Collateral Reuse in Shadow Banking and Monetary Policy

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Abstract

I study the optimality of the reuse of collateral, a practice which is widespread in the shadow banking system. Rehypothecation is the direct repledging of the collateral received in a debt contract by the lender, while securitisation (pyramiding) is the use of the debt contract itself as collateral. With securitisation, the value of the repledgeable collateral, and hence the amount that can be borrowed, is limited by the face value of the original debt contract. Rehypothecation of the collateral enables borrowing of an amount that is typically greater than the face value of the debt backed by that collateral. The rehypothecating lender effectively borrows from the borrower the excess value of the collateral over the face value of the debt. I show that when the lender's cash flows are sufficiently pledgeable to the borrower, rehypothecation is a Pareto improvement on securitisation. I also investigate the implications of rehypothecation for monetary policy. I show that when a central bank removes collateral from the system through an open market operation, the collateral stays idle with the central bank, leading to tighter collateral constraints down the rehypothecation chain and retarded borrowing and investment. I find empirical evidence for this channel in the cost of borrowing in the bilateral repo market. The policy implications of this result include conducting open market operations when collateral is idle. It also suggests that using interest on reserves may be more effective as a policy tool compared to open market operations when collateral is scarce.

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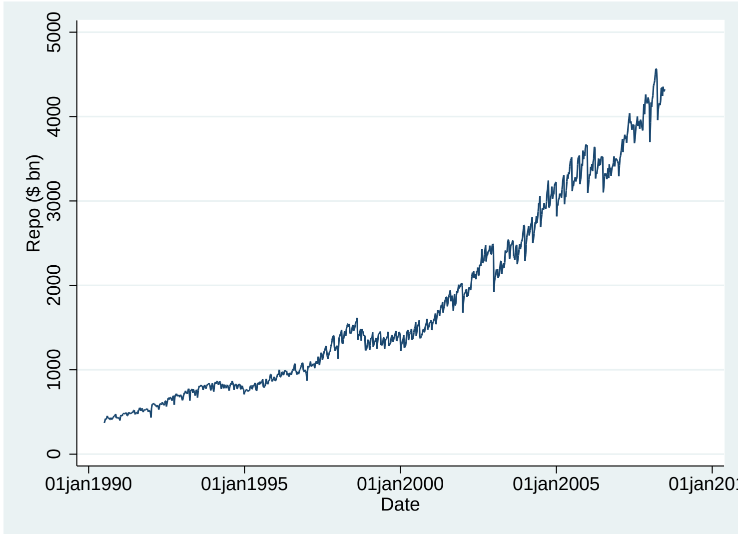

Figure 1: Growth in the market for Repurchase Agreements, representative of the growth in the shadow banking system. (Source: Federal Reserve Bank of New York)

1 Introduction

The last couple of decades have seen a spectacular growth in the shadow banking sector. As defined by Pozsar et al. (2012), this is the network of financial intermediaries other than deposit taking banks that has no explicit access to central bank credit or public sector guarantees. Pozsar et al. (2012), Cetorelli, Mandel and Mollineaux (2012), among others, describe this complex network of credit and maturity transformation that includes the traditional banking sector as just one node packaging and selling various types of retail loans into this network. Financial intermediation has grown beyond plain-vanilla banking, in which banks take deposits and lend them out to firms. Gorton (2009) suggests that this growth was the result of decreased regulation from the mid-1980s onwards and financial innovation such as securitisation.

The growth of shadow banking was naturally accompanied by a growth in short-term financing needs, which led to a rapid growth in the market for repurchase agreements, or "repos", analogous to deposits at banks. The lack of FDIC and other guarantees meant that this type of financing would have to be collateralised. Figure 1 shows the multi-fold growth in the repo market since 1990. Growth in collateralised financing led to an increase in the demand for collateral, and in the reuse of collateral. Rehypothecation and securitisation are the two ways in which this is achieved.

Rehypothecation is the practice of the direct reuse by lenders of the collateral posted with them to borrow on their own account. This is different from securitising the cash flows from the debt and and using them as collateral, or "pyramiding" as it is called by Geanakoplos and Zame (2009), Kilenthong and Townsend (2011) etc. In rehypothecation, the received collateral itself is directly repledged. In securitisation, the entire debt contract is posted as collateral, which includes the cash flows from the repayment secured by the underlying collateral. Rehypothecation of collateral by lenders to a third party creates the lenders' bankruptcy risk−if the lender defaults on his obligation to the third party, the collateral is confiscated by the third party and the original borrower does not get it back even if he is willing and able to fulfil his obligations. The possibility of rehypothecation is common knowledge at the time of contracting, and in most jurisdictions and types of borrowing contracts, a clause can be inserted limiting the amount of rehypothecable collateral or preventing rehypothecation altogether by keeping the collateral in a segregated account. Rehypothecation is common in prime brokerage and bilateral repo markets. Prime-brokers rehypothecate the collateral they receive from their hedge fund clients, and repo lenders reuse the collateral received in a repo contract. A number of recent IMF working papers by Singh and Aitken (2009, 2010) and Singh (2011, 2012) have pointed out its scale, and its implications during the financial crisis. Increased apprehension about the solvency of the lender can make the borrower hold the collateral in a segregated account. In the aftermath of the bankruptcy of Lehman Brothers, panic in the financial markets saw the availability of rehypothecable collateral collapse precipitously. Data from 10-Q forms filed with the SEC indicates that the amount of collateral that was allowed to be repledged declined from about $4.5 trillion in November 2007 to about $2.1 trillion in December 2009 (Singh and Aitken (2010)).

Securitisation is another way to use received collateral. Small banks and savings and loans associations borrow against their mortgages by issuing debt contracts called mortgage backed securities (MBS) or covered bonds. These are purchased by large investment banks who may use them as collateral in the repo market. Alternatively, the investment banks may pool together MBS's and borrow against a portfolio of them by issuing collateralised debt obligations (CDO). As is commonly understood, securitisation involves two steps. The first step is pyramiding, or converting a loan secured by the mortgage into a marketable instrument. The second is tranching, or pooling together several mortgages and creating instruments of different risk profiles. In this paper, I focus on the pyramiding part of securitisation.

In this paper, I study a unified model of rehypothecation and securitisation with the prime brokerage, repo and mortgage markets in mind. In my setup, both the borrower and the lender have variable scale investment projects. The lender can either securitise the debt or rehypothecate the collateral to borrow from a third party to invest in his project. With securitisation, the amount that the lender can borrow is restricted by the face value of the securitised debt, no matter how much of the collateral asset underlies the debt. Even if the lender wishes to borrow for the short term, the amount that he can borrow depends on the face value of the original debt. With rehypothecation, however, the amount that the lender can borrow is typically greater than the face value of the debt. There can be two ways in which this is achieved. One, if the lender wishes to borrow for the short term, he can borrow more by getting lower haircuts[1] since the risk of the collateral is lower in the short term. Two, if the degree of overcollateralisation in the original debt contract is high (higher than that needed to account for the risk of the collateral), the lender can borrow much more than the face value of the original debt by rehypothecating the collateral. I show that rehypothecation achieves the constrained efficient outcome of maximising the investments of both the borrower and the lender, by allowing the lender to make the most efficient use of the collateral and squeeze a large amount of liquidity from it. Rehypothecation comes at a cost to the original provider of the collateral. The lender may fail to deliver the collateral on the settlement date, following which the borrower may have trouble recovering the collateral haircut from the bankrupt or credit-constrained lender. The lender compensates the borrower for this risk by offering him a lower interest rate.

I consider the role of the borrower-lender relationship in making rehypothecation feasible. When a lender rehypothecates the collateral, he effectively borrows from the borrower the difference between the value of the collateral and the face value of the debt. If the lender's income is sufficiently pledgeable to the borrower, for example, in a close relationship characterised by effective monitoring, rehypothecation is optimal. In an arm's length relationship, the borrower's collateral constraint and the lender's pledgeability constraint cannot be simultaneously satisfied and rehypothecation becomes infeasible. Securitisation, then, obtains as the equilibrium outcome.

The long term and multi-contractual nature of prime brokerage relationships between hedge-funds and their prime-brokers, and the hedge fund managers' familiarity with the practices of their brokers, allows them to effectively monitor and discipline them. Similarly, the borrowers in the bilateral repo market are sophisticated hedge funds or other large dealers. The high pledgeability of the lenders in these markets makes rehypothecation feasible and optimal. On the other hand, in the mortgage market, the borrowing is done by relatively unsophisticated agents−a household borrows from a small bank by pledging the house, and a small bank borrows from a large investment bank by pledging the mortgage loan. The unsophisticated nature of the borrowers and their limited contractual arrangements with their lenders renders them ineffective at disciplining their lenders. Hence, rehypothecation becomes infeasible and securitisation obtains. The small bank uses the mortgage loan (secured by the house) as collateral to borrow from a large investment bank by issuing an MBS. The large investment bank uses the MBS (secured by the mortgage) as collateral to borrow in the repo market. Thus, I propose an explanation for the prevalence of securitisation in mortgage markets, and of rehypothecation in prime brokerage markets.

Rehypothecation is a key feature of the bilateral repo market. A number of papers, most

[1] Haircut $= 1 - \frac{\text{Amount of Borrowing}}{\text{Value of Collateral}}$.

notably Gorton and Metrick (2013), document the run-like situation in the bilateral repo market during the crisis, characterised by high rates and haircuts. In view of the documented facts, I study the comparative statics of a rehypothecation contract with respect to the lender's risk and pledgeability, and the collateral risk. I find that an increase in the risk of the collateral increases the haircut, a channel which was at work during the crisis in the repo market. An increase in the risk of the lender's project (keeping the return constant) lowers the haircut and the interest rate. If the pledgeability of the cash flows of the lender falls, the haircut falls and the interest rate rises. If the net present value of the lenders' investment becomes negative or if the lenders' pledgeability is too low, rehypothecation is prevented by borrowers, as was the case during the crisis.

Following the Quantitative Easing actions of the Federal Reserve, concerns have been raised by market participants about the "illiquidity" in the treasury market.[2] Singh (2013) has also documented a fall in the "velocity" (number of times a piece of collateral is pledged and repledged) of treasury collateral after QE-2 and QE-3. Motivated by this, I use the optimal rehypothecation contract to study the intervention of a central bank in the repo market and show that the central bank's actions may be ineffective. During an expansionary open market operation to stimulate the economy, the central bank removes high quality collateral from the system, tightens collateral constraints down the rehypothecation chain and crowds out private lending. The collateral removed from the system sits with the central bank and does no work. The reason for this is a pecuniary externality−counterparties end up selling too much collateral to the central bank without taking into account the effect on the collateral constraints down the rehypothecation chain. The private market cannot out-bid the central bank to borrow the collateral due to limited pledgeability. As a result, the effectiveness of the expansionary operation is reduced. Conversely, a contractionary operation has the side effect of stimulating private lending as the collateral, which was hitherto locked up with the central bank, is released and loosens borrowing constraints. In the data, I find evidence that an increase in the supply of treasury collateral outstanding (net of the amount held at the Fed) increases the spread between the treasury collateral repo rate and the risk-free rate and increases the financing activity using treasury collateral among primary dealers.

This has a number of policy implications. The result shows that open market operations must be conducted when collateral is abundant and idle (an indicator of which is a high spread between the repo rate and the risk-free rate). When collateral is idle, moving some to the central bank or adding some from the central bank to the idle pool does not disturb the equilibrium in the inter-dealer collateralised borrowing market and enables the central bank to better achieve its objective. In view of the imminent reversal of many of the world's central banks' quantitative easing policies, too, it has important implications. First, it suggests that tools such as interest on reserves, which do not affect the supply of collateral, would be

[2] See, e.g. Bloomberg (2015), Financial Times (2015).

more effective than open market operations when collateral is scarce. Indeed, an intention in this direction was signalled in the FOMC meeting of July 2014 (Federal Reserve (2014)). Second, it prescribes that when the central bank, with a contractionary intent, borrows cash against collateral through facilities such as the Overnight Reverse Repo Program (ON-RRP), it should deny rehypothecation permissions to the counterparties receiving the collateral, as rehypothecating the collateral may generate additional financing activity.

Related Literature

There has been very little work on modelling rehypothecation which is so widespread in the financial system. There has been considerable empirical work on the repos markets. Krishnamurthy, Nagel and Orlov (2014) and Copeland, Martin and Walker (2014) study the tri-party repo market and find little evidence of a run there during the crisis. Gorton and Metrick (2013) look at the bilateral repo market and find significant increases in haircuts and interest rates during the crisis. Martin, Skeie and Von Thadden (2014) model repo markets and show that repo runs can occur due to self-fulfilling expectations. They also note that the tri-party repo market may be fragile due to some of its peculiar mechanisms such as the morning unwind. However, they do not model rehypothecation.

Bottazzi, Luque, and Páscoa (2012) study the existence of equilibria with limited and unlimited rehypothecation, but they abstract from considerations of the lender's default risk. In their recent papers, Eren (2014) and Infante (2014) specifically model rehypothecation and view it as a way for the intermediate broker to obtain liquidity by offering differential haircuts: higher haircuts to the cash borrower and lower haircuts to the lender on rehypothecation. Maurin (2014) considers a general equilibrium model with collateral constraints and rehypothecation in a frictionless setting, and finds that rehypothecation can at best be a substitute for complete markets. Andolfatto, Martin and Zhang (2015) focus on the liquidity creating role of rehypothecation, and argue that limiting it may be desirable in increasing the demand for cash balances in economies away from the Friedman rule. Lee (2015) studies the effect of collateral reuse on repo spreads. My paper is most closely related to Eren (2014), Infante (2014) and Maurin (2014). It differs from the literature by looking at the optimality of rehypothecation and showing that rehypothecation can be a major improvement over securitisation in the presence of market incompleteness arising out of moral hazard. This paper also studies the effects of monetary policy actions in the presence of rehypothecation chains.

The model of this paper also relates to the models of money which is valuable as a medium of exchange by different individuals at different points in time. Clearly, rehypothecation increases liquidity in the financial system since the same collateral can be used to finance multiple investment opportunities at different financial institutions at different points of time. Collateral that is allowed to be repledged acquires some features of money reminiscent of

Townsend (1980), in which intrinsically useless money acquires value as a medium of exchange between spatially separated agents who have good and bad endowments alternately as they move across space. Similarly with rehypothecation, a given amount of collateral can be passed from agent to agent to finance multiple investments at multiple points in time. Analogous to the velocity of money, collateral also acquires velocity, which is the number of times a piece of collateral is pledged and repledged.

The results on the impact of central bank intervention on repo markets relate to several papers that compare the different tools of monetary policy. Goodfriend (2002) suggests that the market interest rate and the level of reserves can be set independently, while Keister, Martin and McAndrews (2008) describe a floor system to divorce the two. Martin et al. (2013) analyse the effectiveness of the various tools of the Fed in maintaining the floor on rates. Stein (2012) prescribes the use of interest on reserves to avoid excessive short-term debt creation, while Kashyap and Stein (2012) advocate the use of a combination of open market operations, reserve requirements and open market operations depending on the nature of the financial system. Cochrane (2014) suggests that a regime with a large central bank balance sheet and interest paying reserves creates financial stability as the interest paying reserves reduce the incentives to create run-susceptible "inside money". Ewerhart and Tapking (2008) study the optimal choice of collateral in repurchase agreements, and examine the welfare implications of the central bank's purchases of different types of collateral. Araújo, Schommer and Woodford (2013) study the effect of open market operations on borrowing constraints. While in their paper, asset purchases may tighten or relax collateral constraints depending on the risk of the asset, in my model, asset purchases tighten collateral constraints due to an externality.

The effect of treasury supply on their prices is investigated by Krishnamurthy and Vissing-Jorgensen (2012) who find that treasury securities enjoy a scarcity premium. Krishnamurthy and Vissing-Jorgensen (2013) argue that depriving the economy of liquid treasury bonds may have adverse welfare consequences. D'Amico, Fan and Kitsul (2013) find that purchases of specific CUSIP's by the Fed leads to an increase in the corresponding special repo spread as the supply decreases. Fleming, Hrung and Keane (2010) similarly find that the repo spread between treasury and non-treasury collateral narrowed due to the TSLF which lent treasuries against other assets. Caballero (2006) and Caballero, Farhi and Gourinchas (2006) discuss the macroeconomic and international implications of collateral shortages, and Caballero and Farhi (2013) highlight the benefits of supplying safe assets to the system.

Outline:

In Subsection 1.1, I provide an overview of the mechanics of the prime brokerage and mortgage markets. This subsection motivates the model by identifying the common features in these markets, and can be skipped without losing the essentials of the model and the results. Section

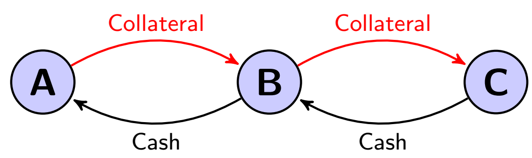

Figure 2: Flow of cash and collateral.

2 describes the model, the contracts and the timing. Section 3 analyses the securitisation and rehypothecation contracts individually. Section 4 looks at the constrained social planner's problem and shows that the rehypothecation allocations are constrained efficient. Section 5 looks at a crisis scenario and discusses the effects of lender health and collateral quality on haircuts and interest rates. Section 6 looks at a central bank intervention in a market characterised by rehypothecation. Section 7 concludes.

1.1 Overview of Markets and Practices

In this section, I describe the market practices which motivate my model. The reader may skip this section without losing the crux of the model and the results. The key mechanics of the model are described by a simple borrowing chain as in Figure 2. Borrower A borrows from lender B, who in turn securitises that debt or rehypothecated the collateral to borrow from C.

1.1.1 Prime brokerage and securities lending

The practice of rehypothecation is quite common in the shadow banking system in the United States, and especially in the United Kingdom. Broker-dealers routinely rehypothecate the collateral received from their hedge fund clients in a repo or a derivative contract. Rehypothecation is also a feature of repo contracts−in a repo contract, the permission to rehypothecate is implicit as the collateral is effectively sold on the origination date to the lender who can do with it what he wishes. Monnet (2011) discusses the legal minutiae of the ownership of the collateral in a repo transaction in different jurisdictions. In tri-party repos, the collateral sits with the clearing bank and is not rehypothecated. The total size of the US repo market is estimated to be about \$3 trillion in 2012, with the tri-party segment making up for about half of it[3]. A survey by Financial Services Authority (2012) indicates that repos make up 40-60% of the borrowing by hedge funds.

The basic steps that result in the creation of the smallest rehypothecation chain of length 2 are as follows. A hedge fund (or a bank) "A" needs to borrow cash from its prime broker (or another bank) "B" for a certain duration. The hedge fund receives the cash and posts a

[3] http://libertystreeteconomics.newyorkfed.org/2012/06/mapping-and-sizing-the-us-repo-market.html

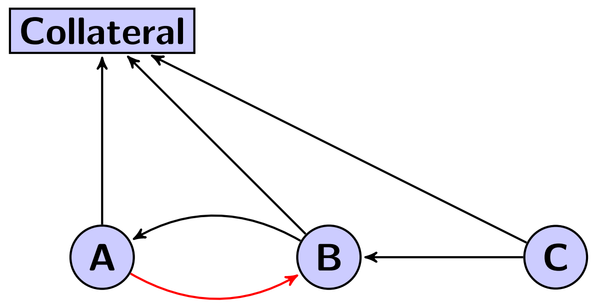

Figure 3: Payoff dependencies with **Rehypothecation**. An arrow from x to y indicates that the performance of y affects the payoff of x.

security as collateral. The collateral is usually of high quality, such as treasury bonds, Agency MBS's or highly liquid shares. The lender usually takes a haircut to protect himself against a fall in the value of the collateral at the precise time when the borrower defaults. Often, the borrowing is in the form of margin buying of securities from the prime broker, in which case the purchased security itself serves as the collateral. If the hedge fund does not permit the prime-broker to rehypothecate the collateral, it is placed in a segregated client account and the borrower retains ownership of it. The collateral is not attacked by the creditors in the event of the prime broker's bankruptcy. If the prime-broker is permitted to rehypothecate the collateral (for example, in a repo), it may use it to further borrow from a money market fund "C". This second borrowing transaction may be of a duration shorter than the first, and when the settlement date of the first transaction (between the hedge fund and the prime broker) arrives, the prime broker will have to purchase the collateral of equivalent value to return to the hedge fund.

On the other hand, if the prime broker has rehypothecated the collateral it may be, on the settlement date of the original borrowing transaction, unable to produce the collateral to return to the borrower either due to a liquidity crunch or bankruptcy. If the value of the collateral to be returned is higher than the value of the debt, which it usually is due to a haircut, the borrower then faces the risk of losing that haircut. Thus, with rehypothecation, both the lender and the borrower are exposed to each other's credit and liquidity risks. The crucial point about rehypothecation is that once the collateral is reused, the borrower no longer has first or exclusive access to it, becomes an unsecured creditor and must stand in the queue of creditors in the event of the lender's bankruptcy if he wishes to recover the haircut. This lack of full security for the borrower may lead it to prevent the lender from rehypothecating the collateral if he is pessimistic about the solvency of the lender. This, however, comes at a cost which is reflected in the higher prime brokerage fees.

