Competing for Loyalty: The Dynamics of Political Support*

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Abstract

We consider a class of dynamic collective action problems in which either a single principal or two competing principals vie for the support of members of a group. We focus on the dynamic problem that emerges when agents’ decisions to support an alternative are irreversible, and agents negotiate and commit their support to the principal sequentially. A danger for the agents in this context is that a principal may be able to poach agents to her side by exploiting competition among members of the group. Would agents benefit from introducing competition between opposing principals? We show that when the principals’ policies provide value to the agents, competition generally reduces agents’ welfare. We study applications to endorsements in elections and corporate takeovers.

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1 Introduction

A fundamental question in political economy is how groups decide about, and can be pressured into accepting, new ideas and proposals. Can a principal (a leader, lobbyist or partner) force upon the group an alternative that members of the group do not really want? Alternatively, when a principal brings something of value to the table, to what extent will members of the group be able to benefit from supporting change?

Consider for example the adoption of fracking operations in rural towns across the US. With technological advances that made gas extraction from deep reserves economically feasible, several energy companies were set to secure drilling rights on vast extensions of land, from New York to the Midwest. Because of a concern with potential environmental risks associated with fracking, voters and representatives in several rural towns have taken the issue of whether to allow fracking operations to a public vote in the local council or assembly. By the time the vote takes place, however, gas companies have already sent contractors out to private homes and farms to lease mineral exploration rights from landowners at terms negotiated between the parties, making these constituents partners of the fracking effort. The concern here is that even if fracking were to be environmentally risky, the gas company might be able to overcome attempts to block fracking operations in a public vote offering little compensation for potential environmental costs to only a majority of landowners/voters by exploiting competition between agents.

A key feature of this example is that landowners’ decisions are effectively irreversible: once a landowner chooses to lease her mineral rights, her fate is tied together with that of the company. To varying degrees, this feature is at the core of many significant problems in politics and economics. A party notable who publicly supports a presidential candidate cannot switch his support to the other candidate without cost. A firm that chooses between two alternative technologies (say Blu-ray or HD-DVD, Boeing or Airbus) can only switch technologies at a substantial cost.

The irreversibility of agents’ actions matters because it alters actors’ strategic considerations, introducing a dynamic dimension to the problem. This is because the terms of the deals that supporters can attain will differ depending on whether they commit their support early, when the fight is even, or late, when one candidate is already close to winning. In the context of the race to win the democratic nomination for President, for
example, a party notable will obtain a different amount of political leverage if he throws his supports behind a candidate in the early elections of Iowa or New Hampshire than if he does this after the candidate establishes a significant lead over her opponent after a big Super Tuesday win. Similarly, in the acquisition of mineral rights application, firms can offer different deals to different landowners depending on the number of contracts already signed in order to secure a winning coalition in the local assembly.

The issue here – and the danger for the agents in this environment – is that principals may be able to convince agents to give them their support by exploiting an intertemporal competition among members of the group: “take this now, otherwise I’ll find others to support me and you’ll get nothing”. In this way a principal would be able to force upon the group an undesirable alternative providing only a small compensation for the loss to some of its members.

Possibly the most likely candidate to protect followers from expropriation in this environment is the existence of competition between principals, each championing an alternative project. In some cases (a fracking company unmatched by organized interests in the town, the US Government assembling the “coalition of the willing” to invade Iraq) there is one clear feasible alternative to the status quo. In these cases, the monopolistic principal can attempt to influence the group to move forward offering transfers, effectively competing only with the status quo. In other cases, two distinct alternatives compete for the support of members of the group. Consider for example competition for influence between the United States and the Soviet Union during the cold war, internal party politics in a divided party, or the high definition optical disc format war between Sony’s Blu-ray and Toshiba’s HD DVD. Or suppose, in the context of our fracking example, that an interest group rises in opposition to the energy companies before the town votes.

Would agents benefit from introducing competition between opposing principals?

On the face of it, the answer to this question seems straightforward: since a single principal can exploit agents by picking the group apart, introducing competition must naturally increase agents’ welfare. We show, however, that this intuition can be misleading.

To study this problem, we consider the following model. There are two leaders and a group of \( n \) followers. In each period before an alternative is selected, an uncommitted follower meets with one of the leaders. The follower and leader who meet in each period are selected at random. Upon meeting, the leader can offer the follower the opportunity to commit

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his support to the leader, possibly in exchange of a transfer to or from the leader. If the follower accepts the offer, he commits his support for the leader and receives the transfer. If he rejects, he remains uncommitted. The first leader to obtain the commitment of a majority of the group implements her preferred alternative. Followers get a payoff of $w_\ell$ if and when alternative $\ell = A, B$ wins (zero before). We also consider the analogous model with a single leader.

As it turns out, a key consideration here is whether the principal’s policy provides value to the agents or not; i.e., whether $w_\ell \geq 0$ or not. The first case is what we call a public good environment. In this case agents would prefer the principal’s policy to the status quo. This is the case of a presidential candidate with high changes of winning the general election, a party line favored by backbenchers, alternative formats for the distribution of movies, or a value-creating raider attempting a corporate takeover. In the second, the principal leads to something that the agents do not like relative to the status quo. We call this a costly actions environment. This is the case of fracking, alliances for war efforts, or unpopular legislative policies.

We show that competition between principals generally improves agents’ welfare with costly actions, but reduces agents’ welfare with public goods. In fact, when two leaders stand behind equally good alternatives, agents are always better off when facing a single alternative than two alternatives. In general, with public goods, agents are always better off facing the best alternative by itself than two alternatives, even if the difference in the value of the two alternatives is arbitrarily small. Moreover, when the number of agents is sufficiently large, agents are better off even when they face the worst alternative by itself than when this alternative competes with an arbitrarily better alternative. With costly actions, instead, the results reverse, and competition is indeed beneficial.

The logic for this result is akin to that for under-provision of public goods. In the case of public goods, in fact, in equilibrium the agent actually pays the principal to move forward (agrees to share some of his surplus with the principal). Thus, by rejecting an offer, a follower can hope to free ride, relying on others to pay the bill. This outside opportunity gives each follower some bargaining power over the leaders. The key point to note is how this outside option changes with and without competition.

Consider the problem of the pivotal follower in each case. In the single alternative case, if the pivotal follower refuses an offer by the leader, there is a relatively high chance
that some other uncommitted follower will come to the negotiating table tomorrow and
the game will end. This relatively high free-riding ability gives the pivotal follower a
considerable bargaining power against the leader and protects him from getting fully
expropriated. Now consider the problem of a pivotal follower under competition. The
number of uncommitted followers with competing principals is at most as large as in the
single alternative case (if one principal was completely shut out) and is generally lower.
In fact, if the battle between the leaders is very competitive, a follower could be pivotal
when both leaders need only one additional commitment to win. But in this case there are
no free riding possibilities, and as a result, the pivotal follower has no bargaining power.
The example is extreme, but the logic is the same in all other cases in which a leader is
not shut out; competition between the leaders gives the pivotal follower fewer free-riding
possibilities, and results in a lower equilibrium payoff for the followers.¹

A concern in this context is the possible multiplicity of equilibria. In the single alternative
case, we show that there exists a unique MPE. But in the competitive case, in fact it is
possible that there are multiple equilibria. Are the results valid only for some selection
of equilibria? We show that the result holds for all MPE. To do this we do an induction
argument, exploiting general equilibrium properties.

From a substantive standpoint, this paper relates to two strands of literature. First,
the analysis of the monopolistic case has clear points of contact with the literature on
contracting with externalities (see for example Grossman and Hart (1980), Harrington
and Prokop (1993), Holmstrom and Nalebuff (1992), Neeman (1999)). In particular see
Segal (1999) and Genicot and Ray (2006). These papers focus on sequential contracting
problems with a single principal, and do not address the relationship between monopolistic
leadership and competition.² A second strand of connected literature is the literature on
vote buying (see for example Myerson (1993), Dixit and Londregan (1996), Groseclose
and Snyder (1996), Morgan and Várady (2011) and Dekel, Jackson, and Wolinsky (2008,
2009)). With the exception of Dekel, Jackson, and Wolinsky (2008, 2009)), these papers

¹The intuition for the reversal of the result with costly actions is the mirror image of the public good
case. Since in this case followers receive positive transfers from the principal/s in equilibrium, by refusing
an offer followers risk not receiving an offer at all (not being compensated for a change that will happen
anyways). Because this outside option determines their bargaining power vis a vis the principal, followers
are better off when the chances of this happening are smaller (i.e., under competition).

²See also coalitional bargaining games with externalities (Bloch (1996), Seidmann and Winter (1998),
Ray and Vohra (1999), Ray and Vohra (2001), Gomes (2005), Gomes and Jehiel (2005)). As in this
literature, and for similar reasons, equilibria in our model can be inefficient.
study static models, and therefore do not capture irreversible actions. Moreover, these papers do not study the comparison between competition and monopolistic leadership. A close reference is Dal Bo (2007), who shows that with pivotal contracts a single principal can induce the committee to vote for her preferred policy at a very low cost.

From a modeling standpoint, the paper builds on the sequential bargaining approach of Gul (1989) (see also Iaryczower and Oliveros (2013)) and the literature on races (Reinganum (1981), Reinganum (1982), Harris and Vickers (1985), Harris and Vickers (1987)).

2 The Model

Two leaders, A and B, compete to gather the support of a majority of members (followers) in a group of size \( n \). The first leader to obtain the commitment of \( q \equiv (n + 1)/2 \) members implements its preferred alternative. There is an infinite number of periods, \( t = 1, 2, \ldots \). In each period \( t \) before a leader won, any one of the \( k(t) \) uncommitted followers at time \( t \) meets leader \( \ell = A, B \) with probability \( \pi_\ell/k(t) \). Say that at the time of the meeting, \( \ell \) needs \( m_\ell \) additional followers to win. In the meeting with an uncommitted follower \( i \), leader \( \ell \) offers \( i \) an amount \( p_\ell(m_\ell, m_{-\ell}) \) to secure \( i \)'s support (possibly \(-\infty\)). Follower \( i \) can accept or reject \( \ell \)'s offer. If he accepts, he commits his support for \( \ell \) and receives \( p_\ell(m_\ell, m_{-\ell}) \). If he rejects the offer, \( i \) remains uncommitted.

Leader \( \ell \) gets a payoff of \( v_\ell > 0 \) if and when she wins, and \( v_j \) if and when her opponent \( j \neq \ell \) wins. \( v_\ell > v_j \). In any period before a leader wins, leaders get a payoff of zero (a normalization). Followers get a payoff of \( w_\ell > 0 \) if and when alternative \( \ell \) wins.\(^3\) As with leaders, we normalize followers’ payoffs in any period before an alternative wins to zero. Leaders and followers have discount factor \( \delta \in (0, 1) \). The solution concept is Markov Perfect Equilibrium (MPE). We let \( W(\vec{m}) \) denote the continuation value of an uncommitted follower in state \( \vec{m} \equiv (m_A, m_B) \), \( W_{out}(\vec{m}) \) denote the continuation value of a committed follower in state \( \vec{m} \), and \( V_\ell(\vec{m}) \) denote the continuation value of leader \( \ell \) in state \( \vec{m} \). It will also be useful to define \( \vec{m}^A \equiv (m_A - 1, m_B) \) and \( \vec{m}^B \equiv (m_A, m_B - 1) \).

