interest rates. When $\omega = 1$, (11) reduces to the consumption function of a standard permanent-income household: future disposable income is discounted at rate β, and the marginal propensity to consume (MPC) from both financial wealth and permanent income is $1-\beta$. Relative to this benchmark, $\omega < 1$ maps to both more discounting of future disposable income (and hence of future taxes) and a higher MPC. It is by now well understood how these qualitative properties extend to richer, more realistic, HANK-type models (e.g., see Auclert et al., 2023; Farhi and Werning, 2019; Wolf, 2021); as we will show formally in Section 5.2, our results are driven by these more general qualitative properties of consumer demand, and not by the specific micro-foundations behind them.

Under our baseline monetary policy, $r_t = 0$ for all t, so the last term in (11) drops out. Aggregating across households, and using the fact that aggregate private financial wealth equals total government debt, we reach the following description of the aggregate consumption function (see Appendix A.2):

$$c_t = \left(1-\beta\omega\right) d_t + \left(1-\beta\omega\right) \mathbb{E}_t \left[\sum_{k=0}^{\infty} \left(\beta\omega\right)^k \left(y_{t+k} - t_{t+k}\right) \right]. \tag{12}$$

Next, using (7) to express future taxes as functions of the current public debt and future output, and replacing c_t with y_t (market clearing), we arrive at the following representation of aggregate demand:

$$y_t = \mathscr{F}_1 \cdot (d_t + \varepsilon_t) + \mathscr{F}_2 \cdot \mathbb{E}_t \left[\left(1-\beta\omega\right) \sum_{k=0}^{\infty} \left(\beta\omega\right)^k y_{t+k} \right], \tag{13}$$

where $\mathscr{F}_1 \equiv \frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{1-\omega(1-\tau_d)}$ and $\mathscr{F}_2 \equiv 1 - \frac{(1-\omega)\tau_y}{1-\omega(1-\tau_d)}$.

(13) is a key equation of this paper. The first term captures the direct ("partial equilibrium") effect of fiscal deficits on aggregate demand. We see that $\mathscr{F}_1 > 0$ (i.e., deficits enter positively in aggregate demand) if and only if $\omega < 1$ (no Ricardian equivalence) and $\tau_d < 1$ (no *immediate* financing). Intuitively, $\tau_d < 1$ means that deficits today are financed at least in part with taxes in the future; as long as $\omega < 1$, this means that a deficit today is a real transfer from future cohorts to current cohorts, increasing aggregate demand. The second term captures the general equilibrium feedback between aggregate demand and income—the "intertemporal Keynesian cross." Note in particular that $\mathscr{F}_2$ measures the "slope" of this Keynesian cross, in the following precise sense: if we raise expectations of future spending in all periods by 1, then current spending increases by $\mathscr{F}_2$.

Finally, we briefly consider how equation (13) specializes for $\omega = 1$. In this case, $\mathscr{F}_1 = 0$, $\mathscr{F}_2 = 1$, and thus (13) collapses to

$$y_t = \mathbb{E}_t \left[(1-\beta) \sum_{k=0}^{\infty} \beta^k y_{t+k} \right], \tag{14}$$

which in turn can be rewritten as $y_t = \mathbb{E}_t[y_{t+1}]$. This makes clear two points. First, that our aggregate demand equation is a natural extension of its RANK-PIH counterpart, namely the Euler equation of a representative, infinitely-lived, financially unconstrained consumer. And second, that debt and taxes enter this equation because and only because we have departed from PIH consumer behavior.[13]

[13] A third point is also evident: (14) admits multiple bounded solutions, some of which allow y_t to vary with d_t and/or ε_t, even though those variables do not enter (14). This echoes our point from the Introduction: the FTPL equates fiscal policy to coordination devices. By contrast, by setting $\omega < 1$, we let fiscal policy be a direct determinant of aggregate demand.

3.2 Aggregate supply

By design, the aggregate supply side of our model is *exactly* the same as its familiar RANK counterpart. In particular, log-linearizing (3), we find that household labor supply is given by the usual relation

$$\frac{1}{\varphi}\ell_t = w_t - \frac{1}{\sigma}c_t. \tag{15}$$

Together with market clearing ($c_t = y_t$) and technology ($y_t = \ell_t$), this pins down the real wage as $w_t = \xi y_t$, where $\xi \equiv \frac{1}{\varphi} + \frac{1}{\sigma} > 0$. Firm optimality, on the other hand, pins down the optimal reset price as a function of current and expected future real marginal costs, and thus inflation as a function of current and expected future wages.

Putting everything together, we can reduce the supply-side of the economy to the familiar NKPC:

$$\pi_t = \kappa y_t + \beta \mathbb{E}_t\left[\pi_{t+1}\right]. \tag{16}$$

where $\kappa = \frac{(1-\theta)(1-\beta\theta)\xi}{\theta} \geq 0$ depends on ξ (the pro-cyclicality of real marginal costs) and $1-\theta$ (the Calvo reset probability). Since there is a one-to-one mapping between κ and θ, and since this mapping is invariant to both fiscal and monetary policy, we henceforth treat κ as an exogenous parameter and (re)parameterize the degree of nominal rigidities by it. Further details are provided in Appendix A.1.

3.3 Law of motion for public debt

The remaining third equilibrium relation comes from combining the government's flow budget constraint 5 with the fiscal rule (7). This yields the following law of motion for public debt:

$$d_{t+1} = \beta^{-1}\big(d_t + \varepsilon_t - \underbrace{\tau_d\left(d_t + \varepsilon_t\right)}_{\text{fiscal adjustment}} - \underbrace{\tau_y y_t}_{\text{tax base}}\big) - \underbrace{\frac{D^{ss}}{Y^{ss}}\left(\pi_{t+1} - \mathbb{E}_t\left[\pi_{t+1}\right]\right)}_{\text{debt erosion}}. \tag{17}$$

with initial condition[14]

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0. \tag{18}$$

Finally, recall that the sequence of government flow budget constraints must be complemented with the usual no-Ponzi restriction $\lim_{k\to\infty}\mathbb{E}_t\left[\beta^k d_{t+k}\right] = 0$.

3.4 Equilibrium definition and characterization

A standard equilibrium definition combines (i) individual optimality for consumers, (ii) individual optimality for firms, (iii) market clearing, and (iv) budget balance for the government (together with the no-Ponzi constraint). The preceding analysis has log-linearized the model and has reduced the

[14]Note that $D_0 = B_0/P_0$, with $B_0 = B^{ss}$ predetermined. We thus arrive at (18).

first three requirements to equations (13) and (16), and the last requirement to equation (17). These equations, like the log-linearization itself, make sense only insofar the economy remains in a neighborhood of the steady state. Accordingly, our notion of equilibrium is as follows.

Definition 1. An equilibrium is a stochastic path $\{y_t, \pi_t, d_t\}_{t=0}^{\infty}$ for output, inflation, and the real value of public debt that is bounded in the sense of Blanchard and Kahn (1980) and that satisfies aggregate demand (13), aggregate supply (16), and the law of motion for public debt (17), along with the initial condition (18) and the no-Ponzi game condition $\mathbb{E}_t\left[\lim_{k\to\infty}\beta^k d_{t+k}\right] = 0$.

We can now state our first main result.

Proposition 1. *Suppose that $\omega < 1$ and $\tau_y > 0$, and let fiscal policy follow our baseline rule (7). There exists a unique (bounded) equilibrium. Along this equilibrium, real output and real public debt satisfy*

$$y_t = \chi\left(d_t + \varepsilon_t\right) \qquad \textit{and} \qquad \mathbb{E}_t\left[d_{t+1}\right] = \rho_d\left(d_t + \varepsilon_t\right), \tag{19}$$

for some $\chi > 0$ and $\rho_d \in (0,1)$. These coefficients solve the following fixed point problem:

$$\chi = \frac{\mathscr{F}_1}{1 - \mathscr{F}_2 \frac{1-\beta\omega}{1-\beta\omega\rho_d}} \qquad \textit{and} \qquad \rho_d = \beta^{-1}\left(1 - \tau_d - \tau_y\chi\right). \tag{20}$$

Finally, inflation satisfies $\pi_t = \frac{\kappa}{1-\beta\rho_d} y_t$.

Condition (19) contains two relations. The first relation expresses the equilibrium level of output as a proportion χ of the private sector's real financial wealth (which itself equals d_t) and the fiscal transfer (the deficit shock ε_t). Note that $\chi > 0$—i.e., deficits trigger booms. As emphasized previously, this is due to two key features of our environment: the failure of Ricardian equivalence, which allows deficits to stimulate aggregate demand; and the nominal rigidity, which lets aggregate demand drive output. The second relation gives the (expected) evolution of the real value of public debt, with ρ_d measuring the (expected) persistence of debt. Since y_t is proportional to d_t, we see that ρ_d here also measures the expected persistence of the Keynesian boom triggered by deficits.

Condition (20) summarizes the fixed-point relation between χ and ρ_d—i.e., the two-way feedback between aggregate demand and fiscal conditions. On the one hand, as long as $\tau_y > 0$, higher aggregate demand contributes to higher output, higher tax revenue and thereby to lower public debt tomorrow. This feedback is reflected in the second part of condition (20), which pins down ρ_d as a function of χ and of the two fiscal policy parameters (τ_d, τ_y). On the other hand, as long as $\omega < 1$, more delay in fiscal adjustment, or more persistence in public debt, will translate to a larger effective transfer from generations in the far future to generations in the present and the near future, thus stimulating aggregate demand both directly (the partial equilibrium effect) and indirectly (the general equilibrium Keynesian cross). This is reflected in the first part of condition (20), which pins down χ as a function of ρ_d

and of the relevant aggregate-demand parameters ($\mathscr{F}_1, \mathscr{F}_2, \beta\omega$). We emphasize that the feedback from deficits to aggregate demand is present only when $\omega < 1$, while the feedback from aggregate demand to tax revenue and thereby to public debt dynamics is present only when $\tau_y > 0$. The combination of these feedbacks lies at the heart of the tax base channel of self-financing. The two-way feedback is also responsible for the uniqueness of the equilibrium, discussed further in Appendix B.1.[15]

Aside: the role of inflation. We close this section with an important remark about the role of inflation. To understand what happens in our economy as we move from $\kappa = 0$ to $\kappa > 0$, it will prove insightful to momentarily consider a variant of our economy in which government debt is indexed to inflation (i.e., entirely real). In that case d_t would be predetermined at the beginning of period t, and so inflation would not enter the fixed point relation between output and debt. Formally, in such an economy, the second part of (19) would hold state-by-state, not just in expectation. It follows that the equilibrium paths of y_t and d_t would be determined independently of the economy's supply block. Intuitively, in that case, the NKPC would still govern the inflationary effects of the fiscally-led boom; however, if public debt is indexed (and the monetary authority pegs the real rate), there is no feedback from inflation to either the government budget or to aggregate demand.

Now return again to our economy with nominal debt. If $\kappa = 0$, then the dynamics of output and debt of course coincide with the indexed-debt economy. If instead $\kappa > 0$, then d_t is no longer predetermined—it depends on the endogenous response of p_t (or, equivalently, of π_t) to ε_t, as reflected in the inflation surprise term in (17). Proposition 1 reveals that this re-scaling of the initial debt d_t is the *only* feedback from inflation back to debt, demand, and output: by (19), impulse responses in our economy with nominal debt coincide with those of an economy with indexed debt, just suitably re-scaled on impact. The slope and shape of the NKPC govern the magnitude of this re-scaling, but otherwise have no effect on the equilibrium dynamics of output and government debt.[16]

4 Self-financing of fiscal deficits

This section presents our headline result on the possibility of self-financing deficits. We first use the intertemporal government budget constraint to provide a quantitative measure of the degree of self-financing. We show that there is *full* self-financing in the limit as fiscal adjustment is delayed further

[15] Under the alternative rule (8), uniqueness requires one of the following minor modifications: (i) a strengthening of the notion of boundedness to $\lim_{k\to\infty} \mathbb{E}_t [y_{t+k}] = 0$, which amounts to saying that expectations "at infinity" are anchored to the steady state; or (ii) the reinterpretation of $\phi = 0$ as the limit of $\phi \to 0$ from above, which is basically a (limit) Taylor principle. The sole role of either one of these modifications is to remove a class of sunspot equilibria that are inherited from the standard New Keynesian model. See Appendix A.3 for details.

[16] A similar point applies to the maturity structure of public debt: the latter naturally alters the mapping from ε_t to d_t through the impact inflation surprise, but otherwise has no effect on the dynamics of d_t and y_t.

and further, and we explain the economics behind this result.

4.1 Sources of fiscal financing

How is a date-0 fiscal deficit shock ε_0 financed in equilibrium? Iterating equations (17) and (18) forward, taking expectations at $t = 0$, and using $\lim_{t\to\infty} \mathbb{E}_0 \left[\beta^t d_t\right] = 0$ (since $\rho_d \in (0,1)$), we obtain the following present-value restriction on fiscal policy:

$$\underbrace{\varepsilon_0}_{\text{deficit}} = \underbrace{\tau_d \left(\varepsilon_0 + \sum_{k=0}^{\infty} \beta^k \mathbb{E}_0 [d_k]\right)}_{\text{fiscal adjustment}} + \underbrace{\tau_y \left(\sum_{k=0}^{\infty} \beta^k \mathbb{E}_0 [y_k]\right)}_{\text{self-financing via tax base}} + \underbrace{\frac{D^{ss}}{Y^{ss}} (\pi_0 - \mathbb{E}_{-1} [\pi_0])}_{\text{self-financing via debt erosion}} . \tag{21}$$

The left-hand side of (21) is the exogenous shock in the initial deficit, while the right-hand side contains the three ways in which this shock will be financed over time: the first term captures the adjustment in current and future taxes triggered by the shock and any resulting accumulation of public debt; the second term collects the present value of the extra tax revenue generated by the deficit-driven boom in real economic activity; and the third term gives the erosion in the real debt burden caused by the associated innovation in date-0 inflation. Put differently, the first term captures the conventional notion of fiscal adjustment—the government actively adjusts its primary surplus to stabilize its debt—while the second and third terms reflect our two sources of self-financing. Finally, we note that a fourth source of financing—monetary accommodation—emerges if the monetary authority depresses real rates in response to deficits. In our main analysis, the assumption that $\phi = 0$ means that this channel is not operative.

We can now define the (overall) degree of self-financing as follows:

Definition 2. The degree of self-financing is the fraction of the initial deficit that is financed by an expansion in the tax base and/or an erosion in the real debt burden:

$$\nu \equiv \frac{\tau_y \left(\sum_{k=0}^{\infty} \beta^k \mathbb{E}_0 [y_k]\right) + \frac{D^{ss}}{Y^{ss}} \pi_0}{\varepsilon_0} \tag{22}$$

This definition applies regardless of whether fiscal policy obeys the baseline rule (7) or the variant rule (8). Note next that the overall degree of self-financing can be decomposed into its two components:

$$\nu \equiv \nu_y + \nu_p$$

where

$$\nu_y \equiv \frac{\tau_y \left(\sum_{k=0}^{\infty} \beta^k \mathbb{E}_0 [y_k]\right)}{\varepsilon_0} \qquad \text{and} \qquad \nu_p \equiv \frac{1}{\varepsilon_0} \frac{D^{ss}}{Y^{ss}} \pi_0 \tag{23}$$

measure, respectively, the tax-base and debt-erosion components of self-financing.

4.2 The self-financing result

We can now state our main theoretical result on the possibility of self-financing.

Theorem 1. *Suppose that $\omega < 1$ and $\tau_y > 0$, and let fiscal policy follow either our baseline rule (7) or the variant rule (8). The equilibrium degree of self-financing, ν, has the following properties:*

1. *It is increasing in the delay of fiscal adjustment; i.e., ν is decreasing in τ_d for the fiscal rule (7) and increasing in H for the fiscal rule (8).*

2. *It converges to 1 as fiscal adjustment is delayed further and further; i.e., $\nu \to 1$ (from below) as $\tau_d \to 0$ (from above) or as $H \to \infty$. These two limits induce the same paths $\{y_t, \pi_t, d_t\}_{t=0}^{\infty}$, and, in this common limit, self-financing is sufficiently strong to return real government debt to steady state (i.e., we have that $\lim_{k\to\infty} \mathbb{E}_t [d_{t+k}] \to 0$ for (7) and $\lim_{H\to\infty} \mathbb{E}_0 [d_H] \to 0$ for (8)).*[17]

Theorem 1 is our core result. Its main implication is that the failure of Ricardian equivalence—here encapsulated in $\omega < 1$—opens the door for fiscal deficits to finance themselves. First, as fiscal adjustment is delayed, the initial fiscal deficit induces a larger and more persistent Keynesian boom, thus increasing the share of self-financing through higher tax revenue and a larger date-0 inflation. Second, the limit as $\tau_d \to 0$ or $H \to \infty$ is one of *full* self-financing: the deficit-driven boom is precisely large enough to cover the cost of the initial fiscal outlay ε_0 and to make sure that public debt returns back to steady state, without any fiscal adjustment.

Before expanding on the intuition behind this result, we complete the picture by clarifying the role played by the slope of the NKPC, or equivalently, by the degree of price flexibility.[18]

Proposition 2. *Suppose that $\omega < 1$ and $\tau_y > 0$, and let fiscal policy follow either our baseline rule (7) or the variant rule (8). The slope of the NKPC, κ, does not affect the monotonicity and limit properties of ν documented in Theorem 1. It only determines the split between tax base and debt erosion components of self financing, with that split given as*

$$\nu_y = \frac{\tau_y}{\kappa \frac{D^{ss}}{Y^{ss}} + \tau_y} \nu \qquad \text{and} \qquad \nu_p = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\kappa \frac{D^{ss}}{Y^{ss}} + \tau_y} \nu. \tag{24}$$

If prices are rigid ($\kappa = 0$), then all self-financing occurs through the tax base ($\nu_y = \nu$); as prices become more flexible (a higher κ), a larger portion of self-financing occurs through debt erosion (a higher ν_p/ν).

[17] The limiting equilibrium path as $\tau_d \to 0$ is furthermore precisely the unique bounded equilibrium when $\tau_d = 0$. In other words, there is no discontinuity at $\tau_d = 0$.

[18] The specific split derived in Proposition 2 relies on the textbook version of the NKPC. The central insight, however, is more broadly applicable: as explained on page 16, the specification of the Phillips curve does not influence either the limiting self-financing result or the fact that more rigid prices result in more self-financing through tax base expansion.

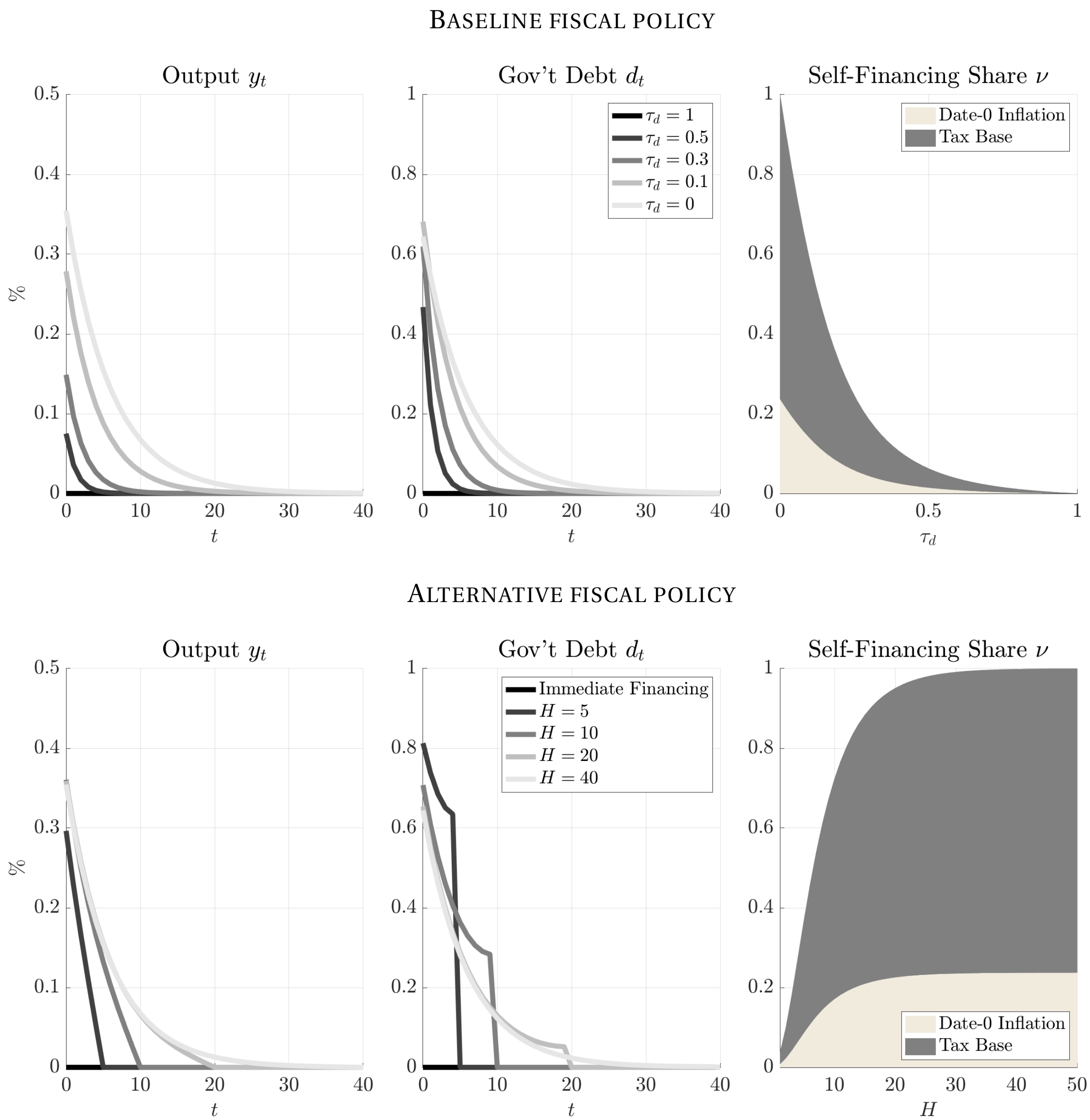

Figure 1: Top panel: impulse responses of output y_t, government debt d_t, and the self-financing share ν to a unit-size shock ε_0 as a function of τ_d. Bottom panel: same as above, but as a function of H.

