

COMPARATIVE ADVANTAGE AND OPTIMAL TRADE POLICY*

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The theory of comparative advantage is at the core of neoclassical trade theory. Yet we know little about its implications for how nations should conduct their trade policy. For example, should import sectors with weaker comparative advantage be protected more? Conversely, should export sectors with stronger comparative advantage be subsidized less? In this article we take a first stab at exploring these issues. Our main results imply that in the context of a canonical Ricardian model, optimal import tariffs should be uniform, whereas optimal export subsidies should be weakly decreasing with respect to comparative advantage, reflecting the fact that countries have more room to manipulate prices in their comparative-advantage sectors. Quantitative exercises suggest substantial gains from such policies relative to simpler tax schedules. *JEL* Codes: F10, F11, F13.

I. INTRODUCTION

Two of the most central questions in international economics are “Why do nations trade?” and “How should a nation conduct its trade policy?” The theory of comparative advantage is one of the most influential answers to the former question. Yet it has had little impact on answers to the latter question. Our goal in this article is to explore the relationship between comparative advantage and optimal trade policy.

Our main result can be stated as follows. The trade taxes that maximize domestic welfare in the models we consider, which we label optimal trade taxes, should be uniform across imported goods and weakly monotone with respect to comparative advantage across exported goods. Examples of optimal trade taxes include (i) a zero import tariff accompanied by export taxes that are

*We thank Kyle Bagwell, Gene Grossman, Elhanan Helpman, Andrés Rodríguez-Clare, Bob Staiger, three anonymous referees, and seminar participants at Brown, Princeton, Sciences Po, ECARES, Dartmouth, and the Cowles Foundation conference for useful comments. Rodrigo Rodrigues Adao provided superb research assistance. Costinot and Donaldson thank the National Science Foundation (under Grant SES-1227635) for research support. Vogel thanks the Princeton International Economics Section for their support.

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The Quarterly Journal of Economics (2015), 659–702. doi:10.1093/qje/qjv007.

Advance Access publication on February 3, 2015.

weakly increasing with comparative advantage, or (ii) a uniform, positive import tariff accompanied by export subsidies that are weakly decreasing with comparative advantage. While the latter pattern accords well with the observation that countries tend to protect their least competitive sectors in practice, in our model larger subsidies do not stem from a greater desire to expand production in less competitive sectors. Rather, they reflect tighter constraints on the ability to exploit monopoly power by contracting exports. Put simply, countries have more room to manipulate world prices in their comparative-advantage sectors.

Our starting point is a canonical Ricardian model of trade. We focus on this model because it is the oldest and simplest theory of comparative advantage as well as the new workhorse model for quantitative work in the field; see Eaton and Kortum (2012). Although this theoretical framework cannot speak to the distributional consequences of trade policy, which are crucial to explain the political economy of trade protection (see, e.g., Grossman and Helpman 1994), it offers a convenient starting point to explore the long-run consequences of targeted trade policy for sectoral specialization and welfare. These normative considerations, which date back to Torrens (1844) and Mill (1844), are the main focus of the present article.

We begin by considering a world economy with two countries, Home and Foreign, one factor of production, labor, a continuum of goods, and constant elasticity of substitution (CES) utility, as in Dornbusch, Fischer, and Samuelson (1977), Wilson (1980), Eaton and Kortum (2002), and Alvarez and Lucas (2007). Labor productivity can vary arbitrarily across sectors in both countries. Home sets trade taxes to maximize domestic welfare, whereas Foreign is passive. In the interest of clarity we assume no other trade costs in our baseline model.

To characterize the structure of optimal trade taxes, we use the primal approach and consider first a fictitious planning problem in which the domestic government directly controls consumption and output decisions. Using Lagrange multiplier methods, we then show how to transform this infinite dimensional problem with constraints into a series of simple unconstrained, low-dimensional problems. This allows us to derive sharp predictions about the structure of the optimal allocation. Finally, we demonstrate how that allocation can be implemented through trade taxes and relate optimal trade taxes to comparative advantage.

Our approach is flexible enough to be used in more general environments that feature non-CES utility and arbitrary neoclassical production functions. In all extensions we demonstrate that our techniques remain well suited to analyzing optimal trade policy and show that our main insights are robust. Perhaps surprisingly, given the leap in generality, our main prediction—that optimal trade taxes are uniform across imported goods and weakly monotone with respect to comparative advantage across goods—is only somewhat more nuanced without CES utility or with arbitrary neoclassical production functions.

The approach developed here can also be used for quantitative work. We apply our theoretical results to study the design of optimal trade policy in a world economy comprising two countries: the United States and the Rest of the World. We consider two separate exercises. In the first exercise, all goods are assumed to be agricultural goods, whereas in the second, all goods are assumed to be manufactured goods.¹ We find that U.S. gains from trade under optimal trade taxes are 20% larger than those obtained under *laissez-faire* for the agricultural case and 33% larger for the manufacturing case. Interestingly, a significant fraction of these gains arises from the use of trade taxes that are monotone in comparative advantage. Under an optimal uniform tariff, gains from trade for both the agriculture and manufacturing exercises would only be 9% larger than those obtained under *laissez-faire*. Although these two-country examples are admittedly stylized, they suggest that the economic forces emphasized in this paper may be quantitatively important as well.

Our article makes two distinct contributions to the existing literature. The first one, at the intersection of international trade and public finance, is related to the classical problem of optimum taxation in an open economy. In his survey of the literature, Dixit (1985) sets up the general problem of optimal taxes in an open economy as a fictitious planning problem and derives the associated first-order conditions. As Bond (1990) demonstrated, such conditions impose very weak restrictions on the structure of

1. In both exercises, we extend our baseline model to incorporate uniform iceberg trade costs between countries. Our main prediction—that optimal trade taxes are uniform across imported goods and weakly monotone with respect to comparative advantage across goods—holds without further qualification in this environment, as we show formally in Online Appendix C.2.

optimal trade taxes. Hence, optimal tariff arguments are typically cast using simple general equilibrium models featuring only two goods or partial equilibrium models.² In such environments, characterizing optimal trade taxes reduces to solving the problem of a single-good monopolist/monopsonist and leads to the prediction that the optimal tariff should be equal to the inverse of the (own-price) elasticity of the foreign export supply curve.³

In this article we go beyond the previous prediction by studying the relationship between comparative advantage and optimal trade taxes in the context of a canonical Ricardian model. In this environment, countries buy and sell many goods whose prices depend on the entire vector of net imports through their effects on wages. Thus the (own-price) elasticity of the foreign export supply curve no longer provides a sufficient statistic for optimal trade taxes. Nevertheless our analysis shows that for any wage level, optimal trade taxes must satisfy simple and intuitive properties. What matters for one of our main results is not the entire schedule of own-price and cross-price elasticities faced by a country acting as a monopolist, which determines the optimal level of wages in a nontrivial manner, but the cross-sectional variation in own-price elasticities across sectors holding wages fixed, which is tightly connected to a country's comparative advantage.

The article most closely related to ours is Itoh and Kiyono (1987), which shows that in a Ricardian model with Cobb-Douglas preferences, export subsidies that are concentrated on "marginal" goods are always welfare-enhancing. Though the logic behind their result is distinct from ours—a point we come back to in Section IV—it resonates well with our finding that at the optimum, export subsidies should be weakly decreasing with comparative advantage, so that "marginal" goods should indeed be subsidized more. Our analysis extends the results of Itoh and Kiyono (1987) by considering a Ricardian environment with general CES utility and, more important, by solving for optimal trade taxes rather than providing examples of welfare-enhancing

2. Feenstra (1986) is a notable exception. It analyzes a general equilibrium trade model with three goods and demonstrates how the introduction of a third good may lead to counterintuitive results, such as welfare-enhancing export subsidies, through demand linkages.

3. This idea is at the center of recent work emphasizing the role of terms-of-trade manipulation in the analysis of optimal tariffs and its implication for the WTO; see Bagwell and Staiger (1999) and Limão (2008).

policies.⁴ Beyond generality, our results also shed light on the simple economics behind optimal trade taxes in a canonical Ricardian model: taxes should be monotone in comparative advantage because countries have more room to manipulate prices in their comparative-advantage sectors.

More broadly, these novel results have implications for the recent debate regarding the consequences of micro-level heterogeneity for the welfare gains from trade; see Helpman (2013). In recent work, Arkolakis, Costinot, and Rodríguez-Clare (2012) have shown that depending on how the question is framed, answers to micro-level questions may be of no consequence for predicting how international trade affects welfare within a broad class of models. These results rely on calibrating certain macro responses, thereby holding them fixed across models. Melitz and Redding (2013) offer a different perspective in which these behavioral responses are not held fixed. Regardless of this methodological debate, our article emphasizes policy margins that bring out the importance of micro structure. Our qualitative results—that trade taxes should be monotone in comparative advantage—and our quantitative results—that such trade taxes lead to substantially larger welfare gains than uniform trade taxes—illustrate that the design of and gains associated with optimal trade policy may crucially depend on the extent of micro-level heterogeneity. Here, micro-level data matter, both qualitatively and quantitatively, for answering a key normative question in the field: how should a nation conduct its trade policy?⁵

The second contribution of our article is technical. The problem of finding optimal trade taxes in a canonical Ricardian model is infinite-dimensional (since there is a continuum of goods), nonconcave (since indirect utility functions are quasi-convex in

4. Opp (2009) also studies optimal trade taxes in a two-country Ricardian model with CES utility, but his formal analysis only allows for import tariffs. Because of the Lerner symmetry theorem, one might have conjectured that this restriction is without loss of generality. Our analysis formally establishes that this is not so. Although the overall level of optimal taxes is indeterminate, whether export instruments are allowed matters in economies with more than two goods. Intuitively, import tariffs and subsidies can be used to manipulate the price of one imported good relative to another good, regardless of whether the other good is imported or exported, but not the relative price of two exported goods.

5. Though we have restricted ourselves to a Ricardian model for which the relevant micro-level data are heterogeneous productivity levels across goods, not firms, the exact same considerations would make firm-level data critical inputs for the design of optimal policy in imperfectly competitive models.

prices), and nonsmooth (since the world production possibility frontier has kinks). To make progress on this question, we follow a three-step approach. First, we use the primal approach to go from taxes to quantities. Second, we identify concave sub-problems for which general Lagrangian necessity and sufficiency theorems apply. Third, we use the additive separability of preferences to break down the maximization of a potentially infinite-dimensional Lagrangian into multiple low-dimensional maximization problems that can be solved by simple calculus. Beyond the various extensions presented herein, the same approach could be used to study optimal trade taxes in economies with alternative market structures such as Bertrand competition, as in Bernard et al. (2003), or monopolistic competition, as in Melitz (2003).

From a technical standpoint, our approach is also related to recent work by Amador, Werning, and Angeletos (2006) and Amador and Bagwell (2013) who have used general Lagrange multiplier methods to study optimal delegation problems, including the design of optimal trade agreements, and to Costinot, Lorenzoni, and Werning (2013), who have used these methods together with the time-separable structure of preferences typically used in macro applications to study optimal capital controls. We briefly come back to the specific differences between these various approaches in Section III. For now, we note that as in Costinot, Lorenzoni, and Werning (2013), our approach relies heavily on the observation, first made by Everett (1963), that Lagrange multiplier methods are particularly well suited for studying “cell problems,” that is, additively separable maximization problems with constraints.⁶ Given the importance of additively separable utility in many field of economics, including international trade, we believe that these methods could prove useful beyond the question of how comparative advantage shapes optimal trade taxes.

