Offshoring and Directed Technical Change*

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Abstract

To study the short-run and long-run implications on wage inequality, we introduce directed technical change into a Ricardian model of offshoring. A unique final good is produced by combining a skilled and an unskilled product, each produced from a continuum of intermediates (tasks). Some of these tasks can be transferred from a skill-abundant West to a skill-scarce East. Profit maximization determines both the extent of offshoring and technological progress. Offshoring induces skill-biased technical change because it increases the relative price of skill-intensive products and induces technical change favoring unskilled workers because it expands the market size for technologies complementing unskilled labor. In the empirically more relevant case, starting from low levels, an increase in offshoring opportunities triggers a transition with falling real wages for unskilled workers in the West, skill-biased technical change and rising skill premia worldwide. However, when the extent of offshoring becomes sufficiently large, further increases in offshoring induce technical change now biased in favor of unskilled labor because offshoring closes the gap between unskilled wages in the West and the East, thus limiting the power of the price effect fueling skill-biased technical change. The unequalizing impact of offshoring is thus greatest at the beginning. Transitional dynamics reveal that offshoring and technical change are substitutes in the short run but complements in the long run. Finally, though offshoring improves the welfare of workers in the East, it may benefit or harm unskilled workers in the West depending on elasticities and the equilibrium growth rate.

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1 Introduction

The rapid rise of offshoring, which involves many production and service tasks previously produced domestically being sourced from abroad, has been one of the most visible trends in the US labor market over the last three decades.\(^1\) The share of imported inputs in total intermediate use in the US manufacturing, for example, has increased from about 6% in 1980 to over 27% today (Feenstra and Jensen, 2009) and by now intermediate inputs accounts for two thirds of world trade. The production structure of Apple’s video iPod gives a glimpse of these trends. Though designed and engineered in the United States, the overwhelming majority of the production jobs created by this product are located abroad (Linden et al. 2011). Despite its prevalence, the implications of offshoring for wages, skill premia and incomes are still debated. The iPod example illustrates its different potential effects. Though most production jobs related to the iPod are offshored, a significant number of high-skill engineering jobs and lower-skill retail jobs are created in the United States, and more than 50% of the value added of the iPod is captured by domestic companies. With more limited offshoring, some of the production jobs may have stayed in the United States, increasing the demand for the services of lower-skill production workers.

The potential negative effects of offshoring on incomes and/or the wages of lower-skill workers in advanced economies (the “West”) have been emphasized by Feenstra and Hanson (1996), Deardorff (2001, 2005), Samuelson (2004) and Hira and Hira (2008), among others. Samuelson, for example, famously pointed out how offshoring could lower Western incomes in a Ricardian trade model if it transfers knowledge to less advanced, lower-wage economies (the “East”), thus eroding the Western technological advantage in a range of tasks. Counteracting this are the efficiency gains due to offshoring, emphasized by several authors including Grossman and Rossi-Hansberg (2008) and Rodriguez-Clare (2010), which potentially benefit both skilled and unskilled workers in the West.

Our focus in this paper is on the impact of offshoring on the direction of technical change and the equilibrium wage structure within the West and between the West and the East.\(^2\) Though there is a vibrant debate on the exact contribution of skill-biased technical change to wage inequality in the United States and other advanced economies, there is a broad consensus that the more rapid rise in the demand for skills than the supply has been at the root of much of it and that more skill-biased technologies, at given factor supplies, tend to increase wage inequality. It is also evident that offshoring opportunities should impact the skill bias of technology. We can illustrate this again with the example of Apple products; without the

\(^1\)Offshoring is not an entirely new phenomenon. The term “production sharing” was coined in 1977 by Peter Drucker to describe activities such as the assembly of electronic equipment in South Korea and Singapore, or cloth processing in Morocco, Malaya and Indonesia. Extensive media coverage of labor market implications of offshoring exploded before the 2004 US presidential election. Current concerns are well captured by what BusinessWeek (2008) defines as one of today’s burning questions: Is American tech supremacy thanks to heavy investments in R&D also benefiting US workers? Or are US inventions mainly creating jobs overseas?

\(^2\)We use the terms “technological change” and “technical change” interchangeably.
opportunity to offshore assembly tasks, it may not have been profitable for Apple to introduce some of the new varieties of iPods because of the higher labor costs it would have faced. This would have reduced the demand for high-skill engineering and design jobs in Apple, corresponding to the “price effect” which creates a positive link between offshoring and skill-biased technical change. Counteracting this, without offshoring opportunities, Apple may have designed iPods differently in order to reduce its dependence on expensive domestic unskilled labor, with potentially adverse effects on the demand for unskilled workers in the United States; this illustrates the “market size effect” which creates a negative link between offshoring and skill-biased technical change. Our objective is to provide a systematic framework to investigate and quantify these economic channels.

In our model, a unique final good is produced by combining a skilled and an unskilled product, each produced from a continuum of intermediates (tasks). Offshoring takes the form of some of these intermediates being transferred to the East and is subject to both fixed and variable costs. Profit-maximizing incentives determine not only how much offshoring will take place in equilibrium, but also the rate at which the productivities of both skilled and unskilled sectors improve. An important implication of offshoring highlighted by our model is the efficiency-enhancing reallocation of production towards countries where wages are lower. This efficiency effect is stronger when there is little offshoring, because the wage gap between the West and the East is greatest in this case. By increasing the demand for labor in the East, greater offshoring closes this gap.\(^3\)

Our main results concern the effects of offshoring on equilibrium technologies. Offshoring encourages skill-biased technical change by increasing the relative price of skill-intensive products. Simultaneously, it encourages unskilled labor-biased (henceforth, with some abuse of grammar, “unskill-biased”) technical change because it expands the market size of technologies complementary to unskilled workers, which can now be used in the East. In the empirically more relevant case where the elasticity of substitution between intermediates (tasks) is greater than the elasticity of substitution between skills and starting from low levels of offshoring, the price effect dominates and greater offshoring opportunities induce skill-biased technical change.\(^4\) The opposite pattern obtains and offshoring induces unskill-biased technical change when the level of offshoring is high. This result turns on the Ricardian features of our model: first, the efficiency gains are strongest when offshoring is limited, and second, offshoring closes

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3 Though our main contribution is on the effects of offshoring on technical change, our model of offshoring with fixed technology has implications on the skill premium that are related to, though different from, those emphasized in the literature. In particular, offshoring tends to increase the skill premium through a labor supply effect and a relative price effect, and tends to reduce it through the efficiency effect. This efficiency effect, which is based on the complementarity between Western and Eastern workers, is related to Grossman and Rossi-Hansberg’s (2008) productivity effect, but also different in a crucial way as highlighted by the fact that it is more pronounced when there is a large wage gap between East and West (and thus little offshoring) and it vanishes as the wage gap falls—a feature that is at the center of many of our key results.

4 Here it is important to interpret offshoring broadly as taking place both in production tasks and intermediates produced by unskilled workers, particularly because, as we discuss in footnote 16 below, most of the estimates on the relevant elasticity of substitution come from trade data on intermediates.
the wage gap between East and West, and when this wage gap is small, the price effect on the
direction of technical change is muted.\(^5\)

This configuration thus yields one of our main qualitative results—an inverse U-shaped rela-
tionship between offshoring and the direction of technical change. In consequence, offshoring
will first increase wage inequality in the West both through its direct effect and by triggering
skill-biased technical change. As offshoring continues, however, technical change will eventu-
ally change direction and become unskill-biased, thus limiting the increase in wage inequality.
Throughout, as expected, offshoring also compresses (unskilled) wage differences between the
East and the West.

Although our model lacks several important factors shaping wage inequality in the United
States (including changes in the domestic supply of skills, institutional factors and other deter-
minants of the types of technologies introduced at different times), it is nonetheless consistent
with the qualitative picture that emerges from several decades of changes in the US wage
structure. The first wave of expansion of offshoring in the 1980s coincided with a sharp decline
in the real wages of unskilled workers, but as offshoring continued to expand in the late 1990s
and 2000s, unskilled wages stabilized and began rising (e.g., Acemoglu and Autor, 2010). Consis-
tently, our results suggest that the impact of offshoring on wage inequality should have been
strongest when the volume of trade in intermediates was limited, as in the 1980s. As such, it
also circumvents the standard criticism directed at trade-based explanations of inequality that
the volume of trade between the United States and developing countries was then too small to
have a meaningful impact on wages.\(^6\)

The dynamics of technology and wages in response to an expansion of offshoring opportu-
nities is also interesting, highlighting that the two activities are substitutes in the short run,
but complements in the long run: immediately after the change in offshoring opportunities,
technical change stops for a while because firms first spend resources to offshore their exist-
ing intermediates/tasks; this is followed by a phase of either skill-biased technical change (for
levels of offshoring below a critical threshold) or unskill-biased technical change (for levels of
offshoring above a critical threshold). Our welfare analysis shows that if the post-offshoring
equilibrium rate of technical change is sufficiently high, then offshoring contributes positively
to the welfare of all workers. However, because offshoring creates a capital loss (by reducing

\(^5\)The impact of offshoring on the direction of technical change is quite different than the impact of trade on the
direction of technical change. For example, in Acemoglu (2003) trade induces skill-biased technical change when
intellectual property rights (IPR) are not enforced internationally, but induces unskill-biased technical change
when they are fully enforced. Here because offshoring is a voluntary, and thus profitable, activity for firms, its
qualitative impact on the direction of technical change is independent of international IPR enforcement, though
changes in IPR has additional implications for the direction of technical change. Other sources of differences
result from the Ricardian aspects of our model which, as explained above, ensure that offshoring closes the wage
gap between East and West, and from the efficiency effect mentioned in footnote 3.

\(^6\)Our model is also broadly consistent both with Bloom et al. (2011), who find that the surge of imports
from China from the late 1990s encouraged investments in information technology across European industries,
and with Autor et al. (2012), who show that it also reduced the demand for labor in US local economies heavily
exposed to this import competition.
the value of existing firms), it can in principle harm workers in the West (especially, unskilled workers) depending on elasticities and growth rates. In particular, if the post-offshoring growth rate is sufficiently high, all workers benefit from offshoring, but otherwise both skilled and unskilled workers in the West can lose out. Our quantitative results suggest that while Eastern workers benefit most and unambiguously, Western unskilled workers are most likely to suffer as a result of offshoring, while Western skilled workers typically obtain limited gains.

The tractability of our framework also enables us to extend it to include the offshoring of skilled intermediates. This more general model confirms the main results discussed so far. It also naturally yields a new result: offshoring can increase wage inequality both in the West and the East simultaneously—a possibility that is generally precluded in standard trade models (see Wood, 1994). This happens because, despite the presence of complete specialization and technological differences across countries, a zero-profit condition implies a form of conditional factor price equalization: if offshoring costs are identical, profit maximization implies that, with offshoring, the skill premium has to be the same in the East as in the West.\(^7\) Finally, we study the transition of the East from low-productivity imitation to higher-productivity offshoring, which leads to a pattern of transition reminiscent of the Chinese process of economic growth over the last three decades, with productivity gains due to reallocations from imitation to offshoring and no wage growth.

Our paper is related to three literatures. First, it is a contribution to the growing literature on offshoring already mentioned above.\(^8\) Our main point of departure from this literature is the endogeneity of the direction of technological change. Glass and Saggi (2001), Naghavi and Ottaviano (2009), Dinopoulos and Segerstrom (2010), Branstetter and Saggi (2011), Rodriguez-Clare (2010) and Jakobsson and Segerstrom (2012) endogenize the overall pace of technological change in models with offshoring, but not its direction. All of our main results derive from the endogeneity of the direction of technological change and are thus not shared by these papers or others in this literature.\(^9\)

Second, our paper is a contribution to the large literature on the theoretical determinants of changes in inequality and wages in the United States and other advanced economies. Our model is closely related to task-based approaches, including Acemoglu and Zilibotti (2001), Autor et al. (2003) and (2008), Costinot and Vogel (2010), and Acemoglu and Autor (2010). The last paper emphasizes the role of technologies replacing tasks previously performed by labor and the similar role of offshoring in this context, but does not model offshoring in detail and does not consider the interplay between offshoring and directed technological change.

Third, our paper builds on and extends models of directed technical change (e.g., Acemoglu,

\(^{7}\)These predictions are broadly consistent with the evidence in Sheng and Yang (2012) who find that processing exports and FDI explain a large fraction of the recent increase in the Chinese college wage premium.

\(^{8}\)Some other recent contributions studying the effect of offshoring on wages include Antràs et al. (2006), Baldwin and Robert-Nicoud (2010), Costinot et al. (2012) and Egger et al. (2012). di Giovanni et al. (2012) provide a quantitative analysis of the global welfare effect of trade opening and productivity growth in China.

\(^{9}\)Goel (2012) studies the effect of offshoring on wages of different workers in a model with capital-skill complementarity, but without directed technical change.
1998, 2002, 2007, Acemoglu and Zilibotti, 2001, Kiley, 1999, Gancia and Zilibotti, 2008), and especially those linking international trade to the direction of new technologies, including Acemoglu (2003), Thoenig and Verdier (2003) and Epifani and Gancia (2008). All three of these papers show how international trade can induce technological changes that further increase the demand for skills, thus amplifying its direct impact on the wage structure.\footnote{Burstein and Vogel (2012) develop and estimate a model based on the interaction between trade and skill-biased technologies, but without endogenous or directed technical change.} This literature has not, to the best of our knowledge, considered offshoring, which has different effects on labor market equilibria and thus on incentives for technical change. These effects include the impact of offshoring on the direction of technical change that is independent of international intellectual property rights (IPR) enforcement (as discussed in footnote 5); the non-monotonic relationship between offshoring and the direction of technical change which crucially depends on the endogeneity of the gap between wages in the East and the West and thus the extent of the price effect, features related to the Ricardian nature of intermediate trade; and the novel result that stronger IPR enforcement, by reducing the cost of offshoring, can trigger skill-biased technical change.\footnote{The recent paper by Chu et al. (2012) studies the effect of changes in the supply of labor in China on the direction of innovation in a model of trade and offshoring. Their results with offshoring mirror those obtained in models with directed technical change under full IPR protection (discussed, for example, in Acemoglu, 2003).}

The rest of the paper is organized as follows. Section 2 presents the basic model of intermediate/task trade and directed technical change and characterizes the effects of offshoring on wages and skill premia for a given level of technology. Section 3 studies the impact of offshoring on the direction of technical change and wages. Sections 4 and 5 study transitional dynamics and the welfare effects of a shock to the cost of offshoring. Section 6 extends the model to include offshoring of high-skill intermediates and imitation. Section 7 concludes. The Appendix contains the omitted proofs.

