Aggregate Dynamics in an Economy with Optimal Long-term Financing

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Abstract

I introduce optimal dynamic contracting between risk-averse investors and firm insiders into a dynamic general-equilibrium model with heterogeneous firms. The model features a time-varying cross-sectional distribution of firms in equilibrium, and I propose a new numerical solution technique to handle the resulting curse of dimensionality. I show that even with moderate levels of insider ownership, agency frictions have significant effects on the dynamics of macro-economic quantities and asset prices. First, the response of risk-premia and key macro-economic quantities depends in a nonlinear manner on the history of aggregate shocks. Accumulation of small shocks results in a disproportionately large decline in aggregate quantities and a rise in risk-premia. Second, inefficiencies resulting from second-best contractual arrangements help amplify the effect of primitive shocks and make the economy more sensitive to negative than to positive shocks. Third, exit rates of firms rise during recessions. Fourth, controlling for current aggregate productivity, firm exit rates contain incremental information about future output growth.

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1 Introduction

What are the aggregate implications of a misalignment of interest between outside investors and the decision makers within a firm? Does imperfect corporate control have sizeable effects on asset prices, such as the market price of risk, and on macro-economic quantities such as aggregate consumption, output, and investment? This paper answers these questions in a production economy setting with heterogenous firms exposed to both aggregate and firm-specific risk.

Frictionless models assume decision makers within a firm maximize the present value of the firm. All positive net present value projects are funded. The choice of financing is irrelevant in this frictionless setting, and investment decisions are completely determined by investment opportunities. There is no need for the firm to maintain financial slack. In reality, financing decisions matter and interact with investment decisions because of the presence of various frictions. In certain circumstances, such frictions have very large economic effects, as illustrated by the recent financial crisis.

To study the effect of financial frictions on a firm’s investment decision, one approach is to fix the financial arrangement (typically single-period debt) between the firm and outside investors. I do not take this path for two reasons. First, I view financing choice as an equilibrium outcome of the interaction between investors and firms. The two parties choose the best possible arrangement to achieve their objectives subject to constraints arising from an agency problem. This approach helps us learn about how financing frictions impact the choices of financial contracts between firms and investors, the joint dynamics of aggregate macro-economic quantities, such as consumption, output and investment, and asset prices – all of which are jointly determined in equilibrium. The second reason for adopting this approach is that in reality, firms use various state-contingent forms of financing. Examples include lines of credit, debt of multiple maturity, convertible debt, and interest rate derivatives.
The dynamics that result from firms using state-contingent financing is expected to be very
different from models in which firms solely use debt-financing.

I integrate a dynamic principal agent model within a stochastic growth model with
investment to determine the joint dynamics of investment, output, and consumption together
with financing policies. The dynamics that is generated from the interaction of this friction
with investment opportunities is quite rich. First, the response of risk-premia and key
macro-economic quantities depends non-linearly on the history of aggregate shocks. In my
model, large drops in aggregate quantities and sharp increases in risk premia are the result of
the accumulation of small shocks. This view of rare disasters is potentially useful because it
gives us a better handle on the probability of deep recessions. Second, the amplification of
primitive shocks is asymmetric – the economy displays a higher sensitivity to negative shocks.
Third, both the level and volatility of firm exit rates increase during recessions. Fourth,
firm exit rates provide additional information about future output growth beyond current
aggregate productivity.

A description of the key elements of my model. Individual firms are operated by insiders
who own a fraction of the firm. I view insiders as composed of at least the CEO and top
level management. The firm needs the insider’s expertise to operate the technology. Insiders
need funding from outside investors. Financing covers operational expenses and also provides
compensation to insiders. The insider’s effort increases expected output, but is costly to
provide. As a result, investors have to provide the right incentives to induce effort. The
financial contract provides incentives to insiders in the form of current and future payments,
together with the threat of possible termination to induce high effort. I assume termination
to be inefficient – insiders receive their outside option which I assume to be a constant
normalized to zero, and outside investors recover only a fraction of the installed capital.
Thus, absent incentive issues, both parties would profit from keeping the match alive. The
financing side is standard in the dynamic contracting literature. In partial equilibrium, the
financial contracts in my model are related to the models of DeMarzo and Fishman (2007a), DeMarzo and Fishman (2007b), and Biais, Mariotti, Plantin, and Rochet (2007) with one key difference. Unlike these papers which consider the investor to be risk-neutral and therefore set the discount rate used to value cash-flows to a constant, in my setting the discount rate is stochastic, and determined endogenously in general equilibrium. The production side of my model is also standard. The particular version I use is from Gomes, Kogan, and Zhang (2003). It is a stochastic growth model with aggregate investment and adjustment cost of capital. This latter feature makes prices more volatile than quantities, a feature of the data.

Some intuition for the main results. History-dependence and non-linear aggregate dynamics arises from the dead-weight loss due to inefficient liquidations and also because a change in the aggregate state involves a non-linear adjustment of contract policies. The result is that the cross-sectional distribution of firms changes shape over time, and the shape is informative about future dynamics of aggregate quantities and risk premia. A sequence of negative shocks is manifested by a larger mass of firms in the left tail, and this is why exit rates are informative about the past history of aggregate shocks. All of the history dependence is endogenously generated by the friction arising from the agency problem.

The reason for amplification of primitive shocks and the behavior of exit rates are related. During booms productivity is high and expected to persist making it optimal for investors to lower the probability of termination during booms. However, to keep the threat of termination alive for incentive purposes, poor past performance leads to contract termination during recessions. In general equilibrium, there is a feedback effect. A higher exit rate during recessions makes these contract poor hedges for the investor, depressing the value of new contracts even more. This lowers investment in recessions increasing consumption risk, which in turn lowers the value of investing even more. Lower current investment leads to lower output in future periods, thus propagating the effect of the bad shock. Recessions are characterized by an increase in consumption volatility, which translates into a higher risk
premium for risky assets.

With exits playing an important role, it is important to accurately track the cross-sectional distribution, especially the left-tail. This poses a challenge. I present a new approach to solve for the equilibrium of this economy. I reduce the infinite dimensional distribution to five parameters by projecting it into a mixture of normal distributions. This turns out to produce sufficiently high level of convergence of the fixed point problem. The current techniques in the literature rely on either approximating the distribution by the mean (Krusell and Smith (1998), or by approximating the entire past history by the last few shocks (Chien and Lustig (2010)). In a model with exits, the mean is not very informative about the shape and dynamics of the distribution near the default boundary. The second approach is also not very convenient in my setting because the dynamics is highly persistent, and would require tracking a prohibitively long sequence of shocks. My solution approach is of independent use in any model with defaults or exits.

Any model with financial frictions is expected to generically produce amplification and persistence of primitive shocks. Prior research on the real effects of financial frictions (for instance the literature following the seminal works of Kiyotaki and Moore (1997) and Bernanke, Gertler, and Gilchrist (1999)) confirms this intuition. What do we gain by allowing firms and investors to choose the form of financing? First, firms do use financing which is intrinsically state-contingent in nature. Lines of credit, debt of multiple maturity, interest rate derivatives are just a few of the many securities that firms use\footnote{Sufi (2009) reports that 85% of firms in his sample obtained a line of credit. This included fully equity financed firms which held no debt.}. The second benefit of this exercise, is that we learn if the effects obtained by fixing the form of financing are robust to allowing firms and investors a much wider financing choice. For instance, with debt financing, insider equity holders are unable to hedge their exposure to aggregate shocks and forced to absorb all risk. Allowing firms to hedge aggregate risk with state-contingent contracts could potentially
soften the impact of the financial frictions. Finally, manager-owned, debt-financed firms is probably a more appropriate description of small, entrepreneurial firms rather than medium or large public firms which have a large pool of outside investors.

A convenient feature of the financial contract in my model is that the fraction of the firm equity owned by insiders is a parameter can be set to a value lower than one. In my numerical analysis, insiders own only 15% of the firm. Finally, a word about manager preferences. Although I report results for a risk-neutral manager in my benchmark setting, I solved the model assuming a risk-averse manager with power utility and risk aversion of 0.5 for low and moderate levels of consumption, and linear utility for high consumption. The financial contract in that setting, additionally provides insurance to the manager. At such low levels of risk-aversion, there is not much quantitative difference in the results. I do not report the results in this paper.

**Literature review**

There is a growing literature which attempts to assess the impact of corporate finance frictions on asset prices. Dow, Gorton, and Krishnamurthy (2005) look at investment and effects on state prices where the friction is free cash flow problem by manager. Investors force repayment by employing a costly auditing technology. Apart from the fact that the financial friction is different, there are three key differences with the economy modeled in this paper. First, in their setting, equilibrium policies and prices deviate from the frictionless setting only in good states of the economy. As a consequence, the market price of risk in their setting decreases with the strength of the friction. In my setting it increases. Second, the assumption of myopic investors in their model implies that the free cash flow problem lasts for a single period. In a different setting Krishnamurthy (2003) and Tella (2013) show firms are able to perfectly hedge aggregate shocks using optimal contracts, and the effects of financial frictions such as amplification and propagation of primitive shocks completely disappear. My setting differs from these two studies because the optimal contract is unable to perfectly hedge the aggregate shock resulting in possible inefficient termination.

