Business Cycle Anatomy

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“One is led by the facts to conclude that, with respect to the qualitative behavior of co-movements among series, business cycles are all alike. To theoretically inclined economists, this conclusion should be attractive and challenging, for it suggests the possibility of a unified explanation of business cycles.” (Lucas 1977)

- **A theorist's ambition:** account for bulk of the business cycle with a single-shock model i.e., multiple triggers but a dominant propagation mechanism
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- A theorist’s ambition: account for bulk of the business cycle with a single-shock model i.e., multiple triggers but a dominant propagation mechanism
- This paper’s contribution: provide an empirical template of it
What We Do

• Estimate a VAR (or VECM) on a few key variables

• Recover shock that has max contribution to volatility of $U$ over BC frequencies

• Repeat exercise by targeting other variables (e.g., TFP) or other frequencies (e.g., LR)
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⇒ "Business Cycle Anatomy" = large collection of one-dimensional cuts of the data
= rich set of restrictions on models of any size and type
Main Findings and their Use

- Establish existence of a “main business cycle (MBC) shock”
  - shocks that target $u$, $Y$, $h$, $l$, and $C$ over BC frequencies produce similar IRFs
  - supports hypothesis of common propagation mechanism

- Document its properties
  - transitory
  - disconnected from TFP at all horizons
  - orthogonal to shock that targets inflation
  - ...

- Use its properties and overall anatomy to guide theory
  - parsimonious, semi-structural perspective
  - fully structural DSGE models
Lessons for Theory

• Good news for parsimonious theories with a dominant shock/propagation mechanism

• Bad news for the following candidates
  
  • technology shocks
    RBC model
  
  • financial, uncertainty, or other shocks that map to TFP fluctuations
    Benhabib and Farmer (1992), Bloom et al (2016)
  
  • news about future TFP
  
  • inflationary demand shocks of the textbook type
  
  • propagation mechanisms in state-of-the-art DSGE models
    Smets & Wouters, Justiniano, Primiceri & Tambalott, Christiano, Motto & Rostagno
Lessons for Theory

• What fits the MBC template best?

• Non-inflationary, non-specialized, demand shocks

• Perhaps they exist (even) outside realm of sticky prices and Philips curves?
  example: our earlier Ecma paper (ACD 2018)
• Empirical Analysis

• Main Findings and Lessons

• Application to Three DSGE Models
Empirical Analysis
Baseline VAR

- Quarterly U.S data: 1955Q1-2017Q4
  - Macro Quantities: Unemployment, GDP, Hours, Invest. (inclusive of durables), Cons.
  - Productivity: util-adjust TFP, NFB labor productivity;
  - Nominal: Inflation (GDP Delator), Federal Fund Rate, Labor Share

- Bayesian VAR, 2 Lags (robust to 4 or 6 lags and VECM)
Baseline VAR

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- Bayesian VAR, 2 Lags (robust to 4 or 6 lags and VECM)

- **What next? Construct the “shock to variable X”**
  Linear combination of the VAR residuals that has the maximal contribution to the volatility of a variable X at the business-cycle frequencies, 6-32 quarters.
Main Business Cycle Shock: Targeting Unemployment

Impulse Response Functions

Variance Contributions, Business-Cycle Frequencies

<table>
<thead>
<tr>
<th></th>
<th>u</th>
<th>Y</th>
<th>h</th>
<th>l</th>
<th>C</th>
<th>TFP</th>
<th>Y/h</th>
<th>Wh/Y</th>
<th>π</th>
<th>R</th>
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<td>5.86</td>
<td>23.91</td>
<td>27.02</td>
<td>6.96</td>
<td>22.27</td>
</tr>
</tbody>
</table>
Main Business Cycle Shock: Alternative Targets

Interchangeable facets of the same shock!

\[ u \text{ shock; } Y \text{ shock; } I \text{ shock; } h \text{ shock; } C \text{ shock; } \text{Shaded area: 68\% HPDI.} \]
## Main Business Cycle Shock: Alternative Targets