2 Model

2.1 Environment

The world economy comprises of two countries, West and East, populated by two types of workers, skilled and unskilled, both in fixed supply. The West is endowed with $L_w$ units of unskilled workers and $H$ units of skilled workers. The East is assumed to be skill scarce. In particular, we assume that the East has $L_e$ unskilled workers and has no skilled workers (we relax this assumption in Section 6). The two countries also differ in the technological capabilities to produce existing intermediates (Krugman, 1979): new technologies are introduced in the West and can be transferred to the East only after paying a fixed offshoring cost. As in earlier models of directed technical change (see, e.g, Acemoglu, 2002), some technologies complement skilled workers while others complement unskilled workers and the evolution of both is endogenous. There are no barriers to trade of goods across countries, but labor is

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immobile. Variables with no country index refer to the world economy.

Infinitely lived households derive utility from the consumption of a unique final good, and supply labor inelastically. Preferences are identical across countries and types of workers, and take a logarithmic form. In consequence, the world economy admits a representative household with preferences at time \( t = 0 \) given by

\[
U = \int_0^\infty e^{-\rho t} \ln C_t dt,
\]

where \( \rho > 0 \) is the discount rate. Logarithmic utility is assumed to simplify the exposition. The representative household sets a consumption plan to maximize utility, subject to an intertemporal budget constraint and a No-Ponzi game condition. The consumption plan satisfies a standard Euler equation,

\[
\frac{\dot{C}_t}{C_t} = r_t - \rho, \tag{1}
\]

where \( r_t \) is the interest rate, as well as a transversality condition which takes the form

\[
\lim_{t \to \infty} \left[ \exp \left( - \int_0^t r_s ds \right) W_t \right] = 0, \tag{2}
\]

where \( W_t \) is the wealth of consumers which, as we will see below, comes from their ownership of firms in the economy.\(^{12}\) In what follows, time indexes will be omitted in what follows as long as this causes no confusion.

The final good, \( Y \), is used for both consumption and investment. The technology to produce it is represented by a production function featuring constant elasticity of substitution (CES) between two sectors (where we suppress the distribution parameter of the CES to simplify notation):

\[
Y = \left( Y_l^{\frac{\epsilon-1}{\epsilon}} + Y_h^{\frac{\epsilon-1}{\epsilon}} \right)^{\frac{\epsilon}{\epsilon-1}}. \tag{3}
\]

Here \( Y_h \) and \( Y_l \) are tradable goods, and \( \epsilon \) is the elasticity of substitution between them. Choosing \( Y \) as the numeraire, profit maximization yields the following inverse demand functions:

\[
P_l = (Y/Y_l)^{1/\epsilon} \quad \text{and} \quad P_h = (Y/Y_h)^{1/\epsilon}, \tag{4}
\]

where \( P_l \) and \( P_h \) are the world prices of \( Y_l \) and \( Y_h \), respectively. Naturally, we also have \( P_h/P_l = (Y_l/Y_h)^{1/\epsilon} \).\(^{13}\)

Production in each of the two sectors requires intermediates, which are in turn manu-

\(^{12}\) In particular, we will have

\[
W_t = \left( \int_0^{A_{h,t}} V_{j,t} dt + \int_0^{A_{l,t}} V_{j,t} dj \right),
\]

where \( V_{j,t} = \int_0^\infty \exp \left[ - \int_t^{s} r_s ds \right] \pi_{j,s} ds \), \( \pi_{j,s} \) is the profits of the firm operating intermediate \( j \) in sector \( s \in \{l, h\} \) as given by (8) below, and \( A_{s,t} \) is the range of active intermediates in sector \( s \).

\(^{13}\) Since \( Y \) is the numeraire, we also have \( (P_l^{1-\epsilon} + P_h^{1-\epsilon})^{\frac{1}{1-\epsilon}} = 1. \)
tured by workers. In particular, the production of intermediates used to make $Y_l$ and $Y_h$, require, respectively, unskilled and skilled workers. The production technologies of the two sectors are represented by the following Dixit-Stiglitz production functions:

$$ Y_l = E_l \left( \int_0^{A_l} x_{l,i}^{\alpha} di \right)^{1/\alpha} \quad \text{and} \quad Y_h = E_h \left( \int_0^{A_h} x_{h,i}^{\alpha} di \right)^{1/\alpha}, $$

(5)

where $x_{l,i}$ (resp., $x_{h,i}$) is the quantity of intermediate $i \in [0, A_l]$ (resp., $i \in [0, A_h]$), and $\sigma \equiv 1/(1-\alpha) > 1$ is the elasticity of substitution between any two of them. As in models of horizontal innovation (e.g., Romer, 1990, see Gancia and Zilibotti, 2005, for a survey), the measures of intermediates, $A_l$ and $A_h$, represents the state of technology in the two sectors which grows endogenously over time. The terms

$$ E_l \equiv (A_l)^{\frac{2\sigma-1}{\alpha}} \quad \text{and} \quad E_h \equiv (A_h)^{\frac{2\sigma-1}{\alpha}} $$

(6)

are technological spillovers introduced to guarantee that the model has balanced growth properties for any $\sigma$.\(^{14}\)

Profit maximization yields the following inverse demand functions:

$$ p_{l,i} = P_l E_l^{a} Y_l^{1-\alpha} x_{l,i}^{\alpha-1} \quad \text{and} \quad p_{h,i} = P_h E_h^{a} Y_h^{1-\alpha} x_{h,i}^{\alpha-1}, $$

(7)

where $p_{l,i}$ ($p_{h,i}$) is the price of the intermediate variety $i$, where $i \in [0, A_l]$ ($i \in [0, A_h]$).

Each intermediate is produced by a single monopolist using a constant returns to scale technology using labor as sole input:

$$ x_{l,i} = l_i \quad \text{and} \quad x_{h,i} = Z h_i, $$

where $l_i$ ($h_i$) is the quantity of unskilled (skilled) labor employed and $Z \geq 1$. Since the demand features a constant elasticity equal to $\sigma \equiv 1/(1-\alpha)$, profit maximizing firms charge prices equal to a markup $1/\alpha$ over the the respective marginal cost: $p_{h,i} = (w_h/Z)/\alpha$ and $p_{l,i} = w_{l,w}/\alpha$, for varieties produced in the West, and $p_{l,i} = w_{l,e}/\alpha$ for varieties produced in the East, where $w_h$ denotes the high-skill wage, and $w_{l,c}$ denotes the low-skill wage in country $c \in \{e, w\}$. As a result, profits are a fraction $(1-\alpha)$ of the value of sales:

$$ \pi_{l,i} = (1-\alpha) p_{l,i} x_{l,i} \quad \text{and} \quad \pi_{h,i} = (1-\alpha) p_{h,i} x_{h,i}. $$

(8)

\(^{14}\)Alternative formulation without such spillovers yield identical results, but complicate the algebra, motivating our choice here. See Gancia and Zilibotti (2009) and Acemoglu, Gancia and Zilibotti (2011) for a more detailed discussion of this formulation.
Equilibrium with Exogenous Technology

In this subsection, we consider the wage effects of offshoring for an exogenous level of technology $(A_l, A_h)$.

As in Krugman (1979), we assume that the West can produce the entire measure of existing intermediates, while the East can only produce a fraction $\kappa < \bar{\kappa} \equiv L_e/(L_e + L_w) < 1$ of them. The restriction that $\kappa < \bar{\kappa}$ guarantees that wages are lower in the East, so that offshoring production to the East, when technologically feasible, is also profitable for Western firms. This assumption also implies that a firm that can offshore will not produce in the West. It follows that in equilibrium a measure $\kappa A_l$ of firms produce in the East and the remaining measure $(1 - \kappa) A_l$ in the West. Note that both the extent of offshoring, $\kappa$, and the skill bias of technology, taken as exogenous here, will be endogenized in the next section (which will also guarantee that $\kappa < \bar{\kappa}$).

Imposing labor market clearing and using the fact that all firms of a given type are identical, we can solve for the quantity produced of any intermediate in the West and the East as:

$$ x_h = \frac{ZH_w}{A_h}, \quad x_{l,w} = \frac{L_w}{(1 - \kappa) A_l} \quad \text{and} \quad x_{l,e} = \frac{L_e}{\kappa A_l}. $$

Next, using (7), we obtain the East-West low-skill wage gap:

$$ \frac{w_{l,w}}{w_{l,e}} = \frac{p_{l,w}}{p_{l,e}} = \left( \frac{x_{l,w}}{x_{l,e}} \right)^{\alpha - 1} = \left( \frac{L_e}{L_w} \frac{1 - \kappa}{\kappa} \right)^{1 - \alpha}, $$

where it is evident that $\kappa < \bar{\kappa}$ implies that $w_{l,w} > w_{l,e}$. As production is relocated to the East (i.e., as $\kappa$ goes up), the demand for unskilled workers falls in the West and increases in the East, thereby compressing the wage gap. Note that conditional on $\kappa$ the elasticity of substitution between unskilled workers in the West and East is also $\sigma \equiv 1 / (1 - \alpha)$.

Substituting (9) into (5), and using (6), we can express the world production of the low-skill good as:

$$ Y_l = A_l \hat{L}, $$

where

$$ \hat{L} \equiv \left( \kappa^{1 - \alpha} L_e^\alpha + (1 - \kappa)^{1 - \alpha} L_w^\alpha \right)^{1/\alpha} $$

is a weighted average of the East’s and the West’s endowments of unskilled workers, with weights depending on the offshoring rate. As in standard models of horizontal innovation, equation (11) shows that production increases linearly in the number of existing varieties, $A_l$. More interestingly, for a given number of varieties, equation (12) shows that production increases in the extent of offshoring:

$$ \frac{d\hat{L}}{d\kappa} = \frac{1 - \alpha}{\alpha} \hat{L}^{1 - \alpha} \left[ \left( \frac{L_e}{\kappa} \right)^\alpha - \left( \frac{L_w}{1 - \kappa} \right)^\alpha \right] > 0, $$

8
where \( \lim_{\kappa \to 0} d\hat{L}/d\kappa = \infty \) and \( \lim_{\kappa \to \bar{\kappa}} d\hat{L}/d\kappa = 0 \). We refer to this as the efficiency effect of offshoring: an increase in \( \kappa \) induces an efficiency-enhancing reallocation of production towards countries where wages are lower. In terms of the production of \( Y_l \), this is equivalent to an increase in the world factor endowment ranging from \( \hat{L} = L_w \), when \( \kappa \to 0 \), to \( \hat{L} = L_w + L_e \), when \( \kappa \to \bar{\kappa} \). Importantly, the efficiency effect is stronger when wages in the East are lower, i.e., when there is little offshoring (low \( \kappa \)) and when the East has a large relative endowment of unskilled workers (high \( L_e/L_w \)). This is intuitive in view of the fact that the efficiency effect exploits the wage gap between East and West, which is inversely related to \( \kappa \).

Consider, next, the skill-intensive sector. Substituting into (5) the expression of \( x_h \) given in (9) and using (6) yields:

\[
Y_h = A_h ZH. \tag{13}
\]

For future reference, it is useful to rewrite the expression of profits given by (8), using (6), (7), (9), (11) and (13):

\[
\pi_h = (1 - \alpha) P_h H, \quad \pi_{l,w} = (1 - \alpha) P_l \hat{L}^{1-\alpha} \left( \frac{L_w}{1 - \kappa} \right)^\alpha, \tag{14}
\]

\[
\pi_{l,e} = (1 - \alpha) P_l \hat{L}^{1-\alpha} \left( \frac{L_e}{\kappa} \right)^\alpha.
\]

### 2.3 Offshoring and Wages with Exogenous Technology

We are now in the position to study the effect of the level of offshoring on wages in the West. We consider, first, the effect of changes in \( \kappa \) on the skill premium, and then on the high- and low-skill wage levels.

Denote the skill premium in the West by \( \omega_w \equiv w_{h,w}/w_{l,w} \). Constant markups then imply that \( \omega_w = Z (p_{nw}/p_{lw}) \). Using (4), (7), (9), (11), and (13) we obtain:

\[
\omega_w = Z \left( \frac{E_h}{E_l} \right)^\alpha \frac{P_h}{P_l} \left( \frac{Y_h x_{l,w}}{Y_l x_{h,w}} \right)^{1-\alpha}
= Z \left( \frac{A_h}{A_l} \right)^{1-1/\epsilon} \left( \frac{L_w}{1 - \kappa} \right)^{1-\alpha} \times \left( \frac{H}{L} \right)^{1/\epsilon} \times \frac{1}{L^{1-\alpha}}, \tag{15}
\]

where, recall, \( \hat{L} \) is an increasing function of \( \kappa \). The first equation shows that the skill premium is increasing in the relative price \( (P_h/P_l) \) and the relative aggregate demand \( (Y_h/Y_l) \) for skill-intensive products, and decreasing in relative firm size \( (x_{h,w}/x_{l,w}) \). The expression in the second line shows that the impact of changes in offshoring (i.e., an increase in \( \kappa \)) on the skill

\hspace{1cm} \text{To illustrate more formally how the efficiency effect depends on the East-West wage gap, consider the impact of offshoring on the world price of } Y_l, P_l. \text{ Using (5) yields } P_l = A_l^{-1} \left[ \kappa p_{l,e}^{1-\sigma} + (1 - \kappa) p_{l,w}^{1-\sigma} \right]^{1/(1-\sigma)}, \text{ implying that } \frac{\partial p_{l,w}}{\partial w_{l,w}} = (A_l \alpha)^{\sigma-1} (w_{l,e}^{1-\sigma} - w_{l,w}^{1-\sigma}). \text{ The expression of the derivative shows that relocating production from the West to the East lowers the production cost of } Y_l \text{ by a factor proportional to the wage gap between the two countries.}
premium can be decomposed into a labor supply effect, \((L_w/(1-\kappa))^{1-\alpha}\), a relative price effect, \((H/\hat{L})^{-1/\epsilon}\), and an efficiency effect, \(\hat{L}^{\alpha-1}\). The first two effects increase the skill premium, whereas the third one reduces it.

We now discuss each effect in more detail. First, offshoring displaces Western unskilled workers who must be rehired by the remaining domestic firms. Holding prices \((P_h/P_l)\) constant, this is analogous to an increase in the supply of unskilled workers in the West which in turn increases the skill premium. Second, offshoring increases low-skill production, raising the relative price of the skill-intensive goods. This relative price effect also increases the skill premium. Third, offshoring raises the overall efficiency of low-skill production, reducing \(Y_h/Y_l\). The increase in the relative demand for the unskilled product raises the demand for the offshored factor in the West. The effect is stronger when the complementarity between unskilled workers in the West and the East is greater \((\text{low } \alpha)\) and when the initial level of offshoring is lower.