Results of this exercise are available on request.
my model, the impact of the financial friction extends over multiple periods. Third, they consider a representative firm, whereas in my model, key features of dynamics just as history-dependence and non-linearity arises from the heterogenous cross-section. Albuquerue and Wang (2008) also considers an agency problem similar to the setting here. The key difference is that they consider investment specific shocks rather than productivity shocks considered here. Moreover, there are no firm exits in their setting. The aggregate effects of imperfect enforcement has been studied in Cooley, Marimon, and Quadrini (2004). Again, the friction is different, and unlike my setting, there are no exits in equilibrium. The sharply contrasting effects of debt financing versus optimal contracts has been shown by Krishnamurthy (2003), and more recently in Tella (2013). The key difference between these papers and the setting here is the presence of inefficient termination in equilibrium. This feature leads to policies which are non-linear in aggregate shocks. This loss of linearity prevents perfect hedging and gives rise to the non-linear and history dependent dynamics.

The literature which considers the impact of debt contracts as borrowing constraints is fairly extensive. Starting with the influential work of Bernanke and Gertler (1989) and Kiyotaki and Moore (1997) (see also Bernanke et al. (1999)) and more recent results by Brunnermeier and Sannikov (2014) demonstrate how productivity shocks are amplified and propagated beyond one period. The bulk of this literature considers single period debt financing. As a result, the default probability is entirely determined by the distribution of firm-specific shocks which are an exogenous input to these models. A second difference is that the cross-sectional distribution of firm characteristics (such as leverage) can be aggregated because policies are linear. This means that cross-sectional heterogeneity plays no role in these models, and they are effectively results for a representative firm borrowing from a representative household. An important recent example of a heterogenous firm model with borrowing constraints is Khan and Thomas (2013). There are no firm liquidations or exits in this model. Gomes and Schmid (2009) is a recent paper which considers the effect of firms
using long-term debt to finance investment. Equity holders issue infinite maturity debt for its tax advantage and default when equity value is zero. Coupon size is determined when the firm enters the economy and held fixed. Gomes, Yaron, and Zhang (2006) consider the impact of a reduced form specification of financing constraints on asset prices.

Financial contracts similar to the one considered here been discussed in a partial equilibrium framework. The financial friction in my model is a dynamic version of the hidden effort model of Holmstrom and Tirole (1997). The financial contract is similar to the discrete-time models of DeMarzo and Fishman (2007a), DeMarzo and Fishman (2007b), and Biais et al. (2007) (see also DeMarzo and Sannikov (2006), who provides a characterization of the contract in a continuous time setting.) Clementi and Hopenhayn (2006) considers the effect of this friction on firm level investment. The friction in these models is that cash flows are privately observed by the manager (see also the two period model by Gertler (1992)). DeMarzo, Fishman, He, and Wang (2012) consider the effects of persistent, publicly observable, productivity shocks on investment in a traditional q-theory model with agency problems. Hoffmann and Pfeil (2010) consider optimal managerial compensation policies in a setting where firms experience persistent productivity shocks. In a different setting, Piskorski and Tchistyj (2010) consider exogenously specified stochastic interest rates to derive the optimal mortgage contract. The key difference of this paper with these studies is that the discount rate used to value cash flows from the contract are stochastic and arise as endogenously from consumption smoothing motives of the risk-averse representative investor. This difference drives most of our results. Costly termination implies investment is partially irreversible. In general equilibrium, this feature adds to the variation in consumption growth and thus to the market price of risk making downturns more severe than booms. I show that the magnitude of the asymmetry depending strongly on the investor’s risk-aversion.

\[4\] A general equilibrium analysis as the one presented in this paper differs from the asset-pricing literate on credit risk which usually assumes an exogenous stochastic discount rate process to price bonds. In contrast, the covariance between defaults and discount rates is endogenously determined in this economy.
risk-premium in this economy is counter-cyclical. A second, less important difference, is that managers in this economy are risk-averse, in contrast to DeMarzo and Fishman (2007a) and DeMarzo and Fishman (2007b), where they are risk-neutral. The interaction of incentive and insurance motives has a long literature – Rogerson (1985), Green (1987), and more recently by Sannikov (2008). Finally, Gromb (1999) and Quadrini (2004) investigate the effect of renegotiation to avoid inefficient project termination.

This paper relates to two strands of existing literatures. One studies the effect of financial frictions on real economic dynamics and asset prices. The second studies optimal dynamic contracting. I contribute to the first literature by making two changes to the standard assumptions. First, financial contracts arise as an optimal response to a more primitive friction thereby avoiding the need to assume particular forms of borrowing constraints. Second, I relax the assumption of full insider ownership and allow the insider to own only a small fraction of the firm. I contribute to the dynamic contracting literature by relaxing the assumption of constant discount rate of the investors. I show that even at the partial equilibrium level, the difference in discount rates of the investor across aggregate states has interesting implications for the financial contract. Terminations are not necessarily monotonic in expected output and depend on the volatility of the discount rate across the states. This generates novel dynamics of risk-premia and aggregate macro-economic quantities, such as consumption, output, investment, and asset prices. In general equilibrium, the discount rate is determined endogenously by the representative household’s consumption and savings decision. This helps us learn about how financing frictions impact the choices of financial contracts between firms and investors, the joint dynamics of aggregate macro-economic quantities (such as consumption, output and investment) and asset prices – all of which are jointly determined in equilibrium.

The rest of this paper is organized as follows. In Section 2 I describe the model. In Section 3 I describe properties of the solution. Section 4 reports results of a numerical
2 The Model

In this section I describe a general equilibrium with a continuum of firms who finance production by entering into long-term contracts with lenders. I begin by describing the production sector with technology and investment, and then describe firm entry and details of the financial contract. Finally, I close the model with a description of the household sector.

2.1 The Environment

Production Sector

There is a continuum of ex-ante identical firms operated by a manager who relies on external financing to operate his technology. The production technology is linear. It produces the same homogenous good, which can be used both for consumption and investment

\[ y(x, z, k) = (x + z)k. \]  

(1)

The variable \( k \) is the firm’s capital/capacity which is installed when the firm begins to operate and is held fixed for the life of the firm. Each period, the capital requires a maintenance cost \( \delta k \). For simplicity, the parameter \( \delta \) is assumed to be constant across firms. \( x \) is an aggregate shock common to all firms and \( z \) is a firm-specific shock. The variable \( k \) is the firm’s capital/capacity which is installed when the firm begins to operate and is held fixed for the life of the firm. \( x \) is a discrete Markov process with transition matrix \( \Gamma \)

\[ \Pr(x_{t+1}|x_t) = \Gamma. \]  

(2)

In the baseline model, for simplicity, I assume only two aggregate states. The aggregate shock \( x \) takes on two values \( x = \{x_G, x_B\} \). The firm-specific shock also takes on two values \( z_+ > z_- \),
and is independently distributed across time and in the cross-section. The probability of a higher realization depends on the effort exerted by the firm manager

\[
\Pr(z_+) = \begin{cases} 
p, & \text{if effort} = 1 \\
 p - \Delta p, & \text{if effort} = 0 \end{cases}
\]  

(3)

Conditional on high effort by the manager, \( z \) is normalized to have zero mean.\(^5\) Providing high effort is costly – shirking (effort= 0) provides the manager with constant private benefit \( B_k \). Although realizations of both shocks \( x \) and \( z \) are publicly observable, investors do not observe the manager’s effort choice and use observations of firm output to give the manager incentives to provide high effort.

Prospective managers have no initial wealth. Starting a firm requires an initial installation cost \( \bar{e}k \) where \( \bar{e} \) is randomly drawn from a uniform measure \( H \), and is revealed to the manager at the beginning of the period. I describe the support of \( H \) and its justification in more detail below. The prospective manager is offered a contract by investors which specifies: (a) payments that he will receive, and (ii) the probability of termination of the contract \( \zeta \). Both of these policies are functions of the history of the individual manager’s output together with the history of output of all the managers in the economy. I assume that the law of large numbers holds, so that the total output in any period is determined by the aggregate shock \( x_t \).

Contract policies depend on the discount rate that the investor uses to value future cash flows. In my setting, the representative household’s equilibrium consumption process determines the discount rate. The possibility of exits makes the household’s future consumption depend on the entire cross-sectional distribution of firms (not just the mean). The aggregate state, which I denote by \( s \), is captured by the the aggregate shock \( x \) and its entire past history. The contract assumes two-sided commitment in which both the manager and the investor agree to abide by the terms of the contract in all possible contingencies, with no possibility

\(^5\)The mean is absorbed by \( x \).

\(^6\)I use the terms project and firm interchangeably.
of renegotiation. While this is a restrictive assumption, it provides a useful benchmark and can be thought of as a limiting case where renegotiation is extremely costly. Such may be the case if the investors consist of a large dispersed pool of individuals.

For simplicity, four additional assumptions are made about the manager: (a) he has limited liability, (b) he cannot save privately, (c) he has a time-invariant outside option normalized to zero\(^7\) and (d) he values a consumption stream \(\{c_t\}\) as \(\sum_t \beta_t c_t\) with time preference parameter much smaller than that of the outside investor \(\beta_e << \beta_l\). As the following proposition shows, provided the gain in expected output from the manager exerting high effort is sufficiently large compared to the private benefit \(B\), the optimal contract elicits high effort.

**Proposition 1** If firm specific shocks satisfy \(\Delta p(z_+ - z_-) >> B\), then there exists an optimal contract in which it is optimal for the manager to exert high effort.

