Job Search with Debt Burden:

Aggregate Implications of Student Loans

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Abstract

This paper evaluates the implication of student loan debt on labor market outcomes. I begin by developing a tractable theoretical framework to analytically demonstrate that individuals under debt burden tend to search less and end up with lower-paid jobs. I then develop and estimate a quantitative search model with risk-averse agents, on-the-job search, and vacancy creation using NLSY97 data to evaluate the proposed mechanism. My model suggests that, under the standard fixed repayment plan, borrowers' consumption is reduced due to debt repayment and lower wage income. The latter indirect effect caused by inadequate job search is potentially larger and more persistent than the direct effect from debt repayment. The income-based repayment plan (IBR) alleviates this distortion; I analytically elucidate the channels and quantitatively evaluate the aggregate and distributional effects of IBR. The model implies that poorer and more indebted borrowers would benefit more from switching to IBR. On average, IBR alleviates the debt burden by about half, among which one-third is attributed to better job matches.

JEL codes: D61, D86, I22, I28, J31, J64.

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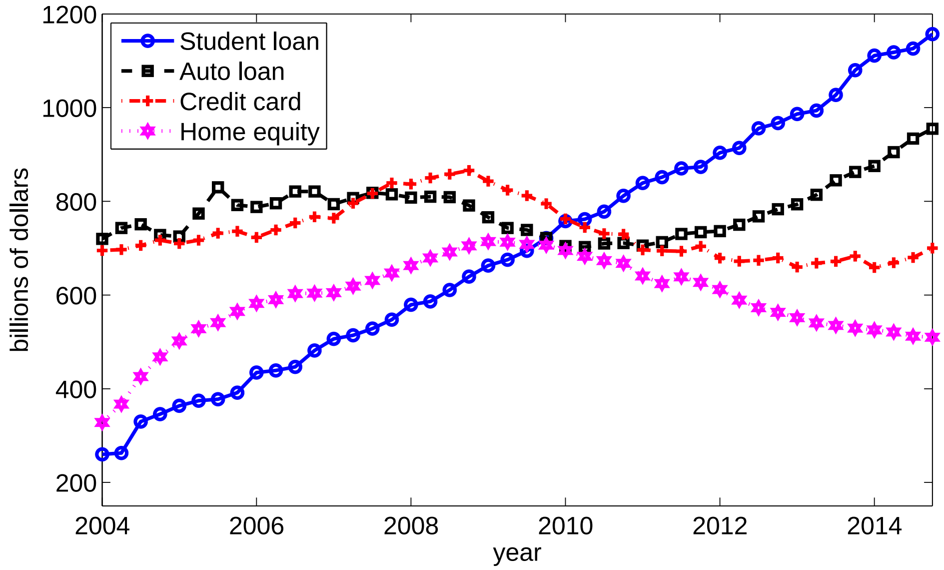
1 Introduction

In the U.S., millions of college students take loans every year, in hopes that their investment will pay back in the future. Unfortunately, it is not uncommon that education loans also leave them in financial hardship after graduation. The ballooning student loan debt over the past decade (see Figure 1) potentially generates a knock-on effect throughout the entire economy.[1] Under the standard fixed repayment plan, student loans are due when borrowers have the least capacity to pay. Although the income-based repayment plan had been proposed as a solution to the "student debt crisis", the side effect of this policy remains a source of debate.[2]



Source: Federal Reserve Bank of New York Consumer Credit Panel (a representative sample drawn from anonymized Equifax credit data).

Figure 1: Non-mortgage balances, 2004Q1-2014Q4.

Concerns about debt repayment presumably affect students' career paths by influencing their job search decisions after college.[3] In this paper, I shed light on the consequences of searching for jobs under the burden of debt. My empirical motivation is the observed correlation between student loan debt and initial labor market outcomes among college graduates of NLSY97. The descriptive evidence suggests that indebted students tend to have shorter initial unemployment duration and lower wage income after college. This effect remains significant after controlling for parental wealth, parental education, and individual test scores.

[1] As discussed by Gale, Harris and Renaud (2014), the increase in outstanding student loan debt over the past decade was attributed to several factors. Skyrocketing college costs, cuts to public funding for higher education, stagnant incomes, and the growth in the college-going population largely contributed to the uptick in outstanding student loans.

[2] Under the fixed repayment plan, borrowers make fixed monthly payments similar to mortgage-style amortization. The repayment under the income-based repayment plan is a fixed fraction of borrowers' income, which arguably provides risk sharing but also distorts labor supply (Dynarski and Kreisman, 2013; Barr, 2014).

[3] A survey by Baum and O'Malley (2003) finds that 17% of respondents' career plans are significantly affected by student loans. Moreover, a 2014 survey from Black Book Market Research shows that 94% of working college graduates reported that their student loan payments are "not manageable", and two-thirds believe that they are earning half or less of what they expected to make.

To rationalize these correlations, I develop a tractable theoretical framework that demonstrates individuals under debt burden tend to search less and end up with lower-paid jobs. I then develop and estimate a quantitative search model with risk-averse agents, on-the-job search, and vacancy creation to evaluate the proposed mechanism. My model suggests that the indirect reduction in consumption due to inadequate job search is potentially larger and more persistent than the direct negative effect from debt repayment. I show that income contingency in repayment provides partial insurance against search risks, alleviating the debt burden. However, as has been argued by the opponents of this scheme, there is an adverse incentive effect that reduces labor supply. My model suggests that the net effect on welfare is positive, and the debt burden is reduced by about half on average when all borrowers in my sample switch to the income-based repayment plan. There is a large distributional effect on welfare from this policy, as poorer and more indebted borrowers would benefit more. Importantly, one-third of the reduction in debt burden is attributed to better job matches due to the increase in reservation wages. I argue that this sizable *reservation wage effect* should be considered when evaluating education financing policies.

My quantitative search model has rich features to match a set of labor market characteristics, and it departs from most of the existing equilibrium search models along three dimensions. First, I consider risk-averse agents searching for jobs in an incomplete market. Second, I model student loan debt as a distinct variable, instead of focusing on net worth, to study the implications of different repayment policies. Third, I introduce elastic labor supply for employed workers. As a result, workers and firms bargain over wage rates instead of wage income.

In the model, agents are heterogeneous in wealth, student loan debt, and efficient labor units. Agents search for jobs in the labor market, and their efficient labor units vary over time to capture the hump-shaped life-cycle earnings profile. On the other side of the labor market, there are firms posting vacancies at a flow cost. Vacancies are heterogeneous in randomly drawn productivity. Agents and vacancies meet each other at endogenous contact rates. Matches are formed if the wage rates determined by Nash bargaining generate positive surpluses on both sides. Jobs separate either exogenously or endogenously due to on-the-job search, which is modeled by Bertrand competition following Postel-Vinay and Robin (2002).