The payoff dependencies in a rehypothecation chain are shown in Figure 3. The payoffs of all three agents depends on the value of the collateral, since it underlies both debt contracts and is liquidated in the event of a bankruptcy. The payoffs of C and B also depend on the performance of the investments of their borrowers B and A respectively. Finally, the risk of losing the collateral haircut due to B's default means that the payoff of A also depends on the performance of his lender B.

Singh and Aitken (2009, 2010) document the regulatory regimes regarding rehypothecation in the United Kingdom and the United States. Here, I summarise the key points. In the United States, broker-dealers are limited to rehypothecating margin securities amounting up to 140% of the debit balance[4] by Rule 15c3-3 of the Securities Investor Protection Act (1970). Non broker-dealers (such as banks) are not covered by this regulation, so that theoretically, there can be unrestricted rehypothecation in the United States. United Kingdom law neither distinguishes between broker-dealers and banks, nor provides for any cap on rehypothecation or customer protection in case of bankruptcy of the rehypothecating party. The regime in the European Union is less clear, but it appears that there is no cap provided for rehypothecation and contracting parties are free to bargain on how much collateral might be permitted to be reused.

Financial institutions like hedge funds have been taking advantage of the regulatory structure concerning rehypothecation by broker-dealers in the UK to borrow from UK-based or UK subsidiaries of US broker-dealers. Singh (2011) documents anecdotal evidence that prime-broker fees in the UK can be as low as LIBOR+50 bps when the broker-dealer is allowed to rehypothecate the collateral, and as high as LIBOR+250 bps otherwise. On the flip side, there is little protection afforded to customers in the event of the broker-dealer's bankruptcy. At the time of Lehman Brothers' filing for bankruptcy in September 2008, its London branch, Lehman Brothers International Europe, had rehypothecated about $22 billion of its clients' assets, most of which were not recovered when British administrators took charge of it (Singh and Aitken (2009)).

1.1.2 Mortgage market

The mortgage market offers an alternative way to reuse collateral. Instead of the lender directly reusing the received collateral for his own borrowing, the lender packages the debt contract it holds into a marketable security and uses that as collateral.

The collateral chain starts with a small retail bank or savings or loan association "A" owning a mortgage on a house. To raise funds for additional investments, the bank sells the mortgage or a pool of mortgages to an investment bank "B" in return for a promise to service the mortgages and channel interest and principal payments from them to the buyer. These are

[4]An example from Singh and Aitken (2009): if $500 of securities are deposited as margin with the broker-dealer and the debit balance (the outstanding amount owed to the broker-dealer) is $200, then the broker-dealer can rehypothecate up to $280 of the collateral (or, naturally, if 140% of the debit balance is higher than the value of the collateral, the entirety of the collateral).

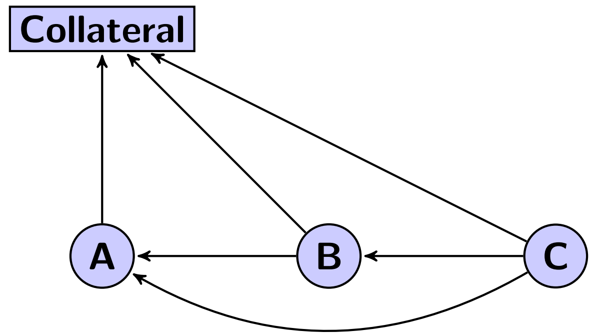

Figure 4: Payoff dependencies with **Type-I securitisation**. An arrow from x to y indicates that the performance of y affects the payoff of x.

called Mortgage-Backed Securities (MBS) or Mortgage Pass-through Securities. In most cases, this is done through an additional intermediary, which is a government-sponsored enterprise (GSE) such as the Fannie Mae, the Freddie Mac or the Ginnie Mae. The GSE securitises the pool of mortgages from the small banks and sells it on to an investment bank, applying its backing to it in the process. The MBS is essentially a debt owed by the originator of the mortgage. If the underlying mortgages start underperforming and the mortgage originator is unable to channel the promised payments, the GSE has recourse to the corporate resources of the mortgage originator. In the event of the mortgage originator's bankruptcy, the GSE has the exclusive right to impound the pool of mortgages underlying the MBS held by the originator.[5] An MBS, thus, essentially represents a debt owed by the mortgage originator to a lender such as an investment bank with the pool of mortgages serving as collateral, and a GSE standing in to securitise the mortgages, market the MBS's, and back the MBS's cash flow after the resources of the mortgage issuer have been exhausted.

Once the investment bank receives these MBS's it can use them as collateral in a repo market to borrow from another bank or a money market fund "C". Notice that the investment bank may use the MBS's as collateral, and does not have the right to use the underlying mortgages as collateral, which may well be residing on the balance sheets of the originators of those mortgages. Figure 4 shows the payoff dependencies with this type of (Type-I) securitisation. A's payoffs only depend on the collateral. B's payoffs depend on the performance of A, and of the collateral. C's payoffs depend on the performance of B, and if B defaults, on the performance of A and the original collateral.

Often, instead of using the MBS's as collateral, the investment bank ring-fences the pool of these MBS's by creating a special purpose vehicle which issues bonds called Collateralised Debt Obligations (CDO). Using the proceeds from the CDO issue, the special purpose vehicle

[5]See, e.g. paragraph 23-3(B) in
http://www.ginniemae.gov/doing_business_with_ginniemae/issuer_resources/MBSGuideLib/Chapter_23.pdf

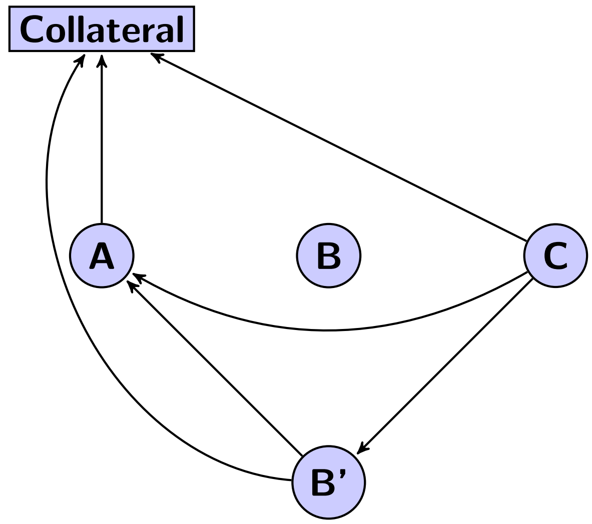

Figure 5: Payoff dependencies with **Type-II securitisation**. An arrow from x to y indicates that the performance of y affects the payoff of x.

purchases these MBS's from the investment bank. The CDO is purely a debt owed by the special purpose vehicle, backed by the assets of the vehicle. The investment bank thus isolates itself from the MBS's, and the CDO investors from the balance sheet of the investment bank. The special purpose vehicle typically has no assets other than the MBS's, and if one abstracts from the tranching of the payoffs from these MBS's to suit the needs of different investors, the arrangement resembles a sale of the MBS's to them. Figure 5 shows the payoff dependencies with these off-balance sheet arrangements (Type-II) securitisation. A's payoffs only depend on the collateral. B has moved the borrowing contract off its balance sheet into a special purpose vehicle B', and is isolated from the contract now. C's payoffs depend on the performance of B', and if B' defaults, on the performance of A and the original collateral.

In the model of this paper, both Type-I and Type-II securitisation are equivalent in terms of payoffs and outcomes. I focus on analysing Type-I securitisation.

2 Model

I describe here a unified model of collateralised borrowing and the reuse of collateral. There are three dates, $t = 1, 2, 3$, and three types of agents, A, B and C. There is a continuum of unit mass of each type. I interpret an agent of type A to be a small bank or a hedge fund that needs to borrow, and an agent of type B to be an investment bank or prime-broker that can lend to the bank/hedge-fund A against collateral, and that can later rehypothecate that collateral. The small bank or hedge fund has a relatively longer term investment (e.g. a mortgage for the bank that can last for years, or an investment strategy for the hedge fund that can last for weeks or months), and the large bank or broker-dealer has a relatively short-term investment (e.g. an overnight liquidity need). Agent C is interpreted as a money market fund that lends to banks B against collateral.

There is one good called cash and an asset called collateral. The cash good is the numeraire. The collateral pays cash at $t = 3$. The expected value at $t = 1$ of the payoff is 1. At every date, the expected value of the payoff evolves as a martingale. At $t = 2$, it goes up by $u > 1$ with probability q, and down by $d < 1$ with probability $1 - q$, such that

$$qu + (1 - q)d = 1.$$

This repeats between $t = 2$ and $t = 3$. Thus, at $t = 3$, the collateral pays off u^2 with probability q^2, ud with probability $2q(1 - q)$ and d^2 with probability $(1 - q)^2$. The evolution of the information about the collateral is shown in Figure 6.

Preferences

Agents of type A, B and C are risk neutral, and consume cash at $t = 2$ and $t = 3$. There is no discounting. The preferences can be summarised as

$$U = E[c_2 + c_3],$$

where the expectation is taken at $t = 1$.

Endowments and Investments

At $t = 1.1$, A has Ω units of the collateral asset, and B has 1 unit of cash. The first period is further divided into two: $t = 1.1$ (beginning of the period) and $t = 1.2$ (end of the period). B cannot store the cash between $t = 1.1$ and $t = 1.2$[6]. B has access to a storage technology between $t = 1.1$ and $t = 3$. I normalise this interest rate to 0. C has a large quantity of cash at each period that can be stored at an interest rate of 0 between any two periods.

At $t = 1.1$, A has access to a variable scale investment opportunity that yields $R_A > 1$ units of cash with certainty at $t = 3$ per unit investment of cash. At $t = 1.2$, B has access to an investment opportunity that yields $R_B > 1$ units of cash at $t = 2$ per unit investment of cash with probability p, and 0 with probability $1 - p$. I assume that the returns to the projects of all agents of type B are independent of each other, and of the value of the collateral.

[6]This assumption is not essential, and only serves to fix the outside option of B.

Borrowing Contracts

I assume that the market is segmented: A and C cannot deal with each other, and must deal with the intermediary B. I examine the consequences of relaxing this constraint in Section 4. To finance their investments, A must borrow from B, and B must borrow from C.

Assumption 1. *The income from A's project is not pledgeable to B or C. The income from B's project is not pledgeable to C. A fraction κ of the income from B's project is pledgeable to A.*

I think of A as a hedge fund or a small bank and of B as a large intermediate bank or broker-dealer. I motivate the lack of the small hedge fund or bank A's pledgeability by possibly extreme belief disagreements, or due to the lack of any recourse. C is a less sophisticated and ill-informed money-market fund, and B's income is not pledgeable to C, again due to extreme belief disagreements or no recourse. If A is an unsophisticated small bank or mortgage originator, I assume that κ is low and B's income is not pledgeable to A as it isn't to C. If A is a sophisticated agent such as hedge fund, I assume that κ is large and B's income is partly pledgeable to A.

Without loss of generality, I consider contracts offered by agents B to A and C. All agents behave competitively. In particular, any agent B offering a contract takes as given the contracts offered by the other agents B. I first consider the contracts $\mathcal{C}_{\mathrm{A,B}} = (r, h, \theta)$ offered by B to A at $t = 1.1$. Here, r is the net interest rate, h is the haircut[7], and $\theta \in \{0, 1\}$ is the rehypothecation permission: B can rehypothecate the collateral if $\theta = 1$ and not otherwise[8]. I define one unit of this contract as lending one unit of cash by B to A under the given terms. I will frequently find it easier to refer to the contracts *per unit of borrowing* in terms of the face value of the debt

$$1 + r,$$

and the number of units of collateral posted

$$\frac{1}{1-h}.$$

At $t = 1.2$, B may borrow from C by offering him a contract $\mathcal{C}_{\mathrm{B,C}} = (\tilde{r}, \tilde{h})$, depending on what contracts he offered at $t = 1.1$. Here, $\tilde{r}$ is the interest rate, and $\tilde{h}$ is the haircut. Again, I define one unit of this contract as lending one unit of cash by C to B. If $\theta = 1$, B can either

[7] Haircut $= 1 - \frac{\text{Amount of Borrowing}}{\text{Value of Collateral}}$.

[8] The binary permission is without loss of generality. The framework does not preclude B offering multiple contracts to A. Some of these contracts may have $\theta = 1$, and others may have $\theta = 0$, so that in aggregate, any fraction between 0 and 1 of the total collateral may be permitted to be rehypothecated. The linear and risk-neutral nature of the framework will, however, ensure a corner solution with only one contract chosen between A and B, and with the aggregate $\theta \in \{0, 1\}$.

directly rehypothecate the received collateral asset, or he can securitise the collateralised debt $\mathcal{C}_{\text{A,B}}$ and post that as collateral. If $\theta = 0$, B can only securitise the collateralised debt.

I make the following assumptions about the contracts.

Assumption 2. *$\mathcal{C}_{\text{A,B}}$ is a long term contract between $t = 1.1$ and $t = 3$. $\mathcal{C}_{\text{B,C}}$ is a short term contract between $t = 1.2$ and $t = 2$.*

Assumption 3. *The collateral in $\mathcal{C}_{\text{A,B}}$ and $\mathcal{C}_{\text{B,C}}$ is sufficient to cover the face value in all states of nature. If $D(s_3)$ is the value of one unit of collateral in $\mathcal{C}_{\text{A,B}}$ in state s_3 at $t = 3$, and if $D(s_2)$ is the value of one unit of collateral in $\mathcal{C}_{\text{B,C}}$ in state s_2 at $t = 2$, then*

$$1 + r \leq \frac{1}{1-h} \min_{s_3} D(s_3), \text{ and } 1 + \tilde{r} \leq \frac{1}{1-\tilde{h}} \min_{s_2} D(s_2).$$

Remark 1. In Appendix E, I use an example of belief heterogeneity to show that if A is more optimistic about the asset's payoff than B, and B is more optimistic than C, then A and B, instead of selling the asset, would prefer to own the asset and borrow against the asset using an optimal debt contract that involves no default, as in Assumption 3. This is similar to the optimal contracts in Geanakoplos (1997). In the same example, A would not borrow short term from B as it would involve either defaulting at $t = 2$ and losing the valuable asset, or unprofitably shoring up liquidity to repay at $t = 2$. For the purpose of the rest of the paper, I simplify and use Assumptions 2 and 3.

The literature has dealt with the problem of repurchase agreements versus asset sales. For example, Dang, Gorton and Holmström (2013) show that repurchase agreements will be preferred to asset sales since the lender (or buyer) will not be required to acquire information at a cost. Monnet and Narajabad (2012) reach the same conclusion by considering a hold-up problem at the repayment date when the lender (or reseller) can extract all the surplus if the borrower has no predetermined right to the collateral which is valuable to him. The focus of this paper is not to understand the trade-off between asset sales and repurchase agreements (collateralised borrowing contracts), but between securitisation and rehypothecation.

At $t = 2$, the returns to B's investment are realised, and then $\mathcal{C}_{\text{B,C}}$ is settled. At $t = 3$, the returns to A's investment are realised, and finally $\mathcal{C}_{\text{A,B}}$ is settled. Figure 6 summarises the timing of the model, the evolution of the expected value of the collateral, and the borrowing contracts.

3 Optimal Contracts

I derive below the optimal contracts $\mathcal{C}_{\text{A,B}}$ and $\mathcal{C}_{\text{B,C}}$. I derive these contracts under the assumption that all agents act competitively, taking the actions of other agents as given. In particular, I assume that while offering the contracts, each agent of type B takes the actions

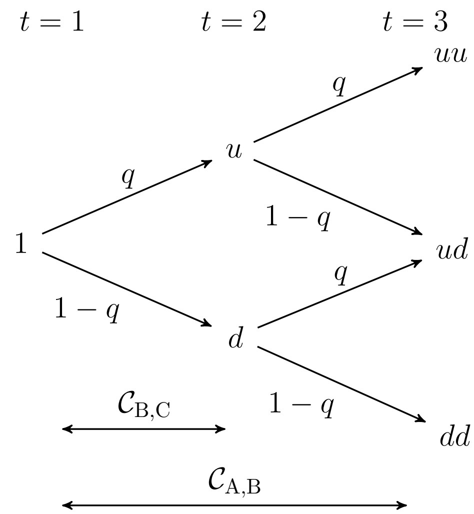

Figure 6: Evolution of the expected value of the payoff of the collateral, and the duration of the borrowing contracts. $\mathcal{C}_{\text{A,B}}$ is the long-term contract through which A borrows from B, and $\mathcal{C}_{\text{B,C}}$ is the short-term contract through which B borrows from C.

of other agents of type B as given. The optimal contracts determine the supply of collateral from A and the demand for it from B, as a function of a "price". This "price" is the net return to A from "lending out" one unit of collateral to B. The market clearing for collateral pins down this equilibrium net return. I look for a symmetric equilibrium where all agents of type B offer the same contract.

Denote by (x_i, z_i), $i \in \{\text{A}, \text{B}\}$ the holdings of cash and collateral respectively at $t = 1.1$, and by $(\tilde{x}_{\text{B}}, \tilde{z}_{\text{B}})$ the holdings of cash and collateral of B at $t = 1.2$.

3.1 Borrowers' Problem

A faces the offer (r, h, θ) from B. Suppose A holds z_{A} units of collateral and commits $\Omega - z_{\text{A}}$ units from his endowment to this contract, and invests the proceeds from the borrowing into his opportunity. Let $1 + \bar{r}$ be the gross cost of borrowing, inclusive of expected losses due to B's default after rehypothecation. (If B does not rehypothecate, $1 + \bar{r} = 1 + r$, the gross interest rate.) A solves

$$\max_{z_{\text{A}} \leq \Omega} \quad \overbrace{(R_A - (1+\bar{r}))}^{\text{Net return per unit of borrowing}} \quad \cdot \overbrace{(1-h)(\Omega - z_{\text{A}})}^{\text{Amount borrowed}} . \tag{1}$$

Definition 1. $\bar{R} = (R_A - (1+\bar{r}))\,(1-h)$.

$\bar{R}$ can be interpreted as the net return earned by A by lending collateral to B, or as the cost faced by B of borrowing collateral from A. Every agent of type B will have to provide at least an excess return of $\bar{R}$ provided by the other competing agents of type B, for the contract to be accepted. Thus, $\bar{R}$ also represents the outside option of A when considering a contract offered by agent of type B. An agent of type B will take $\bar{R}$ as given and offer an optimal contract $(r(\bar{R}), h(\bar{R}), \theta(\bar{R}))$.

A will maximise his payoff by committing all of his endowment of collateral to this contract if $\bar{R} > 0$, and by holding on to the collateral if $\bar{R} < 0$. This gives the optimum holding of collateral

$$z_{\mathrm{A}}(\bar{R}) \begin{cases} = 0, & \text{if } \bar{R} > 0, \\ \in [0, \Omega], & \text{if } \bar{R} = 0, \\ = \Omega, & \text{if } \bar{R} < 0. \end{cases} \tag{2}$$

and the holding of cash[9]

$$x_{\mathrm{A}}(\bar{R}) = (1 - h(\bar{R}))(\Omega - z_{\mathrm{A}}(\bar{R})). \tag{3}$$

3.2 Lenders' Problem: Securitisation vs Rehypothecation

At $t = 1.1$, B can choose from four actions: (i) do nothing and save the endowment of cash into the long-term storage technology and consume it at $t = 3$, (ii) offer a debt contract to A and hold on to it till $t = 3$, (iii) offer a debt contract to A and securitise the debt to borrow from C at $t = 1.2$, or (iv) offer a debt contract to A and rehypothecate the collateral to borrow from C at $t = 1.2$. If B has lent to A at $t = 1.1$, then B must decide whether and how to borrow for the short term to invest in his opportunity at $t = 1.2$. If the return to B from borrowing from C and investing is positive, the linearly scalable nature of his investment opportunity means that he will try to maximise the amount of investment. B's problem at $t = 1.2$ can, therefore, be split into two sub-problems: (i) deciding whether or not to invest, and (ii) maximising the amount of investment. I first look at the problem of maximising the investment, and then check whether the investment is profitable.