We close by remarking on one important joint implication of Theorem 1 and Proposition 2: if fiscal adjustment is delayed further and further (i.e., $\tau_d \to 0$ or $H \to \infty$) while prices are rigid (i.e., $\kappa \to 0$), then the cumulative output multiplier—i.e., $\sum_{k=0}^{\infty} \beta^k \mathbb{E}_0 \left[y_k \right] / \varepsilon_0$—converges to $1/\tau_y$. We will return to this observation in Section 4.3 and in our quantitative analysis of Section 6.

A visual illustration. We provide a visual illustration of Theorem 1 in Figure 1. We will only emphasize *qualitative* features here, relegating a more serious quantitative evaluation to Section 6.[19] The figure shows the effects of a deficit shock ε_0 under different assumptions about fiscal adjustment. The left and middle panels in the top half of the figure begin by showing impulse responses of output y_t and government debt d_t as a function of the fiscal adjustment parameter τ_d in our baseline fiscal rule (7). Consistently with Theorem 1, we see that smaller fiscal response coefficients correspond to larger impact output booms (i.e., larger χ) and more persistent deviations of output and government debt from steady state (i.e., larger ρ_d). This boom then contributes to financing of the initial deficit ε_0 through our two self-financing channels: tax base expansion and date-0 inflation. The top right panel of the figure reports this degree of self-financing ν—as well as the split into ν_y and ν_p—as a function of the fiscal adjustment parameter τ_d. We see that ν is decreasing in the strength of fiscal adjustment τ_d, i.e., it is increasing in the delay of fiscal adjustment. In particular, as τ_d declines towards zero, the degree of self-financing converges to one.

The bottom half of Figure 1 provides a different perspective on the same logic, using instead the alternative fiscal rule (8), in which taxes adjust after some (finite) horizon H to perfectly balance the budget. We see again that ν is increasing in the delay of fiscal adjustment, and in particular again converges to one as fiscal adjustment is delayed further and further ($H \to \infty$). This not only offers a complementary interpretation of what "delay" means, but also proves the following important point: our self-financing result is consistent with fiscal policy being "Ricardian" or "passive" in the strong sense that it commits on bringing debt back to steady state at $t = H + 1$—regardless of the path that the economy has taken up to that point.

4.3 The economics behind the self-financing result

To understand the economics behind Theorem 1 as transparently as possible, we in the majority of this subsection restrict attention to the special case of fully rigid prices (i.e., we will set $\kappa = 0$). Based on the discussion at the end of Section 3, shifting from this rigid-price case to the general sticky-price case will be straightforward, only requiring us to re-scale all impulse responses (without changing their shape). We here also focus on the fiscal "H-rule" (8). As anticipated before, this fiscal rule is pedagogically useful because it makes clear both what we mean by delay in fiscal adjustment and why our results are consistent with fiscal policy being "passive" or "Ricardian".

The policy experiment studied in the remainder of this section is thus as follows: the fiscal authority pays out a lump-sum transfer to households at date 0 and promises to hike taxes at date H in order

[19]For this illustration, we set $\omega = 0.75$—a meaningful departure from Ricardian equivalence—and $\kappa = 0.1$—a rather steep NKPC, allowing a clean visual illustration of our two sources of self-financing.

to return debt to its steady-state value at date $H+1$. The questions of interest are how this policy affects equilibrium outcomes, how large the required tax hike at H turns out to be, and what happens as we increase H. We address these questions in two steps. First, to build intuition, we elaborate on our discussion in the introduction and analyze a two-period example. Second, we show how the intuition from this simple, essentially static example sheds light on the workings of our full dynamic economy.

A simple static example. We consider a two-period economy in which the government pays out a transfer ε to households at $t=0$, generates automatic tax revenue τ_y for every dollar of output, and taxes households to return debt to trend at $t=1$ (as necessary). Prices are fully rigid, so output at $t=0$ is fully demand-determined, and the only source of self-financing will be the tax base channel. We assume that consumer demand in period 0 is given as

$$c = \text{MPC} \cdot y_{\text{disp}}$$

where $\text{MPC} \in (0,1)$ is the marginal propensity to consume and

$$y_{\text{disp}} = (1-\tau_y)y + \varepsilon$$

is disposable income. We note that this set-up embeds a myopia assumption: date-0 consumption is invariant to date-1 outcomes, thus allowing us to characterize the date-0 equilibrium without reference to what happens later. By imposing market clearing ($y=c$), we immediately see that the date-0 equilibrium level of income is given by

$$y = \frac{\text{MPC}}{1-(1-\tau_y)\text{MPC}} \times \varepsilon \tag{25}$$

This equation is just the solution of the familiar, static Keynesian cross: MPC is the partial equilibrium effect of a unit transfer; $(1-\tau_y)\text{MPC}$ is the slope of the Keynesian cross; and $\frac{1}{1-(1-\tau_y)\text{MPC}}$ is the general equilibrium multiplier.

Consider now the government's budget constraint. Since the government hands out the transfer ε and collects taxes $\tau_y y$, the amount of public debt inherited at date 1 is given by

$$\text{debt tomorrow} = R(\varepsilon - \tau_y y), \tag{26}$$

where R is the (fixed) real rate between the two periods. Plugging (25) into (26), we conclude that debt tomorrow $= R(1-\nu)\varepsilon$, where

$$\nu \equiv \frac{\tau_y y}{\varepsilon} = \frac{\tau_y \text{MPC}}{1-(1-\tau_y)\text{MPC}} \tag{27}$$

is the degree of self-financing.

Equation (27) reveals two properties, already highlighted in the Introduction. First, we see that a higher MPC maps both to a larger partial equilibrium effect (numerator) and to a higher general equilibrium multiplier (denominator), and therefore overall to a larger degree of self-financing ν. Second, in the limit as MPC $\to$ 1, the partial equilibrium effect converges to 1, the output multiplier converges to $\frac{1}{\tau_y}$, and ν converges to 1—i.e., there is full self-financing.

Back to the full model. To what extent is the simple—effectively static—example informative about what is actually going on in our full dynamic economy? Note that full self-financing ($\nu \to 1$) in the static model relies on two key properties: first, that the expected date-1 tax hike does not affect date-0 spending behavior; and second, that the date-0 transfer as well as all the additional income it generates are fully spent at date 0 (MPC $\to$ 1), thus generating enough tax revenue to stabilize debt *before* the promised date-1 tax hike. The core intuition is that, as the financing delay H increases, our dynamic economy starts to emulate those two features of the static example.

To see why this is so, we begin by highlighting two important properties of our economy's aggregate consumption function. Formal details are provided in Lemma E.1 in the Online Supplement.

1. *Discounting.* Consider first how consumption demand at some date $t \geq 0$ responds to an anticipated future change in disposable income at some future date $t+\ell$, with $\ell \geq 0$. We write this response as $\mathcal{M}_{t,t+\ell}$—the $(t, t+\ell)$ element of the matrix of intertemporal MPCs studied by Auclert et al. (2023). We evaluate $\beta^{-\ell}\mathcal{M}_{t,t+\ell}$, i.e., the date-$t$ consumption response to a date-$t+\ell$ income change with a date-t value of 1. It is straightforward to see that this response is given as $\beta^{-\ell}\mathcal{M}_{t,t+\ell} = (1-\beta)$ in the PIH benchmark ($\omega = 1$). With finite horizons ($\omega < 1$), on the other hand, it can be shown that $\beta^{-\ell}\mathcal{M}_{t,t+\ell}$ is strictly decreasing in ℓ, and in particular that
$$\lim_{\ell\to\infty} \beta^{-\ell}\mathcal{M}_{t,t+\ell} = 0.$$
In words, as long as $\omega < 1$, an income change of a fixed present value that occurs further and further into the future has a diminishing and eventually vanishing effect on current consumption. We refer to this property of aggregate demand as "discounting."

2. *Front-loading.* Consider next how changes in disposable income at date $t \geq 0$ affect consumption demand at some future date-$t+\ell$, with $\ell \geq 0$. We write this response as $\mathcal{M}_{t+\ell,t}$—the $(t+\ell, t)$ element of the intertemporal MPC matrix. In the PIH benchmark, this response is invariant to ℓ, and in particular, $\mathcal{M}_{t+\ell,t} = \beta^t(1-\beta)$, reflecting perfect consumption smoothing. With finite horizons, we instead have that $\mathcal{M}_{t+\ell,t}$ is decreasing in ℓ, with
$$\lim_{\ell\to\infty} \mathcal{M}_{t+\ell,t} = 0.$$
In words, consumers tend to spend any income receipt faster than in the PIH benchmark, and any income shock today has a vanishing effect on consumption far enough in the future. We refer to this property as "front-loading."

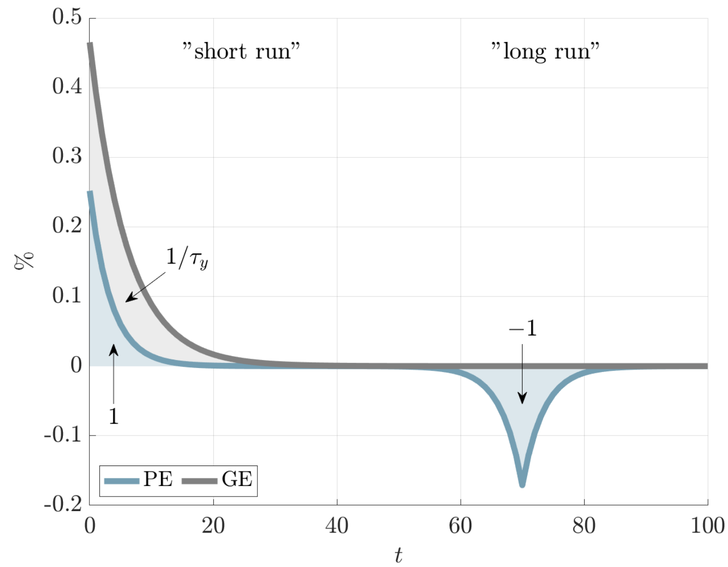

Figure 2: Direct, partial equilibrium demand effect (blue) and full general equilibrium output impulse response (grey) to a date-0 unit-size deficit shock with fiscal financing at date $H = 70$.

These properties allow us to connect our full dynamic economy with the simple two-period example; a visual illustration to accompany the following discussion is provided in Figure 2.

The first object shown in the figure (in blue) is what we will refer to as the partial equilibrium (PE) effect of the policy. It captures the following thought experiment. Suppose the government pays out a transfer of size 1 at date 0, but then counterfactually assume that aggregate output and inflation do not respond at all to this policy. In that case, debt would grow at rate $R^{ss} = \beta^{-1}$ all the way up until date H, when the government hikes taxes to stabilize government debt. The blue line gives the response of consumption demand to this combination of date-0 transfer and date-H tax hike.[20] The discounting and front-loading properties allow us to understand the shape of this partial equilibrium effect. By "discounting", the date-0 cohort is essentially unaffected by the future announced tax hike; by "front-loading", it spends its lump-sum transfer receipt relatively quickly, with the cumulative MPC approaching 1 quickly, and in particular faster than in the PIH benchmark. The cohorts born shortly after date-0 are also essentially unaffected by the future tax hike, so overall aggregate demand is back to trend after around 20 periods. It is only around $t = 60$ that expectations of the future tax hike at $H = 70$ start to depress demand. Our dynamic economy thus echoes the static example: the future tax hike does not affect "short-run" spending behavior, and the "short-run" cumulative MPC approaches 1. The key is to interpret the "short run" as an interval of time that is long enough but also sufficiently distant from the tax hike; in the figure, this corresponds to, roughly, the first 20 quarters.

[20] Mathematically, the blue line plots $\mathscr{M} \cdot \boldsymbol{t}^{PE}$, where $\mathscr{M}$ is the full matrix of intertemporal MPCs and $\boldsymbol{t}^{PE}$ is the "partial equilibrium" tax-and-transfer vector, which equals -1 at date 0, β^{-H} at date H, and 0 otherwise.

The second object shown in the figure (in grey) is the full general equilibrium (GE) effect of our policy experiment—the object we formally characterized in Theorem 1. In general equilibrium, since prices are rigid, the initial increase in consumer demand generates additional income. Importantly, again by the front-loading property of consumer demand, this income is also spent quickly. Since the cumulative short-run MPC approaches 1, we get a front-loaded cumulative Keynesian multiplier of size $\frac{1}{\tau_y}$. Crucially, this Keynesian boom increases tax revenue (in date-0 present value terms) by $\tau_y \times \frac{1}{\tau_y} = 1$, thus returning government debt to trend far before date H (or "the long-run"). As a result, the promised subsequent tax hike at H—together with its negative effect on spending—vanishes, again echoing what happens as MPC $\to 1$ in the simple example above.

While the above discussion focused on the limit as $H \to \infty$, the underlying intuition also readily connects with the monotonicity of ν in H. As H is increased, the effect of the anticipated tax hike on short-run demand decreases, thereby strengthening the initial partial equilibrium demand boom. Similarly, the larger H, the longer the general equilibrium feedback—i.e., the Keynesian cross—can play out before being moderated by the future tax hike. The size of the short-run boom thus is increasing in H, and by extension so is the endogenously raised tax revenue. Finally, we note that the exact same logic also explains why the equilibrium boom becomes larger and more persistent for smaller τ_d under our baseline fiscal rule (7). Either way, as fiscal adjustment is delayed further and further, the short-run Keynesian boom on its own becomes big enough to stabilize debt.

Tax base vs. debt erosion. The intuition developed above throughout assumed rigid prices (i.e., $\kappa = 0$)—or, by the discussion following Proposition 1, that prices can adjust ($\kappa > 0$) but public debt is indexed. When prices can adjust *and* public debt is nominal, the only twist is the rescaling of the initial impulse: the fiscally-led real boom leads to an increase in p_0 and thereby a reduction in d_0, partially offsetting the exogenous increase in ε_0. Compared to the case with rigid prices, this moderates the fiscally-led boom, but does not reduce the scope for self-financing—the Keynesian feedback loop, as articulated above, is now simply smaller because part of the fiscal self-financing is achieved through debt erosion. Formally, as κ increases from 0 towards ∞, the cumulative output multiplier decreases from $1/\tau_y$ to 0, spanning the spectrum between $\nu = \nu_y = 1$ and $\nu = \nu_p = 1$. Our quantitative analysis in Section 6 will shed light on where, between these two extremes, we are likely to be in practice.

A comment on generality. The intuitions offered above transcend the particular OLG economy underlying Theorem 1. We will make this claim precise in Section 5.2, where we generalize our self-financing result to richer aggregate demand structures. In line with the intuition provided above, the key will be to ensure that deficits induce a positive and front-loaded response in aggregate demand. Once this is true, the Keynesian feedback loop—and in particular the property that the cumulative MPC is one—delivers self-financing in exactly the same way as it does in our simple OLG economy.

4.4 Speed of convergence, double limits, and the role of ω

To complete our discussion of the economics underlying the self-financing result, we now dig deeper into the role of ω, investigating in particular what happens for ω close to (but below) 1. We begin by stating comparative statics with respect to ω for our main fiscal policy specification (7).

Proposition 3. *Suppose that $\omega < 1$ and $\tau_y > 0$ and consider the fiscal policy rule* (7).

1. *For any $\tau_d > 0$, a lower ω raises ν and decreases ρ_d; that is, a larger departure from the permanent-income benchmark yields both larger and faster self-financing.*

2. *Let ρ_d^{full} be the limit of ρ_d, i.e., the persistence of government debt and output as $\tau_d \to 0$ (from above). Then, $\rho_d^{full} < 1$ for any $\omega < 1$, but $\rho_d^{full} \to 1$ as $\omega \to 1$.*

The first part of Proposition 3 is straightforward: for any given finite delay in fiscal adjustment (i.e., for any given $\tau_d > 0$), a larger departure from the PIH benchmark (i.e., a smaller ω) implies a larger and quicker Keynesian boom, thus increasing ν and decreasing ρ_d. The second part then zeroes in on how the magnitude of this departure matters for the limit of an infinite delay in fiscal adjustment (i.e., for $\tau_d \to 0$). Provided that $\omega < 1$, we can always achieve full self-financing by pushing the fiscal adjustment sufficiently far in the future. However, the closer ω is to 1, the smaller and less front-loaded the resulting Keynesian boom is, and hence the longer it takes for public debt to return to steady state. This second part reveals that self-financing near the permanent-income limit (i.e., for $\omega \approx 1$) is really about the "order of limits": for any $\omega < 1$, full self-financing ($\nu \to 1$) obtains as $\tau_d \to 0$; however, for any $\tau_d > 0$, self-financing becomes zero ($\nu \to 0$) as $\omega \to 1$. In particular, by taking $\omega \to 1$ and $\tau_d \to 0$ at the same time, *any* value of $\nu \in (0,1)$ can be obtained.[21]

Looking ahead. A practical takeaway from the above discussion is that the empirical relevance of the self-financing mechanism will hinge on how quickly it plays out—i.e., how quickly consumers spend any given transfer—relative to plausible delays in fiscal adjustment. Our quantitative analysis in Section 6 will thus pay particular attention to the available empirical evidence on consumer behavior and the speed of fiscal adjustment.

5 Extensions

This section discusses several extensions of the self-financing result. First, in Section 5.1, we relax our assumption of a "neutral" monetary policy. Second, in Section 5.2, we consider a reduced-form aggre-

[21] The same points also apply to the fiscal "H-rule" (8): for any $\omega < 1$, full self-financing ($\nu \to 1$) obtains as $H \to \infty$, but for any finite H, self-financing becomes zero ($\nu \to 0$) as $\omega \to 1$.

gate demand relation that nests—but also goes materially beyond—our baseline OLG environment. Further extensions are collected in Section 5.3.

5.1 More general monetary policy

We continue within the model environment of Section 2; however, unlike the analysis in Sections 3 and 4, we relax the assumption of a "neutral" monetary policy. Specifically, we now consider the more general monetary rule (10), restated here for convenience:

$$r_t = \phi y_t \tag{28}$$

The $\phi > 0$ scenario can be interpreted as a hawkish monetary authority that leans against the deficit-led boom. The $\phi < 0$ scenario, on the other hand, can help capture two kinds of monetary accommodation: the case of a monetary authority that purposefully lets the government's cost of borrowing fall when fiscal space is tight; and that of a monetary authority constrained by the ZLB.[22]

We now ask how moving to $\phi \neq 0$ affects the self-financing results. For this we will restrict $\phi \in [-1/\sigma, \hat{\phi})$, where $\hat{\phi} \equiv \frac{\tau_y}{\beta \frac{D^{ss}}{Y^{ss}}}$. These bounds have a simple interpretation: when $\phi = -1/\sigma$, the monetary accommodation and the deficit-driven boom are so large that there is *immediate* self-financing; and when $\phi \geq \hat{\phi}$, the increase in the interest-rate costs of public debt is so large that the tax-base channel is fully counteracted. We continue to associate "self-financing" with the same tax base and debt erosion effects as before; the generalized definition of ν is provided in Appendix B.1. We then obtain:

Theorem 2. *Consider our OLG-NK environment with $\omega < 1$ and $\tau_y > 0$, and let fiscal and monetary policy obey, respectively,* (7) *and* (28). *Then there exists a $\bar{\phi} \in \left(0, \frac{\tau_y}{\beta \frac{D^{ss}}{Y^{ss}}}\right)$ such that:*

1. *If $\phi < \bar{\phi}$, as $\tau_d \to 0$ (from above), there exists a unique bounded equilibrium, and it is such that $\nu \to 1$ (from below) and $\lim_{k\to\infty} \mathbb{E}_t[d_{t+k}] \to 0$. That is, full self-financing obtains as fiscal adjustment is indefinitely delayed.*

2. *If $\phi > \bar{\phi}$, there exists a $\bar{\nu}(\phi) \in (0,1)$, such that any bounded equilibrium has a degree of self-financing ν with $\nu < \bar{\nu}(\phi)$.*

The intuition underlying Theorem 2 is straightforward. If $\phi < 0$ (i.e., monetary accommodation), then, in response to the fiscally-led boom, real rates fall, which both helps the government budget and leads to households front-loading their spending even more. The boom is thus even quicker and so debt is even less persistent—i.e., we obtain a smaller ρ_d than in Theorem 1. It follows that, with monetary accommodation, less of a delay in fiscal adjustment is needed to ensure material self-financing.