I assume that borrowers repay under the default 10-year fixed repayment plan, because the participation rate in other repayment plans is low during my sample period. I model default options parsimoniously using a fixed default cost, which is estimated to match the default rate on student loans. Because student loans are non-dischargeable during bankruptcy, default is merely a delay of repayment that partially alleviates the liquidity problem facing borrowers.[4]

I begin by delineating the economic mechanism of job search under debt burden. To analytically highlight the underlying channels and tradeoffs, my theoretical analysis is based on a simplified partial equilibrium search framework that abstracts several realistic features from the quantitative model. In particular, I consider a risk-averse agent endowed with some initial debt. The agent lives in an imperfect credit market and sequentially receives wage offers from an exogenous distribution. In each period,

[4] Dobbie and Song (2015)'s results suggest that consumer bankruptcy protection is an effective way of providing social insurance. However, because student loan debt is practically non-dischargeable under Chapter 7, it is arguably more burdensome compared to other household debt.

the agent decides whether to take the wage offer. Her job search strategy is fully summarized by a reservation wage, above which the wage offer is accepted.

My first key result illustrates and quantifies the mechanism through which debt repayment influences job search decisions. I show that with fixed repayment, the reservation wage decreases with initial debt. This result comes from the fact that search risks are not perfectly insured in an incomplete market. Intuitively, there is a *risk channel* due to the tradeoff between risks and returns because marginally raising the reservation wage increases both expected income and search risks. When debt is higher, the agent becomes more risk averse due to lower consumption, which pushes her to avoid search risks by setting a lower reservation wage. Moreover, because the credit market is imperfect, there also exists a *liquidity channel* from repayment. The liquidity channel reinforces the risk channel and further reduces the reservation wage. The existence of the liquidity channel highlights the tradeoff between present and future returns in job search. Intuitively, by lowering the reservation wage, the agent has a larger chance to accept a wage offer. This increases expected current income at the cost of lowering expected future income.

To evaluate the quantitative implication of this mechanism, I estimate the quantitative model using 1997-2013 panel data from NLSY97. I parametrically estimate the joint empirical distribution of net liquid wealth and student loan debt for college graduates using Maximum Likelihood Estimation (MLE). Because this paper focuses on how student loan debt affects labor market outcomes, my specification here does not consider the fact that borrowing for college study is an endogenous decision. I discuss the implication of endogenizing borrowing decisions in subsection 6.1.

The structural parameters of my quantitative model are estimated using the Method of Simulated Moments (MSM). In particular, parameters of my model are identified to match the duration of employment and unemployment spells; job tenure; wage increase upon job-to-job transitions; the first, second, and third moments of cross-sectional wage income distribution; student loan default rate; and average hours constructed using NLSY97, as well as the life-cycle earnings profile constructed using the Current Population Survey (CPS). The rich setup of my model requires substantial computation power in estimation. By treating the equilibrium job contact rates as parameters, I propose a two-step estimation method, which greatly accelerates the speed of computation as most parameters can be estimated in partial equilibrium.

I validate the model by conducting two sets of out-of-sample tests related to the proposed mechanism. First, I check whether the model can reproduce the differential wage income between borrowers and non-borrowers observed in the data. Second, I check whether the model-implied structural estimates of the elasticities of unemployment duration and re-employment wages with respect to UI benefits and unused credit are in line with the micro estimates in related literature.

I then use the estimated model to evaluate the long-term effect of student loans under the fixed repayment plan through the proposed mechanism. The model simulation results indicate that the impact of debt on wage income is large while borrowers are repaying under the fixed repayment plan. On average, borrowers earn 4.2% less ($2,139) annually compared to non-borrowers in the first 10 years after graduation. Note that average borrowers already need to repay $1,550 every year, which indicates that debt repayment imposes a double burden on consumption. Interestingly, the simulation results also suggest a longer lasting effect on wage income even after debt is repaid, as borrowers are stuck at their old jobs with relatively lower productivity due to ineffective on-the-job search. In my estimation sample, the average duration of employment spells is 2.7 years, and the average duration of job tenure is 2.3 years. The two moments jointly imply a high separation rate and a much lower job-to-job transition rate. In other words, job-to-job transitions in the data are rare, and this is why first jobs matter.[5]

The debt burden potentially has aggregate implications on output and productivity by affecting job search decisions. As the simulation suggests, average output and match quality (measured by job productivity) among borrowers are 2.4% and 1.1% lower compared with non-borrowers. By contrast, The Executive Office of the President of the United States (2016) conjectures that the rising student loan debt has a limited spillover effect on the macroeconomy, attributed to its small overall scale (9% of aggregate income in 2015). My model suggests a relatively large aggregate effect precisely because of the large effect of debt burden on each borrower, attributed to the mismatch in the timing of the benefits and the costs of college attendance, i.e., a steady, well-paying job versus loan payments. One remedy is to insure risks using income contingent loans.

My second key result illustrates and quantifies the effect of the income-based repayment contract on the reservation wage and welfare. In particular, I consider the agent repaying a constant fraction of her income subject to lenders' recoverability constraint. In the theoretical framework, I show that income contingency raises the reservation wage through both the risk channel and the liquidity channel. The potential costs of income contingency come from the distortion on labor supply, due to the canonical tradeoff between insurance and the incentive to work. Despite this tradeoff, the reservation wage matters for welfare. This suggests that the partial insurance provided by income contingency improves welfare directly through better consumption smoothing and indirectly by increasing the reservation wage to enable better job matches.

To understand the quantitative implications of the income-based repayment contract, I apply the quantitative model to assess the income-based repayment plan. Under this realistic plan, borrowers are eligible to repay 15% of their monthly discretionary income, and all the remaining outstanding debt will be forgiven after 25 years of repayment.[6] The model is able to capture two general equilibrium forces after adopting the income-based repayment plan: first, firms create fewer vacancies due to the reduction in profit because more job offers are turned down by borrowers. Second, non-borrowers face lower job contact rates owing to the decrease in the number of vacancies and the higher aggregate search effort.

The simulation results indicate that the income-based repayment plan largely alleviates the debt burden without generating much debt forgiveness for borrowers in my sample due to the small loan size. After switching to this plan, borrowers would on average experience an increase in annual wage

[5]As documented by Menzio and Shi (2011) using the CPS data, in the U.S. labor market, the average unemployment-to-employment rate is 42% per month, and the average employment-to-employment rate is 2.9% per month. The search-matching models usually estimate that search is much less efficient for employed agents compared to unemployed agents (e.g. Postel-Vinay and Robin, 2002; Jarosch, 2015; Lise and Robin, 2016; Gavazza, Mongey and Violante, 2016).

[6]To provide more details, the income-based repayment plan was modified by Congress in 2010, and from then on old borrowers and new borrowers are treated differently. Borrowers who take out their first loans on or after July 1, 2014 are required to repay 10% of their discretionary income. Borrowers who borrowed after July 1, 2014 are eligible for a less generous plan, which requires 15% of discretionary income. Moreover, there is a repayment cap that ensures that borrowers never need to repay more under the income-based repayment plan than what they would repay under the fixed repayment plan. These features are considered in my quantitative analysis.

income by 3.6% ($1,852) due to better job matches in the first 10 years after graduation. The higher wage income already nets out the adverse incentive effect on labor supply, which is small because the positive substitution effect from sorting into more productive jobs partially offsets the negative substitution effect from income contingency.