While offering a debt contract $\mathcal{C}_{\mathrm{A,B}}$ to A, B takes as given the net return $\bar{R}$ per unit of collateral that A gets from other agents B. The contract will, therefore, be a function of $\bar{R}$. Suppose that B has offered a contract $\mathcal{C}_{\mathrm{A,B}}(\bar{R}) = (r(\bar{R}), h(\bar{R}), \theta(\bar{R}))$ to A at $t = 1.1$. Suppose also that A has accepted 1 unit of the contract, i.e. B has lent 1 unit of cash to A. Per unit of $\mathcal{C}_{\mathrm{A,B}}$, the value of the debt held by B is $1 + r$, and the value of the collateral received is $\frac{1}{1-h}$. Now B has a choice of two assets to provide as collateral to borrow from C: the securitised debt or the underlying collateral asset itself. The securitised debt has a safe value $1 + r$, given the assumption that it is fully collateralised. The value of the underlying collateral is risky, and is $\frac{1}{1-h}d$ in the worst case when the short-term contract with C expires at $t = 2$. The following lemma summarises the optimal contract $\mathcal{C}_{\mathrm{B,C}}$ that maximises the amount of

[9] From the budget constraint $z_{\mathrm{A}} + \frac{x_{\mathrm{A}}}{1-h(\bar{R})} = \Omega$.

borrowing from C.

Proposition 1. *Given the contract $\mathcal{C}_{\mathrm{A,B}} = (r, h, \theta)$, the optimal contract $\mathcal{C}_{\mathrm{B,C}}$ is given by*

$$(\tilde{r}, \tilde{h}) = \begin{cases} (0,0), & \textit{if the securitised debt is used as collateral, and} \\ (0, 1-d), & \textit{if } \theta = 1 \textit{ and the underlying collateral is rehypothecated.} \end{cases}$$

The interest rate in each case is zero since C is risk neutral, has a large cash endowment, and access to an outside savings technology with a zero interest rate. The haircut that needs to be provided to C is zero when the collateral is the securitised debt $\mathcal{C}_{\mathrm{A,B}}$ with a safe value, and is positive when the collateral is the rehypothecated asset with a risky value. Note that the degree of collateralisation of $\mathcal{C}_{\mathrm{A,B}}$ is irrelevant to the amount that B can borrow by securitising it, as long as it is over-collateralised−the borrowing of B is only determined by the face value of the debt. Proposition 1 implies that per unit of the contract $\mathcal{C}_{\mathrm{A,B}}$, B can borrow from C the amount

$$\tilde{x}_{\mathrm{B}} = \begin{cases} (1+r), & \text{if the securitised debt is used as collateral, and} \\ \frac{1}{1-h} d. & \text{if } \theta = 1 \text{ and the underlying collateral is rehypothecated.} \end{cases} \tag{4}$$

I now check consider B's decision to invest at $t = 1.2$, and whether to do it by securitising the debt or rehypothecating the collateral. Any contract offered to A must satisfy the assumed full-collateralisation constraint

$$1 + r \leq \frac{1}{1-h} d^2. \tag{5}$$

It must also satisfy A's participation constraint in order for it to be accepted by A.

Consuming Endowment

If B does not lend to A, then at $t = 1.1$ he stores his endowment of 1 unit of cash into his long term storage technology and consumes it at $t = 3$ to get a utility of 1.

Holding on to Debt

When B offers a contract which he intends to hold on to till maturity, without borrowing against it and investing at $t = 1.2$, there is no risk to A of B's default and loss of collateral. The cost of borrowing to A is, therefore, the interest rate r. The contract offered by B must satisfy A's participation constraint

$$(R_A - (1+r))(1-h) \geq \bar{R}. \tag{6}$$

The expected payoff of B is simply $1 + r$ and B solves the problem

$$\max_{r,h} 1 + r, \tag{7}$$

subject to the collateral constraint (5) and the participation constraint (6). Here, a haircut greater than the minimum required serves no purpose, and wastes useful collateral. The solution is to make the collateral constraint bind, and raise the interest rate to the maximum possible so that A's participation constraint binds.

Securitisation

When B securitises the debt contract $\mathcal{C}_{\mathrm{A,B}}$ and posts it as collateral to C, A is unaffected by B's performance. With securitisation, the underlying collateral is not mixed with B's balance sheet, and is kept in a segregated account, possibly with a third party or with A himself, and A has the first rights to it when he repays the debt. The underlying collateral does not pass through B's bankruptcy proceedings. At $t = 2$, if B is solvent, B recovers $\mathcal{C}_{\mathrm{A,B}}$ from C. The collateral asset can then be recovered by A at $t = 3$ by paying off his debt. At $t = 2$, if B is insolvent, C impounds $\mathcal{C}_{\mathrm{A,B}}$. In this case, A pays the interest to C at $t = 3$ and recovers the underlying collateral asset. Thus, the cost of borrowing for A is again simply the interest rate r, and A's participation constraint again takes the form of (6).

Now from Proposition 1, B is able to borrow a total of $1 + r$ cash from C which he invests into his opportunity. I explain in detail the state-by-state cash positions of A, B and C at $t = 2$ and $t = 3$ in Table 3 of Appendix A. Intuitively, if the project succeeds, he gets the cash flow from the project, pays off the debt to C, and receives repayment from A. If the project fails, B gets nothing since $\mathcal{C}_{\mathrm{A,B}}$ is impounded by C.

Thus, B solves

$$\max_{r,h} p\left(\overbrace{R_B(1+r)}^{\text{Cash flow from project}} - \overbrace{(1+r)}^{\text{Repayment to C}} + \overbrace{(1+r)}^{\text{Repayment from A}}\right) = pR_B(1+r), \tag{8}$$

subject to the collateral constraint (5) and the participation constraint (6). The optimal contract offered is the same as in that case of holding on to the debt.

Rehypothecation

From Lemma 1, B is able to borrow a total of $\frac{1}{1-h}d$ from C by rehypothecating the collateral received from A. On borrowing from C, B invests in his opportunity. The effective state-by-state cash positions of B and C after the settlement of $\mathcal{C}_{\mathrm{B,C}}$ are explained in detail in Table 4 in Appendix A. Intuitively, whether or not B's project succeeds, C breaks even, either by getting repaid in full by B, or by liquidating the collateral. (C keeps the amount $\frac{1}{1-h}d$ and returns to B the excess amount as per Assumption 5 in Appendix A.) Thus, when B fails, he defaults on the returning of the collateral to A, A does not repay the debt to B, and instead loses the collateral value $\frac{1}{1-h}d$ to C.

The effective cash positions of A and B after the settlement of $\mathcal{C}_{\mathrm{A,B}}$ are summarised in Table 5 of Appendix A. In summary, the cost of borrowing to A is the interest rate when B is solvent with probability p, and the lost collateral to C when B is insolvent with probability $1-p$. A's participation constraint, thus, takes the form

$$\left(R_A - \overbrace{p(1+r)}^{\text{Normal repayment}} - \overbrace{(1-p)\frac{1}{1-h}d}^{\text{Loss of collateral}} \right)(1-h) \geq \bar{R}. \tag{9}$$

The above expression makes it clear that when B is insolvent, A loses an amount greater than the face value of the debt owed to B. Thus, per unit of borrowing, I can think of $1+r$ as the riskless debt owed to B, and the amount

$$\frac{1}{1-h}d - (1+r),$$

as A's exposure to B's insolvency. This is the amount that B effectively borrows from A.[10]

I assume that B cannot "borrow" indiscriminately from A. Following Holmström and Tirole (1997), I limit the amount owed by B to A as

$$\overbrace{\frac{1}{1-h}d - (1+r)}^{\text{Effective "borrowing" from A}} \leq \kappa \overbrace{\overbrace{R_B \frac{1}{1-h}d}^{\text{Income from investment}}}^{\text{Pledgeable income}}, \tag{10}$$

where the right hand side is the income pledgeable by B to A$-$a fraction κ of the gross cash flow from the investment. I motivate the limited pledgeability of B by the private benefit received by B by shirking on managing the risks of its investment and reducing the probability of success. If the amount owed by B to A is too large, B is highly leveraged and has little stake in the success of his investment, since it is A that stands to lose a lot from B's insolvency. He may then take undue risks leading to inefficient outcomes. As in Holmström and Tirole (1997), I focus on efficient contracts that disincentivise shirking. I discuss this in detail in Appendix B.

[10] Alternatively, the situation can be interpreted as the following. Upon lending to A, B has net worth $1+r$ receivable from A. After borrowing from C, B has assets $\frac{1}{1-h}d$ (cash) and liabilities $\frac{1}{1-h}d-(1+r)$ owed to A, leading to a net worth of $1+r$. As described by Kirk et al. (2014), rehypothecation thus enables B to leverage to invest.

When B succeeds, he earns the cash flow from the project, and repays C's debt at $t = 2$ and recovers the collateral. He then gets repaid by A at $t = 3$, who then recovers the collateral. When B fails, he gets nothing since the collateral is impounded by C, and since A does not repay his debt to B due to B's inability to return the collateral in full. Thus, B solves

$$\max_{r,h} p \left(\overbrace{R_B \frac{1}{1-h} d}^{\text{Cash flow from project}} - \overbrace{\frac{1}{1-h} d}^{\text{Repayment to C}} + \overbrace{1+r}^{\text{Repayment from A}} \right), \tag{11}$$

subject to the collateral constraint (5), the participation constraint (9) and the pledgeability constraint (10).

Combining the collateral constraint (5) and B's incentive compatibility constraint (10), the following two-sided borrowing constraint must be satisfied,

$$\frac{(1 - \kappa R_B)}{1-h} d \leq 1 + r \leq \frac{1}{1-h} d^2, \tag{12}$$

which requires that A should put up sufficient collateral to cover the debt, and that B should not rehypothecate too much collateral. The two constraints will both be satisfied and rehypothecation will be feasible if

$$\kappa \geq \bar{\kappa} \equiv \frac{1-d}{R_B}. \tag{13}$$

Inequality (12) thus captures the two-sided risks inherent in rehypothecation−the risk of A not repaying the debt and the risk of B being irresponsible with the collateral and not being able to return it in full. If both of these risks are not taken care of, then rehypothecation is infeasible.

The following proposition summarises the optimal action of B and the contract $\mathcal{C}_{\mathrm{A,B}}$.

Proposition 2. *There exist cut-offs $\bar{R}^* < \bar{R}^{**} < \bar{R}^{***}$, such that*

1. *If $pR_B \geq 1$, and*
   - *(a) if $\kappa < \bar{\kappa}$,* B *consumes the endowment if $\bar{R} > \bar{R}^{**}$, and lends to* A *and securitises the debt if $\bar{R} \leq \bar{R}^{**}$, and*
   - *(b) if $\kappa \geq \bar{\kappa}$,* B *consumes the endowment if $\bar{R} > \bar{R}^{***}$, and lends to* A *and rehypothecates the collateral if $\bar{R} \leq \bar{R}^{***}$. When $(pR_B - 1)\, d < \bar{R} \leq \bar{R}^{***}$, A's collateral constraint binds. When $\bar{R} < (pR_B - 1)\, d$, B's pledgeability constraint binds.*
2. *If $pR_B < 1$,* B *consumes the endowment if $\bar{R} > \bar{R}^*$, and lends to* A *and holds on to the debt if $\bar{R} \leq \bar{R}^*$.*

The expressions for $\bar{R}^*$, $\bar{R}^{**}$ and $\bar{R}^{***}$ and for the optimal contracts $(r(\bar{R}), h(\bar{R}), \theta(\bar{R}))$ in each case are given in Appendix C.

In the optimal contract when rehypothecation is feasible, exactly one of the collateral constraint (5) and the pledgeability constraint (10) will bind. As I show in the Appendix, when $(pR_B - 1)\, d > \bar{R}$, the net benefit from raising one unit of collateral from A is greater than the cost of raising that collateral. In this case, B's incentive constraint binds. Intuitively, since the net return earned by A on the market is low, the cost of borrowing collateral from A and rehypothecating it is low. The lender B can then demand more collateral than is necessary to satisfy A's collateral constraint, and rehypothecate this large amount of collateral to invest in the profitable opportunity. The amount of overcollateralisation, however, will be limited by B's pledgeability constraint, which binds. When $(pR_B - 1)\, d \leq \bar{R}$, the reverse holds and the net benefit from raising one unit of collateral from A is less than the cost of raising that collateral. Intuitively, the high cost of borrowing collateral means that B is less willing to overcollateralise the debt with A. He demands only as much collateral as is needed to satisfy A's collateral constraint, and B's pledgeability constraint is slack.

Proposition 2 defines the optimal holding of collateral,

$$z_{\mathrm{B}}(\bar{R}) \begin{cases} = 0, & \text{if } \bar{R} > \bar{R}^{cutoff}, \\ \in \left[0, \frac{1}{1-h(\bar{R})}\right], & \text{if } \bar{R} = \bar{R}^{cutoff}, \\ = \frac{1}{1-h(\bar{R})}, & \text{if } \bar{R} < \bar{R}^{cutoff}, \end{cases} \tag{14}$$

where $\bar{R}^{cutoff}$ is the cut-off above which there is no lending in the respective parametric cases of Proposition 2. The optimal holding of cash is then given by[11]

$$x_{\mathrm{B}}(\bar{R}) = 1 - (1 - h(\bar{R}))z_{\mathrm{B}}(\bar{R}). \tag{15}$$

Securitisation vs Rehypothecation: Discussion

Proposition 2 says that rehypothecation is privately optimal whenever it is feasible. The crux of this result and the comparison between securitisation and rehypothecation is seen with an example in a partial equilibrium setting, where B is offering a contract to A, taking $\bar{R}$ as given. To fix ideas, suppose that rehypothecation is feasible ($\kappa \geq \bar{\kappa}$) and desirable ($pR_B \geq 1$). Suppose that $p = 1$, so that A faces no risk of B's default. In this situation, it is easy to see that the binding participation constraints (6) and (9) are the same with securitisation and rehypothecation. If in addition, $\bar{R}$ is large and $(R_B - 1)\, d \leq \bar{R}$ holds, the collateral constraints also bind with both securitisation and rehypothecation from Proposition 2. This implies that the optimal haircuts and interest rates with both securitisation and rehypothecation are exactly the same. However, with securitisation, B can borrow from C an amount equal to

[11]From the budget constraint $x_{\mathrm{B}} + z_{\mathrm{B}}(1 - h(\bar{R})) = 1$.

$1+r$, while with rehypothecation, B can borrow an amount

$$\overbrace{\frac{1}{1-h}d}^{\text{Amt. with Rehypothecation}} \quad > \quad \overbrace{\frac{1}{1-h}d^2 = 1+r}^{\text{Amt. with Securitisation}} \quad .$$

Thus, rehypothecation disconnects the face value of the debt from the amount that can be borrowed by repledging. If B's borrowing requirement is short term and $d < 1$, it is better to rehypothecate since the low risk of the collateral in the short run enables B to borrow more with attractive haircuts. With securitisation on the other hand, the low risk of the collateral in the short term is irrelevant because what is being pledged as collateral is not the underlying collateral asset, but the cash flows from the original long-term debt contract, and the value of these cash flows is limited by the large risk of the collateral in the long run. When $p < 1$, the borrower A faces the risk of losses due to B's insolvency, and the interest rate and haircut are reduced appropriately to compensate A for those expected losses. However, even in this case, the ability to borrow more makes rehypothecation more attractive compared to securitisation.

When $(R_B - 1)\,d > \bar{R}$, B's pledgeability constraint binds, A's collateral constraint is slack, and the collateral posted is strictly greater than what is required if $\kappa > \bar{\kappa}$. Assume that $\bar{R} = 0$, so that the interest rate and the face value of the debt is the maximum possible without violating A's participation constraint. In this case, the higher pledgeability of the lender B makes it possible for him demand larger than necessary haircuts to invest in his very profitable opportunity. Hence, the optimal haircut with rehypothecation is strictly greater than the optimal haircut with securitisation, and B can further borrow a lot more with rehypothecation than with securitisation due to lower collateral risk in the short term. In particular, with securitisation, B can borrow from C an amount equal to $1+r$, while with rehypothecation, B can borrow an amount

$$\overbrace{\frac{1}{1-h}d}^{\text{Amt. with Rehypothecation}} \quad > \frac{1}{1-h}d^2 > \quad \overbrace{1+r}^{\text{Amt. with Securitisation}} \quad .$$

Again, these arguments must be tempered when A faces the risk of B's insolvency, but the attractiveness of rehypothecation relative to securitisation remains.

We can summarise the two channels which make rehypothecation better than securitisation as

$$d < 1, \text{ and}$$

$$\kappa > \bar{\kappa}.$$

When $\bar{R}$ is high, A's collateral constraint binds. In this case, when $d = 1$, the amount of cash that B can borrow from C is the same with securitisation and rehypothecation, and they are equivalent. However, when $d < 1$, it is possible for B to borrow from C strictly more with rehypothecation than with securitisation. When $\bar{R}$ is low, the large pledgeability κ of B allows him to overcollateralise the debt with A and borrow more cash from C by rehypothecating the large amount of collateral thus obtained. This channel operates for any $d \leq 1$.

3.3 Equilibrium

With the optimal contracts of each type at hand, each agent of type B chooses between the four strategies taking $\bar{R}$ as given, and offers contracts $\mathcal{C}_{\mathrm{A,B}}(\bar{R}) = (r(\bar{R}), h(\bar{R}), \theta(\bar{R}))$ and $\mathcal{C}_{\mathrm{B,C}}(\mathcal{C}_{\mathrm{A,B}}(\bar{R})) = (\tilde{r}(\bar{R}), \tilde{h}(\bar{R}))$.

Definition 2. A symmetric competitive equilibrium is the collection of the net return $\bar{R}$ per unit collateral committed by A, the contracts $\mathcal{C}_{\mathrm{A,B}}(\bar{R})$ offered by B to A, the contracts $\mathcal{C}_{\mathrm{B,C}}(\mathcal{C}_{\mathrm{A,B}}(\bar{R}))$ offered by B to C, the allocations $(x_i(\bar{R}), z_i(\bar{R})), i \in \{\mathrm{A, B}\}$ and $(\tilde{x}_{\mathrm{B}}(\bar{R}), \tilde{z}_{\mathrm{B}}(\bar{R}))$ such that

1. *A's optimisation*: Agents A solve the problem (1) at $t = 1.1$ to obtain the demands for cash and collateral $(x_{\mathrm{A}}(\bar{R}), z_{\mathrm{A}}(\bar{R}))$ given by (2) and (3).

2. *B's optimisation*: Agents B choose between the optima of the problems (7), (8) and (11) at $t = 1.1$ to offer optimal contracts $\mathcal{C}_{\mathrm{A,B}}(\bar{R}) = (r(\bar{R}), h(\bar{R}), \theta(\bar{R}))$ given by Proposition 2 and to obtain the demands for cash and collateral $(x_{\mathrm{B}}(\bar{R}), z_{\mathrm{B}}(\bar{R}))$ given by (14) and (15). Agents B maximise their borrowing at $t = 1.2$ from agents C by offering optimal contracts $\mathcal{C}_{\mathrm{B,C}}(\mathcal{C}_{\mathrm{A,B}}(\bar{R})) = (\tilde{r}(\bar{R}), \tilde{h}(\bar{R}))$ given by Proposition 1 to obtain the demands for cash and collateral $(\tilde{x}_{\mathrm{B}}(\bar{R}), \tilde{z}_{\mathrm{B}}(\bar{R}))$ given by (4).

3. *Market clearing*: The collateral market between A and B at $t = 1.1$ clears (the market for cash between A and B at $t = 1.1$ then clears by Walras' law):

$$z_{\mathrm{A}}(\bar{R}) + z_{\mathrm{B}}(\bar{R}) = \Omega. \tag{16}$$

Equilibrium Allocations with Rehypothecation

I describe the equilibrium allocations with rehypothecation, i.e. when $pR_B \geq 1$ and $\kappa \geq \bar{\kappa}$. Figure 7 shows the collateral market clearing between A and B at $t = 1.1$. The demand for collateral $z_B(\bar{R})$ is downward sloping. When $\bar{R} > (pR_B - 1)d$, A's collateral constraint binds, and when $\bar{R} < (pR_B - 1)d$, B's pledgeability constraint binds. When $(pR_B - 1)d = \bar{R}$, B is indifferent and demands any amount of collateral between the two binding constraints. The following proposition summarises the equilibrium allocations of cash and the investments in the projects of A and B.

Proposition 3 (Rehypothecation Allocations)**.** *If $\kappa \geq \bar{\kappa}$ and $pR_B \geq 1$, then in equilibrium*

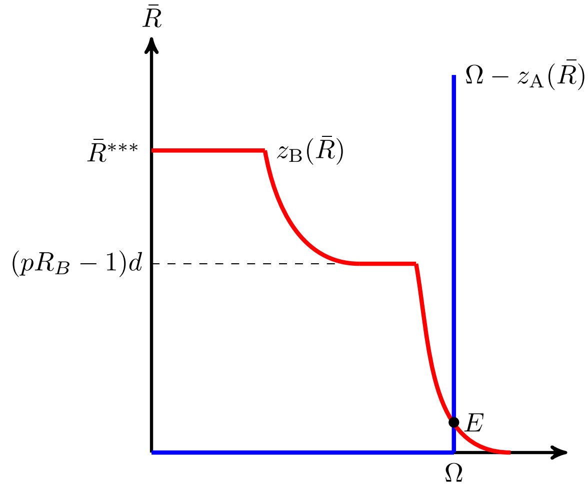

Figure 7: Equilibrium. Collateral market clearing between A and B at $t = 1.1$. The blue curve labelled $\Omega - z_{\mathrm{A}}(\bar{R})$ represents the supply of collateral by A. The red curve labelled $z_{\mathrm{B}}(\bar{R})$ represents the demand for collateral by B.