[22] To understand the latter case, note that, as long as $\kappa > 0$, any deficit-led boom comes together with inflation; and as long as the ZLB binds, such inflation translates one to one to negative real rates, or equivalently to $\phi < 0$.

Conversely, if $\phi > 0$ (i.e., the monetary authority leans against the boom), then real rates rise, leading households to postpone their spending, thus delaying the boom and giving larger ρ_d. The cutoff $\bar{\phi}$ is exactly the point where the monetary policy-induced delay prevents self-financing from returning real government debt to steady state, delivering $\rho_d^{\text{full}}(\bar{\phi}) = 1$. For any strictly more aggressive monetary policy ($\phi > \bar{\phi}$), full self-financing is not possible. In that case, the required fiscal adjustment is bounded away from zero: for any $\phi > \bar{\phi}$, there exists a threshold $\underline{\tau}_d$ such that a (bounded) equilibrium exists and government debt returns to steady state if and only if $\tau_d > \underline{\tau}_d(\phi)$. In particular, in this case, self-financing is necessarily partial, with ν bounded above by $\bar{\nu} < 1$.

The discussion surrounding Theorem 2 is qualitative, demonstrating how the monetary reaction affects the degree of self-financing, in particular identifying conditions under which full self-financing remains possible. The quantitative analysis is provided in Section 6, where we investigate how self-financing varies under empirically relevant assumptions about the nature of the monetary reaction.

5.2 A more general aggregate demand structure

We now extend our results to a more general aggregate demand relation. Doing so allows us to: shed further light on the economics of our result; understand better the kind of environments that allow (or do not allow) for meaningful self-financing; and build a bridge to quantitative HANK models.

Generalizing the demand block. Recall that, in our baseline OLG environment, aggregate consumer demand was given by the following function of current household wealth as well as current and future income net of taxes:

$$c_t = (1-\beta\omega)\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\left(y_{t+k} - t_{t+k}\right)\right]\right). \tag{29}$$

This relation embeds economically meaningful restrictions on consumer behavior: the MPC out of current income and wealth is the same (equal to $1-\beta\omega$), and the MPCs out of the discounted presented value of future disposable income decline at a constant rate ω. We consider a generalized aggregate demand relationship that relaxes these constraints while preserving tractability:

$$c_t = M_d \cdot d_t + M_y \cdot \left(\left(y_t - t_t\right) + \delta \cdot \mathbb{E}_t\left[\sum_{k=1}^{\infty}(\beta\omega)^k\left(y_{t+k} - t_{t+k}\right)\right]\right). \tag{30}$$

Equation (30) generalizes (29) in three meaningful ways. First, it allows for different MPCs out of current income and wealth, denoted respectively by $M_y \in (0,1)$ and $M_d \in (0,1)$, and we will assume that $M_d \leq M_y$. Second, it disentangles the geometric discounting rate ω from the contemporaneous MPCs (now M_y and M_d instead of $1-\beta\omega$). And third, it additionally allows for constant discounting at rate $\delta \in [0,1]$. We will later elaborate on the economics of these extensions and give examples of several familiar and empirically relevant models that can be nested in this general structure.

A general sufficient condition for self-financing. We now revisit our self-financing result with the generalized demand relation (30) replacing the simpler OLG demand block. It turns out that self-financing obtains under two key restrictions on (30), echoing the "discounting" and "front-loading" properties highlighted in Section 4.3.

Assumption 1. *The aggregate consumption function features positive geometric discounting:* $\omega < 1$.

In words, MPCs out of future disposable income relative to MPCs out of current income and wealth decline strictly faster than the rate of interest β. Consistent with our discussion in Section 4.3, this is sufficient to ensure that far-ahead future tax hikes—i.e., tomorrow's tax hike in the analogy in Section 4.3—have vanishingly small effects on current aggregate demand, similar to the baseline OLG model. The fiscal deficit shock will thus lead to a demand boom around date 0.

Assumption 2. *Intertemporal MPCs are sufficiently front-loaded in the particular sense that*

$$M_d + \frac{1-\beta}{\tau_y}\left(1-\tau_y\right) M_y \left(1 + \delta \sum_{k=1}^{\infty} (\beta\omega)^k\right) > \frac{1-\beta}{\tau_y}. \tag{31}$$

For (31) *to hold for all* $\tau_y \in (0,1]$, *the sufficient and necessary condition is*

$$M_d > 1-\beta \quad \text{and} \quad M_y \left(1 + \delta \frac{\beta\omega}{1-\beta\omega}\right) \geq 1. \tag{32}$$

(31) is the condition required to ensure that the persistence of government debt ρ_d in the limiting full self-financing equilibrium is strictly less than 1—i.e., that government debt will return to trend even as the future tax hike becomes vanishingly small. Intuitively, this requires the general equilibrium Keynesian boom to be sufficiently front-loaded, which in turn requires households to spend any income gains sufficiently quickly. If MPCs out of income and wealth are large enough—in the precise sense of (32) or (31)—then household spending is indeed sufficiently fast.[23]

Together, Assumptions 1 and 2 suffice to generalize our self-financing result.

Theorem 3. *Suppose that aggregate demand takes the generalized form* (30) *and that Assumptions 1 and 2 hold. Let fiscal policy follow either our baseline rule* (7) *or the variant rule* (8). *As* $\tau_d \to 0$ *(from above) or* $H \to \infty$, *the equilibrium degree of self-financing* $\nu \to 1$ *(from below).That is, full self-financing obtains as fiscal adjustment is indefinitely delayed. Moreover, self-financing is sufficiently strong to return real government debt to steady state (i.e.,* $\lim_{k\to\infty} \mathbb{E}_t[d_{t+k}] \to 0$ *for* (7) *and* $\lim_{H\to\infty} \mathbb{E}_0[d_H] \to 0$ *for* (8)*).*[24]

[23] Specifically, the first condition in (32) (i.e., that $M_d > 1-\beta$) corresponds to the second property of the consumption function ("front-loaded MPCs") discussed in Section 4.3—that is, it ensures that $\lim_{\ell\to\infty} \mathscr{M}_{t+\ell,t} = 0$. The second condition in (32) guarantees that the general equilibrium Keynesian cross feedback is front-loaded enough to stabilize debt.

[24] For the baseline fiscal rule (7), when $\tau_d \to 0$ (from above), a unique bounded equilibrium of the form

$$y_t = \chi_d d_t + \chi_\varepsilon \varepsilon_t, \quad \mathbb{E}_t[d_{t+1}] = \rho_d d_t + \rho_\varepsilon \varepsilon_t \quad \text{with} \quad \rho_d \in (0,1), \tag{33}$$

Theorem 3 and its underlying assumptions echo our intuitive discussion offered in Section 4.3. First, Assumption 1 guarantees that the future tax hike is *discounted*, so deficits will lead to a short-run boom. Second, Assumption 2 then ensures that any additional income is spent sufficiently quickly to deliver a *front-loaded* boom that raises the required tax revenue before the promised future tax hike becomes necessary. In the remainder of this section we will discuss examples of specific models of household demand that fit into the general form (30) and either satisfy or violate Assumptions 1 - 2.

What environments are consistent with self-financing? The generalized demand block (30) is rich enough to capture the essence of models with occasionally-binding liquidity constraints and informational or behavioral frictions. By Theorem 3, such frictions suffice to deliver our results.

Our baseline OLG model—which is obviously nested in (30)—can be interpreted as a reduced-form representation of occasionally binding liquidity constraints (Farhi and Werning, 2019), with $1-\omega$ giving the probability of the constraint binding. Our generalized aggregate demand relation goes one step further, disentangling the MPC out of wealth M_d and income M_y, thereby allowing us to nest "hybrid" models that combine the OLG block with a margin of hand-to-mouth spenders. Such models have received attention in recent work because they tend to provide a relatively accurate approximation of aggregate demand in richer HANK models (Auclert et al., 2023; Wolf, 2021), and it is straightforward to verify that they satisfy Assumptions 1 and 2.[25] Our quantitative analysis in Section 6 will rely on a particular, calibrated version of this hybrid model; we will furthermore numerically illustrate self-financing in even richer models that go beyond (30), including a quantitative HANK model.

Finally, the demand block (30) can also capture the effects of incomplete information (Angeletos and Lian, 2018), limited rationality (Farhi and Werning, 2019; Vimercati et al., 2021; Angeletos and Sastry, 2021), as well as cognitive discounting (Gabaix, 2020). The common implication of these frictions is to dampen forward-looking behavior; this then translates to lower ω in (30), delivering Assumption 1. As long as MPCs are front-loaded enough to deliver Assumption 2, Theorem 3 again applies.

Adding permanent-income consumers. The canonical PIH model readily fits into our generalized aggregate demand structure with $M_d = M_y = 1 - \beta$ and $\delta = \omega = 1$. It is immediate that Assumptions 1 and 2 are violated: a deficit today induces no boom (since there is no discounting), and even if there was a boom, that boom would never be front-loaded enough, as permanent-income consumers postpone part of their spending to the infinite future.

Next, the self-financing result also fails when there is a margin of *truly* PIH consumers. For example, it is straightforward to see that our generalized aggregate demand relation nests the classical

exists, analogous to (19). For the alternative rule (8), existence and uniqueness follow directly from Footnote 15.

[25] To be precise, such hybrid models are nested in (30) by letting ω equal the survival rate of the OLG households and by setting $M_y = \mu + (1-\mu)(1-\beta\omega)$, $M_d = 1 - \beta\omega$, and $\delta = \frac{(1-\mu)(1-\beta\omega)}{\mu+(1-\mu)(1-\beta\omega)}$, where $\mu \in (0,1)$ is the fraction of hand-to-mouth spenders. As long as $\omega < 1$, both Assumptions 1 and 2 are satisfied, and hence our self-financing result holds.

spender-saver model, populated by hand-to-mouth spenders and PIH consumers (as in Campbell and Mankiw, 1989; Galí et al., 2007).[26] Assumptions 1 - 2 are violated, and the share of self-financing ν is 0: there is a date-0 boom, but that boom is then offset by a subsequent bust of equal size. The intuition is that, since expected real rates are fixed, if the net present value of the output response were not equal to zero, the consumption of PIH consumers would permanently deviate from steady state, and so output would not return to steady state over time. This same logic also applies if, for example, a margin of PIH consumers is added to our baseline OLG environment. Thus, in that case, the share of self-financing ν is *discontinuous* in the margin of PIH consumers, equaling to zero as soon as that margin is strictly positive. We provide a visual illustration of this discussion in Appendix B.2.

How should we interpret this discontinuity? The discussion echoes our earlier point about the order of limits: if we replace the aforementioned margin of *literal* PIH consumers with one of *near*-PIH consumers, then full self-financing is again recovered in the limit as $\tau_d \to 0$ or $H \to \infty$.[27] Even this near-PIH scenario, however, is hard to defend on empirical grounds. First, any positive margin of near-PIH consumers implies that the long-run interest rate elasticity of asset demand is nearly infinite (e.g., see Kaplan and Violante, 2018), an implication grossly inconsistent with empirical evidence (Moll et al., 2022). Second, the available microeconomic evidence (e.g., Fagereng et al., 2021) suggests high MPCs even for rich, liquid consumers. Last but not least, bounded rationality can make households behave *as if* they have shorter horizons (Angeletos and Lian, 2018; Gabaix, 2020; Woodford, 2019). Ultimately, the practical relevance of our self-financing results is an inherently quantitative question: for empirically realistic delays in fiscal adjustment, are households *sufficiently* far from the infinite-horizon, infinite-liquidity, infinite-rationality limit of PIH behavior to deliver meaningful self-financing? We investigate that question in Section 6.

5.3 Further extensions

We finally discuss three further extensions: to alternative assumptions on fiscal policy—distortionary tax hikes and stimulus in the form of government purchases—and to a richer environment that allows for capital accumulation and investment.

Distortionary tax hikes. The analysis thus far has treated the fiscal adjustment margin as non-distortionary. Online Supplement E.2.1 demonstrates that this simplification incurs no serious loss. Intuitively, since the necessary adjustment vanishes in the limit as $\tau_d \to 0$ or $H \to 0$, it is entirely im-

[26] We here have $M_y = \mu + (1-\mu)(1-\beta)$, $M_d = 1-\beta$, and $\delta = \frac{(1-\mu)(1-\beta)}{\mu+(1-\mu)(1-\beta)}$, where $1-\mu \in (0,1)$ is the fraction of PIH consumers.

[27] Theoretically, as an alternative to assumptions on the order of limits, continuity of ν in the size $1-\mu$ of the margin of PIH households can also be restored by modifying the policy mix. For example, if the (delayed, promised) fiscal adjustment falls on permanent-income households, then, for our variant fiscal policy (8) and as $H \to \infty$, we obtain a self-financing share ν that is continuous in μ and converges to 1 (as expected) as $\mu \to 1$. See Appendix B.2 for further details.

material whether this adjustment is distortionary or lump-sum. Furthermore, away from this limit, the distinction matters only through the NKPC: if taxes are distortionary, tax changes show up as a cost-push shock. Consistent with our discussion in Sections 4.3 and 4.4, such cost-push shocks affect the size of the inflation response and thus the split between tax base and debt erosion components of self-financing, but otherwise leave our results unchanged.[28]

Government spending. While we have so far considered a policy of lump-sum transfers to households, it is straightforward to see that our results extend with almost no change to deficit-financed government purchases—such purchases lead to a boom, and the boom can finance the initial expenditure for exactly the same reasons as in our main analysis. See Online Supplement E.2.2.

Investment. Our self-financing insights continue to apply in the presence of firm investment. We sketch the main ideas here and delegate details to Online Supplement E.2.3. Relative to our baseline model, this model extension features a meaningfully generalized supply block, with firms employing both labor and capital, and with capital owned and accumulated by the firms themselves. Firms continue to distribute all their earnings, now net of investment costs, as dividends to the households. It follows that, similar to our baseline model, aggregate firm saving (now net of physical investment) is zero and all public debt is still owned and accumulated by the households. We assume that the fiscal authority taxes household labor income and dividend receipts, as in the baseline environment.

For the purposes of our "self-financing" result, the main insight is that the aggregate demand relation (13) continues to hold, just now in terms of consumption c_t rather than output y_t. Put differently, the Keynesian cross now applies to output *net of investment*. Furthermore, the law of motion for public debt in equation (17) also continues to hold (with c_t instead of y_t). It follows that the equilibrium continues to be characterized as in Proposition 1, with y_t re-interpreted as c_t, and so nothing of essence changes. In particular, the *only* thing that is different is how the dynamics of investment and capital influence real marginal costs, and thereby date-0 inflation—a complication that, as we have seen, does not interfere with the economic intuition at the heart of our self-financing results.[29]

6 Quantitative analysis

We now investigate the practical relevance of our insights. Specifically, we study the importance of self-financing in environments designed to be consistent with empirical evidence on the key ingredi-

[28]The supply-side effects of fiscal adjustment instead play a more central role in Schmitt-Grohé and Uribe (2004, 2007).

[29]Additional subtleties would arise if firms could save or borrow, as this could influence how much of the increase in public debt translates to an increase in household wealth. On the other hand, nothing in our arguments depends on on the abstraction from adjustment costs and variable capacity utilization, the familiar supply-side enrichments found in quantitative business-cycle models (e.g., Smets and Wouters, 2007).

ents of our theory: how far consumers are from the PIH limit; how delayed fiscal adjustment is; how monetary policy reacts; and how strong nominal rigidities are likely to be.

6.1 Model and calibration strategy

We consider a variant of our model in Section 2 with the baseline fiscal rule (7), and with just one minor twist: we add a margin $\mu \in (0,1)$ of hand-to-mouth spenders, along the lines discussed in Section 5.2. These agents receive the same income and pay the same taxes as the perpetual-youth consumers, but only the latter save and hold government debt. Further details are presented in Appendix C.1.

This hybrid spender-OLG model is the ideal environment for our quantitative analysis, for two reasons. First, it remains simple enough to fit into the generalized demand block of Section 5.2. As a result, we can readily invoke Theorem 3 to verify whether full self-financing obtains in the limit as $\tau_d \to 0$. Second, and more importantly, it is rich enough to agree with quantitative HANK models on their implied intertemporal MPCs (Auclert et al., 2023; Wolf, 2021) and to match the relevant empirical evidence.[30] Since the micro-foundations of aggregate demand affect the degree of self-financing only through those intertemporal MPCs (see Online Supplement E.1 for the formal discussion), it follows that our calibrated model is a suitable laboratory for evaluating whether a quantitatively meaningful degree of self-financing can obtain for realistic parameterizations of the intensity of fiscal adjustment, as herein parameterized by τ_d. This is the "race" between empirically relevant delays in fiscal adjustment and distance from PIH consumption behavior that we discussed at the end of Section 5.2.

Our discussion of the model calibration focuses on the model ingredients that matter most for our theory: consumer spending behavior; fiscal adjustment; the monetary policy reaction; and the strength of nominal rigidities. The remaining model parameters are discussed at the end. Each period in the model corresponds to a quarter. A summary of parameters is provided in Table 1.

Consumer spending behavior. Empirical evidence on the household-level consumption response to income gains suggests two salient features of consumer spending behavior (Fagereng et al., 2021). First, the average MPC out of income gains is elevated, with a standard quarterly value of around 0.2. Second, the income gain is spent gradually. Our baseline OLG environment of Section 2 provides a tight joint restriction on the level of the MPC and its dynamics: the level MPC (i.e., entry $\mathcal{M}_{0,0}$ of the matrix of MPCs) is $1-\beta\omega$, while the slope of the spending profile (i.e., the ratio of $\mathcal{M}_{\ell,0}$ to $\mathcal{M}_{\ell-1,0}$) is ω. This model-implied connection between level and slope is, unfortunately, inconsistent with the data; in particular, relative to the impact MPC, income in the data is subsequently spent much more quickly than predicted by the theoretical OLG-implied relationship. The spender-OLG hybrid model instead allows us to disentangle the level and the intertemporal pattern of the spending response to income

[30] We note that the same is not true for our baseline OLG model; see the first point of the discussion on calibration below.

Parameter	Description	Value	Target	Value
Consumer spending				
μ	Share of HtM	0.073	Average MPC	0.2
ω	OLG survival rate	0.865	Short-run MPC slope	0.865
Policy				
τ_d	Tax feedback	$\{0.085, 0.026, 0.004\}$	Literature, see text	
ϕ	Real rate feedback	0	Literature, see text	
Nominal rigidities				
κ	NKPC slope	0.01	Standard	
Rest of the model				
σ	EIS	1	Standard	
β	Discount factor	0.998	Annual real rate	0.01
τ_y	Tax rate	0.33	Average Labor Tax	0.33
D^{ss}/Y^{ss}	Gov't debt	1.04	Liq. wealth holdings	1.04

Table 1: Hybrid OLG-spender model, calibration.

gains in a way that is consistent with empirical evidence; in particular, we choose the spender share μ and the OLG coefficient ω to jointly match (i) impact and (ii) short-run (i.e., up to two years out) spending responses to lump-sum transfer receipt, as estimated by Fagereng et al.. A visual illustration of the implied intertemporal MPCs is provided in the top left panel of Figure 4, and a discussion of several alternative calibration strategies will follow in Section 6.2.

Current and future MPCs out of today's income gains—i.e., the estimand of Fagereng et al.—is of course not all that matters for our theory; for our general equilibrium Keynesian cross, it is similarly important how anticipated income changes in the (far) future affect spending today (i.e., entries $\mathscr{M}_{t,t+\ell}$ for large ℓ in the matrix of MPCs). Given the lack of empirical evidence on such responses to (far-away) income news shocks, our baseline exercise simply extrapolates these spending responses through the structure of the model. Reassuringly, as discussed in Wolf (2021), our spender-OLG model extrapolates in a way very similar to quantitative HANK models. Nevertheless, we later also consider how results change with additional discounting due to limited knowledge or limited rationality, consistent with suggestive evidence from Ganong and Noel (2019).