The rest of the article is organized as follows. Section II describes our baseline Ricardian model. Section III sets up

6. In spite of this mathematical connection, there is no direct relationship between the results of Costinot, Lorenzoni, and Werning (2013), derived in a dynamic endowment economy, and the results of this article, derived in a static Ricardian economy. From an economic standpoint, our predictions about the structure of optimal taxes rely crucially on the endogenous allocation of labor across sectors according to comparative advantage; they therefore have no counterparts in an endowment economy.

and solves the planning problem of a welfare-maximizing country manipulating its terms of trade. Section IV shows how to decentralize the solution of the planning problem through trade taxes and derive our main theoretical results. Section V explores the robustness of our main insights to the introduction of non-CES utility and general production functions. Section VI applies our theoretical results to the design of optimal trade taxes in the agricultural and manufacturing sectors. Section VII offers some concluding remarks. All formal proofs can be found in Online Appendix A.

II. BASIC ENVIRONMENT

II.A. A Ricardian Economy

Consider a world economy with two countries, Home and Foreign, one factor of production, labor, and a continuum of goods indexed by i .⁷ Preferences at home are represented by the CES utility,

$$U(c) \equiv \int_i u_i(c_i) di,$$

where $c \equiv (c_i) \geq 0$ denotes domestic consumption; $u_i(c_i) \equiv \frac{\beta_i(c_i^{1-\frac{1}{\sigma}}-1)}{1-\frac{1}{\sigma}}$ denotes utility per good; $\sigma \geq 1$ denotes the elasticity of substitution between goods; and (β_i) are exogenous preference parameters such that $\int_i \beta_i di = 1$. Preferences abroad have a similar form with asterisks denoting foreign variables. Production is subject to constant returns to scale in all sectors. a_i and α_i^* denote the constant unit labor requirements at home and abroad, respectively. Labor is perfectly mobile across sectors and immobile across countries. L and L^* denote labor endowments at home and abroad, respectively.

II.B. Competitive Equilibrium

We are interested in situations in which the domestic government imposes ad valorem trade taxes-cum-subsidies, $t \equiv (t_i)$, whereas the foreign government does not impose any

7. All subsequent results generalize trivially to economies with a countable number of goods. Whenever the integral sign \int appears, one should simply think of a Lebesgue integral. If the set of goods is finite or countable, \int is equivalent to \sum .

tax. Each element $t_i \geq 0$ corresponds to an import tariff if good i is imported or an export subsidy if it is exported. Conversely, each element $t_i \leq 0$ corresponds to an import subsidy or an export tax. Tax revenues are rebated to domestic consumers through a lump-sum transfer, T . Here we characterize a competitive equilibrium for arbitrary taxes. Next we will describe the domestic government's problem that determines optimal taxes.

At home, domestic consumers choose consumption to maximize utility subject to their budget constraints; domestic firms choose output to maximize profits; the domestic government balances its budget; and the labor market clears:

$$(1) \quad c \in \operatorname{argmax}_{\tilde{c}_i \geq 0} \left\{ \int_i u_i(\tilde{c}_i) di \mid \int_i p_i(1 + t_i)\tilde{c}_i di \leq wL + T \right\},$$

$$(2) \quad q_i \in \operatorname{argmax}_{\tilde{q}_i \geq 0} \{ p_i(1 + t_i)\tilde{q}_i - wa_i\tilde{q}_i \},$$

$$(3) \quad T = \int_i p_i t_i (c_i - q_i) di,$$

$$(4) \quad L = \int_i a_i q_i di,$$

where $p \equiv (p_i) \geq 0$ is the schedule of world prices, $w \geq 0$ is the domestic wage, and $q \equiv (q_i) \geq 0$ is domestic output. Similarly, utility maximization by foreign consumers, profit maximization by foreign firms, and labor market clearing abroad imply

$$(5) \quad c^* \in \operatorname{argmax}_{\tilde{c}_i^* \geq 0} \left\{ \int_i u_i^*(\tilde{c}_i^*) di \mid \int_i p_i \tilde{c}_i^* di \leq w^* L^* \right\},$$

$$(6) \quad q_i^* \in \operatorname{argmax}_{\tilde{q}_i^* \geq 0} \{ p_i \tilde{q}_i^* - w^* a_i^* \tilde{q}_i^* \},$$

$$(7) \quad L^* = \int_i a_i^* q_i^* di,$$

where $w^* \geq 0$ is the foreign wage and $q^* \equiv (q_i^*) \geq 0$ is foreign output. Finally, good market clearing requires

$$(8) \quad c_i + c_i^* = q_i + q_i^*.$$

In the rest of this article we define a competitive equilibrium with taxes as follows.

DEFINITION 1. A competitive equilibrium with taxes corresponds to a schedule of trade taxes $t \equiv (t_i)$, a lump-sum transfer, T , a pair of wages, w and w^* , a schedule of world prices, $p \equiv (p_i)$, a pair of consumption schedules, $c \equiv (c_i)$ and $c^* \equiv (c_i^*)$, and a pair of output schedules, $q \equiv (q_i)$ and $q^* \equiv (q_i^*)$, such that conditions (1)–(8) hold.

By Walras's law, competitive prices are only determined up to a normalization. For expositional purposes, we set prices throughout our analysis so that the marginal utility of income in Foreign, that is the Lagrange multiplier associated with the budget constraint in equation (5), is equal to 1. Hence the foreign wage, w^* , also represents the real income of the foreign consumer.

II.C. The Domestic Government's Problem

We assume that Home is a strategic country that sets ad valorem trade taxes $t \equiv (t_i)$ and a lump-sum transfer T to maximize domestic welfare, whereas Foreign is passive.⁸ Formally, the domestic government's problem is to choose the competitive equilibrium with taxes, $(t, T, w, w^*, p, c, c^*, q, q^*)$, that maximizes the utility of its representative consumer, $U(c)$. This leads to the following definition.

DEFINITION 2. The domestic government's problem is $\max_{t, T, w \geq 0, w^* \geq 0, p \geq 0, c, c^*, q, q^*} U(c)$ subject to conditions (1)–(8).

The goal of the next two sections is to characterize how unilaterally optimal trade taxes, that is, taxes that prevail at a solution to the domestic government's problem, vary with Home's comparative advantage, as measured by the relative unit labor requirements $\frac{a_i^*}{a_i}$. To do so we follow the public finance literature and use the primal approach as in, for instance, Dixit (1985).⁹ Namely, we first approach the optimal policy problem of the domestic government in terms of a relaxed planning problem in which domestic consumption and domestic output can be

8. In other words, we focus on the best response of the domestic government to zero taxes abroad. We conjecture that many of our qualitative predictions about the domestic government's best response would extend to other foreign policy vectors, and in turn, apply to the Nash equilibrium of a game in which both countries are strategic. Of course, from a quantitative standpoint, welfare at Home may be much lower in such a Nash equilibrium.

9. An early application of the primal approach in international trade can be found in Baldwin (1948).

chosen directly (Section III). We then establish that the optimal allocation can be implemented through trade taxes and characterize the structure of these taxes (Section IV).¹⁰

III. OPTIMAL ALLOCATION

III.A. Home's Planning Problem

Throughout this section we focus on a fictitious environment in which there are no taxes and no competitive markets at home. Rather, the domestic government directly controls domestic consumption, c , and domestic output, q , subject to the resource constraint,

$$(9) \quad \int_i a_i q_i di \leq L.$$

In other words, we ignore the equilibrium conditions associated with utility and profit maximization by domestic consumers and firms, we ignore the government's budget constraint, and we relax the labor market clearing condition into inequality (9). We refer to this relaxed maximization problem as Home's planning problem.

DEFINITION 3. Home's planning problem is $\max_{w^* \geq 0, p \geq 0, c \geq 0, q^* \geq 0, q^*}$
 $U(c)$ subject to conditions (5)–(9).

To prepare our discussion of optimal trade taxes, we focus on the foreign wage, w^* , net imports $m \equiv c - q$, and domestic output, q , as the three key control variables of the domestic government. To do so, we first establish that the conditions for an equilibrium in the Rest of the World—namely, foreign utility maximization, foreign profit maximization, and good and labor market clearing—can be expressed more compactly as a function of net imports and the foreign wage alone.

10. As will become clear, our main results do not hinge on this particular choice of instruments. We choose to focus on trade taxes-*cum*-subsidies for expositional convenience because they are the simplest tax instruments required to implement the optimal allocation. It is well known that one could allow for consumption taxes, production taxes, or import tariffs that are not accompanied by export subsidies. One would then find that constraining consumption taxes to be equal to production taxes or import tariffs to be equal to export subsidies, that is, restricting attention to trade taxes-*cum*-subsidies has no effect on the allocation that a welfare-maximizing government would choose to implement.

LEMMA 1. (w^*, p, m, c^*, q^*) satisfies conditions (5)–(8) if and only if

$$(10) \quad p_i = p_i(m_i, w^*) \equiv \min\{u_i^{*'}(-m_i), w^* a_i^*\},$$

$$(11) \quad c_i^* = c_i^*(m_i, w^*) \equiv \max\{-m_i, d_i^*(w^* a_i^*)\},$$

$$(12) \quad q_i^* = q_i^*(m_i, w^*) \equiv \max\{0, m_i + d_i^*(w^* a_i^*)\},$$

for all i , with $d_i^*(\cdot) \equiv u_i^{*'}(\cdot)$, $u_i^{*'}(-m_i) \equiv \infty$ if $m_i \geq 0$, and

$$(13) \quad \int_i a_i^* q_i^*(m_i, w^*) di = L^*,$$

$$(14) \quad \int_i p_i(m_i, w^*) m_i di = 0.$$

According to Lemma 1, when Home’s net imports are high, $m_i + d_i^*(w^* a_i^*) > 0$, foreign firms produce good i , the world price is determined by their marginal costs, $w^* a_i^*$, and foreign consumers demand $d_i^*(w^* a_i^*)$.¹¹ Conversely, when Home’s net imports are low, $m_i + d_i^*(w^* a_i^*) < 0$, foreign firms do not produce good i , foreign consumption is equal to Home’s net exports, $-m_i$, and the world price is determined by the marginal utility of the foreign consumer, $p_i(m_i, w^*) = u_i^{*'}(-m_i)$. Equations (13) and (14), in turn, derive from the foreign labor market clearing condition and the foreign consumer’s budget constraint.

Let $p(m, w^*) \equiv (p_i(m_i, w^*))$, $c^*(m, w^*) \equiv (c_i^*(m_i, w^*))$, and $q^*(m, w^*) \equiv (q_i^*(m_i, w^*))$ denote the schedule of equilibrium world prices, foreign consumption, and foreign output as a function of Home’s net imports and the foreign wage. Using Lemma 1, we can characterize the set of solutions to Home’s planning problem as follows.

LEMMA 2. Suppose that $(w^{0*}, p^0, c^0, c^{0*}, q^0, q^{0*})$ solves Home’s planning problem. Then $(w^{0*}, m^0 = c^0 - q^0, q^0)$ solves

$$(P) \quad \max_{w^* \geq 0, m, q \geq 0} \int_i u_i(q_i + m_i) di$$

11. Recall that good prices are normalized so that the marginal utility of income in Foreign is equal to 1.

subject to

$$(15) \quad \int_i a_i q_i di \leq L,$$

$$(16) \quad \int_i a_i^* q_i^*(m_i, w^*) di \leq L^*,$$

$$(17) \quad \int_i p_i(m_i, w^*) m_i di \leq 0.$$

Conversely, suppose that (w^{0*}, m^0, q^0) solves equation (P). Then there exists a solution to Home's planning problem, $(w^{0*}, p^0, c^0, c^{0*}, q^0, q^{0*})$, such that $p^0 = p(m^0, w^{0*})$, $c^0 = m^0 + q^0$, $c^{0*} = c^*(m^0, w^{0*})$, and $q^{0*} = q^*(m^0, w^{0*})$.