**Proof.** Same as in Appendix 1, Proposition 13 of Biais, Mariotti, Plantin, and Rochet (2004).

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**The Investor’s problem and timing**

The financial contract between the outside investor and the firm manager serves three purposes: it provides for initial installation costs, covers maintenance costs (in the event of low realizations of \(z\)), and also provides insurance to the risk-averse manager. Following Spear and Srivastava (1987) and Green (1987), I use the dynamic programming approach to solve for the optimal contract. In this recursive formulation, the present discounted value of the future payments to the manager, which I denote by \(V\), is a sufficient statistic for the entire past history of firm-specific realizations of \(z\). In the presence of aggregate risk, i.e. time varying \(x\), the distribution of continuation values promised to existing managers varies over

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\(^7\)This assumption could be relaxed in a richer model where the manager has an outside option of becoming a worker. The lower bound of \(V\) is then determined by the rental rate of labor. I leave this extension for the future.
time, and depends on the realized sequence of aggregate shocks. Therefore, in addition to the current aggregate shock \( x \), the aggregate state of this economy includes the cross-sectional measure of continuation values of all managers in the economy, which I denote by \( \mu \). The aggregate state \( s = \{ x, \mu \} \) therefore includes both the current aggregate shock \( x \) and the cross-sectional distribution \( \mu \). In addition to depending on \( V \), contracts are conditioned on the entire aggregate state \( s \) – in particular, each contract implicitly depends on the entire history of aggregate shocks, and also on the history of output of all existing firms through \( \mu \).

The assumption of linear technology allows firm capital \( k \) to be scaled out. The investor’s problem is to offer a contract to the manager that maximizes the present discounted value of future cash flows (scaled by \( k \))

\[
\pi_t(s) F_t(V; s) = \max_{\zeta(s'), d^\pm(s'), V^\pm(s')} E^\Gamma \left[ \pi_{t+1}(s') \left( \zeta(s') \left( (1 - \chi)(1 - \delta) \right) + (1 - \zeta(s)) E^p \left( -\delta + x' + z_\pm - d_\pm(s') + F_{t+1}(V_\pm(s'); s') \right) \right) \right]
\]

\[
V = E^\Gamma \left[ (1 - \zeta(s')) E^p \left( d_\pm(s') + \beta^e V'_\pm(s') \right) \right], \quad \forall s'
\]

\[
B/\Delta p \leq \left( d_+(s') + \beta^e V'_+(s') \right) - \left( d_-(s') + \beta^e V'_-(s') \right), \quad \forall s',
\]

\[
\left\{ \zeta, d^\pm(s'), V^\pm(s') \right\} \in [0, 1] \times \mathbb{R}_+^4.
\] (4)

The choice variables are the termination probability of the manager \( \zeta \), the manager’s payments \( d^\pm(V,s) \) and continuation value \( V^\pm(V,s) \). Each of these depend on the aggregate state \( s \), the current continuation value of the manager, and whether \( z_\pm \) is realized. The expectation \( E^\Gamma[\cdot] \) is computed assuming the transition probability matrix \( \Gamma \) for aggregate shocks \( x \), while the expectation \( E^p[\cdot] \) is computed assuming that the agent exerts high effort. This means that \( z_+ \) is realized with probability \( p \). The assumption of manager’s limited liability implies \( V'_\pm(s) \geq 0 \). An interpretation of the lower bound of the manager’s continuation value is that his outside option is normalized to zero. The law of motion of the aggregate state \( s \)
includes the law of motion of the cross-sectional distribution of manager continuation values \( \mu \). This law of motion, \( \mathcal{H}(x', \mu) \), depends both on \( \Gamma \) and on contract policies \( V'_\pi \). In solving the contracting problem, agents take this law of motion as given. In a rational expectations equilibrium, the law of motion used by agents is the realized in the aggregate. Likewise, investors and the manager take the discount rate \( \pi(s') \) as given. In general equilibrium, however, the latter is determined by the representative household’s consumption and savings decisions, and is determined by the household’s marginal utility evaluated at the equilibrium aggregate consumption level \( \pi(s')/\pi(s) = \beta^l (C^*_t(s')/C^*_t(s))^{-\gamma^l} \), where \( C^* \) is the equilibrium aggregate consumption of the household.

Several comments are in order. First, each period, investors receive a (potentially negative) payment which is firm output net of maintenance expenditure and the manager’s payment \( d \) till the contract is terminated. When the contract is terminated, the investor takes over the firm and recovers \((1 - \chi)(1 - \delta)k\) of the capital installed \((0 < \chi < 1)\). Termination is therefore costly. Second, in valuing contracts and determining optimal policies, the only stochastic component of cash flows priced by investors are those associated with systematic market-wide risk factors. We can either view the representative household as holding a well-diversified portfolio consisting of all the contracts and a risky-free asset in zero net supply, or picture identical, individual investors each entering into a long-term contract with a single manager. In the latter case, individual investors act as pass-throughs – they collect payments and pass them on to the representative household. Since they belong to the same risk-sharing household, all investors use the same discount rate to value cash flows. Both these pictures have identical pricing implications: the idiosyncratic component of cash flows of individual contracts can be completely diversified away, and are therefore not priced. The household’s aggregate consumption process is the only systematic risk factor used in pricing risky cash flows.

Timing in this economy is shown in Fig. 16. At the beginning of each period, the aggregate
shock $x$ is observed. In accordance with the contractual agreement, manager continuation values (discounted utility of future payments) are adjusted depending on the realized aggregate state. As will be shown below, the aggregate state depends not only on the realized value of the aggregate shock $x$, but also on the cross-sectional distribution of the continuation values of all managers in the economy. A public lottery for termination of the project is held in which firms with low continuation values might exit. Maintenance is paid for continuing firms. Production takes place next, i.e. firms realize $z$ and produce output. Finally, managers are paid, new contracts are initiated, and the representative household consumes.

The expectation $E^\Gamma[\cdot]$ is computed assuming the transition probability matrix $\Gamma$ for aggregate shocks $x$, while the expectation $E^p[\cdot]$ is computed assuming that the agent exerts effort $e = 1$. This means that $z_+$ is realized with probability $p$. The assumption of manager’s limited liability $d_{\pm}(s) \geq 0$, together with the form of the manager’s utility function assumed implies $V'_{\pm}(s) \geq 0$. An interpretation of the lower bound of the manager’s continuation value is that his outside option is normalized to zero. The law of motion of the aggregate state $s$ includes the law of motion of the measure over cross-sectional continuation values $\mu$. In a rational expectations equilibrium, all agents correctly forecast this law of motion $H$. In determining optimal policies, investors and the manager take the discount rate $\pi(s')$ as given. In equilibrium, $\pi(s')/\pi(s) = \beta^l(C_{t+1}(s')/C_t(s))^{-\gamma^l}$.

To summarize, at the beginning of each period, after observing $x$, the lottery for termination is held. Conditional on survival, production takes place. The lender chooses the manager’s payments $d_{\pm}(V, s)$ and future promised continuation values $V'_{\pm}(V, s)$, based on high/low output, and conditioned on the aggregate state $s = \{x, \mu\}$. Managers who have their contracts terminated, have future consumption set to zero. They could be thought of as having exited the economy. Optimal policies of the financial contract, such as the manager’s compensation and payments made to the lender, depend on the path of realized cash flows. This means that even though firms are ex-ante identical, there is ex-post cross-sectional
heterogeneity.

**Firm entry and investment**

Aggregate investment takes place through entry of new firms\(^8\). Each period, there is a mass of potential managers with projects ready to enter the economy. To begin production, a one-time fixed cost \(\tilde{e}k\) has to be paid. This random cost is drawn from a uniform measure \(H\), and is revealed to potential managers at the beginning of each period. To ensure balanced growth, I assume the mass of potential entrants is proportional to the measure of existing firms: \(H = hN_t\), where \(N_t = \int d\mu_t\) is the total number of firms in existence at the beginning of period \(t\) before any entry and exit in that period. The constant of proportionality \(h\) is a measure of the investment opportunity set. At the aggregate level, this entry cost mimics adjustment costs and has the effect of making asset prices more volatile.

A new firm is born if the manager is able to secure financing for the project by entering into a long-term financial contract with an outside investor. I assume perfectly competitive lending markets so that investors break even. Managers have all the bargaining power and choose the highest possible payoff subject to the investor’s participation constraint

\[
V_0 = \sup\{V : F(V) \geq \tilde{e}k\}. \quad (5)
\]

Panel A of Figure 2 shows an example. The contract pays for the initial installation cost, and in subsequent periods, conditional on continuation, the investor commits to paying the maintenance cost \(\delta k\), and compensates the manager according to his past and present performance. Production begins from the period subsequent to entry. Projects with high entry costs are unable to secure funding and expire worthless. Panel B of Figure 2 shows this graphically.

\(^8\)I borrow this modeling technique from Gomes et al. (2003)
Households

I assume that the economy is populated by a single representative household. Individual investors are assumed to be members of this single risk-sharing household and therefore, they all share the same stochastic discount factor. This household derives utility from consumption of the single good \( C_t \) and has standard time-separable power utility

\[
E_0 \sum_{t=0}^{\infty} \beta_t^t C_t^{1-\gamma} \frac{1}{1-\gamma_t},
\]

where \( \gamma_t \) is the household’s risk-aversion, and \( \beta_t \) is the time preference parameter. The household derives income from accumulated wealth, and makes consumption and investment decisions to maximize expected lifetime utility subject to the budget constraint. By assumption, the household is assumed to be more patient than managers (\( \beta_e < \beta_t \)). In addition to investing in firms through the financial contracts, the household also invests in a single risk-free asset which is in zero net supply. The household takes the risk-free rate \( r \) and the market price of each of the financial contracts \( F^i \) as given, and chooses consumption \( C_t \) and the portfolio of risk-free asset and financial contracts to maximize utility \( U_t \) subject to the budget constraint.