Finally, and consistent with the theoretical discussion in Section 5.2, we also emphasize that, while not targeted, our model also matches empirical evidence on long-run household asset demand elasticities. A detailed analysis is provided in Online Supplement E.5.

Policy. For our quantitative analysis we focus on fiscal rule (7), in line with a large literature following Leeper (1991). Conveniently, because prior work has estimated this type of rule, we can draw from it to discipline our calibration of τ_d; we consider a range of values taken from Galí et al. (2007), Bianchi and Melosi (2017), and Auclert and Rognlie (2020), as displayed in the middle part of Table 1. The first value ($\tau_d = 0.085$) reflects very rapid fiscal adjustment: a one-dollar increase in public debt surplus leads to an 8.5-cent quarterly increase in tax revenue. The second value ($\tau_d = 0.026$) still corresponds to a quite meaningful quarterly tax response. Finally, our third value ($\tau_d = 0.004$) indicates relatively slow—but importantly still passive, in the language of the FTPL literature (i.e., we have that $\tau_d > 1 - (R^{ss})^{-1}$)—fiscal adjustment.[31]

For monetary policy, we as our baseline consider the case of an acyclical real interest rate—i.e., $\phi = 0$, just as in Sections 3 - 4. In addition to being theoretically convenient, this specification is also arguably empirically relevant. First, in response to identified fiscal shocks in the literature, nominal interest rates tend to move rather little (see Ramey, 2011; Caldara and Kamps, 2017; Wolf, 2023) while inflation increases, suggesting that real interest rates, if anything, decrease. Second, following the post-covid fiscal stimulus, real interest rates only started rising with a substantial delay, and initially even declined. We will nevertheless also evaluate the scope for self-financing under a more aggressive monetary reaction function.

Nominal rigidities. We set the slope of the NKPC to 0.01, a value consistent with the related empirical and quantitative literatures (e.g., see Galí and Gertler, 1999; Christiano et al., 2005; Hazell et al., 2022; Barnichon and Mesters, 2020). In one of our robustness exercises, we will consider a slope of 0.1—an ad hoc, materially larger value that could be relevant in the inflationary, post-COVID environment driven by supply constraints.

Rest of the model. The remaining model parameters are set to standard values. First, we set $\sigma = 1$, giving log preferences. Second, we set the discount factor to hit a steady-state real rate of interest of one per cent (annual). Third, we set $\tau_y = 0.33$, in line with DeLong and Summers (2012): for every dollar of additional output, we assume that the primary surplus automatically rises by 33 cents.

6.2 Are stimulus checks plausibly self-financed? And if so, how?

We now use our calibrated model to study the propagation of a date-0 "deficit shock"—i.e., uniform stimulus checks sent to all households. It is straightforward to verify that the model satisfies the suf-

[31] A detailed literature review is provided in Auclert et al. (2020). Our three values for τ_d correspond to their lower bound, preferred estimate, and upper bound for the fiscal adjustment coefficient (ψ in their notation). See Online Supplement E.4 for further details. Translated to our alternative non-Markovian "H"-rule, these three coefficients correspond—in terms of the self-financing share ν that they imply (see Figure 3)—to $H = \{12, 23, 43\}$, where H is now in quarters.

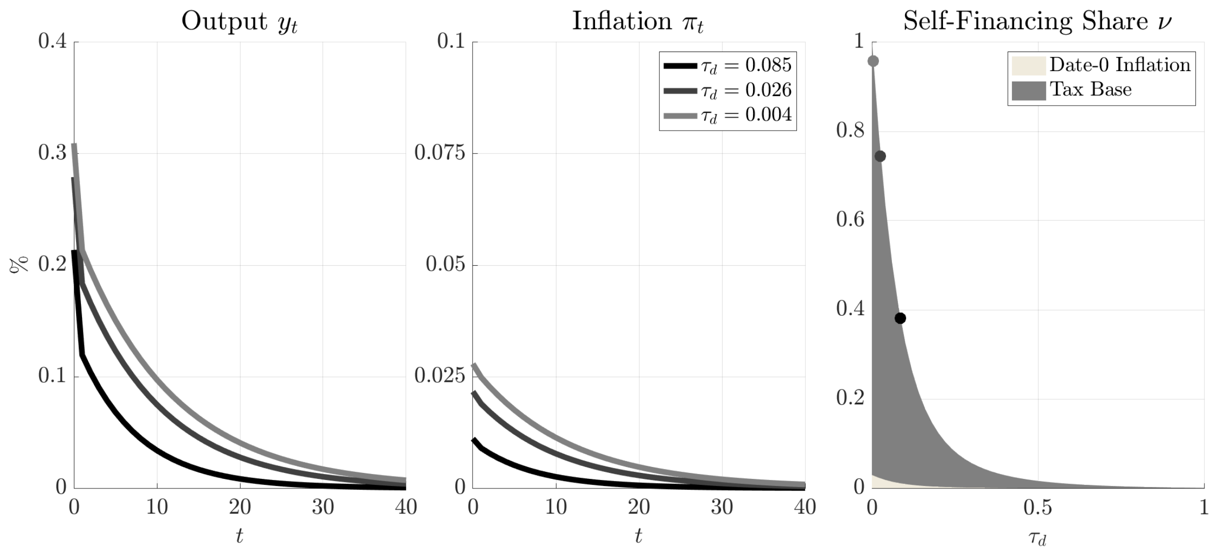

Figure 3: Impulse responses of output y_t, inflation π_t, and the total self-financing share ν to a unit-size shock ε_0 as a function of τ_d. The left and middle panels show the impulse responses for the three values of τ_d discussed in Section 6.1. In the right panel these three points are marked with circles.

ficient conditions identified in Theorem 3 required for full self-financing to be possible in the limit as $\tau_d \to 0$. What remains is to ascertain: (i) how close we are to this limit for the range of τ_d suggested by the relevant empirical literature; (ii) how the resulting self-financing is split between the tax base and debt erosion channels. Results are displayed in Figure 3.

The figure shows output (left panel) and inflation (middle panel) impulse responses to the date-0 deficit shock for our three fiscal adjustment coefficients τ_d (shades of grey); the right panel furthermore shows the degree of self-financing ν as a function of τ_d over the entire unit interval. As expected, the share of self-financing ν is decreasing in τ_d and approaches one as $\tau_d \to 0$, consistent with Theorem 3. For the purposes of the quantitative analysis here, the key takeaway is that we are already quite close to this limit for the values of τ_d estimated in the aforementioned works, with ν up to 0.95. Furthermore, given our assumed—empirically disciplined—flat NKPC, almost all of this self-financing is coming through the tax base channel rather than through inflation.[32]

In the remainder of this section we discuss how these results change under alternative assumptions on the key ingredients of our theory: consumer spending behavior; the monetary reaction; and the strength of nominal rigidities. Throughout, unless otherwise indicated, we maintain our empirically disciplined assumptions on the speed and strength of fiscal adjustment.

[32]The implied cumulative fiscal multipliers are broadly consistent with empirical evidence. That evidence—for example Ramey (2011)—tends to find cumulative fiscal multipliers of around 1 for fiscal expansions that are financed with higher taxes *after just a couple of years*—i.e., relatively quick fiscal adjustment. Our first calibration of $\tau_d = 0.085$ corresponds to such relatively quick adjustment, and delivers a cumulative multiplier of 1.11. Evidence on transfer multipliers that is consistent with *full* self-financing is furthermore provided in Bouscasse and Hong (2023).

Consumer behavior. As remarked in Section 6.1, our calibration strategy for consumer spending behavior—and thus the distance from the PIH limit—relied on two standard but potentially material assumptions: first, we disciplined the slope of the spending profile (i.e., entries $\mathscr{M}_{\ell,0}$) from evidence on relatively *short-run* spending behavior (small ℓ); and second, we relied on model structure to extrapolate from responses to contemporaneous income changes to spending behavior following income news shocks (i.e., entries $\mathscr{M}_{t,t+\ell}$). We now investigate how our conclusions change with alternative calibration strategies and modeling assumptions.

- *Spending responses in the tails.* As we emphasized throughout Sections 4 and 5.2, sufficiently front-loaded intertemporal MPCs are important for our self-financing results—they are what ensures a front-loaded Keynesian boom. We thus now consider alternative calibration strategies that try to more directly leverage information on spending responses to lump-sum transfer receipt in the *tails* (i.e., $\mathscr{M}_{\ell,0}$ for large ℓ). These tail responses are crucial to determine how fast cumulative MPCs converge to 1 and thus how front-loaded the Keynesian boom turns out to be.

  Results are reported in Figure 4. The two panels correspond to two different models: the calibrated model that we have worked with so far (top panel) and an alternative model (bottom panel). Each panel shows: model-implied intertemporal MPCs (left); cumulative MPCs in model vs. data, with the data targets again taken from Fagereng et al. (Figure 2, 2021), corresponding to 95 per cent confidence intervals (middle); and the self-financing share ν as a function of τ_d (right). The main takeaway from the figure is that material self-financing obtains across calibrations. Consider first the top panel. By construction, this model matches MPCs—and thus also cumulative MPCs—over the first couple of years after income receipt. As discussed in Online Supplement E.5, that model is also consistent with empirical evidence on household asset demand elasticities. However, as revealed by the middle part of the top panel, the implied cumulative MPC appears to converge to 1 somewhat too quickly, reflecting intertemporal MPCs that converge to 0 relatively fast, at rate ω. In the bottom panel, we consider an extended model—one featuring spenders together with *two* types of OLG blocks, with heterogeneous ω_1 and ω_2—that is rich enough to provide a tight fit to the entire dynamic profile of cumulative MPCs, up to five years out. The left part of the bottom panel reveals that this model looks rather similar to our baseline calibration in the periods around income receipt, but then has somewhat flatter tail MPCs, reflecting the larger ω_2. Even though this model delivers an asset demand elasticity much larger than empirical estimates (again see Online Supplement E.5), we nevertheless still see material—though less pronounced—self-financing.

- *Spending response to news about future income.* All models discussed above are calibrated to be consistent with evidence on consumer spending behavior in response to income shocks *to-*

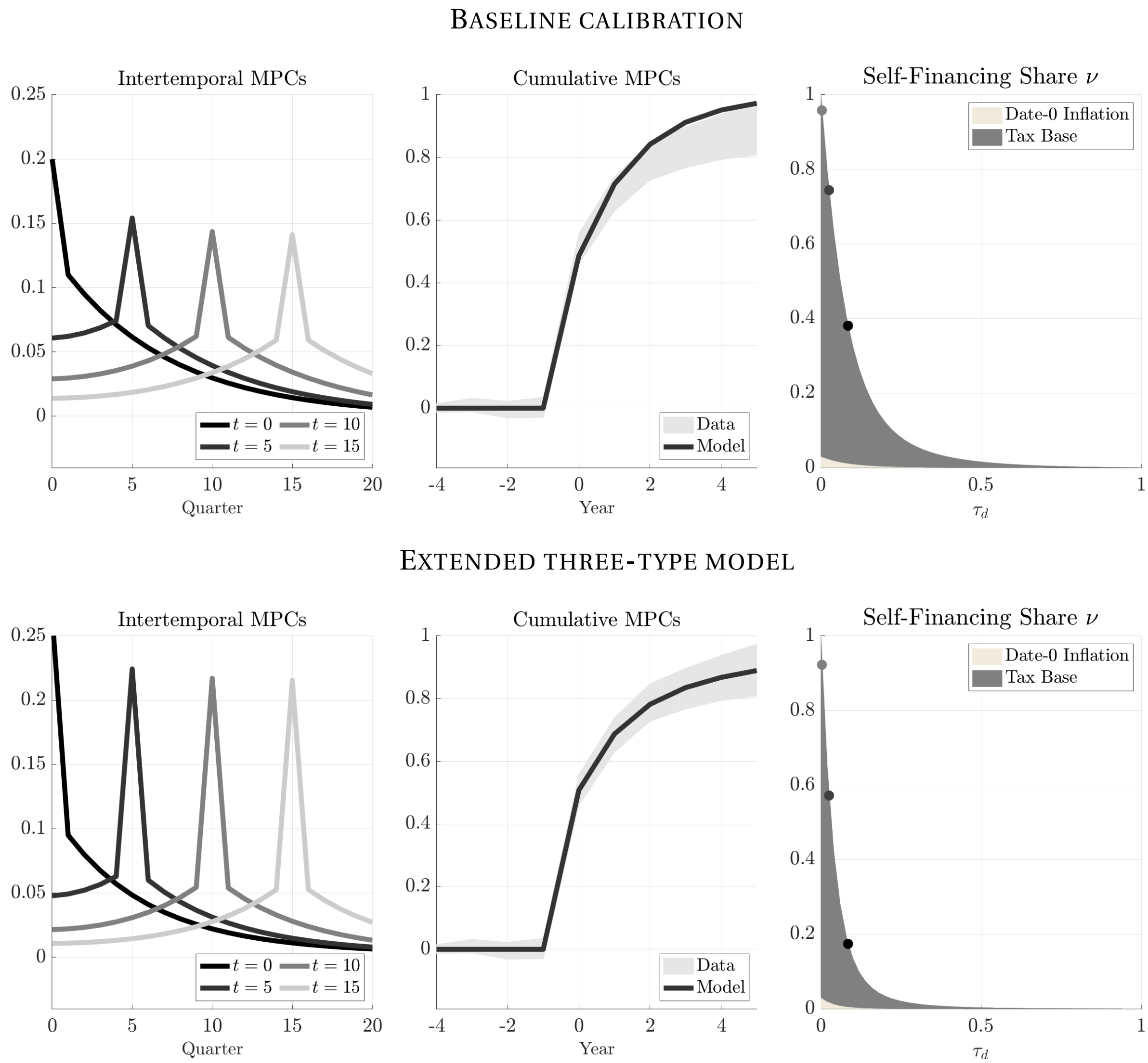

Figure 4: Top panel: iMPCs, cumulative MPCs, and self-financing share ν (as a function of τ_d) in the baseline quantitative model. Bottom panel: same as above, but for a model calibrated to (also) match far-ahead tail MPCs. Parameter values are reported in Online Supplement E.5.

day; the similarly important consumption response to "news shocks" (i.e., $\mathcal{M}_{t,t+\ell}$), on the other hand, is extrapolated through the model structure.

We here briefly discuss results from two additional models that extrapolate somewhat differently, with details in Online Supplement E.6.1 and E.6.2. First, we consider a fully-fledged HANK model. Consistent with Auclert et al. (2023) and Wolf (2021), we find that this model extrapolates MPCs across horizons in almost the same way as our simple spender-OLG hybrid model, so our conclusions on self-financing are largely unchanged. Second, we consider a variant of our spender-OLG model with cognitive discounting. We expect this behavioral friction to introduce two offsetting effects on ν—the future tax hike is discounted by more, but higher future

κ \ ψ	1	1.25	1.5
0.01	1	1	0.96
0.02	1	0.72	0.43
0.1	1	0.22	0.13

Table 2: Maximal degree of self-financing ν_{max} as a function of (ψ, κ).

income feeds back less strongly to the present. Our model simulations confirm this intuition: we find that ν is higher than in our baseline model for intermediate values of τ_d, but converges to the full self-financing limit somewhat more gradually.

Nominal rigidities. In the analysis above, self-financing occurs almost exclusively through the tax base channel, reflecting the fact that our assumed NKPC was—consistent with empirical evidence and most quantitative modeling—very flat. As discussed in Proposition 2, larger values of the slope coefficient κ move the split of self-financing from the tax base to the debt erosion channel.

In Appendix C.2, we repeat our analysis with more flexible prices, setting $\kappa = 0.1$. This value lies at the upper end of recent empirical estimates pertaining to the inflationary post-COVID episode (e.g., see Gagliardone et al., 2023, and the references therein). We find that with this alternative calibration, approximately 20 percent of self-financing is achieved through date-0 inflation.[33]

Monetary reaction. Finally, we investigate what happens if the monetary reaction is more aggressive. To facilitate comparison with the literature, we switch to a conventional Taylor-type rule, with nominal interest rates set to counteract contemporaneous increases in inflation:

$$i_t = \psi \pi_t, \tag{34}$$

for $\psi \geq 1$. Here, a more hawkish monetary rule (higher ψ) or a steeper NKPC (higher κ) mean that the monetary authority leans more aggressively against the fiscal boom. Either way, real rates will increase by more, and this in turn—as discussed in Section 5.1—limits the scope for self-financing. We now investigate how the *maximal* possible degree of self-financing, herein denoted by ν_{max}, varies over a plausible range of values for ψ and κ. [34]

Results are reported in Table 2. Full or near full self-financing remains possible if the monetary

[33]This number would change further if we replaced our standard NKPC with a hybrid one (as required to fit inflation data), or if we allowed for a more realistic maturity structure for government debt. We defer the investigation of these features to future work.

[34]Formally, ν_{max} is defined as the maximal value of ν over the values of $\tau_d \in [0,1]$ that are consistent with a unique, bounded equilibrium. Numerically, we solve for perfect-foresight transition sequences as a function of τ_d on a fine grid, ensuring that we only consider τ_d such that $\lim_{t\to\infty} y_t \to 0$ (up to numerical tolerance). We compute the implied ν and then record the largest one.

reaction is sufficiently weak or if the NKPC is sufficiently flat. Consistent with Theorem 2, the table furthermore shows that, the more aggressive the monetary reaction—due to higher ψ or κ—,the smaller the maximal degree of self-financing. Intuitively, if the monetary authority implements a sufficiently large increase in real rates, then it both arrests the Keynesian boom *and* directly increases the cost of government debt service. The scope for self-financing is thus limited, and fiscal adjustment needs to be large enough and fast enough to prevent debt from following an explosive path. A further visual illustration of these results is displayed in Appendix C.3.

Table 2—in conjunction with our headline analysis in Figure 3—contains two lessons. First, for deficit shocks without aggressive monetary offset, meaningful self-financing is plausible. The historical experience on the *conditional* comovement between output, inflation, and interest rates in response to fiscal shocks suggests that this case is practically relevant. The second and third columns of Table 2 then show clearly that this moderate monetary policy reaction is in fact critical: with a more aggressive offset, the scope for self-financing is meaningfully curtailed.

7 Conclusion

Can the government run a deficit now without having to hike taxes in the future? We addressed this question in model environments with two empirically relevant features: (i) nominal rigidities and (ii) a violation of Ricardian equivalence due to finite lives and liquidity constraints. In this class of environments, deficits naturally contribute to their own financing by stimulating aggregate demand. The demand-led boom in real economic activity allows the fiscal authority to collect more tax revenue without a hike in tax rates, while the accompanying surge in inflation helps reduce the real tax burden of the outstanding government debt.

Our main contribution is to clarify the theoretical properties and the quantitative relevance of such self-financing. On the theoretical front, we established that, unless monetary policy leans heavily against the deficit-driven boom, the following properties hold: (i) the extent of self-financing increases as the fiscal authority delays adjustment in taxes; (ii) as fiscal adjustment is delayed more and more, the initial deficit is fully self-financed; and (iii) this convergence is faster the larger the departure from permanent-income consumption behavior, or the higher the short-run MPCs. On the quantitative front, we showed that a sizable degree of self-financing obtains when the theory is disciplined with the available evidence on intertemporal MPCs, the delays in fiscal adjustment, the strength of nominal rigidities, and the relation between real rates and fiscal shocks.

These lessons, together with an eclectic perspective on the applicability of the New Keynesian model, lead us to the following conclusion: we expect self-financing—and in particular self-financing via tax base expansion—to be a real-world possibility when there is sufficient slack in the economy in the dual sense of (i) aggregate employment and output being demand-determined, and (ii) increases in aggregate demand translating to a real boom without significant inflationary pressures. When the first condition is violated, the applicability of the New Keynesian model becomes questionable. And when the second condition is violated, a given fiscal stimulus is likely to trigger a more aggressive monetary response, limiting the scope for a Keynesian boom and hence for self-financing.

Our analysis suggests at least two avenues for future research. First, to assess the likely empirical relevance of our results, we disciplined the theory with the best available evidence on consumer behavior, the slope of the Phillips curve, and the policy reaction. A more direct, less theory-based, empirical test could compare the dynamic causal effects of equally large but differentially financed changes in government spending, using the standard macroeconometric toolkit on identified shocks (e.g., Ramey, 2016). Second, our analysis here has been entirely positive, not normative. Within our model, if the economy starts at an efficient steady-state, a fiscal stimulus is never optimal—it only reduces welfare by pushing output above its first-best level and generating socially undesirable inflation. The question of how to structure or finance the optimal stimulus is then moot. When instead the economy is depressed (output is inefficiently low), a fiscal stimulus becomes desirable, and the question of how to finance it becomes relevant.