As a benchmark, we also consider the case in which there is only one alternative to the status quo. The model is the same as before, with \( \pi_\ell = 0 \) for some \( \ell = A, B \). Because there is a single leader, the state is just the number \( m \) of additional followers the leader

\(^3\)In Section 5.1 we consider the case in which \( w_\ell < 0 \) for some \( \ell \in \{A, B\} \), possibly both.
needs to win. The price is $p(m)$, the value of an uncommitted follower $w(m)$, the value of a committed follower $w_{\text{out}}(m)$, and the leader’s value $v(m)$.

3 A Single Alternative

We begin by solving the model when there is a single alternative. For convenience, we denote the game with a single alternative and initial position $m$, $\Gamma^s(m)$. We show that in this case there is a unique MPE, in which the leader makes an offer in every meeting until she collects a majority of committed followers, and the uncommitted followers who meet the leader accept these offers. This result allows us to pin down the equilibrium payoff for a follower at the beginning of the game.

Let $\beta_0(m)$ denote the probability that a random follower meets the leader when the leader has to secure the support of $m$ additional followers. Note that in this case there are $n - 1 + 2m$ uncommitted followers, so $\beta_0(m) \equiv 2/(n - 1 + 2m)$.

Proposition 3.1 The game $\Gamma^s(q)$ has a unique MPE. In this equilibrium, the payoff of a follower is given by

$$\bar{w} \equiv w(q) = \left( \prod_{m=1}^{q} r(m) \right) \delta^q w \quad \text{for} \quad r(m) \equiv \frac{1 - \beta_0(m)}{1 - \delta \beta_0(m)}$$

Proof. The proof is in the Appendix. ■

The intuition for the proof can be seen in two steps. First, fix the proposed equilibrium. Since $v > 0$ and $w > 0$, when the leader needs to collect the support of only one additional follower ($m = 1$), the leader and the follower can create and capture a positive surplus by moving forward. Thus, given full information, there is a price at which this transaction occurs. Now consider the situation in which there are $m$ followers remaining. Since in equilibrium there is trade whenever the leader needs to secure the support of $t < m$ additional followers, then in state $m$ there is also a positive surplus for the leader and the selected follower to obtain if they move forward, and then again a price at which this happens. This shows that the proposed strategy profile is an equilibrium. By the same logic, in any equilibrium there must be a transaction when $m = 1$. So suppose
that in equilibrium there is trade whenever the leader needs to secure the support of 
$t < m$ additional followers. Recall that in state $m$, in the proposed equilibrium there 
is a positive surplus for the follower and the leader. Then if in state $m$ the leader does 
not make an offer with positive probability or the follower does not accept the offer with 
positive probability, leader and follower would obtain a lower payoff in this state, and 
thus the gain from moving forward would be higher. It follows that the leader will make 
an offer, which the follower will accept.

Proposition 3.1 implies that in equilibrium the leader cannot extract all surplus from the 
followers. The reason for this is similar to the logic behind under-provision of a public 
good. Note that since the followers benefit from implementing the alternative to the 
status quo when $w > 0$, the leader actually charges them to move on. By rejecting the 
offer, however, a follower can rely on others to pay the bill. The possibility of free riding 
others generates an outside option that gives each follower some bargaining power over 
the leader. Since the cost of deferring implementation of the proposal decreases with 
$\delta$, the value of the outside option is increasing in $\delta$, and so is the followers’ equilibrium 
payoff. In fact, as $\delta$ approaches 1, $r(m) \rightarrow 1$ and $w \rightarrow w$.

Note also that $w(m)/w(m-1) = r(m)\delta < 1$. Thus the equilibrium payoff of uncommitted 
followers increases as the leader gets closer to achieving the majority. This is not just due 
to the effect of being closer to completion. Note that the value of committed followers is 
given by $w_{out}(m) = \delta^m w$, so that $w_{out}(m)/w_{out}(m-1) = \delta$. It follows that the rate of 
growth of the value for uncommitted followers (as the leader gets closer to achieving a 
majority) exceeds that of the committed followers.

We illustrate these results with a simple example.

Example 3.2 Suppose $n = 3$, $w = 100$, and $\delta = 0.75$. Consider first the state $m = 1$, 
reached after exactly one follower already committed his support to the leader. Note that 
a follower meeting the leader in state $m = 1$ gives her support to the leader only if 
$p(1) + \delta w \geq \delta w(1)$. Then the leader offers $p(1) = -\delta[w - w(1)]$, in which case the 
follower gets $\delta w(1)$ independently of whether he accepts the offer or not. Thus the value 
of an uncommitted follower in state $m = 1$ is

$$w(1) = 0.5\delta w(1) + 0.5\delta w \Rightarrow w(1) = \left(\frac{0.5\delta}{1 - 0.5\delta}\right) w = 60$$

The value of a committed follower, on the other hand, is $w_{out}(1) = \delta w = 75$. Now consider
the initial state \( m = 2 \), where all three followers are uncommitted. As before, the payoff of a follower who is meeting the leader is \( \delta w(2) \), independently of whether he accepts the offer or not. Thus the value of a follower is

\[
    w(2) = (1/3)\delta w(2) + (2/3)\delta w(1) \Rightarrow w(2) = \frac{8}{9} \approx 53,
\]

\[\square\]

4 Competition and Main Result

Consider now the game with two alternatives A and B, and initial position \( \vec{m} = (q,q) \), \( \Gamma^c(q,q) \). By refusing to commit his support to \( \ell \) in state \( \vec{m} \), a follower \( i \) obtains two potential benefits. One, as before, is free riding: \( i \) can count on the possibility of not being called again to the negotiation table. Now, however, there is a second motive not to sell himself cheap to \( \ell \). By refusing to commit his support for \( \ell \), \( i \) is not only delaying a possible win by \( \ell \), but also making a win by her opponent more likely. This second element will be more attractive to \( i \) the better is the alternative championed by \( \ell \)'s competitor. But the value of competition will also depend on anticipated equilibrium behavior: when do leaders effectively compete for support, what do they offer when they do, how likely are they to win, and how fast will they win. In a given state \( \vec{m} \), and for any given anticipated play, the value of competition for a follower will depend on the marginal contribution of the follower to a leader in that state.

Consider an uncommitted follower \( i \), meeting a leader \( \ell = A,B \). Recall that we have defined \( \vec{m}^A \equiv (m_A - 1, m_B) \) and \( \vec{m}^B \equiv (m_A, m_B - 1) \). Follower \( i \) accepts an offer \( p_\ell(\vec{m}) \) from \( \ell \) if and only if \( p_\ell(\vec{m}) + \delta W_{\text{out}}(\vec{m}^\ell) \geq \delta W(\vec{m}) \). Thus in equilibrium, if \( \ell \) makes an offer, she offers

\[
    p_\ell(\vec{m}) \equiv -\delta[W_{\text{out}}(\vec{m}^\ell) - W(\vec{m})].
\]

The offer by \( \ell \) has to compensate \( i \) from the outside opportunities of refusing to commit his support for \( \ell \), taking him – along with the entire group – back to a position \( \vec{m} \). Leader \( \ell \) is willing to make this offer if \( p_\ell(\vec{m}) \leq \delta[V_\ell(\vec{m}^\ell) - V_\ell(\vec{m})] \), or substituting \( p_\ell(\vec{m}) \) from (1), if the surplus for \( i \) and \( \ell \) for making \( \ell \) one step closer to the goal is nonnegative; i.e.,

\[
    S_\ell(\vec{m}) \equiv [V_\ell(\vec{m}^\ell) - V_\ell(\vec{m})] + [W_{\text{out}}(\vec{m}^\ell) - W(\vec{m})] \geq 0.
\]
The positive surplus condition (2) for \( i \) with leader \( \ell \) is not always satisfied, of course. Under some conditions (e.g., if the game is too asymmetric) there is no mutually agreeable price at which the transaction can occur. On the other hand, given any parameter values, if the leaders’ value of winning is large enough the strategy profile in which all meetings result in transactions will be an equilibrium (Proposition 4.3 below). We begin our analysis of the competitive environment focusing on this case; i.e., we focus on a MPE of \( \Gamma^c(q,q) \) in which leaders make relevant offers in all states \( \vec{m} \). We call these equilibria fully competitive.

**Definition 4.1** Say that an equilibrium \( \sigma^* \) of \( \Gamma^c(q,q) \) is fully competitive if both leaders make relevant offers in all states \( \vec{m} \) such that \( m_A, m_B \leq q \).

We start our analysis of fully competitive equilibria analyzing the simplest possible example, where there is only one follower. In this situation the follower is pivotal, and can carry both A and B to a victory. However, because there are no other uncommitted followers, the follower cannot free-ride others. This effectively removes all his bargaining power.

**Example 4.2** Suppose \( n = 3 \). The initial game is therefore \( \Gamma^c(2,2) \) where both leaders need to obtain the commitment of two followers. We solve this game by backward induction. Consider the state \( \vec{m} = (1,1) \), reached after one follower has committed for each leader. From (1), leader \( \ell = A, B \) offers the follower \( p_\ell(1,1) = \delta[W(1,1) - w_\ell] \), which the follower accepts. Thus

\[
W(1,1) = \pi_A (p_A(1,1) + \delta w_A) + \pi_B (p_B(1,1) + \delta w_B).
\]

Substituting \( p_\ell(1,1) \), we find \( W(1,1) = 0 \); i.e., all surplus is extracted by the leaders.

Consider next the state \( \vec{m} = (1,2) \). As before, the prices are such that all incremental surplus is extracted, which implies that a follower meeting the leader gets a payoff \( \delta W(1,2) \), and thus

\[
W(1,2) = \frac{1}{2} \delta W(1,2) + \frac{1}{2} (\pi_A \delta w_A + \pi_B \delta W(1,1)).
\]

Note that the follower benefits in the event that he doesn’t meet the leader, in which case he free rides the other members, and obtains a payoff \( w_A \) (if A gets an extra commitment and wins) or moves to the state \( \vec{m} = (1,1) \) (if B moves forward). Since \( W(1,1) = 0 \), however,
the follower only profits if A wins, and we get $W(1, 2) = (\delta \pi_A / (2 - \delta)) w_A > 0 = W(1, 1)$. Proceeding in the same manner we have that $W(2, 1) = (\delta \pi_B / (2 - \delta)) w_B$ and

$$W(2, 2) = \frac{2}{3 - \delta} \left( \frac{(\delta \pi_A)^2}{2 - \delta} w_A + \frac{(\pi_B \delta)^2}{2 - \delta} w_B \right).$$

\[ \square \]

Note first that since $w(1) > 0$ from Proposition 3.1, the previous example implies that $W(1, 1) = 0 < w(1)$. Note also that $\lim_{\delta \to 1} W(2, 2) = (\pi_A)^2 w_A + (\pi_B)^2 w_B$, while in the monopolistic case we showed that $\lim_{\delta \to 1} w(2) = w$. Suppose without loss of generality that $w_A > w_B$. Since $(\pi_A)^2 w_A + (\pi_B)^2 w_B < w_A$, it follows that for $\delta$ close to one, followers are better off facing a single leader A than competition between A and B. Moreover, $\lim_{\delta \to 1} W(2, 2) < w_B$ if and only if $\frac{\pi_A}{1 + \pi_B} < \frac{w_B}{w_A}$. Thus, for some parameters followers are better off facing either A or B in isolation rather than competition between A and B. The example illustrates two general lessons. First, because $\delta \to 1$, the reason why followers are better off under single leadership than under competition is not rooted on the fact that under competition followers can take longer to select a winner. In particular it cannot be due to a destruction of surplus brought by delay. Second, because sometimes the worst leader is better for followers than two leaders, the reason behind the followers’ welfare ordering cannot be due to the “risk” of collectively ending up selecting the worst choice available.