The simulation results also indicate that the income-based repayment plan would alleviate the debt burden by about half when measured using wealth compensation. Moreover, there are large distributional effects: borrowers who are poorer and more indebted benefit more after switching to the income-based repayment plan. By contrast, non-borrowers or borrowers with less debt experience a welfare loss when the whole economy adopts the income-based repayment plan. This is because relative to the small benefit from a more flexible repayment plan, non-borrowers or less indebted borrowers suffer more from the lower job contact rates caused by fewer vacancies and higher aggregate search effort. This suggests that adopting the income-based repayment plan could help alleviate consumption inequality, because poorer and more indebted people arguably consume less relative to non-borrowers. This also suggests that my findings provide a lower bound for the benefits of the income-based repayment plan, because the average borrower in my sample owed about $11,873 in student loans, which is well below the amount owed by current borrowers.

I also use the model to quantify the reservation wage effect of the income-based repayment plan. I compare the simulation results of the income-based repayment plan to a counterfactual economy in which agents cannot adjust their reservation wages. I find that one-third of the reduction in debt burden is attributed to better job matches resulting from the increase in reservation wages. This sizable reservation wage effect implies that the insurance provided by the income-based repayment plan is more desirable in terms of raising welfare when individuals are facing search risks.

Finally, I would like to point out that the income-based repayment contract, however, is not constrained efficient because repayment is a constant fraction of income. To uncover the tradeoffs in optimal insurance provision, I analytically characterize the second-best contract, with repayment being a nonlinear function of income. I show that the optimal contract considers the reservation wage, as expected repayment is affected by the borrower's job search decision. Relative to an environment without search frictions, the optimal contract should provide more insurance to the borrower because this also raises her reservation wage.

Related Literature This paper contributes to the growing literature on student loans (see Lochner and Monge-Naranjo, 2015, for a recent survey). An extensive body of this literature focuses on the impact of financial aid during college (e.g., Keane and Wolpin, 2001; Abbott et al., 2016). However, much less is known about the impact of student loans on labor market outcomes after college. Field (2009) examines the influence of psychological responses to debt on career choices for NYU law school admits. Based on a natural experiment in an elite university, Rothstein and Rouse (2011) find that indebted students receive higher initial wages as they are more likely to work in finance, banking, and consulting industries. This implication, however, cannot be easily reconciled with the high default rates of student loan borrowers. As documented by Looney and Yannelis (2015), most of the increase in default is associated with the rise in the number of non-traditional borrowers, who graduated from weak institutions and experienced poor labor market outcomes after leaving school.[7] Students in the sample of Rothstein and Rouse (2011) are highly selective and have flexible skill sets that allow them to find jobs in high-paid sectors.[8] But these conditions may not apply in a nationally representative sample. The mechanism I propose might be more relevant for students from less-selective institutions, and its prediction that students under debt burden are more likely to end up in lower-paid jobs due to inadequate search is also consistent with the high default rates.[9]

The proposed mechanism includes a risk channel and a liquidity channel. The risk channel is related to the work of Danforth (1979) on job search with risk-averse agents and the work of Guler, Guvenen and Violante (2012) on the job search problem of couples. The liquidity channel is related to a large literature on unemployment insurance (UI) benefits. Acemoglu and Shimer (1999) show that UI provides liquidity to workers, encouraging workers to seek for higher productivity jobs. Several studies have explored the optimal UI with liquidity constraints using simulations of calibrated search models (Hansen and Imrohoroglu, 1992; Acemoglu and Shimer, 2000; Hagedorn et al., 2013). Chetty (2008) finds that the liquidity effect accounts for 60% of the impact of UI. The mechanism is also related to several other papers that shed light on job search problems (Bloemen and Stancanelli, 2001; Algan et al., 2003; Lentz and Tranas, 2005; Silvio, 2006; Browning, Crossley and Smith, 2007; Krueger and Mueller, 2011; Kaplan, 2012; Herkenhoff, 2015; Herkenhoff, Phillips and Cohen-Cole, 2016). My paper is different because I clarify how these channels would affect the job search decision when the repayment contract differs.

The edited volume of Stiglitz, Higgins and Chapman (2014) discusses the risk-sharing benefit and potential costs of income contingent loans.[10] My analysis contributes to the understanding of income contingent loans by elucidating the channels of income contingency on the outcome of job search. My quantitative analysis evaluates the potential aggregate and distributional impact of the income-based repayment plan. To my knowledge, this is the first empirical study of the income-based repayment plan using a search-matching model. There are studies using structural models to assess income-driven repayment plans (Dearden et al., 2008; Ionescu, 2009; Mattana and Joensen, 2014; Ionescu and Ionescu, 2014), but none of them account for search frictions in the labor market, which is the main point of this paper. My analytical and quantitative results indicate that the insurance provided by income contingency is more valuable in an environment with search risks due to the positive response in reservation wages. This reflects Shimer and Werning (2007, 2008)'s insight that the after-tax reservation wage encodes all the relevant information about individuals' welfare.

The optimal repayment contract solves a problem similar to the optimal income taxation problem

[7] Looney and Yannelis (2015) document that about 21% of non-traditional borrowers required to start repayment on loans in 2011 defaulted within two years, in contrast to the 8% default rate among traditional undergraduate borrowers.

[8] We can propose an alternative model to rationalize the findings of Rothstein and Rouse (2011). Suppose students face entry costs when searching for jobs in sectors that do not match their majors or skill sets. The entry costs are heterogeneous; the more talented students are also fast learners and arguably face lower entry costs. When talented students are indebted, they choose to incur the entry cost and search for jobs in high-paid sectors because of the monetary incentive. However, less talented students face high entry costs and thereby choose to search for jobs in the sectors that match their skill sets.

[9] As in 2014, about 58% of student loan borrowers are from non-selective for-profit institutions, 2-year public and private institutions (e.g. community colleges), or non-selective 4-year public and private institutions (Looney and Yannelis, 2015). Cellini and Turner (2016) find that students experience a decline in earnings after attendance in for-profit institutions.

[10] A related paper by Fuster and Willen (2011) evaluates the insurance benefits of income-linked assets, with an emphasis on both intra-temporal and inter-temporal consumption smoothing.

of Mirrlees (1971). The difference lies in the existence of search decisions, which allows the job seeker to control the level of income risks by varying her reservation wage. Using a perturbation approach inspired by Saez (2001), I illustrate a novel reservation wage effect whose presence dictates more insurance provision through unemployment subsidy and a more progressive (loosely speaking) repayment schedule. Therefore, my analysis has implications on optimal income taxation in an economy with search risks. This result is related to the insight of Golosov, Maziero and Menzio (2013), who use a directed search model to argue that the optimal redistribution policy should partially insure search risks to encourage riskier search.