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Online Appendix for:
Can Deficits Finance Themselves?

This online appendix contains supplemental material for the article "Can Deficits Finance Themselves?". We provide: (i) supplementary discussion of our baseline model and results in Sections 2-4; (ii) details for the extensions considered in Section 5; (iii) additional analysis and alternative results for our quantitative investigation in Section 6. The end of this appendix contains all proofs. Further supplementary materials are provided in the Supplement Angeletos et al. (2024).

Any references to equations, figures, tables, assumptions, propositions, lemmas, or sections that are not preceded by "A."—"D." refer to the main article.

A Supplementary details

We here provide some additional discussion of our baseline model. Appendix A.1 characterizes labor supply and explains how our model's supply block reduces to the standard NKPC, and Appendix A.2 derives the aggregate demand relation. Appendix A.3 then explains how we characterize equilibria under the alternative fiscal policy rule (8).

A.1 The supply block

Recall that the optimal labor supply relation is given as (3), re-stated here for convenience:

$$(1-\tau_y)W_t = \frac{\chi L_t^{\frac{1}{\varphi}}}{\int_0^1 C_{i,t}^{-1/\sigma}di}.$$

Log-linearizing, we obtain (15).

Optimal firm pricing decisions as usual give inflation as a function of real marginal costs. With a standard constant-returns-to-scale, labor-only production function this gives (e.g., see the textbook derivations in Woodford, 2003a; Galí, 2008)

$$\pi_t = \tilde{\kappa} w_t + \beta \mathbb{E}_t [\pi_{t+1}], \tag{A.1}$$

where $\tilde{\kappa} = \frac{(1-\theta)(1-\beta\theta)}{\theta}$ is a function of θ (one minus the Calvo reset probability). Combining (A.1) with (15) and imposing that $c_t = \ell_t = y_t$, we obtain

$$\pi_t = \underbrace{\tilde{\kappa}\left(\frac{1}{\varphi} + \frac{1}{\sigma}\right)}_{\equiv\kappa} y_t + \beta\mathbb{E}_t\left[\pi_{t+1}\right], \tag{A.2}$$

as required.

A.2 Derivation of the aggregate demand relation (13)

From the fiscal policy in (7), we have that

$$\begin{aligned}
\mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k d_{t+k}\right] &= d_t + \mathbb{E}_t\left[\sum_{k=1}^{+\infty}(\beta\omega)^k \frac{1}{\beta}\left((1-\tau_d)\, d_{t+k-1} - \tau_y y_{t+k-1}\right)\right] + (1-\tau_d)\,\omega\varepsilon_t \\
&= d_t + \omega(1-\tau_d)\,\mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k d_{t+k}\right] - \omega\tau_y\mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k y_{t+k}\right] + (1-\tau_d)\,\omega\varepsilon_t \\
&= \frac{1}{1-\omega(1-\tau_d)} d_t - \frac{\omega\tau_y}{1-\omega(1-\tau_d)}\mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k y_{t+k}\right] + \frac{(1-\tau_d)\,\omega}{1-\omega(1-\tau_d)}\varepsilon_t,
\end{aligned}$$

and

$$\begin{aligned}
\mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k t_{t+k}\right] &= \tau_d\mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k d_{t+k}\right] + \tau_y\mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k y_{t+k}\right] - (1-\tau_d)\,\varepsilon_t \\
&= \frac{\tau_d}{1-\omega(1-\tau_d)} d_t + \frac{\tau_y(1-\omega)}{1-\omega(1-\tau_d)}\mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k y_{t+k}\right] - \frac{(1-\tau_d)(1-\omega)}{1-\omega(1-\tau_d)}\varepsilon_t
\end{aligned}$$

Together with the aggregate demand relation (12) and market clearing ($c_t = y_t$), we get (13):

$$y_t = \frac{\left(1-\beta\omega\right)(1-\omega)(1-\tau_d)}{1-\omega(1-\tau_d)}(d_t+\varepsilon_t) + \left(1 - \frac{(1-\omega)\,\tau_y}{1-\omega(1-\tau_d)}\right)\mathbb{E}_t\left[\left(1-\beta\omega\right)\sum_{k=0}^{+\infty}(\beta\omega)^k y_{t+k}\right].$$

A.3 Equilibrium characterization with the alternative fiscal rule

We characterize the equilibrium in our OLG-NK environment with $\omega < 1$, $\tau_y > 0$, and the alternative fiscal rule (8) here. The aggregate demand relation (12) together with monetary policy (9) and market clearing $y_t = c_t$ lead to the following recursive aggregate demand relation:

$$y_t = \frac{\left(1-\beta\omega\right)(1-\omega)}{\beta\omega}(d_t - t_t) + \mathbb{E}_t\left[y_{t+1}\right], \tag{A.3}$$

where we use that $\mathbb{E}_t\left[d_{t+1}\right] = \frac{1}{\beta}(d_t - t_t)$ from (5).

We characterize the bounded equilibrium path through backward induction. Given the alternative fiscal rule (8), we know that, for $t \geq H$,

$$d_t - t_t = 0 \Longrightarrow y_t = \mathbb{E}_t\left[y_{t+1}\right].$$

We focus on the equilibrium with $y_t = 0$ for $t \geq H$. As discussed in Footnote 15, this equilibrium selection can be justified in two ways: strengthening the boundedness requirement in the equilibrium definition or considering limits to $\phi = 0$ from above. The sole role of any of these modifications is to remove a class of sunspot equilibria that are inherited from the standard New Keynesian model. Given this selection we use (A.3) to find the equilibrium path of $\{y_t, d_t\}_{t=0}^{H-1}$ through backward induction starting from

$$y_H = \chi_0 d_H \quad \text{with} \quad \chi_0 = 0. \tag{A.4}$$

For $t \leq H-1$, substitute the alternative fiscal rule (8) into (A.3), giving

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}(d_t+\varepsilon_t) + \frac{1}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\mathbb{E}_t\left[y_{t+1}\right].$$

As a result, for $t \leq H-1$,

$$y_t = \chi_{H-t}(d_t+\varepsilon_t) \quad \text{with} \quad \chi_{H-t} = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y} + \frac{\frac{1}{\beta}\left(1-\tau_y\chi_{H-t}\right)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\chi_{H-t-1}, \tag{A.5}$$

where, according to (8), $\varepsilon_t = 0$ for all $t \neq 0$. Rearranging terms, we find the following recursive formula for the χs:

$$\chi_{H-t} = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi_{H-t-1}}{\beta}}{1+\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi_{H-t-1}}{\beta}\right)\tau_y} = g\left(\chi_{H-t-1}\right), \tag{A.6}$$

where

$$g\left(\chi\right) = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi}{\beta}}{1+\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi}{\beta}\right)\tau_y} \quad \text{and} \quad g'\left(\chi\right) = \frac{1}{\beta}\frac{1}{\left(1+\tau_y\left(\frac{(1-\beta\omega)(1-\omega)}{\beta\omega} + \frac{\chi}{\beta}\right)\right)^2} \geq 0 \quad \forall \chi \geq 0. \tag{A.7}$$

We thus know that

$$\chi_k \in (0, \frac{1}{\tau_y}) \quad \forall k \geq 1 \quad \text{and} \quad \chi_k \text{ increases in } k. \tag{A.8}$$

From (5), $r_t = 0$, (8), and (A.5), we also know that, for $t \leq H$,

$$\mathbb{E}_0\left[d_t\right] = \frac{1}{\beta^t}\Pi_{j=0}^{t-1}\left(1-\tau_y\chi_{H-j}\right)(d_0+\varepsilon_0). \tag{A.9}$$

To further characterize the equilibrium it is useful to consider an alternative economy with rigid prices ($\kappa = 0$) but otherwise identical to the baseline economy. Let v' denote the self financing share in this alternative economy, i.e.,

$$\nu' \cdot \varepsilon_0 = \nu'_y \cdot \varepsilon_0 \equiv \sum_{k=0}^{\infty} \tau_y \beta^k \mathbb{E}_0 \left[y_k \right].$$

In this economy, there is no $t = 0$ price level jump and so the real value of public outstanding at $t = 0$, $d_0 = b_0 = 0$ is pre-determined. From (A.5) and (A.9), we have that

$$\nu' = \sum_{t=0}^{H-1} \prod_{j=0}^{t-1} \left(1 - \tau_y \chi_{H-j}\right) \tau_y \chi_{H-t} = 1 - \prod_{j=0}^{H-1} \left(1 - \tau_y \chi_{H-j}\right). \tag{A.10}$$

We can now return to the general case with $\kappa \geq 0$. From the NKPC (16) as well as the definitions in (22) – (23), we have that

$$\nu_p = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y} \nu_y = \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu \tag{A.11}$$

Finally, from the formula of d_0 (18), we know

$$d_0 = -\nu_p \varepsilon_0 \quad \text{and} \quad \nu_y = \left(1 - \nu_p\right) \nu'.$$

Together, we have

$$\nu = \frac{\nu'}{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} + \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'} \quad \nu_y = \frac{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} + \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}, \quad \text{and} \quad \nu_p = \frac{\frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}{\frac{\tau_y}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} + \frac{\kappa \frac{D^{ss}}{Y^{ss}}}{\tau_y + \kappa \frac{D^{ss}}{Y^{ss}}} \nu'}. \tag{A.12}$$

This completes our characterization of the equilibrium.

B Details on model extensions

We elaborate on the various extensions considered in Section 5: equilibria under more general monetary rules and the richer aggregate demand block are discussed here, while the remaining extensions are presented in the Online Supplement.

B.1 More general monetary policy

Here we supplement the discussion of monetary policy in Subsection 5.1. First, we explain how we measure the degree of self-financing when real rates are variable. Second, we investigate the model's determinacy regions, extending Leeper (1991).

Measuring the degree of self-financing when $\phi \neq 0$. With variable expected real rates, the government's intertemporal budget constraint in (21) has to be re-written as follows:

$$\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{k=0}^{+\infty} \beta^{k+1} \mathbb{E}_0 \left[r_k\right] = \tau_d \left(\varepsilon_0 + \sum_{k=0}^{+\infty} \beta^k \mathbb{E}_0 \left[d_k\right]\right) + \sum_{k=0}^{+\infty} \tau_y \beta^k \mathbb{E}_0 \left[y_k\right] + \frac{D^{ss}}{Y^{ss}} \left(\pi_0 - \mathbb{E}_{-1} \left[\pi_0\right]\right).$$

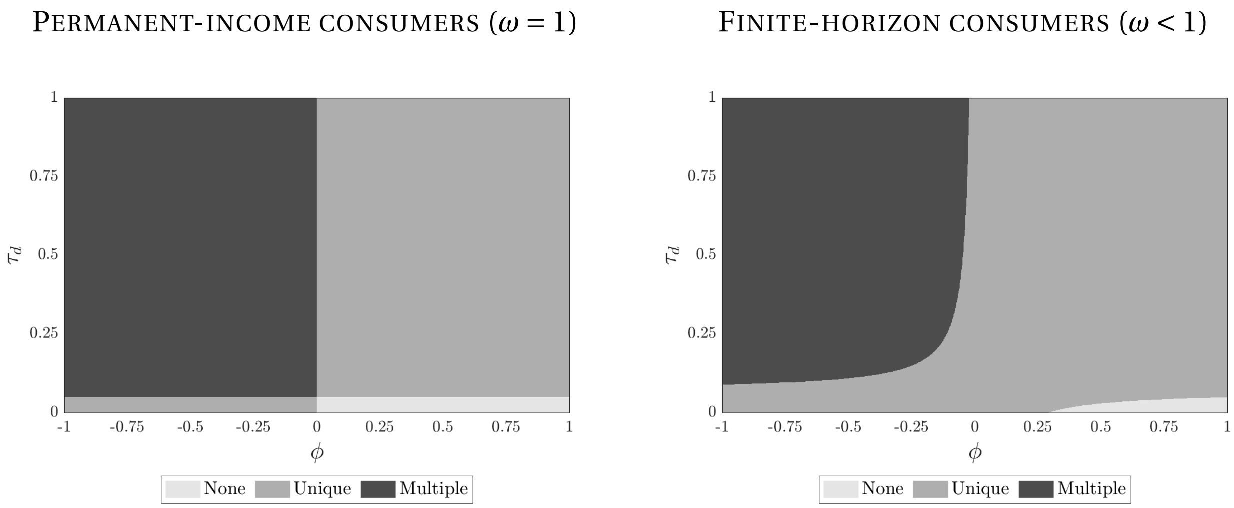

Figure B.1: Determinacy regions for $\omega = 1$ (left panel) and $\omega < 1$ (right panel) as a function of the fiscal and monetary policy rule coefficients τ_d (y-axis) and ϕ (x-axis).

The right hand side is the same as before, while the new term on the left-hand side, $\frac{D^{ss}}{Y^{ss}} \sum_{k=0}^{+\infty} \beta^{k+1} \mathbb{E}_0 [r_k]$, captures how the time-varying expected real interest rate in (10) changes the costs of servicing the outstanding public debt. We interpret this new term as analogous to the deficit shock itself and accordingly define the share of self-financing as

$$\nu \equiv \frac{\sum_{k=0}^{\infty} \tau_y \beta^k \mathbb{E}_0 \left[y_k \right] + \frac{D^{ss}}{Y^{ss}} \pi_0}{\varepsilon_0 + \frac{D^{ss}}{Y^{ss}} \sum_{k=0}^{+\infty} \beta^{k+1} \mathbb{E}_0 \left[r_k \right]} \tag{B.1}$$

This reduces to the original definition when $\phi = 0$.

Determinacy regions. We begin by providing a visual illustration of equilibrium determinacy—i.e., the famous Leeper (1991) regions—in our OLG model. The two panels in Figure B.1 show whether a bounded equilibrium (i.e., the standard solution concept of Blanchard and Kahn (1980)) exists and is unique under different assumptions on monetary policy (ϕ) and fiscal policy (τ_d in (7)), for a standard permanent-income model (left panel) and our OLG economy (right panel).

The figure reveals that the determinacy properties of the two economies are materially different. Results for the permanent-income model are well-known and require little explanation: equilibrium uniqueness requires that fiscal policy is passive ($\tau_d > 1 - \beta$) and monetary policy is active ($\phi > 0$), or vice-versa; if both rules are active then no bounded equilibrium exists, and if both are passive then there are multiple bounded equilibria. With discounting on the household side (i.e., $\omega < 1$), the regions of equilibrium determinacy look rather different. Perhaps most importantly, the benchmark monetary rule of $\phi = 0$ now induces unique bounded equilibrium for any τ_d, consistent with Proposition 1. Intuitively, with $\omega < 1$, determinacy comes from the fact that public debt directly enters the aggregate demand relation. Moreover, existence of a bounded equilibrium is related to $\tau_y > 0$, which implies that output directly affects the government budget. As a result, self-financing is now strong enough to pull debt as well as spending towards zero, even if interest rates do not provide any further Euler equation tilting. This automatic stabilization of government debt also shrinks the equilibrium non-existence region in the bottom right corner of the figure.

B.2 A more general aggregate demand relation

In Section 5.2 we showed explicitly how several popular models of the household consumption-savings problem can be written in our general form (30). We here elaborate further on: (i) our discussion of the well-known spender-saver model; (ii) what happens in general with a margin of permanent-income households; and (iii) the model of cognitive discounting of Gabaix (2020).

Self-financing in the spender-saver model. We provide a visual illustration of self-financing—or the lack thereof—in Figure B.2. The top panel shows impulse responses and the self-financing share in the spender-saver model, while the bottom panel does the same for a spender-OLG hybrid model.

The top panel reveals that, in the spender-saver model, the self-financing share ν is always 0. In particular, we see that the date-0 boom is always exactly offset (in present-value terms) by a bust at date H—the date of the delayed fiscal adjustment. The intuition is that the presence of permanent-income households breaks our discounting and front-loading properties: the date-H tax hike invariably affects date-0 demand, and part of the deficit-financed boom is delayed to the infinite future. The empirically testable flipside of this "connection at infinity" is an infinitely large elasticity of household asset demand to interest rates (e.g., see the discussion in Kaplan and Violante, 2018). With $\omega < 1$, we break this unrealistic feature of the model, return to our discounting and front-loading properties, and thus see that the date-H bust endogenously gets smaller and smaller as we again converge to full self-financing (right panel).

Adding a margin of permanent-income consumers. We now elaborate further on what happens to our self-financing results in the presence of a margin of (at least near-)permanent-income consumers. We proceed in two steps. First, we elaborate on the discontinuity of ν in the presence of such a permanent-income margin, connecting with our discussion in Section 4.4 on the "order of limits." Second, we investigate what happens under alternative configurations of policy.

We begin by considering what happens to the self-financing share ν in our baseline model studied in Sections 3 - 4, but when a margin—here one per cent—of near-PIH consumers is added, with some $\bar{\omega} > \omega$. Throughout we focus on our "H-policy" (8). Results are displayed in Figure B.3, which shows the self-financing share ν for different H (shaded lines), as a function of $\bar{\omega}$ (x-axis). Consistent with the

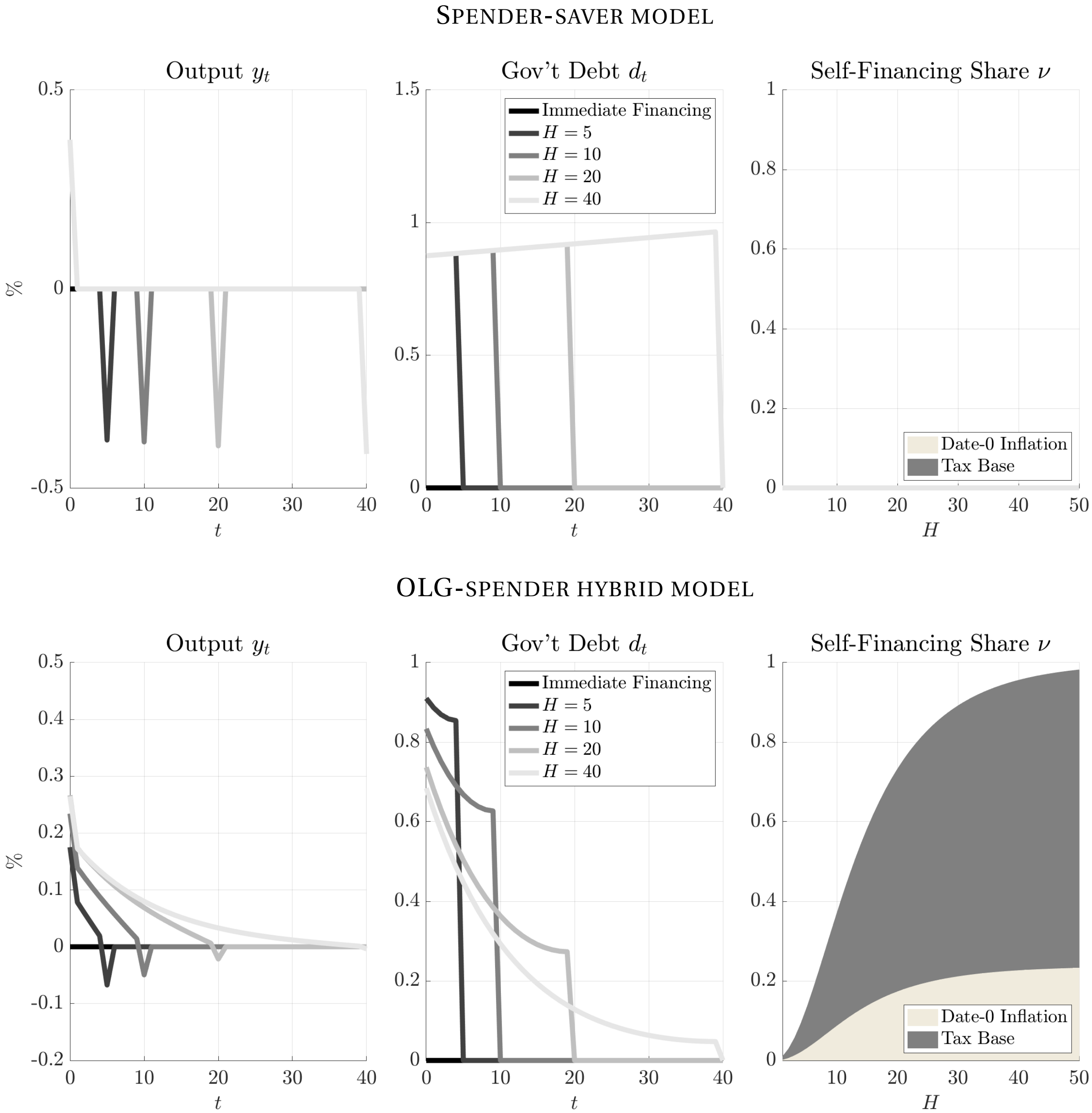

Figure B.2: Top panel: impulse responses of output y_t, government debt d_t, and the total self-financing share ν to a unit-size shock ε_0 as a function of H in a spender-saver model. Bottom panel: same as above, but in an OLG-spender hybrid economy.

discussion in Section 5.2, we see that, as $\bar{\omega} \to 1$, the self-financing share ν converges to zero. However, for every $\bar{\omega} < 1$, as H is increased, the self-financing share increases, eventually converging to one—our "order of limits" discussion. Ultimately, the practical relevance of our results is thus a quantitative question, addressed by the analysis in Section 6.

