Preferences Over Representatives*

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Abstract

We present a theory of preferences over representatives who will vote on one’s behalf. Our theory accepts as given that legislators are generally incapable of unilaterally implementing their platforms. Rather, they vote on things. Based on this, the theory predicts that, when choosing between candidates who will vote over a legislative agenda that is even partially beyond the candidates’ control, voters’ preferences over candidates’ platforms will generally become asymmetric even if their preferences over policy outcomes are symmetric. We show that these induced preferences can prefer either polarization or moderation in the resulting legislature, depending on the legislative agenda. Taking the electoral process as given, we show that elite polarization can occur without parties, without elections, without primaries, without gerrymandering, and without mass polarization.

Work in progress. Comments very welcome.

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Most ideological (or spatial) theories of how people vote presume that voters reduce candidates to an ideological position.\textsuperscript{1} This is a convenient and productive simplification, but it is a simplification: few, if any, political offices in a democracy allow the officeholder to unilaterally impose his or her will by fiat. Rather, the official must work through an institutionalized process in order to have some effect on public policy.\textsuperscript{2}

In this article, we focus on one aspect of such processes: the opportunities to make or influence policy changes is often exogenous. From legislators to executives to judges, most officials with decision-making authority spend most of their time making decisions about issues that were chosen by someone else. For example, in legislatures, the question of which bills will be voted on is itself a collective decision. Accordingly, the issues that any given legislator ends up voting on will be at least partially, if not totally, outside of the legislator’s control.\textsuperscript{3} At the end of the day, such a legislator can implement his or her “platform” only through voting on bills that are not necessarily representative of the policies that he or she would implement if given unilateral authority. Both internal procedures and external events, such as disasters, force legislators to vote on options other than their most-preferred policies.\textsuperscript{4}

We explore the implications of this reality in this article. The key finding is that incorporating this exogeneity into a voter’s strategic calculation about which candidates to vote for generally induces asymmetric preferences. Specifically, the voter’s expectations about what decisions (e.g., votes) a member will confront affect how the voter views ideological positions distinct from his or her own. We show that, in many cases, these expectations create a general preference for more extreme candidates, where “extremity” is relative to the agenda: if the voter tends to be (say) to the right of the alternatives brought up, then a more extreme candidate is one whose platform is even more likely to be to the right of the alternatives brought up on the agenda.

We demonstrate that this tendency can be reversed when the agenda is focused

\textsuperscript{1}As well as, possibly, a “valence” term (Groseclose (2001), Schofield (2004), Carter and Patty (2015)). We ignore this possibility in this article and focus instead on an alternative view of how legislators’ ideological platforms affect voters.

\textsuperscript{2}The institutionally imposed divergence between goals and actions is treated very generally in Penn, Patty and Gailmard (2011) and Gailmard, Patty and Penn (2008).

\textsuperscript{3}Obviously, the same logic applies to executives and judges, particularly those on collegial courts.

\textsuperscript{4}We mention and set to the side for future work the fact that electoral incentives within a legislature could have similar effects, to the degree that some individuals seek to stake out positions on issues through dilatory tactics or other forms of obstruction (Patty16).
or targeted and the voter is generally predisposed to favor the bills proposed on the agenda. Such voters will prefer moderate candidates: candidates whose platforms are more likely to “fall between” the alternatives brought up on the agenda. Finally, our theory illustrates that asymmetric, or partisan, gatekeeping procedures (e.g., Crombez, Groseclose and Krehbiel (2006), Patty (2007)) can induce strong asymmetries between the preferences of voters whose preferences are on opposite sides of the political spectrum.

We note at the outset that we are not offering a theory of electoral competition: that is, we are not attempting to explain how candidates choose the platforms they offer to voters. Rather we note at the outset that, as a high profile example, there is very little evidence in favor of policy convergence in Congressional or presidential elections in the United States. With that introduction in hand, we now present our theory.

1 The Model

We denote the set of potential policies by \(X = \mathbb{R}\) and denote the set of voters by \(N = \{1, \ldots, n\}\). A voter \(i\) is characterized by an ideal point, \(v_i \in \mathbb{R}\), and his or her preferences are represented by a policy payoff function \(u(x, v_i)\), where \(u(x, v_i)\) is a quasi-concave (and hence, “single-peaked”) function of \(x\). Two canonical examples of such a function are

\[
\begin{align*}
    u(x, v_i) &= -|x - v_i|, \text{ and} \\
    u(x, v_i) &= -(x - v_i)^2.
\end{align*}
\]

In line with each of these two examples, it is useful (but not necessary) to assume that \(u\) is a strictly decreasing function of the distance between \(x\) and \(v_i\):

**Assumption 1** For any ideal point \(v\) and pair of policies \(x\) and \(y\),

\[u(x, v) > u(y, v) \iff |x - v| < |y - v|.
\]

With Assumption 1 in hand, Figure 1 displays the voter’s optimal vote choice for all pairs of bills and status quos.

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5 We discuss such a model in Patty and Penn (2016).
6 We discuss generalizing the model to multidimensional policy spaces in Section 5.
Figure 1: The Voter’s Preferred Voting Behavior
Candidates and Platforms. The set of candidates is denoted by $C = \{1, 2\}$. The platform of candidate $c \in C$ is denoted by $p_c \in \mathbb{R}$. We will analyze three different motivations for the candidates: maximizing probability of winning, implementing the candidate’s preferred policy (i.e., a “citizen-candidate model”), and maximizing expected social welfare (which is equivalent to a single candidate election).