1. $x_A = 1$, $\tilde{x}_B = \frac{R_A}{1-\kappa p R_B}$, *if* $\Omega \in \left[\frac{1}{d}\frac{R_A}{1-\kappa p R_B}, \infty\right)$,
2. $x_A = 1$, $\tilde{x}_B = \Omega d$, *if* $\Omega \in \left[\frac{1}{d}\frac{1}{pR_B(1-\bar{\kappa})}, \frac{1}{d}\frac{R_A}{1-\kappa p R_B}\right]$,
3. $x_A = \Omega d p R_B(1-\bar{\kappa})$, $\tilde{x}_B = \Omega d$, *if* $\Omega \in \left[0, \frac{1}{d}\frac{1}{pR_B(1-\bar{\kappa})}\right]$.

When collateral is abundant, A is always able to secure all of B's cash endowment to invest. If collateral is too abundant, B is unable to charge very high haircuts and rehypothecate since A is not willing to take on the risk of losing a large quantity of collateral. In this case, some collateral sits idle with A. When collateral is scarce, B reduces credit to A. The more useful the collateral is to B (pR_B is high), the more willing B is to provide credit to A as even smaller haircuts enable B to generate revenue to break even. In the next section, I show that these decentralised equilibrium allocations are constrained efficient.

4 Social Planner's Problem

The social planner is assumed to maximise the sum of the expected output (sum of the expected consumptions of A, B and C) at $t = 2$ and $t = 3$. It can be easily shown that the objective function is

$$Y(\theta|d,\kappa) = \max_{x_{\mathrm{A}}, \tilde{x}_{\mathrm{B}}, \mathcal{C}_{\mathrm{A,B}}, \mathcal{C}_{\mathrm{B,C}}} (R_A - 1)x_{\mathrm{A}} + (pR_B - 1)\tilde{x}_{\mathrm{B}}, \tag{17}$$

modulo a constant. Here $\theta = 1$ indicates that the planner is permitted to rehypothecate, and $\theta = 0$ indicates that he is not. I focus on the parameters d and κ which provide the two channels by which rehypothecation is preferred to securitisation. The social planner must respect three types of constraints:

1. Resource constraints,
2. Collateral and pledgeability constraints, and
3. Participation ("Implementability") constraints.

The form of each of these constraints is described in Appendix D. I assume that the social planner can choose the type of contracts between A, B and C at $t = 1$, but cannot enforce repayment or force B to efficiently manage his investment after $t = 1$, which is why the collateral constraints for A and B and the pledgeability constraint for B are required. I also assume that the contracts designed by the social planner are preferred by each agent to autarky−these are the participation or implementability constraints.

Theorem 1 (Constrained Optimality)**.** *The decentralised equilibrium allocations are constrained efficient.*

In particular, when rehypothecation is feasible ($\kappa \geq \bar{\kappa}$) and desirable ($pR_B \geq 1$), the equilibrium allocations outlined in Proposition 3 are the same as those achieved by the constrained social planner maximising (17), subject to collateral, pledgeability and participation constraints. This is hardly surprising, since B internalises the impact of $\mathcal{C}_{\mathrm{A,B}}$ on $\mathcal{C}_{\mathrm{B,C}}$.

Definition 3. Define the partial ordering "$\succeq$" on Y (defined in (17)) such that (i) $Y(\theta|d,\kappa) \succeq Y(\theta'|d',\kappa')$ if and only if $\forall\Omega$ $Y(\theta|d,\kappa) \geq Y(\theta'|d',\kappa')$ and $\exists\Omega$ such that $Y(\theta|d,\kappa) > Y(\theta'|d',\kappa')$; (ii) $Y(\theta|d,\kappa) \succ Y(\theta'|d',\kappa')$ if and only if $\forall\Omega$ $Y(\theta|d,\kappa) > Y(\theta'|d',\kappa')$; and (iii) $Y(\theta|d,\kappa) \sim Y(\theta'|d',\kappa')$ if and only if $\forall\Omega$ $Y(\theta|d,\kappa) = Y(\theta'|d',\kappa')$.

The following theorem compares the optimal allocations with rehypothecation and securitisation.

Theorem 2 (Optimality of Rehypothecation)**.** *If $pR_B \geq 1$,*

1. $Y(\theta = 1|d < 1, \kappa = \bar{\kappa}) \succ Y(\theta = 0|d < 1, \kappa = \bar{\kappa})$,

2. $Y(\theta = 1|d = 1, \kappa > \bar{\kappa}) \succeq Y(\theta = 0|d = 1, \kappa > \bar{\kappa})$, *and*

3. $Y(\theta = 1|d = 1, \kappa = \bar{\kappa}) \sim Y(\theta = 0|d = 1, \kappa = \bar{\kappa})$.

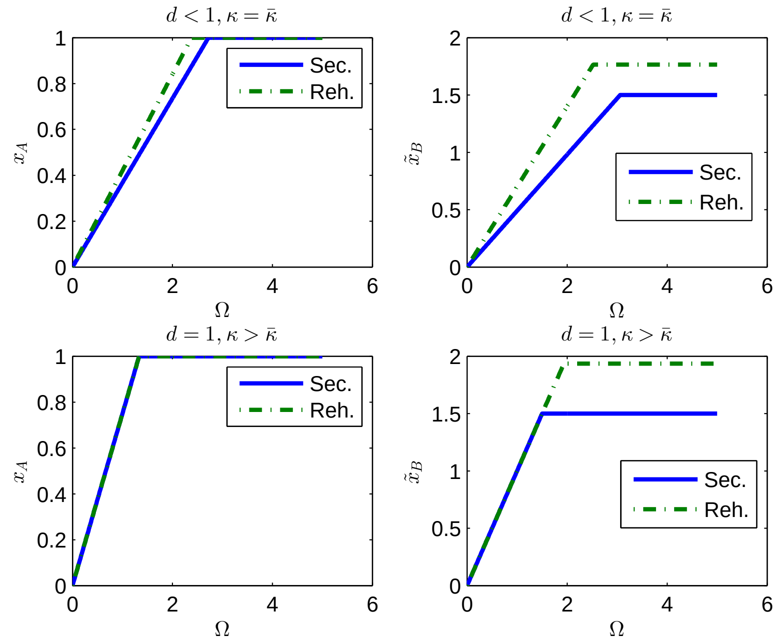

Figure 8: Rehypothecation is better than securitisation. The top two panels plot x_{A} and $\tilde{x}_{\mathrm{B}}$ for $d < 1$ and $\kappa = \bar{\kappa}$. The bottom two panels plot x_{A} and $\tilde{x}_{\mathrm{B}}$ for $d = 1$ and $\kappa > \bar{\kappa}$.

The result again establishes the significance of both d and κ in making rehypothecation better than securitisation. While Proposition 2 shows the private optimality of rehypothecation, Theorem 2 establishes the social optimality. The intuition for this result can be seen using Figure 8.

When $\kappa = \bar{\kappa}$ and $d < 1$, $\mathcal{C}_{\mathrm{A,B}}$ cannot be overcollateralised and A's collateral constraint always binds with rehypothecation so that $1+r = \frac{1}{1-h}d^2$. For every unit of collateral allocated to B, the planner can raise d for B with rehypothecation. But the planner can raise a maximum of $d^2 < d$ with securitisation, and the possibility of exploiting the low short-term risk of the collateral is wasted. Furthermore, when collateral is in short supply so that all the cash of B cannot be allocated to A, the planner would like to minimise the haircut without violating B's participation constraint. The planner finds it easier to do it with rehypothecation than with securitisation because of the better payoff to B with rehypothecation.

When $\kappa > \bar{\kappa}$ and $d = 1$, the amount that B can borrow with securitisation is given by the face value $1 + r$ of $\mathcal{C}_{\mathrm{A,B}}$. Since A must participate, this face value cannot be greater than R_A. Hence, even if the collateral endowment is greater than R_A which is sufficient to sustain an equal face value with $d = 1$, the planner cannot raise the face value and the extra collateral is wasted. However, since $\kappa > \bar{\kappa}$, the amount of borrowing by B with rehypothecation does not depend on the face value $1 + r$. The planner can efficiently use an endowment of collateral larger than R_A to secure more borrowing for B, while compensating A with a lower interest rate.

The two channels allow the intermediate lender B to squeeze as much liquidity as possible from the collateral. A does not have access to C's cash. B's special position in the middle of the chain allows it access to A's collateral and C's cash, and rehypothecation allows B to make the most efficient use of the collateral to obtain the borrowing from C.

Allowing A to borrow from C

So far, I have assumed that A is not allowed to borrow from C, but can only borrow from B. Suppose now that A can borrow directly from C and well as B. In a decentralised equilibrium, A will typically end up borrowing from both B and C. Some of the collateral will end up with C, where it will not be put to use.

Theorem 3. *If* A *can contract with both* B *and* C*, the decentralised equilibrium is inefficient compared to the case when* A *can only contract with* B*, if* $pR_B - 1 > (R_A - 1)d$*.*

If B's project is sufficiently valuable, then the social planner will direct that A should borrow only from B and not from C, since the intermediate lender B will receive all the collateral to rehypothecate and invest in his valuable project. The intuition behind this result is that when A can contract with C as well as B, the equilibrium net returns from lending collateral to B and C are equalised. A gives away some of the collateral to C who demands it to guard against A's default. But the collateral does not produce additional surplus in the hands of C. If instead A borrows only from B, B is able to make good use of it by rehypothecating it and investing in his profitable project. The inefficiency of the decentralised equilibrium is the result of a pecuniary externality. The presence of an alternative but safe cash lender C bids up the net return $\bar{R}$ to A from lending out collateral. The intermediate lender B, due to limited pledgeability, cannot out-bid C even though his project is more profitable. I shall revisit this externality in Section 6 on the intervention by a central bank in a rehypothecation chain.

4.1 Repo vs Mortgage Markets

I differentiate between the repo and mortgage markets by the different pledgeabilities of the intermediate lender B's income to the borrower A. I propose that the difference between the prime brokerage and the mortgage markets is due to the costs of monitoring the lender. The original borrowers in the prime brokerage market, the hedge funds, are sophisticated investors. The fund managers are well versed with the practices of their brokers, the types of investments they make and how they manage the risks. Quite often, the fund managers themselves have had a stint in the brokerage, investment banking or risk management areas in the past, and tend to have close relationships with many market participants. This gives them a dual advantage−they understand the practices and investment tendencies of their brokers well, and are well informed about the general trends in the market, if not about the specific investments of their brokers. Furthermore, as mentioned above, the funds have long-term contracts with their brokers that go beyond these individual debt contracts, and the brokers compete for their services. If the fund manager feels that his broker is taking undue risks with his collateral, he can threaten to renegotiate or terminate the brokerage contract and move to another competing broker. This acts as a disciplining device for the brokers who would then be more conscientious about their risk taking. I interpret these factors in the prime brokerage market as a high pledgeability κ of the lenders' incomes to the borrowers. This makes rehypothecation feasible and optimal.

In the mortgage market, on the other hand, the original borrowers are small banks and savings and loan associations. Their repertoire is generally limited to the plain-vanilla market of home loans. They are not generally informed about the ways and practices of their lenders, the investment banks. They also do not generally have relationships with the investment banks outside of the individual mortgage pass-through securities. Such narrow contractual arrangements prevents these mortgage originators from being able to influence their lenders to be more scrupulous about their risk taking. I interpret these factors in the mortgage market as a low pledgeability κ of the lenders' incomes to the small banks. The low pledgeability arising from the large cost of monitoring makes rehypothecation infeasible. This market, therefore, sees a securitisation of mortgages into MBS's. A similar argument shows us why the unsophisticated homeowners do not allow their lenders (small banks) to rehypothecate the house collateral, which leads the lenders to use the mortgage loans as collateral against which to borrow from the large investment banks.

5 Comparative Statics

I study the behaviour of the repo and prime-brokerage markets, which are characterised by rehypothecation, during a crisis. The collateralised lending market experienced falling rehypothecation, as documented in Singh (2011, 2012) and Singh and Aitken (2009, 2010), and rising haircuts and borrowing rates, as documented in Gorton and Metrick (2013). The seminal work by Gorton and Metrick (2013) was the first to document the effects of the crisis on the bilateral repo market. Rehypothecation is a key feature of this market, and any theoretical analysis of it must incorporate the two sides−supply and demand−of the market for collateral, and how they determine the "price" of collateral. Below, I look at the comparative statics with respect to the variables which were affected by during crisis.

In view of the documented facts, I look at what happens when (i) the lenders' projects become more risky (a fall in p to $p' < p$) with the expected return remaining constant, (ii) the lenders' pledgeability falls (a fall in κ to $\kappa' < \kappa$), and (iii) the risk of the collateral increases (a fall in d to $d' < d$). I restrict attention to parameter values where rehypothecation is still feasible and optimal, i.e. $p'R_B \geq 1$ and $\kappa \geq \frac{1-d'}{R_B} = \bar{\kappa}'$. This will enable me to relate to the facts documented by Gorton and Metrick (2013) who look at rates and haircuts in the bilateral repo market.

When the intermediate lender B becomes more risky, A needs a larger compensation for taking on a higher risk of B's default, leading to a fall in the interest rate. Lower haircuts provide an additional benefit to A by reducing the quantum of loss due to B's default.

Proposition 4 (Lender Risk)**.** *Let (r, h) and (r', h') be the equilibrium rates and haircuts when the probability of success of B's project is p and $p' < p$ respectively. If R_B rises proportionately to $R'_B > R_B$ to keep $pR_B = p'R'_B$ constant, then $r' \leq r$ and $h' \leq h$.*

When the pledgeability of the lenders falls, possibly because of a more severe moral hazard problem, lenders are unable to demand a high haircut and borrow too much collateral. The limited ability of the lenders to invest in their profitable opportunity means that they have less surplus to share with the borrowers, and the interest rate increases.

Proposition 5 (Moral Hazard)**.** *Let (r, h) and (r', h') be the equilibrium rates and haircuts when the pledgeability of B's project is κ and $\kappa' < \kappa$ respectively. Then $h' \leq h$ and $r' \geq r$.*

When the risk of the collateral increases, the haircut increases as more collateral is needed to secure one unit of cash lending by A. The increased demand for the collateral means that borrowers need to be compensated with a lower interest rate.

Proposition 6 (Collateral Risk)**.** *Let (r, h) and (r', h') be the equilibrium rates and haircuts when the lowest value of the collateral at $t = 1$ is d and $d' < d$ respectively. Then $h' \geq h$, and if $d' < \bar{g}(d)$ where $\bar{g}'(\cdot) > 0$, $r' \leq r$.*

Gorton and Metrick (2013) document a rise in repo rates and haircuts across a wide variety of collateral classes during the crisis. The model provides three channels for the changes in haircuts and rates. According to the model, haircuts face a downward pressure as borrowers do not want to hand over too much rehypothecable collateral to risky or unreliable lenders, and face an upward pressure as the collateral becomes riskier. The second channel appears to have dominated during the crisis in the case of highly risky collateral such as corporate bonds. Interest rates face a downward pressure as lenders and the collateral become riskier, and an upward pressure as the limited pledgeability of the lenders also limits their ability to share the surplus. The first channel does not seem to have been dominant during the crisis. Gorton and Metrick (2013) suggest that the rise in the repo rates across a wide variety of collateral classes was to compensate for the increased default risk of the borrowers−a channel not modelled here.

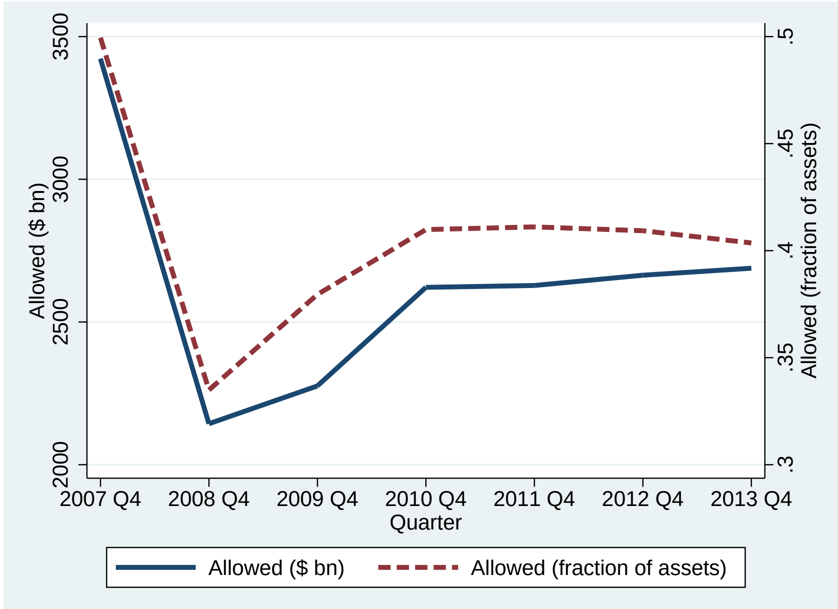

Figure 9: Crash in repledgeable collateral at the top five US broker-dealers.

When the expected net present value of B's investment becomes negative ($pR_B < 1$), rehypothecation becomes inefficient. Borrowers then prevent rehypothecation and lenders simply hold on to the debt. Similarly, when the pledgeability falls below the cutoff ($\kappa < \bar{\kappa}$), rehypothecation becomes infeasible.[12] This is consistent with the observed fall in the repledgeable collateral as a fraction of assets (Figure 9).

6 Central Bank Intervention

The collateral reuse chain was, so far, working without outside interference. I now look at what happens when a central bank intervenes in an existing chain to exchange cash for collateral. I restrict attention to a rehypothecation chain since it is collateral like treasury securities, which are commonly rehypothecated, that is directly affected by such an intervention. I show that a monetary policy action of the central bank may lose bite as the taking out or putting in of collateral in exchange for cash will affect collateral constraints down the chain in a way that will be counterproductive to the objective of the central bank.

The channel I highlight here works through changing quantities of treasury collateral available with borrowers to pledge to private lenders, and is independent of the monetary policy channels that work through changing bond prices and interest rates. Although this

[12] Although rehypothecation becomes infeasible when the lenders' pledgeability falls a lot, securitisation may still be desirable if the lenders' projects are profitable. One reason why the switch to securitisation of repo contracts was not observed during the crisis could be the large fixed costs involved in developing a market and the associated legal framework for it.

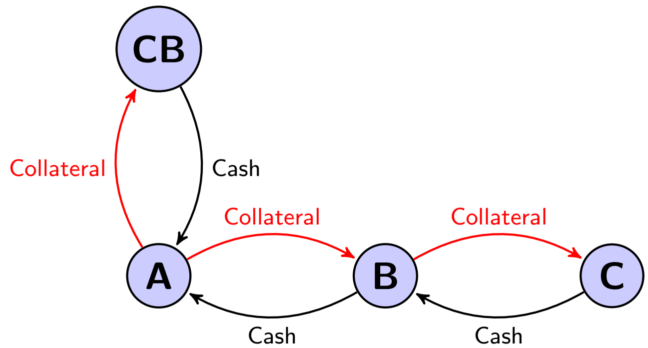

Figure 10: Expansionary operation−central bank lending against collateral.

channel will be operating in normal situations, a useful way to think of the model is in a situation in which large scale purchases or sales of treasury collateral are carried out by the central bank. Such purchases were undertaken by central banks in advanced economies in response to the global financial crisis. There is also a possibility of large scale sales of collateral being carried out in the near future as these economies emerge out of the recession. The large quantities involved make this channel quantitatively important.

I assume a repo-style intervention by the central bank−the central bank borrows or lends cash against collateral, rather than buying or selling it. This is in keeping with the tenor of the model, and does not lose generality. Purchases and sales of collateral will have the same effect on collateral constraints down a rehypothecation chain as the collateral enters or leaves a central bank. I also assume that the central bank lends cash to or borrows cash from A against collateral. To recap, A can be thought of as a hedge fund or a dealer, B as another dealer, and C as a money fund. Figure 10 shows the flow of cash and collateral as the central bank lends cash to A against collateral. The choice of A as the point of entry has two reasons. First, the unintended effects on collateral constrains arise when the entry is at A, and not when it is at B or C. Second, this is quantitatively relevant: Carpenter et al. (2015) show that hedge funds tend to be the largest buyers and sellers of treasury securities, far larger than broker dealers. Their analysis estimates that of the $600 billion of treasuries purchased by the Fed during LSAP-2, about 60% were sold by hedge funds[13]. They propose a preferred habitat explanation for this evidence.