An inspection of (15) shows that the efficiency effect is dominated by the price effect whenever \(\sigma > \epsilon\) \(\text{ (i.e., } 1-\alpha < 1/\epsilon\)\). That is, if the elasticity of substitution between intermediates produced in the East and in the West \(\text{(or between unskilled workers in the East and in the West)}\) is greater than the elasticity of substitution between high- and low-skill workers, then offshoring necessarily increases the skill premium in the West. Given the estimates of elasticities in the empirical literature, this case seems empirically plausible and will be the one we emphasize in the rest of the paper.\(^{16}\)

If, instead, intermediates were more complementary than high- and low-skill workers \((\sigma < \epsilon \text{ or } 1-\alpha > 1/\epsilon)\), then the efficiency effect would dominate the price effect. Whether it would also dominate the labor-supply effect depends on the level of offshoring. Since \(\lim_{\kappa \to 0} d\hat{L}/d\kappa = \infty\), for sufficiently low levels of \(\kappa\), the efficiency effect is so strong that offshoring raises the relative reward to the offshored factor. For sufficiently high levels of offshoring, however, only the labor-supply effect remains \(\text{(recall, } \lim_{\kappa \to \hat{\kappa}} d\hat{L}/d\kappa = 0\)\). The relationship between \(\omega_w\) and \(\kappa\) in the two cases is depicted in Figure 1.

The efficiency effect is similar to the productivity effect of trade in tasks in Grossman and Rossi-Hansberg (2008). In their model, offshoring requires a per-unit cost which varies across tasks, there is no substitutability across tasks and the foreign wage is exogenously given. Under these assumptions, they show that a fall in offshoring costs increases the “effective productivity” of the offshored factor and in some cases its wage. Our results differ in three important respects.

\(^{16}\)Most estimates of the elasticity of substitution between skilled and unskilled workers are in the range \([1.5, 2]\) \(\text{ (e.g., } \text{ Ciccone and Peri 2005)}\). There is also a large literature estimating substitutability across traded varieties, including intermediate inputs. The vast majority of these estimates are above 3. For example, on the basis of existing studies, Anderson and Van Wincoop (2004) conclude that this elasticity is likely to be from 5 to 10. Broda and Weinstein (2006) find an average import demand elasticity of 4 across three-digit SITC sectors, 12 across ten-digit, HTS sectors. Structural estimates in Eaton et al. (2011) suggests an average elasticity around 3.

A final argument suggesting that \(\sigma > \epsilon\) is the plausible case is as follows: when \(\epsilon > \sigma\) an increase in \(L_e\) would reduce the skill premium in the West, which appears implausible and somewhat counterfactual.
First, by taking into account the general equilibrium adjustment of Eastern wages to the higher demand for their services, our model suggests that the efficiency effect becomes endogenously weaker as more offshoring takes place and will eventually vanish once wages have converged worldwide.

Second, differently from Grossman and Rossi-Hansberg (2008), our model allows for substitutability between intermediates, and shows that the extent of such substitution, as captured by the elasticity of substitution between intermediates, changes the strength of the efficiency effect. In fact, our results show that assuming no task/intermediate substitutability, as in several existing models of offshoring, might provide only a partial picture of the implications of offshoring.

Third, Grossman and Rossi-Hansberg (2008) emphasize the beneficial effect of a reduction in the unit cost of offshoring on all offshored tasks (intensive margin), while we focus on the benefit of offshoring additional intermediates and tasks (the extensive margin). In both cases, the efficiency/productivity effect exists, but its determinants are different.

In the rest of this section, we study the effect of offshoring on wage levels. It is easy to establish that wages of both high-skill workers and low-skill Eastern workers increase unambiguously with offshoring (see Proposition 1 and its proof below). The behavior of the wage of low-skill workers in the West is more complex and deserves some discussion. It is also especially interesting since the effect of offshoring on low-skill wages in the West is the subject of an intense debate.

In the model with exogenous technology discussed in this section, offshoring always reduces
low-skill wages when the initial level of offshoring is high (i.e., as $\kappa \to \bar{\kappa}$). But its impact at low initial level of offshoring is ambiguous. More formally, the low-skill wage is given by $w_{l,w} = \alpha p_{l,w} = \alpha P_t E_t^\alpha (Y_t/x_{t,i})^{1-\alpha}$. Using (3), (4), (6), (9), (11) and (13) yields:

$$w_{l,w} = \alpha \left(1 + \frac{A_h Z H}{A_l L} \right)^{\frac{1}{\sigma-\epsilon}} \times A_l \hat{L}^{1-\alpha} \times \left(1 - \frac{\kappa}{L_w} \right)^{1-\alpha}.$$ \hspace{1cm} (16)

The effect of $\kappa$ on $w_{l,w}$ can again be decomposed into a relative price effect, an efficiency effect and a labor supply effect. The interpretation is similar to the discussion above concerning the skill premium: offshoring decreases the low-skill wage via the price and labor supply effects, whereas it increases $w_{l,w}$ via the efficiency effect.

Using (3), (4), (6), (9), (11) and (13) yields:

$$w_{l,w} = \alpha \left(1 + \frac{A_h Z H}{A_l L} \right)^{\frac{1}{\sigma-\epsilon}} \times A_l \hat{L}^{1-\alpha} \times \left(1 - \frac{\kappa}{L_w} \right)^{1-\alpha}.$$ \hspace{1cm} (16)

We have argued above that $\sigma > \epsilon$ is the empirically relevant case. Condition (17) is also plausible. For example, in the US economy the ratio of college to high-school graduates is greater than one and the skill premium greater than 1.5. With $\omega_w H/L_w = 1.5$ and $\epsilon = 1.6$, offshoring necessarily lowers the real wage of unskilled workers in the West whenever the elasticity of substitution across varieties is greater than 2.66, a value comfortably below the empirical estimates in the trade literature (see footnote 16). Thus, under two conditions that we regard as empirically realistic, low-skill wages are uniformly decreasing with offshoring.

The relationship between offshoring and the three wage levels is depicted in Figure 2 for the relatively conservative case $\epsilon = 1.6$ and $\sigma = 3.33$.

When either $\sigma < \epsilon$ or condition (17) is reversed, then the relationship between offshoring and low-skill wages is hump shaped. In particular, as $\kappa \to \bar{\kappa}$, $d\hat{L}/d\kappa \to 0$, both the price and the efficiency effects vanish, and $dw_{l,w}/d\kappa < 0$, unambiguously. However, as $\kappa \to 0$, $d\hat{L}/d\kappa \to \infty$, and the sign of the total effect turns positive.

The next proposition summarizes the impact of offshoring with exogenous technology on the skill premium in the West and wages in the West and the East (proof in the Appendix).

**Proposition 1** With exogenous technology, an increase in offshoring, parameterized by $\kappa$:

- increases the skill premium, $\omega_w$, if $\sigma > \epsilon$ (i.e., if the elasticity of substitution between intermediates is greater than the elasticity of substitution between skills);
- decreases the skill premium, $\omega_w$, for low initial values of $\kappa$ and increases the skill premium for high initial values of $\kappa$ if $\sigma < \epsilon$;
- increases the real wage of skilled workers in the West, $w_{h,w}$, and the real wage of unskilled workers in the East, $w_{l,e}$;
Figure 2: Offshoring and Wages. The figure shows the relationship between offshoring ($\kappa$) and wage levels, $w_{h,w}$ (red), $w_{l,w}$ (solid) and $w_{l,e}$ (dashed), for the case $\epsilon = 1.6$ and $\sigma = 3.3$. See Section 5 for the remaining parameters.

- decreases the real wage of unskilled workers in the West, $w_{l,w}$, if $\sigma > \epsilon$ and $\omega_w H_w/L_w > \epsilon/(\sigma - \epsilon)$, where $\omega_w$ is given by (15). Otherwise, its impact on $w_{l,w}$ is hump-shaped: it increases $w_{l,w}$ for low initial values of $\kappa$, and decreases $w_{l,w}$ for high initial values of $\kappa$.

In the rest of the paper, we characterize the effects of offshoring opportunities on technology. Our analysis characterizes these effects for all values of the various elasticities of substitution, but in light of the discussion above, we emphasize the empirically relevant case where $\sigma > \epsilon$.

3 ENDOGENOUS OFFSHORING AND TECHNOLOGY

In this section, we endogenize technical change and offshoring, focusing on balanced growth paths (BGP). Transitional dynamics will be studied in the next section.

We assume that innovation is subject to a fixed cost $\mu$, assumed for simplicity to be the same in both the high- and low-skill sector. In addition, by paying an additional set-up cost, $f$, a firm has the option to offshore the production of its variety to a partner firm in the East. However, in this case the Western firm only appropriates a share $\tilde{\lambda} \leq 1$ of the profit flow (all our results apply when $\tilde{\lambda} = 1$). This can be motivated by a variety of contractual imperfections, for example by assuming that the offshoring firm is subject to a hold up problem forcing it to transfer part of its profits to a partner company in the East or has only imperfect IPR protection.

Let $V_h$ be the value of a skill-intensive firm. This value must satisfy the usual Hamilton-
Jacobi-Bellman (HJB) equation:

\[ rV_h = \pi_h + V_h. \]  

(18)

In words, the instantaneous return from owning the firms is equal to the profit rate plus any capital gain or losses. Similarly, let \( V^o_i \) be the value of a firm that has paid the offshoring cost, and \( V_I \) the value of a low-skill firm that produces in the West. These value functions are determined by the following HJB equations:

\[
\begin{align*}
    rV^o_i &= \max \{ \pi_{l,w}, \tilde{\lambda} \pi_{l,e} \} + V^o_i, \\
    rV_I &= \max \{ \pi_{l,w} + V_I, r(V^o_i - f) \}.
\end{align*}
\]

(19)

The max operator in the first HJB equation takes into account the fact that the firm will produce in the most profitable location. Typically, a firm that has paid the offshoring cost will find optimal to produce in the East, thus, \( \tilde{\lambda} \pi_{l,e} > \pi_{l,w} \). The max operator in the second HJB equation captures the option for the non-offshored firm to pay the cost \( f \), offshore its production, and change its value to \( V^o_i \).

Free-entry implies that the value of introducing a new intermediate and the value of offshoring the production of an existing intermediate cannot exceed their respective costs: \( V^o_i - V_I \leq f \), \( V_I \leq \mu \), and \( V_h \leq \mu \). In a BGP with positive innovation and offshoring, all free-entry conditions must hold as equalities, so that

\[ V_I = V_h = \mu, \text{ and } V^o_i = f + \mu. \]

(20)

These conditions, together with the HJB equations above, pin down the BGP interest rate:

\[ r = \frac{\tilde{\lambda} \pi_{l,e} - \pi_{l,w}}{f} = \frac{\pi_{l,w}}{\mu} = \frac{\pi_h}{\mu}. \]

(21)

As we now show, this equation, which requires the return from offshoring and from any innovations to be equalized, determines the BGP level of offshoring (\( \kappa \)) and the skill-bias of technology (\( A_h/A_l \)).

To find the BGP level of offshoring, let us define \( \lambda \equiv \tilde{\lambda}/(f/\mu+1) \), which varies between zero and one, and can be interpreted as an index of “offshoring opportunities”. The first equality in (21) implies that \( \lambda \pi_{l,e} = \pi_{l,w} \). Substituting in the expressions of profits given by (14) yields the BGP level of offshoring as a function of \( \lambda \):

\[ \kappa = \left(1 + \lambda^{-\alpha} L_w/L_e\right)^{-1}. \]

(22)

Intuitively, better IPR (\( \tilde{\lambda} \)) and lower offshoring costs (\( f \)), i.e., a higher \( \lambda \), and a greater supply of labor in the East (\( L_e \)) make offshoring more attractive.

Substituting (22) into (10) and (12) yields the BGP East-West wage gap and the effective
world labor supply of unskilled workers:

\[
\frac{w_lw}{w_l,e} = \lambda^{\alpha - 1} \quad \text{and} \quad \hat{L} = \left( \frac{\lambda^{(1-\alpha)/\alpha} L_e + L_w}{L_w + \lambda^{1/\alpha} L_e} \right)^{1/\alpha}.
\]

To determine the direction of technical change, we next turn to the relative BGP value of innovation in the two sectors. First note that (18), (19) and (20) together imply that \( V_h = V_l = h = l \). Then, substituting profits from (14) yields:

\[
\frac{V_h}{V_l} = \frac{P_h}{P_l} \times \frac{Z H_w}{\hat{L}^{1-\alpha}} = \left( \frac{A_l \hat{L}}{A_h H_w} \right)^{1/\epsilon} \times \frac{Z H_w}{\hat{L}^{1-\alpha}} \left( \frac{L_w}{L_e} \right)^{\alpha},
\]

where the last equality follows by substituting for \( P_h/P_l \) from (4) and then using (11) and (13).

As in the canonical model of directed technical change (e.g., Acemoglu, 2002), the relative value for new innovations depends on a price and on a market size effect. However, the market size effect here takes a novel form: while the market size for skill-complementary innovations is simply \( H \), the effective market size in the other sector depends on \( L_w, L_e \) and the extent of offshoring, \( \kappa \).

Consider now the effect of an increase in \( \lambda \) (which also increases \( \kappa \)) on the direction of technical change to illustrate these price and market size effects. First, the improvement in the allocation of labor worldwide leads (via an increase in \( \hat{L} \)) to an increase in the production of the low-skill good, \( Y_l \). This raises the relative price of the high-skill good and increases the relative profitability of high-skill innovation. In other words, it induces a price effect favoring skill biased technologies. Second, the increase in \( \lambda \) also triggers two different types of market size effects. On the one hand, as more tasks and sectors are offshored to the East, each low-skill intermediate still produced in the US employs more workers and is produced in greater quantity. We refer to this effect, which favors unskill-biased technologies in response to greater offshoring, as a direct market size effect. This effect is captured in (23) by the term \( \frac{L_w}{\hat{L}^{1-\alpha}} \) and is clearly increasing in \( \kappa \) and thus in \( \lambda \). On the other hand, an increase in \( \kappa \) also raises \( \hat{L} \), raising the total market size of unskill-biased technologies. Intuitively, the market size depends (for given prices of the final goods and the size of the production of each firm in the US) on how efficiently the labor force is allocated worldwide. Increasing the efficiency of the allocation yields a larger effective market size. The extent of the complementarity across intermediates is crucial for this effect. As \( \alpha \rightarrow 1 \), the intermediates are perfect substitutes, and the effective market size becomes independent of \( \hat{L} \). Conversely, this effect is maximized when \( \alpha \) is small, and intermediates are highly complementary. We refer to this novel effect as a complementary market size effect.