Consider now a group of $n$ followers. Let $\beta(\bar{m})$ denote the probability that any given uncommitted follower meets with one of the leaders. Note if $\ell$ has to secure the support of $m_{\ell}$ more followers there are $(n + 1) - m_A - m_B$ committed followers, and $m_A + m_B - 1$ uncommitted followers. Then $\beta(\bar{m}) = 1/(m_A + m_B - 1)$. As in the examples, (1) implies that the expected payoff of a follower after meeting one of the leaders is $\delta W(\bar{m})$ independently of whether he accepts the proposal or not. This is a crucial property, for it allows us to decouple the system of partial difference equations for $W(\bar{m})$ and $V_\ell(\bar{m})$, $\ell = A, B$, considerably simplifying the task of solving this system. Then

$$W(\bar{m}) = \left( \frac{1}{m_A + m_B - 1} \right) \delta W(\bar{m}) + \left( \frac{m_A + m_B - 2}{m_A + m_B - 1} \right) \delta \sum_\ell \pi_\ell W(\bar{m}^\ell), \quad (3)$$

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so that letting $C(k) \equiv \frac{k-2}{k-(1+\delta)}$, we have

$$W(\vec{m}) = C(m_A + m_B) \delta \sum \pi_\ell W(\vec{m}^\ell)$$  \hspace{1cm} (4)

The coefficient $C(m_A + m_B) = \left(\frac{1-\beta(\vec{m})}{1-\beta(\vec{m})^B}\right)$ is a correction term that reflects the fact that a follower can only obtain a rent if he is able to free ride other followers.

Equation (4) is a partial difference equation with initial values $W(m_A, 0) = w_B$ for $m_A > 0$ and $W(0, m_B) = w_A$ for $m_B > 0$, which we can solve to obtain the general solution

$$W(\vec{m}) = \sum_{j=A,B} (\delta \pi_j)^{m_j} \times \left[ \sum_{l=0}^{m_j-2} \left( \prod_{k=0}^{m_j-1+l} C(m_{-j} + m_j - k) \right) \times \left( \frac{m_j - 1 + l}{m_j - 1} \right) \times (\delta \pi_{-j})^l \right] \times w_j,$$  \hspace{1cm} (5)

where we have adopted the convention that for any $f(\cdot)$, $\sum_{k=a}^b f(k) = 0$ whenever $b < a$.

Equation (5) has a straightforward interpretation. It reflects the value of free riding opportunities. This value depends on the path of play, and thus $W(\vec{m})$ must track the probability of alternative paths leading from one point to a terminal state. In addition, it includes the (state specific) correction terms $C(m_{-j} + m_j - k)$ to reflect the probability of successfully free-riding though the entire path.

To make this more clear, consider two possible paths leading to a win by B. Consider first the initial state $\vec{m} = (3, 4)$ and the terminal state $\vec{m} = (3, 0)$. These points are represented as red dots in the left panel of Figure 4. Because B wins, the relevant part of (5) is that for $j = B$ in the summation, leading to $w_B$. First, since B wins, we know it must take $m_B$ steps in the B direction. Thus the first part $(\delta \pi_B)^{m_B}$. Now, in general, from any interior point to a terminal point, there are various paths. In this case, however, there is only one history taking us from $(3, 4)$ to $(3, 0)$: offers by $(B, B, B, B)$. Thus $\ell = 0$ in the second summation, and $(m_{B-1})^B = 1$, to reflect that there is only one such path. The term $\prod_{k=0}^{m_B-1} C(m_A + m_B - k)$ then takes care of the probability of free-riding along that one path.

Now consider the same initial state $\vec{m} = (3, 4)$ but a different terminal node, $\vec{m} = (2, 0)$. Here A makes one move along the way, so $\ell = 1$. Since it takes $m_B$ moves by B and one move by A to arrive at the terminal state, we have $[(\delta \pi_B)^{m_B} \delta \pi_A]$. Moreover, differently than in the previous case, here there are several paths taking us from $(3, 4)$ to $(2, 0)$. The figure on the right shows the sequence of offers $(B, B, B, A, B)$, but there are three others:
(ii) (B,B,A,B,B), (iii) (B,A,B,B,B), (iv) (A,B,B,B,B).

Figure 4 plots $W(\cdot)$ in two examples, with $\pi_A = 0.5$ (left) and $\pi_A = 0.3$ (right) (in both cases $\delta = 0.99$, and $w_A = w_B = 100$). As it is apparent from the figure, the followers’ value function with competition is not monotonic. Thus, getting closer to implementing something is not necessarily better for followers, as in the single alternative case. In other words, staying uncommitted for longer does not necessarily improve a follower’s payoff. Say that $\ell = A, B$ is the leading alternative at $\vec{m}$ if $\ell$ is most likely to win at $\vec{m}$. Then going in the direction of $-\ell$ has a negative impact on $W(\cdot)$. In the left panel both leaders have the same probability of attracting followers, and are thus equally effective at moving forward. In this case, $\ell$ is the leading alternative if it had a more successful history. Thus $W(\cdot)$ decreases as we move toward the diagonal, eliminating the previous advantage. In the right panel, $\pi_B > \pi_A$, so the future looks better for $B$. Thus, starting from a balanced history, $W(\cdot)$ decreases as we move towards $A$. However, if the history of commitments is sufficiently unbalanced in favor of $A$ then the “good past” balances the “bad future” for $A$. In the example, a balanced situation is achieved at $\vec{m} = (3, 9)$. Moving away from this increases $W(\cdot)$.

There are only four histories because we are counting all sequences that go from $(3, 4)$ to $(2, 0)$ ending with a move by $B$. Thus we are focusing on the first four moves, and counting all sequences with three moves by $B$ and one move by $A$ leading to the last move by $B$. Thus we have $\binom{m_B}{m_A-1} = 4$. 

Figure 1: Two histories resulting in a win by $B$, from initial point $\vec{m} = (3, 4)$.
Having obtained the expression for $W (\vec{m})$ in terms of the fundamentals, we can write down equilibrium prices. Note that once the follower is committed, all strategic considerations are brushed aside, as a committed follower just needs to wait for a leader to form a majority. Thus

$$W_{\text{out}} (\vec{m}) = \sum_{j=A,B} (\delta \pi_j)^{m_j} \times \left( \sum_{l=0}^{m_{-j}-1} \frac{m_j - 1 + l}{l} \right) \times (\delta \pi_{-j})^l \times w_j \tag{6}$$

From equation (1), expressions (5) and (6) pin down equilibrium prices $p_\ell (\vec{m})$ in terms of the fundamentals. This in turn allows us to solve for the value of the leaders, which is given, in recursive form, by

$$V_\ell (\vec{m}) = \pi_\ell \left( \delta V_\ell (\vec{m}^\ell) - p_\ell (\vec{m}) \right) + (1 - \pi_\ell) \delta V_\ell (\vec{m}^{-\ell}) \tag{7}$$

Once we write the price function in terms of the primitives, (7) becomes a stand alone partial difference equation, which we can solve as we did with $W$ and $W_{\text{out}}$. This allows us to prove the next result.

**Proposition 4.3** All else equal, if the leaders’ payoff for winning is sufficiently high (either $\pi_\ell$ or $\pi_\ell - \nu_\ell$), the game $\Gamma(q,q)$ has a fully competitive equilibrium.
We are now ready to state our first main result. In this theorem, we will assume that the sufficient condition for the existence of a fully competitive equilibrium is satisfied, and compare the value of uncommitted followers when there is a single alternative with the value under competition. The theorem establishes that followers are better-off when only the best alternative is available than under competition. This holds for any difference in the value of the alternatives, and in fact also when the two alternatives give followers the same value; i.e., \( w_A = w_B = w \). Moreover, if the initial number of uncommitted followers is sufficiently large, then followers are better-off when only one alternative is available than under competition, irrespective of which alternative this is.

**Theorem 4.4** Consider a fully competitive equilibrium of the game with two alternatives and initial position \((q, q)\), where \( q \equiv (n + 1)/2 \). Then \( W(q, q) < \max\{w_A(q), w_B(q)\} \). Moreover, there exists \( \pi \) such that in a fully competitive equilibrium of the game with \( q = (n + 1)/2 > \bar{n} \), \( W(q, q) < \min\{w_A(q), w_B(q)\} \).

How can competition between alternatives be bad for followers? While the result might be surprising, the logic is straightforward. As we noted in the case of a single alternative, to understand the incentive problem for followers in this context it can be useful to trace a parallel to the problem of contributions for the provision of a public good. By rejecting an offer, a follower delays change, but can also rely on others to pay the bill. This outside opportunity gives each follower some bargaining power over the leaders. The key to understand the result is to understand how this bargaining power changes across the two games.

Consider the problem of the pivotal follower when there is a single alternative (the \( q^{th} \) member to meet the leader). If the pivotal follower refuses an offer by the leader, he will be able to free ride on others with probability \((q - 1)/q = (n - 1)/(n + 1)\). This relatively high free-riding ability gives the pivotal follower bargaining power against the leader and protects him from getting fully expropriated. Now consider the problem of a pivotal follower under competition (a follower who could give one of the leaders a win). The number of uncommitted followers in this situation is at most as large as in the single alternative case and generally lower. In fact, if the battle between the leaders is even until the end, the pivotal follower could meet the leader in state \( \bar{m} = (1, 1) \). But here there are no free riding possibilities; i.e., if the follower rejects an offer he meets a leader
again for sure. As we saw before, this gives the pivotal follower no bargaining power, so \( W(1, 1) = 0 \). This situation is extreme, but the logic generalizes to other states in which the number of uncommitted followers is lower than in the single alternative case. In most states, competition between the leaders gives the pivotal follower fewer free-riding possibilities, and results in a lower equilibrium payoff.

