The Dynamics of Multi-Project Collaborations Online Appendix

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This appendix contains the proof of Proposition 1, calculations for all the figures in the text, and proofs for all the extensions.

Proof of Proposition 1. Recall that after a deviation in period t, players set $P_i^t = \emptyset$ and $b_{i,-i}^t = 0$ if not already chosen. In subsequent periods, they revert to the static equilibrium with zero transfers and no selected projects.

The proof proceeds in four steps: (i) we show that it is without loss of optimality to restrict attention to relational contracts that are surplus-maximizing following every on-path history h^t; (ii) we provide a necessary and sufficient condition for the existence of a relational contract that implements a given project selection rule $\mathbf{P}(\cdot)$; (iii) we show that this condition is independent of the division of surplus between the players; and (iv) we show that, for any two histories that generate the same beliefs, selecting the same continuation equilibrium is without loss of optimality.

Step 1 We show that it is without loss of optimality to restrict attention to relational contracts that are surplus-maximizing following every on-path history h^t. To see this, suppose that there exists an on-path history h^t such that the continuation equilibrium starting in period t, denoted by e^1, has lower total surplus than an alternative continuation equilibrium e^2. Thus, if we define $\mathcal{C}_i^k$ to be the continuation value to player i in equilibrium e^k, then $\sum_i \mathcal{C}_i^1 < \sum_i \mathcal{C}_i^2$. For the rest of Step 1, we omit the superscript $t-1$ in our notation, as we are solely concentrating on period $t-1$ objects.

Let us modify the players' relational contract such that play in and after period t is dictated by e^2 and the period $t-1$ $b_{i,j}(\cdot)$ transfers associated with history h^t

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(and, thus, corresponding to a specific realization of $\mathbf{v}^{t-1}$) are adjusted so that: (i) player 2's expected payoff following the realization of $\mathbf{v}^{t-1}$ is the same as under the original equilibrium and (ii) player 1's expected payoff following the realization of $\mathbf{v}^{t-1}$ increases by $\sum_i \mathcal{C}_i^2 - \sum_i \mathcal{C}_i^1$. Specifically, take the vector of transfers $\mathbf{b}_1 = (b_{1,2}^1, b_{2,1}^1)$ associated with the original equilibrium and create a new vector of transfers $\mathbf{b}_2 = (b_{1,2}^2, b_{2,1}^2)$ such that:

$$\mathcal{C}_1^2 + b_{2,1}^2 - b_{1,2}^2 > \mathcal{C}_1^1 + b_{2,1}^1 - b_{1,2}^1, \tag{1}$$

$$\mathcal{C}_2^2 + b_{1,2}^2 - b_{2,1}^2 = \mathcal{C}_2^1 + b_{1,2}^1 - b_{2,1}^1. \tag{2}$$

Because $\sum_i \mathcal{C}_i^2 - \sum_i \mathcal{C}_i^1 > 0$, finding payments that satisfy $b_{1,2}^2 \leq \mathcal{C}_1^2$ and $b_{2,1}^2 \leq \mathcal{C}_2^2$ is always feasible.

Note that these changes have no impact on player 1's choices of actions made in any period $t' \leq t-1$ because all actions are observable, and hence choosing a different action from the proposed equilibrium would be labeled a defection. If defections were deterred in the original equilibrium, which had a strictly smaller continuation value for player 1, then they are also deterred in the new equilibrium. The same logic applies to player 2 since they obtain the same expected payoff in period $t-1$ (compared to the original equilibrium), and thus also have the same continuation values in all periods $t' < t-1$. Finally, note that surplus from a date 0 perspective is strictly higher under the new equilibrium.