Finally, the quantitative analysis is conducted using an equilibrium search-matching model and thus is related to the extensive literature on search-theoretic models of the labor market (see Rogerson, Shimer and Wright, 2005, for a survey). The most closely related quantitative models are those of Krusell, Mukoyama and Sahin (2010); Lise and Robin (2016); Lise, Meghir and Robin (2016); Bagger et al. (2014); Herkenhoff (2015); Herkenhoff, Phillips and Cohen-Cole (2016). My model includes several features from these models, but differs from them by introducing repayment plans and on-the-job labor supply.

Layout The remainder of this paper is structured as follows. Section 2 introduces the federal student loan program. Section 3 introduces data and descriptive evidence. Section 4 develops a tractable theoretical framework to understand one potential mechanism driving the effect of debt burden and to study income contingency. Section 5 extends the theoretical framework to a quantitative search-matching model for empirical analyses. Section 6 estimates the model. Section 7 applies the model to evaluate the long-term effects of debt burden and the income-based repayment plan. Section 8 provides several robustness checks, and Section 9 concludes.

Appendix A presents the proofs of propositions. Appendix B discusses the welfare implications of the reservation wage effect of the income-based repayment contract. Appendix C derives the formula for the optimal repayment contract. Appendix D presents the estimation procedure and numerical algorithm. Online Appendix A details the construction of moments and variables in my empirical analyses and presents additional regression results. Online Appendix B presents additional simulation results, and Online Appendix C presents additional model details. Online Appendix D presents a detailed background introduction for the federal student loan program. Online Appendix E displays robustness check tables.

2 Program Description

In the U.S., student loans play a very significant role in higher education. About 60% of college students borrow student loans to help cover costs. In 2014, the number of borrowers surpassed 43 million, with an average balance of about $27,000.[11] Student loans are basically split into federal loans and private loans, with the former constituting 80% of the total volume. This paper focuses on the federal student loan program because of its importance. The federal student loan programs include the William D. Ford

[11]Source: Federal Reserve Bank of New York, Consumer Credit Panel (a representative sample drawn from anonymized Equifax credit data).

Federal Direct Loan Program, the Federal Family Education Loan (FFEL) Program, and the Federal Perkins Loan Program.

Student loans are arguably more burdensome compared to other loans because repayment usually starts immediately after students leave college, aside from a 6-month grace period offered by Federal Stafford Loans. Moreover, student loans can only be discharged through bankruptcy if borrowers prove "undue hardship" through a court determination. The undue hardship standard is generally difficult to meet, making student loans practically non-dischargeable through bankruptcy. Under certain circumstances, borrowers can receive a deferment or forbearance that allows them to temporarily postpone or reduce their federal student loan payments. However, because applying for deferment and forbearance involves bureaucratic hurdles and detailed paper work, many borrowers do not use these options (Cunningham, Alisa F. and Gregory S. Kienzl, 2011).[12]

Student loans become delinquent the first day after borrowers miss a payment. Default occurs when borrowers are delinquent for 270 days. At this point, the debt will be put into collections and payment will be required from collection agencies. The consequence of default is severe, including tax withholding and wage garnishment. Moreover, student loan debt will increase by up to 25% of the unpaid balance because of the late fees, additional interest, court costs, collection fees, attorney's fees, and any other costs associated with the collection process (see Online Appendix D for more detailed information).

2.1 Repayment Plans

Both the Direct Loan Program and the FFEL Program allow borrowers to choose from among different repayment plans: standard repayment plan, graduated repayment plan, extended repayment plan, and income-driven repayment plan.

The standard repayment plan is the default option for student loan borrowers. Under this plan, monthly payments are fixed and made for up to 10 years for all loan types except Direct Consolidation Loans and FFEL Consolidation Loans. As of 2013, 88% of federal direct loan borrowers repay their debt under the standard repayment plan (Dynarski and Kreisman, 2013).

Under the graduated repayment plan, monthly payments start out low and increase every two years. The repayment period is 10 years for all loan types except for Direct Consolidation Loans and FFEL Consolidation Loans, which allow an extension of the repayment period to 30 years depending on the amount of total education loan indebtedness. By 2012, only fewer than 5% students enroll in the graduated repayment plan (The Institute for Higher Education Policy, 2014).

Under the extended repayment plan, monthly payments are either fixed or graduated. The repayment period can be extended up to 25 years. As a result, monthly payments are generally lower than those made under the standard or graduated repayment plans. To qualify for the extended repayment plan, borrowers must have had no outstanding balance on a Direct Loan/FFEL Loan as of October 7, 1998, or on the date they obtained a Direct Loan/FFEL Loan after October 7, 1998. Moreover, borrowers must have more than $30,000 in outstanding Direct Loans or in FFEL Loans.

[12] In my model, student loan borrowers do not have access to deferment or forbearance because these options are not frequently used in reality.

The goal of income-driven repayment plans is to help make borrowers' monthly payments more affordable by basing them on their income and family size. The income-based application now includes four different income-driven repayment plans: income-contingent repayment plan (ICR), income-based repayment plan (IBR), pay as you earn repayment plan (PAYE), and revised pay as you earn repayment plan (REPAYE). These plans are different from each other in terms of enrollment eligibility, repayment rates, the length of repayment period, and interest capitalization. The main feature of these plans is that borrowers make payments contingent on their income instead of the balance of outstanding debt, and the remaining debt is forgiven after a certain number of payments.[13]

Although the first income-driven repayment plan (i.e., ICR) has been made available since 1994, the take-up rate was below 1% until 2008 due to the detailed paperwork required and long processing times, among various other reasons. As suggested by The Executive Office of the President of the United States (2016), continuing to expand enrollment in income-driven repayment plans remains a key priority for the administration. In fact, the administration has used several tools to increase enrollment, such as behavioral "nudges", improved loan servicer contract requirements, efforts associated with the President's Student Aid Bill of Rights, a student debt challenge to gather commitments from external stakeholders, and increased and improved targeted outreach to key borrower segments who would benefit from income contingency. The participation rate in income-driven repayment plans has quadrupled over the last four years, from 5% in 2012 to 20% in 2016. In April 2016, the administration announced a series of new actions to further expand enrollment in income-driven repayment plans.[14]

This paper seeks to understand the implications of the standard fixed repayment plan and the income-driven repayment plan because the former covers most student loan borrowers, and the latter has potential side effects that remain a source of debate.[15]

3 Data

My empirical analysis uses panel data from the National Longitudinal Survey of Youth 1997 (NLSY97). This is a nationally representative survey conducted by the Bureau of Labor Statistics. In round 1, 8,984 youths were initially interviewed in 1997. Follow-up surveys were conducted annually. Almost 83% (7,423) of the round 1 sample were interviewed in round 15 (2011-2012). Youths were born between 1980

[13] All of these plans are different from the first attempt at income contingent loans in the U.S. in 1971—the Yale Tuition Postponement Option (TPO). The main difference is that under these plans borrowers do not need to repay more than the amount borrowed. However, there is cross-subsidization under TPO as participants are required to make payments until the debt of an entire "cohort" is repaid.

