Motivation (I)

- Recent economic recession has reopened the debate on industrial policy.
- In October 2008, the US government bailed out GM and Chrysler. (Estimated cost, $82 Billion)
- Similar bailouts in Europe: Estimated cost €1.18 trillion in 2010, 9.6% of EU GDP.
- Many think that this was a success from a short-term perspective, because these interventions
  - protected employment, and
  - encouraged incumbents to undertake greater investments,
More generally, what are the implications of “industrial policy” for R&D, reallocation, productivity growth, and welfare?

Bailouts or support for incumbents could increase growth if there is insufficient entry or if they support incumbent R&D.

- In fact, this is recently been articulated as an argument for industrial policy.

They may reduce growth by

- preventing the entry of more efficient firms and
  - slowing down the reallocation process.

Reallocation potentially important, estimated sometimes to be responsible for up to 70-80% of US productivity growth.
Motivation (III)

- What’s the right framework?
  1. endogenous technology and R&D choices,
  2. rich from dynamics to allow for realistic reallocation and matched the data (and for selection effects),
  3. different types of policies (subsidies to operation vs R&D),
  4. general equilibrium structure (for the reallocation aspect),
  5. exit for less productive firms/products (so that the role of subsidies that directly or indirectly prevent exit can be studied).

- Starting point: Klette and Kortum’s (2004) model of micro innovation building up to macro structure.
Motivating Facts

- R&D intensity is independent of firm size.
- The size distribution of firms is highly skewed.
- Smaller firms have a lower probability of survival, but those that survive tend to grow faster than larger firms. Among larger firms, growth rates are unrelated to past growth or to firm size.
- Younger firms have a higher probability of exiting, but those that survive tend to grow faster than older firms.
- Gibrat’s law holds approximately (but not exactly): firm growth rate roughly independent of size, though notable deviations from this at the top and the bottom.
Model I

- Representative household maximizes

\[ U = \max \int_0^\infty e^{-\rho t} \log C_t \, dt \]

- All expenses are in terms of labor. Hence \( C_t = Y_t \).

- The household owns all the firms including potential entrants. Therefore the total income is

\[ Y_t = w_t L + r_t A_t \]

where \( A \) is the total asset holdings and \( r_t \) is the rate of return on these assets.
Model II

- Final good production

\[
\ln Y_t = \int_0^1 \ln y_{jt} \, dj
\]

- \(y_j\): quantity of intermediate \(j\)

- A fixed mass \(L\) of labor

\[
L_P + S_E + S_I = L
\]

- \(L_P\): production
- \(S_E\): scientists working for outsiders
- \(S_I\): scientists working for incumbent firms.

- All workers receive \(w_t\)

- Normalize the price of the final good to 1.
A firm is defined as a **collection of product lines.**

**Figure 3: Example of a Firm**
Profits II

- $n$ will denote the number of product lines that the firm operates.
- Each intermediate is produced with a linear technology

$$y_{jt} = A_{jt} l_{jt}$$

- This implies that the marginal cost is

$$w_t / A_{jt}$$

where $w_t$ is the wage rate in the economy at time $t$.
- Innovations in each product line improves the productivity by $\lambda > 0$ such that

$$A_{jt+\Delta t} = \begin{cases} 
(1 + \lambda) A_{jt} & \text{if successful innovation} \\
A_{jt} & \text{otherwise}
\end{cases}$$
Bertrand competition $\implies$ previous innovator will charge at least her marginal cost: 
$$\frac{(1+\lambda)w_t}{A_{jt}}.$$ 

Hence the latest innovator will charge the marginal cost of the previous innovator

$$p_{jt} = \frac{(1 + \lambda)w_t}{A_{jt}}.$$ 

Recall that the expenditure on each variety is $Y_t$ (since $P_t = 1$). 

Then the profit is

$$\pi_j = y_j (p_j - MC_j)$$

$$= \frac{A_{jt} Y_t}{(1 + \lambda) w_t} \left( \frac{(1 + \lambda) w_t}{A_{jt}} - \frac{w_t}{A_{jt}} \right)$$

$$= \pi Y_t$$

where $\pi \equiv \frac{\lambda}{1+\lambda}$. 

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Innovations are undirected across product lines.

Innovation technology

\[ X_i = \left( \frac{S_i}{\zeta} \right)^{1-\gamma} n^\gamma \]

where \( \gamma < 1 \), \( X_i \) is the innovation flow rate, \( S_i \) is the amount of R&D workers, \( n \) is the number of product lines to proxy for the firm specific (non-transferable, non-tradable) knowledge stock.
Alternatively, the cost of innovation:

\[ C(X, n) = wS_i \]

\[ = \zeta wn \left[ \frac{X_i}{n} \right]^{\frac{1}{1-\gamma}} \]

\[ = \zeta wn x_i^{\frac{1}{1-\gamma}} \]

where \( x_i \equiv X_i / n \) is the innovation intensity (per product line).

Let \( x \) denote the aggregate innovation rate in the economy.

Innovation rate by entrants is \( x_e \).

Aggregate innovation rate is

\[ \tau = x_i + x_e. \]
When a firm is successful in its current R&D investment, it innovates over a random product line $j' \in [0, 1]$.

1. Then, the productivity in line $j'$ increases from $A_{j'}$ to $(1 + \lambda)A_{j'}$.
2. The firm becomes the new monopoly producer in line $j'$ and thereby increases the number of its production lines to $n + 1$.