I abstract from formalising the objective of the central bank. I assume that when the central bank lends cash, it *intends* to stimulate the economy and increase output, and when it borrows cash, it *intends* to cool down the economy and decrease output. The central bank targets a supply of cash m to be infused into the system. (I abstract from the reasons why the bank would target a particular m.) This is without loss of generality. In the model, targeting

[13]While hedge funds are not counterparties to the Fed's open market operations, they can sell securities to the Fed through their broker-dealers, who are.

a money supply is equivalent to targeting the interest rate between A and B.

Before the market between A and B opens at $t = 1.1$, the central bank at $t = 0$ lends $m \in \mathbb{R}$ units of cash to A until $t = 3$, at the terms (r^{CB}, h^{CB}) set by the market in equilibrium. $m > 0$ indicates an expansionary operation and $m < 0$ indicates a contractionary operation. I assume that the central bank makes the loan fully secured with $1 + r^{CB} = \frac{1}{1-h^{CB}} d^2$, so that only r^{CB} is left to be determined by the market equilibrium. The central bank rebates lump-sum all profits to A, B or C at $t = 3$. As A borrows from the central bank, it moves collateral away from B by an amount

$$\Delta\Omega = m \frac{1}{1 - h^{CB}}. \tag{18}$$

The changed supply of collateral affects the equilibrium between A and B. If $m > 0$, the central bank takes away collateral and increases $\bar{R}$ and reduces the interest rate between A and B. If $m < 0$, the central bank supplies collateral, decreases $\bar{R}$ and increases the interest rate. In the new equilibrium, A is indifferent between borrowing from B and borrowing from the central bank, so that

$$\left(R_A - (1 + r^{CB})\right)(1 - h^{CB}) = \bar{R}. \tag{19}$$

Definition 4. An equilibrium with central bank intervention of $m \in \mathbb{R}$ units of cash is the collection of the net return $\bar{R}$ per unit collateral committed by A, the terms (r^{CB}, h^{CB}) of borrowing from (or lending to) the central bank, the contracts $\mathcal{C}_{\mathrm{A,B}}(\bar{R})$ offered by B to A, the contracts $\mathcal{C}_{\mathrm{B,C}}(\mathcal{C}_{\mathrm{A,B}}(\bar{R}))$ offered by B to C, the allocations $(x_i(\bar{R}), z_i(\bar{R})), i \in \{\mathrm{A}, \mathrm{B}\}$ and $(\tilde{x}_{\mathrm{B}}(\bar{R}), \tilde{z}_{\mathrm{B}}(\bar{R}))$ such that

1. *Optimisation at $t > 0$*: Conditions (1)-(3) in Definition 2 are satisfied for the optimisation of A, B and C at $t > 0$.

2. *Central Bank's haircut choice assumption*: The central bank secures the debt fully and does not overcollateralise, so that $1 + r^{CB} = \frac{1}{1-h^{CB}}$.

3. *A's optimisation at $t = 0$*: Agents A are indifferent between using the collateral to borrow from B and accepting the central bank's offer, so that Equation (19) holds.

4. *Market clearing*: The collateral market between A and B at $t = 1.1$ clears:

$$z_{\mathrm{A}}(\bar{R}) + z_{\mathrm{B}}(\bar{R}) = \Omega - \Delta\Omega,$$

where $\Delta\Omega$ is given by Equation (18).

Define the total output Y as the sum of the expected consumptions of A, B and C at $t = 2$ and $t = 3$. I have the following proposition:

Proposition 7. *In the equilibrium with central bank intervention described in Definition 4, the marginal effect due to the intervention of $m \in \mathbb{R}$ units of cash is given by*

$$\left.\frac{\partial Y}{\partial m}\right|_{m=0} = \begin{cases} (R_A - 1) & \text{if } \bar{R} = 0, \\ (R_A - 1) - (R_B - 1) d \frac{1}{1-h^{CB}} & \text{if } \bar{R} \in \left(0, \bar{R}^{***}\right), \\ (R_A - 1) - (R_B - 1) d \frac{1}{1-h^{CB}} - (R_A - 1)(1-h) \frac{1}{1-h^{CB}} & \text{if } \bar{R} = \bar{R}^{***}. \end{cases} \quad (20)$$

The proposition says that the action by the central bank may have unintended consequences that work against the central bank's objective. For example, in the case of an expansionary operation with $m > 0$, the decreased supply of collateral my tighten the collateral constraint between B and C, having an adverse effect on output. Figure 11 shows the equilibria in the three regions which react varyingly to an expansionary operation with $m > 0$.

In the first region, when collateral is abundant and $\bar{R} = 0$, the action of the central bank will have the desired effect of increasing output as A invests the extra cash. There will be no effect on the equilibrium between A and B as A will use the idle collateral to borrow from the central bank.

However, when collateral is scarce in the second region and $\bar{R} > 0$, useful collateral is taken away from B through a lower haircut, and B cannot borrow as before by rehypothecating the collateral to C. This effect is particular to a rehypothecation chain, and would be absent if the central bank had entered at B. It is clear that the tightening of B's collateral constraint causes the monetary policy action to lose its bite. In fact, as detailed in Appendix G, it could possibly have the opposite effect and cause aggregate expected output to decrease.

Corollary 1. *If $R_B > R_A$, there exists an $\bar{R}^{\#} > 0$ such that $\left.\frac{\partial Y}{\partial m}\right|_{m=0} < 0$ for $\bar{R} \in (0, \bar{R}^{\#})$.*

The reason why output may even fall due to the expansionary intervention is reminiscent of the pecuniary externality in Section 4. A may end up giving too much collateral to the central bank, without taking into account the adverse effects of doing so on the collateral constraint between B and C. The central bank is only contracting with A, and A cannot internalise the loss due to the tightening of B's collateral constraint. The collateral sits idle with the central bank, when otherwise it could have been used profitably by B. Even though the collateral is more useful to B, B cannot outbid the central bank to borrow it due to limited pledgeability.[14]

In the third region, $\bar{R} = \bar{R}^{***}$. B is not lending to A its full endowment of cash, and the central bank's entry causes A to simply substitute to borrowing from the central bank instead

[14] This assumes that the cash (reserves) received by A from the central bank cannot be pledged to B. In Appendix G.2, I relax this assumption and show that the result about the fall in expected output still holds. The intuition is that when B is rehypothecating a A's collateral at bad terms for A (low $\bar{R}$), A chooses to invest the cash received from the central bank in its project instead of lending it to B. B's limited pledgeability prevents it from borrowing more from A under the same terms.

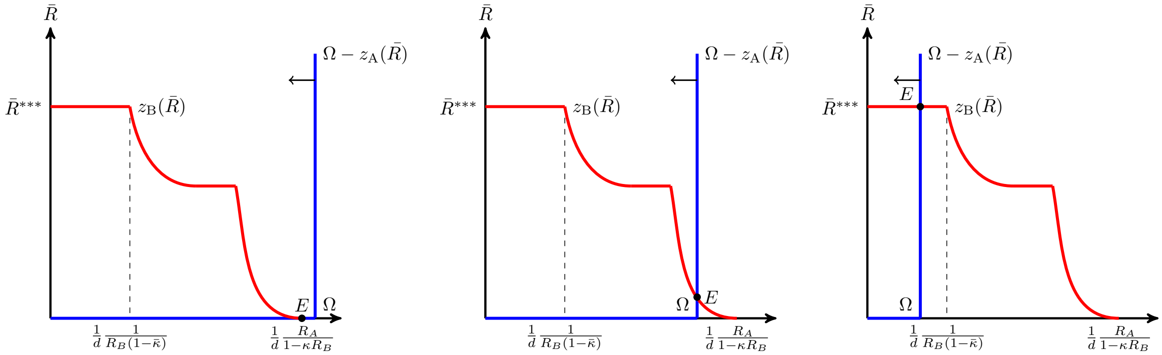

Figure 11: Equilibrium with an expansionary open market operation.

of from B. A's borrowing and investment will increase by the difference in the haircuts h and h^{CB} offered by B and the central bank respectively, and B will consume the idle cash. This effect is not particular to a rehypothecation chain, and would have existed even if the central bank had entered at B.

The case of contractionary operations is nearly symmetric, with an additional feature: the central bank must decide whether to allow A to rehypothecate the received collateral. With a sale of the collateral, the rehypothecation permission in implicit. With lending collateral, however, the permission needs to be made explicit.

Corollary 2 (Contractionary Operation)**.** *The marginal effect due to a contractionary operation ($m < 0$) is given by Proposition 7 if rehypothecation by* A *is permitted, and by* $\left.\frac{\partial Y}{\partial m}\right|_{m=0} = R_A - 1$, $\forall \bar{R} \in [0, \bar{R}^{***}]$ *if rehypothecation by* A *is not permitted.*

If collateral is not permitted to be rehypothecated by A, it just sits with A and does not circulate to generate more borrowing and investment.

6.1 Empirical Evidence

The model predicts that an increase in the supply of collateral lowers the return $\bar{R}$ from lending collateral and increases the repo rate r in the bilateral repo market between A and B.[15] I consider the weekly regression

$$\Delta s_t = \alpha + \rho_1 \Delta s_{t-1} + \rho_2 s_{t-1} + X_t' \gamma + \beta_1 \Delta T_t + \beta_2 I_{t-1} \cdot \Delta T_t + \epsilon_t. \tag{21}$$

Here, $s_t = r_t - y_t^{3mo}$ is the overnight borrowing rate with treasury collateral, normalised by the short term risk free rate, the three month treasury yield. X_t are various market controls

[15] While in the simple model with homogeneous agents, $\bar{R}$ does not change for small changes in the supply of collateral when $\bar{R} \in \{\bar{R}^{***}, (pR_B - 1)d\}$, it can be shown that for a continuum of agents B with project returns in some interval $[R_{B,L}, R_{B,U}]$, $\frac{\partial \bar{R}}{\partial m} < 0$ and $\frac{\partial r}{\partial m} > 0$ when collateral is scarce and $\bar{R} > 0$, and $\frac{\partial \bar{R}}{\partial m} = \frac{\partial r}{\partial m} = 0$ when collateral is idle and $\bar{R} = 0$.

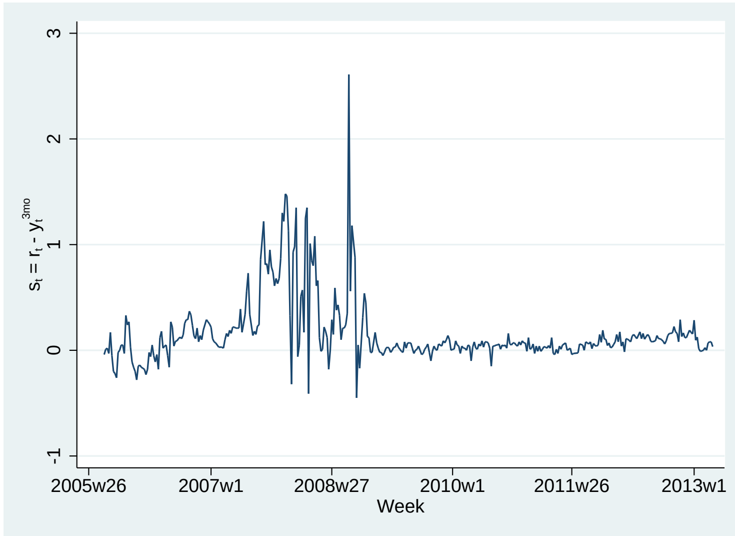

Figure 12: Spread between overnight treasury collateral repo rates and the 90-day treasury yield.

including the lagged equity market, the VIX, short and long term treasury yields, and their changes. ΔT_t are the weekly changes in outstanding treasury securities. The product $I_{t-1} \cdot \Delta T_t$ of a measure of collateral idleness and the change in treasury supply is included to test for the hypothesis that the interest rate reacts less to changing collateral supply when collateral is idle.

The model also predicts that an increase in the supply of collateral increases financing activity between A and B and between B and C as the investment in B's project increases, and more so when collateral is scarce. I consider the weekly regressions

$$\Delta F_t = \alpha + \rho_1 \Delta F_{t-1} + \rho_2 F_{t-1} + D_t' \delta + X_t' \gamma + \beta_1 \Delta T_t + \beta_2 I_{t-1} \cdot \Delta T_t + \epsilon_t. \tag{22}$$

ΔF_t is the weekly change in aggregate overnight borrowing or lending reported weekly by Primary Dealers. I run the regressions for ΔF_t equal to the change in the aggregate borrowing, the aggregate lending, and the net borrowing (borrowing minus lending) using treasury collateral. D_t are weekly dummies at end-of-quarter weeks.[16]

For the index of collateral idleness, I use $I_t = s_t$ itself, since the model predicts that when collateral is abundant and idle, the interest rate is high. The model predicts that in both regressions, $\beta_1 > 0$ and $\beta_2 < 0$.

Data

The data for F_t are obtained from the aggregate FR 2004 reports published by the Federal Reserve Bank of New York. These data are for aggregate (repo and other) borrowing and lending by the set of primary dealers using treasury collateral. The series for T_t are obtained by subtracting the treasury holdings of the System Open Market Account of the FRBNY from the federal debt held by the public[17]. The series for r_t is the index of overnight US treasury collateral repo rates USRG1T obtained from Bloomberg. The data are weekly from 2005Q2 to 2013Q1. I use high frequency weekly data to minimise lower frequency fluctuations and other secular trends caused by regulatory changes. I use overnight financing data to increase the power of the test, since it is overnight financing, which needs to be rolled over every night, that will be impacted most by the changing supply of collateral.

Figure 12 shows the spread between the overnight treasury collateral repo rates and the three month treasury yield. The fact that the spread was positive in the years leading up to the crisis, and has been close to zero since, seems to indicate that collateral has been less idle since the crisis.

Results

The identification assumption in the regressions is that the unobserved factors affecting the dependent variables are uncorrelated with the weekly changes in treasury supply. The issuance of bonds by the Treasury is according to a preset timetable, which does not interact with the unobserved factors at a weekly frequency.

The results for regression (21) are in table 1. The spread between the repo rate and the risk-free rate increases significantly as the treasury supply increases, but not so much when the spread is already high and collateral is idle. Since the time series for the spread indicates that collateral has been less idle since the crisis, I rerun the regression for before and after 1 January 2009. I find that the effect of treasury supply on the spread is significant only after that date, when collateral was scarce. Figure 13 shows this conditional correlation before and after that date.

The results for regression (22) are in Table 2. I find that an increase in the supply of collateral significantly increases the amount of financing by broker-dealers. Most importantly, in the third column I find that net borrowing by primary dealers from outside their set also

[16] These are included because there is an observed drop in financing at the end of the quarter due to cyclical factors.

[17] The weekly data for the federal debt held by the public are obtained from www.treasutydirect.gov. These data are the face value of all US treasury securities. The ideal variable to look at would be the market value of all outstanding treasury securities, the data for which is not publicly available at a weekly frequency to the best of my knowledge. The use of the face value data will only underestimate the significance of my results. An increase is the supply of collateral will reduce the price of the collateral. The measurement error in the increase in the supply of collateral will bias the coefficient downwards.

	$\Delta s_t/10^{-6}$			
	Whole period		Before 01-01-09	After 01-01-09
ΔT_t	0.37^{*}	1.06^{**}	0.29	0.31^{**}
	(0.15)	(0.40)	(0.65)	(0.12)
$s_{t-1} \cdot \Delta T_t$		-3.08^{***}		
		(0.65)		
Controls	Yes	Yes	Yes	Yes
R^2	0.28	0.32	0.31	0.37
N	383	383	179	204

Table 1: Regressing repo-treasury spread on treasury supply. HAC standard errors.

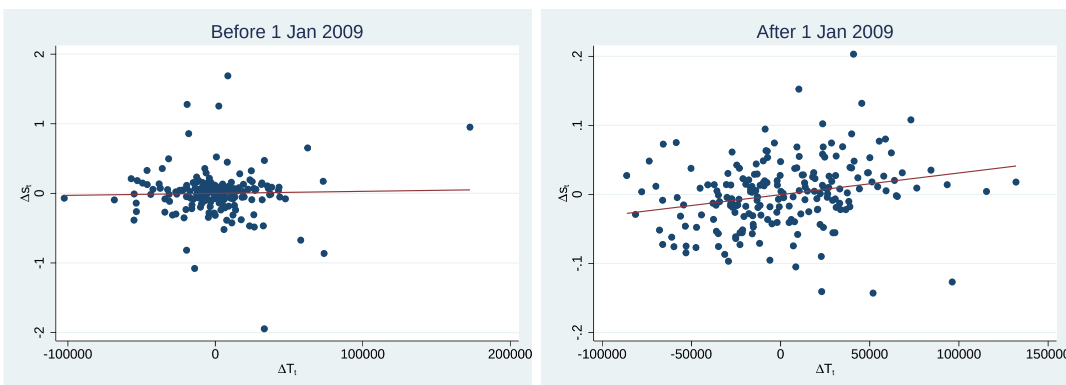

Figure 13: Conditional correlation between treasury supply and the repo-treasury spread, before and after 1 January 2009. Conditioning on the control variables.

	ΔF_t					
	Borrowing		Lending		Net Borrowing	
ΔT_t	0.31^{***}	0.30^{***}	0.15^{*}	0.16^{*}	0.16^{***}	0.15^{***}
	(0.06)	(0.07)	(0.06)	(0.07)	(0.03)	(0.03)
$s_{t-1} \cdot \Delta T_t$		-0.02		-0.05		0.07
		(0.08)		(0.09)		(0.04)
Dummies	Yes	Yes	Yes	Yes	Yes	Yes
Controls	Yes	Yes	Yes	Yes	Yes	Yes
R^2	0.49	0.50	0.42	0.43	0.25	0.25
N	392	392	392	392	392	392

Table 2: Regressing aggregate overnight borrowing, lending and net borrowing on treasury supply. HAC standard errors.

increases. However, this may be consistent with the absence of a rehypothecation chain, and the primary dealer simply borrowing from a money fund to purchase the treasury security from the central bank. This concern is mitigated by the fact that lending by primary dealers also goes up significantly, which indicates that the dealers receive treasury collateral to do with as they wish. The coefficient on the interaction term is not significant. The reason for this is that the quantities are not disaggregated between hedge fund-dealer/inter-dealer ($\mathcal{C}_{\text{A,B}}$) financing and dealer-money fund ($\mathcal{C}_{\text{B,C}}$) financing. Idle collateral (high s_{t-1}) in the bilateral market could mean cheaper funding from money funds for the dealers, and this would show up in their increased borrowing. The effect on the bilateral market and rehypothecation, especially when collateral is idle, can be better identified by looking at the bilateral repo rate regression (21).

6.2 Policy Implications

The trade-off between cash and collateral created by open market operations leads to several policy implications. The fact that the removal of collateral during an expansionary open market operation can contract financing and investment suggests that the ideal way to conduct such operations is for the central bank to not demand collateral for the cash it supplies. The potential losses to the central bank due to default may then be covered by increased taxes on the profits of the financial sector. Similarly, the effective way to conduct contractionary contractionary operations would be to not provide collateral in exchange for cash, and use instruments like reserve requirements and interest on reserves. Other tools such as the Term Deposit Facility (TDF) could be expanded. So could the coverage of reserve requirements across institutions and liability types, in order for reserves to be effectively manipulated on a large scale using IOR without resorting to open market operations. This conclusion relates to Stein (2012) and Kashyap and Stein (2012), who advocate using a combination of interest on reserves and open market operations to optimally conduct monetary policy. In their framework which focusses on financial stability, active adjustment of the level of reserves prevents excessive short-term debt creation. My model highlights the role of externalities down the rehypothecation chain when open market operations are used when collateral is scarce.

If open market operations are to be conducted, they must be conducted in a way that does not affect the supply of collateral that can be profitably rehypothecated. One potential variable to look at would be the spread between the collateralised borrowing rate and the short term risk free rate. A high value of this spread would mean that collateral is abundant and idle and that it is safe to conduct an open market operation. Also, cash must be provided in exchange for bad assets that can no longer serve as collateral. While this prescription resembles that in Kiyotaki and Moore (2012), the channel is different. The purchases of distressed assets are meant to shore up their prices, according to Kiyotaki and Moore (2012), while they are meant to preserve the good collateral in the system in my model. The Term Securities Lending Facility (TSLF), with which the Fed lent high quality treasury collateral against other less valuable assets, is one way to resupply good collateral to the system. In combination with the large scale purchases of treasury securities, the TSLF effectively provided cash in exchange for bad collateral. With contractionary open market operations, it is much easier to ensure that the collateral provided does not enter into circulation by denying rehypothecation rights. The Fed's Overnight Reverse Repo Programme (ON-RRP) is conducted through a tri-party arrangement wherein the collateral stays with the tri-party bank and is not rehypothecated. It may be tempting to consider statutorily limiting rehypothecation to provide a greater control to the central bank over the outcomes of its monetary policy actions. However, this may not be ideal since it will prevent collateral from flowing to where it is needed the most.