The first inequality, equation (15), corresponds to the resource constraint at Home and does not merit further comment. The final two inequalities, equations (16) and (17), are the counterparts of equations (13) and (14) in Lemma 1. One can think of inequality (17) as Home's trade balance condition. It characterizes the set of feasible net imports. If Home were a small open economy, then it would take $p_i(m_i, w^*)$ as exogenously given and the solution to equation (P) would coincide with the free trade equilibrium. Here, in contrast, Home internalizes the fact that net import decisions affect world prices, both directly through their effects on the marginal utility of the foreign consumer and indirectly through their effects on the foreign wage, as reflected in inequality (16).

Two technical aspects of Home's planning problem are worth mentioning at this point. First, in spite of the fact that the foreign consumer's budget constraint and the foreign labor market clearing condition must bind in a competitive equilibrium, as shown in Lemma 1, the solution to Home's planning problem can be obtained as the solution to a new relaxed problem (P) that only features inequality constraints. This will allow us to invoke Lagrangian necessity theorems in Section III.B. Second, Home's planning problem can be decomposed into an inner and an outer problem. Define \mathcal{W}^* as the set of values for w^* such that there exist import and output levels $m, q \geq 0$ that satisfy

equations (15)–(17). The inner problem takes $w^* \in \mathcal{W}^*$ as given and maximizes over import and output levels,

$$(P_{w^*}) \quad V(w^*) \equiv \max_{m, q \geq 0} \int_i u_i(q_i + m_i) di$$

subject to equations (15)–(17). The outer problem then maximizes the value function from the inner problem over the foreign wage,

$$\max_{w^* \in \mathcal{W}^*} V(w^*).$$

It is the particular structure of the inner problem (P_{w^*}) that will allow us to make progress in characterizing the optimal allocation. In the next two subsections, we take the foreign wage w^* as given and characterize the main qualitative properties of the solutions to (P_{w^*}). Since such properties will hold for all feasible values of the foreign wage, they will hold for the optimal one, $w^{0*} \in \operatorname{argmax}_{w^* \in \mathcal{W}^*} V(w^*)$, and so by Lemma 2 they will apply to any solution to Home’s planning problem.¹² Of course, for the purposes of obtaining quantitative results we also need to solve for the optimal foreign wage, w^{0*} , which we do in Section VI.

Two observations will facilitate our analysis of the inner problem (P_{w^*}). First, as we formally demonstrate, (P_{w^*}) is concave, which implies that its solutions can be computed using Lagrange multiplier methods. Second, both the objective function and the constraints in (P_{w^*}) are additively separable in (m_i, q_i) . In the words of Everett (1963), (P_{w^*}) is a “cell-problem.” Using Lagrange multiplier methods, we therefore are able to transform an infinite dimensional problem with constraints into a series of simple unconstrained, low-dimensional problems.

12. This is a key technical difference between our approach and the approaches used in Amador, Werening, and Angeletos (2006), Amador and Bagwell (2013), and Costinot, Lorenzoni, and Werning (2013). The basic strategy here does not consist of showing that the maximization problem of interest can be studied using general Lagrange multiplier methods. Rather, the core of our approach lies in finding a subproblem to which these methods can be applied. Section V illustrates the usefulness of this approach by showing how our results can easily be extended to environments with non-CES utility and arbitrary neoclassical production functions.

III.B. Lagrangian Formulation

The Lagrangian associated with (P_{w^*}) is given by

$$\begin{aligned} \mathcal{L}(m, q, \lambda, \lambda^*, \mu; w^*) \equiv & \int_i u_i(q_i + m_i)di - \lambda \int_i \alpha_i q_i di \\ & - \lambda^* \int_i \alpha_i^* q_i^*(m_i, w^*) di - \mu \int_i p_i(m_i, w^*) m_i di, \end{aligned}$$

where $\lambda \geq 0$, $\lambda^* \geq 0$, and $\mu \geq 0$ are the Lagrange multipliers associated with constraints (15)–(17). As alluded to above, a crucial property of \mathcal{L} is that it is additively separable in (m_i, q_i) . This implies that to maximize \mathcal{L} with respect to (m, q) , one simply needs to maximize the good-specific Lagrangian,

$$\begin{aligned} \mathcal{L}_i(m_i, q_i, \lambda, \lambda^*, \mu; w^*) \equiv & u_i(q_i + m_i) - \lambda \alpha_i q_i - \lambda^* \alpha_i^* q_i^*(m_i, w^*) \\ & - \mu p_i(m_i, w^*) m_i, \end{aligned}$$

with respect to (m_i, q_i) for almost all i . In short, cell problems can be solved cell by cell, or in the present context, good by good.

Building on the previous observation, the concavity of (P_{w^*}) , and Lagrangian necessity and sufficiency theorems—Theorem 1 and Theorem 1 in Luenberger (1969), respectively—we obtain the following characterization of the set of solutions to (P_{w^*}) .

LEMMA 3. For any $w^* \in \mathcal{W}^*$, (m^0, q^0) solves (P_{w^*}) if and only if (m_i^0, q_i^0) solves

$$\max_{m_i, q_i \geq 0} \mathcal{L}_i(m_i, q_i, \lambda, \lambda^*, \mu; w^*)(P_i)$$

for almost all i , with the Lagrange multipliers $(\lambda, \lambda^*, \mu) \geq 0$ such that constraints (15)–(17) hold with complementary slackness.

Let us take stock. We started this section with Home's planning problem, which is an infinite dimensional problem in consumption and output in both countries as well as world prices and the foreign wage. By expressing world prices, foreign consumption, and foreign output as a function of net imports and the foreign wage (Lemma 1), we then transformed it into a new planning problem (P) that only involves the schedule of domestic net imports, m , domestic output, q , and the foreign wage, w^* , but remains infinitely dimensional (Lemma 2). Finally, in this subsection we have taken advantage of the concavity and the additive separability of the inner problem (P_{w^*}) in (m_i, q_i) to go

from one high-dimensional problem with constraints to many two-dimensional, unconstrained maximization problems (P_i) using Lagrange multiplier methods (Lemma 3).

The goal of the next subsection is to solve these two-dimensional problems in (m_i, q_i) taking the foreign wage, w^* , and the Lagrange multipliers, $(\lambda, \lambda^*, \mu)$, as given. This is all we will need to characterize qualitatively how comparative advantage affects the solution of Home's planning problem and, as discussed in Section IV, the structure of optimal trade taxes. Once again, a full computation of optimal trade taxes will depend on the equilibrium values of $(\lambda, \lambda^*, \mu)$, found by using the constraints (15)–(17) and the value of w^* that maximizes $V(w^*)$, calculations that we defer until Section VI.

III.C. Optimal Output and Net Imports

Our objective here is to find the solution (m_i^0, q_i^0) of

$$\max_{m_i, q_i \geq 0} \mathcal{L}_i(m_i, q_i, \lambda, \lambda^*, \mu; w^*) \equiv u_i(q_i + m_i) - \lambda a_i q_i - \lambda^* a_i^* q_i^*(m_i, w^*) - \mu p_i(m_i, w^*) m_i.$$

We proceed in two steps. First, we solve for the output level $q_i^0(m_i)$ that maximizes $\mathcal{L}_i(m_i, q_i, \lambda, \lambda^*, \mu; w^*)$, taking m_i as given. Second, we solve for the net import level m_i^0 that maximizes $\mathcal{L}_i(m_i, q_i^0(m_i), \lambda, \lambda^*, \mu; w^*)$. The optimal output level is then simply given by $q_i^0 = q_i^0(m_i^0)$.

Figure I describes how the shape of $\mathcal{L}_i(m_i, q_i^0(m_i), \lambda, \lambda^*, \mu; w^*)$ varies with the relative unit labor requirement $\frac{a_i}{a_i^*}$. Formal derivations can be found in Online Appendix A.4. $M_i^I \equiv -d_i^*(w^* a_i^*) < 0$ and $M_i^{II} \equiv d_i(\lambda a_i) > 0$ denote the net import levels at which Foreign starts producing good i and Home stops producing good i , respectively. In turn, $m_i^I \equiv -\left(\frac{\sigma^* - 1}{\sigma^* - 1} \frac{\lambda a_i}{\mu \beta_i^*}\right)^{-\sigma^*} < 0$ denotes Home's optimal export level when it is the only potential producer of good i , whereas $m_i^{II} \equiv d_i((\lambda^* + \mu w^*) a_i^*) > 0$ denotes Home's optimal import level when Foreign is the only potential producer of that good.

When Home has a strong comparative advantage, $\frac{a_i}{a_i^*} < A^I \equiv \frac{\sigma^* - 1}{\sigma^*} \frac{\mu w^*}{\lambda}$, the good-specific Lagrangian is maximized at $m_i = m_i^I$, as illustrated in Figure I, Panel a. When Foreign has a strong comparative advantage, $\frac{a_i}{a_i^*} > A^{II} \equiv \frac{\sigma^* - 1}{\sigma^*} \frac{\mu w^*}{\lambda}$, the good-specific

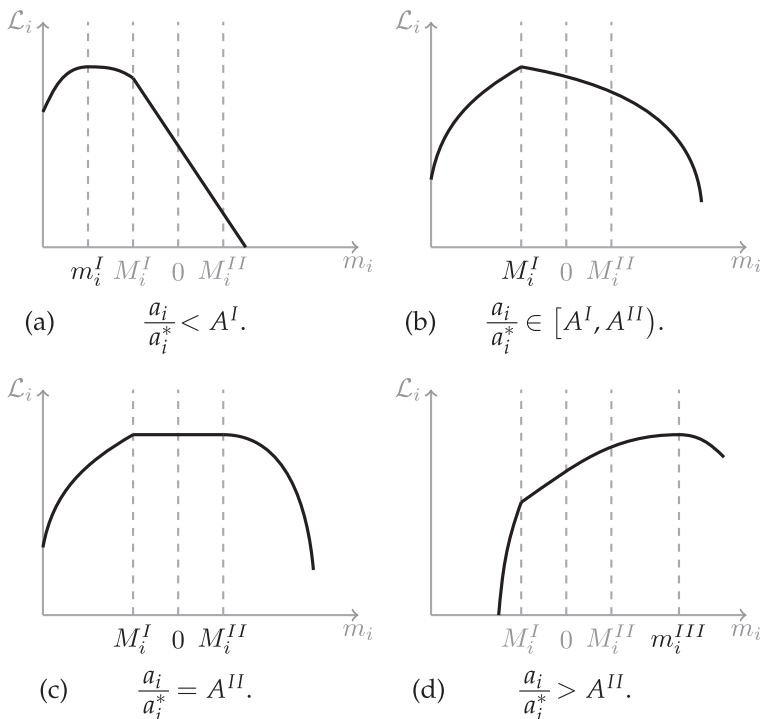


FIGURE I
Optimal Net Imports

Lagrangian is maximized at $m_i = m_i^{II}$, as illustrated in Figure I, Panel d. When relative unit labor requirements are in the intermediate range $[A^I, A^{II})$, Home's optimal export level is equal to M_i^I , as illustrated in Figure I, panel b. For these goods, Foreign is at a tipping point: it would start producing if Home's exports were to go down by any amount. In the knife-edge case, $\frac{a_i}{a_i^*} = A^{II}$, the Lagrangian is flat between M_i^I and M_i^{II} so that any import level between M_i^I and M_i^{II} is optimal, as illustrated in Figure I, Panel c. In this situation, either Home or Foreign may produce and export good i .

We summarize these observations in the following proposition.