This logic is also at the heart of the second result. Consider again the single alternative case. As we noted above, the probability that the pivotal uncommitted follower will be able to free ride after refusing an offer from the leader is \( (n - 1)/(n + 1) \). This probability goes to one as \( n \to \infty \). On the other hand, the game with competition can be very close. Thus, even if \( n \) is large, the pivotal player can be one of only a few uncommitted followers, and as a result have few free-riding opportunities, and therefore less bargaining power. This effect trickles down all the way through to the beginning of the game, and then even a monopoly of the worst alternative is preferred to competition. Interestingly, this result holds for any difference in the value of the alternatives to the followers, as long as the committee is sufficiently large.

Theorem 4.4 shows that if the conditions are such that both leaders compete for support in every state, introducing competition reduces followers’ welfare. As we have seen, for this to happen it is enough to assume that both leaders have a lot to gain from winning \((v_\ell - v_s)\) sufficiently large). Now, for some parameters, the equilibrium will not be fully competitive. Suppose, for example, that \( w_j = 1000, \tau_j = 100, v_j = 0 \) for \( j = A, B \), that \( \delta = 0.95 \) and \( \pi_A = 0.5 \). The game has a fully competitive equilibrium for \((m_A, m_B) \leq (2, 2)\), but not in state \((3, 1)\). The reason is that in state \((3, 1)\), \( S_B > 0 \) if only one principal makes an offer upon meeting a follower, but \( S_B < 0 \) if both principals do, as they are supposed to in a fully competitive equilibrium. Thus, the equilibrium is in mixed strategies. Note that anticipated behavior in this state must make the state \((3, 1)\) less attractive in order to increase the gain from advancing through \( B, S_B \). To achieve this, we need either \( B \) to propose only some of the times she meets a follower in state \((3, 1)\), or followers to accept \( B \)'s offers in this state only some of the time.

Thus, it would be important to know if the previous result extends to other possible equilibria as well. Our next result addresses this issue. Instead of considering one equilibrium at a time, the result uses incentive compatibility conditions that must hold in any equilibrium to show that followers are better off when there is a single alternative to the
status quo than when there is competition between this and an inferior alternative, even if this is also preferred to the status quo. This is the main result of the paper.

**Theorem 4.5** In any equilibrium of the game with two alternatives and initial position \((q, q)\), \( W(q, q) < \max\{w_A(q), w_B(q)\} \).

The result follows immediately for \(q = 1\) from the fact that in any equilibrium \( W(1, 1) = 0 \), as it was the case in the fully competitive equilibrium (while \( w_j(1) > 0 \) for \( j = A, B \) per Proposition 3.1). The proof for \( q \geq 2 \) proceeds by induction. Lemma 7.3 establishes the base case. It shows that \( W(m_A, 2) \leq \max\{w_A(m_A), w_B(2)\} \) for all \( m_A \geq 2 \) (and similarly for \( B \)). Lemma 7.4 establishes the induction step for any state \( \vec{m} \geq (3, 3) \). This shows that if \( W(m_A, 2) \leq \max\{w_A(m_A - 1), w_B(m_B)\} \) and similarly \( W(m_B) \leq \max\{w_A(m_A), w_B(m_B - 1)\} \), then \( W(\vec{m}) \leq \max\{w_A(m_A), w_B(m_B)\} \). Iterative application of the induction step covers the entire state space and establishes the result.

**Initial Advantages and q-rules.** Up to this point we assumed for simplicity that both leaders need to obtain the support of a majority of followers to win; i.e., the initial game is \( \Gamma(n + 1/2, n + 1/2) \). In general, however, we might need to analyze a situation in which one of the alternatives (say \( A \)) has an initial advantage, by which it wins if it obtains the support of \( q_A < n + 1/2 \) followers (i.e., the initial game is \( \Gamma(q_A, n + 1/2) \)). Our next result shows that the presence of initial advantages does not alter our results. Remark 4.6 follows directly from the proof of Theorem 4.5. In essence, the comparison has nothing to do with the symmetry of the initial position.

**Remark 4.6** Consider any equilibrium of the game \( \Gamma(q_A, q_B) \), where \( q_A < q_B \). Then (i) \( W(q_A, q_B) \leq \max\{w_A(q_A), w_B(q_B)\} \). Moreover, (ii) there exists \( \bar{n} \) s.t. if \( \bar{n} < q_A < q_B \), in a competitive equilibrium of \( \Gamma(q_A, q_B) \), \( W(q_A, q_B) \leq \min\{w_A(q_A), w_B(q_B)\} \).

The model with an initial advantage for one of the leaders also allows us to extend the benchmark model to arbitrary \( q \)-rules. Suppose that \( B \) needs to obtain the support of a supermajority \( q_B > (n + 1)/2 \) of members to implement a reform leading to a value of \( w_B > 0 \) for the followers, while \( A \) can block the reform by getting the support of \( q_A = n - q_B \) members, leading to \( w_A = 0 \). We might also want to consider a case in which \( B \) only needs a minority to implement a reform, so that \( q_B < (n + 1)/2 \).
The model for an arbitrary non-unanimous $q$-rule is formally equivalent to introducing initial advantages, with the exception that $w_A = 0$, while Remark 4.6 applies to the public goods environment, where $w_A, w_B > 0$. Part (i) of Remark 4.6 applies to this case as stated, even if $w_A = 0$. Part (ii), however, doesn’t. With $w_A > 0$, $w_A(q_A) > 0$ as well. In the proof of Remark 4.6 we show that for any $w_A(q_A) > 0$ we can choose $q_A, q_B$ large enough so that the fully competitive equilibrium value $W(q_A, q_B)$ can be sandwiched in $(0, w_A)$. When $w_A = 0$, however, $w_A(q_A) = 0$ as well, and then $W(q_A, q_B) > w_A(q_A)$.

5 Applications and Extensions

Up to this point, we have studied a bare bones model, in order to focus on the main mechanism under study. To take the theory to applications, we must adapt the model to incorporate various new features. In the next sections we do this, illustrating each case with a clear application. In Section 5.1 we extend our results to the case of costly actions (coalition of the willing). In Section 5.2 we show that the results remain qualitatively unaltered if cash transfers are replaced by promises (candidate endorsements). In Section 5.3 we allow payoffs to differ for insiders, outsiders and rivals of a given winner (corporate takeovers). For simplicity, throughout this section, we focus on a fully competitive equilibrium.

5.1 Costly Actions: The Coalition of the Willing

Up to this point, we have assumed that the alternatives championed by the leaders are a public good, in the sense that followers prefer the outcome associated with a victory of A or B to the status quo; i.e., $w_A, w_B > 0$. In some instances, though, it is natural to assume that followers prefer the status quo to any of the options proposed by the leaders. This would be the case, for example, with legislators of the party in government being asked to support austerity measures, in fracking without royalties, or in a coalition for war.

Consider for example President Bush’s “Coalition of the Willing”. In September of 2002, President Bush put forth a defense of the use of military force against Saddam Hussein’s
government in his address to the UN Security Council.\footnote{http://georgewbush-whitehouse.archives.gov/news/releases/2002/09/20020912-1.html} Since the key international actors were divided on this issue, it became important for the US government to have the support of a relatively broad coalition of countries. While population in most of the countries that were targeted by the US was generally opposed to an invasion, the US managed to gather a \textit{coalition of the willing} by – using the words of former Secretary of State James Baker – “cajoling, extracting, threatening and occasionally buying votes”:\footnote{See Baker III (1995). The sentence makes reference to his experience in the U.N. Security Council vote authorizing the Gulf War in 1990.}

“Turkey has been offered $6 billion in direct aid, plus billions more in loans, if it will allow the U.S. to base soldiers there in advance of an invasion. But promises are flowing to nations far from the war front. A no vote by Chile could jeopardize a bill now pending in Congress for increased trade access; a measure worth billions of dollars over time. For Cameroon, a proposed 670-mile oil pipeline from Chad to be built by Exxon Mobil and Chevron Texaco is at stake. Poland stands to win $3.8 billion in loans for military aircraft. Bulgaria has no doubt heard hints that it could win a chance to host a new U.S. military base, which would inject millions into its economy. Guinea’s army rangers continue to need U.S. training to prevent attacks from neighboring Liberia.”\footnote{“Coalition of the billing – or unwilling?”, by Laura McClure. Published online in Salon.com, March 12 2003 (http://www.salon.com/2003/03/12/foreign_aid/).}

Differently than in our benchmark case, in this case action is costly for followers, so $w < 0$ (similarly, if Russia had actually put pressure on governments to actively oppose the US, this would also have been costly. Thus $w_A < 0$ and $w_B < 0$. Here we extend our analysis to consider this case, which we refer to as \textit{costly actions}. As before, we begin with the case in which there is only one alternative to the status quo. We show that as long as the leader’ payoff for winning is sufficiently high, or players are sufficiently patient, the game $\Gamma^s(q)$ has a unique equilibrium, in which the leader makes an offer whenever she meets an uncommitted follower up to the point in which she wins.

\textbf{Proposition 5.1} \textit{Let $w < 0$. All else equal, if the leader’s payoff for winning is sufficiently high or players are sufficiently patient, the equilibrium characterized in Proposition 3.1 remains the unique MPE of the game $\Gamma^s(q)$, and followers’ equilibrium payoff is given
Proposition 5.1 says that if followers are sufficiently patient, the leader can implement a policy that is arbitrarily bad for followers with the support of a majority of the group. This of course is terrible news for those agents who are not compensated.

The reason for this result is that the leader uses uncommitted followers against each other. Upon meeting, both leader and follower know that if the follower rejects the leader’s offer, the next follower will accept it. Thus, the follower can only delay the implementation of the costly action for one period, if he forgoes any compensation. To prevent this delay, the leader can offer to compensate the follower for this differential (and not for the full cost that implementation will bring to the follower). Note that since the power of the follower stems from his ability to delay implementation, it is larger the more the followers discount the future (i.e., the lower $\delta$ is). In fact, from Proposition 3.1, $w(q)$ attains a lower bound of $w < 0$ as $\delta \to 1$ and an upper bound of zero as $\delta \to 0$.

Thus, with costly actions, a monopolistic principal can be really costly for followers. Would they benefit from introducing competition in this context? As one might anticipate, the answer is yes: with costly alternatives, competition increases followers’ welfare. The intuition is straightforward. With costly actions, followers can only cut their loses if they are part of the coalition that supports the leader. Thus, the same logic in Theorems 4.4 and 4.5 now reverses the result: a pivotal agent that refuses the offer with a single alternative will have a much lower chance of being brought back to the table of negotiations than a pivotal follower in a competitive environment, and as a result can demand a higher transfer in exchange of his support. Formally, the proof of this result is step by step the same proof as of Theorems 4.4 and 4.5, reversing inequalities due to $W(\cdot) < 0$.