[14] Dynarski and Kreisman (2013) propose to make a variant of income-driven repayment plans as the default option for student loan borrowers, which is arguably the most direct way to expand enrollment rate in income-driven repayment plans. In fact, income contingent student loans have already been widely adopted in other countries including Australia, New Zealand, Ethiopia, England, Hungary, South Africa and South Korea. In Australia, all student loan payments became income contingent since 1989. In the UK, repayment was completely transformed from a fixed-term "mortgage-style" system to several variants of income contingent repayment schemes for student loans taken out after September 1998.

[15] When evaluating the income-driven repayment plan, I focus specifically on IBR, which is the most widely available income-driven repayment plan. As of the third quarter of 2014, 70% of participants in income-driven repayment plans are enrolled in IBR (The Institute for Higher Education Policy, 2014). For the graduated repayment plan and the extended repayment plan, I provide a theoretical discussion in subsection 4.2.2 and a preliminary quantitative evaluation in Online Appendix B.4.

and 1984. Their ages ranged from 12 to 18 in round 1 and were 26 to 32 in round 15. The survey contains extensive information on each youth's labor market behavior and documents the amount of education loans borrowed during college, which makes NLSY97 an ideal data set for studying the implication of student loan debt on job search decisions.

My analysis focuses on youths who earned a bachelor's degree. I drop youths who have ever served in the military or attended graduate schools because they are not in the same position as the other youths in my sample when it comes to making labor market decisions. I also drop youths who received the bachelor's degree before 1997 due to the lack of labor market information upon college graduation. This leaves me with a sample of 1,261 youths. Below, I describe the variables used to provide descriptive evidence; the other variables used in structural estimation are detailed in Online Appendix A.

The survey documents each youth's college enrollment status in each month since 1997. Based on this information, I obtain the last date enrolled in college for each youth. I consider the youth as being in the labor market after this date is passed. The survey also documents each youth's weekly employment status since 1997 and the associated employer number. I construct the duration of unemployment spells by tracking the period until an unemployed youth finds a job.

The survey asks each youth to provide income received from wages, salary, commission, or tips from all jobs in past year, before deductions for taxes or anything else. I use the answer to this question to construct each youth's annual wage income.[16] When constructing annual wage income, I follow Rubinstein and Weiss (2006) by excluding the youths whose hourly wage rates are below $4 or higher than $2,000 and who worked fewer than 35 weeks or fewer than 1,000 annual hours.

The survey asks each youth the amount of loans borrowed in government-subsidized loans or other types of loans while the youth attended schools in each term and at each college. Together with the records on enrollment information, I construct the amount of student loans taken out in each year and the total amount of student loans borrowed before college graduation. Unfortunately, information on repayment is not available in the data. Because students rarely repay debt during college, I consider the total amount of student loans borrowed as the amount of outstanding student loan debt upon college graduation.

3.1 Descriptive Evidence

I group youths into those who never borrowed student loans during college and those who borrowed. Among the youths with student loans, I create a high-loan group for borrowers who borrowed more than the median loan value ($8,821). Table 1 presents summary statistics.

The table shows that about 78.1% of non-borrowers find full-time jobs within six months after college graduation, and it is 79.8% and 82.7% for borrowers and high-loan borrowers. Moreover, the average duration of the first unemployment spell after college graduation is 17.7 weeks for non-borrowers, and it is 16.4 and 14.1 weeks for borrowers and high-loan borrowers. It is also shown that the mean hourly wage income paid by the first job for non-borrowers is about 3.3 dollars higher than borrowers. These

[16] An alternative method to construct annual wage income is to use the information on hours and hourly wage rate. The two methods usually provide different numbers due to measurement errors. I do not use the second method because the hourly wage rate is constructed by BLS staff based on several discretionary assumptions.

Table 1: Summary statistics of the sample from NLSY97.

	Non-borrowers	Borrowers all	Borrowers high-loan	All
Observations	484	777	388	1,261
Age	23.4	23.8	23.7	23.7
Gender (male=0, female=1)	0.51	0.58	0.57	0.55
AFQT	66.9	66.1	66.2	66.4
Mean liq. wealth upon graduation ($)	6,877	3,001	2,914	4,432
Mean student loans ($)	0	11,873	18,970	7,316
Frac. with jobs within 6 months	78.1%	79.8%	82.7%	79.1%
Mean uemp. duration (1st spell, week)	17.7	16.4	14.1	16.9
Mean hourly wage rate (1st job, $)	20.0	16.7	16.4	17.9
Finance, consulting, banking (1st job)	21.5%	16.1%	17.6%	18.1%
Mean annual wage income (1st year, $)	40,110	35,915	34,951	37,531
Mean annual wage income (2nd year, $)	43,699	38,644	38,086	40,609
Mean annual wage income (3rd year, $)	49,839	42,676	43,041	45,592
Mean annual wage income (4th year, $)	48,854	44,831	45,601	46,620
Mean annual wage income (5th year, $)	54,122	46,018	46,171	49,311

Note: This table presents summary statistics by the total amount of student loan debt. The sample is from NLSY97, restricted to youths with a bachelor's degree. The sample size is 1261 after dropping youths who have ever served in the military or attended graduate schools and those who received the bachelor's degree before 1997. Youths are grouped into non-borrowers (i.e., those who never borrowed student loans during college), borrowers (i.e., those who have borrowed student loans during college), and high-loan borrowers (i.e., those whose outstanding loan is larger than the median loan value of $8,821). Variables' construction is explained in Online Appendix A.

summary statistics suggest that student loan borrowers spend less time in job search after graduation and are more likely to end up in lower-paid jobs.

The relative longer impact on wage income also indicates that borrowers earn less. The mean annual wage income is consistently lower for borrowers relative to non-borrowers in the first five years after college graduation. The difference is significant at the 1% level in the first three years.[17] Table 1 shows that about 18.1% of students in my sample work in finance, banking, or consulting industries as compared to about 60% in the elite sample of Rothstein and Rouse (2011).

To explore the suggestive effect of student loans on job search decisions, I regress the duration of the first unemployment spell (Dur_i) after college graduation on the amount of student loan debt (s_i) and control variables X_i including parental wealth, parental education, gender, race, AFQT score, marital status, the cubic age polynomials, and the county of residence in graduation year:

$$Dur_i = \alpha + \beta_1 s_i + \beta_2 X_i + \varepsilon_i. \tag{3.1}$$

Table 2 shows that a $10,000 increase in the amount of student loans reduces unemployment duration by about 2 weeks. To explore the implication on wage income, I regress the annual wage income in the

[17]Using a different dataset, Looney and Yannelis (2015, Figure 11) report that borrowers with larger student loans earn more. Note that they do not restrict the sample to graduates with only college degrees. As they explain, the main driver of their result is that "students with larger debts tend to have been enrolled longer, achieved higher levels of educational attainment, pursued higher levels of post-secondary education (such as a BA instead of a certificate or a graduate degree), and have attended 4-year institutions where borrowing amounts are greatest, which tend to be the more selective 4-year institutions. For these reasons, borrowers with more debt tend to earn much more."