We next show that alternative policy mixes deliver a self-financing share ν that is continuous in the margin of exact permanent-income consumers. Specifically, we consider a PIH-OLG hybrid economy

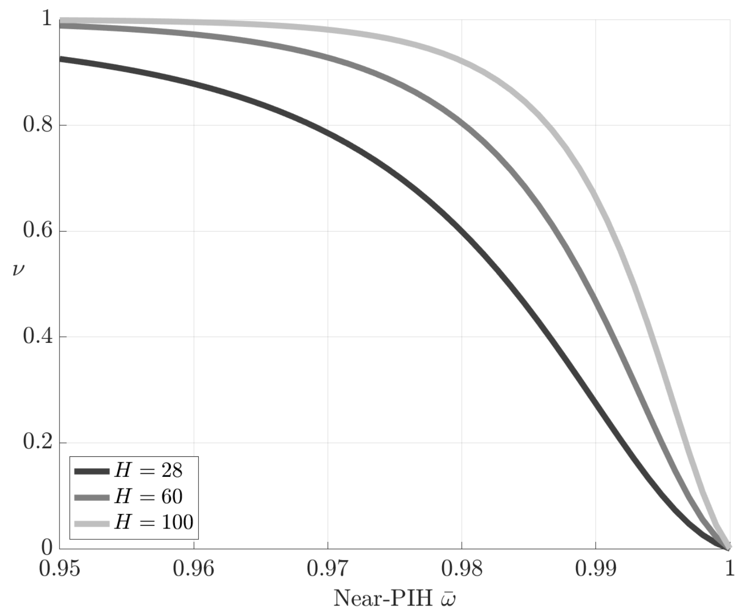

Figure B.3: Self-financing share in an augmented version of the model of Sections 3 - 4, featuring a margin (1 per cent) of households with higher OLG survival coefficient $\bar{\omega}$. ν is plotted as a function of $\bar{\omega}$ (x-axis) and H (shaded lines).

with a share μ of OLG consumers (with a survival rate ω) and a residual share $1-\mu$ of PIH consumers. Note that this environment nests the classical spender-saver model with $\omega = 0$. We consider a variant of fiscal policy (8), where the tax burden of fiscal adjustment at date H is imposed in a group-specific manner, reducing the post-tax financial wealth of both permanent-income consumers and spenders to its steady-state value 0. That is, for a PIH consumer i, $t_{i,t} = t_t^{\text{PIH}} = \tilde{a}_t^{\text{PIH}}$ for $t \geq H$, while for an OLG consumer i, $t_{i,t} = t_t^{\text{OLG}} = \tilde{a}_t^{\text{OLG}}$, where superscripts PIH and OLG capture group-specific averages and $\tilde{a}_t$ captures real financial wealth. For $t < H$, the fiscal policy is exactly the same as (8)

$$t_{i,t} = t_t = \begin{cases} \tau_y y_{i,t} - \varepsilon_0 & t = 0 \\ \tau_y y_{i,t} & t \in \{1, \cdots, H-1\} \end{cases}.$$

Given the fiscal policy for $t \geq H$ and similar to the main analysis (see Footnote 15), for $t \geq H$,

$$c_t^{\text{PIH}} = c_t^{\text{OLG}} = y_t = 0.$$

Since expected real interest rates are fixed, consumption of PIH consumers for $t \leq H-1$ is given by:

$$c_t^{\text{PIH}} = 0.$$

As a result, for $t \leq H-1$,

$$y_t = \mu c_t^{\text{OLG}} \quad \text{and} \quad t_t^{\text{OLG}} = \tau_y y_t = \mu \tau_y c_t^{\text{OLG}}. \tag{B.2}$$

Consumption for OLG consumers for $t \leq H-1$ is given by:

$$\begin{aligned} c_t^{\mathrm{OLG}} &= (1-\beta\omega)\left(\tilde{a}_t^{\mathrm{OLG}} + \mathbb{E}_t\left[\sum_{k=0}^{+\infty}(\beta\omega)^k\left(y_{t+k} - t_{t+k}^{\mathrm{OLG}}\right)\right]\right) \\ &= (1-\beta\omega)\left(\tilde{a}_t^{\mathrm{OLG}} + (1-\tau_y)\,\mu\mathbb{E}_t\left[\sum_{k=0}^{H-t-1}(\beta\omega)^k c_{t+k}^{\mathrm{OLG}}\right] - (\beta\omega)^{H-t}\tilde{a}_H^{\mathrm{OLG}}\right). \end{aligned} \tag{B.3}$$

Together with OLG consumers' budget,

$$\begin{aligned} \tilde{a}_{t+1}^{\mathrm{OLG}} &= \frac{1}{\beta}\left(\tilde{a}_t^{\mathrm{OLG}} + y_t - t_t^{\mathrm{OLG}} - c_t^{\mathrm{OLG}}\right) \\ &= \frac{1}{\beta}\left(\tilde{a}_t^{\mathrm{OLG}} - \left(1-(1-\tau_y)\,\mu\right)c_t^{\mathrm{OLG}} + \varepsilon_0\mathbb{1}_{\{t=0\}}\right), \end{aligned} \tag{B.4}$$

(B.3) and (B.4) fully characterize $\{c_t^{\mathrm{OLG}}\}_{t=0}^{H-1}$ and hence $\{y_t^{\mathrm{OLG}}\}_{t=0}^{H-1}$ given $\tilde{a}_0^{\mathrm{OLG}} = d_0$.

Note that when $\mu = 1$ (all consumers are OLG consumers), (B.3) and (B.4) fully characterize the equilibrium in our baseline economy studied in Sections (3) and (4). That is, for $t \leq H-1$,

$$c_t = (1-\beta\omega)\left(d_t + (1-\tau_y)\,\mathbb{E}_t\left[\sum_{k=0}^{H-t-1}(\beta\omega)^k c_{t+k}\right] - (\beta\omega)^{H-t} d_H\right), \tag{B.5}$$

with

$$d_{t+1} = \frac{1}{\beta}\left(d_t - \tau_y c_t + \varepsilon_0\mathbb{1}_{\{t=0\}}\right), \tag{B.6}$$

Now, let $\{c_t^{\mathrm{Baseline}}\}_{t=0}^{H-1}$ denote the path characterized by (B.5) and (B.6) given d_0,with $\tau_y^{'} = 1-(1-\tau_y)\,\mu$. From (B.3) and (B.4), we know that, for all $t \leq H-1$,

$$c_t^{\mathrm{OLG}} = c_t^{\mathrm{Baseline}} \quad \text{and} \quad y_t = \mu c_t^{\mathrm{Baseline}}.$$

Now first of all suppose that prices are fully rigid (i.e., we have that $\kappa = 0$). Then $d_0 = b_0 = 0$, and so the self-financing share of the OLG-PIH economy is given by

$$\nu = \frac{\mu\tau_y}{1-(1-\tau_y)\,\mu}\nu^{Baseline},$$

where

$$\nu \equiv \frac{\tau_y\sum_{t=0}^{t-1}\beta^t y_t}{\varepsilon_0} \quad \text{and} \quad \nu^{Baseline} \equiv \frac{\tau_y^{'}\sum_{t=0}^{t-1}\beta^t c_t^{Baseline}}{\varepsilon_0}.$$

As $H \to \infty$, $\nu^{Baseline} \to 1$, and so we find that

$$\nu = \frac{\mu\tau_y}{1-(1-\tau_y)\,\mu}$$

Note that ν is continuous in μ and limits to 1 as the margin of PIH consumers vanishes. Away from the rigid-price case, from (D.10), the limiting self-financing share is given by

$$\nu = \frac{\frac{\mu\tau_y}{1-(1-\tau_y)\mu}}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}} + \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\frac{\mu\tau_y}{1-(1-\tau_y)\mu}},$$

again continuous in the margin of exact PIH consumers.

Cognitive discounting. Under cognitive discounting, a shock h periods in the future is additionally discounted by a factor of θ, with $\theta = 1$ corresponding to the standard full-information, rational-expectations model and $\theta = 0$ corresponding to myopic households. It is immediate that cognitive discounting added to our baseline OLG model gives the adjusted aggregate demand relation

$$c_t = (1-\beta\tilde{\omega})\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\tilde{\omega}\theta)^k(y_{t+k}-t_{t+k})\right]\right), \tag{B.7}$$

where $\tilde{\omega}$ is the survival rate. This fits into our demand structure (30) with $M_y = M_d = 1-\beta\tilde{\omega}$, $\delta = 1$ and $\omega = \tilde{\omega}\theta$. It is immediate that, for $\tilde{\omega} < 1$ and $\theta < 1$, Assumption 1 holds. Differently from the baseline OLG case, however, Assumption 2 does not hold automatically; plugging in to (31) and re-arranging we find that we need

$$\tau_y > \frac{\tilde{\omega}(1-\theta)}{1-\tilde{\omega}\theta}\frac{1-\beta}{1-\beta\tilde{\omega}} \tag{B.8}$$

This relation holds automatically for $\theta = 1$, but need not hold for $\theta < 1$; intuitively, as already discussed in the main text, $\theta < 1$ dampens demand spillovers from the future to the present and thus slows down the Keynesian boom. (B.8) is, however, a very mild condition: even for $\theta = 0$, as long as β is close to one and for values of $\tilde{\omega}$ as considered in Section 6, Assumption 2 holds even for small τ_y.

C Quantitative analysis

This section supplements our quantitative analysis in Section 6. We first provide some missing details on the specification of our spender-OLG hybrid model in Appendix C.1, and then consider model variants with more flexible prices and more aggressive monetary reactions in Appendices C.2 and C.3. Several further related results are presented in the Online Supplement.

C.1 Further details on the hybrid model

We first elaborate on the model environment and discuss in greater detail the model's implications for household consumption behavior, contrasting it in particular with the predictions of quantitative HANK-type models.

Model. The only change relative to our baseline model of Section 2 is that we generalize the household block to also feature a margin μ of spenders—that is, households who do not hold any assets and immediately spend any income they receive. The remaining fraction $1-\mu$ of households are exactly as described in Section 2.1. Both groups of households receive labor income as well as dividends and pay taxes, but only the OLG block holds government bonds.

We will make assumptions ensuring that both groups of households receive the same labor and dividend income, pay the same taxes (up to a between-group steady-state transfer), and have identical steady-state consumption. First, we assume that unions assign identical hours worked to both groups, and that dividends also accrue equally to both. Second, we assume that the government in lump-sum fashion redistributes between the two groups to ensure identical steady-state consumption; given that government bonds are held by the OLG block, this will generally require lump-sum transfers to spenders. Under those assumptions, it is first of all immediate that the supply block of the economy—notably (16)—is unchanged. Next, the demand block of the economy generalizes (29) as follows:

$$c_t = (1-\beta\omega)\cdot d_t + \left[\mu + (1-\mu)(1-\beta\omega)\right]\cdot\left((y_t - t_t) + \frac{(1-\mu)(1-\beta\omega)}{\mu+(1-\mu)(1-\beta\omega)}\mathbb{E}_t\left[\sum_{k=1}^{\infty}(\beta\omega)^k\left(y_{t+k}-t_{t+k}\right)\right]\right). \tag{C.1}$$

Replacing (29) by (C.1) is the only difference between our baseline OLG economy and the generalized hybrid model. Relative to (29), the most important change in (C.1) is that we allow the MPC out of income to be larger than that out of wealth. As we discuss next, this minimal departure from our baseline OLG model is all that is needed to ensure (approximate) consistency with consumption-savings behavior even in quantitative HANK-type models.

Household consumption-savings behavior. By our discussion in Online Supplement E.1 we know that the role of the household consumption-savings decision in driving our self-financing result is fully governed by the matrix of intertemporal marginal propensities to consume. The left top panel of Figure 4 provides a visual illustration of this matrix in our spender-OLG hybrid model, as implied by the generalized demand block (C.1).

The figure plots the spending response over time to (anticipated) income gains at different dates. We emphasize two takeaways. First, the response to a date-0 income gain—that is, the first column of $\mathcal{M}$—agrees with prior empirical evidence (Fagereng et al., 2021), as discussed in Section 6.1. Second, the higher-order columns are qualitatively and quantitatively similar to those implied by HANK-type models. This observation has been made previously in Auclert et al. (2023) and Wolf (2021). For our purposes, the important takeaway is that our analysis is indeed quantitatively relevant—as far as our question of self-financing is concerned, our model will have very similar predictions as richer quantitative HANK-type models. We further illustrate this observation in Online Supplement E.6.1.

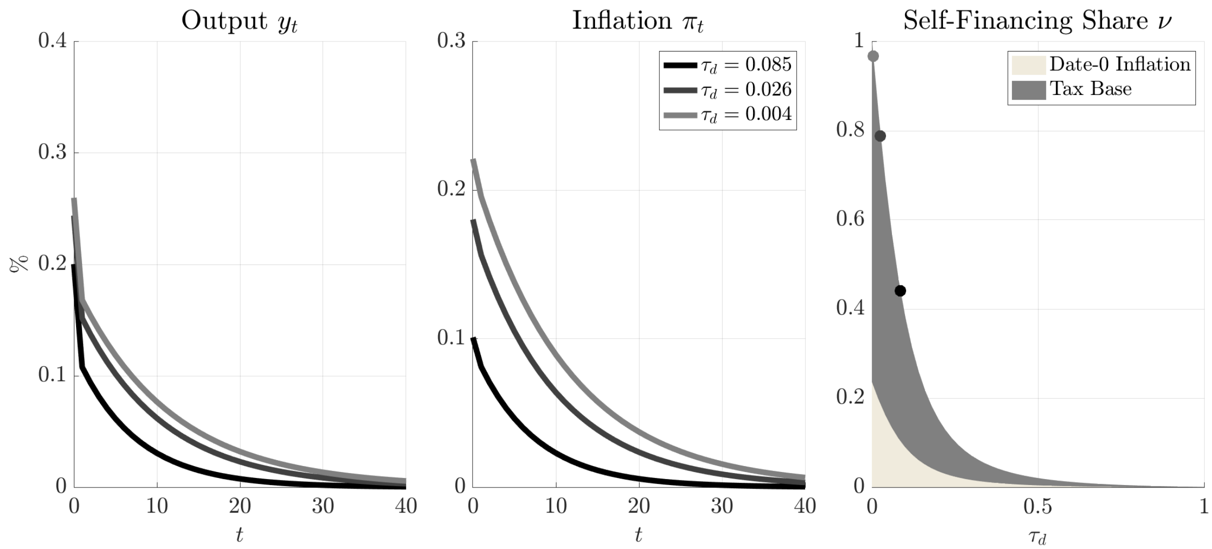

Figure C.1: Impulse responses of output y_t, inflation π_t, and the total self-financing share ν to a unit-size shock ε_0 as a function of τ_d, with more flexible prices. The left and middle panels show the impulse responses for the three particular values of τ_d discussed in Section 6.1. In the right panel these three points are marked with circles.

C.2 Self-financing with more flexible prices

Figure C.1 repeats our analysis of Section 6.2 in a variant of the baseline model with more flexible prices, setting $\kappa = 0.1$—a value at the large end of recent empirical evidence and arguably more relevant for the inflationary post-covid environment. With more flexible prices, a non-trivial share—here around 20 per cent—of self-financing comes from adjustments in prices rather than quantities. We note that alternative assumptions on the shape of the NKPC (e.g., a hybrid NKPC) or on government debt maturity could further impact that split; we leave this investigation for future work.

C.3 Active monetary reaction

Figure C.2 shows the degree of self-financing ν as a function of τ_d under the active monetary (Taylor-type) rule (34), for $\psi \in \{1, 1.25, 1.5\}$, with the three panels corresponding to the three displayed values of ψ. In the right panel the (barely visible) red region for τ_d very close to zero indicates equilibrium non-existence for the underlying policy mix.

The figure provides a visual illustration that complements Table 2. The left and middle panels reveal that, for $\psi = 1$ and $\psi = 1.25$, full self-financing is possible; in particular, this panel is qualitatively and quantitatively similar to our headline results in Figure 3. The right panel then shows a more aggressive monetary reaction for which full self-financing is not possible. We nevertheless see that, for small values of τ_d outside of the (very small) non-existence region, there can still be meaningful

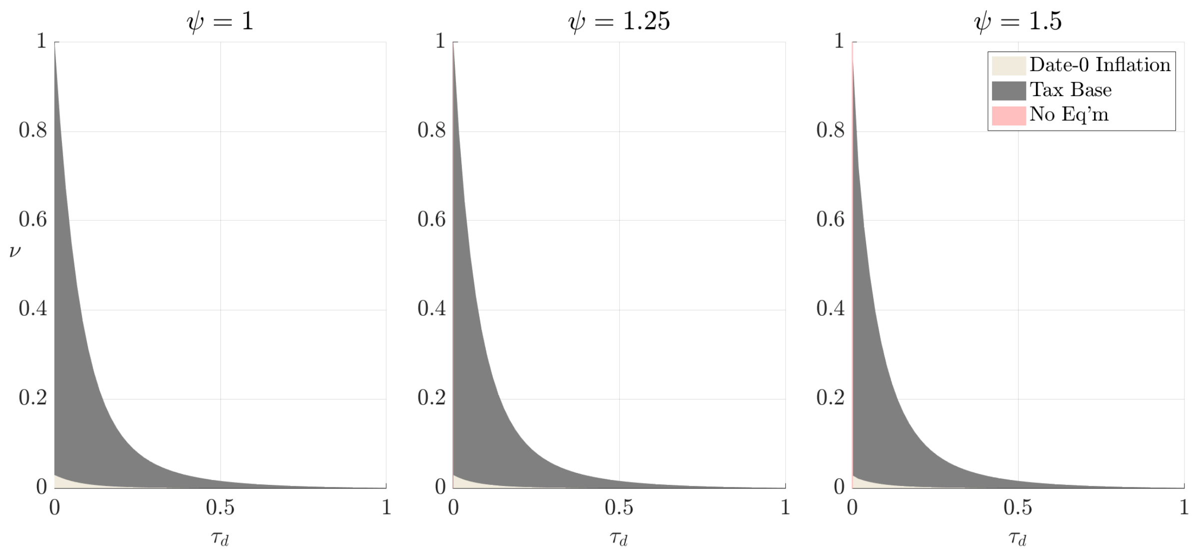

Figure C.2: Total self-financing share ν in response to a unit-size shock ε_0 as a function of τ_d, for the standard nominal Taylor-type monetary policy rule $i_t = \psi\pi_t$, with $\psi \in \{1, 1.25, 1.5\}$. The (barely visible) red area in the right panel indicates (bounded) equilibrium non-existence.

degrees of self-financing, even for a central bank that quite aggressively leans against the fiscal boom.