**Proposition 5.2 (Costly Actions)**  
(i) Consider any equilibrium of the game with two costly alternatives inducing payoffs $w_A, w_B < 0$ for the followers, and initial position $(q,q)$, where $q \equiv (n+1)/2$. Then $W(q,q) \geq \min \{w_A(q), w_B(q)\}$. Moreover, (ii) there exists $\pi$ such that in a competitive equilibrium of the game with $q = (n+1)/2 > \pi$, $W(q,q) \geq \max \{w_A(q), w_B(q)\}$.

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8By the same logic, it follows that if one of the leaders proposes a public good and one of them champions a costly action, say $w_A > 0 \geq w_B$, then followers prefer $\Gamma_A(q) \geq \Gamma_A(q) \geq \Gamma_B(q)$. 

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The results shed some light on the effect for third countries of having a unipolar world order or a bipolar world order in international relations. When international policy decisions concern mostly public good issues (e.g., environmental policy, international trade, coordination of monetary policies, etc) third countries would benefit from having a unipolar world order. When instead international policy decisions are mostly about conflict, third countries are better off under a bipolar world order.

5.2 Promises: Endorsements in Presidential Primaries

So far we have assumed that the leader offers an instantaneous cash transfer in exchange for a commitment of support. Transfers that occurred in the past are sunk and, hence, do not affect the incentives in subsequent periods. Alternatively one can assume that the leader and the follower agree on a contingent transfer in exchange for support when they meet; a “partnership” offer instead of a buyout. This in fact seems the most appropriate assumption in some applications, as in the case of endorsements by party elders in presidential primaries. Consider the following passage from “HRC”, the book on Hillary Clinton by Politico’s Jonathan Allen and The Hill’s Amie Parnes:

“They carefully noted who had endorsed Hillary, who backed Barack Obama, and who stayed on the sidelines – standard operating procedure for any high-end political organization. But the data went into much more nuanced detail . . . . For Hillary, whose loss was not the end of her political career, the spreadsheet was a necessity of modern political warfare, an improvement on what old-school politicians called a favor file. It meant that when asks rolled in, she and Bill would have at their fingertips all the information needed to make a quick decision – including extenuating, mitigating, and amplifying factors – so that friends could be rewarded and enemies punished.”

In this case candidates negotiate with party elders their support, but they do so in exchange of future promises. How would this change our previous analysis and conclusions? Do party elders prefer a contested primary or a shoo-in candidate?

The analysis of the promise game has some differences with the cash game, but also substantial similarities. Let \( \tilde{W}_{out}(\tilde{m}|p_j(\tilde{m})) \) denote the value in state \( \tilde{m}' = (m'_A, m'_B) \) of a committed follower locked with a promise \( p_j(\tilde{m}) \) acquired towards leader \( j \) in state
\( \bar{m} = (m_A, m_B) \). Note that

\[
\tilde{W}_{out}(\bar{m}'|p_A(\bar{m})) = \sum_t \Pr (j \text{ wins in } t \text{ periods } | \bar{m}') \delta^t [w_j + p_j(\bar{m})] \\
+ \sum_t \Pr (\ell \text{ wins in } t \text{ periods } | \bar{m}') \delta^t w_\ell \\
= W_{out}(\bar{m}') + \sum_t \Pr (j \text{ wins in } t \text{ periods } | \bar{m}') \delta^t p_j(\bar{m}),
\]

where \( W_{out}(\bar{m}') \) denotes the value of a committed follower in state \( \bar{m}' \) in the cash game, and \( \tilde{p}_j(\bar{m}', \bar{m}) \) gives the expected value of the contingent transfer \( p_j(\bar{m}) \) in state \( \bar{m}' \). Note then that the value function \( \tilde{W}_{out}(\bar{m}'|p_\ell(\bar{m})) \) is separable in transfers and the value derived from implementing the alternative. Thus when \( \ell \) meets an uncommitted follower in state \( \bar{m} \), she offers a contingent payment \( p_\ell(\bar{m}) \) such that

\[
\tilde{p}_\ell(\bar{m}', \bar{m}) + \delta W_{out}(\bar{m}') = \delta \tilde{W}(\bar{m})
\]

This implies that the continuation payoff of a follower after he meets one of the leaders is \( \delta \tilde{W}(\bar{m}) \) no matter what, and then the recursive representation of \( \tilde{W}(\bar{m}) \) is given by (3) as in the “cash” game, so that \( \tilde{W}(q, q) = W(q, q) \); i.e., the value of the uncommitted follower at the beginning of the promises game is equal to the value in the cash game. This moreover implies in (9) that the expected value of the payment in the promise game is the same as in the cash game.

So all of this is very much alike. Now, to evaluate existence, we need to consider the value of the leader. And in this regard there is in fact a crucial difference. Note that since promises are executed if and only when the leader wins, present exchanges now affect the incentives for future exchanges and must be incorporated on the value function. In particular, the relevant state in the promises game is not only the number of followers that each leader needs to win, but also the stock of promises that a leader brings to the table when meeting another follower. This technical difference does not alter our main results. To see this note that after a leader wins, she obtains a payoff from this success and also a transfer from/to all committed followers. These are, indeed, additively separable when realized. Moreover, this property still holds recursively, which implies that the value function of the leader in any state (now with the stock of promises as part of the state) is also additively separable in the utility for winning and the promises collected
if she wins. It follows immediately that Proposition 4.3 extends to this case and a fully competitive equilibrium exists for sufficiently high $\bar{v}$ or $\bar{v} - \underline{v}$. A similar argument holds for the monopoly case, and the welfare comparison in the paper holds.

5.3 Insider and Outsider Payoffs: Corporate Takeovers

Up to this point, we have maintained the assumption that, excluding transfers, the payoff that followers obtain when an alternative wins is independent of whether they have committed their support for the winning alternative, the losing alternative, or remained uncommitted. The assumption that insiders, rivals, and outsiders obtain the same payoff allowed us to focus on the main mechanism at work in the comparison between competition and a single alternative without distractions. From a practical standpoint, thought, this assumption is restrictive. In some applications, it can be more natural to allow differences in the payoffs of insiders, outsiders and rivals. In others, it is a necessity.

Consider for example corporate takeovers. In its simplest form, a raider (an individual or company attempting to gain control of a target company) acquires shares of a target company in order to control its board of directors. This attempt by the raider to acquire a majority of shares can be a hostile takeover, if it is resisted by the incumbent management, or a friendly takeover, when it is not. Can shareholders benefit from a hostile takeover?

In principle, the application seems to fit our benchmark model nicely. In a hostile takeover there are two principals (the raider and incumbent) and $n$ followers (the shareholders). The incumbent provides a status quo value of zero for shareholders. The raider, if effective, can generate additional value. There is, however, one crucial distinction: once the raider (or the incumbent) buys out a shareholder, the shareholder can’t appropriate the additional value generated by the raider.\(^9\) This creates a difference in the payoff of insiders and outsiders. A similar distinction can be made in other applications. This would be the case in the coalition of the willing if the supporters of the United States have to provide combat troops, or in endorsements to political candidates, if there is a

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\(^9\)Hostile corporate takeovers can take several forms. In a proxy fight, a raider tries to persuade a majority of shareholders to replace the management with a new one which will approve a takeover resisted by the incumbent management. This variant fits the benchmark model without changes. A raider can also try to acquire the target company through a tender offer. Here the acquiring company makes a public offer at a fixed price above the current market price. This variant is outside the scope of our model.
payoff of being on the winner’s side.

Here we extend the model to consider these applications. In order to do this, we need to differentiate payoffs for insiders \((z_i)\), rivals \((y_i)\) and outsiders \((w_i)\); i.e., \(z_i\) denotes the payoff that an individual who gave her support to \(i\) gets when \(i\) wins, \(y_i\) denotes the payoff that an individual who gave her support to \(j \neq i\) gets when \(i\) wins, and as before, \(w_i\) denote the payoff an uncommitted follower gets if \(i\) wins.

We want to solve for the followers’ value function that accounts for insider/outsider payoffs, \(\hat{W}\). This is actually very simple. The point is that the amount leader \(\ell\) will offer to a follower when meeting in a state \(\vec{m}\) satisfies

\[
\hat{p}_\ell(\vec{m}) + \delta \hat{W}^\ell_{out}(\vec{m}^\ell) = \delta \hat{W}(\vec{m}),
\]

(10)

where here \(\hat{W}^\ell_{out}(\vec{m}^\ell)\) is the value in state \(\vec{m}^\ell\) of a follower who committed his support to leader \(\ell\), and \(\hat{p}_\ell(\vec{m})\) is the cash transfer \(\ell\) proposes in \(\vec{m}\). Because of this, the recursive representation of the followers value \(\hat{W}\) is given by (4), exactly as before, and thus \(\hat{W}(\vec{m}) = W(\vec{m})\). This says that the followers’ value only depends on the outsider payoffs \(w_i\), with all the new elements going into the “revised” cash transfer \(\hat{p}_\ell(\vec{m})\). It follows that to determine whether we are in a “public good” or a “costly action” model, we only need to consider the outsider payoffs \(w_i\): if \(w_i < 0\) competition is beneficial for followers, and if \(w_i > 0\), competition is detrimental for followers.

Now consider the transfers. Note that from (10), substituting \(\hat{W}(\vec{m}) = W(\vec{m})\), we have

\[
\hat{p}_\ell(\vec{m}) = \delta \left[ W(\vec{m}) - \hat{W}^\ell_{out}(\vec{m}^\ell) \right].
\]

In the benchmark model we had \(W_{out}(\vec{m}^\ell) = \sum_\ell J_\ell(\vec{m})w_\ell\). Now we have \(\hat{W}^\ell_{out}(\vec{m}^\ell) = J_\ell(\vec{m}^\ell)z_\ell + J_i(\vec{m}^\ell)y_i\) for \(i \neq \ell\), which we can write as

\[
\hat{W}^\ell_{out}(\vec{m}^\ell) = W_{out}(\vec{m}^\ell) + J_\ell(\vec{m}^\ell)(z_\ell - w_\ell) + J_i(\vec{m}^\ell)(y_i - w_i)
\]

Then substituting,

\[
\hat{p}_\ell(\vec{m}) = p_\ell(\vec{m}) - \delta \left[ J_\ell(\vec{m}^\ell)(z_\ell - w_\ell) + J_i(\vec{m}^\ell)(y_i - w_i) \right]
\]

(11)

So we can now revisit cash transfers in our applications in light of (11):
**Corporate Takeovers.** Suppose the raider (B) adds value to the company, but the incumbent (A) does not attain extraordinary rents due to mismanagement or inferior business strategies. Then \( w_B > 0 \) and \( w_A = 0 \). If the incumbent or the raider buys a shareholder out, the (ex) shareholder is excluded from any benefits the company can produce, so \( z_i = y_i = 0 \). Then \( \hat{p}_B(m) > 0 \) even when in the benchmark model we would have \( p_B(m) = 0 \). In this case competition from the incumbent management is bad for shareholders, and prices are positive.

![Diagram](image)

Figure 3: Here \( y_\ell = w_\ell \) for \( \ell = A, B \).