Table 2: The duration of the first unemployment spell after college graduation.

	Duration of the first unemployment spell	
	(1)	(2)
Loan amount	-1.54**	-2.08***
(in $10,000)	(0.66)	(0.68)
Parental wealth	-0.02	-0.00
(in $10,000)	(0.06)	(0.07)
Parental education	0.36	0.68
	(0.41)	(0.53)
Observations	884	771
County fixed effect	√	√
R^2	0.0057	0.0183

Note: This table examines the impact of student loan debt on the duration of the first unemployment spell after graduation. A $10,000 increase in the amount of student loans reduces the duration of the first unemployment spell by about 2 weeks. Each observation is at the individual level. The dependent variable is the number of weeks elapsed from the college graduation date to the date of starting the first full-time job (i.e., work more than 35 hours per week for at least two consecutive weeks). The dependent variable is regressed on the total amount of student loan debt borrowed during college study, recorded in units of $10,000. All regressions control for parental wealth, parental education, and the county of residence in the graduation year. Column (2) also controls for gender, race, AFQT score, marital status, and the cubic age polynomials. Standard errors are clustered at the county level. ***, **, and * indicate significance at the 1, 5, and 10 percent level.

first three years after college graduation on the amount of student loans:[18]

$$Wage_{i,t} = \alpha_t + \beta_{1,t} s_i + \beta_{2,t} X_i + \varepsilon_{i,t}, \quad \text{for } t = 1, 2, 3. \tag{3.2}$$

Table 3 shows that a $10,000 increase in the amount of student loans lowers the annual wage income by about $2,000. The effect is similar in magnitude but insignificant in the fourth and fifth year (not reported). Slightly larger effects are found if the regression also controls for industry, college major, and the duration of college study (see Online Appendix A).

The OLS regressions suggest that indebted students could be less picky in job search, which is why they end up in lower-paid jobs. However, the estimates are probably biased due to the existence of unobservables that are correlated with both the amount of student loans and the dependent variable of interest. The selection biases could go either way. For example, students who borrowed may be more likely to be from low-income families; as a result, they presumably received poorer pre-school education, which could lead to poorer performance in the labor market. On the other hand, students who borrowed could be more talented and confident about their future labor market outcomes and their ability to repay the loans. As a result, they end up with higher wages. In a companion work in progress (Ji and Yannelis, 2016), we explore causal implications using the administrative data on federal student loans and de-identified tax records.

[18]The regression is implicitly restricted to the sample of youths whose wage income is reported. There are several reasons for missing wage income: first, youths could be non-interviewed in that year. Second, youths could be unemployed or out of labor force. Third, youths could forget or refuse to report their wage income. Fourth, youths graduating in later cohorts only started working in recent years. All of them would generate selection biases.

Table 3: The impact of student loan debt on post-graduation wage income.

	First year		**Second year**		**Third year**	
	(1)	(2)	(1)	(2)	(1)	(2)
Loan amount	-1,830**	-2,067**	-1,812**	-2,152**	-2,009*	-2,619**
(in $10,000)	(770)	(890)	(789)	(865)	(1,117)	(1,309)
Parental wealth	100*	94*	91	106	53	33
(in $10,000)	(56)	(55)	(70)	(84)	(85)	(83)
Parental education	19	-376	290	-364	611	-29
	(305)	(380)	(389)	(523)	(538)	(623)
Observations	671	596	588	518	483	427
County fixed effect	√	√	√	√	√	√
R^2	0.0175	0.0651	0.0221	0.0733	0.0185	0.0713

Note: This table examines the impact of student loan debt on wage income in the first three years after college graduation. A $10,000 increase in the amount of student loans reduces the annual wage income by about $2,000. Each observation is at the individual level. The dependent variable is wage income in the t-th year ($t = 1, 2, 3$) after college graduation. The dependent variable is regressed on the total amount of student loan debt borrowed during college study, recorded in units of $10,000. All regressions control for parental wealth, parental education, and the county of residence in the graduation year. Column (2) also controls for gender, race, AFQT score, marital status, and the cubic age polynomials. Standard errors are clustered at the county level. ***, **, and * indicate significance at the 1, 5, and 10 percent level.

4 Mechanism and Channels

In this section, I propose one economic mechanism that drives the empirical correlations observed in Table 2 and 3. I build a partial equilibrium model based on McCall (1970) with several simplifying assumptions to shed light on the mechanism of debt burden and to analyze the implications of different repayment contracts. These assumptions will be made more realistic when conducting quantitative analyses in the next section.

4.1 Environment

Consider an agent who is born at $t = 0$ and sequentially searches for a job. Time is discrete and there is no aggregate uncertainty. The agent maximizes lifetime utility from consumption, $E \sum_{t=1}^{\infty} \beta^t u(c(t))$ with subjective rate of time preference β. The per-period utility function, $u(x)$, is bounded from above, strictly increasing, concave, and twice continuously differentiable, i.e., $\lim_{x\to\infty} u(x) = M, u'(x) > 0, u''(x) < 0$.

The agent can either be unemployed or employed. For now, suppose that the agent supplies one unit of labor inelastically when being employed. Starting from $t = 1$, if the agent is unemployed, the agent receives UI benefits $\theta > 0$, and wage offers w from an exogenous cumulative distribution function $F(w)$ in each period, which is differentiable on the support $[\theta, \overline{w}]$.

The agent needs to decide immediately whether to accept the wage offer upon receiving it. There is no recall of past wage offers. Consumption is chosen after the realization of wage offers. If the agent rejects the offer, she continues to search. Otherwise, she gets employed at wage w forever.

The credit market is imperfect in the sense that savings are constrained to be non-negative, $s_t \geq 0$, for all t.[19] The interest rate on savings is r. For simplicity, I assume $\beta(1 + r) = 1$ so that the agent has no

[19] While student loan balances may not have a large effect on credit scores, they can affect borrowers' eligibility to take out other loans. For example, mortgage lenders often look at borrowers' debt-to-income ratio. If this ratio is too high, meaning

incentive to transfer wealth across periods.[20]

The agent is born with outstanding debt S whose repayment schedule is specified in the contract. The interest rate on debt is equal to the interest rate on savings. In the following, I analyze the implication of debt burden on job search decisions for different repayment contracts.

4.2 Fixed Repayment Contract

In this subsection, I analyze job search decisions under the fixed repayment contract. To obtain a stationary result, I consider indefinite fixed payment flows such that the present value of this perpetuity covers the initial outstanding debt S.