Imposing the BGP technology market clearing condition, \( V_l = V_h \), into (23) yields the BGP
Figure 3: Offshoring and Directed Technical Change. The figure shows the relationship between offshoring ($\kappa$) and $A_h/A_l$ for the case $\epsilon = 1.6$, $\sigma = 5$ (dashed) and $\sigma = 3.33$ (solid). See Section 5 for the remaining parameters.

ratio of technologies:

$$
\frac{A_h}{A_l} = (ZH_w)^{\epsilon-1} \hat{L}^{1-\epsilon+\epsilon\alpha} \left( \frac{1-\kappa}{L_w} \right)^{\epsilon\alpha},
$$

(24)

where

$$
\frac{d \ln (A_h/A_l)}{d\lambda} = \left[ (1 - \epsilon + \epsilon\alpha) \frac{d \ln \hat{L}}{d\kappa} - \frac{\epsilon\alpha}{1-\kappa} \right] \frac{d\kappa}{d\lambda}.
$$

(25)

When $\sigma > \epsilon$, (25) is positive for small values of $\kappa$, because in this case the price effect is strong and the market size effect is limited (recall that $d \ln \hat{L}/d\kappa \to \infty$ as $\kappa \to 0$). It turns negative for high values of $\kappa$ because now the direct market size effect is more pronounced (recall that $d \ln \hat{L}/d\kappa \to 0$ as $\kappa \to \hat{\kappa}$). We define, for future reference, the threshold value $\hat{\lambda}$ such that, at this threshold, (25) is equal to zero (recall that $\kappa$ is monotonically increasing in $\lambda$) as:

$$
\hat{\lambda} = \phi^{-1}\left( \frac{\epsilon\alpha^2}{(1 - \epsilon + \epsilon\alpha)(1-\alpha)} \right),
$$

(26)

where $\phi(\lambda) \equiv (\hat{L}(\lambda))^{-\alpha} \left( (\kappa(\lambda))^{-\alpha} L^\alpha_e - (1 - \kappa(\lambda))^{-\alpha} L^\alpha_w \right) (1 - \kappa(\lambda))$, which is monotonically decreasing in $\lambda$. Intuitively, $\hat{\lambda}$ is the threshold value of $\lambda$ such that, if $\lambda < \hat{\lambda}$ (resp., $\lambda \geq \hat{\lambda}$), then $d (A_h/A_l)/d\lambda > 0$ (resp., $d (A_h/A_l)/d\lambda \leq 0$). Note that a lower $\epsilon$ strengthens the price effect, which tends to induce skill-biased technical change, and thus implies a higher $\hat{\lambda}$. The relationship between the BGP level of $A_h/A_l$ and $\kappa$ for the case $\sigma > \epsilon$ and two plausible values of $\sigma$ is represented in Figure 3.

To better understand the intuition for the hump-shaped pattern of Figure 3, recall that, as
discussed above, the effect of an increase of $\kappa$ on $\tilde{L}$ becomes very large as $\kappa \to 0$, and vanishes as $\kappa \to \tilde{\kappa}$. Since the direct market size effect does not vanish as $\kappa \to \tilde{\kappa}$, it dominates all other effects when $\kappa$ is large. This guarantees that a reduction in offshoring costs leads unambiguously to unskill-bias technical change (UBTC) when $\kappa$ is large (sufficiently close to $\tilde{\kappa}$). When $\kappa$ is small, we have the converse situation where the direct market size effect becomes negligible relative to the effects operating through $\tilde{L}$ (i.e., the price effect and the complementary market size effect). However, since these two effects work in opposite directions, their net impact is still ambiguous. When $\sigma > \epsilon$, as shown in the figure, the price effect is large and dominates the complementary market size effect. In this case, the increase in offshoring (or offshoring opportunities) generates a race between the price effect (net of the complementary market size effect) that pulls towards skill-biased technical change (SBTC) and the direct market size effect that pushes towards UBTC. When wages in the East are low, the effect of more offshoring opportunities is a large increase in the aggregate production of low-skill goods ($Y_l$), causing a significant increase in the relative price of high-skill good. Thus offshoring induces SBTC in this case. On the contrary, when wages in the East are already high, the effect of offshoring opportunities on $Y_l$ is limited, and is dominated by the fact that each low-skill firm in the US employs more workers and serves a larger market. Thus, over this range, offshoring induces UBTC. This pattern contrasts with the case where $\sigma < \epsilon$, which involves the price effect being dominated by the complementary market size effect. In this case, an increase in $\kappa$ unambiguously induces UBTC. As Figure 3 shows, the effect of offshoring on the skill bias of technology can be significant, particularly when the elasticity of substitution between intermediates is high (dashed line).

It is useful to compare these results with those obtained in models focusing on the impact of trade on the direction of technological progress, such as Acemoglu (2003), Acemoglu and Zilibotti (2001) and Gancia, Muller and Zilibotti (2011). In those models, the equation for the relative profitability of skill-complement innovations, (23), simplifies to:

$$\frac{V_h}{V_l} = \frac{\pi_h}{\pi_l} = \frac{P_h Z H}{P_l L},$$

where $H$ and $L$ are the relevant endowments. This corresponds to the autarky equilibrium in the present model. The effect of trade integration on the relative profitability of high- and low-skill innovation depends then on assumptions about the total skill endowment in the free-trade equilibrium and the extent of international protection of Intellectual Property Rights (IPR). Consider the opening to trade of a large skill-scarce country. Without global IPR, the market size for new technologies does not change. Then, the only effect will be an increase in the world price of skill-intensive products ($P_h/P_l$), which will induce SBTC. With global IPR protection, the market size dominates the price effect and the larger endowment of unskilled workers in the world economy would make it profitable to invest in UBTC. When $\sigma > \epsilon$, our model of offshoring nests these two extreme scenarios and predicts an endogenous switch from
SBTC to UBTC as integration increases. The reason is that the relative strength of the price effect varies with the level of offshoring: it dominates when wages in the East are low and the efficiency effect is strong, but it disappears as more offshoring eliminates the cost differences between the East and the West.

What are the implications for the skill premium in the West? Substituting (24) into (15), we can find the BGP skill premium as:

$$
\omega_w = \kappa^{-1} H_w^{1-\epsilon} \hat{L}^{1-\epsilon^\alpha} \left( \frac{L_w}{1-\kappa} \right)^{1-\epsilon^\alpha}.
$$

(27)

In the extreme cases of prohibitive offshoring costs (implying $\kappa = 0$) and zero offshoring costs (implying $\kappa = \kappa$), the skill premium is a function of the relative endowment of skilled labor in the West and in the entire world, respectively:

$$
\omega_w \mid \lambda = 0 = \kappa^{-1} \left( \frac{H}{L_w} \right)^{1-\epsilon^2}, \quad \omega_w \mid \lambda = 1 = \kappa^{-1} \left( \frac{H}{L_w + L_e} \right)^{1-\epsilon^2}.
$$

As in standard models of directed technical change (e.g., Acemoglu, 2002), the relationship between the skill premium and the relative supply of skill is increasing whenever $\epsilon > 2$. In intermediate cases where $\lambda \in (0,1)$, the effect of offshoring on the skill premium is generally non-monotonic and depends crucially on $\epsilon$ and $\alpha$. This can be seen by differentiating (27) with respect to $\lambda$:

$$
d \ln \omega_w \over d\lambda = \left( (1-\epsilon+\epsilon^\alpha) \frac{\partial \ln \hat{L}}{\partial \kappa} + \frac{1-\epsilon^\alpha}{1-\kappa} \frac{\partial \kappa}{\partial \lambda} \right)\partial \lambda.
$$

(28)

For low levels of offshoring, the efficiency effect ($\hat{L}$) is the dominant force. Focusing again on the case $\sigma > \epsilon$, the skill premium will increase with $\lambda$ for two reason. The first is the static effect presented in the previous section. The second reason is the induced SBTC which we have just discussed. For high levels of offshoring ($\lambda \to 1$), however, the efficiency effect disappears ($d \ln \hat{L} / d\alpha = 0$) and UBTC tends to lower wage inequality. Equation (28) then shows that the skill premium will fall with $\lambda$ whenever $\epsilon > 2$. In intermediate cases where $\lambda \in (0,1)$, the effect of offshoring on the skill premium is generally non-monotonic and depends crucially on $\epsilon$ and $\alpha$. This can be seen by differentiating (27) with respect to $\lambda$:

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$$

(28)

For low levels of offshoring, the efficiency effect ($\hat{L}$) is the dominant force. Focusing again on the case $\sigma > \epsilon$, the skill premium will increase with $\lambda$ for two reason. The first is the static effect presented in the previous section. The second reason is the induced SBTC which we have just discussed. For high levels of offshoring ($\lambda \to 1$), however, the efficiency effect disappears ($d \ln \hat{L} / d\alpha = 0$) and UBTC tends to lower wage inequality. Equation (28) then shows that the skill premium will fall with $\lambda$ whenever $\epsilon > 2$. In intermediate cases where $\lambda \in (0,1)$, the effect of offshoring on the skill premium is generally non-monotonic and depends crucially on $\epsilon$ and $\alpha$. This can be seen by differentiating (27) with respect to $\lambda$:

$$
\frac{d \ln \omega_w}{d \lambda} = \left( (1-\epsilon+\epsilon^\alpha) \frac{\partial \ln \hat{L}}{\partial \kappa} + \frac{1-\epsilon^\alpha}{1-\kappa} \frac{\partial \kappa}{\partial \lambda} \right)\partial \lambda.
$$

(28)

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$$

(28)
Figure 4: Offshoring and the Skill Premium. The figure shows the relationship between offshoring (κ) and the skill premium in the West (ω) for endogenous technology (solid) and exogenous technology (dashed). The main parameters are ε = 1.6, σ = 3.33 and the others are described in Section 5.

Figure 4 shows the relationship between the skill premium and offshoring for ε = 1.6 and σ = 3.33. The monotonic schedule (dashed line) is obtained by holding technology constant at the autarky level, while the black line represents the BGP skill premium with endogenous skill bias. As the figure makes clear, the endogenous reaction of technology provides a strong amplification of the impact of offshoring on the skill premium for low levels of integration, while this effect is reverted for high levels of offshoring.\(^{18}\) Thus, the combination of offshoring together with directed technical change can explain a large surge in the skill premium even for low levels of trade between the West and the East.

Finally, let us turn to the effect of offshoring on innovation and growth in the long run. The Euler equation for consumption, \(g = r - \rho\), links the BGP interest rate and growth rate. The interest rate is uniquely pinned down by the free-entry condition for innovation, \(r = \pi_h/\mu\). Substituting for \(\pi_h\) from (14) and using (3), (4), (11), (13) and (22) yields:

\[
r = \frac{1 - \alpha}{\mu} \left\{ \left[ \frac{1}{L_w^{1-\alpha}} \left( L_w + \lambda^{1/\alpha} L_w \right)^{\alpha-1} + (\lambda H_w)^{\epsilon-1} \right] \right\}^{\frac{1}{1-\epsilon}}. \tag{29}
\]

Since \(d\bar{L}/d\lambda > 0\), an increase in offshoring opportunities (λ) increases the BGP interest rate and thus the BGP growth rate. Usual arguments then establish that consumption, \(Y, Y_h\),

\(^{18}\)The pattern presented in Figure 3 also suggests that the amplification effect would be even stronger for higher, but still plausible, values of σ.
\(Y_t, A_h, \text{and } A_l\) all grow at the common rate \(g\), which is strictly positive provided that \(\rho \sigma \mu < \min\{L_w, ZH_w\}\).

The next proposition summarizes the main results of this section (proof in the text).

**Proposition 2** Suppose \(\sigma > \epsilon > 1\) and \(\rho \sigma \mu < \min\{L_w, ZH_w\}\). Then there exists a unique BGP with growth \(g > 0\). In this BGP, an increase in offshoring opportunities, parameterized by \(\lambda\):

- increases the offshoring rate, \(\kappa\);
- increases the equilibrium interest rate, \(r\), and the growth rate, \(g\);
- induces a hump-shaped response in the skill bias of technology [i.e., it induces SBTC, or higher \(A_h/A_l\), for low initial \(\lambda\), and UBTC, or lower \(A_h/A_l\), for high initial \(\lambda\)];
- increases the skill premium, \(\omega_w\), if \(\frac{\sigma}{\sigma - 1} > \epsilon\), and induces a hump-shaped reaction in the skill premium, \(\omega_w\), if \(\epsilon > \frac{\sigma}{\sigma - 1}\) [i.e., it increases \(\omega_w\) for low initial \(\lambda\), and decreases \(\omega_w\) for high initial \(\lambda\)];
- reduces the wage gap between unskilled workers in the East and in the West, \(w_{1,e}/w_{1,w}\).

This proposition thus summarizes the rich pattern of interactions between offshoring and the endogenous direction of technology. Greater offshoring opportunities first induce SBTC and tend to raise wage inequality, but will ultimately induce UBTC and, in this case, will limit or even reduce inequality. Figure 4 above, which also plots the relationship between offshoring and wage inequality with endogenous technology, illustrates that the non-monotonic relationship is indeed driven by the endogenous response of technology.

Proposition 2 is stated under the assumption that \(\sigma > \epsilon\) which we argued above to be the empirically relevant case. For completeness, we state the analogous results in Proposition 4 for the case of \(\sigma < \epsilon\) in the Appendix. As discussed in the text, the main differences are that, in the low-\(\sigma\) case, an increase in offshoring opportunities necessarily induces UBTC, and generates either a U-shaped response or a monotonically decreasing response in the skill premium.

## 4 TRANSITIONAL DYNAMICS

In this section, we consider the implications of a small (unexpected) increase in offshoring opportunities (henceforth, an *offshoring shock*) on the entire equilibrium path of the economy.

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19 Since in BGP, \(V_t = V_h = \mu\) and \(V_t^o = \mu + f\) (from (20)), the transversality condition (2) becomes

\[
\lim_{t \to \infty} \exp \left( - \int_0^t r_s ds \right) \left( A_{h,t} \mu + A_{l,t} (\mu + \kappa f) \right) = 0.
\]

As \(A_l\) and \(A_h\) grow at the rate \(g\), and \(r = \rho + g > g\), this is automatically satisfied in the unique candidate BGP.
To simplify the discussion, we assume that before the shock the economy is in the BGP. The shock may be caused by either an increase in $\lambda$ or a decrease in $f$. This shock increases the BGP offshoring rate, $\kappa$, and also impacts the skill bias of technology as indicated in Proposition 2. The next proposition characterizes the transitional dynamics of $\kappa$, $A_h$ and $A_l$. A formal proof and a complete characterization of the dynamical system is provided in the Appendix.