If elected, candidate $c \in C$’s platform, $p_c$, will determine the candidate’s subsequent voting behavior as follows. For any pair of bill and status quo, $(b, q)$, a candidate with realized platform $p$ will vote for the bill, $b$, if and only if $u(b, p) > u(q, p_c)$.

That is, if elected, the candidate will vote for a bill if and only if the bill is closer to the candidate’s realized platform than the status quo. For any realized platform $p$, this voting behavior is represented formally by the following function:

$$V(b, q, p) = \begin{cases} b & \text{if } u(b, p) > u(q, p), \\ q & \text{otherwise.} \end{cases}$$

For any voter $i$ with ideal point $v_i \in X$ and platform $p \in X$, let

$$D(p, v_i) = \{(b, q) : V(b, q, p) \neq V(b, q, v_i)\}$$

denote voter $i$’s disagreement set with respect to the platform $p$. This region is illustrated in Figure 2.

1.1 The Legislative Agenda

We represent the legislative agenda (or, more simply, “agenda”) by a distribution on $X^2$ described by a probability measure $\sigma : X^2 \rightarrow [0, 1]$, and we denote the support of $\sigma$ by $\text{Support}(\sigma)$.\footnote{Formally, a pair $(b, q)$ is in the support of $\sigma$ if $\sigma(Y) > 0$ for every open set $Y$ containing $(b, q)$.} Given $\sigma$ and ideal point $v$, the voter’s expected payoff from a candidate who with a realized platform of $p$ is

$$EU(p, v_i) = \int_{X^2} u(V(b, q, p), v_i) d\sigma(b, q). \quad (1)$$

Our first result is that the voter’s expected payoff is maximized by a representative whose platform is equal to the voter’s ideal point.

**Theorem 1** For any ideal point $v_i \in X$ and any agenda $\sigma$, the function $EU(p, v_i)$ is maximized at $p = v_i$. 
Figure 2: Voter and Candidate Disagreement Region
**Proof:** Fix an agenda $\sigma$ and an ideal point $v_i$. Then consider any platform $p > v_i$ (the case of $p < v_i$ is symmetric).

$$EU(v, v_i) - EU(p, v_i) = \int_{X^2} u(V(b, q, v), v_i) d\sigma(b, q) - \int_{X^2} u(V(b, q, p), v_i) d\sigma(b, q),$$

$$\geq 0.$$

Furthermore, the inequality is strict if $\sigma(D(p, v_i)) > 0$. Thus, any platform other than the voter’s ideal point yields an expected payoff that is no greater than the voter’s expected payoff from a platform equal to his or her ideal point, as was to be shown.

The next theorem extends Theorem 1 by establishing that the expected payoff function is single-plateaued.

**Theorem 2** For any ideal point $v_i \in X$ and any agenda $\sigma$, the function $EU(p, v_i)$ is single plateaued: if $p \leq p \leq v_i$ then $EU(p, v_i) \leq EU(p, v_i)$ and, if $p \geq p \geq v_i$ then $EU(p, v_i) \leq EU(p, v_i)$.

**Proof:** Let $p < p < v_i$. We will show that every time candidates at $p$ and $p$ vote differently, voter $i$ prefers the vote of the candidate at $p$. Suppose that $(b, q)$ is a $(\text{bill}, \text{status quo})$ pair for which a candidate at $p$ and $p$ vote differently, with $b \neq q$.

First, if a candidate at $p$ votes for $b$ while $p$ votes for $q$ then it must be the case that $b > q$. This is because $|p - b| \leq |p - q|$ and $|p - b| \geq |p - q|$ can be rewritten $b^2 - 2bp + p^2 \leq q^2 - 2qp + p^2$ and $b^2 - 2bp + p^2 \geq q^2 - 2qp + p^2$. This reduces to $2b(p - p) \geq 2q(p - q)$, or $b > q$.

Second, if $b, q > p$ or if $b, q < p$, then $p$ and $p$ will vote the same way. Therefore if they vote differently then either (a) $b \geq p \geq q \geq p$, or (b) $p \geq b \geq q \geq p$, or (c) $p \geq b \geq p \geq q$.

Our results are proved for the cases of (b) and (c) by the assumption that $u(x, v_i)$ is single peaked, as $v_i > b \geq q$ in these cases. For case (a) we know that $b - p < p - q$.

Theorems 1 and 2 jointly establish a weak version of the “ally principle” (Bendor and Meirowitz (2004)) in this setting. Theorem 1 establishes that each voter
should, if he or she can, appoint his or her ideological clone to vote on his or her behalf. Theorem 2 goes a step farther and implies that, when choosing among candidates whose platforms are all on the same side of the voter’s ideal point (i.e. \( p_c \leq v_I \) for all \( c \in C \) or \( p_c \geq v_I \) for all \( c \in C \)), then the voter should appoint the candidate whose platform is closest to his or her ideal point. These establish a “weak version” of the ally principle because they do not imply that “all else equal, a rational boss should choose her closest ally as an agent.” \(^8\) Specifically, the theorems do not address situations in which at least one candidate is offering a platform strictly larger, and another candidate is offering a platform that is strictly lower, than the voter’s ideal point.