Another way of minimising the unintended consequences of open market operations is to conduct them with counterparties who are at the lending end of the chain. Contractionary operations which lend collateral to money funds will be more effective since they are not natural rehypothecators and will hold on to it. The ON-RRP's expanded counterparties include such entities. In fact, primary dealers rarely avail of this facility and most of the volumes are transacted with money market funds.

7 Conclusion

I construct a unified model of rehypothecation and securitisation with the prime brokerage, repo and mortgage markets in mind. I find that rehypothecation allows the lender to borrow for the short term against the full value of the collateral received by him, which is typically larger than the face value of the debt contract. I consider the borrowers' ability to monitor and discipline their lenders into not taking too many risks with their collateral. As long as the costs of monitoring are small and the health of the lender is good, rehypothecation will be preferred to securitisation. I argue that the nature of the prime brokerage market and its participants makes it easier for the borrowers to monitor their lenders, and rehypothecation obtains in these markets. On the other hand, the unsophisticated nature of mortgage originators and their limited contractual arrangements with their lenders renders them ineffective at disciplining their lenders, and securitisation obtains in these markets. I show that open market operations of the central bank can backfire as high quality collateral removed from the system can tighten collateral constraints down the rehypothecation chain and impair financing and investment activity. I discuss a number of implications for policy.

A direction for future work would be to explore longer rehypothecation chains. Such chains will feature network externalities as in Acemoglu, Ozdaglar and Tahbaz-Salehi (2013), as the parties at one end of the rehypothecation chain will not internalise the effect of their contract on the parties at the other end of the chain. The network externalities will impair financial stability by allowing inefficient investment. A full analysis of the general chain will allow the formalisation of the concept of collateral velocity and provide additional insights into monetary and macro-prudential policy.

Another direction for future research could be to further explore the quantitative effects of open market operations on collateral constraints and financing. Special repo spreads provide a more accurate measure of the discrepancy between the supply and demand for collateral. Examining the impact of treasury supply on financing using treasury collateral when the spreads are high will clarify the quantitative importance of these effects.

A further direction would be to analyse the desirability of bilateral, over the counter markets, over anonymous, centralised markets. Over the counter markets enable the exploitation of pledgeability through relationships with known counterparties. The analysis in this paper shows that having the intermediate lender B in the middle of the chain is desirable as his cash flows are pledgeable to the borrower. Given a distribution of agents with varying pledgeabilities, the optimal network and institutional arrangement can be studied.

References

Acemoglu, D., Ozdaglar, A., & Tahbaz-Salehi, A. (2013). Systemic risk and stability in financial networks (No. w18727). National Bureau of Economic Research.

Andolfatto, D., Martin, F. M., & Zhang, S. (2015). Rehypothecation and Liquidity. Federal Reserve Bank of St. Louis Working Paper Series, (2015-003).

Araújo, A., Schommer, S., & Woodford, M. (2013). Conventional and unconventional monetary policy with endogenous collateral constraints (No. w19711). National Bureau of Economic Research.

Bloomberg (2015). The Treasury Market's Legendary Liquidity Has Been Drying Up. February 1, 2015. http://www.bloomberg.com/news/articles/2015-02-02/if-trading-bonds-is-hard-think-about-the-pain-when-rates-rise

Bottazzi, J. M., Luque, J., & Páscoa, M. R. (2012). Securities market theory: Possession, repo and rehypothecation. Journal of Economic Theory, 147(2), 477-500.

Caballero, R. J. (2006). On the macroeconomics of asset shortages (No. w12753). National Bureau of Economic Research.

Caballero, R. J., & Farhi, E. (2013). A model of the safe asset mechanism (sam): Safety traps and economic policy (No. w18737). National Bureau of Economic Research.

Caballero, R. J., Farhi, E., & Gourinchas, P. O. (2006). An equilibrium model of" global imbalances" and low interest rates (No. w11996). National Bureau of Economic Research.

Carpenter, S., Demiralp, S., Ihrig, J., & Klee, E. (2015). Analyzing Federal Reserve asset purchases: From whom does the Fed buy?. Journal of Banking & Finance, 52, 230-244.

Cetorelli, N., Mandel, B. H., & Mollineaux, L. (2012). The evolution of banks and financial intermediation: framing the analysis. Federal Reserve Bank of New York Economic Policy Review, 18(2), 1-12.

Cochrane, J. H. (2014). Monetary policy with interest on reserves. Journal of Economic Dynamics and Control, 49, 74-108.

Copeland, A., Martin, A., & Walker, M. (2014). Repo Runs: Evidence from the Tri-Party Repo Market. The Journal of Finance, 69(6), 2343-2380.

D'Amico, S., Fan, R., & Kitsul, Y. (2013). The scarcity value of Treasury collateral: Repo market effects of security-specific supply and demand factors. Federal Reserve Bank of Chicago, WP 2013-22.

Dang, T. V., Gorton, G., & Holmström, B. (2013). Haircuts and repo chains. Working paper.

Eren, E. (2014). Intermediary Funding Liquidity and Rehypothecation as Determinants of Repo Haircuts and Interest Rates. Working Paper, Stanford University.

Ewerhart, C., & Tapking, J. (2008). Repo markets, counterparty risk and the 2007/2008 liquidity crisis (No. 0909). European Central Bank.

Federal Reserve. (2014). Minutes of the July 17-18 2014 Federal Open Market Committee Meeting, http://www.federalreserve.gov/monetarypolicy/fomcminutes20140618.htm.

Financial Services Authority (2012). Assessing the possible sources of systemic risk from hedge funds, Report.

Financial Times (2015). Treasury volumes raise liquidity concerns. June 11, 2015. http://www.ft.com/intl/cms/s/0/9a5b334e-0fb5-11e5-94d1-00144feabdc0.html

Fleming, M. J., Hrung, W. B., & Keane, F. M. (2010). Repo market effects of the term securities lending facility. Federal Reserve Bank of New York Staff Report, 426, 1-25.

Geanakoplos, J. (1997). Promises Promises. In Santa Fe Institute Studies in the Sciences of Complexity, Volume 27, 285-320. Addison Wesley Publishing Co.

Geanakoplos, J. (2009). The Leverage Cycle. NBER Macroeconomics Annual, 24(1), 1-66.

Geanakoplos, J., & Zame, W (2009). Collateralized security markets. Working Paper.

Goodfriend, M. (2002). Interest on reserves and monetary policy. Federal Reserve Bank of New York Economic Policy Review, 8(1), 13-29.

Gorton, G. B. (2009). Slapped in the Face by the Invisible Hand: Banking and the Panic of 2007. Available at SSRN 1401882.

Gorton, G., & Metrick, A. (2013). Securitized banking and the run on repo. Journal of Financial Economics, 104(3), 425-451.

Holmström, B., & Tirole, J. (1997). Financial intermediation, loanable funds, and the real sector. Quarterly Journal of Economics, 112(3), 663-691.

Holmström, B., & Tirole, J. (1998). Private and Public Supply of Liquidity. Journal of Political Economy, 106(1), 1-40.

Infante, S. (2014). Money for Nothing: The Consequences of Repo Rehypothecation. Working Paper, Federal Reserve Board.

Kashyap, A. K., & Stein, J. C. (2012). The Optimal Conduct of Monetary Policy with Interest on Reserves. American Economic Journal: Macroeconomics, 4(1), 266-282.

Keister, T., Martin, A., & McAndrews, J. (2008). Divorcing money from monetary policy. Federal Reserve Bank of New York Economic Policy Review, 14(2), 1-16.

Kilenthong, W. & Townsend, R. (2011). Market Based, Segregated Exchanges with Default Risk. Working Paper.

Kirk, A. M., McAndrews, J., Sastry, P., & Weed, P. (2014). Matching collateral supply and financing demands in dealer banks. Federal Reserve Bank of New York Economic Policy Review, Forthcoming.

Kiyotaki, N., & Moore, J. (2012). Liquidity, business cycles, and monetary policy (No. w17934). National Bureau of Economic Research.

Krishnamurthy, A., Nagel, S., & Orlov, D. (2014). Sizing up repo. The Journal of Finance, 69(6), 2381-2417.

Krishnamurthy, A., & Vissing-Jorgensen, A. (2012). The aggregate demand for treasury debt. Journal of Political Economy, 120(2), 233-267.

Krishnamurthy, A., & Vissing-Jorgensen, A. (2013). The Ins and Outs of LSAPs. In Economic Symposium Conference Proceedings. Jackson Hole, WY: Federal Reserve Bank of Kansas City.

Lee, J. (2015). Collateral circulation and repo spreads. Available at SSRN 2548209.

Martin, A., McAndrews, J., Palida, A., & Skeie, D. R. (2013). Federal Reserve Tools for Managing Rates and Reserves. Federal Reserve Bank of New York Staff Report, 642, 1-34.

Martin, A., Skeie, D., & Von Thadden, E. L. (2014). Repo runs. Review of Financial Studies, 27(4), 957-989.

Maurin, V. (2014). Re-using the collateral of others: A General Equilibrium Model of Rehypothecation. Working Paper, European University Institute.

Monnet, C. (2011). Rehypothecation. Business Review, Federal Reserve Bank of Philadelphia, (Q4), 18-25.

Monnet, C., & Narajabad, B. (2012). Why rent when you can buy? A theory of repurchase agreements. In 2012 Meeting Papers (Vol. 647).

Pozsar, Z., Adrian, T., Ashcraft, A. B., & Boesky, H. (2010). Shadow banking. Federal Reserve Bank of New York Staff Report, 458, 1-35.

Singh, M. (2011). Velocity of Pledged Collateral: Analysis and Implications. IMF Working Papers, 11/256, 1-24.

Singh, M. (2012). The (Other) Deleveraging. IMF Working Papers, 12/179, 1-22.

Singh, M. & Aitken, J. (2009). Deleveraging After Lehman−Evidence from Reduced Rehypothecation. IMF Working Papers, 9/42, 1-12.

Singh, M. & Aitken, J. (2010). The (sizable) Role of Rehypothecation in the Shadow Banking System. IMF Working Papers, 10/172, 1-15.

Singh, M. (2013). Collateral and Monetary Policy. IMF Working Papers, 13/186, 1-17.

Stein, J. C. (2012). Monetary Policy as Financial Stability Regulation. The Quarterly Journal of Economics, 127(1), 57-95.

Townsend, R. (1980). Models of Money with Spatially Separated Agents. In Models of Monetary Economies, ed. by J. H. Kareken and Neil Wallace, Minneapolis: Federal Reserve Bank of Minneapolis, 265-304.

Appendix

A State-by-state Cash Positions with Securitisation and Re-hypothecation

This appendix describes the mechanism of the settlement of contracts between A, B and C by listing out the state-by-state cash flows and positions. At $t = 2$, the returns to B's investment are realised, and then $\mathcal{C}_{\mathrm{B,C}}$ is settled. At $t = 3$, the returns to A's investment are realised, and finally $\mathcal{C}_{\mathrm{A,B}}$ is settled.

When $\mathcal{C}_{\mathrm{B,C}}$ is being settled and B defaults, C is entitled to the collateral only up to the value of the debt, and returns any value from the liquidation in excess to the value of the debt back to B. This is in accordance with the legal practice for OTC derivatives and repos.[18,19]

Assumption 4. *During the settlement of* $\mathcal{C}_{\mathrm{B,C}}$*, if* B *defaults and* C *liquidates the collateral to get* $\tilde{v}$*,* C *gives the excess value from the liquidation* $\tilde{v} - (1 + \tilde{r})\tilde{b} \geq 0$ *back to* B*, where* $\tilde{b}$ *is the amount borrowed.*

When $\mathcal{C}_{\mathrm{A,B}}$ is settled, I assume that the loan is repaid only if the collateral is returned in full. I motivate this by the delivery versus payment systems.[20] I emphasise that even in the place of such systems, the borrower may be unsecured because the size of the loan may be smaller than the value of the collateral due to a high haircut. Since the lender is always fully secured, there will be default only when B is unable to return the collateral. In such an eventuality, I assume that A becomes an unsecured creditor of B, and receives from B the excess of the value of the collateral over the face value of the debt.

Assumption 5. *During the settlement of* $\mathcal{C}_{\mathrm{A,B}}$*, if* B *has value* c *on its balance sheet less than the value* v *of the collateral owed back to* A*, then* B *returns to* A *the amount* $\min\{c, v-(1+r)b\}$*, where* b *is the amount borrowed.*

State of the World	Cash position of A	Cash position of B	Cash position of C
(R_B, u^2)	$R_A - (1+r) + \frac{1}{1-h}u^2$	$R_B(1+r) - (1+r) + (1+r)$	$1+r$
(R_B, ud)	$R_A - (1+r) + \frac{1}{1-h}ud$	$R_B(1+r) - (1+r) + (1+r)$	$1+r$
(R_B, d^2)	$R_A - (1+r) + \frac{1}{1-h}d^2$	$R_B(1+r) - (1+r) + (1+r)$	$1+r$
$(0, u^2)$	$R_A - (1+r) + \frac{1}{1-h}u^2$	0	$1+r$
$(0, ud)$	$R_A - (1+r) + \frac{1}{1-h}ud$	0	$1+r$
$(0, d^2)$	$R_A - (1+r) + \frac{1}{1-h}d^2$	0	$1+r$

Table 3: Effective cash positions of A, B and C after the settlement of $\mathcal{C}_{\mathrm{B,C}}$ at $t = 2$ and $\mathcal{C}_{\mathrm{A,B}}$ at $t = 3$. The position of A is per unit of borrowing from B.

[18] http://www.sec.gov/Archives/edgar/data/1065696/000119312511118050/dex101.htm

[19] http://www.icmagroup.org/assets/documents/Legal/GMRA-2011/GMRA-2011/GMRA%202011_2011.04.20_formular.pdf

[20] E.g. Fedwire. Neither party faces settlement risk (also known as principal risk)

Table 3 lists out the cash positions of A, B and C with securitisation at at $t = 2$ and $t = 3$. At $t = 2$, if B is solvent, he repays the total debt to C amounting to $\tilde{x}_{\text{B}}(1 + \tilde{r}) = 1 + r$ and recovers the securitised contract $\mathcal{C}_{\text{A,B}}$. If B is insolvent at $t = 2$, C impounds the contract $\mathcal{C}_{\text{A,B}}$ and holds it up to $t = 3$ when he gets the safe payment $1+r$. (C may also sell it at $t = 2$ at the price $1 + r$ to get the same utility.) When $\mathcal{C}_{\text{A,B}}$ is being settled at $t = 3$, regardless of who is holding the contract, A must pay $1+r$ to the holder and recover all of the collateral. If B was insolvent at $t = 2$, he gets nothing at either $t = 2$ or $t = 3$. If B was solvent at $t = 2$, he would earn a total of $R_B(1+r)$ from his investment, would have to repay B's debt amounting to $1 + r$, and would be paid $1 + r$ by A at $t = 3$. Thus, net of the collateral position, A's expected payoff per unit of collateral committed is

$$(R_A - (1 + r))(1 - h),$$

and B's expected payoff is

$$pR_B(1 + r).$$

State of the World	Cash position of B	Cash position of C
(R_B, u)	$R_B \frac{1}{1-h}d - \frac{1}{1-h}d + \frac{1}{1-h}u$	$\frac{1}{1-h}d$
(R_B, d)	$R_B \frac{1}{1-h}d - \frac{1}{1-h}d + \frac{1}{1-h}d$	$\frac{1}{1-h}d$
$(0, u)$	$-\frac{1}{1-h}d + \frac{1}{1-h}u$	$\frac{1}{1-h}d$
$(0, d)$	$-\frac{1}{1-h}d + \frac{1}{1-h}d$	$\frac{1}{1-h}d$

Table 4: Effective cash positions of B and C after the settlement of $\mathcal{C}_{\text{B,C}}$ at $t = 2$.

Table 4 lists out the cash positions of B and C with rehypothecation at $t = 2$. If B is solvent, B pays C the total amount $\tilde{x}_{\text{B}}(1 + \tilde{r}) = \frac{1}{1-h}d$, and recovers the collateral which has value $\frac{1}{1-h}v_1$, where $v_1 \in \{u, d\}$. When B fails, C liquidates the collateral to get $\frac{1}{1-h}v_1$ in cash, keeps the amount $\tilde{x}_{\text{B}}(1 + \tilde{r}) = \frac{1}{1-h}d$ that is owed to him, and returns the remainder to B or its bankruptcy administrator. Thus, whether or not B fails, he always has cash amounting to $\frac{1}{1-h}v_1 - \frac{1}{1-h}d$. This is the excess value of the pledged collateral after liquidation, and the value of collateral lost to C due to B's insolvency is always $\frac{1}{1-h}d$.

State of the World	Cash position of A	Cash position of B
(R_B, u^2)	$R_A - (1+r) + \frac{1}{1-h}u^2$	$R_B\frac{1}{1-h}d - \frac{1}{1-h}d + 1 + r$
(R_B, ud)	$R_A - (1+r) + \frac{1}{1-h}ud$	$R_B\frac{1}{1-h}d - \frac{1}{1-h}d + 1 + r$
(R_B, du)	$R_A - (1+r) + \frac{1}{1-h}ud$	$R_B\frac{1}{1-h}d - \frac{1}{1-h}d + 1 + r$
(R_B, d^2)	$R_A - (1+r) + \frac{1}{1-h}d^2$	$R_B\frac{1}{1-h}d - \frac{1}{1-h}d + 1 + r$
$(0, u^2)$	$R_A + \left(-\frac{1}{1-h}d + \frac{1}{1-h}u\right)$	0
$(0, ud)$	$R_A + \left(-\frac{1}{1-h}d + \frac{1}{1-h}u\right)$	0
$(0, du)$	$R_A + \left(-\frac{1}{1-h}d + \frac{1}{1-h}d\right)$	0
$(0, d^2)$	$R_A + \left(-\frac{1}{1-h}d + \frac{1}{1-h}d\right)$	0

Table 5: Effective cash positions of A and B after the settlement of $\mathcal{C}_{\mathrm{A,B}}$. The position of agent i of type A is per unit of borrowing b_i from the agent of type B under consideration.

Table 5 lists out the cash positions of A and B with rehypothecation at $t = 3$. When B is solvent, he recovers all of the underlying collateral asset from C by repaying his debt amounting to $\frac{1}{1-h}d$, and can pass it on to A when A repays the debt. On the other hand, when B fails, there is an early termination of $\mathcal{C}_{\mathrm{A,B}}$. His cash position then is $\frac{1}{1-h}v_1 - \frac{1}{1-h}d$ since C has liquidated some of the collateral. B owes to A a net amount $\frac{1}{1-h}v_1 - (1+r)$, which it would have effectively transferred to A after the repayment of A's debt and the return of the collateral to A. It is easy to see that since $1 + r \leq \frac{1}{1-h}d^2 \leq \frac{1}{1-h}d$, what B owes to A is always more than what B has. Thus, the amount $\frac{1}{1-h}v_1 - \frac{1}{1-h}d$ that B has is recovered by A, who is now effectively an unsecured creditor of A. Thus, A's expected payoff per unit of collateral committed, net of the collateral position is

$$\left(R_A - p(1+r) - (1-p)\frac{1}{1-h}d\right)(1-h),$$

and B's expected payoff is

$$\begin{aligned}
& p\left(R_B\frac{1}{1-h}d + \left(\frac{1}{1-h} - \frac{1}{1-h}d\right) - \left(\frac{1}{1-h} - (1+r)\right)\right) \\
& +(1-p)\left(\frac{1}{1-h} - \frac{1}{1-h}d - \min\left\{\frac{1}{1-h} - \frac{1}{1-h}d, \frac{1}{1-h} - (1+r)\right\}\right) \\
= \;& p\left((R_B - 1)\frac{1}{1-h}d + 1 + r\right).
\end{aligned}$$

The above expression makes clear the cash flows when B is solvent and when he is not. When B is solvent, it gets the cash from its investment, receives the expected value of the collateral back from C after repaying his debt, and must return the collateral back to A when A repays his debt to B. When B is insolvent, he receives the excess value of the collateral after liquidation from C as per Assumption 4, and must repay the net value owed to A according to Assumption 5. Thus, B owes A the excess value of the collateral over the value of the debt, $\frac{1}{1-h} - (1+r)$. Of this, the expected value $\frac{1}{1-h} - \frac{1}{1-h}d$ is safe and is always recovered by A. The remaining amount can be thought of as A's debt to B.

B Moral Hazard

Suppose that B can either manage his risks well so that the probability of success of his investment is p_H, or he can shirk and get a private benefit b per unit of investment. In the latter case, the probability of success of the project becomes $p_L < p_H$. I assume that shirking is always socially suboptimal, i.e. $p_H R_B > p_L R_B + b$, and wish to implement only the efficient contracts where B is sufficiently incentivised to manage his risks well. From (11), this requires

$$p_H \left((R_B - 1)\frac{1}{1-h}d + 1 + r \right) \geq p_L \left((R_B - 1)\frac{1}{1-h}d + 1 + r \right) + b\frac{1}{1-h}d.$$

Rearranging, I get

$$\frac{1}{1-h}d - (1+r) \leq \left(1 - \frac{b}{R_B \Delta p}\right) R_B \frac{1}{1-h}d. \tag{23}$$

Denoting $\kappa = \left(1 - \frac{b}{R_B \Delta p}\right)$ gives (10).