At the same time, each of its $n$ current production lines is subject to the creative destruction $\tau$ by new entrants and other incumbents.

Therefore during a small time interval $dt$,

1. the number of production units of a firm increases to $n + 1$ with probability $X_i dt$, and
2. decreases to $n - 1$ with probability $n \tau dt$.

A firm that loses all of its product lines exits the economy.
From Micro to Macro Innovation: Klette-Kortum

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Relevant firm-level state variable: number of products in which the firm has the leading-edge technology, $n$.

Then the value function of a firm as a function of $n$ is

$$\dot{V}_t(n) = \max_{x_i \geq 0} \left\{ n\pi_t - w_t \zeta n^{\frac{1}{1-\gamma}} + nx_i [V_t(n+1) - V_t(n)] + n\tau [V_t(n-1) - V_t(n)] \right\}$$

This can be rewritten as

$$\rho v = \pi - \tau v + \max_{x_i \geq 0} \left\{ x_i v - \omega \zeta x_i^{\frac{1}{1-\gamma}} \right\}$$

where $v \equiv V_t(n)/nY_t$ is normalized per product value and $\omega \equiv w_t/Y_t$ is the labor share and constant in steady state.
Value Function II

- First-order condition of R&D choice gives:
  \[ x_i = \left( \frac{\nu}{\eta \zeta \omega} \right)^{\frac{1-\gamma}{\gamma}}. \] (1)

- Or substituting it back:
  \[ \nu = \frac{\pi - \zeta \omega x_i^{\frac{1}{1-\gamma}}}{\rho + \tau - x_i}. \] (2)
Proposition  
Per-product line value of a firm $v$ can be expressed as a sum of production value $v_P$ and R&D “innovation option” value $v_R$:

$$v = v_P + v_R$$

where

$$v_p = \frac{\pi}{\rho + \tau}$$

and

$$v_R = \frac{1}{(\rho + \tau)} \max_{x_i \geq 0} \left\{ x_i (v_R + v_P) - \omega \zeta x_i^{\frac{1}{1-\gamma}} \right\}.$$
Entry I

- A mass of potential entrants.
- In order to generate 1 unit of arrival, entrants must hire a team of $\psi$ researchers, i.e., production function for entrant R&D is
  \[ x_e = \frac{S_E}{\psi}. \]
- The free-entry condition equates the value of a new entry $V_t(1)$ to the cost of innovation $\psi w_t$ such that
  \[ v = \omega \psi. \]
- Thus, together with (1) and (2):
  \[ x_e = \frac{\pi}{\omega \psi} - (1 - \gamma) \left( \frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} - \rho \quad \text{and} \quad x_i = \left( \frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}}. \]


**Labor Market Clearing I**

- **Production workers**

\[ L_P = \frac{Y_t}{A_j p_j} = \frac{1}{(1 + \lambda) \omega} \]

- **Incumbent R&D workers**

\[ S_I = \zeta \left( \frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} \]

- **Entrant R&D workers**

\[ S_E = \frac{\pi}{\omega} - \zeta \left( \frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} - \psi \rho \]
Therefore labor market clearing determines the normalized wage rate

\[
L = \frac{1}{(1 + \lambda)\omega} + \zeta \left( \frac{(1 - \gamma)\psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} \\
+ \frac{\pi}{\omega} - \zeta \left( \frac{(1 - \gamma)\psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} - \psi\rho \\
\implies \\
\omega = \frac{1}{L + \rho\psi}
\]
Recall the final good production function

\[
\ln Y_t = \int_0^1 \ln y_{jt} \, dj
\]

\[
= \int_0^1 \ln A_{jt} l_{jt} \, dj
\]

\[
= \ln \frac{Y_t}{(1 + \lambda) w_t} + \int_0^1 \ln A_{jt} \, dj
\]

\[
= \ln \frac{L + \rho \psi}{1 + \lambda} + \int_0^1 \ln A_{jt} \, dj
\]
Define

\[ Q_t \equiv \exp \left( \int_0^1 \ln A_{jt} \, dj \right) \]

\[ \ln Q_t \equiv \int_0^1 \ln A_{jt} \, dj \]

Thus

\[ g = \frac{\dot{C_t}}{C_t} = \frac{\dot{Q_t}}{Q_t} \]
Moreover

\[
\ln Q_{t+\Delta t} = \int_0^1 \left[ \tau \Delta t \ln(1 + \lambda) A_{jt} + (1 - \tau \Delta t) \ln A_{jt} \right] dj + o(\Delta t)
\]

\[
= \tau \Delta t \ln(1 + \lambda) + \ln Q_t + o(\Delta t)
\]

\[\iff\]

\[g = \tau \ln(1 + \lambda)\]

Hence

\[
g = \left[ \left( \frac{\lambda}{1 + \lambda} \right) \frac{L}{\psi} + \frac{1 - \gamma}{\gamma} \left( \frac{(1 - \gamma) \psi}{\zeta} \right)^{\frac{1-\gamma}{\gamma}} - \frac{\rho}{1 + \lambda} \right] \ln(1 + \lambda)
\]
Moments

- Consider a firm with \( n \) product lines. The “approximate” growth rate is

\[
\dot{n}_t = \frac{n}{n_t} \Delta t = x_i - \tau
\]