For any agenda \( \sigma \), we define the notion of the span of \( \sigma \) as follows.

**Definition 1** For any ideal point \( v_i \in X \), \( v_i \) is in the span of \( \sigma \) if there exists \((b, q)\) such that \( b < v_i < q \) or \( q < v_i < b \) such that \((b, q) \in \text{SUPPORT}(\sigma)\). We denote the span of \( \sigma \) by \( \text{SPAN}(\sigma) \) and the convex hull of the span by \( \text{SPAN}^H(\sigma) \). \(^9\)

Substantively, if a voter \( i \)'s ideal point \( v_i \) is in the span of \( \sigma \), then voter \( i \) has a positive probability of observing a vote between a bill \( b \) and status quo \( q \) such that either \( b < v_i < q \) or \( q < v_i < b \). The following theorem states that voters whose ideal points are outside the span of the agenda (in the sense of not being inside the convex hull of the span of \( \sigma \)) are indifferent between an infinitely long interval of platforms and these platforms all maximize the voter’s expected payoff.

**Proposition 1** For any agenda \( \sigma \) and any ideal point \( v_i \in X \) such that \( v_i \notin \text{SPAN}^H(\sigma) \), there exists a value \( e_i \in X \) such that \( EU(p, v_i) \) is constant and maximized on either \((−\infty, e_i] \) or \([e_i, \infty)\).

**Proof:** [Sketch.] Fix an agenda \( \sigma \) and ideal point \( v_i \) such that \( v_i \notin \text{SPAN}^H(\sigma) \). Then it is the case that for any \((b, q) \in \text{SUPPORT}(\sigma)\), either \( \max[b, q] < v_i \) or \( v_i < \min[b, q] \). Without loss of generality, suppose that \( v_i < \min[b, q] \): voter \( i \) is “to the left of” the most liberal policies that might come up for a vote. Then, for any pair of platforms \( p, p' \in (−\infty, v_i] \) and any \((b, q) \in \text{SUPPORT}(\sigma)\),

\[
V(b, q, p) = V(b, q, p') = V(b, q, v_i),
\]

which implies that \( EU(p, v_i) = EU(p', v_i) \) and, by Theorem 1, \( EU(p, v_i) = \max_{\tilde{p}} EU(\tilde{p}, v_i) \).

\(^8\)Bendor and Meirowitz (2004), p. 300.

\(^9\)The convex hull of a set \( Y \), \( Y^H \), is the smallest convex set (by set inclusion) containing \( Y \).
In colloquial terms, Proposition 1 implies that a voter with extreme preferences has no incentive to choose representatives with platforms that are more moderate than the voter’s.

The next theorem generalizes Proposition 1 by including situations in which the voter’s ideal point is outside the span, but “inside” the convex hull of the span, of σ. This can only occur when the span of the agenda is not convex: this occurs when there is a positive probability of observing a vote satisfying \( \max[b, q] < v_i \) and a positive probability of observing a vote satisfying \( \min[b, q] > v_i \), but zero probability of observing a vote with \( \min[b, q] < v_i < \max[b, q] \).

**Theorem 3** For any agenda σ and any ideal point \( v_i \in X \) such that \( v_i \notin \text{SPAN}(\sigma) \), there exists values \( \ell_i < h_i \) such that \( E(U(p, v_i)) \) is constant and maximized on either \([\ell_i, h_i]\).

**Proof:** [Sketch.] Fix an agenda σ and ideal point \( v_i \) such that \( v_i \notin \text{SPAN}(\sigma) \). Proposition 1 covers the case where \( v_i \notin \text{SPAN}^H(\sigma) \), so suppose that \( v_i \in \text{SPAN}^H(\sigma) \). This implies that there is an open set of \( X \) containing \( v_i \), \( O_i \subset X \), such that \( \sigma(O_i \times O_i) = 0 \). Accordingly, any pair of platforms \( p, p' \in O_i \) will vote identically over any pair \((b, q) \in \text{SUPPORT}(\sigma)\), implying that \( E(U(p, v_i)) = E(U(p', v_i)) \), as was to be shown.

### 1.2 Specific Agenda Processes

We consider two types of agenda processes. The first of these, the \( F \)-naive agenda model, assumes that \( b \) and \( q \) are independently and identically distributed according to a probability distribution with cumulative distribution function \( F : [0, 1] \to [0, 1] \). In the second, the \((m, \mu)\)-setter model, the status quo \( q \) is distributed according to the Uniform\([0, 1]\) distribution and the bill \( b \) is a function of the realized status quo and two exogenous parameters, \( m \) and \( \mu \), with \( m \leq \mu \):

\[
b(q; m, \mu) = \begin{cases} 
\mu & \text{if } |m - \mu| \leq |m - q|, \\
2m - q & \text{otherwise}.
\end{cases}
\]

This model mimics the seminal agenda-setting model due to Romer and Rosenthal (1978). The two families of agenda processes are formally defined as follows.