Suppose that b is too large and B's pledgeable income is too small to effect a rehypothecation contract. B's lender A can then choose to monitor his operations so that the private benefit from shirking reduces. Monitoring can take the form of closely following the investment activity and threatening costly renegotiation if it is perceived that risk is not being managed well. Suppose that A incurs a cost c of monitoring per unit of B's investment. In order that it is incentive compatible for A to monitor B, I must have from (9)

$$R_A - p_H(1+r) - (1-p_H)\frac{1}{1-h}d - c\frac{1}{1-h}d \geq R_A - p_L(1+r) - (1-p_L)\frac{1}{1-h}d.$$

Rearranging, I get

$$\frac{1}{1-h}d - (1+r) \geq \frac{c}{\Delta p}\frac{1}{1-h}d. \tag{24}$$

In order that monitoring is useful, conditions (23) and (24) together with A's collateral constraint must define a feasible region. Hence, I must have

$$\begin{aligned} \frac{c+b}{\Delta p} &\leq R_B, \text{ and,} \\ \frac{c}{\Delta p} &\leq 1 - d. \end{aligned}$$

Thus, if the cost of monitoring $c \leq \min\{\Delta p R_B - b, \Delta p(1-d)\}$ is sufficiently small, the rehypothecation contract is feasible.

C Proof of Proposition 2

Consuming the endowment gives B a utility

$$U^{CE}(\bar{R}) = 1.$$

Lending to A may give him a higher utility. There are two cases.

$\kappa < \bar{\kappa}$:

I first consider the case when $\kappa < \bar{\kappa}$ and rehypothecation is infeasible. Suppose that B offers one unit of the contract $(r, h, \theta = 0)$ to A, and securitises a fraction w and holds on to a fraction $1 - w$ of the contract. Write $1 + r = X$ and $\frac{1}{1-h} = Z$. B thus solves a combination of the two problems (7) and (8):

$$\max_{X,Z} pR_B Xw + X(1-w), \text{ s.t.}$$

$$\begin{aligned} \text{Collateral const.:} \quad & X \leq Zd^2, \\ \text{Participation const.:} \quad & R_A - X \geq \bar{R}Z, \end{aligned}$$

The collateral and participation constraints occurs in both problems and remain unchanged.

Since the participation constraint will always bind, I can rewrite the objective function as

$$[1 + (pR_B - 1)w](R_A - \bar{R}Z).$$

If $pR_B < 1$, we immediately get $w = 0$ and B will hold on to the debt. Since any additional collateral over the minimum required by the collateral constraint is wasteful and costs A a utility of $\bar{R}$ per unit, B will try to minimise the collateral demanded and the collateral constraint will bind in the optimum. This gives the optimal contract

$$1 + r = \frac{R_A}{1 + \frac{\bar{R}}{d^2}}, \quad \frac{1}{1-h} = \frac{1}{d^2}\frac{R_A}{1 + \frac{\bar{R}}{d^2}}, \quad \theta = 0.$$

This gives B a utility of

$$U^{HO}(\bar{R}) = \frac{R_A}{1 + \frac{\bar{R}}{d^2}}.$$

B will stop lending and consume the endowment when $\bar{R} > \bar{R}^*$ such that $U^{HO}(\bar{R}^*) = 1$. this gives

$$\bar{R}^* = (R_A - 1)d^2.$$

This proves part (1) of the proposition for $\kappa < \bar{\kappa}$.

If $pR_B \geq 1$, $w = 1$ and B will securitise the debt. Again, since collateral is costly, B will try to minimise it and the collateral constraint will bind. This gives the optimal contract

$$1 + r = \frac{R_A}{1 + \frac{\bar{R}}{d^2}}, \quad \frac{1}{1-h} = \frac{1}{d^2} \frac{R_A}{1 + \frac{\bar{R}}{d^2}}, \quad \theta = 0.$$

This gives B a utility of

$$U^{SE}(\bar{R}) = \frac{pR_B R_A}{1 + \frac{\bar{R}}{d^2}}.$$

B will stop lending and consume the endowment when $\bar{R} > \bar{R}^*$ such that $U^{SE}(\bar{R}^{**}) = 1$. this gives

$$\bar{R}^{**} = (pR_B R_A - 1)d^2.$$

This proves part (2a) of the proposition.

$\kappa \geq \bar{\kappa}$**:**

Now I consider the case when $\kappa \geq \bar{\kappa}$ and rehypothecation is feasible. Suppose that B offers one unit of the contract $(r, h, \theta = 1)$ to A, and rehypothecates a fraction w_1, securitises a fraction w_2 and holds on to a fraction $1 - w_1 - w_2$ of the contract. Write $1 + r = X$ and $\frac{1}{1-h} = Z$. B thus solves a combination of the three problems (7), (8) and (11):

$$\max_{X,Z} p\left((R_B - 1)Zd + X\right)w_1 + pR_B X w_2 + X(1 - w_1 - w_2), \text{ s.t.}$$

$$\begin{aligned}
\text{Collateral const.:} \quad & X \leq Zd^2, \\
\text{Participation const.:} \quad & R_A - (pX + (1-p)Zd)w_1 - Xw_2 - X(1 - w_1 - w_2) \geq \bar{R}Z, \\
\text{Pledgeability const.:} \quad & Zd - X \leq \kappa R_B Z d.
\end{aligned}$$

The collateral constraint occurs in all three problems and remains unchanged. The pledgeability constraint also remains unchanged as B's project is linearly scalable and the condition on the proportion of the face value X and the amount of collateral Z to overcome moral hazard with rehypothecation is unchanged. The participation constraint changes to reflect that a smaller fraction of the collateral is rehypothecated and the expected loss of collateral is proportionately smaller.

Since the participation constraint will always bind, I can rewrite the objective function as

$$\left[(pR_B - 1)dw_1 - \bar{R}\right]Z + (pR_B - 1)Xw_2 + R_A.$$

If $pR_B < 1$, we immediately get $w_1 = w_2 = 0$ and B will hold on to the debt. This proves part (1) of the proposition for $\kappa \geq \bar{\kappa}$.

When $pR_B \geq 1$, $1 - w_1 - w_2 = 0$ as there is always a non-negative utility from securitisation. The derivative w.r.t w_1 is

$$(pR_B - 1)dZ,$$

and that w.r.t w_2 is

$$(pR_B - 1)X.$$

Now by the collateral constraint, $X \leq Zd^2 < Zd$, the derivative w.r.t w_1 is always larger. B can borrow more from C by rehypothecation at a given $\bar{R}$, and rehypothecation is always preferred to securitisation. Thus, we have $w_1 = 1$. Now there are two cases:

Case I: $0 \leq (pR_B - 1)d \leq \bar{R}$. In this case, the first term in the modified objective function is negative. The benefit from rehypothecating collateral is less than the cost of raising that collateral. B will minimise Z so that the collateral constraint binds. The optimal contract is

$$1 + r = \frac{R_A}{1 + \frac{\bar{R}}{d^2}\frac{(1-p)(1-d)}{d^2}}, \quad \frac{1}{1-h} = \frac{1}{d^2}\frac{R_A}{1 + \frac{\bar{R}}{d^2} + \frac{(1-p)(1-d)}{d^2}}, \quad \theta = 1,$$

and the utility is

$$U^{RE}(\bar{R}) = \frac{p\left(1 + \frac{R_B - 1}{d}\right)R_A}{1 + \frac{\bar{R}}{d^2} + \frac{(1-p)(1-d)}{d}}.$$

Case II: $\bar{R} < (pR_B - 1)d$. In this case, the first term in the modified objective function is positive. The benefit from rehypothecating collateral is greater than the cost of raising that collateral. B will maximise Z so that the pledgeability constraint binds. The optimal contract is

$$1 + r = \frac{R_A(1 - \kappa R_B)}{1 + \frac{\bar{R}}{d} - \kappa p R_B}, \quad \frac{1}{1-h} = \frac{1}{d}\frac{R_A}{1 + \frac{\bar{R}}{d} - \kappa p R_B}, \quad \theta = 1,$$

and the utility is

$$U^{RE}(\bar{R}) = \frac{pR_B(1-\kappa)R_A}{1 + \frac{\bar{R}}{d} - \kappa p R_B}.$$

I guess and verify an $\bar{R}^{***}$ such that $\bar{R}^{***} > (pR_B - 1)d$ and $U^{RE}(\bar{R}^{***}) = 1$. From the expression for the utility in the guessed range of $\bar{R}^{***}$, I have

$$\bar{R}^{***} = p(R_B - 1 + d)(R_A - 1)d + (pR_B - 1)d.$$

This proves part (2b) of the proposition.

D Social Planner's Problem

I describe and solve the social planner's problem with securitisation and rehypothecation. For ease of exposition, write the contracts $\mathcal{C}_{\mathrm{A,B}}$ and $\mathcal{C}_{\mathrm{B,C}}$ as $1+r=X$, $\frac{1}{1-h}=Z$, $1+\tilde{r}=\tilde{X}$ and $\frac{1}{1-\tilde{h}}=\tilde{Z}$. Here, $\tilde{Z}$ is the number of units of the collateralised debt $\mathcal{C}_{\mathrm{A,B}}$ (if $\theta=0$) or the underlying collateral asset (if $\theta=1$) provided to C as collateral per unit of borrowing by B.

D.1 $\theta = 0$.

I first describe the planner's problem with securitisation. The sum of the expected consumptions of A, B and C at $t=2$ and $t=3$ is

$$\overbrace{\Omega + R_A x_{\mathrm{A}} - X x_{\mathrm{A}}}^{\text{A's cons.}} + \overbrace{pR_B\tilde{x}_{\mathrm{B}} + 1 - x_{\mathrm{A}} + X x_{\mathrm{A}} - \tilde{X}\tilde{x}_{\mathrm{B}}}^{\text{B's cons.}} + \overbrace{\tilde{X}\tilde{x}_{\mathrm{B}} - \tilde{x}_{\mathrm{B}}}^{\text{C's cons.}},$$

which simplifies to (17) modulo a constant. The resource constraints are

$$\begin{aligned} x_{\mathrm{A}} &\leq 1, \\ x_{\mathrm{A}} Z &\leq \Omega, \\ \tilde{x}_{\mathrm{B}}\tilde{Z} &\leq x_{\mathrm{A}} X. \end{aligned}$$

The first two constraints say that the cash invested by A cannot be greater than the endowment of B, and the collateral provided to B cannot be greater than the endowment of A. The third constraint says that the value of the collateral provided to C cannot be greater than the face value of the debt held by B against A.

The collateral constraints are, as usual,

$$\begin{aligned} x_{\mathrm{A}} X &\leq x_{\mathrm{A}} Z d^2, \\ \tilde{x}_{\mathrm{B}}\tilde{X} &\leq \tilde{x}_{\mathrm{B}}\tilde{Z}, \end{aligned}$$

where the second constraint says that the face value of the debt held by C cannot be greater than the value of the collateral provided to C.

Finally, the implementability constraints are

$$\begin{aligned} R_A x_{\mathrm{A}} - X x_{\mathrm{A}} &\geq 0, \\ pR_B\tilde{x}_{\mathrm{B}} + 1 - x_{\mathrm{A}} + X x_{\mathrm{A}} - \tilde{X}\tilde{x}_{\mathrm{B}} &\geq 1, \\ \tilde{x}_{\mathrm{B}}\tilde{X} &\geq \tilde{x}_{\mathrm{B}}. \end{aligned}$$

It can be easily shown that C's participation constraint, B's collateral constraint and the resource constraint for collateral at $t = 1.2$ always bind, so that $\tilde{X} = \tilde{Z} = 1$ and $\tilde{x}_{\mathrm{B}} = x_{\mathrm{A}} X$.

Simplifying the objective function and the constraints and writing the Lagrangian,

$$\begin{aligned}\mathcal{L} &= (R_A - 1)x_{\mathrm{A}} + (pR_B - 1)x_{\mathrm{A}}X - \lambda_1(x_{\mathrm{A}} - 1) - \lambda_2(x_{\mathrm{A}}Z - \Omega) \\ &\quad -\lambda_3(X - Zd^2) + \lambda_4(R_A - X) + \lambda_5(pR_BX - 1).\end{aligned}$$

The first order conditions are

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial x_{\mathrm{A}}} &= R_A - 1 + (pR_B - 1)X - \lambda_1 - \lambda_2 Z = 0, \\ \frac{\partial \mathcal{L}}{\partial X} &= (pR_B - 1)x_{\mathrm{A}} - \lambda_3 - \lambda_4 + \lambda_5 pR_B = 0, \\ \frac{\partial \mathcal{L}}{\partial Z} &= -\lambda_2 x_{\mathrm{A}} + \lambda_3 d^2 = 0.\end{aligned}$$

Case I: $\Omega \in \left[\frac{R_A}{d^2}, \infty\right)$**.**

It can be easily verified that $\lambda_1 > 0$, $\lambda_2 \geq 0$, $\lambda_3 \geq 0$, $\lambda_4 > 0$ and $\lambda_5 = 0$, together with $x_{\mathrm{A}} = 1$, $R_A = X$, $Z \leq \Omega$ and $X \leq \Omega d^2$ satisfy all the conditions. This gives $\tilde{x}_{\mathrm{B}} = R_A$, and

$$Y(\theta = 0|d, \kappa) = (R_A - 1) + (pR_B - 1)R_A.$$

Case II: $\Omega \in \left[\frac{1}{pR_Bd^2}, \frac{R_A}{d^2}\right]$**.**

It can be easily verified that $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 > 0$, $\lambda_4 = 0$ and $\lambda_5 = 0$, together with $x_{\mathrm{A}} = 1$, $Z = \Omega$ and $X = \Omega d^2$ satisfy all the conditions. This gives $\tilde{x}_{\mathrm{B}} = \Omega d^2$, and

$$Y(\theta = 0|d, \kappa) = (R_A - 1) + (pR_B - 1)\Omega d^2.$$

Case III: $\Omega \in \left[0, \frac{1}{pR_Bd^2}\right]$**.**

It can be easily verified that $\lambda_1 = 0$, $\lambda_2 > 0$, $\lambda_3 > 0$, $\lambda_4 = 0$ and $\lambda_5 > 0$, together with $x_{\mathrm{A}} = pR_B\Omega d^2$, $Z = \Omega$ and $X = \frac{1}{pR_B}$ satisfy all the conditions. This gives $\tilde{x}_{\mathrm{B}} = \Omega d^2$, and

$$Y(\theta = 0|d, \kappa) = (R_A - 1)pR_B\Omega d^2 + (pR_B - 1)\Omega d^2.$$

D.2 $\theta = 1$.

I now describe the planner's problem with rehypothecation. The sum of the expected consumptions of A, B and C at $t = 2$ and $t = 3$ is

$$\overbrace{\Omega + R_A x_{\mathrm{A}} - pXx_{\mathrm{A}} - (1-p)\tilde{X}\tilde{x}_{\mathrm{B}}}^{\text{A's cons.}} + \overbrace{p(R_B\tilde{x}_{\mathrm{B}} - \tilde{X}\tilde{x}_{\mathrm{B}} + Xx_{\mathrm{A}}) + 1 - x_{\mathrm{A}}}^{\text{B's cons.}} + \overbrace{\tilde{X}\tilde{x}_{\mathrm{B}} - \tilde{x}_{\mathrm{B}}}^{\text{C's cons.}},$$

which simplifies to (17) modulo a constant. The resource constraints are

$$\begin{aligned} x_{\mathrm{A}} &\leq 1, \\ x_{\mathrm{A}} Z &\leq \Omega, \\ \tilde{x}_{\mathrm{B}} \tilde{Z} &\leq x_{\mathrm{A}} Z. \end{aligned}$$

The first two constraints are the same as with securitisation. The third constraint says that the value of the collateral provided to C by B cannot be greater than the value of the collateral provided to B by A.

The collateral constraints are, as usual,

$$\begin{aligned} x_{\mathrm{A}} X &\leq x_{\mathrm{A}} Z d^2, \\ \tilde{x}_{\mathrm{B}} \tilde{X} &\leq \tilde{x}_{\mathrm{B}} \tilde{Z} d, \end{aligned}$$

where the second constraint says that the face value of the debt held by C cannot be greater than the value of the collateral provided to C in the worst case. the pledgeability constraint is

$$\tilde{x}_{\mathrm{B}} \tilde{X} - x_{\mathrm{A}} X \leq \kappa R_B \tilde{x}_{\mathrm{B}},$$

which says that the the excess amount lost by A when B defaults, or the excess haircut owed to A by B, cannot be greater than a fraction of the gross return from investing. This enables B to have low leverage and enough skin in the game to efficiently manage the investment.

Finally, the implementability constraints are

$$\begin{aligned} R_A x_{\mathrm{A}} - pX x_{\mathrm{A}} - (1-p)\tilde{X}\tilde{x}_{\mathrm{B}} &\geq 0, \\ p(R_B \tilde{x}_{\mathrm{B}} - \tilde{X}\tilde{x}_{\mathrm{B}} + X x_{\mathrm{A}}) + 1 - x_{\mathrm{A}} &\geq 1, \\ \tilde{x}_{\mathrm{B}} \tilde{X} &\geq \tilde{x}_{\mathrm{B}}. \end{aligned}$$

It can be easily shown that C's participation constraint, B's collateral constraint and the resource constraint for collateral at $t = 1.2$ always bind, so that $\tilde{X} = 1$, $\tilde{Z} = \frac{1}{d}$ and $\tilde{x}_{\mathrm{B}} = x_{\mathrm{A}} Z d$. Simplifying the objective function and the constraints and writing the Lagrangian,

$$\begin{aligned} \mathcal{L} = {} & (R_A - 1)x_{\mathrm{A}} + (pR_B - 1)x_{\mathrm{A}} Z d - \lambda_1(x_{\mathrm{A}} - 1) - \lambda_2(x_{\mathrm{A}} Z - \Omega) - \lambda_3(X - Zd^2) \\ & -\lambda_4(Zd - X - \kappa R_B Z d) + \lambda_5(R_A - pX - (1-p)Zd) + \lambda_6(p((R_B - 1)Zd + X) - 1). \end{aligned}$$

The contrast with securitisation is clear that now B's investment depends on the value of the collateral, rather than on the face value of the debt with securitisation. The first order conditions are

$$\begin{aligned}
\frac{\partial \mathcal{L}}{\partial x_{\mathrm{A}}} &= R_A - 1 + (pR_B - 1)Zd - \lambda_1 - \lambda_2 Z = 0, \\
\frac{\partial \mathcal{L}}{\partial X} &= -\lambda_3 + \lambda_4 + -\lambda_5 p + \lambda_6 p = 0, \\
\frac{\partial \mathcal{L}}{\partial Z} &= (pR_B - 1)x_{\mathrm{A}}d - \lambda_2 x_{\mathrm{A}} + \lambda_3 d^2 - \lambda_4(1-\kappa R_B)d - \lambda_5(1-p)d + \lambda_6 p(R_B - 1)d = 0.
\end{aligned}$$

Case I: $\Omega \in \left[\frac{R_A}{d(1-\kappa p R_B)}, \infty\right)$**.**

It can be easily verified that $\lambda_1 > 0$, $\lambda_2 = 0$, $\lambda_3 = 0$, $\lambda_4 > 0$, $\lambda_5 > 0$ and $\lambda_6 = 0$, together with $x_{\mathrm{A}} = 1$, $Z = \frac{R_A}{d(1-\kappa p R_B)}$ and $X = \frac{R_A(1-\kappa R_B)}{1-\kappa p R_B}$ satisfy all the conditions. This gives $\tilde{x}_{\mathrm{B}} = \frac{R_A}{1-\kappa p R_B}$, and

$$Y(\theta = 1|d,\kappa) = (R_A - 1) + (pR_B - 1)\frac{R_A}{1-\kappa p R_B}.$$

Case II: $\Omega \in \left[\frac{R_A}{pR_B d(1-\kappa)}, \frac{R_A}{d(1-\kappa p R_B)}\right]$**.**

It can be easily verified that $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 = 0$, $\lambda_4 = 0$, $\lambda_5 = 0$ and $\lambda_6 = 0$, together with $x_{\mathrm{A}} = 1$, $Z = \Omega$ and $X = \Omega d(1-\kappa R_B)$ satisfy all the conditions. This gives $\tilde{x}_{\mathrm{B}} = \Omega d$, and

$$Y(\theta = 1|d,\kappa) = (R_A - 1) + (pR_B - 1)\Omega d.$$

Case III: $\Omega \in \left[\frac{R_A}{pR_B d(1-\bar{\kappa})}, \frac{R_A}{pR_B d(1-\kappa)}\right]$**.**

It can be easily verified that $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 = 0$, $\lambda_4 = 0$, $\lambda_5 = 0$ and $\lambda_6 = 0$, together with $x_{\mathrm{A}} = 1$, $Z = \Omega$ and $X \in \left[\Omega d(1-\kappa R_B), \Omega d^2\right]$ satisfy all the conditions. This gives $\tilde{x}_{\mathrm{B}} = \Omega d$, and

$$Y(\theta = 1|d,\kappa) = (R_A - 1) + (pR_B - 1)\Omega d.$$

Case IV: $\Omega \in \left[\frac{1}{pR_B d(1-\bar{\kappa})}, \frac{R_A}{pR_B d(1-\bar{\kappa})}\right]$**.**

It can be easily verified that $\lambda_1 > 0$, $\lambda_2 > 0$, $\lambda_3 = 0$, $\lambda_4 = 0$, $\lambda_5 = 0$ and $\lambda_6 = 0$, together with $x_{\mathrm{A}} = 1$, $Z = \Omega$ and $X = \Omega d^2$ satisfy all the conditions. This gives $\tilde{x}_{\mathrm{B}} = \Omega d$, and

$$Y(\theta = 1|d,\kappa) = (R_A - 1) + (pR_B - 1)\Omega d.$$

Case V: $\Omega \in \left[0, \frac{1}{pR_B d(1-\bar{\kappa})}\right]$**.**

It can be easily verified that $\lambda_1 = 0$, $\lambda_2 > 0$, $\lambda_3 > 0$, $\lambda_4 = 0$, $\lambda_5 = 0$ and $\lambda_6 > 0$, together with $x_{\mathrm{A}} = pR_B d(1-\bar{\kappa})\Omega$, $Z = \frac{1}{pR_B d(1-\bar{\kappa})}$ and $X = \frac{1}{pR_B d(1-\bar{\kappa})}d^2$ satisfy all the conditions. This gives $\tilde{x}_{\mathrm{B}} = \Omega d$, and

$$Y(\theta = 1|d, \kappa) = (R_A - 1)pR_B d(1 - \bar{\kappa})\Omega + (pR_B - 1)\Omega d.$$

D.2.1 Allowing A to borrow from C

Suppose $(pR_B - 1) > (R_A - 1)d$. Consider first the decentralised equilibrium. If A were to borrow from C, C would be willing to lend any amount of cash to A between $t = 1.1$ and $t = 3$ at a haircut of $1 - d^2$ and an interest rate of 0. The equilibrium net return faced by A would then be at least

$$(R_A - (1 + r))(1 - h) = (R_A - 1)d^2.$$

B would be required to match this $\bar{R}$ or exceed it. Define $\Omega^* = \frac{R_A}{(1-\kappa pR_B)d+(R_A-1)d^2}$.