**Real Estate Developers.** Consider a real estate developer who wants to buy properties in a block to build a mall. She could possibly face competition from another real estate developer who wants to buy the properties to build a stadium. Houses are assumed to be homogeneous. We assume that the projects can be built if and only if the developer has at least half of the space in the block. A house without a mall or stadium in front has a value of 1M (this will be the status quo, which we will normalize to zero). If an owner does not sell the house and the mall (A) is built, the house has a value of 500K. Thus (after the normalization) \( w_A = -500K \). If an owner does not sell the house and the stadium (B) is
built, the house has a value of 100\text{K}. Thus $w_B = -900\text{K}$. If an owner sells the house, he
doesn’t care what happens with the neighborhood. Thus $y_A = z_A = y_B = z_B = -1\text{M}$. In
this case competition is favorable for the owners, and prices are positive.

**Coalition of the Willing.** Since here $w_\ell < 0$, the equilibrium of the benchmark model
implied positive transfers. Now suppose that if a country supports the US, it has to
send soldiers to combat operations, and that if a country supports Russia, it loses trade
opportunities with the West. Then a plausible configuration of parameters is $z_{US} <
w_{US} \leq 0 < y_{US}$, and $y_R < w_R < \min\{0, z_F\}$. In this case the US has to pay more, and
Russia has to pay less than in the benchmark model, possibly even a negative transfer.

**Endorsements in Elections.** Consider the endorsement game we discussed above. It
might be reasonable to consider the following structure of payoffs. If candidate $i$ wins
without the support of a given party notable, the party notable gets $w_i > 0$. If the
party notable supported the candidate $i$ he obtains $z_i > w_i$ if $i$ wins, and $y_j < w_i$
(being excluded from all the benefits that accrue to other party members). In this case,
party members would prefer to have a shoo-in candidate (competition is detrimental for
followers’ welfare), and transfers are positive if the cost of supporting the losing candidate
is sufficiently large, negative otherwise.

As the examples illustrate, in the model with insiders, outsiders and rivals the equivalence
between the effect of competition and the direction of the transfers breaks down. This
might seem disconcerting. Since the intuition for the result in the benchmark model relied
heavily on free riding, it can be difficult at first glance to reconcile competition being bad
for followers with positive transfers from the leaders to the followers.

As we will show, however, the main intuition is unchanged. The reason for this is straight-
forward. In the original model, with no “bonus payments” (no insider/outside differential)
the only possible compensation to/from followers is in cash. Thus when followers
receive a positive cash transfer, meeting the leader is good for followers, and when follow-
ers give in equilibrium a transfer to the leader avoiding the leader is good for followers.
But when the payoff of insiders, outsiders and rivals differ, the total compensation (or
taxation) is going to be composed of both cash and a “bonus” (the insider/outside differential).
Thus focusing on the cash component of total compensation is not informative,
as this can go in any direction, but the total compensation (cash and bonus) will have the same informational content as before: when total compensation is positive, followers want to meet the leader, and when total compensation is negative, they want to avoid them. To see this more clearly, note that from (10),
\[ \hat{p}_\ell(\vec{m}) + \delta \hat{W}_{out}^\ell(\vec{m}^\ell) = \delta \hat{W}(\vec{m}), \]
and from (5.3)
\[ \hat{W}_{out}^\ell(\vec{m}^\ell) = W_{out}(\vec{m}^\ell) + J_\ell(\vec{m}^\ell)(z_\ell - w_\ell) + J_i(\vec{m}^\ell)[y_i - w_i] \quad \text{for } i \neq \ell. \]
Thus, substituting, we have that
\[ \delta [\hat{W}(\vec{m}) - W_{out}(\vec{m}^\ell)] = \left[ \hat{p}_\ell(\vec{m}) + J_\ell(\vec{m}^\ell)(z_\ell - w_\ell) + J_i(\vec{m}^\ell)(y_i - w_i) \right] T_\ell(\vec{m}) \]
(12)
\[ T_\ell(\vec{m}) \]
is the total “transfer” – the increment in value that is not directly related to the project itself – that \( \ell \) gives to the follower to get his support, in cash and goodies. Here \( \hat{p}_\ell(\vec{m}) \) is the cash component, and \( J_\ell(\vec{m}^\ell)(z_\ell - w_\ell) + J_i(\vec{m}^\ell)(y_i - w_i) \) is the discounted net expected increment in goodies, \( (z_\ell - w_\ell) \) being the insider’s gain (the increment in goodies the follower gets when being with \( \ell \) if \( \ell \) wins, relative to being uncommitted), and \( (y_i - w_i) \) is the rival’s loss.

The key point here is that since \( \hat{W}(\vec{m}) = W(\vec{m}) \) from our previous argument, it follows that the total transfer \( T_\ell(\vec{m}) \) is exactly the same as the value of the cash transfer with no goodies are available. What matters is not the direction of the cash transfers but the total compensation, which is pinned down by \( \delta [\hat{W}(\vec{m}) - W_{out}(\vec{m}^A)] \), and is therefore unrelated with \( z_\ell \) and \( w_i \). This generalizes in a natural way our previous intuition. In a public good environment the total transfer \( T_\ell(\vec{m}) \) will be negative, and followers extract a free rider surplus when they can avoid meeting the leaders. Thus competition, which reduces the free riding opportunities is not beneficial for followers. In a costly actions environment, instead, meeting the leaders allows followers to extract a positive total transfer, and therefore competition increases their equilibrium payoff.

6 Conclusion

We consider a class of dynamic collective action problems in which either a single principal or two competing principals vie for the support of members of a group. We focus on
the dynamic problem that emerges when agents’ decisions to support an alternative are irreversible, and agents negotiate and commit their support to the principal sequentially. We show that competition between principals generally improves agents’ welfare with costly actions, but reduces agents’ welfare with public goods. In fact, when two leaders stand behind equally good alternatives, agents are always better off when facing a single alternative than two alternatives. In general, with public goods, agents are always better off facing the best alternative by itself than two alternatives, even if the difference in the value of the two alternatives is arbitrarily small. With costly actions, instead, the results reverse, and competition is indeed beneficial.

The logic for this result is akin to that for under-provision of public goods. In the case of public goods, in fact, in equilibrium the agent actually pays the principal to move forward (agrees to share some of his surplus with the principal). Thus, by rejecting an offer, a follower can hope to free ride, relying on others to pay the bill. This outside opportunity gives each follower some bargaining power over the leaders. The key point to note is that in any MPE, under competition followers have fewer free-riding opportunities (and thus less bargaining power) than under a single leader.

We apply the analysis to study a number of cases of interest. We conclude (i) that countries such as Turkey and Chile would have benefited if Russia had maintained a more active role buying votes against the attempts by the United States to form the coalition of the willing, (ii) that party notables providing their endorsement to candidates in presidential primary elections would rather have a shoo-in candidate than face competition between candidates, and that (iii) shareholders facing a valuable raider would prefer the incumbent management not to resist the takeover attempt by the raider.
References


7 Appendix

Proof of Proposition 3.1. We begin allowing a MPE in mixed strategies. When the leader meets follower $i$ in state $m$, she makes an offer $p(m)$ with probability $\gamma_m \in [0, 1]$. The follower accepts the offer with probability $\alpha_m \in [0, 1]$. Note that the follower $i$ meeting the leader in state $m$ accepts only if $\delta w_{out}(m - 1) + p(m) \geq \delta w(m)$, and accepts with probability one if this inequality holds strictly. Note that since $i$ accepts offers $p(m) > -\delta [w_{out}(m - 1) - w(m)]$ with probability one, then any such proposal cannot be offered in equilibrium, for $L$ could make a lower offer and still get accepted. Thus, whenever $L$ meets a follower $i$ in state $m$, she offers

$$p(m) = \begin{cases} -\delta [w_{out}(m - 1) - w(m)] & \text{if (14) holds} \\ -\infty & \text{otherwise.} \end{cases} \quad (13)$$

$L$ is willing to make the offer is state $m$ if

$$\alpha_m [\delta v(m - 1) - p(m)] + (1 - \alpha_m) \delta v(m) \geq \delta v(m),$$

which boils down to

$$p(m) \leq \delta [v(m - 1) - V(m)],$$

as before. Thus the leader obtains a non-negative payoff from making an offer if and only if

$$s(m) \equiv [v(m - 1) - v(m)] + [w_{out}(m - 1) - w(m)] \geq 0 \quad (14)$$

Now suppose that in equilibrium (14) holds strictly in state $m$. Then the follower meeting the leader in state $m$ must accept all such offers; i.e., $\alpha_m = 1$. This is because since the follower accepts any offer higher than $-\delta [w_{out}(m - 1) - w(m)]$, if $\alpha_m < 1$ the leader would increase the offer slightly, getting a discrete gain in payoffs. Thus, if in equilibrium the follower rejects the leader’s offer with positive probability in state $m$, (14) must hold with equality in state $m$; i.e., if $\alpha_m < 1$, then

$$s(m) = [v(m - 1) - v(m)] + [w_{out}(m - 1) - w(m)] = 0$$

The value of an uncommitted follower in state $m$ is

$$w(m) = \left(\frac{2}{n + 2m - 3}\right) \delta w(m) + \left(\frac{n + 2m - 3}{n + 2m - 1}\right) \delta [\gamma_m \alpha_m w(m - 1) + (1 - \gamma_m \alpha_m)w(m)],$$

or equivalently,

$$w(m) = H_m \delta w(m - 1),$$

where

$$H_m \equiv \left(\frac{(n + 2m - 3) \gamma_m \alpha_m}{n + 2m - 1 - 2 \delta - (n + 2m - 3) \delta (1 - \gamma_m \alpha_m)}\right)$$
Thus
\[ w(m) = \left[ \prod_{k=1}^{m} H_m \right] \delta^m w \] (15)

The value of a committed follower in state \( m \) is
\[ w_{\text{out}}(m) = \gamma_m \alpha_m \delta w_{\text{out}}(m - 1) + (1 - \gamma_m \alpha_m) \delta w_{\text{out}}(m) \]
or
\[ w_{\text{out}}(m) = \left( \frac{\gamma_m \alpha_m \delta}{1 - \delta(1 - \gamma_m \alpha_m)} \right) w_{\text{out}}(m - 1) \]
so that
\[ w_{\text{out}}(m) = \left[ \prod_{k=1}^{m} \left( \frac{\gamma_k \alpha_k}{1 - \delta(1 - \gamma_k \alpha_k)} \right) \right] \delta^m w. \] (16)

The value for the leader in state \( m \) is
\[ v(m) = \gamma_m \alpha_m (\delta v(m - 1) - p(m)) + (1 - \gamma_m \alpha_m) \delta v(m) \]
or
\[ v(m) = \left( \frac{\gamma_m \alpha_m \delta}{1 - \delta(1 - \gamma_m \alpha_m)} \right) (v(m - 1) + w_{\text{out}}(m - 1) - w(m)), \] (17)