PROPOSITION 1. If (m_i^0, q_i^0) solves equation (P_i) , then optimal net imports are such that: (i) $m_i^0 = m_i^I$, if $\frac{a_i}{a_i^*} < A^I$; (ii) $m_i^0 = M_i^I$,

if $\frac{a_i}{a_i^*} \in [A^I, A^{II})$; (iii) $m_i^0 \in [M_i^I, M_i^{II}]$ if $\frac{a_i}{a_i^*} = A^{II}$; and (iv) $m_i^0 = m_i^{II}$, if $\frac{a_i}{a_i^*} > A^{II}$, where $m_i^I, M_i^I, m_i^{II}, M_i^{II}, A^I$, and A^{II} are the functions of w^* and $(\lambda, \lambda^*, \mu)$ defined above.

Proposition 1 highlights the importance of comparative advantage, that is, the cross-sectoral variation in the relative unit labor requirement $\frac{a_i}{a_i^*}$, for the structure of optimal imports. Thus Proposition 1 implies that Home is a net exporter of good i , $m_i^0 < 0$, only if $\frac{a_i}{a_i^*} \leq A^{II}$. Using Lemmas 2 and 3 to go from equation (P_i) to Home's planning problem, this leads to the following corollary.

COROLLARY 1. At any solution to Home's planning problem, Home produces and exports goods in which it has a comparative advantage, $\frac{a_i}{a_i^*} < A^{II}$, whereas Foreign produces and exports goods in which it has a comparative advantage, $\frac{a_i}{a_i^*} > A^{II}$.

According to Corollary 1, there will be no pattern of comparative advantage reversals at an optimum. Like in a free trade equilibrium, there exists a cut-off such that Home exports a good only if its relative unit labor requirement is below the cut-off. Of course, the value of that cut-off as well as the export levels will in general be different from those in a free trade equilibrium.

Finally, note that all goods are traded at the solution to Home's planning problem. This stands in sharp contrast to what would occur if only uniform trade taxes were available, as in Dornbusch, Fischer, and Samuelson (1977) or Alvarez and Lucas (2007). We come back to this issue in more detail in Sections IV.C and VI.A.

IV. OPTIMAL TRADE TAXES

We now demonstrate how to implement the solution of Home's planning problem using trade taxes in a competitive equilibrium.

IV.A. Wedges

Trade taxes cause domestic and world prices to differ from one another. To prepare our analysis of optimal trade taxes, we start by describing the wedges, τ_i^0 , between the marginal

utility of the domestic consumer, $u'_i(c_i^0) = \beta_i(c_i^0)^{-\frac{1}{\sigma}}$, and the world price, p_i^0 , that must prevail at any solution to Home's planning problem:

$$(18) \quad \tau_i^0 \equiv \frac{u'_i(c_i^0)}{p_i^0} - 1.$$

By Lemma 1, we know that if $(w^{0*}, p^0, c^0, c^{0*}, q^0, q^{0*})$ solves Home's planning problem—and hence satisfies conditions equations (5)–(8)—then $p_i^0 = p_i(m_i^0, w^{0*})$. By Lemma 2, we also know that if $(w^{0*}, p^0, c^0, c^{0*}, q^0, q^{0*})$ solves Home's planning problem, then $(w^{0*}, m^0 = c^0 - q^0, q^0)$ solves equation (P). In turn, this implies that $(m^0 = c^0 - q^0, q^0)$ solves (P_{w^*}) for $w^* = w^{0*}$, and by Lemma 3, that (m_i^0, q_i^0) solves equation (P_i) for almost all i . Accordingly, the good-specific wedge can be expressed as

$$\tau_i^0 = \frac{u'_i(q_i^0(m_i^0) + m_i^0)}{p_i(m_i^0, w^{0*})} - 1,$$

for almost all i , with $p_i(m_i^0, w^{0*})$ and $q_i^0(m_i^0)$ given by equations (10) and (39) and m_i^0 satisfying conditions (i)–(iv) in Proposition 1. This further implies

$$(19) \quad \tau_i^0 = \begin{cases} \frac{\sigma^* - 1}{\sigma^*} \mu^0 - 1, & \text{if } \frac{a_i}{a_i^*} < A^I \equiv \frac{\sigma^* - 1}{\sigma^*} \frac{\mu^0 w^{0*}}{\lambda^0}; \\ \frac{\lambda^0 a_i}{w^{0*} a_i^*} - 1, & \text{if } A^I < \frac{a_i}{a_i^*} \leq A^{II} \equiv \frac{\mu^0 w^{0*} + \lambda^0}{\lambda^0}; \\ \frac{\lambda^0}{w^{0*}} + \mu^0 - 1, & \text{if } \frac{a_i}{a_i^*} > A^{II}. \end{cases}$$

Since $A^I < A^{II}$, we see that good-specific wedges are (weakly) increasing with $\frac{a_i}{a_i^*}$. For goods that are exported, $\frac{a_i}{a_i^*} < A^{II}$, the magnitude of the wedge depends on the strength of Home's comparative advantage. It attains its minimum value, $\frac{\sigma^* - 1}{\sigma^*} \mu^0 - 1$, for goods such that $\frac{a_i}{a_i^*} < A^I$ and increases linearly with $\frac{a_i}{a_i^*}$ for goods such that $\frac{a_i}{a_i^*} \in (A^I, A^{II})$. For goods that are imported, $\frac{a_i}{a_i^*} > A^{II}$, wedges are constant and equal to their maximum value, $\frac{\lambda^0}{w^{0*}} + \mu^0 - 1$.

IV.B. Comparative Advantage and Trade Taxes

Let us now demonstrate that any solution $(w^{0*}, p^0, c^0, c^{0*}, q^0, q^{0*})$ to Home's planning problem can be implemented by constructing a schedule of trade taxes, $t^0 = \tau^0$, and a lump-sum transfer, $T^0 = \int_i p_i \tau_i^0 m_i^0 di$. Since the domestic government's budget constraint is satisfied by construction and the resource constraint (9) must bind at any solution to Home's planning problem, equations (3) and (4) trivially hold. Thus we only need to check that we can find a domestic wage, w^0 , such that the conditions for utility and profit maximization by domestic consumers and firms at distorted local prices $p_i^0(1 + t_i^0)$, that is, conditions (1) and (2), are satisfied as well.

Consider first the problem of a domestic firm. At a solution to Home's planning problem, we argued in the Online Appendix that for almost all i ,

$$u'_i(q_i^0 + m_i^0) \leq \lambda^0 a_i, \text{ with equality if } q_i^0 > 0.$$

By definition of τ^0 , we also know that $u'_i(q_i^0 + m_i^0) = u'_i(c_i^0) = p_i^0(1 + \tau_i^0)$. Thus if $t_i^0 = \tau_i^0$, then

$$(20) \quad p_i^0(1 + t_i^0) \leq \lambda^0 a_i, \text{ with equality if } q_i^0 > 0.$$

This implies that condition (2) is satisfied with the domestic wage in the competitive equilibrium given by the Lagrange multiplier on the labor resource constraint, $w^0 = \lambda^0$.

Let us turn to the domestic consumer's problem. By definition of τ^0 , if $t_i^0 = \tau_i^0$, then

$$u'_i(c_i^0) = p_i^0(1 + t_i^0).$$

Thus for any pair of goods, i_1 and i_2 , we have

$$(21) \quad \frac{u'_{i_1}(c_{i_1}^0)}{u'_{i_2}(c_{i_2}^0)} = \frac{1 + t_{i_1}^0 p_{i_1}^0}{1 + t_{i_2}^0 p_{i_2}^0}.$$

Hence, the domestic consumer's marginal rate of substitution is equal to the domestic relative price. By Lemma 1, we know that trade must be balanced at a solution to Home's planning

problem, $\int_i p_i^0 m_i^0 di = 0$. Together with $T^0 = \int_i p_i \tau_i^0 m_i^0 di = \int_i p_i t_i^0 m_i^0 di$, this implies

$$(22) \quad \int_i p_i^0 (1 + t_i^0) c_i^0 di = \int_i p_i^0 (1 + t_i^0) q_i^0 di + T^0.$$

Since conditions (4) and (20) imply $\int_i p_i^0 (1 + t_i^0) q_i^0 di = \lambda^0 L$, equation (22) implies that the domestic consumer's budget constraint must hold for $w^0 = \lambda^0$. Combining this observation with equation (21), we can conclude that condition (1) must hold as well.

At this point, we have established that any solution $(w^{0*}, p^0, c^0, c^{0*}, q^0, q^{0*})$ to Home's planning problem can be implemented by constructing a schedule of trade taxes, $t^0 = \tau^0$, and a lump-sum transfer, $T^0 = \int_i p_i \tau_i^0 m_i^0 di$. Since Home's planning problem, as described in Definition 3, is a relaxed version of the domestic government's problem, as described in Definition 2, this immediately implies that $(t^0, T^0, w^0, w^{0*}, p^0, c^0, c^{0*}, q^0, q^{0*})$ is a solution to the original problem. Conversely, suppose that $(t^0, T^0, w^0, w^{0*}, p^0, c^0, c^{0*}, q^0, q^{0*})$ is a solution to the domestic's government problem, then $(w^{0*}, p^0, c^0, c^{0*}, q^0, q^{0*})$ must solve Home's planning problem and, by condition (1), the optimal trade taxes t^0 must satisfy

$$t_i^0 = \frac{u'_i(c_i^0)}{\nu p_i^0} - 1,$$

with $\nu > 0$ the Lagrange multiplier on the domestic consumer's budget constraint. By equation (18), this implies that $1 + t_i^0 = \frac{1}{\nu}(1 + \tau_i^0)$. Combining this observation with equation (19), we obtain the following characterization of optimal trade taxes.

PROPOSITION 2. At any solution to the domestic government's problem, trade taxes, t^0 , are such that (i) $t_i^0 = (1 + \bar{t})\left(\frac{A^I}{A^H}\right) - 1$, if $\frac{a_i}{a_i^*} < A^I$; (ii) $t_i^0 = (1 + \bar{t})\left(\frac{a_i}{a_i^* A^H}\right) - 1$, if $\frac{a_i}{a_i^*} \in [A^I, A^H]$; and (iii) $t_i^0 = \bar{t}$, if $\frac{a_i}{a_i^*} > A^H$, with $\bar{t} > -1$ and $A^I < A^H$.

Proposition 2 states that optimal trade taxes vary with comparative advantage as wedges do. Trade taxes are at their lowest values, $(1 + \bar{t})\left(\frac{A^I}{A^H}\right) - 1$, for goods in which Home's comparative

advantage is the strongest, $\frac{a_i}{a_i^*} < A^I$; they are linearly increasing with $\frac{a_i}{a_i^*}$ for goods in which Home's comparative advantage is in some intermediate range, $\frac{a_i}{a_i^*} \in [A^I, A^{II}]$; they are at their highest value, \bar{t} , for goods in which Home's comparative advantage is the weakest, $\frac{a_i}{a_i^*} > A^{II}$.

Since only relative prices and hence relative taxes matter for domestic consumers and firms, the overall level of taxes is indeterminate. This is an expression of Lerner symmetry, which is captured by the free parameter $\bar{t} > -1$ in the previous proposition. Figure II illustrates two polar cases. In Panel a, there are no import tariffs, $\bar{t} = 0$, and all exported goods are subject to an export tax that rises in absolute value with comparative advantage. In Panel b in contrast, all imported goods are subject to a tariff $\bar{t} = \frac{A^{II}}{A^I} - 1 \geq 0$, whereas exported goods receive a subsidy that falls with comparative advantage. For expositional purposes, we focus in the rest of our discussion on the solution with zero import tariffs, $\bar{t} = 0$, as in Figure II, Panel a.