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$Y$</th>
<th>$h$</th>
<th>$I$</th>
<th>$C$</th>
<th>TFP</th>
<th>$Y/h$</th>
<th>$Wh/Y$</th>
<th>$\pi$</th>
<th>$R$</th>
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<td>22.27</td>
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<tr>
<td>$Y$</td>
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<td>44.73</td>
<td>67.13</td>
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<td>4.24</td>
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<td>40.20</td>
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<td>$h$</td>
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<td>22.61</td>
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<tr>
<td>$I$</td>
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<td>66.60</td>
<td>45.20</td>
<td>80.29</td>
<td>19.01</td>
<td>3.81</td>
<td>33.74</td>
<td>36.44</td>
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<td>$C$</td>
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<td>20.15</td>
<td>17.10</td>
<td>68.30</td>
<td>1.57</td>
<td>12.93</td>
<td>10.31</td>
<td>9.93</td>
<td>4.50</td>
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The Main Business Cycle Shock: Alternative Targets
PCA on Business Cycle Frequencies

First Principal Component, Business Cycle Frequencies

<table>
<thead>
<tr>
<th></th>
<th>u</th>
<th>Y</th>
<th>h</th>
<th>l</th>
<th>C</th>
<th>TFP</th>
<th>Y/h</th>
<th>wh/Y</th>
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<th>R</th>
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<td>2.33</td>
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<tr>
<td>VAR-Based</td>
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<td>87.33</td>
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<td>29.19</td>
<td>14.16</td>
<td>0.68</td>
<td>8.10</td>
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</table>

- Similar message about variance contributions: $\text{MBC} \approx 1\text{st PC}$
- But our approach adds info about (i) IRFs and (ii) footprint on other frequencies
The Main Long-Run Shock

<table>
<thead>
<tr>
<th>Target</th>
<th>Y</th>
<th>I</th>
<th>C</th>
<th>TFP</th>
<th>Y / h</th>
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<tbody>
<tr>
<td>Y</td>
<td>99.59</td>
<td>95.94</td>
<td>99.47</td>
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<td>97.82</td>
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<tr>
<td>I</td>
<td>96.88</td>
<td>97.83</td>
<td>96.41</td>
<td>91.62</td>
<td>93.02</td>
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<tr>
<td>C</td>
<td>99.34</td>
<td>95.63</td>
<td>99.53</td>
<td>95.39</td>
<td>97.59</td>
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<tr>
<td>TFP</td>
<td>97.39</td>
<td>92.55</td>
<td>97.40</td>
<td>98.43</td>
<td>98.46</td>
</tr>
<tr>
<td>Y / h</td>
<td>98.52</td>
<td>93.36</td>
<td>98.67</td>
<td>97.70</td>
<td>99.25</td>
</tr>
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</table>
## Disconnect Between the Short Run and the Long Run

<table>
<thead>
<tr>
<th></th>
<th>$u$</th>
<th>$Y$</th>
<th>$h$</th>
<th>$l$</th>
<th>$C$</th>
<th>TFP</th>
<th>$Y/h$</th>
</tr>
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<tbody>
<tr>
<td>MBC shock $\rightarrow$ Long Run</td>
<td>20.83</td>
<td>4.64</td>
<td>5.45</td>
<td>5.16</td>
<td>4.13</td>
<td>4.09</td>
<td>3.88</td>
</tr>
<tr>
<td>LR TFP shock $\rightarrow$ Short Run</td>
<td>9.63</td>
<td>24.78</td>
<td>11.01</td>
<td>17.56</td>
<td>15.58</td>
<td>22.01</td>
<td>21.89</td>
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</table>

MBC shock $\rightarrow$ TFP at different horizons

![TFP chart](chart.png)
MBC Shock: Main Properties and Prelim Lessons

- Explains bulk of BC volatility in key quantities
- Realistic business cycle, with $u$, $h$, $Y$, $I$, $C$ moving in tandem
- Interchangeability: same IRFs regardless of target
  - support for parsimonious theories
- $\approx 0$ comovement with TFP at BC frequencies
  - rules out technology and financial, uncertainty or other shocks that map to TFP fluctuations
- $\approx 0$ footprint on the Long Run (and conversely LR has small footprint on BC)
  - hard to reconcile with Beaudry & Portier (2006)
- Disconnect from inflation (coming soon)
More on News Shocks: a Semi-structural Exercise