- R&D spending/intensity

\[
\frac{R&D}{Sales} = \frac{\zeta wnx_i^{1-\gamma}}{n} = \zeta wx_i^{1-\gamma}
\]

- Both of these are independent of firm size (consistent with “Gibrat’s law”).
Firm Size Distribution

- Firm size distribution: fraction of firms with $n$ leading-edge products, $\mu_n$, given by:

\[
\begin{align*}
\text{Outflow} & \quad \mu_1 \tau = x_e \\
\text{Entry & exit:} & \\
\text{1-product:} & (x_i + \tau) \mu_1 = \mu_2 2\tau + x_e \\
\text{n-product:} & (x_i + \tau) n\mu_n = \mu_{n+1} (n+1) \tau + \mu_{n-1} (n-1) x_i
\end{align*}
\]

- This implies the following simple firm size distribution:

\[
\begin{align*}
\mu_1 &= \frac{x_e}{\tau} \\
\mu_2 &= \frac{x_e}{2\tau^2} x_i \\
\mu_3 &= \frac{x_e x_i}{3\tau^3} \\
\cdots &= \cdots \\
\mu_n &= \frac{x_e x_i}{n\tau^n}
\end{align*}
\]
What’s Missing?

- A nice and tractable model, but:
  - *no reallocation* (all firms that previously in equilibrium are equally good at using all factors of production);
  - *no endogenous exit* of less productive firms;
  - limited heterogeneity (see next slide).

- All of these together imply very little room for endogenous selection which could be impacted by policy.
- We now consider a model that extended this framework to introduce these features.
Why Heterogeneity Matters

1A: Transition Rates

1B: R&D Intensity

1C: Sales Growth

1D: Employment Growth
Baseline Model: Preferences

- Simplified model (abstracting from heterogeneity and non-R&D growth).
- Infinite-horizon economy in continuous time.
- Representative household:

\[
U = \int_0^\infty \exp\left(-\rho t\right) \frac{C(t)^{1-\theta} - 1}{1-\theta} dt.
\]

- Inelastic labor supply, no occupational choice:
  - Unskilled for production: measure 1, earns \(w^u\)
  - Skilled for R&D: measure \(L\), earns \(w^s\).

- Hence the budget constraint is

\[
C(t) + \dot{A}(t) \leq w^u(t) + w^s(t) \cdot L + r(t) \cdot A(t)
\]

- Closed economy and no investment, resource constraint:

\[
Y(t) = C(t).
\]
Final Good Technology

- Unique final good $Y$:

$$Y = \left( \int_{\mathcal{N}} y_j \frac{\varepsilon - 1}{\varepsilon} \, dj \right)^{\frac{\varepsilon}{\varepsilon - 1}}.$$

- $\mathcal{N} \subset [0, 1]$ is the set of *active* product lines.
- The measure of $\mathcal{N}$ is less than 1 due to
  1. exogenous destructive shock
  2. obsolescence
Intermediate Good Technology

- As usual, each intermediate good is produced by a **monopolist**:
  \[ y_{j,f} = q_{j,f} l_{j,f}, \]
  
  \(q_{j,f} : \) worker productivity, \(l_{j,f} : \) number of workers.
- Marginal cost:
  \[ MC_{j,f} = \frac{w^u}{q_{j,f}}. \]
- Fixed cost of production, \(\phi\) in terms of skilled labor.
- Total cost
  \[ TC_{j,f} (y_{j,f}) = w^s \phi + w^u \frac{y_{j,f}}{q_{j,f}}. \]
Definition of a Firm

- A firm is defined as a collection of product qualities as in Klette-Kortum

\[
\text{Firm } f = Q_f \equiv \{ q_f^1, q_f^2, \ldots, q_f^n \}.
\]

\( n_f \equiv |Q_f| : \text{is the number of product lines of firm } f. \)
Relative Quality

- Define *aggregate quality* as

\[ Q \equiv \left( \int_{\mathcal{N}} q_j^{\varepsilon-1} \, dj \right)^{\frac{1}{\varepsilon-1}}. \]

- In equilibrium,

\[ Y = C = Q, \]

- Define *relative quality*:

\[ \hat{q}_j \equiv \frac{q_j}{w^u}. \]
R&D and Innovation

- Innovations follow a “controlled” Poisson Process

\[ X_f = n_f^\gamma h_f^{1-\gamma}. \]

- \( X_f \): flow rate of innovation
- \( n_f \): number of product lines.
- \( h_f \): number of researchers (here taken to be regular workers allocated to research).

- This can be rewritten as per product innovation at the rate

\[ x_f \equiv \frac{X_f}{n_f} = \left( \frac{h_f}{n_f} \right)^{1-\gamma}. \]

- Cost of R&D as a function of per product innovation rate \( x_f \):

\[ w^s G(x_f) \equiv w^s n_f x_f^{\frac{1}{1-\gamma}}. \]
Innovation by Existing Firms

- Innovations are again *undirected* across product lines.
- Upon an innovation:
  1. firm $f$ acquires another product line $j$
  2. if technology in $j$ is active:
     \[
     q(j, t + \Delta t) = (1 + \lambda) q(j, t).
     \]
  3. if technology in $j$ is not active, i.e., $j \notin \mathcal{N}$, a new technology is drawn from the steady-state distribution of relative quality, $F(\hat{q})$.  