To gain intuition about the economic forces that shape optimal trade taxes, consider first the case in which foreign preferences are Cobb-Douglas, $\sigma^* = 1$, as in Dornbusch, Fischer, and Samuelson (1977). In this case, $A^I = 0$ so that the first region, $\frac{a_i}{a_i^*} < A^I$, is empty. In the second region, $\frac{a_i}{a_i^*} \in [A^I, A^{II}]$, there is limit pricing: Home exports the goods and sets export taxes $t_i^0 < \bar{t} = 0$ such that foreign firms are exactly indifferent between producing and not producing those goods, that is, such that the world price satisfies $p_i^0 = \frac{\lambda^0 a_i}{v(1+t_i^0)} = w^{0*} a_i^*$. The less productive are foreign firms relative to domestic firms, the more room Home has to manipulate prices, and the bigger the export tax (in absolute value). Finally, in the third region, $\frac{a_i}{a_i^*} > A^{II}$, relative prices are pinned down by the relative unit labor requirements in Foreign. Since Home has no ability to manipulate these relative prices, a uniform import tariff (here normalized to 0) is optimal.

In the more general case, $\sigma^* \geq 1$, as in Wilson (1980), Eaton and Kortum (2002), and Alvarez and Lucas (2007), the first region, $\frac{a_i}{a_i^*} < A^I$, is no longer necessarily empty. The intuition, however, remains simple. In this region the domestic government has incentives to charge a constant monopoly markup,

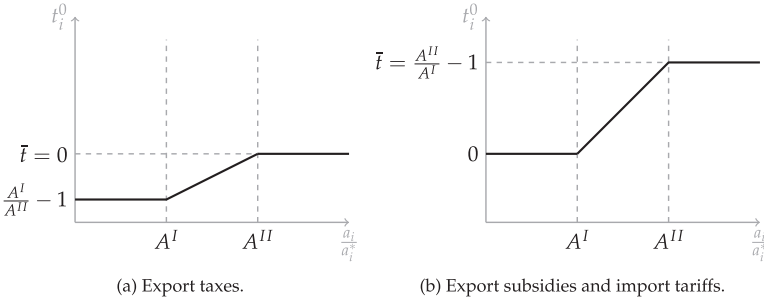


FIGURE II
Optimal Trade Taxes

proportional to $\frac{\sigma^*}{\sigma^*-1}$. Specifically, the ratio between the world price and the domestic price is equal to $\frac{1}{1+t_i^0} = \frac{\sigma^*}{\sigma^*-1} \frac{v}{\mu^0}$. In the region $\frac{a_i}{a_i^*} \in [A^I, A^{II}]$, limit pricing is still optimal. But because A^I is increasing in σ^* , the extent of limit pricing, all else equal, decreases with the elasticity of demand in the foreign market.

IV.C. Discussion

Proposition 2 accords well with the observation that governments often protect a small number of less competitive industries. Yet in our model, such targeted subsidies do not stem from a greater desire to expand production in these sectors. On the contrary, they reflect tighter constraints on the ability to exploit monopoly power by contracting exports. According to Proposition 2, Home can only charge constant monopoly markups for exported goods in which its comparative advantage is the strongest. For other exported goods, the threat of entry of foreign firms leads markups to decline together with Home’s comparative advantage.

An interesting issue is whether our results could be interpreted more generally in terms of some hypothetical “no-nontraded-goods-should-be-created-by-the-optimal-tax” rule. As we already pointed out in Section III.C, all goods are traded at the optimal allocation. This suggests that the domestic government’s desire to keep all goods traded may be the key driver of the relationship between comparative advantage and optimal trade taxes uncovered in Proposition 2. To clarify this issue, we

consider in Online Appendix B an alternative version of the domestic government's problem in which tax instruments are restricted to import taxes and subsidies. If the hypothetical no-nontraded-goods-should-be-created-by-the-optimal-tax rule were operative, we would expect Home to impose different import taxes on different goods depending on Home's comparative advantage. Instead, we show that Home always finds it optimal to impose a uniform tariff, thereby creating nontraded goods. This casts doubt on the existence of a no-nontraded-goods-should-be-created-by-the-optimal-tax rule. In our model, limit pricing per se is the economic channel that creates a link between comparative advantage and optimal trade taxes.

Another interesting issue is whether the structure of optimal trade taxes described in Proposition 2 crucially relies on the assumption that domestic firms are perfectly competitive. Since Home's government behaves like a domestic monopolist competing à la Bertrand against foreign firms, one may conjecture that if each good were produced by only one domestic firm, then Home would no longer have to use trade taxes to manipulate prices: domestic firms would already manipulate prices under *laissez-faire*. This conjecture, however, is incorrect for two reasons.

The first reason is that although the government behaves like a monopolist, the domestic government's problem involves nontrivial general equilibrium considerations. Namely, it internalizes the fact that by producing more goods at home, it lowers foreign labor demand, which must cause a decrease in the foreign wage and an improvement of Home's terms of trade. These considerations are captured by the foreign resource constraint equation (16) in Home's planning problem. As we discuss in more detail in Section VI.A, provided that the Lagrange multiplier associated with that constraint, λ^{0*} , is not zero, the optimal level of the markup charged by the domestic government will be different from what an individual firm would have charged, that is, $\frac{\sigma^*}{\sigma^*-1}$.

The second reason is that to manipulate prices, Home's government needs to affect the behaviors of both firms and consumers: net imports depend on both supply and demand. If each good were produced by only one domestic firm, Home's government would still need to impose good-varying consumption taxes that mimic the trade taxes described above (plus output subsidies that reflect general equilibrium considerations). Intuitively, if each good were produced by only one domestic firm, consumers would face monopoly markups in each country,

whereas optimality requires a wedge between consumer prices at home and abroad.

As mentioned in Section I, our findings are related to the results of Itoh and Kiyono (1987). They have shown that in the Ricardian model with Cobb-Douglas preferences considered by Dornbusch, Fischer, and Samuelson (1977), export subsidies may be welfare enhancing. A key feature of the welfare-enhancing subsidies that they consider is that they are not uniform across goods; instead, they are concentrated on “marginal” goods. This is consistent with our observation that at the optimum, export taxes should be weakly decreasing (in absolute value) with Home’s relative unit labor requirements, $\frac{a_i}{a_i^*}$, so that “marginal” goods should indeed be taxed less. The economic forces behind their results, however, are orthogonal to those emphasized in Proposition 2. Their results reflect the general equilibrium considerations alluded to before: by expanding the set of goods produced at home, the domestic government can lower the foreign wage and improve its terms-of-trade. In contrast, the heterogeneity of taxes across goods in Proposition 2 derives entirely from the structure of the inner problem (P_{w^*}), which takes the foreign wage as given. This implies, in particular, that Proposition 2 would still hold if Home were a “small” country in the sense that it could not affect the foreign wage.¹³

V. ROBUSTNESS

In this section we incorporate general preferences and technology into the Ricardian model presented in Section II. Our goal is twofold. First, we want to demonstrate that Lagrange multiplier methods, in particular our strategy of identifying concave cell problems, remain well suited to analyzing optimal trade policy in these alternative environments. Second, we want to explore the extent to which the predictions derived in Section IV hinge on the assumption of CES utility or the fact that comparative advantage derives from differences in technologies alone. To save space, we focus on sketching alternative environments and summarizing their main implications.

13. The observation that optimal trade policy may not converge toward zero as the economy becomes arbitrarily small is related to the point made by Gros (1987) in a monopolistically competitive model and Alvarez and Lucas (2007) in a Ricardian model.

V.A. Preferences

While the assumption of CES utility is standard in the Ricardian literature—from Dornbusch, Fischer, and Sameulson (1977) to Eaton and Kortum (2002)—it implies strong restrictions on the demand side of the economy: own-price elasticities and elasticities of substitution are both constant and pinned down by a single parameter, σ . In practice, price elasticities may vary with quantities consumed and substitution patterns may vary across goods.

Here we relax the assumptions of Section II by assuming (i) that Home’s preferences are weakly separable over a discrete number of sectors, $s \in \mathcal{S} \equiv \{1, \dots, S\}$, and (ii) that subutility within each sector, U^s , is additively separable, though not necessarily CES. Specifically, we assume that Home’s preferences can be represented by the following utility function,

$$(23) \quad U = F(U^1(c^1), \dots, U^S(c^S)),$$

where F is a strictly increasing function; $c^s \equiv (c_i)_{i \in \mathcal{I}^s}$ is the consumption of goods in sector s , with \mathcal{I}^s the set of goods that belongs to that sector; and U^s is such that

$$U^s(c^s) = \int_{i \in \mathcal{I}^s} u_i^s(c_i) di.$$

Foreign’s preferences are similar, and asterisks denote foreign variables. Section II corresponds to the special case in which

there is only one sector, $S = 1$, and U^s is CES, $u_i^s(c_i) \equiv \frac{\beta_i^s \left(c_i^{1-\frac{1}{\sigma^s}} - 1 \right)}{1 - \frac{1}{\sigma^s}}$.¹⁴

For expositional purposes, let us start by considering an intermediate scenario in which utility is not CES while maintaining the assumption that there is only one sector, $S = 1$. It should be clear that the CES assumption is not crucial for the results derived in Sections II.B–III.B. In contrast, CES plays a key role in determining the optimal level of net imports, $m_i^I = - \left(\frac{\sigma^s \lambda a_i}{\sigma^s - 1 \mu \beta_i^s} \right)^{-\sigma^s}$, in Online Appendix Section A.4 and, in turn, in establishing that trade taxes are at their lowest values, $(1 + \bar{t}) \left(\frac{A^I}{A^H} \right) - 1$, for goods in which Home’s comparative advantage is the strongest in

14. The analysis in this section trivially extends to the case in which only a subset of sectors have additively separable utility. For this subset of sectors, and this subset only, our predictions would remain unchanged.

Section IV.B. Absent CES utility, trade taxes on imported goods would still be uniform, but trade taxes on exported goods, like the optimal monopoly markup, would now also vary with the elasticity of demand.

Now let us turn to the general case with multiple sectors, $S \geq 1$. With weakly separable preferences abroad, one can check that foreign consumption in each sector, $c^{s*} \equiv (c_i^*)_{i \in \mathcal{I}^s}$, must be such that

$$c^{s*} \in \operatorname{argmax}_{\tilde{c} \geq 0} \left\{ U^{s*}(\tilde{c}) \mid \int_{i \in \mathcal{I}^s} p_i^s \tilde{c}_i di \leq E^{s*} \right\},$$

Accordingly, by the same argument as in Lemma 1, we can write the world price and foreign output for all $s \in \mathcal{S}$ and $i \in \mathcal{I}^s$ as

$$(24) \quad p_i^{s*}(m_i, w^*, E^{s*}) \equiv \min \{ u_i^{s*/'}(-m_i) v^{s*}(E^{s*}), w^* a_i^* \},$$

and

$$(25) \quad q_i^{s*}(m_i, w^*, E^{s*}) \equiv \max \{ m_i + d_i^{s*}(w^* a_i^* / v^{s*}(E^{s*})), 0 \},$$

where $v(E^{s*})$ is the Lagrange multiplier associated with the constraint $\int_{i \in \mathcal{I}^s} p_i^s \tilde{c}_i di \leq E^{s*}$.