Step 2 We show that there exists a relational contract that implements a project selection rule $\mathbf{P}(\cdot)$ if and only if the following inequality holds for all t and for all histories $h^t \in \mathcal{H}^t$:

$$\sum_{p \in \mathbf{P}^t} \sum_{i=1,2} \max\Big(0, c - \mathbb{E}(v_{i,p}|h^t)\Big) \leq \mathcal{C}(h^t), \tag{3}$$

where $\mathcal{C}(h^t)$ is the continuation value.

To show that (3) is a necessary and sufficient condition, consider a set of transfers $b_{i,-i}(\mathbf{v}^t) \geq 0$ to be paid on path given a vector of realized values $\mathbf{v}^t$.

Given an equilibrium project selection $\mathbf{P}^t$, note that it is without loss of generality to assume that $P_1^t = P_2^t = \mathbf{P}^t$. Thus, for each player and for each $p \in \mathbf{P}^t$, the player must weakly prefer to include p in P_i^t, rather than excluding it. Let $\alpha_i(\mathbf{v}^t)$ denote player i's share of $\mathcal{C}(h^t \sqcup \mathbf{v}^t)$ as a function of $\mathbf{v}^t$. Hence, the condition for selecting

$\mathbf{P}^t$ is:

$$\sum_{p\in\mathbf{P}^t} \max\left(c - \mathbb{E}(v_{i,p}|h^t), 0\right) \leq \mathbb{E}\Big(b_{-i,i}(\mathbf{v}^t) - b_{i,-i}(\mathbf{v}^t) + \alpha_i(\mathbf{v}^t)\mathcal{C}(h^t \sqcup \mathbf{v}^t)\Big), \quad \forall i, \tag{4}$$

$$b_{i,-i}(\mathbf{v}^t) \leq \alpha_i(\mathbf{v}^t)\mathcal{C}(h^t \sqcup \mathbf{v}^t), \quad \forall \mathbf{v}^t, \forall i. \tag{5}$$

Expectations are taken over the project valuations realizations $\mathbf{v}^t$ and $h^t \sqcup \mathbf{v}^t$ denotes the players' updated beliefs after observing $\mathbf{v}^t$.[1] The first expression states that the promised transfers and the expected share of the total continuation value must be enough to prevent a player from shirking on any subset of the projects. The second expression states that the each player is willing to pay the other player the necessary transfer.

To show necessity: Note that since Equation (4) must hold for a fixed i, the inequality also holds summing over all i. Further, all transfers cancel out when summing over i. Finally, by definition, $\mathbb{E}(\mathcal{C}(h^t \sqcup \mathbf{v}^t)) = \mathcal{C}(h^t)$. Hence, we are left with Equation (3).

To show sufficiency: We will show this result in two substeps.

SubStep 1: We show it is necessary and sufficient to replace Equation (5) by its expectation. This new expression is as follows:

$$\mathbb{E}(b_{i,-i}(\mathbf{v}^t)) \leq \mathbb{E}\Big(\alpha_i(\mathbf{v}^t)\mathcal{C}(h^t \sqcup \mathbf{v}^t)\Big) \quad \forall i. \tag{6}$$

We first show that if there is a solution to Equations (6) and (4), then there exists a solution to Equations (5) and (4).

Take a set of transfers $b_{i,-i}(\mathbf{v}^t)$ that satisfy Equations (6) and (4). Define:

$$b'_{i,-i}(\mathbf{v}^t) = \alpha_i(\mathbf{v}^t)\mathcal{C}(h^t \sqcup \mathbf{v}^t) - \Bigg(\mathbb{E}\Big(\alpha_i(\mathbf{v}^t)\mathcal{C}(h^t \sqcup \mathbf{v}^t) - b_{i,-i}(\mathbf{v}^t)\Big)\Bigg). \tag{7}$$

Since Equation (6) holds, the term in the expectation of Equation (7) is positive and thus Equation (5) holds for all realizations of $\mathbf{v}^t$ under the set of transfers $b'_{i,-i}(\mathbf{v}^t)$. Finally, $\mathbb{E}(b'_{i,-i}(\mathbf{v}^t)) = \mathbb{E}(b_{i,-i}(\mathbf{v}^t))$ so Equation (6) continues to hold.