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Entry and Exit

- A set of potential entrants invest in R&D.
- Exit happens in three ways:
  1. **Creative destruction.** Firm $f$ will lose each of its products at the rate $\tau > 0$ which will be determined endogenously in the economy.
  2. **Obsolescence.** Relative quality decreases due to the increase in the wage rate, at some point leading to exit.
  3. **Exogenous destructive shock** at the rate $\varphi$. 
Static Equilibrium

- Drop the time subscripts.
- Isoelastic demands imply the following monopoly price and quantity

\[ p_{j,f}^* = \left( \frac{\epsilon}{\epsilon - 1} \right) \frac{1}{\hat{q}_j} \quad \text{and} \quad c_{j}^* = \left( \frac{\epsilon - 1}{\epsilon} \hat{q}_j \right)^\epsilon Y \]

- Gross equilibrium (before fixed costs) profits from a product with relative quality \( \hat{q}_j \) are:

\[ \pi (\hat{q}_{j,f}) = \hat{q}_j^{\epsilon-1} \left( \frac{(\epsilon - 1)^{\epsilon-1}}{\epsilon^\epsilon} \right) Y. \]
\hat{q} = \frac{q}{w}$
\[ \hat{q} = \frac{q}{w} \]
\[ \hat{q} = \frac{q}{w} \uparrow \]
Without a fixed cost

\[ \hat{q} = \frac{q}{w} \]
With fixed cost $\phi > 0$

$$\hat{q} = \frac{q}{w}$$

Exit

$\hat{q}_{\text{min}}$
Simplified Model

Implications for Firm Size Distribution

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In equilibrium, 

\[ Y = C = Q \]

and

\[ w^u = \frac{\varepsilon - 1}{\varepsilon} Q. \]

Let us also define *normalized values* as

\[ \tilde{V} \equiv \frac{V}{Y}, \quad \tilde{\pi} (\hat{q}_j, f) = \frac{\pi (\hat{q}_j, f)}{Y}, \quad \tilde{w}^u \equiv \frac{w^u}{Y} \quad \text{and} \quad \tilde{w}^s \equiv \frac{w^s}{Y}. \]
Dynamic Equilibrium (continued)

\[ r^* \tilde{V}(\hat{Q}_f) = \left[ \sum_{\hat{q}_{j,f} \in \hat{Q}_f} \right] \max_{x_f} \left\{ \begin{array}{c} \tilde{\pi}(\hat{q}_{j_f}) - \tilde{w}^s \phi_j \\ + \tilde{\pi} \left( \hat{Q}_f \setminus \{ \hat{q}_{j_f} \} \right) - \tilde{V}(\hat{Q}_f) \\ + \tau \left[ \tilde{V}(\hat{Q}_f \setminus \{ \hat{q}_{j_f} \}) - \tilde{V}(\hat{Q}_f) \right] \\ - \tilde{w} G(x_f) \\ + x_f \left[ E_{\hat{q}} \tilde{V}(\hat{Q}_f \cup (1 + \lambda) \hat{q}_{j',f}) - \tilde{V}(\hat{Q}_f) \right] \\ + \phi \left[ 0 - \tilde{V}(\hat{Q}_f) \right] \end{array} \right\} \]

\[ \tau: \text{creative destruction rate in the economy.} \]
Dynamic Equilibrium (continued)

\[ r^* \tilde{V}(\hat{Q}_f) = \sum_{\hat{q}_{j,f} \in \hat{Q}_f} \left\{ \tilde{\pi}(\hat{q}_{jf}) - \tilde{w}^s \phi_j + \tau \left[ \tilde{V}(\hat{Q}_f \setminus \{\hat{q}_{jf}\}) - \tilde{V}(\hat{Q}_f) \right] \right\} + \max_{x_f} \left\{ \tilde{V}(\hat{Q}_f \cup (1 + \lambda) \hat{q}_{j',f}) - \tilde{V}(\hat{Q}_f) \right\} \]

\[ + \varphi \left[ 0 - \tilde{V}(\hat{Q}_f) \right] \]

\( \tau \): creative destruction rate in the economy.
Franchise and R&D Option Values

**Lemma** The normalized value can be written as the sum of franchise values:

\[ \tilde{V}(\hat{Q}_f) = \sum_{\hat{q} \in \hat{Q}_f} Y(\hat{q}), \]

where the franchise value of a product of relative quality \( \hat{q} \) is the solution to the differential equation (iff \( \hat{q} \geq \hat{q}_{\text{min}} \)):

\[ rY(\hat{q}) - \frac{\partial Y(\hat{q})}{\partial \hat{q}} \frac{\partial \hat{q}}{\partial w^u(t)} \frac{\partial w^u(t)}{\partial t} = \tilde{\pi}(\hat{q}) - \tilde{w}^u \phi + \Omega - (\tau + \phi) Y(\hat{q}), \]

where \( \Omega \) is the R&D option value of holding a product line,

\[ \Omega \equiv \max_{x_f \geq 0} \left\{ -\tilde{w}^s G(x_f) + x_f \left( \mathbb{E}_{\hat{q}} \tilde{V}(\hat{Q}_f \cup (1 + \lambda) \hat{q}_{j_f}) - \tilde{V}(\hat{Q}_f) \right) \right\}, \]

Moreover, exit follows a cut-off rule: \( \hat{q}_{\text{min}} \equiv \pi^{-1} (\tilde{w}^s \phi - \Omega) \).
Equilibrium Value Functions and R&D