**Definition 2** For any cumulative distribution function \( F : X \to [0, 1] \), the agenda is a \( F \)-naive agenda model if \( b \) and \( q \) are independently and uniformly distributed according to \( F \).
Definition 3 For any real numbers $m$ and $\mu$ with $\mu > m$, the agenda is a $(m, \mu)$-setter model if $q$ is uniformly distributed over $[0, 1]$ and $b$ is distributed according to the following function:

$$b(q; m, \mu) = \begin{cases} 
\mu & \text{if } |m - \mu| \leq |m - q|, \\
2m - q & \text{otherwise.}
\end{cases}$$

2 Analysis

In this section, we analyze each of the two families of agenda processes in turn, beginning with the naive agenda model. The focus throughout is on the question of under what conditions a voter would have a strict preference over two candidates whose platforms are on opposite sides of, and equidistant from, the voter’s ideal point, $v_i$. That is, we are interested in the voter’s preference between two candidates, $L$ and $R$, whose platforms satisfy the following: $p_L = v_i - \delta$ and $p_R + \delta$ for some $\delta > 0$. Extending Figure 2, Figure 3 compare

2.1 The Naive Agenda Model

Before proving a few results for the general family of naive agenda processes, we provide some intuition by considering a baseline case, the “Uniform Naive Model,” in which the bill and status quo are each independently distributed according to the Uniform $[0, 1]$ distribution. In this setting, if $v_i < \frac{1}{2}$, then the voter always prefers extremism to moderation. Figure 4 illustrates this. Specifically, it highlights a voter’s payoffs from two example platforms, $p_L$ and $p_R$, equidistant from the voter’s ideal point. Because the ideal point is less than $\frac{1}{2}$ (specifically, $v = 0.35$), the platform of candidate $R$, $p_R$, represents the “moderate” platform and that of candidate $L$, $p_L$, represents the “extreme” platform. The voter receives a higher expected payoff from the extreme platform than he or she would receive from the moderate platform.

The preference for extreme platforms illustrated in Figure 4 is a general conclusion in the Uniform Naive Model, as we state formally in the next proposition.

Proposition 2 Suppose that $\sigma$ is the Naive Agenda Model. Then, for any ideal point $v_i$ and any $\delta > 0$, the voter receives a weakly higher expected payoff from
Figure 3: Disagreement Regions for Both Candidates $L$ and $R$
Voter’s expected utility

\[ EU(p, v) \]

\[ EU(p_L, v) \]

\[ EU(p_R, v) \]

\[ \delta \]

\[ \delta \]

Figure 4: Expected Payoffs In Uniform Naive Model
the extreme platform than from the moderate one:

\[ v_i \leq \frac{1}{2} \iff EU(v_i - \delta, v_i) \geq EU(v_i + \delta, v_i), \]
\[ v_i \geq \frac{1}{2} \iff EU(v_i + \delta, v_i) \geq EU(v_i - \delta, v_i). \]

**Furthermore, the inequality is strict whenever** \( v_i \neq \frac{1}{2} \).

The logic behind this result is illustrated in Figure 5. The two dark triangles represent \((b, q)\) pairs in which the moderate platform will vote against the voter’s interests and for which there is no analogous pair in which the extremist would vote against the voter’s interests. That is, for every point in the voter’s disagreement region with respect to the extremist candidate’s platform, a unique point with exactly the same disutility for the voter exists in his or her disagreement region with the moderate candidate’s platform, but the converse does not hold whenever \( v \neq \frac{1}{2} \). Because every \((b, q)\) pair is equally likely, this establishes the conclusion of Proposition 2.

We can say a few things about general naive agenda processes. The first is a straightforward generalization of Proposition 2.

**Proposition 3** Suppose that \( F \) is symmetric about \( x_m \in X \) and \( \text{SUPPORT}(F) = R \). Then,

\[ v_i \leq x_m \iff EU(v_i - \delta, v_i) \geq EU(v_i + \delta, v_i), \]
\[ v_i \geq x_m \iff EU(v_i + \delta, v_i) \geq EU(v_i - \delta, v_i). \]

**Furthermore, the inequality is strict whenever** \( v \neq x_m \).

### 2.2 The Setter Agenda Model

The setter agenda model is parameterized by two locations: \( \mu \), which represents the “unconstrained” target of the bills, and \( m \), which represents the constraint on the targeting of \( \mu \): the bill can be no farther away from the status quo than \( |q - m| \).

We remain agnostic about the origins of \( m \) and \( \mu \): they might represent members of the legislature (as in Cox and McCubbins (1993, 2005)), or either or both of the parameters might represent agenda-setting processes outside of the legislature (e.g., the other chamber in bicameral settings, the executive in separation of powers systems, a constitutional court responsible for reviewing legislation, etc.).

Unlike in the naive agenda model, the voter can have an induced preference for moderation. We discuss the logic for this below, but the intuition is that voters
Figure 5: Why Extremists Are Preferred in Uniform Naive Model
who are more predisposed to prefer the items on the are hurt less by platforms that are biased toward the agenda than are voters who are less favorably disposed toward the bills that will be brought forward.

To understand the logic behind preferences for moderation, we first overlay an example of the \((m, \mu)\)-setter model on the disagreement regions between a voter and two platforms \(p_L = v - \delta\) and \(p_R = v + \delta < m < \mu\). This is displayed in Figure 7.