SubStep 2: Using substep 1, it suffices to show that Equation (3) implies a

[1] The history also includes the project selections, and both the upfront and end-of-period transfers. However, for notational convenience we only include the realized valuations as every other object can be inferred on path from the realized valuations.

solution to Equations (4) and (6). To simplify all the notation with expectations, Equation (4) can be re-expressed as:

$$\beta_i - \gamma_i \leq (\tilde{b}_{-i,i} - \tilde{b}_{i,-i}), \tag{8}$$

where $\tilde{b}_{i,-i}$ is the expected transfer from i to $-i$, $\beta_i = \sum_{p\in\mathbf{P}^t} \max\left(0, c - \mathbb{E}(v_{i,p}|h^t)\right)$, and $\gamma_i = \mathbb{E}(\alpha_i(\mathbf{v}^t)\mathcal{C}(h^t \sqcup \mathbf{v}^t))$. Equation (6) can thus be re-written as:

$$\tilde{b}_{i,-i} \leq \gamma_i. \tag{9}$$

Rearranging Equation (3) implies $\sum_i(\beta_i - \gamma_i) \leq 0$. One can now show that $\tilde{b}_{i,-i} = \max(0, \beta_{-i} - \gamma_{-i})$ satisfies Equation (9). Further, Equation (8) holds because:

$$\begin{aligned} \beta_i - \gamma_i \leq \max(0, \beta_i - \gamma_i) - \max(0, \beta_{-i} - \gamma_{-i}) && (10)\\ \iff \max(0, \beta_{-i} - \gamma_{-i}) - \min(0, \gamma_i - \beta_i) \leq 0 && (11)\\ \Longleftarrow \sum_i(\beta_i - \gamma_i) \leq 0, && (12) \end{aligned}$$

where the final step follows from noting that $\beta_1 - \gamma_1$ and $\beta_2 - \gamma_2$ cannot both be positive and analyzing the remaining three cases based on the signs of $\beta_i - \gamma_i$.

Finally, Equation (9) reduces to

$$\begin{aligned} \max(0, \beta_{-i} - \gamma_{-i}) \leq \gamma_i \Longleftarrow \beta_{-i} - \gamma_{-i} \leq \gamma_i && (13)\\ \Longleftarrow \sum_i(\beta_i - \gamma_i) \leq 0, && (14) \end{aligned}$$

where the final implication is due to β_i being weakly positive.

Step 3: We show that any relational contract that implements a given project selection rule can be replaced by an alternative relational contract that implements the same project selection rule and yields no surplus to player 2.[2] First, note that the way the players share their continuation value does not affect Equation (2) from the main text. Hence, for any period t where player 2's expected payoff is positive, $w_{2,1}$ can be increased until player 2's expected payoff is zero. Player 2 is willing to make this transfer because not doing so would be seen as a deviation, resulting in a

[2]Of course, one could take the relational contract derived from Steps 3 and 4 and choose to redistribute the surplus by an up-front payment every period from player one combined with reducing the expected payment from player one at the end of each period.

payoff of 0 for player 2.

Step 4: We now show that, for any two histories h_1^t and $h_2^{t'}$ that generate the same beliefs μ, selecting the same continuation equilibrium is without loss of optimality. Take a relational contract r that is surplus-maximizing at all on-path histories and has two histories h_1^t and $h_2^{t'}$ prescribing different (surplus-maximizing) continuation equilibria under the same beliefs μ. Recall from Step 3 that one can consider relational contracts in which player 2 obtains an expected payoff equal to 0 in every period. In this case, since the two continuation equilibria are both optimal and both give all the surplus to player 1, switching from one continuation equilibrium to the other does not change the players' incentives as both prescribe the exact same payoffs to the players. Hence, when focusing on relational contracts that specify the same continuation equilibrium following histories that induce the same beliefs, one can replace $\mathcal{C}(h^t)$ with $\mathcal{C}(\mu^t)$. □