2.2 General Equilibrium and Aggregation

The equilibrium concept is a Markov perfect competitive equilibrium. The state space of this problem includes the cross-sectional distribution of manager continuation values, which is infinite dimensional. The formal definition of the equilibrium is given below:

**Definition 1** Recursive equilibrium – A recursive competitive equilibrium is defined as a set of functions for

(i) contract policies \( \Phi(V, x, \mu) = \{V_\pm(k, V, x, \mu), d_\pm(k, V, x, \mu), \zeta(k, V, x, \mu)\} \),
(ii) initial contract state \( V_0(x, \mu) \),
(iii) consumption policies of the representative household \( C(x, \mu) \), and
(iv) law of motion of states \( s' \sim \{x', \mu'\} \), such that

(i) individual contracts are
optimal, (ii) the initial state is such that the lender breaks even (Eq. 5), (iii) the representative household’s policies are optimal (Eq. 4)

(iv) the goods market clears

\[ C^*(s) = \int [xzk - d(s)]d\mu - I(s) - L(s). \]  

(7)

where aggregate investment I and loss from default L are given by

\[ I(s) = \int_0^{\bar{e}(s)} ekdH + \int \delta kd\mu - D(s)k = \left[ \frac{h}{2} e^2(s) + \delta - D(s) \right] k \int d\mu, \]  

(8)

\[ L(s) = \chi(1 - \delta)kD(s) \int d\mu. \]  

(9)

D(s) is the rate of termination of contracts,

(v) the market for contracts clears

\[ W_{ct}^* = \int F_t(V)d\mu_t \]  

(10)

where \( W_{ct}^* \) is the household’s wealth invested in financial contracts, (vi) the bond market clears

\[ W_{bt}^* = 0 \]  

(11)

where \( W_{bt}^* \) is the household’s wealth invested in the risk-free asset, and (vi) the law of motion of the cross-sectional distribution of continuation values, \( \mathcal{H}(x', \mu) \) is consistent with individual contract policies and the stochastic process for \( z \).

The following proposition summarizes properties of asset returns and the risk-free rate in the economy.

**Proposition 2** The equilibrium stochastic discount factor in this economy is defined by \( \pi_{t+1}/\pi_t = \beta_t \left( \frac{C^*_t + 1}{C^*_t} \right)^{-\gamma_t} \), where \( C^*_t \) is the equilibrium consumption of the representative household. All gross returns \( R^i \) in this economy, satisfy the no-arbitrage relation \( E_t \left[ \frac{\pi_{t+1}}{\pi_t} R^i_{t+1} \right] = 1. \)
The risk-free rate, in particular is given by 

\[ 1 + r_t = 1/E_t \left[ \frac{\pi_{t+1}}{\pi_t} \right]. \]

**Proof.** See Appendix. \[\blacksquare\]

### 3 Model Solution

I first describe the competitive equilibrium with no moral hazard. In the presence of moral hazard, aggregate consumption growth depends on the cross-sectional distribution of continuation values of all managers in the economy. Manager payment policies change non-linearly when the aggregate state changes, and as a consequence, the cross-sectional distribution moves over time. The situation is similar to models with aggregate risk, where non-linearity of policies makes it impossible for the heterogenous cross-section to be aggregated in a tractable manner (for example Krusell and Smith (1998)). Since the cross-sectional distribution is infinite dimensional, it is infeasible to solve for prices and policies exactly, and an approximation approach is necessary to proxy the cross-sectional distribution. The presence of exits complicates matters, since the mass of managers with low values of \( V \) and close to the default boundary have to be tracked accurately to be able to reliably compute exit rates. I approximate the cross-sectional distribution by a mixture of two normal distributions (see Appendix for the computational algorithm). The aggregate state \( s \) is approximated by four numbers: \((x, \eta, \mu_1, \mu_2)\), where the last three parameters are defined on a discrete grid. \[\blacksquare\]

This approach is of independent use in any model of default where it is crucial to track the shape of the distribution near the left-tail.

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\footnote{For the contract \( i \), the return \( R^i = (\tau^i_{t+1} + \tilde{F}^i_{t+1})/\tilde{F}^i_t \), where \( \tilde{F}^i \) is the value of the contract after maintenance cost and manager payments have been made, i.e. the ex-dividend price.}

\footnote{As a robustness check I included the variances of the normal distributions as state variables, but the added computation cost did not produce significant improvements in convergence of the fixed point problem.}
3.1 No agency problem

When the manager receives no benefit from shirking ($B = 0$), the agency problem disappears. All positive NPV projects are funded. By assumption, the manager is sufficiently impatient so that he values the firm less than outside investors. Managers with positive NPV projects immediately sell all of firm equity to outside investors. Investors pay for the initial set-up cost $\check{c}k$ and, thereafter, pay per-period maintenance cost $\delta k$ and receive all cash future flows.

**Proposition 3** (Equilibrium allocations) The competitive equilibrium is characterized by aggregate consumption $C_t(x) = c_t(x) \int d\mu_t$ where $\int d\mu_t$ is the total number of firms in the economy. Aggregate output $Y_t(x) = x_t \int d\mu_t$, and investment $I_t(x) = \left[ \frac{h \check{e}^2(x_t)}{2} + \delta \right] k \int d\mu_t$ where

$$\check{e}(x_t) = c(x_t)^{\gamma_2} f(x_i), \quad c(x_t) = x_t - h \check{e}^2(x_t) - \delta,$$

and where $f(x_i)$ is

$$f(x_i) = E_0 \left[ \sum_{s=1}^{\infty} \beta^s \left( c(x) \prod_{j=1}^{s} (1 + h \check{e}(x_j)) \right)^{-\gamma_2} x_s | x_0 \right].$$

The manager is assumed to be sufficiently impatient so that $\beta_e$ satisfies both of the following conditions

$$\check{e}_1 \geq E_0 \left[ \sum_{t} \beta^t e_t x_t | x_0 \right] = \frac{\beta_e x_2 \zeta_1 + x_1 (1 - \beta_e (1 - \zeta_1))}{(1 - \beta_e)(1 - \beta_e (1 - \zeta_1 - \zeta_2))},$$

$$\check{e}_2 \geq E_0 \left[ \sum_{t} \beta^t e_t x_t | x_0 \right] = \frac{\beta_e x_1 \zeta_2 + x_2 (1 - \beta_e (1 - \zeta_1))}{(1 - \beta_e)(1 - \beta_e (1 - \zeta_1 - \zeta_2))}.$$

**Proof.** See Appendix. ■

With no moral hazard, there are no exits and as seen from Proposition 3 the current aggregate state $x$ and the total number of firms are sufficient statistics for aggregate output, investment, consumption, and asset prices. Details of the cross-sectional distribution do not matter for quantities and prices. Primitive shocks do not propagate and there is no history.
dependence.  

3.2 With moral hazard

With positive private benefit to the manager from shirking, investors need to induce high effort by providing incentives using the contract described in Section 2.1. Without aggregate uncertainty \( x_G = x_B \), the investor’s consumption growth rate is a constant in the steady-state, and therefore, the investor’s discount rate \( \beta_t (C_{t+1}/C_t)^{-\gamma} \) is constant. This is the setting considered in DeMarzo and Fishman (2007a), DeMarzo and Fishman (2007b), and also in Biais et al. (2007). I quickly review the features of the optimal contract in this simple case. The reader is referred to these papers for more details.

The form in which incentives are provided depend on the manager’s continuation utility. Figure 3 shows the investor’s value of future cash flows from the contract as a function of the manager’s continuation value. It is weakly concave. At very low values, increasing the manager’s continuation value lowers the probability of inefficient termination of the firm. This leads to an initial increase in the value (to the investor). At very high continuation value, increasing the manager’s continuation value leads to a decrease in value to the investor because it constitutes a wealth transfer from the investor to the manager. Figure 4 shows optimal policies. The panel on the left shows the manager’s continuation value after realization of firm output. His continuation value changes from \( V \) to \( V_+ \) for high output (solid, blue curve), and to \( V_- \) (dashed, red curve) for low output. The spread provides incentives for him to provide high effort. At very low levels of manager’s continuation value, this spread cannot be provided, and incentives are provided by threatening termination. The panel on the right, shows cash policies of this contract. From the figure, we see that cash is paid only after a sufficiently high continuation value is achieved by the manager. The solid, blue curve shows payments

\[11\] Note that although there is no additional propagation of primitive aggregate shocks, consumption growth is persistent because the aggregate productivity shocks are persistent.
received by the manager after a high cash flow realization, while the dashed, red curve shows
his cash compensation for low output. To sum up, for very high continuation values, the
manager is compensated by cash payments. At intermediate values, he is compensated by
future promises. At very low continuation values, incentives are provided through the threat
of terminating the contract. Figure 5 shows the the fractional loss in firm value due to the
borrowing constraints arising from the moral hazard problem. The solid, black curve shows
firm value defined as the sum of the investor’s value and the manager’s continuation utility.
Firm value is lower than the first-best value because of the cost of providing incentives to the
manager and because of the possibility of an inefficient termination lowers the value of the
firm. The dotted, blue curve plots the stationary distribution of manager continuation values.
In this example, about half of the firms suffer a loss greater than 10%.