In this situation, Home's planning problem can still be decomposed into an outer problem and multiple inner problems, one for each sector. At the outer level, the government now chooses the foreign wage, w^* , together with the sectoral labor allocations in Home and Foreign, L^s and L^{s*} , and the sectoral trade deficits, D^s , subject to aggregate factor market clearing and trade balance. At the inner level, the government treats L^s , L^{s*} , D^s , and w^* as constraints and maximizes subutility sector by sector. More precisely, Home's planning problem can be expressed as

$$\max_{\{L^s, L^{s*}, D^s\}_{s \in \mathcal{S}}, w^* \in \mathcal{W}^*} F(V^1(L^1, L^{1*}, D^1, w^*), \dots, V^S(L^S, L^{S*}, D^S, w^*))$$

subject to

$$\begin{aligned} \sum_{s \in \mathcal{S}} L^s &= L, \\ \sum_{s \in \mathcal{S}} L^{s*} &= L^*, \\ \sum_{s \in \mathcal{S}} D^s &= 0, \end{aligned}$$

where the sector-specific value function is now given by

$$V^s(L^s, L^{s*}, D^s, w^*) \equiv \max_{m^s, q^s \geq 0} \int_{i \in T^s} u_i^s(m_i + q_i) di$$

subject to

$$\begin{aligned} \int_{i \in T^s} a_i q_i di &\leq L^s, \\ \int_{i \in T^s} a_i^* q_i^{s*}(m_i, w^*, w^* L^{s*} - D^s) di &\leq L^{s*}, \\ \int_{i \in T^s} p_i^s(m_i, w^*, w^* L^{s*} - D^s) m_i di &\leq D^s. \end{aligned}$$

Given equations (24) and (25), the sector-specific maximization problem is of the same type as in the baseline case (program 2). As in Section III.B, we can therefore reformulate each infinite-dimensional sector-level problem into many two-dimensional, unconstrained maximization problems using Lagrange multiplier methods. Within any sector with CES utility, all of our previous results hold exactly. Within any sector in which utility is not CES, our previous results continue to hold subject to the qualification about monopoly markups discussed before.

What about the variation in trade taxes across sectors when utility functions satisfy equation (23)? A common special case in the literature consists in assuming that upper-level utility functions are Cobb-Douglas and that lower-level utility functions are CES with goods being “differentiated” by country of origin. Formally, this corresponds to

$$U = \sum_s \beta^s \ln \left[(c_h^s)^{\frac{\sigma-1}{\sigma}} + (c_f^s)^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

where c_h^s and c_f^s denote the consumption of the domestic and foreign varieties of good s , $\sigma > 1$ denotes the elasticity of substitution between domestic and foreign varieties, and (β_s) are exogenous preference parameters such that $\sum_s \beta_s = 1$. In this so-called Armington model, one can show that our results still apply across sectors without further qualification; see Online Appendix C.1. Namely, optimal import tariffs are uniform across goods and optimal export taxes are decreasing (in absolute

value) with $\frac{\alpha^s}{\alpha^{s*}}$.¹⁵ As before, this reflects the fact that in sectors in which Home's comparative advantage is stronger, that is, $\frac{\alpha^s}{\alpha^{s*}}$ is lower, it has more room to manipulate prices, which leads to bigger export taxes. In short, none of our qualitative insights hinge on the assumption that domestic and foreign goods are perfect substitutes.

V.B. Technology

As is well known, the Ricardian model can always be interpreted as a neoclassical model with multiple factors of production under the restriction that all goods use factors of production in the same proportions. In practice, however, factor intensities do differ across sectors. Thus, relative production costs depend not only on productivity differences—as in our baseline model—but also on factor intensities and factor prices. Here we generalize the model of Section II.A to allow for arbitrary neoclassical production functions. In spite of the generality of this new environment, we find that our main prediction survives: due to limit pricing, taxes remain weakly monotone with respect to relative cost across exported goods.

The new environment can be described as follows. There are multiple factors of production indexed by n , each of which are perfectly mobile across goods and immobile across countries. We now denote by $L \equiv (L_n) \geq 0$ the exogenous vector of factor endowments at home. Production of each good is subject to constant returns to scale. If $l_i \equiv (l_{in})$ units of each factor are employed in sector i at home or abroad, then total output is given by $f_i(l_i)$ or $f_i^*(l_i)$. The Ricardian model considered in Section II.A corresponds to the special case in which $f_i(l_i) = \frac{l_i}{a_i}$.

The competitive equilibrium can still be described as in Section II.B. Besides the fact that multiple factor markets must now clear at home and abroad, the only additional equilibrium condition is that firms must choose unit factor requirements to minimize their costs,

$$a_i(w) \equiv (a_{in}(w)) = \operatorname{argmin}_{a_i \equiv (a_{in})} \{a_i \cdot w | f_i(a_i) \geq 1\},$$

15. As σ goes to infinity, optimal taxes converge to those derived in Section IV.B. Because of the assumption of Cobb-Douglas upper-level utility functions, though, all goods fall in the region with limit pricing. We conjecture that similar results extend to the more general case with nested CES utility functions.

where $w \equiv (w_n)$ is now the vector of factor prices and \cdot denotes the scalar product. A similar condition holds abroad, with asterisks denoting foreign factor prices and foreign production functions.

Regarding Home's planning problem, the same argument as in Lemma 1 implies that the world price, foreign consumption, foreign unit factor requirements, and foreign production can be expressed more compactly as functions of net imports and foreign factor prices alone. Similarly, Home's planning problem can still be decomposed into an outer problem and an inner problem. At the inner level, the government treats the vector of foreign factor prices, $w^* \equiv (w_n^*)$, as given and chooses net imports and output, $m, q \geq 0$, as well as domestic unit factor requirements, $a \geq 0$. At the outer level, the government then chooses the vector of foreign factor prices, w^* . Using the same approach as in Section III.B and substituting for the optimal unit factor requirements, we can then reduce Home's problem to maximizing with respect to (m_i, q_i) the good-specific Lagrangian,

$$\mathcal{L}_i(m_i, q_i, \lambda, \lambda^*, \mu; w^*) \equiv u_i(q_i + m_i) - \lambda \cdot a_i(\lambda)q_i - \lambda^* \cdot a_i^*(w^*)q_i^*(m_i, w^*) - \mu p_i(m_i, w^*)m_i,$$

where $\lambda \equiv (\lambda_n)$ and $\lambda^* \equiv (\lambda_n^*)$ now denote the vectors of Lagrange multipliers associated with the domestic and foreign resource constraints, respectively.

Solving for optimal output and net imports follows similar arguments as in Online Appendix Section A.4. First, we find optimal output, $q_i^0(m_i)$, as a function of net imports. Second, we find the value of m_i that maximizes \mathcal{L}_i . As in the baseline model, the Lagrangian has two kinks. The first one occurs at $m_i = M_i^I \equiv -d_i^*(w^* \cdot a_i^*(w^*)) < 0$, when Foreign starts producing good i , and the second occurs at $m_i = M_i^{II} \equiv d_i(\lambda \cdot a_i(\lambda)) > 0$, when Home stops producing good i . Letting $\gamma_i \equiv \lambda \cdot a_i(\lambda)$ and $\gamma_i^* \equiv w^* \cdot a_i^*(w^*)$ denote the unit costs of production in the two countries and considering separately the three regions partitioned by the two kinks, we obtain the following generalization of Proposition 1.

PROPOSITION 3. Optimal net imports are such that (i) $m_i^0 = m_i^I \equiv -\left(\frac{\sigma^*}{\sigma^* - 1} \frac{\gamma_i}{\mu \beta_i^*}\right)^{-\sigma^*}$, if $\frac{\gamma_i}{\gamma_i^*} < \mu \frac{\sigma^* - 1}{\sigma^*}$; (ii) $m_i^0 = M_i^I$, if $\frac{\gamma_i}{\gamma_i^*} \in \left[\mu \frac{\sigma^* - 1}{\sigma^*}, \frac{\lambda^* \cdot a_i^*(w^*)}{\gamma_i^*} + \mu\right)$; (iii) $m_i^0 \in [M_i^I, M_i^{II}]$ if $\frac{\gamma_i}{\gamma_i^*} = \frac{\lambda^* \cdot a_i^*(w^*)}{\gamma_i^*} + \mu$; and (iv) $m_i^0 = m_i^{II} \equiv d_i(\lambda^* \cdot a_i^*(w^*) + \mu \gamma_i^*)$, if $\frac{\gamma_i}{\gamma_i^*} > \frac{\lambda^* \cdot a_i^*(w^*)}{\gamma_i^*} + \mu$.

Starting from Proposition 3 and using the same arguments as in Section IV.A, one can show that wedges should be constant and equal to $\frac{\sigma^* - 1}{\sigma^*} \mu - 1$ in region (i) and linear in $\frac{Y_i}{Y_i^*}$ in region (ii). This implies that optimal taxes on exported goods should be constant for goods in which Home's relative cost is the lowest (region i) and monotone in relative costs for other goods (region ii). Like in our baseline Ricardian model, the previous monotonic relationship reflects the constraints on the ability of the domestic government to manipulate world prices in that region.

Proposition 3, however, differs from Proposition 1 in two important respects. First, it establishes a relationship between imports and relative costs, not a relationship between imports and comparative advantage. Since unit costs of production—unlike unit labor requirements in a Ricardian model—are endogenous objects, it is a priori possible that no goods may fall into regions (i), (ii), and (iv). This would happen if all goods are produced by both countries, perhaps because in the short run there are factors of production specific to each sector in all countries, as assumed in the Ricardo-Viner model. In such a situation, all goods would fall in region (iii).

An important question, therefore, is whether there are other canonical models for which Proposition 3 can be restated in terms of primitive assumptions about technology and factor endowments. Another prominent example for which this is the case is the Heckscher-Ohlin model considered by Dornbusch, Fischer, and Samuelson (1980). In an economy with a continuum of goods, CES utility, two factors (capital and labor), and identical technologies across countries, one can show that at the solution of the planning problem, the capital-abundant country should have relatively lower costs in the capital-intensive sectors. Accordingly, optimal taxes should be constant on the most capital-intensive goods and monotone in capital intensity for other exported goods, that is, monotone in comparative advantage.

The second difference between Propositions 1 and 3 is more subtle and relates to optimal taxes on imported goods. Compared to the baseline Ricardian model, the domestic government may now manipulate the relative price of its imports. In the Ricardian model, those relative prices were pinned down by relative unit labor requirements abroad. In this more general environment, the domestic government may affect the relative price of its imports by affecting relative factor demand abroad and, in turn,

relative factor prices. Whenever the vector of Lagrange multipliers associated with the foreign resource constraint, λ^* , is not collinear with the vector of foreign factor prices, w^* , it will have incentives to do so, thereby leading to taxes on imported goods that may now vary across goods.

Based on the quantitative results presented in Section VI, we expect the manipulation of relative factor demand abroad to be relatively unimportant in practice. Even when considering a country as large as the United States, we find that λ^* is close to 0. This is suggestive of most countries being “small” in the sense that they cannot affect factor market clearing conditions in the rest of the world. Yet as our analysis demonstrates, this does not imply that most countries would not want to manipulate world prices to their advantage. Even if λ^* is equal to 0—so that λ^* is trivially collinear with w^* —Proposition 3 shows that a small but strategic country would like to impose taxes on exported goods that are weakly monotone in relative costs. Put differently, whereas the ability to affect factor prices in the rest of the world clearly depends on country size, the incentives for limit pricing do not.

VI. APPLICATIONS

To conclude, we apply our theoretical results to two sectors: agriculture and manufacturing. Our goal is to take a first look at the quantitative importance of optimal trade taxes for welfare, both in an absolute sense and relative to the simpler case of uniform import tariffs.