**Proposition 3** Suppose $\sigma > \epsilon$, the economy is initially in the pre-shock BGP, and there is a (positive) offshoring shock at time $t = 0$. Then, the dynamic equilibrium path converges in finite time to a new BGP with a higher offshoring rate. Moreover:

(i) If $\lambda < \tilde{\lambda}$ (where $\tilde{\lambda}$ defined in (26)), then the offshoring shock induces a two-stage transition whereby, for some $T$ and $\bar{T}$ such that $0 < T < \bar{T} < \infty$, we have: (stage 1) $\dot{\kappa}_t > 0$, $\dot{A}_{l,t} = \dot{A}_{h,t} = 0$ for all $t \in [0,T]$; (stage 2, SBTC) $\dot{\kappa}_t > 0$, $\dot{A}_{h,t} > 0$, and $\dot{A}_{l,t} = 0$ for all $t \in [T,\bar{T}]$. The economy attains the new BGP at $t = \bar{T}$. In new BGP, the technology is more skill biased, i.e., $A_h/A_l$ is higher, than in the initial BGP.

(ii) If $\lambda > \tilde{\lambda}$, then the offshoring shock induces a two-stage transition such that for some $T$ and $\bar{T}$ (0 < $T < \bar{T} < \infty$), we have: (stage 1) $\dot{\kappa}_t > 0$, $\dot{A}_{l,t} = \dot{A}_{h,t} = 0$ for all $t \in [0,T]$; (stage 2, UBTC) $\dot{\kappa}_t = 0$, $\dot{A}_{h,t} > 0$, and $\dot{A}_{l,t} > 0$ for all $t \in [T,\bar{T}]$. The economy attains the new BGP at $t = \bar{T}$. In the new BGP, the technology is less skill biased, i.e., $A_h/A_l$ is lower, than in the initial BGP.

Upon impact, the increase in $\lambda$ triggers a wave of offshoring investments, which in turn causes a discrete increase in the interest rate. The initial stage of the transition which goes on over to interval $[0,T]$ (stage 1) features a continuous increase in $\kappa$ (hence, $V_l^o - V_l = f$) but no innovation. The intuition for why innovation is temporarily paused is as follows: offshoring opportunities cause the interest rate to jump up, and at this higher interest rate, we have both $V_h < \mu$ and $V_l < \mu$, making innovation unprofitable. Nevertheless, the increasing offshoring rate ultimately restores the profitability of innovation, so that either $V_h \leq \mu$ or $V_l \leq \mu$ starts holding again with equality, at which point innovation restarts.

Which type of innovation is restored at this point depends on the initial $\lambda$. As discussed above, if offshoring was initially low (i.e., if $\lambda < \tilde{\lambda}$), the price effect dominates the market size effect, and the shock triggers SBTC. More formally, in the second stage of the transition we have $V_l^o - V_l = f$, $V_h = \mu$ and $V_l < \mu$, and consequently, there is both offshoring and high-skill innovation, but no low-skill innovation. Over time, the price adjustment reduces the gap between $\pi_h$ and $\pi_l$, and the economy eventually attains the new BGP where low-skill innovation is also restored. On the contrary, if offshoring was initially high ($\lambda > \tilde{\lambda}$), the market size effect dominates the price effect, and the shock triggers UBTC in the second stage of the transition (more formally, $V_l^o - V_l = f$, $V_l = \mu$ and $V_h < \mu$). Note that in this case $\kappa$ reaches

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20The proposition discusses the effect of small changes in $\lambda$. When $\lambda < \tilde{\lambda}$, a large increase in $\lambda$ could take us above $\tilde{\lambda}$ and would thus have ambiguous effects (in particular due to the inverse-U shaped relationship between $\lambda$ and $A_h/A_l$ as shown in equation (24)).
the new BGP level already at the end of the first stage of the transition. During stage 2, offshoring continuous but the offshoring rate, $\kappa$, remains constant.

Changes in the offshoring rate and technology affect wages in both the West and the East. During the first stage, characterized by no innovation and a steep increase in offshoring, the wage differential between low-skill workers in the West and East shrinks. The absolute level of the low-skill wage in the West also typically falls during this stage.\(^{21}\) The wage of skilled workers, instead, goes up. During the second stage, the wage dynamics depend on the nature of technical change as described in Proposition 2.

![Figure 5: Transitional dynamics after an increase in $\lambda$. The figure shows the response of high-skill wages (top), low-skill wages in the West (medium) and low-skill wages in the East (bottom). Panel a, on the left, shows the response of wages to a shock that induces SBTC (low initial $\kappa$). Panel b, on the right, shows the response of wages to a shock that induces UBTC (high initial $\kappa$). Dashed lines show the corresponding no-shock counterfactual.](image)

Figure 5 shows the transitional dynamics of wages in two numerical cases corresponding to low (Panel a) and high (Panel b) initial offshoring, respectively.\(^{22}\) In particular, it shows

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\(^{21}\) In principle, there may exist a range of initially low offshoring such that all wages in the West increase as offshoring increases. However, when condition (17) is satisfied, offshoring decreases the low-skill wage at all initial levels of offshoring.

\(^{22}\) The parameter choices for these figures are discussed in the next section below.
how the wages of the different types of workers (from top to bottom, high- and low-skill in the West, and low-skill workers in the East) evolve over time during the transition relative to the counterfactual wage dynamics under no shock (dashed lines). The high-skill wage is in both cases higher than in the no-shock counterfactual throughout the whole transition. The low-skill wage in the West exhibits, in both cases, a U-shaped transitional dynamics. In Panel a where the second stage of the transition \( t \in [T, \bar{T}] \) is characterized by SBTC, the low-skill wage remains below the no-shock counterfactual throughout the whole transition. In Panel b where the second stage is characterized by UBTC, it surpasses the no-shock counterfactual at the end of the transition. Finally, the low-skill wage in the East is significantly higher with the offshoring shock than without.

In all cases we know from Proposition 2 that the new BGP has a higher growth rate, and this implies that all workers will earn higher wages in a sufficiently far future. In consequence, low-skill workers in the West may face a trade-off between short-run wage losses and long-run wage gains. The welfare consequences of the increase in offshoring and the resolution of this trade-off are discussed in the next section.

5 Welfare Analysis

In this section, we study the welfare effects of the offshoring shock discussed in the previous section. We compute the discounted utility of different types of agents. Using the Euler equation (1), agent \( i \)'s discounted utility evaluated at time \( t = 0 \) can be written as:

\[
U_{i,0} = \int_0^\infty e^{-\rho t} \ln C_{i,t} dt = \frac{\ln C_{i,0}}{\rho} + \int_0^\infty e^{-\rho t} \left( \int_0^t r_s ds - \rho t \right) dt. \tag{30}
\]

The initial consumption, \( C_{i,0} \), can be found by combining the Euler equation and the lifetime budget constraint:

\[
C_{i,0} = \rho \left( \int_0^\infty w_{i,t} \exp \left( - \int_0^t r_s ds \right) dt + a_{i,0} \right),
\]

where \( w_{i,t} \) is agent \( i \)'s wage and \( a_{i,0} \) is the value of his asset holdings at \( t = 0 \). The welfare effect of the offshoring shock can be decomposed into an impact effect, i.e., the instantaneous jump in the level of consumption \( C_{i,0} \), and a growth effect, including both the effect of the change in the growth rate during transition and on the new BGP. The offshoring shock affects \( C_{i,0} \) by changing both the present value of wages and the value of the initial assets. To understand the latter, note that the only assets in positive net supply in the economy are claims to the profit flow of existing firms. The present value of firm \( j \) evaluated at time \( t = 0 \) is given by:

\[
V_{j,0} = \int_0^\infty \exp \left( - \int_0^t r_s ds \right) \pi_{j,t} dt.
\]

Along a BGP, \( V_{j,0} = \mu \). However, during the first stage of the transition where there is only offshoring and no innovation, we have \( V_{j,0} < \mu \), and so the offshoring shock causes a capital
loss to asset owners by increasing the world interest rate.

In the no-shock counterfactual, discounted utility at $t = 0$ along the old BGP would instead be given by $U_{i,0}^* = (\ln C_{i,0}^* + g_0^*/\rho)$, where $C_{i,0}^* = w_{i,t}^* + \rho a_{i,0}^*$ and starred variables denote BGP values assuming no offshoring shock.

Since there are no closed form solutions for the transitional wage and interest rate trajectories, in this section we rely on numerical analysis.\textsuperscript{23} We calibrate the model economy so as to be broadly consistent with some salient facts of the recent development of the global economy. We identify the West with the US and the East with China, the two largest economies among the industrializing and emerging markets as well as the most important actors in the process of globalization and technology offshoring. We normalize the size of the unskilled labor in the West to $L_w = 1$. The labor force of China is set to $L_e = 4.7$, to match the average relative size of the Chinese urban labor force over the last decade.\textsuperscript{24}

We set $H_w = 1.2$ so as to match the relative skill endowment (as measured by the share of workers with college degree or more) in the US in 2000. We set the elasticity of substitution between high- and low-skill workers to $\epsilon = 1.6$, consistent with the estimates reported by Ciccone and Peri (2005). Since the overwhelming majority of estimates of the elasticity of substitution between traded goods is greater than 3 (see footnote 16), we consider two possible values for this parameter: $\sigma = 3.33$ (corresponding to $\alpha = 0.7$) and $\sigma = 5$ (corresponding to $\alpha = 0.8$). Crucially, in both cases, we have $\sigma > \epsilon > \frac{\sigma}{\sigma-1}$, so the BGP responses of the skill premium to $\lambda$ is hump-shaped (see Proposition 2). Finally, we set $\rho = 0.04$ which, when combined with a 2% long-run growth rate, implies a rate of return on equity about 6%, which is in the ballpark of the return on equity in post-war US. The initial offshoring cost parameter, $f$, and the parameter $Z$ are set to match, respectively, the PPP-adjusted wage gap between Chinese and low skill US workers ($w_{l,e}/w_{l,w} = 0.16$), and the skill premium in the US ($\omega_w = 1.9$) in year 2000.\textsuperscript{25} This yields $\kappa_0 = 0.01$ and $Z = 1.65$ in the case of $\alpha = 0.7$, and $\kappa_0 = 0.0005$ and $Z = 1.49$ in the case of $\alpha = 0.8$. In order to generate the wage gap between the US and China, the model requires significant technological differences, which in the context of our model implies high offshoring costs (since offshoring closes the technology differences). The innovation cost, $\mu$, is chosen to ensure that the post-shock equilibrium annual BGP

\textsuperscript{23}Unless otherwise stated, the calibration described in this section was also used to produce all the figures in the paper.

\textsuperscript{24}The average size of the unskilled US labor force is 61 millions. This is derived from the total number of non-agricultural workers in the US, which is 135 millions (source: Current Population Survey). Of these, 61 millions are classified as unskilled (“high school graduates or less”) and 74 millions are classified as skilled (“some college or more”) workers. The average number of urban workers in China over the last decade is 286 millions (source: China Statistical Yearbook). Consistent with the model, we make the simplifying assumption that all Chinese workers are employed in the low-skill sector (see the next section for a generalization where we allow offshoring to the high-skill sector).

\textsuperscript{25}The wage gap is calculated using the ratio between the average wage in the US and the average urban wage in China (from the China Statistical Yearbook). This is adjusted to yield the ratio between the average Chinese urban wage and the low skill wage in the US (own calculation). The PPP is from the Penn World Table. The US skill premium is from Acemoglu and Autor (2010).
growth rate is equal to the average growth rate of the US economy between 1950 and 2010, approximately 2%. Motivated by the recent slowdown in the world growth rates, we also consider an alternative low-growth scenario where $\mu$ is larger and consistent with a 1% annual growth rate, close to the average growth rate of the US economy between 1995 and 2010 (all growth rates from the Penn World Table 7.1).

Finally, welfare effects also depend on the initial asset distribution. Since observing the exact asset holdings of different types of workers is challenging, for this exercise we assume that the initial share of world assets held by each group of workers is proportional to the present value of their wages in the initial BGP. This assumption implies that before the offshoring shock high- and low-skill workers in the West own, respectively, 56.5% and 24.8% of the world assets, while Chinese workers own 18.6% of world assets. For simplicity, we also set $\tilde{\lambda} = 1$ (full IPR protection) so that we do not have to consider profit stealing, and let the offshoring shock take the form of a reduction in $f$.

The size of the offshoring shock is chosen to generate an increase in the skill premium in the West comparable to the empirical observation for the US between 2000-08.\textsuperscript{26} Consistent with the results in Proposition 3, with both $\alpha = 0.7$ and $\alpha = 0.8$, the transitional dynamics feature a pure offshoring stage followed by a stage in which there is SBTC.

In Table 1, we report the effect of the offshoring shock on the relative wage in China ($w_{l,c}/w_{l,w}$), the growth rate ($g$) and on welfare of all workers, expressed as the equivalent change in their level of consumption in the old BGP ($\Delta c^*_{h,w}$, $\Delta c^*_{l,w}$ and $\Delta c^*_{l,e}$). The first four columns refer to the case of $\alpha = 0.7$, and show that the effect of the increase in offshoring on wages in China is quite significant. In all four cases, wage of Chinese workers relative to US unskilled workers grows from the initial level of 0.16 to 0.22. This is approximately 43% of the catch-up observed between 2000 and 2008, indicating that, according to our model, offshoring of Western technology to China accounts for close to half of Chinese wage growth during this recent period.

In column (1) we consider the benchmark 2% growth scenario, in which offshoring increases the BGP growth rate of the world economy from 1.8% to 2%. The last three rows of the table show that the shock has important distributional effects.\textsuperscript{27} In particular, Chinese workers gain considerably (+32%), followed by the skilled workers in the US (+10%). Unskilled workers in the West also gain, but only a modest +1.8%. In column (2) we consider the same experiment in the alternative low-growth scenario where offshoring increases the BGP growth rate of the world economy from 0.83% to 1%. In this case, all welfare gains are smaller and unskilled

\textsuperscript{26}Note that in order to focus on the implications of changes in offshoring in a transparent manner, we keep the relative supply of skilled workers constant in our simulations. Over the last decade the average educational attainment of the US labor force has grown from to $H_w/L_w = 1.2$ in 2000 to 1.37 in 2008, and there has been an increase in the skill premium from 1.9 to 2. We first compute, given our elasticity estimates, what the skill premium would have been had $H_w/L_w$ remained at 1.2, which is $\omega_w = 2.1$. We then choose the value of offshoring shock such that the skill premium increases to 2.1.

\textsuperscript{27}We refer to these as “distributional effects” since the fictitious world representative consumer would always benefit from increased offshoring.
workers in the US start to lose out. The fact that gains are smaller when the growth potential of the world economy is lower is a reflection of the long-run complementarity between innovation and offshoring we mentioned in the introduction. Recall that offshoring increases the BGP growth rate, and in addition, a high innovation potential speeds up the transition so that the long-run benefits from offshoring materialize faster.

