Cooperation with Network Monitoring: Corrigendum

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There is an error in the proof of Theorem 1, and the correction requires a slight strengthening of the stated assumption on players’ utility functions.\(^1\) The relevant assumption is that, for every pair of players \(i, j\), the function \(f_{i,j}\) measuring \(i\)'s benefit from \(j\)'s action is either strictly concave or identically 0. The proof of Theorem 1 is correct if all of the \(f_{i,j}\) are non-zero, but an extra assumption is needed to allow \(f_{i,j} = 0\). A simple sufficient assumption for the case of a fixed monitoring network \(L\) is the following.

**Assumption** If \(f_{i,j} \neq 0\) and player \(k \neq i, j\) lies on a shortest path from \(i\) to \(j\) in \(L\), then \(f_{k,j} \neq 0\).

To see that Theorem 1 can fail without this assumption, suppose there are three players with the fixed monitoring network \(l_{1,2} = l_{2,1} = l_{2,3} = l_{3,2} = 1, l_{1,3} = l_{3,1} = 0\) (i.e., players 1 and 2 see each other’s actions, as do players 2 and 3, but not players 1 and 3), and benefit functions \(f_{1,3} \neq 0, f_{3,1} \neq 0, f_{1,2} = f_{2,1} = f_{2,3} = f_{3,2} = 0\). Then player 2 will never play \(x_2 > 0\), so players 1 and 3 will not find out if the other shirks (recall that players need not observe their own payoffs), and therefore \(x^*_1 = x^*_2 = x^*_3 = 0\), while Theorem 1 may state that \(x^*_1\) and \(x^*_3\) are positive.

The error in the proof of Theorem 1 is that “news” about a deviation cannot be spread by a player whose equilibrium action is already 0 (like player 2 here). Formally, the mistake is in the first paragraph on p. 424. One must show that \(\sigma_j^* (h_j^*) = 0\) whenever \(j \in D(\tau, t, i)\), \(\Pr (j \in D(\tau, t, i)) > 0\), and \(f_{i,j} \neq 0\) (this last condition is missing in the published version).

\(^1\)I thank Shengwu Li for finding the error.
The key error is in the third-to-last sentence of this paragraph: the conclusion in this sentence that $\hat{x}_k > 0$ is valid only if $f_{k,j} \neq 0$. The player $k$ referenced in this sentence may be taken to lie on a shortest path from $i$ to $j$. Thus, the argument in this paragraph is valid under the above extra assumption.

The analysis of all applications in the paper remains valid. In particular, the only application that involves $f_{i,j} = 0$ for some $i, j$ is the “local public goods” case in Section 5. The extra assumption is trivially satisfied in this application, as if $f_{i,j} \neq 0$ then $j \in N_i$, so the only players on a shortest path from $i$ to $j$ are $i$ and $j$ themselves.