Calculations for Figure 1. Throughout, the notation $V(\cdot), \mathcal{C}(\cdot)$ will denote the net-present value and continuation value of a given project selection rule, respectively. Further, recall that we set $c = 1$. One can solve for the optimal project selection rule recursively starting from $n = m = 3$. When $n = 3$, the players exploit all three projects because Assumption 1 implies that exploitation is an equilibrium. Recall that Assumption 1 states:

$$1 \leq \frac{\delta}{1-\delta}(v-2). \tag{15}$$

By Proposition 3, $f(2) \geq 2$. When the project selection rule is such that the relationship scope is stochastically maximal, the players choose to exploit two projects and explore zero projects (if the players could explore a third project, then they would do so). It must thus be the case that:

$$3c > 2\mathcal{C}(\text{exploit}) + \mathcal{C}(\text{explore}), \tag{16}$$

where $\mathcal{C}(\text{explore}) = q\frac{\delta}{1-\delta}(v-2) + (1-q)\delta(qv - 2 + \mathcal{C}(\text{explore}))$. We can reduce this constraint to:

$$
\begin{aligned}
3 &> 2\frac{\delta}{1-\delta}(v-2) + \mathcal{C}(\text{explore}) \\
&= \frac{\delta}{1-\delta}(v-2)\Big(2 + \frac{q}{1-\delta(1-q)}\Big) + \frac{(1-q)\delta(qv-2)}{1-\delta(1-q)}.
\end{aligned} \tag{17}
$$

When this constraint does not hold, $f(2) = 2$. Since the players will never explore projects in a domain where a suitable project has been identified, the players have two choices: exploit two projects, or exploit one project and explore another. Choosing $f(n) = n$ is always feasible, and hence both project selection rules are feasible. Thus, the players must prefer to exploit 2 projects as opposed to exploiting a single project for the stochastically maximal project selection rule to be optimal. This condition for such a preference is stated below:

$$
\begin{aligned}
&2V(\text{exploit}) \geq V(\text{exploit}) + V(\text{delayed exploit}) + V(\text{explore}) \\
\iff &V(\text{exploit}) \geq V(\text{delayed exploit}) + V(\text{explore}),
\end{aligned} \tag{18}
$$

where,

$$
V(\text{delayed exploit}) = 0 + \delta(qV(\text{exploit}) + (1-q)V(\text{delayed exploit})) \tag{19}
$$

$$
V(\text{explore}) = (qv-2) + \delta(qV(\text{exploit}) + (1-q)V(\text{explore})). \tag{20}
$$

Hence, we can reduce Equation (18) to:

$$
\begin{aligned}
V(\text{exploit}) &\geq \frac{qv - 2 + \delta q V(\text{exploit})}{1-(1-q)\delta} + V(\text{exploit})\frac{\delta q}{1-\delta(1-q)} \\
\iff V(\text{exploit})(1-\delta-\delta q) &\geq qv - 2 \\
\iff \frac{v-2}{1-\delta}(1-\delta-\delta q) &\geq qv - 2
\end{aligned} \tag{21}
$$

To complete the proof, we must additionally prove that, given the choices dictated by the project selection rule when $n = 2, 3$, $f(0) > 0$ and hence the relational contract is non-empty. To do so, we consider the project selection rule when $n = 1, 0$. Proposition 5 implies that the scope reaches its maximum of 3 with positive probability, and thus the players must explore precisely two projects. Finally, $f(1) \leq f(2) = 2$, implying that when $n = 1$, the players exploit zero projects.