In both applications, we compute optimal trade taxes as follows. First, we use Proposition 1 to solve for optimal imports and output given arbitrary values of the Lagrange multipliers, $(\lambda, \lambda^*, \mu)$, and the foreign wage, w^* . Second, we use constraints (15)–(17) to solve for the Lagrange multipliers. Finally, we find the value of the foreign wage that maximizes the value function $V(w^*)$ associated with the inner problem. Given the optimal foreign wage, w^{0*} , and the associated Lagrange multipliers, $(\lambda^0, \lambda^{0*}, \mu^0)$, we compute optimal trade taxes using Proposition 2.

For both sectors, we also explore how our quantitative results change in the presence of exogenous iceberg trade costs, $\delta \geq 1$, such that if 1 unit of good i is shipped from one country to another, only a fraction $\frac{1}{\delta}$ arrives. Theoretical results used for simulations in this richer environment can be found in Online Appendix C.2.

VI.A. Agriculture

In many ways, agriculture provides an ideal environment in which to explore the quantitative importance of our results. From a theoretical perspective, the market structure in this sector is arguably as close as possible to the neoclassical ideal. From a measurement perspective, the scientific knowledge of agronomists provides a unique window into the structure of comparative advantage, as discussed in Costinot and Donaldson (2011). Finally, from a policy perspective, agricultural trade taxes are pervasive and one of the most salient and contentious global economic issues, as illustrated by the World Trade Organization's current, long-stalled Doha Round.

1. Calibration. We start from the Ricardian economy presented in Section II.A and assume that each good corresponds to 1 of 39 crops for which we have detailed productivity data, as we discuss shortly. All crops enter utility symmetrically in all countries, $\beta_i = \beta_i^* = 1$, and with the same elasticity of substitution, $\sigma = \sigma^*$. Home is the United States and Foreign is an aggregate of the rest of the world (ROW). The single factor of production is equipped land.

The parameters necessary to apply our theoretical results are: (i) the unit factor requirement for each crop in each country, a_i and a_i^* ; (ii) the elasticity of substitution, σ ; (iii) the relative size of the two countries, $\frac{L^*}{L}$; and (iv) trade costs, δ , when relevant. For setting each crop's unit factor requirements, we use data from the Global Agro-Ecological Zones (GAEZ) project from the Food and Agriculture Organization (FAO); see Costinot and Donaldson (2011).¹⁶ Feeding data on local conditions—for example, soil, topography, elevation, and climatic conditions—into an agronomic model, scientists from the GAEZ project have computed the yield

16. While there are 43 crops in the original GAEZ database, we collapse this to 39 because of our need to merge the GAEZ crops with those in the FAOSTAT data (on trade flows and land usage). In doing so, we combine wetland rice and dryland rice, pearl millet and foxtail millet, and phaseolus bean and gram, in each case taking the maximum predicted GAEZ yield as the yield of the combined crop. In addition, we dropped jatropha from the analysis because this crop is not tracked in the FAOSTAT data.

that parcels of land around the world could obtain if they were to grow each of the 39 crops we consider in 2009.¹⁷ We set a_i and a_i^* equal to the average hectare per ton of output across all parcels of land in the United States and ROW, respectively.

The other parameters are chosen as follows. We set $\sigma = 2.9$ in line with the median estimate of the elasticity of substitution across our 39 crops in Broda and Weinstein (2006).¹⁸ We set $L = 1$ and $L^* = 10.62$ to match the relative acreage devoted to the 39 crops considered, as reported in the FAOSTAT data in 2009. Finally, in the extension with trade costs, we set $\delta = 1.72$ so that Home's import share in the equilibrium without trade policy matches the U.S. agriculture import share—that is, the total value of U.S. imports over the 39 crops considered divided by the total value of U.S. expenditure over those same crops—in the FAOSTAT data in 2009, 11.1%.

2. *Results.* The left and right panels of Figure III report optimal trade taxes on all traded crops i as a function of comparative advantage, $\frac{a_i}{a_i^*}$, in the calibrated examples without trade costs, $\delta = 1$, and with trade costs, $\delta = 1.72$, respectively.¹⁹ The region between the two vertical lines in the right panel corresponds to goods that are not traded at the solution of Home's planning problem.

17. The GAEZ project constructs output per hectare predictions under different assumptions on a farmer's use of complementary inputs (e.g., irrigation, fertilizers, and machinery). We use the measure that is constructed under the assumption that a "high" level of inputs (fertilizers, machinery, etc.) is available to farmers and that water supply is rain-fed.

18. The elasticity of substitution estimated by Broda and Weinstein (2006) are available for five-digit SITC sectors. When computing the median across our 39 crops, we restrict ourselves to five-digit SITC codes that can be matched to raw versions of the 39 FAO crops.

19. We compute optimal trade taxes, throughout this and the next subsection, by performing a grid search over the foreign wage w^* so as to maximize $V(w^*)$. Since Foreign cannot be worse off under trade than under autarky—whatever world prices may be, there are gains from trade—and cannot be better off than under free trade—since free trade is a Pareto optimum, Home would have to be worse off—we restrict our grid search to values of the foreign wage between those that would prevail in the autarky and free trade equilibria. Recall that we have normalized prices so that the Lagrange multiplier associated with the foreign budget constraint is equal to one. Thus w^* is the real wage abroad.

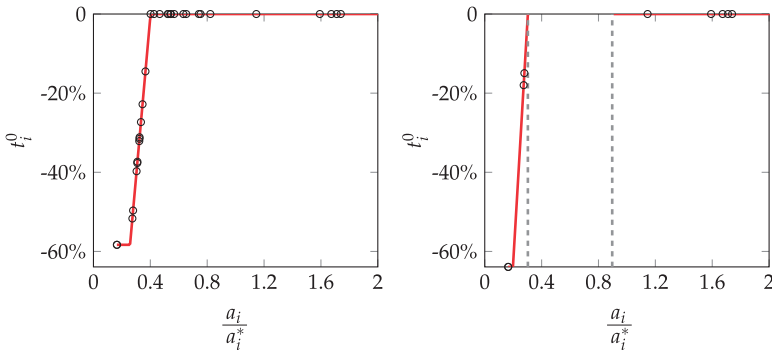


FIGURE III

Optimal Trade Taxes for the Agricultural Case

The left panel assumes no trade costs, $\delta = 1$. The right panel assumes trade costs, $\delta = 1.72$.

As discussed in Section IV.B, the overall level of taxes is indeterminate. Figure III focuses on a normalization with zero import tariffs. In both cases, the maximum export tax is close to the optimal monopoly markup that a domestic firm would have charged on the foreign market, $\frac{\sigma}{\sigma-1} - 1 \simeq 52.6\%$. The only difference between the two markups comes from the fact that the domestic government internalizes the effect that the net imports of each good have on the foreign wage. Specifically, if the Lagrange multiplier on the foreign resource constraint, λ^{0*} , were equal to 0, then the maximum export tax, which is equal $\frac{A^j}{A^H} - 1$, would simplify to the firm-level markup, $\frac{\sigma}{\sigma-1} - 1$. In other words, such general equilibrium considerations appear to have small effects on the design of optimal trade taxes for goods in which the U.S. comparative advantage is the strongest. In light of the discussion in Section IV.C, these quantitative results suggest that if domestic firms were to act as monopolists rather than take prices as given, then the domestic government could get close to the optimal allocation by only using consumption taxes that mimic the optimal trade taxes.

The first column of Table I displays U.S. welfare gains from trade, that is, the percentage change in total income divided by the CES price index relative to autarky, in the baseline model with no trade costs, $\delta = 1$. Three rows report the values of these gains from trade under each of three scenarios: (i) a laissez-faire regime with

TABLE I
GAINS FROM TRADE FOR THE AGRICULTURAL CASES

	No trade costs ($\delta = 1$)		Trade costs ($\delta = 1.72$)	
	U.S.	ROW	U.S.	ROW
Laissez-faire	39.15%	3.02%	5.02%	0.25%
Uniform tariff	42.60%	1.41%	5.44%	0.16%
Optimal taxes	46.92%	0.12%	5.71%	0.04%

no U.S. trade taxes, (ii) a U.S. optimal uniform tariff, and (iii) U.S. optimal trade taxes as characterized in Proposition 2.²⁰ In this example, optimal trade taxes that are monotone in comparative advantage increase U.S. gains from trade in agriculture by 20% ($\frac{46.92}{39.15} - 1 \simeq 0.20$) relative to the laissez-faire case. Interestingly, we also see that more than half of the previous U.S. gains arise from the use of nonuniform trade taxes since a uniform tariff would increase U.S. gains by only 9% ($\frac{42.60}{39.15} - 1 \simeq 0.09$).

The third column of Table I revisits the previous three scenarios using the model with trade costs, setting $\delta = 1.72$. Not surprisingly, as the U.S. import shares goes down from around 80% in the model without trade costs to its calibrated value of 11.1% in the model with trade costs, gains from trade also go down by an order of magnitude, from 39.15% to 5.02%. Yet the relative importance of trade taxes that vary with comparative advantage remains fairly stable. Even with trade costs, gains from trade for the United States are 14% larger under optimal trade taxes than in the absence of any trade taxes ($\frac{5.71}{5.02} - 1 \simeq 0.14$) and, again, slightly less than half of these gains arise from the use of nonuniform trade taxes ($\frac{5.44}{5.02} - 1 \simeq 0.08$).

At the world level, the welfare impact of nonuniform relative to uniform trade taxes is theoretically ambiguous. By construction, the solution to the inner problem (P_{w^*}) maximizes domestic welfare conditional on w^* , that is, foreign welfare. Thus holding w^* fixed, allowing for nonuniform trade taxes must improve world efficiency. This is reflected in the fact that unlike in the case of a

20. Scenarios (i) and (ii) are computed using the equilibrium conditions (1)–(8) in Section II.B. In scenario (i), we set $t_i = 0$ for all goods i . In scenario (ii) we set $t_i = t$ for all imported goods, we set $t_i = 0$ for other goods, and we do a grid search over t to find the optimal tariff.

uniform tariff, all goods are traded at the optimal taxes. The real foreign wage, however, may not remain fixed. Since nonuniform taxes make it less costly for the domestic government to manipulate its terms of trade, it may end up lowering further the real foreign wage and exacerbating inefficiencies at the world level.²¹ The second and fourth columns illustrate the quantitative importance of this second force. In the absence of trade costs, for instance, we see that nonuniform taxes decrease ROW's gains from trade by 96% ($1 - \frac{0.12}{3.02} \simeq 0.96$) relative to the laissez-faire case and by 91% ($1 - \frac{0.12}{1.41} \simeq 0.91$) relative to the case of a uniform import tariff.

VI.B. Manufacturing

There are good reasons to suspect that the quantitative results from Section VI.A may not generalize to other tradable sectors. In practice, most traded goods are manufactured goods and the pattern of comparative advantage within those goods may be very different than within agricultural products. We now explore the quantitative importance of such considerations.

1. Calibration. As in the previous subsection, we focus on the baseline Ricardian economy presented in Section II.A and the extension to iceberg trade costs presented in Online Appendix C.2. Home and Foreign still correspond to the United States and ROW, respectively, but we now assume that each good corresponds to 1 of 400 manufactured goods that are produced using equipped labor.²²

Compared to agriculture, the main calibration issue is how to set unit factor requirements. Since one cannot measure unit factor requirements directly for all manufactured goods in all countries, we follow the approach pioneered by Eaton and Kortum (2002) and assume that unit factor requirements are independently drawn across countries and goods from an extreme value distribution whose parameters can be calibrated to match a few key moments in the macro data. In a two-country setting,

21. We thank Bob Staiger for pointing out this trade-off.

22. The number of goods is chosen to balance computational burden against distance between our model and models with a continuum of goods such as Eaton and Kortum (2002). We find similar results with other numbers of goods.