The welfare results highlighted thus far are partly driven by the effect of offshoring on growth, which is arguably unrealistic. In columns (3) and (4), we neutralize the growth effect by changing simultaneously the cost of offshoring \((f)\) and of innovation \((\mu)\) so as to keep the BGP growth rate constant before and after the shock. This strategy can be viewed as a way to isolate the redistributive effects of technology, which is the main focus of the paper, while remaining agnostic on the determinants of long-run growth. It also captures the essence of models of semi-endogenous growth (e.g., Jones, 1995), as well as of recent models suggesting that offshoring may directly increase innovation costs due to, for example, coordination problems (as in Naghavi and Ottaviano, 2009). As columns (3) and (4) show, once growth effects are neutralized, the welfare gains of all agents are reduced, and can turn into significant losses for the unskilled workers in the US (-2.3% and -3.21% in the high- and low-growth scenarios respectively).

Columns (5)-(8) replicate the same exercises for the case \(\alpha = 0.8\). Overall, the results are very similar, although the effect of the shock on Chinese wages and welfare is slightly smaller (the wage gap of China increases from 0.16 to 0.21). As in the previous case, the Chinese gain the most, the US skilled workers also experience positive welfare gains, ranging from +7% in the most pessimistic scenario (low growth without growth effects) to +10.4% in the most optimistic scenario (high growth with growth effects), while unskilled workers in the US gain (marginally) only in the most optimistic scenario.

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<td>25.60%</td>
<td>25.70%</td>
<td>23.70%</td>
</tr>
</tbody>
</table>

Notes: shock calibrated to generate a change in the skill premium from 1.9 to 2.11

Table 1: Welfare Effects, 2000-08

We complete this section by presenting additional numerical simulations starting from higher levels of offshoring, i.e., higher levels of \(\kappa_0\). The goal of this exercise is to explore the
implications of the model in a wider range of parameters—not to claim that these scenarios are relevant for what the world economy has so far experienced. Nevertheless, if offshoring continues to grow, these scenarios might become relevant for the future evolution of the global economy. We focus on the benchmark calibration for $\alpha = 0.7$, and change the initial offshoring rate, $\kappa_0$, which yields a higher relative wage in China (28%, 40%, 50% and 60% of the unskilled wage in the US). In each experiment, the offshoring shock is chosen so as to generate an increase in the Chinese wage of 6 percentage points relative to the corresponding level in the US (i.e., to 34%, 46%, 56% and 66%), as it was the case in Table 1. Finally, for the reasons discussed above and to improve comparability across experiments, we neutralize any growth effect of offshoring by changing the cost of innovation so as to keep $g = 2\%$.

Table 2 reports the results of this exercise. It depicts the direction of technological progress along the transition (SBTC or UBTC), the Chinese wage gap ($w_{t,e}/w_{t,w}$) and the skill premium ($\omega$) both before and after the transition, and the welfare effect for all workers, expressed as again in consumption-equivalent changes ($\Delta c^*_h,w$, $\Delta c^*_t,w$ and $\Delta c^*_t,e$). Column (1) replicates column (3) in Table 1 as a benchmark for comparison. Column (2) shows that, when workers in China earn 28% of the Western wage initially, the offshoring shock induces SBTC, brings about a higher skill premium and hurts the unskilled workers in the US. Yet, when the initial wage in China is sufficiently high, as in columns (3)-(5), the shock induces UBTC, and this switch in the direction of technical change has important distributional implications favoring the unskilled workers in the US. Starting from high levels of offshoring, unskilled workers in the US start to gain (see columns (4) and (5)), and end up gaining even more than the skilled workers in column (5).

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$w_{t,e}/w_{t,w}$</td>
<td>0.16</td>
<td>0.28</td>
<td>0.40</td>
<td>0.50</td>
<td>0.60</td>
</tr>
<tr>
<td>$w_{t,e}/w_{t,w}$</td>
<td>0.22</td>
<td>0.34</td>
<td>0.46</td>
<td>0.56</td>
<td>0.66</td>
</tr>
<tr>
<td>$\omega_{t}$</td>
<td>1.9</td>
<td>2.3</td>
<td>2.57</td>
<td>2.71</td>
<td>2.75</td>
</tr>
<tr>
<td>$\omega_{t+T}$</td>
<td>2.11</td>
<td>2.45</td>
<td>2.66</td>
<td>2.74</td>
<td>2.74</td>
</tr>
<tr>
<td>$\Delta c^*_h,w$</td>
<td>6.14%</td>
<td>0.93%</td>
<td>0.64%</td>
<td>1.25%</td>
<td>1.83%</td>
</tr>
<tr>
<td>$\Delta c^*_t,w$</td>
<td>-2.32%</td>
<td>-4.19%</td>
<td>-2.10%</td>
<td>0.17%</td>
<td>1.96%</td>
</tr>
<tr>
<td>$\Delta c^*_t,e$</td>
<td>26.63%</td>
<td>11.86%</td>
<td>9.60%</td>
<td>9.60%</td>
<td>10%</td>
</tr>
</tbody>
</table>

Notes: $\sigma = 3.33$, $g=2\%$, no growth effects

Table 2: Welfare Effects, Counterfactuals

In conclusion, the results in this section suggest that the welfare effects of increased offshoring are highly asymmetric. For the realistic parameter values used in Table 1, our results suggest that low-skill workers in the West may lose even in a dynastic world (where discounted utility is the relevant criterion) with no credit market imperfections (that would prevent consumption smoothing). In models with short-lived, non-altruistic agents, or in models where agents cannot borrow against future wages, the welfare losses can become more pronounced.
An interesting implication of our analysis is that fostering innovation may be important at counterbalancing the negative distributional effects of offshoring because losses are less likely in the high-growth scenario. Finally, and perhaps more importantly, our analysis also shows that adverse distributional effects of offshoring may become less pronounced, or even subside, as the technological gap between China and the West declines—because of the change in the direction of technical change induced by offshoring.

6 Extensions

We now extend our benchmark model in two directions. First, we allow for offshoring in skill-intensive intermediates/tasks. Second, we allow Eastern firms to transfer technology from the West also by imitating Western technologies.

6.1 High-Skill Offshoring

To study the effect of offshoring skill-intensive intermediates, we assume that the East is endowed with $H^e$ units of skilled labor, but maintain that the West is skill abundant: $H^w/L^w > H^e/L^e$. For simplicity, we restrict the analysis to the BGP.

It is immediate to verify that, for given technology $(A_h, A_l)$ and offshoring rates $(\kappa_h, \kappa_l)$, the skill premia in the West and East are:

$$\omega_w = \left( \frac{ZA_h}{A_l} \right)^{1-1/\epsilon} \left( \frac{\hat{H}}{L} \right)^{-1/\epsilon} \left( \frac{\hat{H}}{L} \right)^{1-\alpha} \left( \frac{L^w 1 - \kappa_h}{H^w 1 - \kappa_l} \right)^{1-\alpha}$$

and

$$\omega_e = \left( \frac{ZA_h}{A_l} \right)^{1-1/\epsilon} \left( \frac{\hat{H}}{L} \right)^{-1/\epsilon} \left( \frac{\hat{H}}{L} \right)^{1-\alpha} \left( \frac{L^e \kappa_h}{H^e \kappa_l} \right)^{1-\alpha},$$

where $\hat{H} \equiv (\kappa^{1-\alpha} H^a + (1- \kappa)^{1-\alpha} H^a)^{1/\alpha}$. The comparative statics of changes in $(\kappa_h, \kappa_l)$ follows directly from the baseline case.

More interesting results can be derived when offshoring is endogenous. We start from the simpler case in which offshoring costs are the same in the two sectors. Then, the equilibrium offshoring rate is pinned down by the conditions $\lambda \pi_{l,e} = \pi_{l,w}$ and $\lambda \pi_{h,e} = \pi_{h,w}$. As in our benchmark model with only low-skill offshoring, the BGP profit gap between domestic and offshored firms must be equal to the offshoring cost, $1/\lambda$. Substituting in the expressions of profits yields:

$$\kappa_l = \left( 1 + \lambda^{-1/\alpha} L^w/L^e \right)^{-1}$$
$$\kappa_h = \left( 1 + \lambda^{-1/\alpha} H^w/H^e \right)^{-1}.$$

Since the East is skill-scarce, it is easy to see that the relative extent of offshoring, $\frac{\kappa_l}{\kappa_h}$, declines
monotonically from \( \frac{H_w}{L_w}/L_e \) to \( \frac{1+H_w}{1+L_w}/H_e \) as \( \lambda \) increases. Interestingly, offshoring is endogenously more prevalent in the low-skill sector. This is intuitive: the relative abundance of unskilled labor in the East induces Western firms to offshore production relatively more in the unskilled sector. As \( \lambda \) increases, however, offshoring increases relatively more in the lagging skilled sector. This pattern accords well with the available evidence: for example, the volume of trade in services, which are relatively skill-intensive, is lower than the volume of trade in intermediate products, but it has recently grown at a faster rate (World Trade Report, 2008).

Next, the indifference conditions between domestic and offshore production in both sectors imply that the international wage gap in both sectors is given by:

\[
\frac{w_{l,w}}{w_{l,e}} = \frac{w_{h,w}}{w_{h,e}} = \lambda^{\frac{\alpha+1}{\alpha}}.
\]

It follows immediately that the skill premium is the same in both countries. This is an important result: offshoring generates conditional factor price equalization, even if the two countries are fully specialized and have different technological capabilities. This result is driven by the assumption that the cost of offshoring is the same in both sectors, which in turn implies that the value of offshoring, which is proportional to the East-West wage difference, must also be equalized. This is accomplished by a higher offshoring rate in the unskilled sector, so as to increase the relative demand and hence the wage for unskilled workers in the East.

The BGP skill premium, \( \omega = \omega_w = \omega_e \), is now:

\[
\omega = Z^{\epsilon-1} \left( \frac{L}{H} \right)^{1-\epsilon+\epsilon_\alpha} \left( \frac{1 - \kappa_h L_w}{1 - \kappa_l H_w} \right)^{1-\epsilon_\alpha} \left( \frac{L_w + \lambda^{1/\alpha} L_e}{H_w + \lambda^{1/\alpha} H_e} \right)^{1-\epsilon_\alpha}.
\]

The fact that \( H_w/L_w > H_e/L_e \) implies that an increase in \( \lambda \) raises both terms in parenthesis. Intuitively, offshoring has a larger impact in the unskilled sector because the East has a relatively larger endowment of unskilled workers. It follows that the comparative statics to changes in \( \lambda \) are similar to the baseline case. In particular, depending on the elasticities \( \epsilon \) and \( \alpha \), the relationship between \( \lambda \) and \( \omega_w \) is still likely to be non-monotonic. Figure 6 plots the relationship between \( \omega \) and the extent of offshoring, \( \lambda \), using the calibration of Section 5. The graph shows both the previously studied case in which \( H_e = 0 \) (solid line) and the case in which 10% of workers in the East are skilled (dashed line). As the figure makes clear, adding high-skill offshoring does not change the qualitative relationship between the skill premium in the West and offshoring: the shape of the two lines is similar, with the only difference that, with a larger skill-endowment in the East, the effect of offshoring on the skill premium is smaller (the red line is below the black line). Interestingly, for sufficiently low levels of offshoring, a fall in offshoring costs raises skill premia both in the origin and destination countries. These predictions are broadly consistent with the evidence reported in Ge and Yang (2012), who find
that the college premium in China increased from around 1.3 in 1992 to more than 1.6 in 2007, and in Sheng and Yang (2012), who find that processing exports and FDI can account for 75% of the increase in the Chinese college wage premium between 2000 and 2006.

The results are easily generalized to the case in which offshoring costs are different in the two sectors. In this case, the BGP skill premium would also vary across locations. In particular, if the cost of offshoring was larger for high-skill jobs ($\lambda_h < \lambda_l$), then there would be less H-offshoring, resulting in lower demand for skilled workers in the East and a lower skill premium compared to the West:

$$\omega_e = \omega_w \left( \frac{\lambda_h}{\lambda_l} \right)^{1-\alpha}. $$

Overall, the generalized model of offshoring can explain why, despite its scarcity of skilled labor, the skill premium in China is lower than in the United States and why it has increased in both countries.

6.2 Imitation, Trade and Offshoring

So far, the only mode of technology transfer from West to East has been offshoring. In this section, we add the possibility for local firms in the East to imitate Western technologies. Imitation is modelled as an inferior form of technology transfer: the labor productivity for producing an intermediate is lower with imitation than under offshoring, for example because tacit knowledge of Western firms prevents perfect imitation. However, imitation entails no
payment of monopoly rents to the innovating firms in the West. We show that in this environment, two regimes emerge in equilibrium: as long as offshoring costs are sufficiently high, technology transfer occurs only through imitation. However, when offshoring costs become sufficiently low, offshoring starts prevailing and less productive local imitating firms gradually disappear.

More specifically, we assume that Eastern firms can copy existing intermediates at a small cost and become local monopolists. However, technology transfer via imitation is imperfect: imitated intermediates are produced with a worse technology, with labor productivity equal to \( \varphi < 1 \). There is free trade in final goods, \( Y_h \) and \( Y_l \). Intermediates can also be traded, but foreign trade entails a small flow cost to be paid independently of the quantity exported. As a result, trade in final goods will equalize prices in both countries and there will be no trade in individual intermediates.\(^{28}\)

To simplify the analysis further, in this extension we focus on the case where \( \varphi < \alpha \). Then, the monopoly price charged by a firm that offshores production to the East is lower than the marginal cost of a local imitator. In this case, offshoring would drive imitated intermediates out of business, and the equilibrium conditions of the benchmark model without imitation would continue to hold for offshored intermediates.

Let us now start with the benchmark without offshoring (but with imitation), which is similar to the world economy with trade but no IPR studied in Acemoglu (2003). Then Eastern firms will imitate all varieties, and there will be trade in \( Y_h \) and \( Y_l \) only. The relative (world) price of these goods will be:

\[
\frac{P_h}{P_l} = \left( \frac{A_h Z H_w}{A_l L_w + \varphi L_e} \right)^{-1/\epsilon} \tag{31}
\]

The skill bias of the technology is determined by the incentive to innovate in the West. The relative profitability of skill-complementary technologies is:

\[
\frac{V_h}{V_l} = \frac{\pi_{h,w}}{\pi_{l,w}} = \frac{P_h Z H_w}{P_l L_w}.
\]

Along the BGP, all types of innovations must be equally profitable, thus \( V_h = V_l \). This

\(^{28}\)The assumption of (small) trade costs, which is quite realistic, avoids complications arising from two producers being active in the same market. More formally, the equilibrium can be described by the following game. There are two producers (Eastern and Western monopolist) of the same variety. The Eastern producer has a technological disadvantage, but this is perfectly offset in equilibrium by a lower wage. The infinitesimal trade cost keeps the two market segmented. The Eastern producer knows that, if it paid the trade cost, it would enter a stage game in the Western market in which Bertrand competition would drive profits to zero. The same argument keeps the Western producer from entering the Eastern market. Therefore, in equilibrium, each producer serves the local market. See Acemoglu, Gancia and Zilibotti (2012) for details. Note that we could alternatively rule out Eastern export of varieties to the West by assuming that these would constitute a violation of IPR that are protected in the West.
condition combined with (31) yields BGP relative technologies as:

$$\frac{A_h}{A_l} = \left( \frac{ZH_w}{L_w} \right) \epsilon L_w + \varphi L_e L_H.$$  \hfill (32)

Intuitively, in a world with no offshoring, imitation affects the direction of technological progress in the West through the price effect—there is no market size effect because of lack of IPR. Better imitations (higher $\varphi$) leads to greater production of unskilled goods in the East and so to a higher relative price of skilled goods. This induces SBTC.