Lastly, when $n = 0$, the players have no projects to exploit. In any non-empty optimal relational contract, $f(0) \geq 1$. Further, $f(0) \leq f(1) = 2$. The players explore either one or two projects when $n = 0$. It is sufficient to show that exploring one project when $n = 0$ is feasible, given the project selection rule determined above when $n = 1, 2, 3$. Below we show the constraints for feasibility at period 0.

Hence, if the stochastically maximal project selection rule is feasible for $n = 0$ and $n = 1$ and it satisfies Equations (16), (18), and (21), then such a project selection rule is optimal. To write down the feasibility constraints for $n = 0$ and $n = 1$, one must compute the expected continuation value when $n = 1$, $\mathcal{C}(1)$:

$$\begin{aligned}\mathcal{C}(1) &= \delta\Big(q^2 3\frac{v-2}{1-\delta} + 2q(1-q)2\frac{v-2}{1-\delta} + (1-q)^2(2(qv-2) + \mathcal{C}(1))\Big) \\ &= \frac{\delta\Big((4q-q^2)\frac{v-2}{1-\delta} + 2(1-q)^2(qv-2)\Big)}{1-\delta(1-q)^2}.\end{aligned} \tag{22}$$

Thus, the constraint for feasibility when $n = 1$ is:

$$2 \leq \frac{\delta\Big((4q-q^2)\frac{v-2}{1-\delta} + 2(1-q)^2(qv-2)\Big)}{1-\delta(1-q)^2} = \mathcal{C}(1). \tag{23}$$

Further, the expression for $\mathcal{C}(0)$ is:

$$\begin{aligned}\mathcal{C}(0) &= \delta\Big(q(2(qv-2) + \mathcal{C}(1)) + (1-q)((qv-2) + \mathcal{C}(0))\Big) \\ &= \frac{\delta q(2(qv-2) + \mathcal{C}(1)) + (1-q)\delta(qv-2)}{1-\delta(1-q)}.\end{aligned}$$

Thus, the constraint for feasibility when $n = 1$ boils down to:

$$1 \leq \frac{\delta q(2(qv-2) + \mathcal{C}(1)) + \delta(1-q)(qv-2)}{1-\delta(1-q)}. \tag{24}$$

One can plot these constraints to check whether they can jointly be satisfied. If there exists a set of parameter values where all the constraints hold with a strict inequality, then there exists an open set of parameter values where the inequalities hold strictly. Finally, any value in the interior of the region of Figure 1 in which the stochastically maximal project selection rule is reported as optimal satisfies the inequalities strictly.[3] $\square$

Proof of Figure 2. When $s_p \sim \text{Exp}(\lambda)$ and benefits are symmetric, we can compute the threshold s^0 such that the players are indifferent between exploitation and exploration:

$$\frac{s^0}{1-\delta} = \mathbb{E}(s_p) + \frac{\delta}{1-\delta}\mathbb{E}\left(\max\{s_p, s^0\}\right) \tag{25}$$

$$\iff \frac{s^0}{1-\delta} = \frac{1}{\lambda} + \frac{\delta}{1-\delta}\left(e^{-\lambda s^0}\left(s^0 + \frac{1}{\lambda}\right) + \left(1 - e^{-\lambda s^0}\right)s^0\right). \tag{26}$$

The left-hand side corresponds to the exploitation surplus. The right-hand side corresponds to the expected surplus when exploring one more time and subsequently exploiting the best project found until then. The second step utilizes the expected value of the exponential and computes the expected value of the maximum operator conditional on whether $s^0 < s_p$ or $s_p < s^0$, respectively.