Dekle, Eaton, and Kortum (2007) have shown that this approach is equivalent to assuming

$$(26) \quad a_i = \left(\frac{i}{T}\right)^{\frac{1}{\theta}} \text{ and } a_i^* = \left(\frac{1-i}{T^*}\right)^{\frac{1}{\theta}},$$

with θ the shape parameter of the extreme value distribution, which is assumed to be common across countries, and T and T^* the scale parameters, which are allowed to vary across countries. The goods index i is equally spaced between $\frac{1}{10,000}$ and $1 - \frac{1}{10,000}$ for the 400 goods in the economy.

Given the previous functional form assumptions, we choose parameters as follows. We set $\sigma = 2.5$ to match the median estimate of the elasticity of substitution among five-digit SITC manufacturing sectors in Broda and Weinstein (2006), which is very close to the value used in the agricultural exercise.²³ We set $L = 1$ and $L^* = 19.2$ to match population in the United States relative to ROW, as reported in the 2013 World Development Indicators for 2009. Since the shape parameter θ determines the elasticity of trade flows with respect to trade costs, we set $\theta = 5$, which is a typical estimate in the literature; see, for example, Anderson and Van Wincoop (2004) and Head and Mayer (2013). Given the previous parameters, we then set $T = 5,194.8$ and $T^* = 1$ so that in the equilibrium without trade policy Home’s share of world GDP matches the U.S. share, 26.2%, as reported in the World Development Indicators for 2009.²⁴ Finally, in the extension with trade costs, we now set $\delta = 1.44$ so that Home’s import share in the equilibrium without trade policy matches the U.S. manufacturing import share—that is, total value of U.S. manufacturing imports divided by total value of U.S. expenditure in manufacturing—as reported in the OECD Structural Analysis (STAN) database in 2009, 24.7%.

23. SITC manufacturing sectors include “Manufactured goods classified chiefly by material,” “Machinery and transport equipment,” and “Miscellaneous manufactured articles.”

24. Specifically, using equation (26), one can show that in the equilibrium without trade policy, the following relationship must hold

$$\frac{T}{T^*} = \left(\frac{wL}{w^*L^*}\right)^{1+\theta} \left(\frac{L}{L^*}\right)^{-\theta}.$$

Given values of θ , $\frac{L}{L^*}$, and $\frac{wL}{w^*L^*}$, the previous equation pins down $\frac{T}{T^*}$.

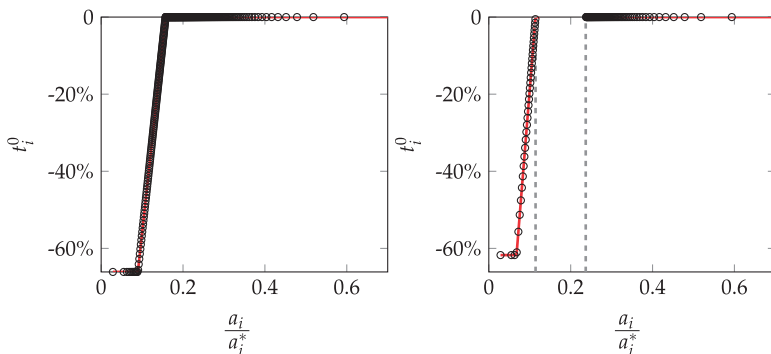


FIGURE IV

Optimal Trade Taxes for the Manufacturing Case

The left panel assumes no trade costs, $\delta = 1$. The right panel assumes trade costs, $\delta = 1.44$.

2. Results. Figure IV reports optimal trade taxes as a function of comparative advantage for manufacturing. As before, the left and right panels correspond to the models without and with trade costs, respectively, under a normalization with zero import tariffs. Like in the agricultural exercise of Section VI.A, we see that the maximum export tax is close to the optimal monopoly markup that a domestic firm would have charged on the foreign market, $\frac{\sigma}{\sigma-1} - 1 \simeq 66.7\%$, suggesting that the United States remains limited in its ability to manipulate the foreign wage.

Table II displays welfare gains in the manufacturing sector. In the absence of trade costs, as shown in the first two columns, gains from trade for the United States are 33% larger under optimal trade taxes than in the absence of any trade taxes ($\frac{36.85}{27.70} - 1 \simeq 0.33$) and 86% smaller for the ROW ($1 - \frac{0.93}{6.59} \simeq 0.86$). This again suggests large inefficiencies from terms-of-trade manipulation at the world level. Compared to our agricultural exercise, the share of the U.S. gains arising from the use of nonuniform trade taxes is now even larger: more than two thirds ($\frac{30.09}{27.70} - 1 \simeq 0.09$ as compared to 0.33).

As before, although the gains from trade are dramatically reduced by trade costs—they go down to 6.18% and 2.02% for the United States and the ROW, respectively—the importance of nonuniform trade taxes relative to uniform tariffs remains broadly stable. In the presence of trade costs, gains from trade for the United States, reported in the third column, are 49%

TABLE II
GAINS FROM TRADE FOR THE MANUFACTURING CASE

	No trade costs ($\delta = 1$)		Trade costs ($\delta = 1.44$)	
	U.S.	ROW	U.S.	ROW
Laissez-faire	27.70%	6.59%	6.18%	2.02%
Uniform tariff	30.09%	4.87%	7.31%	1.31%
Optimal taxes	36.85%	0.93%	9.21%	0.36%

larger under optimal trade taxes than in the absence of any trade taxes ($\frac{9.21}{6.18} - 1 \simeq 0.49$), and more than half of these gains (since $\frac{7.31}{6.18} - 1 \simeq 0.18$) arise from the use of trade taxes that vary with comparative advantage.

In contrast to the equivalence results of Arkolakis, Costinot, and Rodríguez-Clare (2012), the present results speak well to the importance of micro-level heterogeneity for the design of and gains from trade policy. In this example, the functional form assumption imposed on the distribution of unit labor requirements—equation (26)—implies that the model satisfies a gravity equation, as in Eaton and Kortum (2002). Conditional on matching the same trade elasticity and observed trade flows, the welfare changes associated with any uniform trade tax would be the same as in a one-sector Armington or Krugman (1980) model.²⁵ This equivalence is reflected in the fact that the optimal uniform tariff in the present example is equal to the inverse of the trade elasticity multiplied by the share of foreign expenditure on foreign goods, as established by Gros (1987) in the context of the Krugman (1980) model. Since the United States is small compared to the rest of the world, this is roughly $\frac{1}{\theta} \simeq 20\%$, both in the exercises with and without trade costs. In contrast, Figure IV shows that the optimal export tax is around 60% and slowly decreasing in absolute value with Foreign's relative unit labor requirements. As shown in Table II, these differences in design are

25. The basic argument is the same as the one used by Arkolakis, Costinot, and Rodríguez-Clare (2012) to establish that the welfare changes associated with any movement in iceberg trade costs are the same in the one-sector Armington, Krugman (1980), and Eaton and Kortum (2002) models. This equivalence extends to the Melitz (2003) model if one further assumes that import tariffs are imposed before firm-level markups, as discussed in Costinot and Rodríguez-Clare (2013).

associated with significant welfare effects, at least within the scope of this simple calibrated example.²⁶

Intuitively, the equivalence emphasized by Arkolakis, Costinot, and Rodríguez-Clare (2012) builds on the observation that at the aggregate level, standard gravity models are equivalent to endowment models in which countries exchange labor and relative labor demand curves are isoelastic. Hence, conditional on the shape of these demand curves, the aggregate implications of uniform changes in trade costs, that is, exogenous demand shifters, must be the same in all gravity models. For those particular changes, the micro-level assumptions through which isoelastic demand curves come about—either CES utility functions in the one-sector Armington model or an extreme value distribution in the Eaton and Kortum (2002) model—are irrelevant. Trade taxes, however, are imposed on goods, not labor. When heterogeneous across goods, such taxes no longer act as simple labor demand shifters and the equivalence in Arkolakis et al. (2012) breaks down. This is precisely what happens when trade taxes are chosen optimally.

VII. CONCLUDING REMARKS

Comparative advantage is at the core of neoclassical trade theory. In this article we have taken a stab at exploring how comparative advantage across nations affects the design of optimal trade policy. In the context of a canonical Ricardian model of international trade, we have shown that optimal trade taxes should be uniform across imported goods and weakly monotone with respect to comparative advantage across exported goods. Specifically, export goods featuring weaker comparative advantage should be taxed less (or subsidized more) relative to those featuring stronger comparative advantage, reflecting the fact

26. We have explored the consequences of varying σ and θ for the share of U.S. gains arising from the use of nonuniform trade taxes, holding all other model parameters constant. In our numerical simulations, this share is always declining in σ . Intuitively, a lower value of σ increases the range of goods for which limit pricing is optimal, while leaving the elasticity of aggregate trade flows, and hence the benefit from a uniform tariff, unchanged. The relationship between θ and the relative gains from nonuniform taxes is less clear-cut. Although a lower value of θ creates more heterogeneity, which tends to increase the welfare gains from nonuniform trade taxes, it also makes trade flows less elastic, which tends to increase the welfare gains from a uniform tariff.

that countries have more room to manipulate world prices in their comparative-advantage sectors.

Though the focus here is primarily normative, the previous results also have positive implications. Like perfectly competitive models providing a benchmark to identify market failures, we view our model with a welfare-maximizing government as a useful benchmark to assess the importance of political-economy considerations in practice. Previous empirical work by Broda, Limão, and Weinstein (2008) and Bagwell and Staiger (2011) suggests that terms-of-trade considerations do affect observed trade policies across countries and industries. In future work it would be interesting to explore empirically the extent to which comparative advantage shapes trade taxes across import-oriented and export-oriented sectors. The extent to which it does may provide a new window on the preferences and constraints faced by policy makers, including in agriculture, where protectionism remains a salient global policy issue.²⁷

At a technical level, characterizing optimal trade taxes in a Ricardian model is nontrivial. As mentioned, the maximization problem of the country manipulating its terms of trade is infinite-dimensional, nonconcave, and nonsmooth. A second contribution of this article is to show how to use Lagrange multiplier methods to solve such problems. Our basic strategy can be sketched as follows: (i) use the primal approach to go from taxes to quantities; (ii) identify concave subproblems for which general Lagrangian necessity and sufficiency theorems apply; and (iii) use the additive separability of preferences to break the Lagrangian into multiple low-dimensional maximization problems that can be solved by simple calculus. Although we have focused on optimal trade taxes in a Ricardian model, our approach is well suited to other additively separable problems. For instance, one could use these tools to compute fully optimal policy in the Melitz (2003) model, extending the results of Demidova and Rodríguez-Clare (2009) and Felbermayr, Jung, and Larch (2013). It would also be interesting to extend our analysis to the case of trade wars, in which both countries behave strategically, and to environments featuring a large number of countries and a rich geography of trade costs, as in Ossa (2014).

27. Interestingly, export restrictions, which play a central role in our theory, have also become a serious concern in recent years for trade in agricultural goods and natural resources; see Karapinar (2012).

Finally, we studied the quantitative implications of our theoretical results for the design of unilaterally optimal trade taxes in agricultural and manufacturing sectors. In our applications, we have found that trade taxes that vary with comparative advantage across goods lead to substantially larger welfare gains than optimal uniform trade taxes. In spite of the similarities between welfare gains from trade across models featuring different margins of adjustment—see, for example, Atkeson and Burstein (2010) and Arkolakis, Costinot, and Rodríguez-Clare (2012)—this result illustrates that the design of and the gains associated with optimal trade policy may crucially depend on the extent of heterogeneity at the micro level.

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SUPPLEMENTARY MATERIAL

An Online Appendix for this article can be found at QJE online (qje.oxfordjournal.org).

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