Now suppose that in equilibrium \( L \) makes a relevant offer in every \( m > 1 \). We will solve for the equilibrium values and then come back and verify that (14) holds for all \( m \) to check that this is an equilibrium. First, note that since \( L \) makes a relevant offer in every meeting, (15) boils down to
\[ w(m) = \left[ \prod_{k=1}^{m} \left( \frac{n + 2k - 3}{n + 2k - 1 - 2\delta} \right) \right] \delta^m w = \left[ \prod_{k=1}^{m} r(k) \right] \delta^m w \] (18)
and (16) boils down to
\[ w_{\text{out}}(m) = \delta^m w. \] (19)

Substituting (18) and (19) in (17), we have
\[ v(m) = \delta v(m - 1) + \left( 1 - \delta \prod_{k=1}^{m} r(k) \right) \delta^m w \]

Recursively we have that
\[ v(m) = \delta^m v + \left[ \sum_{l=1}^{m} \left( 1 - \delta \prod_{k=1}^{l} r(k) \right) \right] \delta^m w \] (20)
Then note that
\[
v(m-1)-v(m) = \delta^{m-1}(1-\delta)v + \delta^{m-1}w \left\{ (1-\delta) \sum_{l=1}^{m-1} \left( 1 - \delta \prod_{k=1}^{l} r(k) \right) - \delta \left( 1 - \delta \prod_{k=1}^{m} r(k) \right) \right\}
\]
and
\[
w(m) - w_{out}(m-1) = - \left[ 1 - \delta \prod_{k=1}^{m} r(k) \right] \delta^{m-1}w
\]
so substituting, (14) is
\[
s^*(m) = (1-\delta)\delta^{m-1} \left[ v + w \sum_{l=1}^{m} \left( 1 - \delta \prod_{k=1}^{l} r(k) \right) \right] \geq 0
\]
which is satisfied if and only if
\[
v + w \sum_{l=1}^{m} \left( 1 - \delta \prod_{k=1}^{l} r(k) \right) \geq 0
\]
Because this always holds for \( v > 0 \) and \( w > 0 \), it follows that this is an equilibrium.

Next we show that this is the unique equilibrium with an induction argument. First note from (15) and (17) that for all \( m \geq 1 \), \( v(m) \) and \( w(m) \) are maximized when \( \gamma_m = \alpha_m = 1 \). Then \( s^*(1) \geq 0 \) implies \( s(1) = [v-v(1)] + [w-w(1)] > 0 \) whenever \( \gamma_1 \alpha_1 < 1 \). It follows that in state \( m = 1 \) the leader makes a proposal with probability one; i.e., \( \gamma_1 = 1 \). But then \( \alpha_1 = 1 \) as well. For suppose \( \alpha_1 \in (0,1) \). Then \( s(1) > 0 \) and the leader would gain by increasing the offer slightly, getting it accepted with probability one. Now suppose that in equilibrium \( \gamma_t = \alpha_t = 1 \) for all \( t < m \). Consider the surplus in state \( m \). Note that \( v(m-1) \) and \( w_{out}(m-1) \) are exactly as in the equilibrium characterized above. Since \( v(m) \) and \( w(m) \) are maximized when \( \gamma_m = \alpha_m = 1 \), then \( s^*(m) \geq 0 \) implies \( s(m) > 0 \) whenever \( \gamma_m \alpha_m < 1 \). Thus \( \gamma_m = 1 \). As before, then also \( \alpha_1 = 1 \), for otherwise \( s(m) > 0 \) and the leader would gain by increasing the offer slightly, getting it accepted with probability one.

**Proof of Proposition 4.3.** We will show that for any \( j = A, B \) there is a \( v^* \in \mathbb{R}_+ \) such that if \( v_j \geq v^* \), when all players play the proposed equilibrium strategies, \( S_j(\vec{m}) \geq 0 \) for all \( \vec{m} \).
Consider the surplus expression (2). Note that (5) and (6) imply that \( W_{\text{out}}(\vec{m}^j) - W(\vec{m}) \) does not depend on \((\vec{v}_A, \vec{v}_B, \vec{v}_B, \vec{v}_B)\), and is therefore a constant. It follows that \( \vec{v}_{-j} \) and \( \vec{u}_{-j} \) do not affect \( S_j(\vec{m}) \), and \( \vec{v}_j \) and \( \vec{u}_j \) enter \( S_j(\vec{m}) \) only through the term \( V_j(\vec{m}^j) - V_j(\vec{m}) \).

Now, note that having expressed \( p_j(\vec{m}) \) in terms of the primitives of the model, we can solve (7) as a stand alone partial difference equation, to obtain

\[
V_j(\vec{m}) = (\delta \pi_j)^{m_j} \left[ \sum_{l=0}^{m_j-1} \binom{m_j - 1}{l} (\delta \pi_{-j})^l \right] \vec{v}_j \\
+ (\delta \pi_{-j})^{m_j} \left[ \sum_{l=0}^{m_{-j}-1} \binom{m_{-j} - 1}{l} (\delta \pi_j)^l \right] \vec{u}_j - H(\vec{m}).
\]

where \( H(\vec{m}) \) is a function of prices \( p_j(r, s) \) for \( r \leq m_j, s \leq m_{-j} \), which are constant in \((\vec{v}_j, \vec{u}_j, \vec{v}_{-j}, \vec{u}_{-j})\) by (5) and (6). Thus \( V_j(\vec{m}^j) - V_j(\vec{m}) \) is given by

\[
\left\{ (\delta \pi_j)^{m_j-1} \left[ \sum_{l=0}^{m_j-1} \binom{m_j - 2 + l}{l} (\delta \pi_{-j})^l \right] - (\delta \pi_j)^{m_j} \left[ \sum_{l=0}^{m_{-j}-1} \binom{m_{-j} - 1 + l}{l} (\delta \pi_j)^l \right] \right\} \vec{v}_j \\
- (\delta \pi_{-j})^{m_{-j}} \binom{m_A + m_B - 2}{m_j - 1} (\delta \pi_j)^{m_j-1} \vec{u}_j \\
+ H(\vec{m}) - H(\vec{m}^j).
\]

We will show that this expression can be made arbitrarily large by increasing \( \vec{v}_j \) or reducing \( \vec{u}_j \). The last line is a constant. From the second line it follows that all else equal, there is a \( \vec{v}^* \) such that if \( \vec{v}_j < \vec{v}^* \), then \( S_j(\vec{m}) > 0 \). Next, after some algebra, the bracket in the first line can be written as

\[
(\delta \pi_j)^{m_j-1} \left[ (1 - \delta) \sum_{l=0}^{m_j-1} \binom{m_j - 1 + l}{l} (\delta \pi_{-j})^l + \binom{m_A + m_B - 2}{m_j - 1} (\delta \pi_{-j})^{m_j} \right] > 0.
\]

Thus, all else equal, there is a \( \vec{v}^* \) such that if \( \vec{v}_j > \vec{v}^* \), then \( S_j(\vec{m}) > 0 \).

**Proof of Theorem 4.4.** The first statement follows as a corollary of Theorem 4.5.

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Now consider the second part. From expression (5), we have

\[ W(q, q) = \sum_{l=0}^{q-2} \left( \prod_{k=0}^{q-1+l} C(2q-k) \right) \times \left( q - \frac{1+l}{l} \right) \times \left[ (\delta \pi_A)^q (\delta \pi_B)^l w_A + (\delta \pi_B)^q (\delta \pi_A)^l w_B \right] \]  \tag{23}

On the other hand, with a single alternative, \( w(q) = \left( \prod_{q=1}^{q} C(q+k) \right) \delta^q w \). Now, since \( r(k) = \frac{n+2k-3}{n+2k-(1+2q)} \) by definition and \( n = 2q - 1 \), we have \( r(k) = C(q+k) \). Thus

\[ w(q) = \left( \prod_{k=1}^{q} C(q+k) \right) \delta^q w = \left( \prod_{k=0}^{q-1} C(2q-k) \right) \delta^q w \]  \tag{24}

Suppose without loss of generality that \( w_A > w_B \). We want to show that for sufficiently large \( q \) the equilibrium payoff of an uncommitted follower in the game with a single alternative yielding value \( w_B \) is larger than his (competitive) equilibrium payoff in the game with two alternatives yielding value \( w_A \) and \( w_B \). Suppose not. Then making \( w = w_B \) in (24), and dividing (23) by (24),

\[ U(q) \equiv \sum_{l=0}^{q-2} \left( \prod_{k=q}^{q-1+l} C(2q-k) \right) \times \left( q - \frac{1+l}{l} \right) \times \left[ (\pi_A)^q (\delta \pi_B)^l + (\pi_B)^q (\delta \pi_A)^l \left( \frac{w_B}{w_A} \right) \right] \geq \frac{w_B}{w_A} \]

Now, since \( \delta \leq 1 \), \( \prod_{k=q}^{q-1+l} C(2q-k) \leq 1 \), and \( w_B/w_A < 1 \), for any integer \( q \), we have

\[ U(q) < \sum_{j=A,B} \sum_{l=0}^q \left( \frac{\pi_j}{q} \right)^q \sum_{l=0}^{q-2} \frac{\Gamma(q+l)}{\Gamma(q)} \frac{(\delta \pi_j)^l}{l!} \equiv U(q) \]

where for any integer \( k \), we define \( \Gamma(k) \equiv (k-1)! \). Now define the function

\[ F(a, b, c, z) \equiv \sum_{l=0}^{\infty} \frac{(a+l)(b+l)(c+l)}{(c+l)!} z^l \]

and note that we can write

\[ U(q) = \sum_{j=A,B} \left( \frac{\pi_j}{1+\delta \pi_j} \right)^q - \left( \frac{\pi_j}{1+\delta \pi_j} \right)^q \sum_{l=q-1}^{\infty} \frac{\Gamma(q+l)}{\Gamma(q)} \frac{(\delta \pi_j)^l}{l!} \]  \tag{25}
where the equality follows from the fact that \( F(a, b, b, z) = (1 - z)^{-a} \) (see Property 15.1.8 for hypergeometric functions in Abramowitz and Stegun (2012); p.556). Noting that

\[
\left( \frac{\pi_j}{1 - \delta \pi_j} \right) = \left( \frac{1 - \pi_j}{1 - \delta \pi_j} \right) < 1
\]
as long as \( \delta < 1 \), it follows that for any \( \varepsilon > 0 \) there is a \( Q \) such that if \( q > Q \), then \( U(q) < \varepsilon \). Thus, for any \( \pi_B/\pi_A \), there is a \( Q \) such that \( U(q) < \pi_B/\pi_A \) whenever \( q > Q \).

\[\text{Proof of Theorem 4.5.}\] Let \( \gamma_j(\bar{m}) \) be the probability that leader \( j = A, B \) makes an offer in state \( \bar{m} \), \( \alpha_j(\bar{m}) \) be the probability that an uncommitted follower accepts an offer from leader \( j = A, B \) in state \( \bar{m} \), and \( \mu_j(\bar{m}) \equiv \gamma_j(\bar{m}) \alpha_j(\bar{m}) \). Then

\[
W(\bar{m}) = \left( \frac{1}{m_A + m_B - 1} \right) \delta W(\bar{m}) + \left( \frac{m_A + m_B - 2}{m_A + m_B - 1} \right) \sum_{j=A,B} \pi_j \left( \mu_j(\bar{m}) \delta W(\bar{m}) + (1 - \mu_j) \delta W(\bar{m}) \right).
\]  

(26)

For \( j = A, B \), define

\[
\xi_j(\bar{m}) \equiv \frac{\delta \pi_j \mu_j(\bar{m})}{\left( \frac{m_A + m_B - 1}{m_A + m_B - 2} \right) (1 - \delta) + \delta \sum_{j=A,B} \pi_j \mu_j(\bar{m})}
\]

whenever \( \bar{m} \neq (1, 1) \), and \( \xi_j(1, 1) \equiv 0 \). Then we can write (26) as

\[
W(\bar{m}) = \sum_{j=A,B} \xi_j(\bar{m}) W(\bar{m}^j)
\]  

(27)

for all \( \bar{m} \) and \( j = A, B \). Note in particular that the recursion (27) implies that if \( w_A, w_B > 0 \) (as we are assuming here), then \( W(\bar{m}) \geq 0 \) for all \( \bar{m} \).