Working through this example, consider the \((m, m)\)-setter model and a voter with \(v \in \left(\frac{m}{2}, m\right)\) and platforms \(p_L = v - \delta\), and \(p_R = v + \delta\), for \(\delta = \frac{m - v}{2}\). The voter disagrees with the extreme candidate, \(L\), when the status quo is between \(v - 2\delta\) and \(v\), and disagrees with the moderate candidate, \(R\) when the status quo is between \(v\) and \(v + 2\delta\). In the former case, the voter prefers the bill \(m\) while \(L\) prefers the
Figure 7: Disagreement Regions in the $(m, \mu)$-setter Model
status quo, $q$. In the latter case the voter prefers the status quo, $q$, while $R$ prefers the bill, $m$. While these regions of disagreement are both of length $2\delta$ (and hence equally likely to occur), the voter is not indifferent between $p_L$ and $p_R$. This is because the disagreement between the voter and $R$ is less costly for the voter than the disagreement between the voter and $L$. To see this, note that the voter’s payoff from $L$’s choice is:

$$EU(v, p_L) = \int_0^{2\delta} -(v - s) ds = 2\delta(\delta - v),$$

while the voter’s payoff from $R$’s choice is (substituting $m = 2p_L = 2(v - \delta)$ into the voter’s payoff function):

$$EU(v, p_R) = \int_{2\delta}^{4\delta} -(2(v - \delta) - v) ds = 2\delta(2\delta - v).$$

In this case, the voter receives an additional $2\delta^2$ from the choices of the more moderate candidate, $p_R$.

The logic of this example, and the basic logic behind Proposition ??, is displayed in Figure 8. In the figure, both $L$ and $R$ have the same probability of disagreeing with the voter (because the status quo $q$ is assumed to drawn Uniformly) but the disutility from any disagreement between the voter and the extreme candidate is strictly larger than the analogous disagreement between the voter and the moderate. The size of this disagreement is represented by the size of the light triangle within the extreme candidate’s disagreement region.

In the general $(m, \mu)$-setter model with $m \neq \mu$, the voters’ expected payoff function becomes discontinuous at $m$, and constant between $m$ and $\mu$. These characteristics, as well as the sensitivity of the function to the relative locations of $v$, $m$, and $\mu$ is illustrated in Figures 9 and 10. Two qualitative characteristics of the expected payoff function in the $(m, \mu)$-setter model (that distinguish it from the payoffs in the naive agenda model) are presented in the following proposition.

**Proposition 4** In the $(m, \mu)$-setter model, and any ideal point $v$, $EU(p, v)$ is

1. Constant for all $p \in [m, \mu]$, and
2. Discontinuous at $m$.

Considering the comparison between equidistant platforms, the following fact is simple to derive:
Figure 8: Why Moderates Can Be Preferred in the Setter Model
Figure 9: Expected Payoffs in the Setter Model, $v < m < \mu$

Figure 10: Expected Payoffs in the Setter Model, $m < \mu < v$
Fact 1 Consider the \((m, \mu)\)-setter model. For any voter with ideal point \(v \neq m\), there exists a value \(\hat{\delta} > 0\) such that

\[
EU(v + \delta, v) < EU(v - \delta, v) \text{ for all } \delta > \hat{\delta}.
\]

In light of Fact 1, the next definition clarifies our language when talking about a voter’s preference with respect to candidates closer to, or farther away from, the agenda process.

Definition 4 In the \((m, \mu)\)-setter model, a voter with ideal point \(v < m\) prefers moderation if, for all \(\delta \in (0, m - v)\),

\[
EU(v + \delta, v) > EU(v - \delta, v),
\]

and he or she prefers extremism if, for all \(\delta > 0\),

\[
EU(v + \delta, v) < EU(v - \delta, v),
\]

Similarly, a voter with ideal point \(v > m\) prefers moderation if, for all \(\delta \in (0, m - v)\),

\[
EU(v + \delta, v) < EU(v - \delta, v),
\]

and he or she prefers extremism if, for all \(\delta > 0\),

\[
EU(v + \delta, v) > EU(v - \delta, v),
\]

Proposition 5 In the \((m, \mu)\)-setter model, a voter with ideal point \(v < m\) prefers moderation if and only if \(v \in \left(\frac{\mu}{2}, m\right)\).

Proof: Supposing that the status quo is distributed uniformly, the net expected payoff from \(p_R = v + \delta\) (relative to \(c^* = v\)) is

\[
EU(p_R, v) = \left(\frac{p_R - p_L}{2}\right)\frac{2(\mu - p_R) + 2\mu - p_L - p_R}{2},
\]

\[
= 2\delta \left(2\mu - p_R - \frac{p_R + p_L}{2}\right),
\]

\[
= 2\delta (2\mu - 2v - \delta),
\]

the net expected disutility from \(p_L = v - \delta\) (relative to \(c^* = v\)) is

\[
EU(p_R, v) = \left(\frac{p_L + p_R - \mu}{2}\right)\frac{2\mu - p_L - p_R}{2},
\]

\[
= (2v - \mu) \left(\frac{3\mu}{2} - \frac{p_R + p_L}{2}\right),
\]

\[
= (2v - \mu) \left(\frac{3\mu}{2} - v\right),
\]
Thus, given the normalization that $\mu > v$, the net expected disutility from $p_L$ is greater than that from $p_R$ (and hence $v$ wants moderation) for all sufficiently small values of $\delta > 0$ if

$$v > \frac{\mu}{2}.$$

Thus, if $v \in \left(\frac{\mu}{2}, m\right)$, the voter has a strict preference for moderation for small $\delta \leq m - v$. Conversely, if $v$ is distant enough from $\mu$, then the voter will prefer polarization for all values of $\delta \in (0, m-v)$, as was to be shown.