Are there more inefficient terminations if the manager’s private benefit is higher? From
Eq. 4, incentive compatibility requires an increase in the spread of manager’s continuation
value for higher $B$. This is intuitive, and means that a stronger moral hazard problem
necessitates a higher level of insider ownership. A higher volatility of continuation values
increases the risk of termination. However, in this case, it is optimal for the investor to
postpone payments even longer and instead increase the manager’s continuation value at a
higher rate away from the termination boundary. To compare the termination rates in the
steady-state, in Figure 6 I plot the stationary distributions corresponding to a high and low
value of $B/\Delta p(z_+ - z_-)$. The dotted black curve is for a lower value of $B/\Delta p(z_+ - z_-) = 0.20$,
while the solid blue curve is for higher $B/\Delta p(z_+ - z_-) = 0.40$. A higher value of $B$ is
characterized by a distribution which is wider (because of higher volatility), but the core of
the distribution is also pushed to higher values of $V$ reflecting the contract’s precautionary
motive to reduce the risk of termination from a more volatile $V$. The net result of these
opposing forces is that the increase in exit rate is very gradual. [12]

[12]I find the increase to be non-monotonic in $B/\Delta p(z_+ - z_-)$, but this could be a numerical artifact.
Implementation of the optimal contract is not unique. DeMarzo and Fishman (2007b) show how to implement this contract using a combination of inside and outside equity, long-term debt, and a line of credit. In this implementation, the manager keeps \( B/\Delta p(z^+ - z^-) \) of firm equity, while outsiders hold the remaining fraction of equity, together with long term debt with fixed coupon payments, and a line of credit. None of the results presented in this paper depend on the particular form of implementation. Therefore, I do not discuss this further, and refer the interested reader instead to DeMarzo and Fishman (2007b) and Biais et al. (2007).

The discussion above focused on the contract under no aggregate uncertainty. Next I discuss dynamics under aggregate uncertainty.

**Aggregate Uncertainty**

With aggregate shocks, the distribution of continuation values (and therefore manager payments), changes over time. There is no steady-state distribution to which the economy settles into, and this gives rise to rich dynamics which depends on the cross-sectional distribution. For this reason, proving the existence of an equilibrium is difficult. This is not unique to the set-up here and occurs in problems with incomplete markets which feature aggregate uncertainty.

As Proposition 4 shows, the introduction of public lotteries makes the investor’s value function concave in the manager’s continuation utility. The marginal cost of providing incentives, \(-F'(V)\) is bounded from above by 1.

**Proposition 4** The investor’s value function \( F \) is a concave function of the manager’s promised utility \( V \). The slope is bounded by \( F'(V) \geq -1 \).

**Proof.** See Appendix. ■

How does the dynamics of risk premia and macro-quantities compare between the frictionless economy and the one with an agency problem? The frictionless economy does not
feature exits. Risk premia and growth rates of consumption, output, and investment are
given by the aggregate productivity shock. In the economy with the friction turned on, the
dynamics is a lot more interesting.

In the presence of aggregate productivity shocks, contract policies such as termination,
depend on two forces. I call them the profitability effect and the discount rate effect. These
forces act in opposite directions which is best illustrated by an example. Suppose the aggregate
state changes from $x_B$ to $x_G$. By assumption, aggregate shocks are persistent. This means,
that the present value of future cash flows to the investor is now higher than it was before.
Investors will now want to reduce the probability of termination and capture higher profits.
They achieve this by increasing the continuation values of managers, thereby lowering their
risk of termination, should they draw a low firm-specific shock $z$. However, to keep the threat
of termination real, managers will have their continuation values lowered when the aggregate
state changes from $x_B$ to $x_G$. Figure 7 shows this behavior for low $V$. In fact, managers
with $V$ below a threshold, have their continuation values set to zero, and are immediately
terminated. However, there is another force at play which has to do with the change in the
investor’s discount rate. In the low productivity state, the discount rate of the investor is
higher than in the state $x_G$. In other words, relatively speaking, the manager is more patient.
This makes it cheaper to delay paying the manager in this state, in exchange for higher
payments in the future. This behavior is seen in Figure 7 for high values of $V$.

In general, whether the profitability effect, or the discount rate effect dominates, depends
on the volatility of the stochastic discount rate of the investor, the aggregate volatility of cash
flows, and the dead-weight loss from terminations. In aggregate states with high volatility
of the stochastic discount rate, the discount rate of the investor is much higher in the bad
state compared to the good state. In this situation, investors will optimally terminate poorly
performing managers in the good aggregate state. This feature of the contract makes it
distinctly different from pure long-term debt where the default rate is always counter-cyclical.
Having said that, in the numerical analysis presented in the next section, I choose parameters which result in counter-cyclical exits.

With this choice of parameters, the dynamics of risk premia is counter-cyclical and highly non-linear. To fix ideas, suppose the economy starts from the stochastic steady-state. Realization of a low aggregate shock $x_B$ results in manager’s continuation values being adjusted as shown in Figure 7. Managers with low continuation values are either immediately terminated, or, have their continuation values adjusted downward. This leads to an increase in the mass of firms near the default boundary, as shown in Figure 8. The dotted, blue curve is the stochastic steady-state distribution, and the solid, red curve is the resulting distribution after realization of $x_B$. If a low aggregate shock is realized in the following period, the exit rate will increase because there is a higher number of managers close to the termination boundary. Figure 9 shows the increase in exit rates after two successive realizations of $x_B$. Risk premium follows a similar pattern. Higher exits in states with high marginal utility of the representative household lowers the value of investment. As a result, fewer firms enter the economy. Lower investment raises the consumption risk. This causes the market price of risk to go up, as shown in Figure 10.

The result of several bad shocks is more severe. With each successive realization of $x_B$, more firms are pulled closer towards the termination boundary. This causes the exit rate to increase sharply as shown in Figure 11. A much higher rate of terminations when the household’s marginal utility is high causes a sharp drop in the value of potential entrants leading to a big drop in investment. In general equilibrium, this causes consumption to be very volatile leading to the sharp increase in the market price of risk shown in Figure 12.

The upshot of this is that risk-premia is not only counter-cyclical, but also depend on the history of realized aggregate shocks. It is important to point out that the behavior is also not guaranteed to be monotonic in aggregate shocks. With very high discount rate of
the household, termination behavior could switch.\textsuperscript{13} In general, the dynamics of investment, output, consumption, and asset prices in this economy is a highly non-linear, path-dependent function of aggregate shocks.

4 Quantitative Results

I summarize results comparing the frictionless model ($B = 0$) with moderate and high values of $B$. The first set of results are for unconditional moments of the aggregate macroeconomic quantities and asset prices. The purpose is mainly to make sure that the values of parameters chosen are reasonable and that results are not due to unreasonable parameter choices. Next I report how aggregate consumption, investment, and output vary over the business cycle. The model is solved using the approach outlined in the Appendix.

4.1 Parameters

There are 12 parameters in the model. Although the model is calibrated and computed at quarterly frequency, I annualize all parameters for easy interpretation. The two preference parameters of the representative investor are $\beta_l = 0.998$, risk-aversion of $\gamma_l = 10$. The manager’s time-preference parameter is set to be 0.92. The technology parameters consist of the transition matrix $\Gamma$, with the (annual) transition intensity out of the state $x = x_G$ set at $\zeta_G = 0.33$, and the transition intensity out of state $x = x_B$ is set to $\zeta_B = 0.70$. Aggregate shocks $x$ are thus assumed to be persistent. The ratio $x_G/x_B = 1.091$ and is chosen to match the unconditional volatility of output growth. With this choice of $x_{B,G}$ and the transition matrix, both aggregate states have the same amount of consumption risk and therefore any time variation in conditional volatility arising from constraints is purely the result of the friction arising from the agency problem. It might appear though, that with this choice of transition matrix, recessions and booms are much shorter lived than in the data. However,\textsuperscript{13}In the current calibration, this probably requires many more realizations of $x_B$.
because the drop in aggregate output from $x_G$ to $x_B$ is relatively small, the model counterpart of recessions in the data are two or more successive realizations of $x_B$. The firm-specific shocks $z_\pm$ are chosen to occur with equal probability ($p = 0.5$), and the size of $|z_\pm|$ is chosen to correspond to an annual volatility of 40%. The depreciation rate of capital is set to 5% per year, which corresponds to $\delta = 0.0125$. Finally, the loss from termination of contracts $\chi$ is assumed to be such that the value of the firm after default is 0.6 times the first-best value of the firm. This amounts to setting $\chi = 0.35$. The entry rate $h$ is chosen so that the annual growth rate of consumption in the model with financing friction is 1.5%.

The strength of the moral hazard problem depends on $B/\Delta p(z_+ - z_-) = 0.15$. In this numerical exercise, I set $B/\Delta p(z_+ - z_-) = 0.15$, which corresponds to insiders holding 15% of the equity of the firm in the steady-state (with no aggregate uncertainty). Jenter and Lewellen (2013) reports average CEO ownership in public firms is 5.5%. I choose a higher value because insiders in the model is anyone who has significant influence over decisions made within a firm. For the frictionless model $B = 0$. I simulate 1000 independent panels of $N = 10^5$ firms over a time period of 300 quarters. All reported values are averages over these 1000 independent simulations.

4.2 Aggregate Macro and Asset Prices

Unconditional averages

Table 2 show unconditional moments of aggregate macroeconomic quantities and asset prices. The top panel shows investment, output, consumption, and exits (in this stylized simple model, termination of contracts correspond to exits). The model produces aggregate investment levels of 20% of output, which is close to that observed in the data. This is not a prediction, rather parameters were chosen to hit this target. With the same parameters, the frictionless model predicts a much higher investment rate, because, without costly exits, the value of entering is much higher for both realizations of the aggregate shock. The first
two moments of the household’s consumption growth process is also close to the empirical counterparts. The frictionless model features much less consumption risk. The risk of costly termination during recessions magnifies consumption volatility considerably. Although the model is able to match the empirical volatility of consumption and output growth, it is unable to generate sufficient investment volatility. This could be remedied by including investment specific shocks which increase the availability of good projects during good times ($h_G > h_B$). The frictionless model has an even lower investment volatility. The amplification in investment volatility with agency frictions arises due to a higher exit rate during recessions when the household’s marginal utility is high. This lowers the value of entering even more in the low state ($x = x_B$).