We need to show that \( W(q, q) < \max \{ w_A(q), w_B(q) \} \). The proof follows from three lemmas. Lemma 7.2 establishes the result for \( q = 1 \) and shows an additional result for all boundary states which is used in Lemma 7.3. The proof for interior states is by induction. Lemma 7.3 establishes the base case, and Lemma 7.4 the induction step. Iterative application of the induction step covers the entire state space and establishes the result. We begin with Lemma 7.1, which establishes an intermediate result that is used in the proof of Lemmas 7.2 and 7.4.\[\text{\blacksquare}\]
Lemma 7.1 (Bound)  In any MPE of the game $\Gamma(\vec{m})$, 

$$W(\vec{m}) \leq \max_{j \in \{A,B\}} \{ \delta r(m_j) W(\vec{m}^j) \}$$  

Proof of Lemma 7.1.  Note that for all $m_A \geq 2, m_B \geq 2$ we have

$$W(\vec{m}) \leq \xi_A(\vec{m}) W(\vec{m}^A) + \xi_B(\vec{m}) W(\vec{m}^B)$$  

Thus we need to show that

$$\sum_{j=A,B} \xi_j(\vec{m}) W(\vec{m}^j) \leq \max \{ r(m_A) W(\vec{m}^A), r(m_B) W(\vec{m}^B) \}$$

(a) Assume first that $W(\vec{m}^A) \geq W(\vec{m}^B)$ so it is sufficient if

$$\left[ \sum_{j=A,B} \xi_j(\vec{m}) \right] W(\vec{m}^A) \leq \max \{ r(m_A) W(\vec{m}^A), r(m_B) W(\vec{m}^B) \}$$

Note that since $r(m) = \frac{n+2m-3}{n+2m-(1+2\delta)}$, then

$$\left[ \sum_{j=A,B} \xi_j(\vec{m}) \right] = \frac{\delta [\pi_A \mu_A(\vec{m}) + \pi_B \mu_B(\vec{m})]}{\frac{m_A+m_B-1}{m_A+m_B-2} (1-\delta) + \delta [\pi_A \mu_A(\vec{m}) + \pi_B \mu_B(\vec{m})]} \leq \delta \min \{ r(m_A), r(m_B) \}.$$ (28)

Then it is sufficient if

$$\delta \min \{ r(m_A), r(m_B) \} W(\vec{m}^A) \leq \max \{ r(m_A) W(\vec{m}^A), r(m_B) W(\vec{m}^B) \}$$

which is true when either $r(m_A) W(\vec{m}^A) \geq r(m_B) W(\vec{m}^B)$ or the opposite holds.

(b) Suppose instead that $W(\vec{m}^A) \leq W(\vec{m}^B)$. Then it is sufficient if

$$\left[ \sum_{j=A,B} \xi_j(\vec{m}) \right] W(\vec{m}^B) \leq \max \left\{ \begin{array}{c} r(m_A) W(\vec{m}^A), \\ r(m_B) W(\vec{m}^B) \end{array} \right\}$$

and again using (28) it is sufficient if

$$\delta \min \{ r(m_A), r(m_B) \} W(\vec{m}^B) \leq \max \{ r(m_A) W(\vec{m}^A), r(m_B) W(\vec{m}^B) \}$$

which is true. This completes the proof. £
Lemma 7.2 (Boundaries)

\[ W(m_A, 1) < w_B(1) \text{ for all } m_A \geq 1 \quad \text{and} \quad W(1, m_B) < w_A(1) \text{ for all } m_B \geq 1 \]

Proof of Lemma 7.2. The result for state \( \vec{m} = (1, 1) \) follows immediately from the fact that \( W(1, 1) = 0 \). Now consider the remaining boundary states (states adjacent to terminal states). Solving the recursion (27) for the boundaries, we obtain

\[
W(m_A, 1) = \left( \sum_{l=1}^{m_A-1} \xi_B(m_A - l, 1) \prod_{k=0}^{l-1} \xi_A(m_A - k, 1) + \xi_B(m_A, 1) \right) w_B \quad \text{(29)}
\]

\[
W(1, m_B) = \left( \sum_{l=1}^{m_B-2} \xi_A(1, m_B - l) \prod_{k=0}^{l-1} \xi_B(1, m_B - k) + \xi_A(1, m_B) \right) w_A \quad \text{(30)}
\]

for all \( m_A, m_B \geq 1 \).

Consider \( \vec{m} = (2, 1) \). Note that since

\[
W(2, 1) = \frac{\delta \pi_B \mu_B(2, 1)}{2(1 - \delta)} w_B < \frac{(n - 1)\delta}{n + 1 - 2\delta} w_B = r(1)\delta w_B = w_B(1)
\]

By the same argument, \( W(1, 2) < w_A(1) \). Next, consider \( W(m_A, 1) \) for \( m_A \geq 3 \). We have

\[
W(m_A, 1) = \left( \sum_{l=0}^{m_A-3} \xi_B(m_A - (1 + l), 1) \prod_{k=0}^{l} \xi_A(m_A - k, 1) + \xi_B(m_A, 1) \right) w_B
\]

and since \((\xi_B(m_A - (1 + l), 1) + \xi_A(m_A - (1 + l), 1)) \leq 1\), it follows that

\[
W(m_A, 1) \leq (\xi_B(m_A, 1) + \xi_A(m_A, 1)) w_B < \delta r(1) w_B = w_B(1)
\]

Analogously, we have that \( W(1, m_B) < w_A(1) \). ■

Lemma 7.3 (Base Case)

\[ W(m_A, 2) < \max \{ w_A(m_A), w_B(2) \} \text{ for all } m_A \geq 2 \]

and

\[ W(2, m_B) < \max \{ w_A(2), w_B(m_B) \} \text{ for all } m_B \geq 2 \]
Proof of Lemma 7.3. First, note that

\[
W(2, 2) \leq \xi(4) \max \{w_B(1), w_A(1)\}
\]

\[
< \delta \left(\frac{2}{3 - \delta}\right) \max \{w_B(1), w_A(1)\}
\]

\[
< \delta r(2) \max \{w_B(1), w_A(1)\} = \max \{w_B(2), w_A(2)\}
\]

Next consider \(W(m, 2)\). By successive application of Lemma 7.1

\[
W(m, 2) \leq \max \{\delta r(m) W(m, 2), \delta W(1, 2)\}
\]

\[
\leq \max \left\{\delta^2 \prod_{j=0}^1 r(m - j) W(m, 2), \delta^2 \prod_{j=0}^0 r(m - j) W(m, 1) r(2), \delta W(1, 2)\right\}
\]

\[
\leq \max \left\{\delta^m \prod_{j=0}^{m-2} r(m - j) W(1, 2), \max_{k \leq m-2} \left\{\delta^{k+1} \prod_{j=0}^{k-1} r(m - j) W(m, 1) r(2)\right\}, \delta W(1, 2)\right\}
\]

Now, we have shown in Lemma 7.2 that \(W(m, 1) < w_B(1)\) for all \(m \geq 1\), and \(W(1, m) < w_A(1)\) for all \(m \geq 1\). Using these results in the RHS of the expression above, we get

\[
W(m, 2) < \max \left\{\delta^m \prod_{j=0}^{m-2} r(m - j) w_A(1), \max_{k \leq m-2} \left\{\delta^{k+1} \prod_{j=0}^{k-1} r(m - j) w_B(1)\right\} r(2), \delta r(2) w_B(1)\right\}
\]

Using (18) we get that

\[
\max_{k \leq m-2} \left\{\delta^{k+1} \prod_{j=0}^{k-1} r(m - j) w_B(1)\right\} = \delta^2 r(m) w_B(1),
\]

so

\[
W(m, 2) < \max \left\{\delta^m \prod_{j=0}^{m-2} r(m - j) w_A(1), \delta r(2) w_B(1)\right\}.
\]

Therefore, using equation (18) and Lemma 7.2 one more time, we have

\[
W(m, 2) < \max \{w_A(m), w_B(2)\}.
\]

By the same logic, \(W(2, m) < \max \{w_B(m), w_A(2)\}\).
Lemma 7.4 (Induction Step) Consider any state $\vec{m} \geq (3, 3)$. If
\[ W(\vec{m}^B) \leq \max \{ w_A(m_A), w_B(m_B - 1) \} \] (31)
and
\[ W(\vec{m}^A) \leq \max \{ w_A(m_A - 1), w_B(m_B) \} \] (32)
then
\[ W(\vec{m}) \leq \max \{ w_A(m_A), w_B(m_B) \} \] (33)

Proof of Lemma 7.4. By Lemma 7.1,
\[ W(\vec{m}) \leq \max \{ \delta r(m_A) W(\vec{m}^A), \delta r(m_B) W(\vec{m}^B) \} \] (34)

Using (31) and (32), and then noting that $w_j(m_j) = \delta r(m_j) w_j(m_j - 1)$ for $j = A, B$, and substituting, (34) becomes
\[ W(\vec{m}) \leq \max \left\{ \begin{array}{c}
\max \{ w_A(m_A), \delta r(m_A) w_B(m_B) \}, \\
\max \{ \delta r(m_B) w_A(m_A), w_B(m_B) \}
\end{array} \right\} \leq \max \{ w_A(m_A), w_B(m_B) \} \]

Proof of Proposition 5.1. The proof of Proposition 3.1 implies that the equilibrium characterized in Proposition 3.1 is still the unique MPE if and only if
\[ v \geq -w \sum_{i=1}^{m} \left( 1 - \delta \prod_{k=1}^{i} r(k) \right) \]
Thus, for any given $w < 0$, there is a $\overline{v}(w)$ such that this condition is satisfied if $v > \overline{v}(w)$. Similarly, for any given $w < 0$ there is a $\overline{\delta}(w) \in (0, 1)$ such that this condition is satisfied if $\delta > \overline{\delta}(w)$. ■