Proposition 5 speaks to the case of a voter who is on the opposite side of $m$ relative to $\mu$. The next proposition considers the case where the voter is on the opposite side of $\mu$ relative to $m$. The proof is omitted, because it is analogous to that of Proposition 5.

**Proposition 6** In the $(m, \mu)$-setter model, a voter with ideal point $v > \mu$ prefers moderation if and only if $v \in \left(\mu, \frac{1+\mu}{2}\right)$.

The next proposition merely restates that, for $v \in (m, \mu)$, the voter’s expected payoff function is constant for all platforms in an open neighborhood of $v$.

**Proposition 7** In the $(m, \mu)$-setter model, a voter with ideal point $v \in (m, \mu)$ prefers neither moderation nor extremism: for sufficiently small $\delta$, $EU(v - \delta, v) = EU(v + \delta, v)$.

With the theory laid out, we now turn to the two central components of our theory: the interaction between voters and candidates and the legislative agenda.

### 3 Knowledge, Voting, and Responsiveness

How voters evaluate candidates and parties when making their vote choices is a broad, deep, and long-studied question. For our purposes, there are three key questions at hand: whether voters know how their representatives vote and, if so, whether voters use this knowledge when making their vote choices.

**Do voters know how their representatives vote?** Political scientists have examined, and in general been pessimistic about, the levels of political knowledge most voters possess. Fortunately, however, there is a growing body of evidence that many voters are at least somewhat aware of how their representatives vote. While this information is generally confined to high profile (or “salient”) matters,
such as the Affordable Care Act (Nyhan, McGhee, Sides, Masket and Greene (2012)), voters tend to have correct impressions about their representative’s voting record (Ansolabehere and Jones (2010)).

The premises of our theory are bolstered by the fact that voters are incompletely aware of their representatives’ voting records. As we discuss in more detail later in the article, our theory indicates that the possibility that the voter misperceives a candidate’s platform (or an incumbent’s record) creates an incentive for candidates to adopt a position other than the median voter’s most-preferred policy. This linkage is also in line with recent work considering how candidates attempt to explain their positions (e.g., Grose, Malhotra and Van Houweling (2014), Peterson and Simonovits (2014), Broockman and Butler (2014)). The ubiquity of explanations indicates that candidates can not communicate a given ideological position to constituents with certainty.

Do voters vote based on incumbents’ voting records? There is substantial evidence that voters can and do punish incumbents for their voting records. For example, incumbents are occasionally taken to task for deviations on highly salient votes (Bovitz and Carson (2006)). More generally, legislators tend to pay an electoral penalty when their voting records get too far “out of step” with their constituents’ interests (Ansolabehere, Snyder, Jr. and Stewart, III (2001), Canes-Wrone, Brady and Cogan (2002), Hollibaugh, Rothenberg and Rulison (2013)), Overall, while factors such as individual partisanship (Jessee (2009, 2010)), incumbency advantage, and electoral tides appear to mitigate both the individual and collective abilities of voters to hold representatives accountable for their voting records, voters nonetheless respond to their representatives’ voting records in a manner that is consistent with the spatial theory of political preferences.

While current empirical estimates of the electoral effect of both challengers’ and incumbents’ positions are small, at best (e.g., Montagnes and Rogowski (2014), Tausanovitch and Warshaw (2014)), our theory provides two explanations for this. First, office-seeking candidates’ strategic incentives will generally induce them to announce positions such that the marginal effect of their position is relatively small. Second, the marginal effect of a candidate’s platform on the median voter’s expected payoff will vary systematically across districts. Our theory indicates that the size of this effect are smaller for districts where the median voter’s ideal point is more distant from the future legislative agenda.

10And, similarly, that voters are incompletely aware of candidates’ platforms in general.
4 The Legislative Agenda

The legislative “agenda” is a nebulous concept. In colloquial terms, the agenda is composed of the issues and policies being formally and informally debated by the legislators. In narrower terms, and within our theory, the agenda is simply a list of roll call votes that the legislators will cast. In spatial terms, this is merely a list of pairs, one pair for each roll call vote. Each pair represents (1) a proposed new location for some policy and (2) the “status quo” current location of that policy.

Even from the relatively privileged standpoint of legislators themselves, and regardless of which conception of the agenda one uses, the legislative agenda is notoriously mercurial and difficult to manage. Partly, this is because the legislature is merely one arena among many in the overall policy process (Kingdon (1984), Baumgartner and Jones (2009)). Partly as a result of this embedding, the legislative agenda is arguably best perceived of as being composed of policy ideas, as opposed to bills, *per se* (Wilkerson, Smith and Stramp (2015)) and evidence demonstrates that any given idea will receive collective attention for a only a brief interval of time (Burstein, Bauldry and Froese (2005)). Successfully affecting public policy from within a legislative chamber requires matching a bill (or bills) with one or more of the policy ideas that is likely to receive attention.