Now consider a reduction in offshoring costs that makes offshoring profitable. In this case, there would be a switch from a BGP with only imitation to one with pure offshoring. To first determine the condition for offshoring to start, note that offshoring will be profitable, starting from a BGP without offshoring, when

$$\lambda \pi_{l,e}^o - \pi_{l,w} \geq f,$$  \hfill (33)

where $\pi_{l,e}^o$ denotes the profit of an individual Western firm that deviates from a no-offshoring equilibrium and offshores production to the Eastern market. Such a deviating firm can pay Eastern workers a wage that is only a fraction $\varphi$ of the Western wage, and yet can use the state-of-the-art technology. $\pi_{l,w}$ is the equilibrium profit in the West under no offshoring, and $r = \pi_{l,w}/\mu$ is the corresponding BGP interest rate. Thus, condition (33) ensures that starting from the BGP with only imitation, offshoring will be profitable. Substituting for profits, (33) can equivalently be rewritten as:

$$\varphi \leq \lambda \frac{1 - \alpha}{\alpha}. $$  \hfill (34)

When condition (34) holds, starting from the BGP with only imitation, Western firms will find it profitable to offshore to the East. Let us now characterize the BGP that emerges after offshoring. The first important observation is that although in a BGP with offshoring only a fraction $\kappa$ of the varieties are offshored, there will be no imitation in the remaining intermediates. The reason is that all Eastern producers now face higher wages: though in the BGP without offshoring, the technological disadvantage of the Eastern producers was offset by the lower wages in the East—enabling local producers with imitated technology to be active in all markets—this is no longer the case with offshoring, and thus low-productivity imitators in the East can no longer survive when Eastern wages are pushed up due to offshoring. As a result, offshoring induces specialization: in the new BGP, the East will export the intermediates produced in the offshored sectors to the West, and the West will produce and export to the East the remaining intermediates. Inferior (imitated) technologies will be abandoned altogether.

The transitional dynamics are particularly interesting. Consider an increase in $\lambda$ triggering the transition from a BGP with only imitation to a BGP with offshoring. After the increase in $\lambda$, we will first have a period of offshoring in which, for reasons we have already discussed in Section 4, there will be no innovation. During this phase, offshoring will also push out low-
productivity imitating firms in the East. Interestingly, however, during this process, wages in the East do not increase until all low-productivity (imitator) firms have exited the market. This perfectly elastic behavior of wages in the East is central for encouraging offshoring. Thus, equilibrium dynamics take the form of rapid growth accompanied by the reallocation of workers from low-productivity firms to high-productivity firms with no wage growth. The intuition for this result is related to Song et al. (2011), who also provide evidence that this is a good description of the process of economic growth in China over the last three decades. Our economic mechanism is different however. In Song et al. (2011), wages are kept low due to the presence of inefficient state-owned enterprises and the credit market constraints slowing the expansion of private sector firms. Here wages are kept low due to imitators and the gradual equilibrium expansion of the offshored sector.

The transitional dynamics enter their second phase only when all low-productivity imitators have exited the market and at this point, wages in the East start growing again. In this second phase, the rest of the transition to BGP is identical to the dynamics described in Section 4: First, there is now a stage characterized by only offshoring. Increasing wages in the East reduce the incentive for offshoring. The first stage of the transition is over when either skilled or unskilled innovation becomes as profitable as offshoring. Then, the second stage of the transition starts, where both offshoring and one type of innovation coexist. Eventually, innovation is restored in both skilled and unskilled activities, and \( \kappa \) stays constant at its BGP level. Whether the second stage features SBTC or UBTC again depends on whether (24) is higher or lower than (32) evaluated at \( \varphi > \lambda^{\frac{1-\alpha}{\alpha}} \).

The extension described in this subsection also has interesting implications about IPR policies. In the pure offshoring regime, better IPR in the East corresponds to a higher \( \lambda \), and triggers technological convergence between the East and the West (and as we have emphasized, in most cases higher wage inequality in the West). The implications are reversed in the region where the equilibrium only involves imitation. In this case, stronger IPR, which correspond to a decrease in \( \varphi \) and thus limit the extent of imitation, also slow down technological convergence and SBTC. In consequence, in the model enriched with imitation, the impact of international IPR on technological convergence (and likely on wage inequality in the West) is non-monotonic.

7 Conclusions

Offshoring of jobs to low-wage countries and skill-biased technical change are among the most prominent and fiercely debated trends of the US labor market. This paper has shown how these

\[ \omega_w = Z \frac{P_h A_h}{P_l A_l} = (Z_H \frac{L_w}{L_w} + \varphi \frac{L_e}{H_w})^{\phi-1}, \]

which is increasing in \( \varphi \).
two phenomena are likely to be strongly interlinked— because of the impact of offshoring on the direction of technical change.

Our theoretical analysis provides several new insights on these interlinkages. Most importantly, we show that a decline in the cost of offshoring intermediates/tasks has in general ambiguous effects on the level of wages, the skill premium and the direction of technical change. Nevertheless, our analysis clearly identifies the contrasting effects and when some dominate the others. In particular, in the most plausible scenario, starting from an equilibrium with a low volume of offshoring, a decline in offshoring costs triggers a transition characterized initially by falling real wages for unskilled workers in the West and followed by rapid skill-biased technical change. These implications highlight why, in contrast to the conventional wisdom, offshoring could have a major impact on wage inequality even when the extent of trade and offshoring is limited. They also suggest that, despite leaving out several important determinants of wage inequality in the US, our model accords fairly well with the available evidence on US labor market trends of the 1980s and early 1990s. The implications of offshoring are very different, however, once its volume reaches a critical level: in this case, further offshoring will induce unskilled-biased technical change and a lower skill premium. This suggests that the future potential distributional effects of offshoring could be quite different than its past impact.

We also characterized the dynamics of wages and technology in the face of increasing offshoring. An interesting result here is that convergence to the new balance growth path equilibrium follows a stage in which offshoring halts innovation in the West, thus indicating that offshoring and innovation are substitutes in the short run. However, they turn out to be complements in the long run, in that profitable offshoring tends to increase the rate of innovation in the long run, and also the welfare implications of offshoring are more positive when the baseline rate of innovation in the economy is greater.

Our framework further permits an investigation of the welfare effects of offshoring. Workers in the East are the clear winners because offshoring enables them to benefit from better technology. Skilled workers in the West typically tend to benefit also (though the opposite could happen), and unskilled workers could end up worse off, even if the future wage growth is fully factored in and consumption smoothing is allowed without any credit constraints.

Finally, the tractable nature of our framework enables several extensions, two of which we have discussed. First, we studied offshoring of both skilled and unskilled intermediates, which naturally leads to a pattern in which there will be greater unskilled offshoring, but skilled offshoring will also take place, at least after a while. An interesting implication of this model is a form of conditional factor price equalization—from the profit maximization of offshoring firms. This result also implies that, in contrast to the standard Stolper-Samuelson theorem, globalization can lead to higher skill premia even in skill-scarce countries. Second, we investigated the transition of the East from low-productivity imitation to higher-productivity offshoring, which leads to a pattern of transition reminiscent of the Chinese process of economic growth over the last three decades, with productivity gains from reallocation associated with
no wage growth.

A Appendix

A.1 Proof of Proposition 1

The effect of $\kappa$ on $\omega_w$ follows from (15) as discussed in the text. To establish the effect of $\kappa$ on $w_{l,w}$, differentiate (16) to obtain:

$$
\frac{d \ln w_{l,w}}{d\kappa} = \eta \frac{d \hat{L}}{d\kappa} - \frac{1 - \alpha}{1 - \kappa},
$$

where $\eta \equiv \frac{1 - \alpha + (1 - \alpha - 1/\epsilon)(A_hZH_w/A_l\hat{L})^{\epsilon-1}}{L + \hat{L}(A_hZH_w/A_l\hat{L})^{\epsilon-1}}$. As $\kappa \to \bar{\kappa}$, $\frac{d \hat{L}}{d\kappa} \to 0$, hence, $w_{l,w}$ decreases unambiguously with offshoring at high level of $\kappa$. As $\kappa \to 0$, $\frac{d \hat{L}}{d\kappa} \to \infty$, hence, the sign of the effect depends on the sign of $\eta$. Note that $\eta$ is positive if $\sigma < \epsilon$ (i.e., $1 - \alpha > 1/\epsilon$). However, if $\sigma > \epsilon$ (i.e., $1 - \alpha < 1/\epsilon$), then $\eta$ is negative provided that

$$
\frac{A_hZH_w}{A_l\hat{L}} > \left(\frac{(1 - \alpha) \epsilon}{1 - \epsilon (1 - \alpha)}\right)^{\frac{\epsilon}{\epsilon - 1}}.
$$

Since $\lim_{\kappa \to 0} \hat{L} = L_w$ and using (15) and $\sigma \equiv 1/(1 - \alpha)$, this condition can be rewritten as (17) in the main text. The real wages of all other workers are $w_{h,w} = \alpha Zp_{h,w}$ and $w_{l,e} = \alpha p_{l,e}$. Using (4), (6), (7), (9), (11) and (13) yields:

$$
w_{h,w} = \alpha \left(\frac{Y}{A_hZH_w}\right)^{\frac{1}{\epsilon}} ZA_h, \text{ and}
$$

$$
w_{l,e} = \alpha \left(\frac{Y}{A_l}\right)^{\frac{1}{\epsilon}} A_l\hat{L}^{1-\alpha - 1/\epsilon} \left(\frac{\kappa}{L_e}\right)^{1-\alpha}.
$$

Both $w_{h,w}$ and $w_{l,e}$ are increasing in $\kappa$ since $\frac{d \hat{L}}{d\kappa} > 0$, and because, as can be readily verified, $\hat{L}^{1-\alpha - 1/\epsilon} \kappa^{1-\alpha}$ is increasing in $\kappa$.

A.2 Statement of Proposition 2 for $\sigma > \epsilon$

**Proposition 4** Suppose $\sigma < \epsilon$ and $\epsilon > 1$. Then the BGP is unique and in the BGP, an increase in offshoring opportunities, parameterized by $\lambda$:

- increases the offshoring rate, $\kappa$;
- increases the equilibrium interest rate, $r$, and the growth rate, $g$;
- reduces the skill bias of technology \( (A_h/A_l) \);
- decreases the skill premium, \( \omega_w \), if \( \frac{\sigma}{\sigma-1} < \epsilon \); induces a U-shaped reaction in the skill premium, \( \omega_w \), if \( \frac{\sigma}{\sigma-1} > \epsilon \) [i.e., it decreases \( \omega_w \) for low initial \( \lambda \), and increases \( \omega_w \) for high initial \( \lambda \)];
- reduces the wage gap between unskilled workers in the East and in the West, \( w_{l,e}/w_{l,w} \).

A.3 Proof of Proposition 3

A.3.1 Preliminary results

Lemma 1 Suppose there are no unanticipated shocks for all \( t \geq s \), and at \( t = s \), \( V_z = \mu \), with \( z = \{h,l\} \). Then \( V_z = \mu \) for all \( t > s \). Similarly, if at \( t = s \) we have \( V_l^\mu - V_l = f \), then \( V_l^\mu - V_l = f \) for all \( t > s \).

Proof. If \( V_z = \mu \) at \( t = s \), but \( V_z < \mu \) later, then it would imply an anticipated capital loss, violating (18) or (19).

Lemma 2 The conditions \( V_l = V_h = \mu \) and \( V_l^\mu - V_l = f \) are both necessary and sufficient for the economy to be in a BGP.

Proof. \( V_h = V_l = \mu \) and \( V_l^\mu - V_l = f \) are simultaneously satisfied only for unique value of \( \kappa \), which in turn defines \( A_h/A_l \) uniquely.

Let us also define

\[ r_{off} = \left( \frac{\lambda \pi_{l,e} - \pi_{l,w}}{f}, \frac{\pi_{h,w}}{\mu}, \frac{\pi_{l,w}}{\mu} \right) \]

Here \( r_{off} \) is the equilibrium interest rate when there is positive offshoring (it follows from \( V_l^\mu - V_l = f \)); \( r_h \) is the equilibrium interest rate when there is positive technical change in the skilled sector (it follows from \( V_h = \mu \)); \( r_l \) is the equilibrium interest rate when there is positive technical change in the unskilled sector (it follows from \( V_l = \mu \)).

Finally, note that, as in Caselli and Ventura (2000), aggregation holds in the world economy, since agents have CRRA utility, and capital markets are perfectly integrated (i.e., agents can invest their savings in both local and foreign assets). Thus agents only differ by their wealth and labor endowments.

A.3.2 General characterization

Given no uncertainty, no arbitrage implies that \( r(A_{h,t}, A_{l,t}, \kappa_t) = \max \{ r_{off}, r_h, r_l \} \). In a BGP, \( r_{off} = r_h = r_l \) (see equation (21)). The world equilibrium path can then be described by the
following system of differential equations:

\[
\frac{\dot{C}_t}{C_t} = r \left( A_{h,t}, A_{l,t}, \kappa_t \right) - \rho \
\]
\[
\mu \dot{A}_{h,t} + \left( \mu + \kappa_t \right) \dot{A}_{l,t} + f A_{l,t} \dot{\kappa}_t = Y \left( A_{h,t}, A_{l,t}, \kappa_t \right) - C_t
\]

(35) (36)

with boundary conditions given by \( \kappa_0, A_{h,0} \) and \( A_{l,0} \) at \( t = 0 \) and the transversality condition (2). Here \( C \) is the consumption of the world representative agent, and \( Y \) is the world GDP, defined as

\[
Y \left( A_{h,t}, A_{l,t}, \kappa_t \right) = \left( 1 + \frac{A_{l,t} \hat{L} \left( \kappa_t \right)}{A_{h,t} Z H_w} \right)^{\frac{1}{1+\alpha}} A_{h,t} Z H_w,
\]

where, recall, \( \hat{L} \left( \kappa_t \right) = \left[ \kappa_t^{1-\alpha} L_e^\alpha + \left( 1 - \kappa_t \right)^{1-\alpha} L_w^\alpha \right]^{1/\alpha} \).