Proposition

*Equilibrium normalized value functions are:*

\[
Y(\hat{q}) = \frac{\tilde{\tau}(\hat{q})}{r + \tau + \varphi + g(\varepsilon - 1)} \left[ 1 - \left( \frac{\hat{q}_{\text{min}}}{\hat{q}} \right)^{\frac{r + \tau + \varphi + g(\varepsilon - 1)}{g}} \right] 
\]

\[
= \frac{\Omega - \tilde{w}^s \phi}{r + \tau + \varphi} \left[ 1 - \left( \frac{\hat{q}_{\text{min}}}{\hat{q}} \right)^{\frac{r + \tau + \varphi}{g}} \right],
\]

*and equilibrium R&D is*

\[
x^*(\hat{q}) = x^* = \left[ \frac{(1 - \gamma) \mathbb{E}_q Y(\hat{q})}{\tilde{w}^s} \right]^{\frac{1 - \gamma}{\gamma}}.
\]
Entry

- Entry by outsiders can now be determined by the free entry condition:

\[
\max_{x^{\text{entry}} \geq 0} \left\{-w^s \phi + x^{\text{entry}} \mathbb{E} \mathcal{V}^{\text{entry}}(\hat{q}, \theta) - w^s G \left(x^{\text{entry}}, \theta^E\right)\right\} = 0
\]

where \( G \left(x^{\text{entry}}, \theta^E\right) \), as specified above, gives a number of skilled workers necessary for a firm to achieve an innovation rate of \( x^{\text{entry}} \) (with productivity parameter \( \theta^E \)).

- \( X^{\text{entry}} \equiv mx^{\text{entry}} \) is the total entry rate where
  - \( m \) is the equilibrium measure of entrants, and
  - \( x^{\text{entry}} \) innovation rate per entrant.
Labor Market Clearing

- Unskilled labor market clearing:
  \[ 1 = \int_{N(t)} l_j (w^u) \, dj. \]

- Skilled labor market clearing
  \[ L^s = \int_{N(t)} \left[ \phi + h (w^s) \right] dj + m \left[ \phi + G \left( x^{entry}, \theta^E \right) \right]. \]
Finally, we need to keep track of the distribution of relative quality → stationary equilibrium distribution of relative quality $F$.

This can be done by writing transition equations describing the density of relative quality.

These are more complicated than in Klette-Kortum because there is no strict Gibraltar’s law anymore.
Preferences and Technology in the General Model

- Same preferences.
- Introduce managerial quality affecting the rate of innovation of each firm.
- Some firms start as more innovative than others, over time some of them lose their innovativeness.
  - Young firms are potentially more innovative but also have a higher rate of failure.
- Introduce non-R&D growth (so as not to potentially exaggerate the role of R&D and capture potential advantages of incumbents).
Definition of a Firm

- A firm is again defined as a pair of technology set and “management quality” $\theta$:

$$\text{Firm } f \equiv (Q_f, \theta_f),$$

where

$$Q_f \equiv \{ q^1_f, q^2_f, \ldots, q^n_f \}.$$  

- $n_f \equiv |Q_f|$ : is the number of product lines owned by firm $f$. 

R&D and Innovation

- Innovations follow a controlled Poisson Process.
- Flow rate of innovation for leader and follower given by

\[ X_f = (n_f \theta_f)^\gamma h_f^{1-\gamma}. \]

- \( n_f \): number of product lines.
- \( \theta_f \): firm type (management quality).
- \( h_f \): number of researchers.
Innovation Realizations

**With R&D**

- Innovations are *undirected* within the industry.
- After a successful innovation, innovation is realized in a random product line $j$. Then:
  1. firm $f$ acquires product line $j$
  2. technology in line $j$ improves

\[
q(j, t + \Delta t) = (1 + \lambda) q(j, t).
\]

**Without R&D**

- Firms receive a product line for free at the rate $\varrho$. 

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\[ \hat{q} \sim F(\hat{q}) \]
Entry and Exit

- There is a measure of potential entrants.
- Successful innovators enter the market.
- At the time of initial entry, each firm draws a management quality $\theta$:

\[
\begin{align*}
\Pr (\theta = \theta^H) &= \alpha \\
\Pr (\theta = \theta^L) &= 1 - \alpha,
\end{align*}
\]

where $\alpha \in (0, 1)$ and $\theta^H > \theta^L > 0$.

- Exit happens in three ways as in the baseline model.
Maturity Shock

- Over time, high-type firms become low-type at the rate $\nu > 0$:
  \[ \theta^H \rightarrow \theta^L. \]

- Convenient to capture the possibility of once-innovative firms now being inefficient (and the use of skilled labor).
Equilibrium

- Equilibrium definition and characterization similar to before (with more involved value functions and stationary transition equations).
Data: LBD, Census of Manufacturing and NSF R&D Data

- Sample from combined databases from 1987 to 1997.
- Longitudinal Business Database (LBD)
  - Annual business registry of the US from 1976 onwards.
  - Universe of establishments, so entry/exit can be modeled.
- Census of Manufacturers (CM)
  - Detailed data on inputs and outputs every five years.
- NSF R&D Survey.
  - Firm-level survey of R&D expenditure, scientists, etc.
  - Surveys with certainty firms conducting $1m or more of R&D.
- USPTO patent data matched to CM.
- Focus on “continuously innovative firms”:
  - I.e., either R&D expenditures or patenting in the five-year window surrounding observation conditional on existence.
Data Features and Estimation