Mirroring this substantive richness, the set of sponsored bills (the “potential legislative agenda”) possesses a more complex structure than does legislative voting (Talbert and Potoski (2002)). Affecting the observed legislative agenda—for example, by sponsoring a bill and having it brought up for a vote—requires individual effort, or “legislative entrepreneurship” (Wawro (2000)). Successfully influencing the agenda requires balancing and tempering one’s legislative behavior (Anderson, Box-Steffensmeier and Sinclair-Chapman (2003)) and policy goals (Woon (2008)). More generally, institutional factors such as legislative rules, party structure and leadership, and committee composition play significant roles in determining the content of the legislative agenda.11

Our theory is based on the assumption that the legislative agenda is essentially independent of which candidate is elected by most districts. That is, for the districts under consideration in our theory, we assume that the collection of bills that will be voted upon in the following session of the legislature is unaffected by the platform of the candidate elected to represent the district. Perhaps more to the point, we assume that the voter believes that the subsequent legislative agenda is

independent of the voter’s decision about which candidate to support. This assumption implies that the voter chooses among competing candidates based on how each candidate would vote on the bills that are likely to be brought to the floor.

5 Extensions and Robustness

Multidimensional Policy Space. It is straightforward to generalize our theory to allow for multiple dimensions. For reasons of space, we briefly consider only the case of the naive agenda model. Let $X = \mathbb{R}^M$ denote the policy space, for $M \geq 2$. Each voter $i$’s ideal point is then a point $v_i \in X$ and the naive agenda process is represented by the uniform distribution on $[0, 1]^{2M}$, where the first $M$ components represent the bill, $b$, and the second $M$ components represent the status quo, $q$. Extending the theory to multiple dimensions allows for a richer set of possible collective choice mechanisms. At one extreme of such mechanisms is a “closed rule” variant in which a representative must vote for either $b$ or $q$ and, at the other extreme is a “dimension-by-dimension” rule in which the representative casts $M$ votes, one for each of the dimensions of the comparison.

The Voter’s Payoff Function. As presented in this article, the voter’s payoff function is based on the votes cast by the representative, rather than outcomes actually implemented. At first, this seems at odds with consequentialism and therefore incompatible with many standard conceptions of “rational” decision-making. This divergence is only in appearance, however, because the voter is unable to affect anything except choose which platform he or she sends to the legislature to cast votes on his or her behalf. Introducing a more explicitly consequentialist account would lead to considering only those cases in which the vote cast by the representative is decisive (or “pivotal”). We present the foundations of such a model in the appendix.

Independence of the Legislative Agenda. We have assumed that the legislative agenda is entirely independent of the voter’s choice of representative. This is obviously a strong assumption. However, it is straightforward to see that the qualitative characteristics of our conclusions will remain true if this independence is relaxed somewhat. For example, suppose that the representative will set policy equal to his or her platform with probability $\phi \in (0, 1)$ and vote on an exogenous pair of policies as assumed in the article with probability $1 - \phi$. In such a model, it
is clear that the voter’s expected payoff function will converge to his or her policy payoff function, $u$, as $\phi \to 1$.

6 Conclusion

We have presented a theory of voting for representatives. The key finding is that, when candidates’ platforms represent how they will vote over an exogenous agenda, the voter’s preferences over these platforms will not generally be the same as his or her preferences over those platforms if they represented the policy that would be implemented by the candidate. Once recognized, this finding is intuitive but, to our knowledge, relatively unaccounted for in theoretical and empirical investigations of voting. We also find that, when the agenda is totally random, voters will always have a preference for a more extreme candidate over a more moderate one, holding the degree of divergence from the voter’s ideal platform constant. This finding is mitigated when the agenda is targeted: in such cases, the voter will have a preference for more moderate platforms when the divergence is not too great and the voter’s most-preferred policy is relatively close to the bills on the agenda.
A Instrumental Motivations and Probabilistic Voting in the Legislature

We consider a minimal model of elections in which a single candidate chooses a platform and the voter chooses whether to elect the representative or choose an unknown challenger.\textsuperscript{12}

**Perceptions and Voting.** The voter does not observe $p$ directly. Rather, the voter observes a perturbation of $p$, denoted by

$$
\hat{p} = p + \theta,
$$

where $\theta$ is a shock that we assume, for simplicity, is drawn from a Uniform\([-\frac{\Theta}{2}, \frac{\Theta}{2}]\) distribution, where $\Theta \geq 0$ is an exogenous parameter of the model. This parameter is inversely related to the precision of the voter’s information about the candidate’s platform (or, similarly, the incumbent’s record). We assume that $\theta$ is independent of $p$.

The voter then votes to elect either the incumbent ($E = I$) or a random replacement ($E = R$) as follows:

$$
E(\hat{p}) = \begin{cases} 
I & \text{if } EU(\hat{p}, v_i) \geq 0, \\
C & \text{otherwise}. 
\end{cases}
$$

(Given the assumption that $\delta$ is independent of $p$, the choice of zero as a threshold is without loss of generality.)