- Could it be that disconnect between SR and LR reflects offsetting effects of (i) expansionary news shocks and (ii) contractionary unanticipated shocks?
- Semi-structural exercise using our anatomy:
  - recover these two shocks from reduced-form shocks that drive TFP in SR and LR
- Explore sensitivity to VAR size

<table>
<thead>
<tr>
<th>VAR 1</th>
<th>VAR 2</th>
<th>VAR 3</th>
<th>VAR 4</th>
<th>VAR 5</th>
<th>VAR 6</th>
<th>VAR 7</th>
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<tbody>
<tr>
<td>0</td>
<td>20</td>
<td>40</td>
<td>60</td>
<td>80</td>
<td>100</td>
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</table>

\[ VAR_1 = \{ u, \text{TFP} \}, \quad VAR_2 = VAR_1 \cup \{ I \}, \quad VAR_3 = VAR_2 \cup \{ Y, C, h \}, \quad VAR_4 = \text{Baseline VAR}, \quad VAR_5 = VAR_4 \cup \{ \text{SP} \_{500} \}, \quad VAR_6 = VAR_5 \cup \{ \text{utilization} \}, \quad VAR_7 = VAR_6 \cup \{ \text{credit spread} \}. \]
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Variance Contribution of News Shock to Unemployment

\[ \text{VAR}_1 = \{u, \text{TFP}\}, \quad \text{VAR}_2 = \text{VAR}_1 \cup \{I\}, \quad \text{VAR}_3 = \text{VAR}_2 \cup \{Y, C, h\}, \quad \text{VAR}_4 = \text{Baseline VAR}, \]
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MBC Shock: Robustness

Robust to

- More lags, VECM
- Varying the sample: Post vs Pre-Volcker era, w/o Great Recession/ZLB ...
- Adding variables: $SP$, $P^i/P^C$, financial variables ...
- ...
- Shifting to time domain rather than frequency domain
### MBC Shock: Robustness

#### Short-Run Variance Contributions

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<tr>
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<td>[1] Benchmark</td>
<td>73.71</td>
<td>58.51</td>
<td>47.72</td>
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<td>20.38</td>
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<td>[4] VECM(2)</td>
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<td>54.99</td>
<td>48.82</td>
<td>53.78</td>
<td>44.93</td>
<td>12.17</td>
<td>19.51</td>
<td>29.71</td>
<td>11.29</td>
<td>19.51</td>
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<tr>
<td>[7] pre-Volcker</td>
<td>74.23</td>
<td>56.75</td>
<td>43.21</td>
<td>61.50</td>
<td>23.43</td>
<td>6.82</td>
<td>30.69</td>
<td>28.43</td>
<td>17.45</td>
<td>27.60</td>
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<td>[8] post-Volcker</td>
<td>73.39</td>
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<td>[10] Financial</td>
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<td>57.56</td>
<td>46.84</td>
<td>59.95</td>
<td>25.94</td>
<td>7.04</td>
<td>27.20</td>
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<td>[11] Chained-Type C&amp;I</td>
<td>81.41</td>
<td>59.04</td>
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### Long-Run Variance Contributions

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<td>$Wh/Y$</td>
<td>$\pi$</td>
<td>$R$</td>
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MBC Shock: Robustness

Robustness of IRFs

Baseline; VECM₁; 1960-2007; Extended; Financial
MBC as a Demand Shock along a Philips curve?

Challenge #1: tiny signal-to-noise ratio (negligible $R^2$)

<table>
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<tr>
<th>Target</th>
<th>$u$</th>
<th>$\pi$</th>
<th>$Wh/Y$</th>
</tr>
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<tbody>
<tr>
<td>Unemployment</td>
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<td>27.02</td>
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Challenge #2: magnitude

Actual inflation response; --- Predicted, textbook NKPC.
Supports parsimonious models with dominant shock/propagation mechanism

Rules out following candidates for that role

- technology shocks
- financial, uncertainty, or other shocks that map to TFP fluctuations
- news about future TFP
- inflationary demand shocks of textbook variety