Exits in my model are counter-cyclical. The model is quite stylized and firm exits correspond to insiders being fired. In the data, these two interpretations are quite different. The level of CEO terminations reported by Kaplan and Minton (2012) is 15.8% (a previous version of their paper which used data till 2004 reported a lower value of 12.8%). The model presented here produces a much lower average termination rate of 2.9%. This is expected, for CEO’s are also terminated for factors different from agency problems, such as investors learning about CEO’s skill. The model presented here does not consider such factors for simplicity. Firm exit rates are also in the same range, although most of the exits are for small firms which typically have much higher insider-ownership than the value used in my numerical analysis (15%). In the data, the volatility of the de-trended rate of CEO termination is about 3.4% which is close to the model prediction of 2.9%. The termination of exits is highly correlated with output growth: the model prediction is 0.88, while Eisfeldt and Rampini (2008) report a value of 0.9. Note, that there are no exits in the frictionless model.

The lower panel of Table 2 shows that asset pricing moments are also close to the empirical counterparts. The equity premium, defined as the expected excess return over the risk-free rate to a claim on aggregate output is lower than in the data. This is because in my
model, I define equity-premium to be the excess return to a claim on aggregate consumption. In the data, however, the stream of payments priced are dividends, which are much more volatile than consumption. The high expected return on aggregate consumption is because of three reasons. First, persistent variations of consumption growth increases the investor’s precautionary savings motive by lowering both the level and the volatility of the risk-free rate. Part of this predictability is inherited from the predictability of productivity shocks. However, successive low realizations of $x$ depletes the number of firms which takes time to rebuild. With exits, the time to rebuild is even longer, which further depresses the risk-free rate and increases the equity premium. The second reason is because exits in the bad aggregate state amplify the volatility of consumption. Finally, a high choice of risk-aversion ($\gamma_l = 10$) also increases the expected return on risky consumption. Costly termination in states in which the investor’s marginal utility is high, raises the expected return producing a higher equity premium and a higher Sharpe ratio.

### 4.3 Business cycle properties

The presence of financial frictions introduces changes both for aggregate quantities and prices, and improves the model’s ability to match the data. There are three important and noticeable changes. First, the asymmetric response of growth rates of output and investment is much more pronounced (by a factor of 1.35 for the numerical example). Second, the market price of risk (Sharpe ratio) is time-varying in the economy with borrowing constraints. Third, there is additional propagation of primitive productivity shocks. I describe each of these in turn, and provide intuition behind the results.

**Asymmetric business cycles**

Aggregate investment is pro-cyclical because of increased exits during recessions and lower levels of entry. Figure [13](#) shows the asymmetric response of investment to positive and
negative shocks. As the solid, blue curve shows, a one-standard deviation positive shock in the aggregate shock $x$, produces a 4.1% rise in investment. The lower dotted, red curve shows that a negative shock of the same magnitude produces a 5.5% drop. Output growth exhibits a similar asymmetry as seen from Figure 14.

The asymmetric response is due to defaults occurring in states in which the investors’s marginal utility is very high. With a risk-averse lender, a negative realization of the aggregate shock, $x$, has two effects. It lowers current output (by lowering productivity), and secondly, because the exit rate goes up precisely when marginal utility is high, the value of entering goes down. This leads to even lower investment. Non-zero risk premia causes investment (and also output) to respond asymmetrically to positive and negative shocks. This intuition is confirmed by Figure 15. The figure plots the amount of asymmetry as measured by the ratio of the magnitude of the drop in investment in response to a negative productivity shock, to the increase when a positive shock of the same magnitude is realized. As expected, the asymmetry is larger if the investor is more risk-averse.

**Time varying risk premia and predictability**

The market price of risk depends on the volatility of aggregate consumption growth. In the frictionless economy, consumption is completely determined by the current aggregate shock $x$ and the total number of existing firms. Consumption growth inherits the Markov nature of the primitive productive shocks. There is no further dependence on past history. The baseline model is calibrated so that the conditional volatility of consumption growth, and hence the market price of risk, in the frictionless setting is the same across the two aggregate states $x_{G,B}$. For the economy with borrowing constraints, however, the exit rate depends both on the aggregate shock, and on the shape of the left-tail of the distribution. For instance, the exit rate is higher when the aggregate state changes from $x_G$ to $x_B$ if the cross-sectional distribution of manager continuation values is skewed more to the left. This introduces two
changes to the conditional volatility of consumption growth with accompanying pricing effects. First, consumption growth is more volatile during recessions than in booms because of an increase in the exit rate during recessions. The second difference from the frictionless setting, is that consumption volatility depends on the sequence of realized aggregate shocks. When a low aggregate shock is realized, it is optimal for the contract to reduce the continuation value of managers with low continuation values. This increases the mass of managers near the default boundary to increase. Consequently, a second low realization of $x$ will lead to an even higher number of exits compared to the previous period. This accelerated exit behavior is shown in Figure 11 together with a similar behavior for the volatility of the stochastic discount rate in 12.

A prediction of the model therefore, is that controlling for current realized aggregate productivity, the past history of aggregate shocks is informative about future growth rates of investment, output, and consumption. Since I approximate the cross-sectional distribution by a mixture of normals, the relative weight of the normal with lower mean acts an approximate store of the past history of shocks. For instance, successive realizations of $x_B$ lead to an increase in the relative weight of the normal distribution with lower mean. Table 3 shows that regression results for output and investment growth after controlling for current aggregate shock. In line with our intuition, the slope is negative and is significant.

5 Conclusion

In this paper, I provide a simple, unified framework to illustrate the interaction of financing frictions on key aggregate quantities and risk premia. The presence of financing frictions lead to rich aggregate dynamics of macro-economic quantities and asset prices, and improves the predictions of neo-classical frictionless models along several dimensions. A key finding of this paper is that in the presence of financing friction, the response of the economy to aggregate shocks is both non-linear and history-dependent. The marginal effect of an aggregate shock
depends on the sequence of shocks that preceded it. This findings of this paper also provides an alternate view of deep recessions. Rather than being the result of a single large shock, they are the consequence of a series of small negative shocks. This is useful because while it is much harder to estimate the probability of a single large rare event, the probability of realization of a long sequence of small shocks which individually occur frequently can be better estimated.

The essential elements of the dynamics induced by firm heterogeneity in this model might carry over to other settings. It would be interesting to see if higher moments of the cross-sectional distribution of firm performance are able to better capture the state of the economy at a given moment, and hence better predict for aggregate dynamics.
References


Appendix

A  Proofs of Propositions

Proof of Proposition 2

The household takes the risk-free rate, $r_t$, and the ex-dividend contract prices $F^i_t$ as given. The household’s Bellman equation is

$$U_t(g, \vec{b}, s) = \max_{C \geq 0, g', \vec{b}'} \left[ u(C_t) + E \left[ \beta^{l+1} U_{t+1}(g', \vec{b}', s') \right] \right], \quad s = \{x, \mu\}, \quad (13)$$

subject to the budget constraint

$$g_t + \sum_{i \in \text{Continue}} b^i_t (F^i_t + y^i_t - d^i_t - \delta) k + \sum_{i \in \text{Default}} (1 - \chi)(1 - \delta) k - \sum_{i \in \text{Entry}} \bar{e}^i k = \frac{g_{t+1}}{1 + r_t} + \sum_{i \in \text{Continue}} b^{i+1}_t F^i_t + C_t \quad (14)$$

where $g$ and $\vec{b}$ are the household’s holding of risk-free asset and the long-term contracts, and $d^i_t$ is the payment received by the manager from contract $i$ in period $t$. The aggregate state $s$ explicitly depends on aggregate shock $x$ and also on the distribution of continuation values of surviving managers. The expectation is over realizations of $x'$ given that the current shock is $x$, with exogenously specified transition probability matrix $\Gamma$. By “continue” I mean only those firms which are still in existence. This excludes new entrants. Finally, $1 - \chi$ is the recovery rate given default has occurred. The law of large numbers is assumed to hold, so that $C_t$ does not depend on individual realizations of firm-specific shock $z$.

Market clearing: In equilibrium, the bond market and the market for financial contracts clear

$$g_t = g_{t+1} = 0, \quad b^i_t = b^{i+1}_t = 1, \quad (15)$$

for continuing firms $i$. Substituting the market clearing conditions into the budget constraint Eq.14, prices ($r_t$ and $F^i_t$) drop out, and the goods market clearing condition becomes

$$C_t + \sum_{i \in \text{Continue}} d^i_t = Y_t - I_t - L_t, \quad (16)$$
where

\[ I_t = \sum_{i \in \text{All}} \delta k + \sum_{i \in \text{Entrants}} \tilde{e}^i k - \sum_{i \in \text{Default}} k, \]

\[ L_t = \chi (1 - \delta) \sum_{i \in \text{Default}} k, \]

where the fractional loss \(0 \geq \chi \geq 1\). This coincides with the relation in the main text Eq. 7 with aggregate investment \(I\) and loss from default \(L_t\). I assume that entering firms start production in the period subsequent to their entry.