- 17,055 observations from 9835 firms.
- Accounts for 98% of industrial R&D.
- Relative to the universal CM, our sample contains over 40% of employment and 65% of sales.
- “Important” small firms also included:
  - of the new entrants or very small firms that later grew to have more than 10,000 employees or more than $1 billion of sales in 1997, we capture, respectively, 94% at 80%.
- We use Simulated Method of Moments on this dataset to estimate the parameters of the model.
Creating Moments from the Data

- We target 21 moments to estimate 12 parameters.
- Some of the moments are:
  - Firm entry/exit into/from the economy by age and size.
  - Firm size distribution.
  - Firm growth by age and size.
  - R&D intensity (R&D/Sales) by age and size.
  - Share of entrant firms.
RESULTS
## Table 1. Parameter Estimates

<table>
<thead>
<tr>
<th>#</th>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>$\varepsilon$</td>
<td>CES</td>
<td>1.701</td>
</tr>
<tr>
<td>2.</td>
<td>$\phi$</td>
<td>Fixed cost of operation</td>
<td>0.032</td>
</tr>
<tr>
<td>3.</td>
<td>$L^S$</td>
<td>Measure of high-skilled workers</td>
<td>0.078</td>
</tr>
<tr>
<td>4.</td>
<td>$\theta^H$</td>
<td>Innovative capacity of high-type firms</td>
<td>0.216</td>
</tr>
<tr>
<td>5.</td>
<td>$\theta^L$</td>
<td>Innovative capacity of low-type firms</td>
<td>0.070</td>
</tr>
<tr>
<td>6.</td>
<td>$\theta^E$</td>
<td>Innovative capacity of entrants</td>
<td>0.202</td>
</tr>
<tr>
<td>7.</td>
<td>$\alpha$</td>
<td>Probability of being high-type entrant</td>
<td>0.428</td>
</tr>
<tr>
<td>8.</td>
<td>$\nu$</td>
<td>Transition rate from high-type to low-type</td>
<td>0.095</td>
</tr>
<tr>
<td>9.</td>
<td>$\lambda$</td>
<td>Innovation step size</td>
<td>0.148</td>
</tr>
<tr>
<td>10.</td>
<td>$\gamma$</td>
<td>Innovation elasticity wrt knowledge stock</td>
<td>0.637</td>
</tr>
<tr>
<td>11.</td>
<td>$\varphi$</td>
<td>Exogenous destruction rate</td>
<td>0.016</td>
</tr>
<tr>
<td>12.</td>
<td>$\varrho$</td>
<td>Non-R&amp;D innovation arrival rate</td>
<td>0.012</td>
</tr>
</tbody>
</table>
### Table 2. Moment Matching

<table>
<thead>
<tr>
<th>#</th>
<th>Moments</th>
<th>model</th>
<th>data</th>
<th>#</th>
<th>Moments</th>
<th>model</th>
<th>data</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.</td>
<td>Firm Exit (small)</td>
<td>0.086</td>
<td>0.093</td>
<td>12.</td>
<td>Sales Gr. (small)</td>
<td>0.115</td>
<td>0.051</td>
</tr>
<tr>
<td>2.</td>
<td>Firm Exit (large)</td>
<td>0.060</td>
<td>0.041</td>
<td>13.</td>
<td>Sales Gr. (large)</td>
<td>-0.004</td>
<td>0.013</td>
</tr>
<tr>
<td>3.</td>
<td>Firm Exit (young)</td>
<td>0.078</td>
<td>0.102</td>
<td>14.</td>
<td>Sales Gr. (young)</td>
<td>0.070</td>
<td>0.071</td>
</tr>
<tr>
<td>4.</td>
<td>Firm Exit (old)</td>
<td>0.068</td>
<td>0.050</td>
<td>15.</td>
<td>Sales Gr. (old)</td>
<td>0.030</td>
<td>0.014</td>
</tr>
<tr>
<td>5.</td>
<td>Trans. large-small</td>
<td>0.024</td>
<td>0.008</td>
<td>16.</td>
<td>R&amp;D/Sales (small)</td>
<td>0.097</td>
<td>0.099</td>
</tr>
<tr>
<td>6.</td>
<td>Trans. small-large</td>
<td>0.019</td>
<td>0.019</td>
<td>17.</td>
<td>R&amp;D/Sales (large)</td>
<td>0.047</td>
<td>0.042</td>
</tr>
<tr>
<td>7.</td>
<td>Prob. small</td>
<td>0.539</td>
<td>0.715</td>
<td>18.</td>
<td>R&amp;D/Sales (young)</td>
<td>0.083</td>
<td>0.100</td>
</tr>
<tr>
<td>8.</td>
<td>Emp. Gr. (small)</td>
<td>0.063</td>
<td>0.051</td>
<td>19.</td>
<td>R&amp;D/Sales (old)</td>
<td>0.061</td>
<td>0.055</td>
</tr>
<tr>
<td>9.</td>
<td>Emp. Gr. (large)</td>
<td>-0.007</td>
<td>0.013</td>
<td>20.</td>
<td>5-year Ent. Share</td>
<td>0.363</td>
<td>0.393</td>
</tr>
<tr>
<td>10.</td>
<td>Emp. Gr. (young)</td>
<td>0.040</td>
<td>0.070</td>
<td>21.</td>
<td>Aggregate growth</td>
<td>0.022</td>
<td>0.022</td>
</tr>
<tr>
<td>11.</td>
<td>Emp. Gr. (old)</td>
<td>0.010</td>
<td>0.015</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
2A: Transition Rates