Remaining possibilities

- demand shocks of DSGE variety (extremely flat Philips curve)
- demand shocks without sticky prices/Philips curves
- ...
Evaluating DSGE Models
Evaluating Two DSGE Models

- **JPT** (Justiniano, Primiceri & Tambalotti, 2010)
  - Same as CEE, SW (but estimation more suitable for our purposes)
  - Sticky prices, Sticky wages, Monetary Policy
  - Standard Bells and Whistles (Habit, Invt Adj Costs, Utilization)
  - Multiple shocks (but 1 shock is most important)

- **ACD** (Angeletos, Collard & Dellas, 2018)
  - RBC with variation in “confidence”
  - Waves of optimism and pessimism about SR economic outlook
  - Example of literature on demand shocks without sticky prices/Philips curves

Q: Do these models match MBC template form the data?
A: Only second meets interchangeability property
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- **Q**: Do these models match MBC template form the data?
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JPT vs ACD: Interchangeability of MBC Facets

MBC facets interchangeable in ACD model (as in data), less so in JPT

⇒ JPT/CEE/SW lacks the “right” propagation mechanism

Note: “factors” refer to reduced-form shocks recovered via our approach, “shocks” to theoretical shocks.
JPT and ACD: Interchangeability of MBC Facets

- Measure of Interchangeability: \( D_v = \frac{1}{4} \sum_{f \in F} \sqrt{\sum_{k=0}^{20} (Z_{v,k}^f - \bar{Z}_{v,k})^2} \)

- Smaller numbers mean more interchangeability

<table>
<thead>
<tr>
<th></th>
<th>Y</th>
<th>C</th>
<th>I</th>
<th>h</th>
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<td>0.52</td>
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<td>ACD</td>
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<td>0.56</td>
<td>1.56</td>
<td>0.22</td>
<td>0.75</td>
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</table>

- Ranking robust to re-estimating both models on the basis of our factors
### Contribution of Theoretical Shocks to Factors

<table>
<thead>
<tr>
<th>Factor</th>
<th>JPT</th>
<th>ACD</th>
</tr>
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<tbody>
<tr>
<td>Y</td>
<td>31%</td>
<td>88%</td>
</tr>
<tr>
<td>I</td>
<td>66%</td>
<td>12%</td>
</tr>
<tr>
<td>C</td>
<td>1%</td>
<td>99%</td>
</tr>
<tr>
<td>h</td>
<td>2%</td>
<td>93%</td>
</tr>
</tbody>
</table>

In JPT, “A shock” a permanent technology shock, “I shock” a transitory investment-specific demand shock, “C shock” a transitory discount-factor; “other” include monetary policy, price, wage markup shocks. In ACD, “beliefs” a transitory shock to higher-order beliefs; “other” include both transitory and permanent technology shocks, news shocks, and I and C shocks of JPT.

- JPT and many other DSGE models: **specialized shocks** ⇒ poor interchangeability
- ACD: **“shotgun” shock** ⇒ great interchangeability
JPT (and many other models): No individual shock resembles the MBC shock in the data;
ACD: the confidence shock does

- needless to say, this doesn’t mean that ours is the “right” model
- but illustrates what the current paradigm misses and what it takes to match MBC template
(a) Data (1985-2011)
Bonus: Christiano, Motto & Rostagno (2014)

(a) Data (1985-2011)

(b) Christiano, Motto & Rostagno (2014)

- Interchangeability: great in terms of $Y$, $h$, $I$, worse in terms of $C$
- Real-financial nexus: misses dynamics of credit spread and credit level
Summary

- Simple and flexible method for dissecting the macroeconomic dynamics
- Supports hypothesis of dominant propagation mechanism
- Provides an empirical template for it ⇒ looks like a non-inflationary AD shock
- Detects defects in propagation dynamics of DSGE models fitted to the data
- Perhaps resolution rests on accommodating demand-driven cycles even without sticky prices
## Business-Cycle Moments