D Proofs

D.1 Proof of Proposition 1

Note that we restrict that $\omega \in (0,1)$, $\tau_y \in (0,1)$, and $\tau_d \in (0,1)$. We first write (13) recursively:

$$\begin{aligned} y_t - \mathscr{F}_1 \cdot (d_t + \varepsilon_t) &= (1-\beta\omega)\mathscr{F}_2 \cdot y_t + \beta\omega\mathbb{E}_t\left[y_{t+1} - \mathscr{F}_1 \cdot (d_{t+1} + \varepsilon_{t+1})\right] \\ &= (1-\beta\omega)\mathscr{F}_2 \cdot y_t + \beta\omega\mathbb{E}_t\left[y_{t+1} - \mathscr{F}_1 \cdot \frac{1}{\beta}\left[(1-\tau_d)(d_t+\varepsilon_t) - \tau_y y_t\right]\right]. \end{aligned}$$

After rearranging terms and using the formula of $\mathscr{F}_1$ and $\mathscr{F}_2$ (as stated after (13)), we have

$$\begin{aligned} y_t &= \frac{(1-\omega(1-\tau_d))\mathscr{F}_1}{1-\omega\tau_y\mathscr{F}_1 - (1-\beta\omega)\mathscr{F}_2}(d_t+\varepsilon_t) + \frac{\beta\omega}{1-\omega\tau_y\mathscr{F}_1 - (1-\beta\omega)\mathscr{F}_2}\mathbb{E}_t\left[y_{t+1}\right] \\ &= \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}(d_t+\varepsilon_t) + \frac{1}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y}\mathbb{E}_t\left[y_{t+1}\right]. \end{aligned}$$

Applying period-t expectations $\mathbb{E}_t[\cdot]$ to (17), we have

$$\begin{pmatrix} \mathbb{E}_t[d_{t+1}] \\ \mathbb{E}_t[y_{t+1}] \end{pmatrix} = \begin{pmatrix} \frac{1-\tau_d}{\beta} & -\frac{\tau_y}{\beta} \\ -\frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{\beta\omega} & 1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y \end{pmatrix} \begin{pmatrix} d_t + \varepsilon_t \\ y_t \end{pmatrix} \tag{D.1}$$

The two eigenvalues of the system are given by the solutions of

$$\lambda^2 - \lambda\left(\frac{1}{\beta}(1-\tau_d) + 1 + \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right) + \frac{1}{\beta}(1-\tau_d) = 0,$$

with

$$\begin{aligned}\lambda_1 &= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)+\sqrt{\left(1+\frac{1}{\beta}(1-\tau_d)+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2-4\frac{1}{\beta}(1-\tau_d)}}{2}\\ &= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)+\sqrt{\left(1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2+4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2}\\ &> \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)+\left|1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right|}{2} \geq 1\end{aligned} \tag{D.2}$$

and

$$\begin{aligned}\lambda_2 &= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)-\sqrt{\left(1+\frac{1}{\beta}(1-\tau_d)+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2-4\frac{1}{\beta}(1-\tau_d)}}{2}\\ &= \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)-\sqrt{\left(1-\frac{1}{\beta}(1-\tau_d)-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2+4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2}\\ &< \frac{\left(\frac{1}{\beta}(1-\tau_d)+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)-\left|\frac{1}{\beta}(1-\tau_d)+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)-1\right|}{2} \leq 1,\end{aligned} \tag{D.3}$$

with $\lambda_2 > 0$ too since $\lambda_1\lambda_2 = \frac{1}{\beta}(1-\tau_d) > 0$. Let $\left(1,\chi_2\right)'$ denote the eigenvector associated with λ_2, where

$$\lambda_2 = \frac{1}{\beta}\left(1-\tau_d-\tau_y\chi_2\right) \quad \text{and} \quad \chi_2 = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y-\lambda_2} > 0. \tag{D.4}$$

This means that any bounded equilibrium path $\{d_t, y_t\}_{t=0}^{+\infty}$ of (D.1)[35] uniquely takes the form of

$$y_t = \chi(d_t+\varepsilon_t) \quad \text{and} \quad \mathbb{E}_t[d_{t+1}] = \rho_d(d_t+\varepsilon_t),$$

where χ and ρ_d are uniquely given by

$$\chi = \chi_2 > 0 \quad \text{and} \quad \rho_d = \lambda_2 \in (0,1). \tag{D.5}$$

In other words, the equilibrium takes the form of (19) and satisfies (20).[36]

[35] Boundedness means that $\lim_{k\to+\infty}\mathbb{E}_t[d_{t+k}]$ and $\lim_{k\to+\infty}\mathbb{E}_t\left[y_{t+k}\right]$ are bounded for any t, $d_t+\varepsilon_t$, and y_t, similar to Blanchard and Kahn (1980).

[36] To see the first part of (20), combine (13) with (19).

To prove equilibrium uniqueness, note that the total amount of nominal public debt outstanding at the start of $t = 0$, $B_0 = B^{ss}$, is given. From (16) and (18), we know d_0 is uniquely pinned down by

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0 = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\mathbb{E}_0\left[y_k\right] = -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}\left(d_0+\varepsilon_0\right) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\varepsilon_0. \tag{D.6}$$

Similarly, for $t \geq 1$,

$$\begin{aligned} d_t - \mathbb{E}_{t-1}\left[d_t\right] &= -\frac{D^{ss}}{Y^{ss}}\left(\pi_t - \mathbb{E}_{t-1}\left[\pi_t\right]\right) = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\left(\mathbb{E}_t\left[y_{t+k}\right]-\mathbb{E}_{t-1}\left[y_{t+k}\right]\right) \\ &= -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}\left(d_t - \mathbb{E}_{t-1}\left[d_t\right]+\varepsilon_t\right) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\varepsilon_t. \end{aligned} \tag{D.7}$$

Together with (16) and (19), this pins down the unique bounded equilibrium path of $\left\{\pi_t, d_t, y_t\right\}_{t=0}^{+\infty}$.

D.2 Proof of Theorem 1

We start with the case based on the baseline fiscal policy (7). We begin by considering an alternative economy with rigid prices ($\kappa = 0$) but otherwise identical to the baseline economy. Let ν' denote the self financing share in this alternative economy, which similarly to (23) is given as

$$\nu'\cdot\varepsilon_0 = \nu'_y\cdot\varepsilon_0 \equiv \sum_{k=0}^{\infty}\tau_y\beta^k\mathbb{E}_0\left[y_k\right].$$

Note that in this alternative economy all self-financing comes from tax base changes. In particular, there is no $t = 0$ price level jump and so the real value of public outstanding at $t = 0$, $d_0 = b_0 = 0$ is pre-determined. From (19) and (23) we know that

$$\nu' = \tau_y\frac{\chi}{1-\beta\rho_d}. \tag{D.8}$$

Now, consider the general case with $\kappa \geq 0$. From NKPC (16) and the definitions in (22) – (23), we have

$$\nu_p = \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y}\nu_y = \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu \tag{D.9}$$

From the formula of d_0 (18), we know

$$d_0 = -\nu_p\varepsilon_0 \quad \text{and} \quad \nu_y = \left(1-\nu_p\right)\nu'.$$

Together, we have

$$\nu=\frac{\nu'}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}+\frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}\quad \nu_y=\frac{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}+\frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'},\quad\text{and}\quad \nu_p=\frac{\frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}{\frac{\tau_y}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}+\frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}}\nu'}. \tag{D.10}$$

From the second part of (20), we know that

$$\frac{\chi}{1-\beta\rho_d}=\frac{\chi}{\tau_d+\tau_y\chi}. \tag{D.11}$$

From (D.3) and (D.5), we know

$$\rho_d=\lambda_2=f(a,b)\equiv\frac{a+b+1-\sqrt{(a+b-1)^2+4b}}{2} \tag{D.12}$$

where $a=\frac{1}{\beta}(1-\tau_d)>0$ and $b=\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)>0$. Since $\frac{\partial f}{\partial a}=\frac{1}{2}-\frac{(a+b-1)}{2\sqrt{(a+b-1)^2+4b}}>0$,we know that ρ_d decreases with τ_d. From (D.4) and (D.5), we then know $\chi=\frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\tau_y-\rho_d}$ also decreases in τ_d. From (D.11), we know $\frac{\chi}{1-\beta\rho_d}$ decreases in τ_d. Finally, from (D.8) and (D.10), we know ν decreases in τ_d. This finishes the proof of Part 1.

For Part 2, from (D.3) and (20), we know that ρ_d and χ are continuous in $\tau_d\in[0,1)$, and

$$\begin{aligned}\rho_d^{\text{full}}\equiv\lim_{\tau_d\to0^+}\rho_d&=\frac{\left(\frac{1}{\beta}+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)-\sqrt{\left(1-\frac{1}{\beta}-\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)\right)^2+4\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)}}{2} \qquad (D.13)\\&<\frac{\frac{1}{\beta}+1+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)-\left|\frac{1}{\beta}+\frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)-1\right|}{2}\le1\\ \chi^{\text{full}}\equiv\lim_{\tau_d\to0^+}\chi&=\frac{1-\beta\rho_d^{\text{full}}}{\tau_y}>0 \qquad (D.14)\end{aligned}$$

From (D.11), we know $\lim_{\tau_d\to0^+}\frac{\chi}{1-\beta\rho_d}=\frac{1}{\tau_y}$. From (D.8) and (D.10), we know $\lim_{\tau_d\to0^+}\nu=1$. Next, the fact that $\lim_{k\to\infty}\mathbb{E}_t[d_{t+k}]\to0$ follows directly from $\rho_d^{\text{full}}<1$. Moreover, the limiting equilibrium path as $\tau_d\to0^+$ is the unique bounded equilibrium when $\tau_d=0$, characterized by (19), (D.6), and (D.7) with $\rho_d=\rho_d^{\text{full}}$ and $\chi=\chi^{\text{full}}$.

Now we turn to the alternative fiscal policy rule in (8), for which we use the equilibrium characterization in Appendix A.3. For the case with rigid prices ($\kappa=0$), one can see from (A.10) that ν' increases in H, which proves Part I. For Part II and to find $\lim_{H\to\infty}\nu'$, first note that, from (A.8), $\{\chi_k\}_{k=0}^{\infty}$ is a bounded, increasing sequence. As a result, there exists $\chi^{\text{full,NM}}$ such that $\lim_{k\to\infty}\chi_k=\chi^{\text{full,NM}}$ and $\chi^{\text{full,NM}}=g\left(\chi^{\text{full,NM}}\right)\in\left(0,\frac{1}{\tau_y}\right)$. From (A.10), we know that $\lim_{H\to\infty}\nu'=1$.

From (D.4) and (D.5), we also know that $g\left(\chi^{\text{full}}\right)=\chi^{\text{full}}$ where χ^{full} defined in (D.14) parametrizes the output response in the full self-financing limit ($\tau_d\to0$) with the baseline fiscal rule (7). From the definition of $g(\cdot)$ in (A.7), we know that there is a unique $\chi>0$ such that $g(\chi)=\chi$ when $\omega<1$ and $\tau_d\in(0,1)$. As a result, $\chi^{\text{full,NM}}=$

$\chi^{\text{full}} < \frac{1}{\tau_y}$ and $\lim_{k\to+\infty}\chi_k = \chi^{\text{full}}$. That is, these two limits ($\tau_d \to 0$ and $H \to \infty$) share the same equilibrium path. Moreover, $\lim_{k\to\infty}\frac{1-\tau_y\chi_k}{\beta} = \frac{1-\tau_y\chi^{\text{full}}}{\beta} = \rho_d^{\text{full}} < 1$. From (A.9), we know that $\lim_{H\to\infty}\mathbb{E}_0[d_H] = 0$. Finally, for the general case with $\kappa \geq 0$, the desired result follows directly from the rigid price case together with (A.12) and (D.10). The final part of the theorem follows directly from (D.15).

D.3 Proof of Proposition 2

Since $\pi_0 = \kappa\sum_{k=0}^{\infty}\beta^k\mathbb{E}_0\left[y_k\right]$ (by the NKPC), it follows that the debt erosion effect is proportional to the tax base effect:

$$\nu_p = \frac{\kappa\frac{D^{ss}}{Y^{ss}}}{\tau_y}\nu_y. \tag{D.15}$$

(24) follows directly from (D.15). The rest of the Proposition 2 follows directly from (24).

D.4 Proof of Proposition 3

Consider the baseline fiscal policy (7). From (D.12), we know

$$\rho_d = \frac{a+b+1-\sqrt{(a+b+1)^2-4a}}{2} = \frac{2a}{a+b+1+\sqrt{(a+b+1)^2-4a}},$$

where $a = \frac{1}{\beta}(1-\tau_d) \in (0,1)$ and $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega) > 0$. From the second part of the equation, we know that ρ_d decreases in $b = \frac{1-\beta\omega}{\beta\omega}\tau_y(1-\omega)$ and increases in ω. From the second half of (20), we then know that χ decreases in ω. From (D.8), (D.10) and (D.11), we know that ν decreases in ω. This proves Part 1.

For Part 2, from Theorem 1, we know that $\rho_d^{\text{full}} < 1$ for any $\omega < 1$. From (D.13), we know that $\lim_{\omega\to 1}\rho_d^{\text{full}} = \frac{\left(\frac{1}{\beta}+1\right)-\sqrt{\left(1-\frac{1}{\beta}\right)^2}}{2} = 1$.

D.5 Proof of Theorem 2

As a preparation, for any bounded equilibrium in the form of (19), from (16) and (18) ,

$$d_0 = -\frac{D^{ss}}{Y^{ss}}\pi_0 = -\kappa\frac{D^{ss}}{Y^{ss}}\sum_{k=0}^{+\infty}\beta^k\mathbb{E}_0\left[y_k\right] = -\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}(d_0+\varepsilon_0) = -\frac{\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}\varepsilon_0.$$

From the definition (B.1), we know

$$\nu = \frac{\frac{\left(\tau_y+\kappa\frac{D^{ss}}{Y^{ss}}\right)\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}}{1+\frac{\beta\frac{D^{ss}}{Y^{ss}}\phi\frac{\chi}{1-\beta\rho_d}}{1+\kappa\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}}} = \frac{\left(\kappa\frac{D^{ss}}{Y^{ss}}+\tau_y\right)\frac{\chi}{1-\beta\rho_d}}{1+(\kappa+\beta\phi)\frac{D^{ss}}{Y^{ss}}\frac{\chi}{1-\beta\rho_d}} = \frac{\left(\kappa\frac{D^{ss}}{Y^{ss}}+\tau_y\right)\chi}{\tau_d+\left(\kappa\frac{D^{ss}}{Y^{ss}}+\tau_y\right)\chi}, \tag{D.16}$$

where we use $\rho_d = \frac{1-\tau_d}{\beta} - \frac{\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}}{\beta}\chi$ from (5).

As mentioned in the main text, we restrict $\phi \in [-1/\sigma, \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}})$. Aggregating individual demand relation (11), together with monetary policy (28), goods and asset market clearing, and the government budget (5) lead to the following aggregate demand relation:

$$y_t = (1-\beta\omega)\left(d_t + \mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k\left(y_{t+k} - t_{t+k}\right)\right]\right) - \left(\sigma\omega - (1-\beta\omega)\frac{D^{ss}}{Y^{ss}}\right)\beta\phi\mathbb{E}_t\left[\sum_{k=0}^{\infty}(\beta\omega)^k y_{t+k}\right]$$
$$= \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}}{1+\sigma\phi - \frac{(1-\beta\omega)(1-\omega)}{\omega}\phi\frac{D^{ss}}{Y^{ss}}}(d_t - t_t) + \frac{1}{1+\sigma\phi - \frac{(1-\beta\omega)(1-\omega)}{\omega}\phi\frac{D^{ss}}{Y^{ss}}}\mathbb{E}_t\left[y_{t+1}\right]$$

Together with the baseline fiscal policy (7) we arrive at the following equation:

$$y_t = \frac{\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}(1-\tau_d)}{1+\sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)}(d_t + \varepsilon_t) + \frac{1}{1+\sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)}\mathbb{E}_t\left[y_{t+1}\right].$$

Applying the period-t expectation operator $\mathbb{E}_t[\cdot]$ to (5), we have

$$\begin{pmatrix} \mathbb{E}_t[d_{t+1}] \\ \mathbb{E}_t\left[y_{t+1}\right] \end{pmatrix} = \begin{pmatrix} \frac{1-\tau_d}{\beta} & -\frac{\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}}{\beta} \\ -\frac{(1-\beta\omega)(1-\omega)(1-\tau_d)}{\beta\omega} & 1+\sigma\phi + \frac{(1-\beta\omega)(1-\omega)\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)}{\beta\omega} \end{pmatrix}\begin{pmatrix} d_t + \varepsilon_t \\ y_t \end{pmatrix} \tag{D.17}$$

The two eigenvalues are given by the solutions of

$$\lambda^2 - \lambda\left(\frac{1-\tau_d}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) + \left(1+\sigma\phi\right)\frac{1-\tau_d}{\beta} = 0. \tag{D.18}$$

From $\phi \in [-1/\sigma, \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}})$ and $\tau_d \in [0,1]$, we know that $\lambda_1 + \lambda_2 \geq 0$ and $\lambda_1\lambda_2 \geq 0$, so $\lambda_1 \geq 0$ and $\lambda_2 \geq 0$.

We first prove Part 1 of Theorem 2. That is, if

$$\phi < \bar{\phi} \equiv \frac{\frac{(1-\beta\omega)(1-\omega)}{\omega}\tau_y}{\sigma\left(1-\beta\right) + \frac{(1-\beta\omega)(1-\omega)}{\omega}\beta\frac{D^{ss}}{Y^{ss}}} < \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}}, \tag{D.19}$$

full self-financing is achieved as the fiscal adjustment is infinitely delayed. That is, there exists a bounded equilibrium of the form (19) with $\lim_{\tau_d\to 0^+}\rho_d \in (0,1)$ and $\lim_{\tau_d\to 0^+}\nu = 1$.

Since the eigenvalue of (D.18) is continuous in τ_d at 0, we have

$$\lim_{\tau_d\to 0^+}\lambda_1 = \frac{\left(\frac{1}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) + \sqrt{\left(\frac{1}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2 - 4\frac{1+\sigma\phi}{\beta}}}{2}$$
$$= \frac{\left(\frac{1}{\beta} + 1 + \sigma\phi + \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right) + \sqrt{\left(1+\sigma\phi - \frac{1}{\beta} - \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2 + 4\left(1+\sigma\phi\right)\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right)}}{2} \tag{D.20}$$

and

$$\lim_{\tau_d\to 0^+}\lambda_2 = \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)-\sqrt{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{1+\sigma\phi}{\beta}}}{2}$$
$$= \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)-\sqrt{\left(1+\sigma\phi-\frac{1}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2+4\left(1+\sigma\phi\right)\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)}}{2}. \tag{D.21}$$

When $\phi \in [-1/\sigma, 0)$, from (D.20) and (D.21),

$$\lim_{\tau_d\to 0^+}\lambda_1 \geq \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\left|\frac{1}{\beta}+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)-1-\sigma\phi\right|}{2} \geq \frac{1}{\beta}+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)>1$$
$$\lim_{\tau_d\to 0^+}\lambda_2 \leq \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)-\left|1+\sigma\phi-\frac{1}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right|}{2} \leq 1+\sigma\phi<1$$

When $\phi \in [0, \bar{\phi})$, from (D.19), we have

$$\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right) > \sigma\phi\left(\frac{1}{\beta}-1\right).$$

Hence

$$\lim_{\tau_d\to 0^+}\lambda_1 \geq \frac{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\left|1+\sigma\phi-\frac{1}{\beta}-\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right|}{2} > 1+\sigma\phi \geq 1$$
$$\lim_{\tau_d\to 0^+}\lambda_2 = \frac{2\frac{1+\sigma\phi}{\beta}}{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)+\sqrt{\left(\frac{1}{\beta}+1+\sigma\phi+\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right)\right)^2-4\frac{1+\sigma\phi}{\beta}}} \tag{D.22}$$
$$< \frac{2\frac{1+\sigma\phi}{\beta}}{\frac{1}{\beta}+1+\frac{\sigma\phi}{\beta}+\sqrt{\left(1+\frac{1+\sigma\phi}{\beta}\right)^2-4\frac{1+\sigma\phi}{\beta}}} = \frac{2\frac{1+\sigma\phi}{\beta}}{1+\frac{1+\sigma\phi}{\beta}+\left|\frac{1+\sigma\phi}{\beta}-1\right|} = 1.$$

Thus, as long as (D.19) holds, and as $\tau_d \to 0^+$, a bounded equilibrium in the form of (19), with $\rho_d = \lambda_2$ (where $\lim_{\tau_d\to 0^+}\rho_d < 1$) and $\chi = \frac{1-\tau_d-\beta\rho_d}{\tau_y} > 0$, will be the unique bounded solution of (D.17). Given (19), one can then back out $\{\pi_t, y_t, d_t\}$ from (16), (D.6), and (D.7). The fact that $\lim_{\tau_d\to 0^+}\nu \to 1$ follows directly from (D.16). The fact that and $\lim_{k\to\infty}\mathbb{E}_t[d_{t+k}] \to 0$ follows directly from $\lim_{\tau_d\to 0^+}\rho_d \in (0,1)$.

For Part 2 of Theorem 2, note that, when $\phi > \bar{\phi}$, we from (D.19) have

$$\frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}\right) < \sigma\phi\left(\frac{1}{\beta}-1\right).$$

Hence, from (D.22),

$$\lim_{\tau_d \to 0^+} \lambda_2 > \frac{2\frac{1+\sigma\phi}{\beta}}{\frac{1}{\beta}+1+\frac{\sigma\phi}{\beta}+\sqrt{\left(1+\frac{1+\sigma\phi}{\beta}\right)^2-4\frac{1+\sigma\phi}{\beta}}} = 1.$$

As a result, there exists no bounded equilibrium if the fiscal adjustment is infinitely delayed (i.e., if $\tau_d \to 0$ from above).