Definition 1. *The fixed repayment contract requires the agent to repay $s = rS$ in each period.*

For tractability, I assume that the agent cannot be delinquent on making payments. Therefore, to avoid the pathological case, I consider $S < \frac{\theta}{r}$ so that the agent can repay the loan, while at the same time maintaining positive consumption, even if she is permanently unemployed.[21]

Denote V as the value function of an unemployed agent, and $W(w)$ as the value function of an employed agent with wage w. Thus,

$$W(w) = \frac{u(w-s)}{1-\beta}. \tag{4.1}$$

When the agent rejects the wage offer, the income in the current period is θ and the value function V can be written as

$$V = u(\theta - s) + \beta \int_{\theta}^{\overline{w}} \max\{W(w), V\} dF(w). \tag{4.2}$$

Equation (4.1) states that the agent accepts the wage offer if it provides a higher value than unemployment. Because $W(w)$ is increasing in w, the optimal job search decision follows a cutoff strategy, and the wage offer is accepted if $w > w^*_{FIX}$, where w^*_{FIX} is the reservation wage under the fixed repayment contract. The agent sets w^*_{FIX} to maximize her welfare, which happens when the value of staying unemployed is equal to the value of being employed at the reservation wage, i.e., $V = W(w^*_{FIX})$:

$$u(w^*_{FIX} - s) = u(\theta - s) + \frac{\beta}{1-\beta} \int_{w^*_{FIX}}^{\overline{w}} [u(w-s) - u(w^*_{FIX} - s)] dF(w). \tag{4.3}$$

The RHS of equation (4.3) captures the per-period utility of rejecting the wage offer. It states that rejecting the wage offer results in a lower current utility $u(\theta - s)$ but preserves the possibility of receiving a higher wage offer in the future. Setting a higher reservation wage implies a smaller chance of being employed but also generates a higher expected employment value. The optimal reservation wage is set to balance these two effects.

borrowers' have too much debt relative to their income, borrowers may be turned down for a loan. Card, Chetty and Weber (2007) and Rothstein and Rouse (2011) also provide empirical support that the younger population enters the labor force with limited liquidity.

[20]When the agent is unemployed, the agent does not save because she expects future income to be higher. When the agent is employed, the agent is indifferent about savings because wage income is flat and $\beta(1+r) = 1$.

[21]If $S > \frac{\theta}{r}$, the agent is involuntarily forced into delinquency either when she is unemployed or when she is employed at wage $w < rS$. Suppose the remaining income is garnished upon delinquency, then we can show how the reservation wage varies with debt depends on whether there is an inada condition on $u(\cdot)$.

4.2.1 The Risk Channel and the Liquidity Channel

Job search is a risky investment that pays off in the future. The agent controls the reservation wage to manage risks, as setting a lower reservation allows the agent to accept a constant wage offer sooner and take fewer search risks. Therefore, we can think of the reservation wage characterized by equation (4.3) as the certainty equivalent payoff of continued job search. More risk-averse agents have a lower certainty equivalent valuation of any risky lotteries, thus they set a lower reservation wage in job search. Because the agent's risk attitude is closely related to her consumption, there is a risk channel from debt repayment as formalized in Proposition 1.

Proposition 1. *Under the fixed repayment contract, the effect of debt depends on how risk aversion varies with consumption. With decreasing absolute risk aversion, w^*_{FIX} is decreasing in debt; with increasing absolute risk aversion, w^*_{FIX} is increasing in debt; with constant absolute risk aversion, w^*_{FIX} is unaffected by debt.*

Proof. see Appendix A.1. □

This proposition is proved using Pratt (1964, Theorem 1). Its implication is not trivial because the proposition suggests that risk aversion alone is not sufficient to deliver an unambiguous effect of debt burden on job search decisions. Proposition 1 essentially states that the effect of debt burden is related to the third derivative of the utility function under the fixed repayment contract. Because decreasing absolute risk aversion is empirically plausible (Friend and Blume, 1975), the agent would set a lower reservation wage to avoid search risks because higher repayment makes her more risk averse. I discuss in the proof that this proposition holds even if the credit market is perfect. However, the quantitative implication would be much smaller because what would matter is the relative value of outstanding debt to total income instead of income in the current period. This implies that there also exists a liquidity channel that reinforces the risk channel by further reducing the reservation wage.

It is worth noting that the risk channel and the liquidity channel result from two different tradeoffs in job search. First, job search is risky. Therefore, an agent who becomes more risk averse due to a higher level of debt would tradeoff risks and returns by adjusting the reservation wage. This is the risk channel. Second, job search encodes an option value that only pays off in the future, at the time of accepting the wage offer. Therefore, the reservation wage implicitly determines the wealth transfer across periods. When the credit market is imperfect, the agent faces an intertemporal tradeoff in job search because a lower reservation wage increases the chance of accepting a wage offer, and thus more wealth is transferred from future periods to the current period. This is the liquidity channel.

4.2.2 Restructuring the Fixed Repayment Contract

The existence of the liquidity channel suggests that the lender can restructure the schedule of repayment to mitigate the debt burden. In reality, the federal student loan system has such features. For example, under the Direct Loan Program and the FFEL Program, borrowers have a 6-month grace period after graduation before payments are due. Moreover, the graduated repayment plan allows borrowers to make smaller payments at first and then increase their payments over time. The extended repayment plan allows qualified borrowers to extend the repayment period up to 25 years.

To formalize the intuition behind these realistic repayment plans, consider a particular contract that requires the agent to repay s_1 at $t = 1$, and s_2 at $t \geq 2$, such that the outstanding debt S is recovered:

$$\frac{s_1}{1+r} + \sum_{t=2}^{\infty} \frac{s_2}{(1+r)^t} = S. \tag{4.4}$$

Proposition 2 shows that back-loading debt payments increases the reservation wage at $t = 1$ through the liquidity channel.

Proposition 2. *Reducing s_1 and increasing s_2 subject to the constraint (4.4) strictly increases the reservation wage at $t = 1$ when the borrowing constraint is binding.*

Proof. see Appendix A.2. □

In contrast to Proposition 1, Proposition 2 holds for any risk-averse agents, but it requires an imperfect credit market. When the borrowing constraint is not binding, the liquidity channel is absent because any change in the repayment schedule only results in a change in savings rather than affecting the job search decisions. When the borrowing constraint is binding, back-loading debt payments affects the reservation wage through two effects. First, reducing s_1 has a direct positive effect on the reservation wage at $t = 1$, because it provides liquidity for continued search. Second, reducing s_1 induces a higher s_2, resulting in a lower reservation wage at $t \geq 2$. The lower future reservation wages reduce the value of continued job search, which in turn indirectly imposes a negative effect on the reservation wage at $t = 1$. When the borrowing constraint is binding, the direct effect dominates the indirect effect. Intuitively, this is because the agent faces a higher marginal utility of consumption in the current period, thus she has the incentive to transfer wealth from future periods by setting a lower reservation wage. Requiring a smaller payment in the current period dampens this incentive by reducing the intertemporal gap in the marginal utility of consumption. As a result, the agent would increase her reservation wage to pursue a higher expected future return.