Solving this expression for s^0 when $\lambda = 1\,/\,3$ yields the equation for s^0 provided in the text. Finally, solving for $\tilde{s}$ was done in the text. $\square$

For the extension considered in Section 5.3, we assume that $s_p \sim F$. However, we also suppose that: with probability $1-q$, both players have valuations $v_{p,1} = v_{p,2} = s_p\,/\,2$; with probability $q\,/\,2$, player 1 values the project at $v_{p,1} = s_p$ and player 2 at $v_{p,2} = 0$; and with the same probability $q\,/\,2$, player 2 values the project at $v_{p,2} = s_p$ and player 1 at $v_{p,1} = 0$. We assume that the distribution of benefits is i.i.d. across projects. Our assumptions regarding the distribution of project values imply that project exploration is an equilibrium of the stage game, as in Section 4.2. For simplicity, we also suppose that $m = 1$. Setting $q = 1$ thus corresponds to the analysis in Section 4.2. By contrast, setting $q = 0$ corresponds to a special case of the benchmark model with symmetric benefits analyzed in Section 3.2.

Proposition A1 (**Symmetric and Asymmetric Benefits**)
In any optimal relational contract, there exist two thresholds, denoted as s_s^* and s_a^*, such that the project selection rule is as follows: the players explore a project if both (i) the highest-valued symmetric-benefits project found so far has a value less than s_s^*, and (ii) the highest-valued asymmetric-benefits project found so far has a value less

[3] The Mathematica code needed to plot these inequalities is available upon request.

than s_a^*. Furthermore, they permanently exploit the first symmetric- or asymmetric-benefits project with a valuation greater than s_s^* or s_a^*, respectively.

Proof of Proposition A1. Note first that the optimal relational contract conditions only on the highest-valued symmetric- and asymmetric-benefits projects found to date. By Proposition 1, these are the only projects that may ever be exploited. Denote the values associated with the highest-valued symmetric- and asymmetric-benefits projects by $\hat{s}_s$ and $\hat{s}_a$, respectively. In any optimal relational contract, the project selection rule of the players can then be summarized as a function mapping $\hat{s}_s, \hat{s}_a$ into one of three choices: (1) exploiting the symmetric-benefits project, (ii) exploiting the asymmetric-benefits project, and (iii) exploration.

Next, note that after exploiting a project, the players' beliefs about the projects do not change, and, hence, if the players exploit a project once, they will permanently exploit that project. Therefore, the continuation value of the players' relationship associated with the permanent exploitation of a project with value s (if the exploitation of a project with value s is feasible) is equal to $\frac{\delta}{1-\delta}(s-2c)$. More specifically, for symmetric-benefits projects, exploitation is an equilibrium of the stage game, and thus the continuation value from exploiting a symmetric-benefits project with value s is always equal to $\frac{\delta}{1-\delta}(s-2c)$. In contrast, the continuation value from exploiting an asymmetric-benefits project with value s is $\frac{\delta}{1-\delta}(s-2c)\mathbf{1}_{c\leq\frac{\delta}{1-\delta}(s-2c)}$, where the condition in the indicator function corresponds to the condition under which the players are able to cooperate in exploiting the project.

Finally, the players never choose to exploit a project p they previously chose not to exploit. To see this, note that the players cannot exploit p in the future even if p is the highest-valued project (since, by assumption, they have chosen not to exploit it in the past). However, by Proposition 1, the players cannot exploit p when it is not the highest-valued project either. Hence, the continuation value from exploration is some constant, which we denote B.

Finally, suppose the highest-valued project found to date has value $\hat{s}$. If this project is a symmetric-benefits project, the players exploit it if and only if $(\hat{s}-2c) / (1-\delta) \geq B$. If this project is an asymmetric-benefits project, the players exploit it if and only if $(\hat{s}-2c) / (1-\delta) \geq B$ and $\delta(\hat{s}-2c) / (1-\delta) \geq c$. It follows from these expressions that the thresholds s_s^* and s_a^* stated in the proposition exist. $\square$

Recall that $\tilde{s} = c(1+\delta) / \delta$. We now characterize the thresholds s_a^* and s_s^*.