<table>
<thead>
<tr>
<th></th>
<th>Data</th>
<th>Experiment 1</th>
<th>Experiment 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\text{st.dev}(y_t))</td>
<td>1.41</td>
<td>1.39</td>
<td>1.01</td>
</tr>
<tr>
<td>(\text{st.dev}(\pi_t))</td>
<td>0.21</td>
<td>0.30</td>
<td>0.25</td>
</tr>
<tr>
<td>(\text{corr}(y_t, y_{t-1}))</td>
<td>0.92</td>
<td>0.91</td>
<td>0.89</td>
</tr>
<tr>
<td>(\text{corr}(y_t, y_{t-2}))</td>
<td>0.70</td>
<td>0.67</td>
<td>0.61</td>
</tr>
<tr>
<td>(\text{corr}(\pi_t, \pi_{t-1}))</td>
<td>0.91</td>
<td>0.89</td>
<td>0.86</td>
</tr>
<tr>
<td>(\text{corr}(\pi_t, \pi_{t-2}))</td>
<td>0.67</td>
<td>0.61</td>
<td>0.49</td>
</tr>
<tr>
<td>(\text{corr}(y_t, \pi_{t-2}))</td>
<td>-0.11</td>
<td>0.11</td>
<td>-0.08</td>
</tr>
<tr>
<td>(\text{corr}(y_t, \pi_{t-1}))</td>
<td>0.06</td>
<td>0.18</td>
<td>-0.15</td>
</tr>
<tr>
<td>(\text{corr}(y_t, \pi_t))</td>
<td>0.22</td>
<td>0.22</td>
<td>-0.17</td>
</tr>
<tr>
<td>(\text{corr}(y_t, \pi_{t+1}))</td>
<td>0.34</td>
<td>0.20</td>
<td>-0.13</td>
</tr>
<tr>
<td>(\text{corr}(y_t, \pi_{t+2}))</td>
<td>0.43</td>
<td>0.13</td>
<td>-0.07</td>
</tr>
</tbody>
</table>

Moments obtained from bandpass-filtered series (6-32 Quarters). The two model-based experiments are those described in the text.
Technicalities

- Consider the VAR
  \[ A(L)X_t = u_t, \]
  with \( A(L) \equiv \sum_{\tau=0}^{p} A_{\tau}L^{\tau}, \quad A(0) = I \) and \( \mathbb{E}(u_t u_t') = \Sigma; \)

- Orthogonalize the residuals as \( u_t = S\varepsilon_t \) where \( \mathbb{E}(\varepsilon_t \varepsilon_t') = I; \)

- Rewrite \( S \) as \( S = \tilde{S}Q \), where \( \tilde{S} \) is the Cholesky decomposition of \( \Sigma \), and \( Q \) is an orthonormal matrix \( (QQ' = I) \)

  \[ \implies \varepsilon_t = S^{-1}u_t = Q'\tilde{S}^{-1}u_t \]

  \[ \implies \text{Each } \varepsilon_t \text{ is associated to a column of } Q. \]
Technicalities

- Let us write the $VMA(\infty)$ representation of the VAR
  \[ X_t = B(L)u_t \]
  where $B(L) = A(L)^{-1}$ is an infinite matrix polynomial of the form $B(L) = \sum_{\tau=0}^{\infty} B_{\tau} L^\tau$.
- Replace \( u_t = \tilde{S} Q \epsilon_t \),
  \[ X_t = C(L)Q\epsilon_t = \Gamma(L)\epsilon_t, \]
  where $C(L) = B(L)\tilde{S}$ and $\Gamma(L) = C(L)Q$ are infinite matrix polynomials.
- The contribution of shock $j$ to the spectral density of variable $k$ over the frequency band $[\omega, \bar{\omega}]$ is given by
  \[ \Upsilon(q; k, \omega, \bar{\omega}) \equiv \int_{\omega \in [\omega, \bar{\omega}]} \left( \overline{C[k](e^{-i\omega})q} \ C[k](e^{-i\omega})q \right) d\omega = q' \left( \int_{\omega \in [\omega, \bar{\omega}]} \overline{C[k](e^{-i\omega})} C[k](e^{-i\omega})d\omega \right) \]
  \( q \) is then determined by maximizing the latter quantity $\implies$ Standard eigenvalue problem.