Consider now the impact effect of a (positive) offshoring shock. Since \( \pi_{l,e}, \pi_{l,w} \) and \( \pi_{h,w} \) (and, hence, \( r_h \) and \( r_l \)) are not affected by changes in either \( f \) or \( \lambda \), while \( r_{off} \) increases if either \( f \) falls or \( \lambda \) increases, then, upon the shock, the following condition must hold:

\[
r \left( A_{h,t}, A_{l,t}, \kappa_t \right) = r_{off} > r_h = r_l.
\]

(37)

Lemma 1 guarantees that offshoring never stops for \( t > 0 \). Thus, for all \( t > 0 \), \( r \left( A_{h,t}, A_{l,t}, \kappa_t \right) = r_{off} \), implying that

\[
r \left( A_{h,t}, A_{l,t}, \kappa_t \right) = \left( \frac{Y \left( A_{h,t}, A_{l,t}, \kappa_t \right)}{A_{l,0} \hat{L} \left( \kappa_t \right)} \right)^{\frac{1}{1}} \frac{\left( 1 - \alpha \right)}{f} \left( \hat{L} \left( \kappa_t \right) \right)^{1-\alpha} \left( \frac{L_e}{L_e} \right)^{\alpha} \left( \frac{L_w}{1 - \kappa_t} \right)^{\alpha}.
\]

A.3.3 The first stage of the transition: pure offshoring

In the first stage of the transition, (37) implies that \( V_1^o - V_t = f \), \( V_h < \mu \) and \( V_l < \mu \). Then, the dynamical system, (35)-(36), simplifies to:

\[
\frac{\dot{C}_t}{C_t} = r \left( A_{h,0}, A_{l,0}, \kappa_t \right) - \rho \
\]
\[
f A_{l,0} \dot{\kappa}_t = Y \left( A_{h,0}, A_{l,0}, \kappa_t \right) - C_t
\]

(38) (39)

where \( \kappa_0 \) is pinned down by the pre-shock BGP condition, \( \kappa_0 = \left( 1 + \lambda_0^{-1/\alpha} L_w/L_e \right)^{-1} \). The assumption that the economy starts from a BGP further implies that

\[
A_{h,0} = \left( Z H_w \right)^{-1} \left( \kappa_0^{1-\alpha} L_e^\alpha + \left( 1 - \kappa_0 \right)^{1-\alpha} L_w^\alpha \right)^{\frac{1+\alpha}{\alpha}} \left( L_w + \lambda_0^{1/\alpha} L_e \right)^{-\alpha} A_{l,0}.
\]
Thus, for given \( A_{t,0}, A_{h,0} \) is uniquely pin down by the BGP requirement.

Next, we prove that the pure offshoring stage of the transition \((r_{off} > r_h \text{ and } r_{off} > r_l)\) must end in finite time, restoring positive innovation. Suppose, to obtain a contradiction, that this is not the case, so there is no innovation thereafter. First, we can rule out that (for any \( \varepsilon > 0 \)) \( r(A_{h,0}, A_{t,0}, \kappa_t) > \rho + \varepsilon \) for all \( t \). If this were true, \( C_t \) would grow unboundedly, which contradicts the fact that with no innovation \( Y(A_{h,0}, A_{t,0}, \kappa_t) \) is bounded (recall, in particular, that \( \kappa_t \leq \bar{\kappa} \), so continuous growth without innovation is not possible). This implies that, without innovation, the dynamical system must converge to a steady state with zero growth and with \( r(A_{h,0}, A_{t,0}, \kappa) = \rho \). But \( r_{h,t} > \rho \) throughout, since \( r_{h,0} > \rho \) and \( r_h \) is increasing in \( \kappa \), which is itself increasing along the transition path. This implies that at some point \( r(A_{h,0}, A_{t,0}, \kappa) = r_{h,t} \), triggering skill-biased innovations, and yielding a contradiction.

We have so far established that the post-shock dynamics cannot lead to a new BGP in which there is no innovation. Next, look at whether the stage of pure offshoring is followed by SBTC or UBTC. Note that, during the pure offshoring stage of transition, \( \frac{r_{l,t}}{r_{h,t}} = \left( \frac{A_{h,0}}{A_{t,0}} \right)^{\frac{1}{\epsilon}} (ZH_w)^{\frac{1}{\epsilon}} \left( \tilde{L}(\kappa_t) \right)^{1-\frac{1}{\epsilon}} \left( \frac{L_w}{1-\kappa_t} \right)^{\alpha} \).

In general, it is ambiguous whether \( r_{l}/r_{h} \) is increasing or decreasing in \( \kappa \). However, it is easy to establish that there exists \( \hat{\kappa} \in (0, \bar{\kappa}) \) such that (i) \( r_{l}/r_{h} \) is decreasing in \( \kappa \) for \( \kappa < \hat{\kappa} \); (ii) \( r_{l}/r_{h} \) is increasing in \( \kappa \) for \( \kappa \geq \hat{\kappa} \). This can be seen from the derivative:

\[
\frac{\partial \ln (r_{l}/r_{h})}{\partial \kappa} = \left( 1 - \alpha - \frac{1}{\epsilon} \right) \frac{\partial \ln \tilde{L}}{\partial \kappa} + \frac{\alpha}{1-\kappa}.
\]

By assumption, \( 1 - \alpha - 1/\epsilon < 0 \). Then, the result follows from the fact that \( \partial \ln \tilde{L}/\partial \kappa \) decreases monotonically from \( \infty \) at \( \kappa \rightarrow 0 \) to \( 0 \) at \( \kappa \rightarrow \bar{\kappa} \). In case (i), the pure offshoring stage is followed by a stage of the transition in which the equilibrium features offshoring and SBTC \((V_{l}^o - V_l = f, V_h = \mu \text{ and } V_l < \mu)\). In case (ii), the stage of pure offshoring is followed by a stage in which the equilibrium features offshoring and UBTC \((V_{l}^o - V_l = f, V_l = \mu \text{ and } V_h < \mu)\). The convergence to the new BGP must be studies separately for each of the two cases.

### A.3.4 Second stage of the transition: offshoring+factor biased technical change

**Case 1: SBTC** \((\kappa < \hat{\kappa})\) We start by pinning down the offshoring rate, \( \kappa^{SBTC} \), that triggers a switch from pure offshoring to SBTC+offshoring \((\tilde{\kappa}_t > 0, A_{h,t} > 0 \text{ and } A_{l,t} = 0)\). \( \kappa^{SBTC} \) is implicitly determined by the condition \( r_{off} = r_h \), which can be rewritten as

\[
\frac{A_{h,0}}{A_{l,0}} = \frac{\tilde{L}(\kappa^{SBTC})^{1-\epsilon+\alpha}}{(ZH_w)^{1-\epsilon}} \left[ \tilde{\lambda} \left( \frac{L_w}{\kappa^{SBTC}} \right)^{\alpha} - \left( \frac{L_w}{1-\kappa^{SBTC}} \right)^{\alpha} \right]^{-\epsilon} \left( \frac{f}{\mu} \right)^{\epsilon}.
\]

38
As proven above, $\kappa_t$ will attain $\kappa^{SBTC}$ in finite time. Let $T > 0$ denote the time in which SBTC+offshoring starts ($\kappa_T = \kappa^{SBTC}$). Note that $T$ can be determined by numerical integration. For all $t \geq T$, the condition $r_{off} = r_h$ must hold, and this yields

$$A_{h,t} = A_h(\kappa_t) = \frac{L_h}{(ZH_w)^{1-\epsilon}} \left[ \lambda \left( \frac{L_w}{\kappa_t} \right)^{\alpha} - \left( \frac{L_w}{1-\kappa_t} \right)^{\alpha} \right]^{-1} \left( \frac{f}{\mu} \right)^{\epsilon} A_{l,0}. \quad (40)$$

The equilibrium dynamics can therefore be expressed as:

$$\frac{\dot{C}_t}{C_t} = r(A_h(\kappa_t), A_{l,0}, \kappa_t) - \rho, \quad (41)$$

$$\left( \mu \frac{d}{d\kappa_t} A_h(\kappa_t) + f A_{l,0} \right) \dot{\kappa}_t = Y(A_h(\kappa_t), A_{l,0}, \kappa_t) - C_t, \quad (42)$$

for $t \geq T$, with the initial condition $\kappa_T = \kappa^{SBTC}$. Note that equation (40) allows us to reduce the number of state variables in the dynamic system to one.

Next, we show that this stage comes to an end (i.e., even the low-skill innovation is restored) in finite time. Suppose, to obtain a contradiction, that the SBTC+offshoring stage never ends. Since $\kappa \leq \hat{\kappa}$, (40) implies that $A_h(\kappa_t)$ and $Y(A_h(\kappa_t), A_{l,0}, \kappa_t)$ are bounded. Thus, the same argument used to prove that the stage of pure offshoring must end in finite time can be used to establish that (i) if the transition featuring SBTC+offshoring continued forever, then $r(A_h(\kappa_t), A_{l,0}, \kappa_t)$ would fall to $\rho$, and the economy would attain a steady state with zero growth; (ii) in converging to a steady state with zero growth, $r$ would decline sufficiently to trigger unskill-biased innovations, yielding a contradiction.

In summary, the argument above establishes that there exists $\tilde{T} < \infty$ such that, for $t \geq \tilde{T}$, $V_i^o - V_l = f$, $V_h = V_l = \mu$, and the economy attains the new BGP. Formally, one can add the terminal condition $\kappa_T = \left( 1 + \lambda^{-1/\alpha} L_w/L_c \right)^{-1}$ (where $\lambda$ is the after-shock offshorability index) to the system (41)–(42), and determine the finite time for such a switch $\tilde{T}$.

**Case 2: UBTC ($\kappa \geq \hat{\kappa}$)** In this case, the conditions $V_i^o - V_l = f$ and $V_l = \mu$ must hold simultaneously, i.e., $r_{off} = r_l$. But because this is the condition that determines the BGP level of offshoring, in this stage $\kappa$ must be at its (after-shock) BGP level (22). Since (22) only depends on exogenous parameters, in this stage there is offshoring, but $\kappa$ remains constant over time. The system of equations characterizing equilibrium simplifies then to

$$\frac{\dot{C}_t}{C_t} = r(A_{h,0}, A_{l,t}, \kappa) - \rho, \quad (43)$$

$$\left( \mu + f \kappa \right) \dot{A}_{l,t} = Y(A_{h,0}, A_{l,t}, \kappa) - C_t. \quad (44)$$
This is a system of autonomous differential equations in $C_t$ and $A_{l,t}$, with the initial condition $A_{l,T} = A_{l,0}$. It is straightforward to show, as in case 1, that this transition cannot go forever, since the technology features decreasing returns to $A_{l,t}$ (holding constant $\kappa$ and $A_h$), and thus $r$ would fall to $\rho$. However, this is impossible, and thus innovation in the skilled sector is restored in finite time. In fact, skill-biased innovation is restored as soon as $r_l = r_h$. This occurs at $t = \tilde{T}$ such that

$$A_{l,\tilde{T}} = A_{h,0} \left( \left( \tilde{L}(\kappa) \right)^{1-\alpha-1/\epsilon} \left( \frac{L_w}{1-\kappa} \right)^{\alpha} \left( ZH_w \right)^{1/\epsilon-1} \right)^\epsilon.$$  

Thereafter the BGP dynamics apply.

**A.3.5 Equilibrium consumption trajectories**

Full equilibrium dynamics can be characterized by solving for equilibrium consumption trajectory, and in particular for $C_0$. At $t = \tilde{T}$, consumption must be consistent with its BGP expression. In particular, the BGP expression of consumption yields

$$\frac{C}{A_l} = \left( \frac{Y (A_h, A_l, \kappa)}{A_l} - \mu g \left( 1 + \frac{A_h}{A_l} \right) \right),$$

$$\frac{C}{A_h} = \left( \frac{Y (A_h, A_l, \kappa)}{A_h} - \mu g \left( 1 + \left( \frac{A_l}{A_h} \right)^{-1} \right) \right),$$

where $g = \left\{ \tilde{L}^{1-\alpha} \left( L_w + \lambda^{1/\alpha} L_e \right)^{\alpha-1} + (ZH_w)^{\epsilon-1} \right\}^{-1} (1 - \alpha) / \mu - \rho$ and, recall, $A_h/A_l$, $Y/A_h$ and $Y/A_l$ are constant in a BGP. Then, in case $1 (\kappa \leq \tilde{\kappa})$,

$$C_{\tilde{T}} = \left( \frac{Y (A_h, A_l, \kappa)}{A_l} - \mu g \left( 1 + \left( \frac{A_l}{A_h} \right)^{-1} \right) \right) \times A_{l,0},$$

whereas, in case $2 (\kappa > \tilde{\kappa})$,

$$C_{\tilde{T}} = \left( \frac{Y (A_h, A_l, \kappa)}{A_h} - \mu g \left( 1 + \left( \frac{A_l}{A_h} \right)^{-1} \right) \right) \times A_{h,0}.$$

In addition, for all $t \leq \tilde{T}$, the time paths of $\kappa$, $A_{h,t}$ and $A_{l,t}$ are fully determine. Note, in particular, that in case 1 $A_{l,\tilde{T}} = A_{l,0}$, and in case 2 $A_{h,\tilde{T}} = A_{h,0}$, which yields the expressions for all other variables at time $\tilde{T}$ (in terms of the BGP expressions of $A_h/A_l$, $Y/A_h$ and $Y/A_l$).

Given the terminal conditions $\{C_{\tilde{T}}, A_{h,\tilde{T}}, A_{l,\tilde{T}}, \kappa_{\tilde{T}}\}$, the system of differential equation (41)-(42) in case 1 and (43)-(44) in case 2 can be integrated backwards to yield a solution for $\{C_T, A_{h,T}, A_{l,T}, \kappa_T\}$, where, recall, $T$ is the the endpoint of the first stage of the transition (pure offshoring). Likewise, one can use $\{C_T, A_{h,T}, A_{l,T}, \kappa_T\}$ as the terminal condition of the
first stage of the transition to integrate backwards the system of differential equations (38)-(39), and find a solution for the initial consumption, $C_0$, given the other initial conditions, $A_{h,0}, A_{l,0}, \kappa_0$. (Recall here that $A_{l,0}$ is an arbitrary initial condition, whereas $A_{h,0}$ and $\kappa_0$ are pinned down by the initial BGP equilibrium condition).

References


[38] Goel, Manisha (2012). “Does Offshoring Lift All Boats? The Role of Induced Technology Adoption and Innovation” mimeo.