**Optimality conditions:** Prices \(F^i\) and \(r_t\) get determined by the household’s first-order conditions

\[ u'(C_t)F^i_t = \beta^t E[u'(C_{t+1})(\tau^i_{t+1} + F^i_{t+1})], \quad 1 = E[\beta^t u'(C_{t+1})/(1 + r_t)], \]

where \(\tau^i_t = -\delta + y^i_{t+1} - d^i_{t+1}\) is the investor’s cash flow (scaled by \(k\)) after paying for maintenance and paying the manager. The stochastic discount factor used to discount cash flows is, therefore, \(\pi_{t+j}/\pi_t = \beta^j u'(C_{t+1})/u'(C_t)\).

**Proof of Proposition 3**

**Proof.** In equilibrium, the firm which just breaks even (zero net present value) has present value of cash flows equal to cost. For this firm, scaling out \(k\),

\[ \tilde{e}_t(x_t) = E_t \left[ \sum_{s=1}^{\infty} \beta^s \left( \frac{C_{t+s}}{C_t} \right)^{-\gamma_t} x_{t+s}|x_t \right]. \]

Conjecture that aggregate household consumption \(C_t = c(x_t)N_t\) where the total number of firms in the economy \(N_t = \int d\mu_t\), and grows at the rate \(h\tilde{e}(x_t)\).

\[ \tilde{e}_t(x_t) = E_t \left[ \sum_{s=1}^{\infty} \beta^s \left( \frac{c(x_{t+s})N_{t+s}}{c(x_t)N_t} \right)^{-\gamma_t} x_{t+s}|x_t \right], \]

\[ = c(x_t)^{\gamma_t} E_t \left[ \sum_{s=1}^{\infty} \beta^s \left( \frac{c(x_{t+s})}{c(x_t)} \prod_{j=1}^{s} (1 + h\tilde{e}(x_j)) \right)^{-\gamma_t} x_{t+s}|x_t \right], \]

which is of the form conjectured in the first of Eq. [12] with

\[ f(x_i) = E_0 \left[ \sum_{s=1}^{\infty} \beta^s \left( c(x_i) \prod_{j=1}^{s} (1 + h\tilde{e}(x_j)) \right)^{-\gamma_i} x_i|x_0 \right]. \]
The second equation in Eq. 12 follows from the goods market clearing condition
\[ C_t(x_t) = c(x_t) \int d\mu_t = \left( x_t - \delta \right) k \int d\mu_t - h\bar{e}^2 \int d\mu_t. \]

where the last term is the consumption of managers who have positive net present value projects and have just entered the economy. Each manager is paid \( \bar{e}k \) and a total of \( h\bar{e} \int d\mu_t \) of them enter in period \( t \). He consumes \( (\bar{e} - \bar{e})k \) immediately. ■

**Proof of Proposition 4**

**Proof.** The bound on \( F'(V) \) follows from the optimality of increasing the manager’s current consumption by \( \epsilon \) accompanied by a decrease in continuation utility satisfying the promise-keeping constraint. For \( F(V) \) to be optimal, \( F(V; s) \geq F(V - \epsilon) + \epsilon. \) This implies that \( F'(V) \geq -1. \)

To prove concavity of \( F, \) decompose into 3 sub-problems. Starting from the end of each period, first, consider the subproblem of maximizing the lender’s value conditional on continuing (not exiting):

**Firm-specific shock:**

\[
F^2_t(V^2, s) = \max_{d_\pm \geq 0, V^2_\pm \geq 0} \left[ -\delta k + p \left( xz_+ - d_+ + F^3_t(V^3_+, s) \right) + (1 - p) \left( xz_- - d_- + F^3_t(V^3_-, s) \right) \right],
\]

\[ V = E^p \left[ d_+(s) + V'_+(s) \right], \quad \forall s \]

\[ B \leq \Delta p \left[ \left( d_+(s) + V'_+(s) \right) - \left( d_-(s) + V'_-(s) \right) \right], \quad \forall s, \]

Let \( T \) be the Bellman operator (i.e. right side of first line). The set of bounded, continuous, weakly concave functions that vanish at \( (1 - \chi)(1 - \delta)k, \) which we denote by \( C'(\mathbb{R}_+) \subset C(\mathbb{R}_+) \) of the complete metric space \( C(\mathbb{R}_+). \) It is sufficient to show that \( T[C'(\mathbb{R}_+)] \) maps this set into itself. To verify this, let \( F^3 \in C'(\mathbb{R}_+), \) and let

\[ V^0 \neq V^1, \quad \theta \in (0, 1), \quad \text{and} \quad V^\theta = \theta V^0 + (1 - \theta)V^1. \]

Let \( (V^i) \) attain \( (TF^3)(V^i) \) for \( i = 0, 1. \) Next, I show that for \( V^\theta, \) the controls \( (d^*_\pm, V^*_\pm) \) are feasible, where they are chosen to be \( V^*_\pm = \theta V^0_\pm + (1 - \theta)V^1_\pm \) and manager payments \( d^*_\pm \) defined so that \( u(d^*_\pm) = \theta u(d^0_\pm) + (1 - \theta)u(d^1_\pm) \) is a feasible choice. The promise-keeping constraint is satisfied for \( V^\theta \) (the manager gets the weighted average of the two utilities). The IC constraint is satisfied for policies \( (d^*_\pm, V^*_\pm) \) because \( C^* = u(d^*_\pm) + \beta \Delta V^*_\pm \) is a feasible choice. The promise-keeping constraint is satisfied for \( V^\theta \) (the manager gets the weighted average of the two utilities). The IC constraint is satisfied for policies \( (d^*_\pm, V^*_\pm) \) because \( C^0 = u(d^*_\pm) + \beta \Delta V^*_\pm + B/\Delta p = \theta C^0 + (1 - \theta)C^1 \geq 0 \) because the constraints \( C^0 \) and \( C^1 \) are satisfied. Since \( F^3 \) is concave, it follows that \( TF^3(V^\theta) \geq \theta TF^3(V^0) + (1 - \theta)TF^3(V^1), \) and thus \( TF^3 \) is concave.
Public randomization:

\[
F^1_t(V^1_t, s) = \max_{\zeta \in (0, 1), V^2_t \geq 0} \left[ (1 - \zeta) F^2_t(V^2_t, s) + \zeta (1 - \chi)(1 - \delta) k \right], \\
V^1_t = (1 - \zeta) V^2_t.
\]

Concavity of \(F^1_t\) follows from the fact that \(F^2_t\) is concave, and the constraint is linear.

Aggregate shock/state change:

\[
\pi_t(s) F^3_t(V^3_t, s) = \max_{V^3_t \geq 0, V^1_{t+1} \geq 0} E_t^R \left[ \pi_{t+1}(s') F^1_{t+1}(V^1_{t+1}, s') \right], \\
V^3_t = \beta_e E_t^R \left[ V^1_{t+1} \right].
\]

Once again, the constraint is linear, and concavity of \(F^3\) follows from concavity of \(F^1\).
B Computational Algorithm

Frictionless economy

The unknowns are \( c(x_i), f(x_i), \) and \( \bar{e}(x_i) \). First, eliminate \( c(x_i) \) using
\[
 c(x_i) = x_i - \frac{h}{2} \bar{e}^2(x_i) - \delta
\]
and \( \bar{e}(x_i) = c(x_i)^\eta f(x_i) \) and from the definition of \( f(x_i) \) we have the system
\[
 f(x_i) = \bar{e}(x_i) \left[ x_i - \frac{h}{2} \bar{e}^2(x_i) - \delta \right]^{-\gamma}, \tag{17}
\]
\[
 f(x_i) = E_0 \left[ \sum_{s=1}^\infty \beta^s \left( c(x_s) \prod_{j=1}^s (1 + h \bar{e}(x_j)) \right)^{-\gamma} x_s | x_0 \right]. \tag{18}
\]

The baseline model has two aggregate states \( x = \{x_1, x_2\} \) on a Markov chain with transition probability \( \Gamma \). Start with a guess for \( f_1 \) and \( f_2 \). The initial guess is obtained by solving the system with no aggregate risk \( x_1 = x_2 = x \). Computing the sum explicitly and cancelling \( c(x) \) solve
\[
 \bar{e} = \frac{x \beta f(1 + h \bar{e})^{-\gamma}}{1 - \beta f(1 + h \bar{e})^{-\gamma}}.
\]

Compute \( f_1 = f_2 \) using the computed value of \( \bar{e} \) and Eq. refappendixprice. The following scheme is used iteratively

1. Given \( f_i^{(n)} \), solve for \( \bar{e}_1^{(n)} \) using Eq. refappendixprice.

2. Compute the conditional expectations to update \( f_{1,2} \).
\[
 f_i^{(n+1)} = E_0 \left[ \sum_{s=1}^\infty \beta^s \left( c(x_s) \prod_{j=1}^s (1 + h \bar{e}(x_j)) \right)^{-\gamma} x_s | x_i \right], \quad c(x_s) = x_s - \frac{h}{2} \bar{e}^2(x_s) - \delta.
\]

3. Go to step 1.

With moral hazard

The distribution of continuation values across firms in the economy is a state variable. With aggregate uncertainty, the distribution is time-varying. I approximate it by a 3 parameter family of a mixture of normal distributions: \((\eta, \mu_1, \mu_2)\), where \(0 < \eta < 1\) is the fraction of the normal distribution with lower mean \( \mu_1 \). The aggregate state \( s \) is approximated by \((x, \eta, \mu_1, \mu_2)\), where the last three parameters are defined on a discrete grid.

The equilibrium is computed iteratively:

1. Start with a guess for the pricing kernel and the law of motion of the three parameters approximating the distribution.
2. Solve for the optimal contract policies.

3. Use obtained policies for continuation values, together with entry and exit to compute the next-period distribution starting from a particular point on the grid for \((\eta, \mu_1, \mu_2)\).