Case I: $\Omega \in [0, \Omega^*)$**.**

For small enough Ω, B would be able to offer $\bar{R} > (R_A - 1)d^2$. A would only borrow from B, and the equilibrium would be the same as in the case without the ability to borrow from C.

Case II: $\Omega \in [\Omega^*, \infty)$**.**

For a large Ω, B would borrow 1 unit of cash from B by providing Ω^* units of collateral, and the rest from C, and $\bar{R} = (R_A - 1)d^2$. In this case, the total output would be

$$Y^{AB,AC} = (R_A - 1)\left(1 + (\Omega - \Omega^*)\, d^2\right) + (pR_B - 1)\Omega^* d.$$

On the other hand, if A was not allowed to contract with C, the total output would be

$$Y^{AB} = (R_A - 1) + (pR_B - 1)\Omega d.$$

This gives

$$Y^{AB} - Y^{AB,AC} = -(R_A - 1)d^2(\Omega - \Omega^*) + (pR_B - 1)d(\Omega - \Omega^*),$$

which is positive if $(pR_B - 1) > (R_A - 1)d$. The social planner would rather make A borrow only from B.

E Belief Disagreements and Optimality of Safe Debt

In this appendix, I provide a partial justification for Assumptions 2 and 3. I assume that there are belief disagreements about the payoff of the collateral between A, B and C. In particular, I assume that A is optimistic about the collateral at $t = 1$ and believes that it will pay off u^2

with probability q and ud with probability $1-q$. C is pessimistic about the collateral at $t=1$ and believes it will pay off ud with probability q and d^2 with probability $1-q$. B's beliefs evolve as in Section 2.

Lemma 1. *In the absence of rehypothecation, there exist parameters for which the long-term optimal contract $\mathcal{C}_{\mathrm{A,B}}$ takes the form $1+r = \frac{1}{1-h}d^2$.*

Proof. Without loss of generality, let B offer the contract to A. Write $1+r = X$ and $\frac{1}{1-h} = Z$. B solves

$$\max_{X,Z} E_B[\min\{X, Zv\}],$$

$$\text{s.t. } R_A - E_A[\min\{X, Zv\}] \geq \bar{R}Z,$$

where v is the payoff of one unit of the collateral at $t=3$, and the expectations $E_A[\cdot]$ and $E_B[\cdot]$ are taken according to the beliefs of A and B respectively.

It is clear that the constraint will always bind since we can increase X otherwise. Due to the linearity of the problem, it is sufficient to compare the values of the maximand at $X \in \{Zu^2, Zud, Zd^2\}$.

$X = Zu^2$: Here, there is always default, and we have $R_A - Zu = \bar{R}Z$. This gives B a payoff of $Z = \frac{R_A}{u+\bar{R}}$.

$X = Zud$: Here, there is default when $v \leq ud$. We have $R_A - X = \bar{R}Z$ since A does not believe he will ever default. This gives $Z = \frac{R_A}{ud+\bar{R}}$. Then B's payoff is

$$(1-(1-q)^2)X + (1-q)^2 Zd^2 = \frac{R_A d}{ud+\bar{R}}(u-(1-q)^2(u-d)).$$

$X = Zd^2$: Here, there is no default, and we have $R_A - X = \bar{R}Z$. This gives $Z = \frac{R_A}{d^2+\bar{R}}$, and B's payoff is $\frac{R_A}{1+\frac{\bar{R}}{d^2}}$.

Now the contract with $X = Zd^2$ will be the most preferred if $\bar{R} \leq \min\left\{\frac{ud^2-1}{1-d^2}, \frac{(u-1)^2}{(u-d)^2-(u-1)^2}\right\}$. The upper bound can be made very large with a large u and a large d, so that in the admissible range of $\bar{R}$ where B would be willing to lend to A instead of holding on to the cash, the contract with $1+r = \frac{1}{1-h}d^2$ is always optimal. □

The safe debt contract is optimal because otherwise, there is default and A loses collateral that he values more than B. This case also nests the case of a sale of the collateral to B, with $1+r > \frac{1}{1-h}u^2$, so that there is always default.

Corollary 3. *Under the parameter restrictions of Lemma 1,* A *always prefers borrowing from* B *against the collateral to selling the collateral to* B*.*

If A were to borrow short term from B, A would have to default in all states of the world since A has no cash flows at $t=2$. This case is also nested above.

Corollary 4. *The optimal borrowing contract between* A *and* B *is long-term.*

Given that C is even more pessimistic than B, A will prefer to borrow from C using safe debt.

Corollary 5. *The optimal contract between A and C takes the form as in Lemma 1.*

Given that B does not value the collateral highly, the optimal contract between B and C would not necessarily be safe.

Lemma 2. B *is indifferent between selling the collateral to* C*, borrowing short term with* $1 + \tilde{r} = \frac{1}{1-\tilde{h}}d$ *and borrowing long term at the same terms.*

F Comparative Statics

Define

$$\begin{aligned}
\Omega_1 &= \frac{1}{d}\frac{1}{pR_B(1-\bar{\kappa})}, \\
\Omega_2 &= \frac{1}{d}\frac{R_A}{pR_B(1-\bar{\kappa})}, \\
\Omega_3 &= \frac{1}{d}\frac{R_A}{pR_B(1-\kappa)}, \\
\Omega_4 &= \frac{1}{d}\frac{R_A}{1-\kappa pR_B}.
\end{aligned}$$

Ω_1 is the cutoff below which the supply of collateral is too low and $\bar{R}$ is too high for B to be willing to supply all of the cash to A. Ω_4 is the cutoff above which the supply of collateral is too high and $\bar{R} = 0$, and A does not use all of the collateral to borrow from B. Between Ω_1 and Ω_2, A's collateral constraint binds, and between Ω_3 and Ω_4, B's pledgeability constraint binds. Between Ω_2 and Ω_3, $\bar{R} = (pR_B - 1)d$ and neither constraint binds.

I provide a graphical proof for the propositions in Section 5. From Propositions 2 and 3 and the proof of Proposition 2, the equilibrium haircuts and interest rates are given by

$$\frac{1}{1-h} = \begin{cases} \Omega_1, & \text{if } \Omega \in [0, \Omega_1], \\ \Omega, & \text{if } \Omega \in [\Omega_1, \Omega_4], \\ \Omega_4, & \text{if } \Omega \in [\Omega_4, \infty), \end{cases}$$

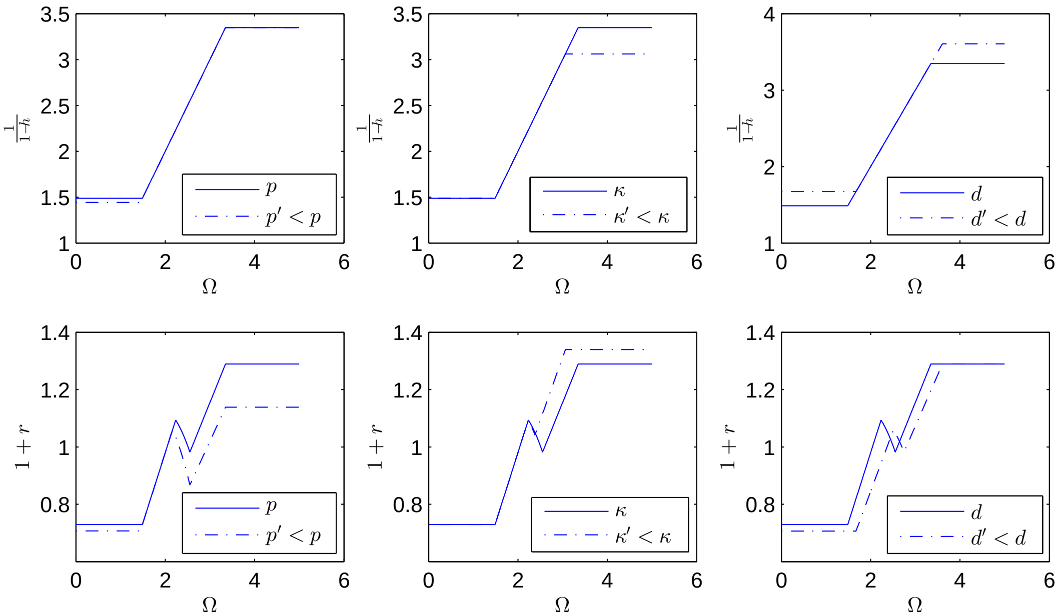

Figure 14: Comparative Statics

$$1+r = \begin{cases} \Omega_1 d^2 & \text{if } \Omega \in [0, \Omega_1], \\ \Omega d^2 & \text{if } \Omega \in [\Omega_1, \Omega_2], \\ \Omega \left[\frac{\Omega_3 - \Omega}{\Omega_3 - \Omega_2} d^2 + \frac{\Omega - \Omega_2}{\Omega_3 - \Omega_2} (1 - \kappa R_B) d \right] & \text{if } \Omega \in [\Omega_2, \Omega_3], \\ \Omega (1 - \kappa R_B) d & \text{if } \Omega \in [\Omega_3, \Omega_4], \\ \Omega_4 (1 - \kappa R_B) d & \text{if } \Omega \in [\Omega_4, \infty) \, . \end{cases}$$

Figure 14 shows the change in the haircuts and interest rates for various values of Ω when (i) the lender's risk increases (keeping the project NPV constant), (ii) the lender's pledgeability decreases, and (iii) the collateral risk increases. It is clear that the haircuts and interest rates are affected as in Section 5. The effect of a decrease in d on the interest rate is ambiguous. However, for a large enough fall in d, the interest rate decreases for all values of Ω. The decrease has to be such that the value of Ω_2 with d' is greater than the value of Ω_3 with d, i.e

$$\frac{1}{d'} \frac{R_A}{p(R_B - 1 + d')} > \frac{1}{d} \frac{R_A}{pR_B(1-\kappa)},$$

$$d'^2 + (R_B - 1)d' - dR_B(1-\kappa) < 0,$$

$$d' < \frac{-(R_B - 1) + \sqrt{(R_B - 1)^2 + 4R_B(R_B - 1)(1-\kappa)d}}{2} \equiv g(d).$$

G Central Bank Intervention

G.1 Proof of Proposition 7

Fix an $m \in \mathbb{R}$. $m > 0$ stands for cash supplied by the central bank in exchange for collateral, and $m < 0$ stands for cash withdrawn by the central bank.[21] I prove the result for $m > 0$. The case for $m < 0$ is exactly symmetric. The total expected output at $t = 2$ and $t = 3$ can be written as

$$\begin{aligned} Y &= \overbrace{\Omega + R_A x_\mathrm{A} - pXx_\mathrm{A}^\mathrm{B} - (1 + r^{CB})m - (1-p)\tilde{X}\tilde{x}_\mathrm{B} + T_A}^{\text{A's cons.}} \\ &\quad + \overbrace{p(R_B\tilde{x}_\mathrm{B} - \tilde{X}\tilde{x}_\mathrm{B} + Xx_\mathrm{A}^\mathrm{B}) + 1 - x_\mathrm{A}^\mathrm{B} + T_B}^{\text{B's cons.}} + \overbrace{\tilde{X}\tilde{x}_\mathrm{B} - \tilde{x}_\mathrm{B} + T_C}^{\text{C's cons.}}, \end{aligned}$$

where T_A, T_B and T_C are the lump sum rebates from the central bank, and $x_\mathrm{A} = x_\mathrm{A}^\mathrm{B} + m$ is the sum of the amounts borrowed from B and the central bank respectively. Since the central bank rebates all profits lump sum,

$$T_A + T_B + T_C = r^{CB}m.$$

Thus, modulo a constant, the total expected output can be written as

$$Y = (R_A - 1)x_\mathrm{A} + (R_B - 1)\tilde{x}_\mathrm{B}.$$

Now in equilibrium, for small enough $|m|$,

$$x_\mathrm{A} = \begin{cases} 1 + m & \text{if } \bar{R} = 0, \\ 1 + m & \text{if } (0, \bar{R}^{***}), \\ (\Omega - \Delta\Omega)(1 - h) + m & \text{if } \bar{R} = \bar{R}^{***}, \end{cases}$$

and

$$\tilde{x}_\mathrm{B} = \begin{cases} \frac{R_A}{1 - \kappa p R_B} & \text{if } \bar{R} = 0, \\ (\Omega - \Delta\Omega)d & \text{if } \bar{R} \in (0, \bar{R}^{***}), \\ (\Omega - \Delta\Omega)d & \text{if } \bar{R} = \bar{R}^{***}. \end{cases}$$

An explanation for these values is in order. In the first region, when the initial endowment of collateral is large, $\bar{R} = 0$ and A gives away idle collateral to the central bank. The investment into A's project increases by m. The investment into B's project is at its highest level possible given the limited pledgeability of B, and is unaffected. In the intermediate

[21] Assume that A starts off with an endowment of cash which he invests in the project.

region, the investment into B's project increases by m, and the investment into B's project falls by $\Delta\Omega d$, since the amount collateral lent by A to B falls by $\Delta\Omega$. Finally, in the region where the initial endowment of collateral is too small, the investment into A's project increases by m, but decreases by $\Delta\Omega(1-h)$ as A substitutes to borrowing from the central bank. I now look at the three regions in turn.

Case I: $\bar{R} = 0$**.**

I can write

$$Y = (R_A - 1) + (R_B - 1)\frac{R_A}{1 - \kappa p R_B} + (R_A - 1)m,$$

which gives

$$\left.\frac{\partial Y}{\partial m}\right|_{m=0} = R_A - 1.$$

Case II: $\bar{R} \in (0, \bar{R}^{***})$**.**

I can write

$$Y = (R_A - 1) + (R_B - 1)\Omega d + (R_A - 1)m - (R_B - 1)\Delta\Omega d.$$

Differentiating w.r.t. m,

$$\frac{\partial Y}{\partial m} \quad = \quad (R_A - 1) - (R_B - 1)d\frac{\partial \Delta\Omega}{\partial m}.$$

Now from Equation (18), $\Delta\Omega = m\frac{1}{1-h^{CB}}$, which gives

$$\frac{\partial \Delta\Omega}{\partial m} = \frac{1}{1 - h^{CB}} + m\frac{\partial}{\partial m}\frac{1}{1 - h^{CB}}.$$

Evaluating the result at $m = 0$,

$$\left.\frac{\partial Y}{\partial m}\right|_{m=0} = (R_A - 1) - (R_B - 1)d\frac{1}{1 - h^{CB}}.$$

Since $\frac{1}{1-h^{CB}} = \frac{1}{d^2}\frac{R_A}{1+\frac{\bar{R}}{d^2}}$, we can have $\left.\frac{\partial Y}{\partial m}\right|_{m=0} < 0$ for $\bar{R} \in (0, \bar{R}^{\#})$, where $\bar{R}^{\#} = \frac{R_B-1}{R_A-1}R_A d - d^2$.

Case III: $\bar{R} = \bar{R}^{***}$**.**

I can write

$$Y = (R_A - 1)\Omega(1-h) - (R_A - 1)\Delta\Omega(1-h) + (R_A - 1)m + (R_B - 1)\Omega d - (R_B - 1)\Delta\Omega d.$$

In this region, h is constant. This gives

$$\left.\frac{\partial Y}{\partial m}\right|_{m=0} = (R_A - 1) - (R_B - 1)d\frac{1}{1-h^{CB}} - (R_A - 1)(1-h)\frac{1}{1-h^{CB}}.$$

G.2 Allowing Reserves to be Pledged

The discussion so far has assumed that A invests the cash received from the central bank. It has implicitly assumed that the cash (reserves) received from the Central Bank is not pledgeable to B. Pledging of the reserves as collateral in order to borrow cash effectively involves lending the haircut to B. The assumption that this is not allowed is reasonable since the mandate of hedge funds generally does not allow this type of direct unsecured lending to intermediaries like prime brokers, even though it is effectively achieved indirectly through rehypothecation. However, I show that even after relaxing this assumption, the result about the possible fall in expected output still holds.

I prove the result for $d = 1$, i.e. the case when the collateral is money-like. In this case, the cash (reserves) and the collateral are equivalent for B. B's demand curve for "collateral", which could either be the original collateral asset or the reserves from the central bank, remains unchanged. However, for A, the cash and the collateral are not equivalent. A is forced to go to B and borrow cash against collateral to invest in his project. On receiving the cash from the central bank, A can either invest it in his project, or borrow against it from B. The net return from investing is $R_A - 1$, and the net return from using it as collateral to borrow from B is $\bar{R}$. Thus, when $\bar{R} < R_A - 1$, A will invest the cash in his project, and when $\bar{R} \geq R_A - 1$, A will borrow against it from B. In view of this, the indifference Equation (19) gets modified to

$$\begin{aligned}(R_A - (1 + r^{CB}))(1 - h^{CB}) &= \bar{R}, \text{ if } \bar{R} < R_A - 1,\\ (1 + \bar{R} - (1 + r^{CB}))(1 - h^{CB}) &= \bar{R}, \text{ if } \bar{R} \geq R_A - 1.\end{aligned}$$

In each case, the right hand side represents the net return to A from borrowing from B and investing the proceeds. In the first case, the left hand side represents the net return to A from borrowing from the central bank and investing the proceeds. In the second case, the left hand side represents the net return to A from borrowing from the central bank and then borrowing against the reserves from B.

Thus, when $\bar{R} \geq R_A - 1$, $r^{CB} = h^{CB} = 0$. The central bank exchanges the collateral for cash one for one, and A extends the cash to B, leaving the equilibrium unchanged. The central bank's intervention is neutral when $\bar{R} \geq R_A - 1$.

But when $\bar{R} < R_A - 1$, A finds it more profitable to invest the cash in the project. The situation in Section 6 holds, and so do Proposition 7 and Corollary 1. $\left.\frac{\partial Y}{\partial m}\right|_{m=0} < 0$ for

$\bar{R} < \bar{R}^{\#}$. Since, $\bar{R}^{\#} = \frac{R_B - 1}{R_A - 1} R_A - 1 > R_A - 1$ for $R_B > R_A$, this is true for all $\bar{R} < R_A - 1$.

When $\bar{R}$ is sufficiently small, A is already lending collateral to B to the binding pledgeability limit. Then A does not get a good enough return from lending a marginal amount to B, and instead chooses to invest in the project.