Proposition A2 (**Optimal Thresholds' Properties**)

1. *$s_a^* = \max\left(s^0, \tilde{s}\right) \geq s^0 \geq s_s^*$.*

2. *s_s^* is monotone increasing in δ and monotone decreasing in q.*

3. *s_a^* is independent of q and U-shaped in δ.*

To prove Proposition A2, we first prove the following lemma.

Lemma A1 (**Continuation Value Comparative Statics**)
Define by $\mathcal{C}(\delta, q)$ the continuation value of the players' relationship following exploration. Then, $\mathcal{C}(\delta, q)$ is decreasing in q and $\mathcal{C}(\delta, q)(1-\delta)$ is increasing in δ.

Proof of Lemma A1. First, recall from the proof of Proposition A1 that the continuation value following exploration in the current period is independent of the values of the projects explored by the players up until and including the previous period. To prove that $\mathcal{C}(\delta, q)$ is decreasing in q, note that, as q decreases, the players are strictly more likely to encounter a symmetric-benefits project. Because the players are always able to exploit symmetric-benefits projects, the continuation value of their relationship weakly increases as q decreases.

Next, consider any two values $\delta_1 < \delta_2$. Note that any project selection rule implementable by an optimal relational contract when the players have discount factor δ_1 must also be implementable in equilibrium when the players have discount factor δ_2, because Equation (2) in the main text is relaxed as δ increases. Thus, given an optimal project selection rule for discount factor δ_1, $\mathbf{P}$, the players' expected continuation value is simply:

$$\mathcal{C}(\delta_1, q) = \delta_1 \pi(t+1, \mathbf{P}) + \delta_1^2 \pi(t+2, \mathbf{P}) + \ldots, \tag{27}$$

where $\pi(\cdot)$ denotes the expected joint surplus in a given period under the project selection rule. Further, because this project selection rule is also feasible with δ_2,

$$\mathcal{C}(\delta_2, q) \geq \delta_2 \pi(t+1, \mathbf{P}) + \delta_2^2 \pi(t+2, \mathbf{P}) + \ldots \tag{28}$$

Combining these observations implies that $\mathcal{C}(\delta, q)(1-\delta)$ is increasing in δ. □

Proof of Proposition A2. **Statement 1:** We first show that $s_a^* = \max\left(s^0, \tilde{s}\right)$. This result was shown in Corollary 1 in the text for the case when $q = 1$. By Lemma A1, for any $q < 1$, the continuation value following exploration weakly increases compared to the case when $q = 1$. Because s_a^* is defined by the players' indifference between exploration and exploitation, the increased continuation value following exploration implies that $s_a^* \geq \max\left(s^0, \tilde{s}\right)$. Finally, s_a^* is not necessarily strictly greater than $\max\left(s^0, \tilde{s}\right)$, because (i) $s_a^* \geq s^0$ implies that, when benefits are symmetric, the players would exploit such a project and (ii) $s_a^* \geq \tilde{s}$ implies that the players are able to replicate the project selection rule of the symmetric-benefits case.

What is left to show is $s^0 \geq s_s^*$. Note that the continuation value following exploitation is the same in this case and the case of symmetric benefits. However, the surplus following exploration is weakly higher under symmetric benefits. Thus, for any value s where exploitation is preferred in the symmetric-benefits benchmark, exploitation is also preferred with asymmetric benefits. Thus, the threshold must be weakly higher compared to the symmetric-benefits benchmark.

Statement 2: Note that the joint surplus associated with the exploitation of a symmetric-benefits project with value s is equal to $(s - 2c) \,/\, (1 - \delta)$. Further, s_s^* represents the value a project must achieve for the players to be indifferent between exploiting the project and exploring. Thus:

$$\frac{s_s^* - 2c}{1 - \delta} = \mathcal{C}(\delta, q) \iff s_s^* - 2c = (1 - \delta)\mathcal{C}\left(\delta, q\right). \tag{29}$$

The statement now follows given the results stated in Lemma A1.

Statement 3: This is immediate from the definition of s_a^*. $\square$