4. Compute the stochastic discount rates for each aggregate state using the goods market clearing condition.

5. Update rules for the law of motion of the distribution and the guess for the pricing kernel.

These steps are iterated till both the pricing kernel and the law of motion of \((\eta, \mu_1, \mu_2)\) converge.
The manager’s time preference parameter is $\beta_e$, private benefit from low effort is $B$, and $\Delta p$ is the amount by which the probability of drawing the high firm-specific shock $z_+$ is reduced when the manager shirks. The investor’s time preference parameter is $\beta_l$ and has power utility with risk-aversion $\gamma_l$. The technology parameters are: firm-specific productivity $z_{\pm}$, with $p$ the probability of drawing $z_+$, and $|z_+| = |z_-|$; aggregate productivity $x_{G,B}$ with Markov transition intensities $\zeta_{G,B}$. $\delta$ is the maintenance cost of capital each quarter. $\chi$ is the fraction of capital that is lost on firm termination.

| $\beta_e$ | $B/\Delta p$ | $\beta_l$ | $\gamma_l$ | $|z_+|$ | $p$ | $x_G$ | $x_B$ | $\zeta_G$ | $\zeta_B$ | $\delta$ | $\chi$ |
|----------|--------------|-----------|-------------|--------|-----|------|------|---------|---------|-------|------|
| 0.92     | 0.03         | 0.998     | 10          | 0.20   | 0.5 | 0.036| 0.033| 0.33    | 0.70    | 0.0125| 0.35 |
Table 2: Unconditional Moments

This table reports unconditional moments of annual growth rates of consumption ($\Delta C = C_{t+1}/C_t - 1$), output ($\Delta Y = Y_{t+1}/Y_t - 1$), and investment ($\Delta I = I_{t+1}/I_t - 1$). The numbers under the Data column are from Campbell, Lo, and McKinlay (1997). The exit rates data reported are CEO exit rates from Kaplan and Minton (2012) which covers the period from 1992 – 2007. The data is de-trended using an HP filter. Model results are generated by 1000 independent panels of $N = 10^5$ firms over a time period of 300 quarters. Results are averages across independent panels. Equity return is the return for a claim on aggregate household’s consumption. All values, except the investment to output ratio and the maximum Sharpe ratio are in percentages.

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Model</th>
<th>Frictionless</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma(\Delta C)$</td>
<td>3.3</td>
<td>2.9</td>
<td>1.83</td>
</tr>
<tr>
<td>$\sigma(\Delta C) / \sigma(\Delta Y)$</td>
<td>0.70</td>
<td>0.65</td>
<td>0.98</td>
</tr>
<tr>
<td>$I/Y$</td>
<td>0.19</td>
<td>0.20</td>
<td>0.33</td>
</tr>
<tr>
<td>$\sigma(\Delta I) / \sigma(\Delta Y)$</td>
<td>4.59</td>
<td>2.90</td>
<td>1.04</td>
</tr>
<tr>
<td>$\rho(\Delta I, \Delta Y)$</td>
<td>0.81</td>
<td>0.85</td>
<td>0.98</td>
</tr>
<tr>
<td>Exit rate</td>
<td>15.8</td>
<td>2.9</td>
<td>0</td>
</tr>
<tr>
<td>$\sigma($Exits$)$</td>
<td>3.4</td>
<td>2.7</td>
<td>0</td>
</tr>
<tr>
<td>$\rho($Exits, $\Delta Y$)</td>
<td>-0.90</td>
<td>-0.82</td>
<td>–</td>
</tr>
<tr>
<td>$E[r^f]$</td>
<td>1.80</td>
<td>1.5</td>
<td>2.8</td>
</tr>
<tr>
<td>$\sigma(r^f)$</td>
<td>3.00</td>
<td>3.2</td>
<td>10.8</td>
</tr>
<tr>
<td>$E[r^e - r^f]$</td>
<td>6.0</td>
<td>4.8</td>
<td>0.8</td>
</tr>
<tr>
<td>Sharpe ratio</td>
<td>0.33</td>
<td>0.29</td>
<td>0.19</td>
</tr>
</tbody>
</table>
Table 3: Predicting output and investment growth

This table reports simulation results showing additional information about the aggregate state beyond current realized productivity. Model results are generated by 1000 independent panels of $N = 10^5$ firms over a time period of 300 quarters. Results are averages across independent panels. t-stats are computed using Newey-West standard errors with 5 lags, and reported in brackets.

<table>
<thead>
<tr>
<th>Exit rate($t$)</th>
<th>$x_t$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td>log($Y_{t+1}/Y_t$)</td>
<td>0.27</td>
<td>1.2</td>
</tr>
<tr>
<td></td>
<td>(2.1)</td>
<td>(3.1)</td>
</tr>
<tr>
<td>log($I_{t+1}/I_t$)</td>
<td>0.76</td>
<td>3.7</td>
</tr>
<tr>
<td></td>
<td>(2.4)</td>
<td>(4.2)</td>
</tr>
</tbody>
</table>
Figure 1: Intra-period time-line of the model. The aggregate shock $x$ is revealed at the beginning of the period, after which the lottery for termination is held. A manager whose contract is terminated receives nothing, while the investor recovers a fraction, $(1 - \chi)$, of the physical assets of the firm. If the manager continues, he receives maintenance cost $\delta k$. Production takes place when the firm-specific shock $z$ is realized. The output $y$ is divided between the manager and investors. The period ends.
Figure 2: Entry condition of new managers into the economy. Panel A shows an example. The solid curve shows the investor’s value of a new contract as a function of promised utility to the manager. The initial installation cost for this project is $e_0k$. The contract is initiated with the manager getting a promised utility of $V_0$. Panel B shows the same for all potential entrants in the economy. A new firm requires a fixed installation cost $\tilde{e}k$ to be paid. This random cost is drawn from a uniform distribution. Managers who draw a project cost less than the maximum contract value to the investor begin operations, while those with more expensive projects are rationed.

Figure 3: Investor’s valuation of contract. The solid curve shows the investor’s value of future cash flows from the contract as a function of the manager’s continuation value. For low continuation value, the curve is increasing because increasing the manager’s continuation value lowers the probability of termination. At high continuation value, increasing the manager’s continuation value is a wealth transfer to the manager. This lowers the investor’s value.
Figure 4: Incentive provision by the optimal contract. The left panel shows the manager’s continuation value after realization of output. His continuation value changes from $V$ to $V_+$ for high output (solid, blue curve), and to $V_-$ (dashed, red curve) for low output. The spread provides incentives for him to provide high effort. At very low levels of manager’s continuation value, this spread cannot be provided, and incentives are provided by threatening termination. The right panel shows payments received by the manager as a function of his continuation value. The solid, blue curve shows his payment $d_+$, for high output. The dashed, red curve shows his payment $d_-$ for low output.
Figure 5: Loss in value of match due to borrowing constraints. The solid, black curve shows the value of match, which is the sum of the investor’s value of the contract, and the manager’s promised utility, as a fraction of frictionless value. The dotted, blue curve shows the cross-sectional distribution of manager continuation values.
Figure 6: The steady-state distribution for two values of insider ownership measured by $B/\Delta p(z_+ - z_-)$. The dotted black curve is for a lower value of $B/\Delta p(z_+ - z_-) = 0.25$, while the solid blue curve is for higher $B/\Delta p(z_+ - z_-) = 0.40$.

Figure 7: The change in continuation value when the state changes from boom to recession.
Figure 8: Dynamics of the distribution of continuation values. The solid, red curve shows the cross-sectional distribution of continuation values of managers after realization of the low aggregate productivity shock, $x_B$. The dotted, blue curve shows the distribution before the shock, and is chosen to be the stochastic steady-state distribution of the economy.

Figure 9: Increase in exit rates after a short recession. The economy starts at the stochastic steady-state and experiences two successive low aggregate shocks $x_B$. 
The maximum Sharpe ratio is the ratio of the volatility of the stochastic discount rate to its mean $\sigma(\pi')/E[\pi']$. The economy starts at the stochastic steady-state and experiences two successive low aggregate shocks $x_B$.

The economy starts at the stochastic steady-state and experiences five successive low aggregate shocks $x_B$.

Figure 10: Increase in the maximum Sharpe ratio after a short recession. The maximum Sharpe ratio is the ratio of the volatility of the stochastic discount rate to its mean $\sigma(\pi')/E[\pi']$. The economy starts at the stochastic steady-state and experiences two successive low aggregate shocks $x_B$.

Figure 11: Increase in exit rates after a long recession. The economy starts at the stochastic steady-state and experiences five successive low aggregate shocks $x_B$.
Figure 12: Increase in the maximum Sharpe ratio after a long recession. The maximum Sharpe ratio is the ratio of the volatility of the stochastic discount rate to its mean $\sigma(\pi')/E[\pi']$. The economy starts at the stochastic steady-state and experiences five successive low aggregate shocks $x_B$.

Figure 13: Response of aggregate investment to a one standard deviation shock in aggregate productivity $x$. The dashed red curve shows the response to a negative shock, while the solid blue curve is the response to a positive shock. Notice the asymmetric response.
Figure 14: Response of aggregate output to a one standard deviation shock in aggregate productivity $x$. The dashed red curve shows the response to a negative shock, while the solid blue curve is the response to a positive shock. Notice the asymmetric response.

Figure 15: Increase in asymmetric response of investment with increase in the representative investor’s risk-aversion. The y-axis plots the ratio of the magnitude of the instantaneous drop on realization of a negative shock to the corresponding increase for a positive shock of the same magnitude.
Figure 16: Intra-period time-line of the model.