2B: R&D Intensity

2C: Sales Growth

2D: Employment Growth
Table 3: Non-targeted Moments

<table>
<thead>
<tr>
<th>Moments</th>
<th>Model</th>
<th>Data</th>
</tr>
</thead>
<tbody>
<tr>
<td>Corr(exit prob, R&amp;D intensity)</td>
<td>0.04</td>
<td>0.05</td>
</tr>
<tr>
<td>Exit prob of low-R&amp;D-intensive firms</td>
<td>0.36</td>
<td>0.32</td>
</tr>
<tr>
<td>Exit prob of high-R&amp;D-intensive firms</td>
<td>0.37</td>
<td>0.34</td>
</tr>
<tr>
<td>Corr(R&amp;D growth, emp growth)</td>
<td>0.48</td>
<td>0.19</td>
</tr>
<tr>
<td>Share firm growth due to R&amp;D</td>
<td>0.77</td>
<td>0.73</td>
</tr>
<tr>
<td>Ratio of top 7.2% to bottom 92.8% income</td>
<td>13.4</td>
<td>9.3</td>
</tr>
</tbody>
</table>
Comparison to Micro Estimates

- Estimates of the elasticity of patents (innovation) to R&D expenditures (e.g., Griliches, 1990):
  - [0.3, 0.6]
  - This corresponds to $1 - \gamma$, so a range of [0.4, 0.7] for $\gamma$.
  - Our estimate is in the middle of this range.

- Use IV estimates from R&D tax credits.
  - US spending about $2 billion with large cross-state over-time variation.
  - Literature estimates:
    $$\log(R&D_{i,t}) = \alpha_i + \beta_t + \gamma \log(R&D\_Cost\_of\_Capital_{i,t})$$

  - Bloom, Griffith and Van Reenen (2002) find -1.088 (0.024) on a cross-country panel. Similar estimates from Hall (1993), Baily and Lawrence (1995) and Mumuneas and Nadiri (1996).

  - In the model, $\ln R&D = \frac{\gamma-1}{\gamma} \ln (c_{R&D}) + \text{constant}$.
  - So approximately $\gamma \approx 0.5$, close to our estimate of $\gamma = 0.637$. 
## Baseline Results

### Table 4. Baseline Model

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,\text{min}}$</th>
<th>$\hat{q}_{h,\text{min}}$</th>
<th>$g$</th>
<th>Wel</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.46</td>
<td>2.80</td>
<td>9.58</td>
<td>73.6</td>
<td>71.16</td>
<td>24.53</td>
<td>13.90</td>
<td>0.00</td>
<td>2.24</td>
<td>100</td>
</tr>
</tbody>
</table>

Note: All numbers except wage ratio and welfare are in percentage terms.

- $g$: growth rate
- $x^{out}$: entry rate
- $x^{low}$: low-type invn rate
- $x^{high}$: high-type invn rate
- $\Phi^{low}$: fraction of low p. lines
- $\Phi^{high}$: fraction of high p. lines
- $\hat{q}_{l,\text{min}}$: low-type cutoff quality
- $\hat{q}_{h,\text{min}}$: high-type cutoff quality
- wel: welfare in cons equiv.
Relative Quality Distribution

Figure 3

- Explains why very little obsolescence of high-type products.
Policy Analysis: Subsidy to Incumbent R&D

**Table 4. Baseline Model**

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
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<td>13.90</td>
<td>0.00</td>
<td>2.24</td>
<td>100</td>
</tr>
</tbody>
</table>

- Use 1% and 5% of GDP, resp., to subsidize incumbents R&D:

**Table 5A. Incumbent R&D Subsidy ($s_i = 15\%$)**

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,\min}$</th>
<th>$\hat{q}_{h,\min}$</th>
<th>$g$</th>
<th>Wel</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.46</td>
<td>3.05</td>
<td>10.56</td>
<td>68.1</td>
<td>70.74</td>
<td>24.96</td>
<td>13.40</td>
<td>0.00</td>
<td>2.23</td>
<td>99.86</td>
</tr>
</tbody>
</table>

**Table 5B. Incumbent R&D Subsidy ($s_i = 39\%$)**

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,\min}$</th>
<th>$\hat{q}_{h,\min}$</th>
<th>$g$</th>
<th>Wel</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.46</td>
<td>3.61</td>
<td>13.04</td>
<td>49.8</td>
<td>69.58</td>
<td>25.97</td>
<td>13.15</td>
<td>0.00</td>
<td>2.16</td>
<td>98.48</td>
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</table>
Policy Analysis: Subsidy to the Operation of Incumbents

**Table 4. Baseline Model**

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
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<th>$\Phi^h$</th>
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<th>$\hat{q}_{h,\min}$</th>
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<tbody>
<tr>
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<td>9.58</td>
<td>73.6</td>
<td>71.16</td>
<td>24.53</td>
<td>13.90</td>
<td>0.00</td>
<td>2.24</td>
<td>100</td>
</tr>
</tbody>
</table>

- Use 1% of GDP to subsidize operation costs of incumbents:

**Table 6. Operation Subsidy ($s_o = 6\%$)**

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,\min}$</th>
<th>$\hat{q}_{h,\min}$</th>
<th>$g$</th>
<th>$\text{Wel}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.46</td>
<td>2.80</td>
<td>9.59</td>
<td>73.7</td>
<td>71.30</td>
<td>24.52</td>
<td>11.74</td>
<td>0.00</td>
<td>2.22</td>
<td>99.82</td>
</tr>
</tbody>
</table>

- Now an important negative selection effect.
Policy Analysis: Entry Subsidy and Selection

### Table 4. Baseline Model

<table>
<thead>
<tr>
<th>$x^{\text{entry}}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,\text{min}}$</th>
<th>$\hat{q}_{h,\text{min}}$</th>
<th>$g$</th>
<th>Wel</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.46</td>
<td>2.80</td>
<td>9.58</td>
<td>73.6</td>
<td>71.16</td>
<td>24.53</td>
<td>13.90</td>
<td>0.00</td>
<td>2.24</td>
<td>100</td>
</tr>
</tbody>
</table>

- Use 1% of GDP to subsidize entry:

### Table 7. Entry Subsidy ($s_e = 5\%$)

<table>
<thead>
<tr>
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<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
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<th>$\hat{q}_{h,\text{min}}$</th>
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<th>Wel</th>
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<tbody>
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<td>2.73</td>
<td>9.30</td>
<td>75.3</td>
<td>71.16</td>
<td>24.41</td>
<td>15.91</td>
<td>0.00</td>
<td>2.26</td>
<td>100.15</td>
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</table>
**Figure 4. Policy effect on Productivity Distributions**

**A. high type**

**B. low type**
### Social Planner’s Allocation

#### Table 4. Baseline Model

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,\text{min}}$</th>
<th>$\hat{q}_{h,\text{min}}$</th>
<th>$g$</th>
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<tbody>
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<td>9.58</td>
<td>73.6</td>
<td>71.16</td>
<td>24.53</td>
<td>13.90</td>
<td>0.00</td>
<td>2.24</td>
<td>100</td>
</tr>
</tbody>
</table>

- What would the social planner do (taking equilibrium markups as given)?

#### Table 8. Social Planner

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
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<th>$\hat{q}_{h,\text{min}}$</th>
<th>$g$</th>
<th>$\text{Wel}$</th>
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</thead>
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<td>10.47</td>
<td>80.9</td>
<td>54.06</td>
<td>27.76</td>
<td>118.6</td>
<td>1.02</td>
<td>3.80</td>
<td>106.5</td>
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</table>
**Optimal Policy (I)**

### Table 4. Baseline Model

<table>
<thead>
<tr>
<th>$x^{\text{entry}}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,\text{min}}$</th>
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<td>73.6</td>
<td>71.16</td>
<td>24.53</td>
<td>13.90</td>
<td>0.00</td>
<td>2.24</td>
<td>100</td>
</tr>
</tbody>
</table>

- Optimal mix of incumbent R&D subsidy, operation subsidy and entry subsidy:

### Table 9. Optimal Policy Analysis and Welfare

**Incumbent & Entry Policies ($s_i = 17\%$, $s_o = -246\%$, $s_e = 6\%$)**

<table>
<thead>
<tr>
<th>$x^{\text{entry}}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,\text{min}}$</th>
<th>$\hat{q}_{h,\text{min}}$</th>
<th>$g$</th>
<th>Wel</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.46</td>
<td>3.04</td>
<td>10.21</td>
<td>75.5</td>
<td>62.19</td>
<td>25.53</td>
<td>96.28</td>
<td>55.88</td>
<td>3.12</td>
<td>104.6</td>
</tr>
</tbody>
</table>
Optimal Policy (II)

Table 4. Baseline Model

<table>
<thead>
<tr>
<th>$x^{entry}$</th>
<th>$x^l$</th>
<th>$x^h$</th>
<th>$m$</th>
<th>$\Phi^l$</th>
<th>$\Phi^h$</th>
<th>$\hat{q}_{l,\text{min}}$</th>
<th>$\hat{q}_{h,\text{min}}$</th>
<th>$g$</th>
<th>Wel</th>
</tr>
</thead>
<tbody>
<tr>
<td>8.46</td>
<td>2.80</td>
<td>9.58</td>
<td>73.6</td>
<td>71.16</td>
<td>24.53</td>
<td>13.90</td>
<td>0.00</td>
<td>2.24</td>
<td>100</td>
</tr>
</tbody>
</table>

- Optimal mix of incumbent R&D subsidy and operation subsidy:

Table 9. Optimal Policy Analysis and Welfare

<table>
<thead>
<tr>
<th>Incumbent Policies ($s_i = 12%, s_o = -264%$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x^{entry}$</td>
</tr>
<tr>
<td>-------------</td>
</tr>
<tr>
<td>8.46</td>
</tr>
</tbody>
</table>
Summing up

- Industrial policy directed at incumbents has negative effects on innovation and productivity growth—though small.
- Subsidy to entrants has small positive effects.
- But not because R&D incentives are right in the laissez-faire equilibrium.
- The social planner can greatly improve over the equilibrium.
- Similar gains can also be achieved by using taxes on the continued operation of incumbents (plus small R&D subsidies).
  - This is useful for encouraging the exit of inefficient incumbents who are trapping skilled labor that can be more productively used by entrants and high-type incumbents.
These results are qualitatively and in fact quantitatively quite robust. The remain largely unchanged if:
- $\gamma = 0.5$.
- $\varphi = 0$.
- entry margin much less elastic.