For $\tau_d > 0$, we have[37]

$$\lambda_2 = f(a,b) \equiv \frac{a+b+1+\sigma\phi-\sqrt{\left(a+b-(1+\sigma\phi)\right)^2+4(1+\sigma\phi)\,b}}{2}$$

where $f(a,b)$, $a = \frac{1}{\beta}(1-\tau_d) > 0$, and $b = \frac{(1-\beta\omega)(1-\omega)}{\beta\omega}\left(\tau_y - \beta\phi\frac{D^{ss}}{Y^{ss}}\right) > 0$. Since $\frac{\partial f}{\partial a} = \frac{1}{2} - \frac{a+b-(1+\sigma\phi)}{2\sqrt{(a+b-(1+\sigma\phi))^2+4(1+\sigma\phi)b}} > 0$, we know that ρ_d decreases with τ_d and $\lim_{\tau_d\to 1^-}\lambda_2 = 0$. As a result, for $\phi > \bar{\phi}$, there exists an $\underline{\tau}_d(\phi) \in (0,1)$ such that $\lambda_2 < 1$ if and only if $\tau_d > \underline{\tau}_d(\phi)$. As a result, for $\phi > \bar{\phi}$, any bounded equilibrium exists if and only if $\tau_d > \underline{\tau}_d(\phi)$. For any $\phi \in \left(\bar{\phi}, \frac{\tau_y}{\beta\frac{D^{ss}}{Y^{ss}}}\right)$, $\chi = \frac{1-\tau_d-\beta\rho_d}{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}} \le \frac{1-\underline{\tau}_d(\phi)}{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}$. From (D.16), $\nu \le \frac{\left(\kappa\frac{D^{ss}}{Y^{ss}}+\tau_y\right)\frac{1-\underline{\tau}_d(\phi)}{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}}{\bar{\tau}_d(\phi)+\left(\kappa\frac{D^{ss}}{Y^{ss}}+\tau_y\right)\frac{1-\underline{\tau}_d(\phi)}{\tau_y-\beta\phi\frac{D^{ss}}{Y^{ss}}}} \equiv \bar{\nu}(\phi) < 1$. This proves Part 2 of Theorem 2.

D.6 Proof of Theorem 3

We start with the baseline fiscal policy (7). We focus on a bounded equilibrium similar to (19), taking the form of

$$y_t = \chi_d d_t + \chi_\varepsilon \varepsilon_t, \quad \mathbb{E}_t[d_{t+1}] = \rho_d d_t + \rho_\varepsilon \varepsilon_t \quad \text{with} \quad \chi_d, \chi_\varepsilon > 0, \;\; \rho_d \in (0,1). \tag{D.23}$$

For (D.23) to be an equilibrium, it needs to satisfy (17) and (30). For (D.23) to satisfy to the government budget (17), we need

$$\rho_d = \frac{1}{\beta}\left(1-\tau_d-\tau_y\chi_d\right) \quad \text{and} \quad \rho_\varepsilon = \frac{1}{\beta}\left(1-\tau_d-\tau_y\chi_\varepsilon\right). \tag{D.24}$$

For (D.23) to satisfy aggregate demand (30) (together with market clearing $c_t = y_t$), we need

$$\chi_d = M_d + M_y\left[\left(1-\tau_y\right)\chi_d - \tau_d\right]\left(1+\delta\sum_{k=1}^{+\infty}\left(\beta\omega\rho_d\right)^k\right) = \frac{M_d - \tau_d M_y\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)}{1-M_y\left(1-\tau_y\right)\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)}. \tag{D.25}$$

and

$$\chi_\varepsilon = M_y\left(1+\left(\left(1-\tau_y\right)\chi_\varepsilon - \tau_d + \delta\left(\left(1-\tau_y\right)\chi_d - \tau_d\right)\sum_{k=1}^{+\infty}(\beta\omega)^k\rho_d^{k-1}\rho_\varepsilon\right)\right). \tag{D.26}$$

[37] The formula for the function f is slightly adjusted compared to (D.12) in the baseline analysis, to accommodate $\phi \neq 0$.

(D.24) and (D.25) together mean that ρ_d needs to be the root of the following equation:

$$h(\rho_d,\tau_d) \equiv \frac{1-\tau_d-\beta\rho_d}{\tau_y} - \frac{M_d - \tau_d M_y \left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)}{1-M_y(1-\tau_y)\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)} = 0.$$

When Assumption 2 holds, we first show that there exists $\rho_d^{\text{full}} \in [0,1)$ such that

$$h\left(\rho_d^{\text{full}},0\right) = \frac{1-\beta\rho_d^{\text{full}}}{\tau_y} - \frac{M_d}{1-M_y(1-\tau_y)\left(1+\frac{\delta\beta\omega\rho_d^{\text{full}}}{1-\beta\omega\rho_d^{\text{full}}}\right)} = 0.$$

Note that that $h(0,0) = \frac{1}{\tau_y} - \frac{M_d}{1-M_y(1-\tau_y)} \geq 0$ because $\tau_y > 0$, $M_y \in (0,1)$ and $M_y \geq M_d$. Then, there are two cases. First, $M_y(1-\tau_y)\left(1+\frac{\delta\beta\omega}{1-\beta\omega}\right) > 1$. In this case, there exists $\bar{\rho} \in (0,1)$ such that

$$M_y(1-\tau_y)\left(1+\frac{\delta\beta\omega\bar{\rho}}{1-\beta\omega\bar{\rho}}\right) = 1.$$

For $\rho_d \in [0,\bar{\rho})$, $h(\rho_d,0)$ monotonically decreases in ρ_d and $\lim_{\rho_d\to(\bar{\rho})^-} h(\rho_d,0) = -\infty$. As a result, there exists a unique $\rho_d^{\text{full}} \in [0,\bar{\rho})$ such that we have $h\left(\rho_d^{\text{full}},0\right) = 0.$[38]

Second, $M_y(1-\tau_y)\left(1+\frac{\delta\beta\omega}{1-\beta\omega}\right) < 1$. From Assumption 2, in this case,

$$h(1,0) = \frac{1-\beta}{\tau_y} - \frac{M_d}{1-M_y(1-\tau_y)\left(1+\frac{\delta\beta\omega}{1-\beta\omega}\right)} < 0,$$

and $h(\rho_d,0)$ monotonically decreases in $\rho_d \in [0,1]$. As a result, there exists a unique $\rho_d^{\text{full}} \in [0,1)$ such that $h\left(\rho_d^{\text{full}},0\right) = 0$.

Note that $h(\rho_d,\tau_d)$ is continuously differentiable in the neighborhood of $\left(\rho_d^{\text{full}},0\right)$. Because $\frac{\partial h\left(\rho_d^{\text{full}},0\right)}{\partial\rho_d} < 0$, we can use the implicit function theorem to show that for each τ_d in a right neighborhood of 0, there exists a unique $\rho_d(\tau_d) \in [0,1)$ such that $h(\rho_d(\tau_d),\tau_d) = 0$ and $\lim_{\tau_d\to 0^+} \rho_d(\tau_d) = \rho_d^{\text{full}} \in [0,1)$. For $\tau_d \to 0^+$, given $\rho_d(\tau_d)$, one can find $\rho_\varepsilon(\tau_d)$ and $\chi_d(\tau_d), \chi_\varepsilon(\tau_d) > 0$ from (D.24) – (D.26), and constitute a bounded equilibrium based on (16)–(18). The fact that $\lim_{\tau_d\to 0^+} \nu = 1$ follows directly from boundedness and $\lim_{\tau_d\to 0^+} \frac{\tau_d\left(\varepsilon_0+\sum_{k=0}^{\infty}\beta^k\mathbb{E}_0[d_k]\right)}{\varepsilon_0} = 0$, using (21) and (22). The fact that $\lim_{k\to\infty}\mathbb{E}_t[d_{t+k}] \to 0$ follows directly from $\rho_d^{\text{full}} \in [0,1)$. This finishes the proof with the baseline fiscal policy (7).

We now turn to the case with (8). Together with market clearing $c_t = y_t$, we first write the aggregate demand

[38] Note that, for $\rho_d \in (\bar{\rho},1)$, $\frac{M_d}{1-M_y(1-\tau_y)\left(1+\frac{\delta\beta\omega\rho_d}{1-\beta\omega\rho_d}\right)} < 0$. As a result, $h(\rho_d,0) > 0$.

in (30) recursively

$$
\begin{aligned}
y_t &= \frac{M_d}{1-M_y} d_t - \frac{M_y}{1-M_y} t_t + \delta\beta\omega \frac{M_y}{1-M_y} \mathbb{E}_t\left[y_{t+1} - t_{t+1}\right] + \beta\omega \mathbb{E}_t\left[y_{t+1} - \frac{M_d}{1-M_y} d_{t+1} + \frac{M_y}{1-M_y} t_{t+1}\right] \\
&= \frac{M_d}{1-M_y} d_t - \frac{M_y}{1-M_y} t_t + \delta\beta\omega \frac{M_y}{1-M_y} \mathbb{E}_t\left[y_{t+1} - t_{t+1}\right] + \beta\omega \mathbb{E}_t\left[y_{t+1} + \frac{M_y}{1-M_y} t_{t+1}\right] - \omega \frac{M_d}{1-M_y} \left(d_t - t_t\right) \\
&= \frac{M_d\left(1-\omega\right)}{1-M_y} d_t - \frac{M_y - \omega M_d}{1-M_y} t_t + \beta\omega \left(\frac{1-\left(1-\delta\right) M_y}{1-M_y}\right) \mathbb{E}_t\left[y_{t+1}\right] + \beta\omega \frac{M_y}{1-M_y} \left(1-\delta\right) \mathbb{E}_t\left[t_{t+1}\right].
\end{aligned} \tag{D.27}
$$

From (8), we know that $t_t = d_t$ for all $t \geq H$. As a result, $d_{t+1} = 0$ for all $t \geq H$. Similar to the argument in Appendix A.3, we can then focus on the case that $y_t = d_t = 0$ for $t \geq H+1$. At $t = H$, from (D.27), we have

$$
y_H = \frac{-\left(M_y - M_d\right)}{1-M_y} d_H = \chi_0 d_H \text{ with } \chi_0 = \frac{-\left(M_y - M_d\right)}{1-M_y}. \tag{D.28}
$$

Similar to the main analysis in Appendix A.3, we will now use (D.27) to find the equilibrium path of $\{y_t, d_t\}_{t=0}^{H-1}$ through backward induction. At $t = H-1$, from (8) and (D.27),

$$
\begin{aligned}
y_{H-1} &= \frac{\frac{M_d(1-\omega)}{1-M_y}}{1+\tau_y \frac{M_y-\omega M_d}{1-M_y}} d_{H-1} + \beta\omega \frac{\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \frac{M_y(1-\delta)}{1-M_y}}{1+\tau_y \frac{M_y-\omega M_d}{1-M_y}} d_H \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y}}{1+\tau_y \frac{M_y-\omega M_d}{1-M_y}} d_{H-1} + \omega \frac{\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \frac{M_y(1-\delta)}{1-M_y}}{1+\tau_y \frac{M_y-\omega M_d}{1-M_y}} \left(d_{H-1} - \tau_y y_{H-1}\right) \\
y_{H-1} &= \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left[\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \frac{M_y(1-\delta)}{1-M_y}\right]}{1+\tau_y \frac{M_y-\omega M_d}{1-M_y} + \omega\tau_y\left[\left(\frac{1-(1-\delta)M_y}{1-M_y}\right)\chi_0 + \frac{M_y(1-\delta)}{1-M_y}\right]} d_{H-1} \\
&= \chi_1 d_{H-1},
\end{aligned} \tag{D.29}
$$

with

$$
\begin{aligned}
\chi_1 &= \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left[\frac{-\delta M_y(1-M_d)+M_d(1-M_y)}{(1-M_y)^2}\right]}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\tau_y\left[\frac{-\delta M_y(1-M_d)+M_d(1-M_y)}{(1-M_y)^2}\right]} \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y} + \frac{\omega M_y}{1-M_y}\left(\frac{M_d}{M_y} - \delta\frac{1-M_d}{1-M_y}\right)}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \frac{\omega M_y}{1-M_y}\left(\frac{M_d}{M_y} - \delta\frac{1-M_d}{1-M_y}\right)\tau_y}.
\end{aligned} \tag{D.30}
$$

For $1 \le t \le H-2$, from (8) and (D.27),

$$\begin{aligned}
y_t &= \frac{\frac{M_d(1-\omega)}{1-M_y} d_t}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \beta\omega \frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} \mathbb{E}_t\left[y_{t+1}\right] \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y} d_t}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \omega \frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} \left(d_t - \tau_y y_t\right)\chi_{H-t-1} \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-t-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-t-1}} d_t \\
&= \chi_{H-t} d_t \;\text{ with }\; \chi_{H-t} = \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-t-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-t-1}}
\end{aligned} \tag{D.31}$$

Finally, for $t=0$, from (8) and (D.27), we know

$$\begin{aligned}
y_0 &= \frac{\frac{M_d(1-\omega)}{1-M_y} d_0 + \frac{M_y-\omega M_d}{1-M_y}\varepsilon_0}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \beta\omega \frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} \mathbb{E}_0\left[y_1\right] \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y} d_0 + \frac{M_y-\omega M_d}{1-M_y}\varepsilon_0}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} + \omega \frac{\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y} \left(d_0+\varepsilon_0 - \tau_y y_0\right)\chi_{H-1} \\
&= \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-1}} d_0 + \frac{\frac{M_y-\omega M_d}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-1}} \varepsilon_0 \\
&= \chi_H d_0 + \chi_H^{\varepsilon}\varepsilon_0 \;\text{ with }\; \chi_H = \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-1}},
\end{aligned} \tag{D.32}$$

and $\chi_H^{\varepsilon} = \frac{\frac{M_y-\omega M_d}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{H-1}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{H-1}}$. Define

$$g(\chi) \equiv \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi} = \frac{1}{\tau_y} - \frac{\frac{M_y-M_d}{1-M_y} + \frac{1}{\tau_y}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi}. \tag{D.33}$$

From (D.30), we know that $\chi_1 = g\left(\chi_0'\right)$ with

$$\chi_0' = \frac{M_y}{1-(1-\tau_y)(1-\delta)M_y}\left(\frac{M_d}{M_y} - \delta\frac{1-M_d}{1-M_y}\right). \tag{D.34}$$

From (D.31) and (D.32) we have $\chi_k = g\left(\chi_{k-1}\right)$ for all $k \in \{2,\cdots,H\}$. We first find the fixed point of $g(\chi)$:

$$\chi_{MSV} = \frac{\frac{M_d(1-\omega)}{1-M_y} + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{MSV}}{1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV}}, \tag{D.35}$$

which is equivalent to

$$\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV}^2 + \chi_{MSV}\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y - \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right) - \frac{M_d(1-\omega)}{1-M_y} = 0. \tag{D.36}$$

Let $\chi_{MSV,1}$ denote the smaller root and $\chi_{MSV,2}$ denote the larger root:

$$\chi_{MSV,1} = \frac{-\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right) - \sqrt{\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)^2 + 4\frac{M_d(1-\omega)}{1-M_y}\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}}{2\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}$$

$$\chi_{MSV,2} = \frac{-\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right) + \sqrt{\left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)^2 + 4\frac{M_d(1-\omega)}{1-M_y}\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}}{2\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y}. \tag{D.37}$$

If Assumption 1 holds ($\omega < 1$), we know that $\chi_{MSV,1}\chi_{MSV,2} < 0$ so $\chi_{MSV,1} < 0$ and $\chi_{MSV,2} > 0$. Note that $1+\frac{M_y-\omega M_d}{1-M_y}\tau_y + \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV,1} > 0$,[39] we have $g(\chi) > \chi$ if $\chi \in (\chi_{MSV,1}, \chi_{MSV,2})$ and $g(\chi) < \chi$ if $\chi \in (\chi_{MSV,2}, +\infty)$. From (D.33), we also know that $g(\chi)$ increases if $\chi \in [\chi_{MSV,1}, +\infty)$.

Moreover, from (D.34), we know that $\chi_0' \geq \chi_{MSV,1}$. To see this, define the left-hand side of (D.36) as

$$h(\chi) = \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi^2 + \left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y - \omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)\chi - \frac{M_d(1-\omega)}{1-M_y}.$$

We have

$$\begin{aligned}
h\left(\chi_0'\right) &= \left(\omega\frac{\tau_y M_y}{1-M_y}\left(\frac{M_d}{M_y}-\delta\frac{1-M_d}{1-M_y}\right)+1+\frac{M_y-\omega M_d}{1-M_y}\tau_y-\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\right)\chi_0' - \frac{M_d(1-\omega)}{1-M_y}\\
&= \left(1+\frac{\tau_y M_y}{1-M_y}\left(1-\omega\delta\frac{1-M_d}{1-M_y}\right)\right)\chi_0' - \omega\frac{M_y}{1-M_y}\left(\frac{M_d}{M_y}-\delta\frac{1-M_d}{1-M_y}\right) - \frac{M_d(1-\omega)}{1-M_y}\\
&= \frac{M_y\left(1+\frac{\tau_y M_y}{1-M_y}\left(1-\omega\delta\frac{1-M_d}{1-M_y}\right)\right)}{1-(1-\tau_y)(1-\delta)M_y}\left(\frac{M_d}{M_y}-\delta\frac{1-M_d}{1-M_y}\right) - \frac{M_d}{1-M_y}\left(1-\omega\delta\frac{M_y}{M_d}\frac{1-M_d}{1-M_y}\right)\\
&= \frac{(1-M_y)^2+\tau_y M_y(1-M_y-\omega\delta(1-M_d))}{1-(1-\tau_y)(1-\delta)M_y}\frac{M_d}{1-M_y}\left(1-\delta\frac{M_y}{M_d}\frac{1-M_d}{1-M_y}\right) - \frac{M_d}{1-M_y}\left(1-\omega\delta\frac{M_y}{M_d}\frac{1-M_d}{1-M_y}\right)
\end{aligned}$$

Since $\frac{(1-M_y)^2+\tau_y M_y(1-M_y-\omega\delta(1-M_d))}{1-(1-\tau_y)(1-\delta)M_y} < 1$ and $1-\delta\frac{M_y}{M_d}\frac{1-M_d}{1-M_y} < 1-\omega\delta\frac{M_y}{M_d}\frac{1-M_d}{1-M_y}$, we know that $h\left(\chi_0'\right) < 0$ so $\chi_0' \geq \chi_{MSV,1}$. The fact that $\chi_0' \geq \chi_{MSV,1}$ together with the aforementioned property of $g(\chi)$ means that $\{\chi_k\}_{k=0}^{\infty}$ is a bounded, monotonic sequence converging to $\lim_{k\to+\infty}\chi_k = \chi_{MSV,2} > 0$.

[39] If $1+\frac{M_y-\omega M_d}{1-M_y}\tau_y+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\tau_y\chi_{MSV,1} < 0$, $\frac{M_d(1-\omega)}{1-M_y}+\omega\left(\frac{1-(1-\tau_y)(1-\delta)M_y}{1-M_y}\right)\chi_{MSV,1} < 0$, and $\chi_{MSV,1} > 0$ from (D.35), a contradiction.

If Assumptions 1 and 2 hold, $\chi_{MSV,2} > \frac{1-\beta}{\tau_y}$. To see this, we have

$$\begin{aligned}
h\left(\frac{1-\beta}{\tau_y}\right) &= \omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\frac{(1-\beta)^2}{\tau_y} + \left(1+\frac{M_y-\omega M_d}{1-M_y}\tau_y - \omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\right)\frac{1-\beta}{\tau_y} - \frac{M_d(1-\omega)}{1-M_y} \\
&= -\beta\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\frac{1-\beta}{\tau_y} + \left(1+\frac{M_y}{1-M_y}\tau_y\right)\frac{1-\beta}{\tau_y} - \frac{(1-\beta\omega)M_d}{1-M_y} \\
&< -\beta\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\frac{1-\beta}{\tau_y} + \left(1+\frac{M_y}{1-M_y}\tau_y - \frac{(1-\beta\omega)\left[1-\left(1-\tau_y\right)M_y\left(1+\delta\frac{\beta\omega}{1-\beta\omega}\right)\right]}{1-M_y}\right)\frac{1-\beta}{\tau_y} \\
&= -\beta\omega\left(\frac{1-\left(1-\tau_y\right)(1-\delta)M_y}{1-M_y}\right)\frac{1-\beta}{\tau_y} + \beta\omega\left(\frac{1-\left(1-\tau_y\right)M_y(1-\delta)}{1-M_y}\right)\frac{1-\beta}{\tau_y} = 0.
\end{aligned}$$

Similar to (A.9),

$$\mathbb{E}_0\left[d_t\right] = \frac{1}{\beta^{t-1}}\Pi_{j=1}^{t-1}\left(1-\tau_y\chi_{H-j}\right)\mathbb{E}_0\left[d_1\right].$$

Since $\lim_{k\to+\infty}\chi_k = \chi_{MSV,2} > \frac{1-\beta}{\tau_y}$, we know that $\lim_{H\to\infty}\mathbb{E}_0\left[d_H\right]\to 0$. From (21) and (22), we also know that $\nu\to 1$ as $H\to\infty$. This finishes the proof with the alternative fiscal policy (8).