The candidate chooses his or her platform so as to maximize the chances of receiving the voter’s vote. This reduces to the following problem:

$$
\max \int
$$

B Instrumental Motivations for the Voter

In this section, we briefly lay out a setting in which the voter’s expected payoffs are entirely instrumental in the sense of being based on the effect the chosen representative will have on the policy chosen by the legislature, as opposed to being

\textsuperscript{12}The model and its predictions are consistent with richer models of electoral competition, but this assumption suffices for our purposes.
based only on the vote choice of the representative. The point of this setting is to establish that the baseline model considered in the text is not only consistent with “expressive” motivations.

Let $X \subseteq \mathbb{R}$ denote a convex policy space and let $L = \{1, 2, \ldots, n\}$ denote a set of legislative districts (and legislators). Then, suppose that each legislator $i$, with platform $c_i \in X$ and confronted by two alternatives $x, y \in X$, votes for $x$ with probability $p(x, y|c_i) \in [0, 1]$, where $p(x, y|c_i)$ is

- weakly decreasing in $|x - c_i|$,
- weakly increasing in $|y - c_i|$,
- symmetric: $p(x, y|c_i) = 1 - p(y, x|c_i)$ for all $x, y \in X$.

A prominent family of functions that satisfies these three properties is that of the “logit” functions, parameterized by $\lambda \geq 0$:

$$p^\text{logit}_\lambda(x, y|c_i) = \frac{e^{-\lambda|x-c_i|}}{e^{-\lambda|x-c_i|} + e^{-\lambda|y-c_i|}}.$$ 

Let $D$ denote the set of decisive coalitions in the legislature. For any pair of alternatives $(x, y) \in X^2$ and any profile of platforms $C = \{c_i\}_{i \in L}$, denote the probability that $x$ is approved by

$$W(x, y|C) = \sum_{D \in D} \prod_{i \in D} p(x, y|c_i) \prod_{j \in L \setminus D} (1 - p(x, y|c_j)).$$

For any $i \in L$, let $D_i$ denote the set of decisive coalitions in which $i$ is pivotal:

$$D_i = \{ D \in D : D \setminus \{i\} \notin D \},$$

and for any pair of alternatives $x, y \in X$, let $\pi(i|x, y, C)$ denote the probability that legislator $i$’s vote is pivotal between $x$ and $y$:

$$\pi(i|x, y, C) = \sum_{D \in D_i} \prod_{k \in D \setminus \{i\}} p(x, y|c_k) \prod_{j \in L \setminus D} (1 - p(x, y|c_j)).$$

Fact 2 For all $i \in L$, all $(x, y) \in X^2$, and all $C \in \mathbb{R}^n$, $\pi(i|x, y, C)$ is independent of $c_i$.

Fact 2 implies that we can write $\pi(x, y|C_{-i}) \equiv \pi(i|x, y, C)$.

Suppose for the moment that the representative elected by voter $i$ is presumed to vote deterministically as a function of the representative’s platform $c_i$. This is a
useful baseline case for comparing the instrumental voting setting with the setting considered earlier.

In this setting, and for any probabilistic voting function $p$ and any agenda process $\sigma : X^2 \to [0, 1]$, let $EU_{p,\sigma}^{C_i}(c_i, v)$ denote the expected policy payoff for a voter with ideal point $v$ who chooses a representative with platform $c_i \in X$, given the collection of the other $n-1$ legislators’ platforms, $C_{-i}$. Letting

$$
\chi_{p,\sigma}^{C_i}(c_i, c_i', v) \equiv EU_{p,\sigma}^{C_i}(c_i, v) - EU_{p,\sigma}^{C_i}(c_i', v),
$$

denote the difference in expected payoffs from a pair of representatives with platforms $c_i$ and $c_i'$, the following fact is simple to derive.\(^{13}\)

**Fact 3** For any probabilistic voting function $p$, any collection of platforms for the other districts’ representatives $C_{-i} = \{c_j\}_{j \neq i}$, any agenda process $\sigma : X^2 \to [0, 1]$, any ideal point $v \in X$, and any pair of platforms $c_i, c_i' \in X$,

$$
\chi_{p,\sigma}^{C_i}(c_i, c_i', v) = \int_{X^2} \pi(x, y|C_{-i}) \left[U(V(b, q, c_i)) - U(V(b, q, c_i'))\right] \, d\sigma(b, q). \tag{2}
$$

Supposing that $\sigma$ is the naive agenda model, where $b$ and $q$ are independently drawn from the Uniform$[0, 1]$ distribution, equation (2) reduces to

$$
\chi_{p,\sigma}^{C_i}(c_i, c_i', v) = \int_0^1 \int_0^1 \pi(x, y|C_{-i}) \left[U(V(b, q, c_i)) - U(V(b, q, c_i'))\right] \, db \, dq. \tag{3}
$$

Assuming that $p_R \leq 1/2$, the area of disagreement between $v$ and $p_R = v + \varepsilon$ for $\varepsilon \geq 0$ is

$$
2\sqrt{2}v\varepsilon + \varepsilon^2
$$

\(^{13}\)If voter $i$ believes that the representative will vote according to $p$ as well (conditional on the representative’s platform $c_i$), the analogue to equation (2) is

$$
\chi_{p,\sigma}^{C_i}(c_i, c_i', v) = \int_{X^2} \pi(x, y|C_{-i}) \left[(p(b, q|c_i) - p(b, q|c_i'))(U(b) - U(q))\right] \, d\sigma(b, q).
$$
References


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