4.2.3 Implication on Expected Income

A lower reservation wage implies that the agent would have less expected income when she is indebted under the fixed repayment contract. To see this, let $I(w^*_{FIX})$ denote the present value of expected income as a function of the reservation wage w^*_{FIX}, and then it can be solved recursively:

$$I(w^*_{FIX}) = F(w^*_{FIX})[\theta + \beta I(w^*_{FIX})] + \int_{w^*_{FIX}}^{\overline{w}} \frac{w}{1-\beta} dF(w). \tag{4.5}$$

Equation (4.5) states that when the agent draws an offer below w^*_{FIX} with probability $F(w^*_{FIX})$, she rejects it and receives UI benefits θ in the current period and the same present value of expected income $I(w^*_{FIX})$ in the next period. When the wage offer is above w^*, she accepts it and gets paid perpetually. The compensation for search risks implies a monotonic relationship between w^*_{FIX} and $I(w^*_{FIX})$:

Proposition 3. *There exists a unique income-maximizing reservation wage $\hat{w}$, determined by*

$$\hat{w} - \frac{\beta}{1-\beta}\int_{\hat{w}}^{\overline{w}} (w - \hat{w}) dF(w) = \theta. \tag{4.6}$$

The present value of expected income is strictly increasing in w^*_{FIX} when $w^*_{FIX} < \hat{w}$, and strictly decreasing in w^*_{FIX} when $w^*_{FIX} > \hat{w}$. Moreover, the optimal reservation wage for any risk-averse agent satisfies $w^*_{FIX} < \hat{w}$.

Proof. see Appendix A.3. □

In fact, the income-maximizing reservation wage $\hat{w}$ is the reservation wage set by risk-neutral agents. In an incomplete market, the existence of uninsured search risks incentivizes risk-averse agents to set a strictly lower reservation wage in order to smooth consumption.

4.3 Income-Based Repayment Contract

The main feature of the income-based repayment plan is that borrowers make payments contingent on their income instead of the balance of outstanding debt. Although a realistic income-based repayment plan also incorporates other auxiliary features like debt forgiveness and payment caps, my theoretical analysis would not explicitly consider them.[22] Instead, I consider the income based-repayment contract that allows the lender to recover all the outstanding debt in expectation conditional on the agent's endogenous job search decisions. Similar to the fixed repayment contract, I assume that the repayment period is indefinite.

Definition 2. *The income-based repayment contract requires the agent to repay a fraction α of her income. The repayment ratio α is set by the lender such that the expected present value of payment flows is just enough to cover the outstanding debt S:*

$$\alpha I(w^*_{IBR}) = \frac{S}{\beta}, \tag{4.7}$$

where w^*_{IBR} is the agent's optimal reservation wage under the income-based repayment contract:

$$u((1-\alpha)w^*_{IBR}) = u((1-\alpha)\theta) + \frac{\beta}{1-\beta}\int_{w^*_{IBR}}^{\overline{w}} \left[u((1-\alpha)w) - u((1-\alpha)w^*_{IBR})\right] dF(w). \tag{4.8}$$

I call equation (4.7) the lender's recoverability constraint. Expected repayment not only depends on the repayment ratio α but also on the agent's reservation wage w^*_{IBR}. Because the reservation wage is unobservable, the income-based repayment contract only specifies the repayment ratio α. The agent optimally chooses her reservation wage according to the indifference equation (4.8), which can be thought of as the incentive compatibility constraint. Since the income-based repayment contract is not specifying the reservation wage, this naturally introduces an inefficiency because the agent does not internalize the effect of her reservation wage on expected repayment. The welfare implication of this inefficiency is discussed in Appendix B.

[22] I incorporate these features in subsection 7.2 when quantitatively evaluating the 2014 income-based repayment plan.

4.3.1 The Reservation Wage Effect

The income-based repayment contract provides insurance and risk sharing for job search, because the agent repays less when income is low. In fact, we can view the fixed repayment contract as a pure debt contract and the income-based repayment contract as an equity contract. Intuitively, the agent should set a relatively higher reservation wage if debt is repaid under the income-based repayment contract, because equity contracts encourage activities with high returns and high risks. This result is summarized in the following proposition.

Proposition 4. *With CRRA utility, the reservation wage under the income-based repayment contract is strictly higher, i.e.,* $w^*_{IBR} > w^*_{FIX}$.

Proof. see Appendix A.4. □

Proposition 4 is limited to CRRA utility, which has decreasing absolute risk aversion. Therefore, with CRRA utility, Propositions 1 and 4 jointly imply that the repayment of debt reduces the reservation wage and the income-based repayment contract alleviates this distortion. I can work out the proof for CRRA because CRRA is a homogeneous function, i.e., the proportional change in utility does not depend on consumption level when consumption is changed proportionately.[23]

To elucidate the exact channels through which the income-based repayment contract influences the reservation wage, let us focus on the disposable wage, which is wage income net of debt repayment. Denote $F_{IBR}(w)$ and $F_{FIX}(w)$ as the disposable wage offer distribution under the income-based repayment contract and the fixed repayment contract; thus

$$F_{IBR}(w - \alpha w) = F_{FIX}(w - s) = F(w), \forall w \in [\theta, \overline{w}] \tag{4.9}$$

Denote $\tilde{w}^*_{IBR}$ and $\tilde{w}^*_{FIX}$ as the associated disposable reservation wages. By definition,

$$\tilde{w}^*_{FIX} = w^*_{FIX} - s, \tag{4.10}$$

$$\tilde{w}^*_{IBR} = (1 - \alpha) w^*_{IBR}. \tag{4.11}$$

The income-based repayment contract has a less risky disposable wage offer distribution because of better risk sharing. Using the monotonicity property of expected income illustrated in Proposition 3 and a single-crossing property of $F_{IBR}(w)$ and $F_{FIX}(w)$, I show that the income-based repayment contract is second-order stochastic dominant over the fixed repayment contract.

Lemma 1. *The disposable wage offer distribution under the income-based repayment contract,* $F_{IBR}(w)$, *strictly*

[23] Within the class of DARA utility, the relative risk aversion could be increasing, constant, or decreasing. When the repayment is proportional to income, we can show that the agent's reservation wage would increase/remain unchanged/decrease due to an increase in repayment ratio when the relative risk aversion is increasing/constant/decreasing. Therefore, Proposition 4 can be made slightly more general, and it holds when the utility has increasing relative risk aversion, or when the utility has decreasing relative risk aversion with the decreasing speed bounded from above by some value. Unfortunately, I cannot generalize the implication of Proposition 4 to the whole class of DARA utility.

second-order stochastically dominates that under the fixed repayment contract, $F_{FIX}(w)$*:*

$$\int_0^x F_{IBR}(w)dw \leq \int_0^x F_{FIX}(w)dw, \forall x. \tag{4.12}$$

Proof. see Appendix A.5. □

By applying integration by parts twice, I decompose the difference in the disposable reservation wage under the two contracts into three channels:

Proposition 5. *The difference in the disposable reservation wage between the income-based repayment contract and the fixed repayment contract is characterized by the following decomposition:*

$$\begin{aligned} u(\tilde{w}^*_{IBR}) - u(\tilde{w}^*_{FIX}) = & \underbrace{(1-\beta)\left[u((1-\alpha)\theta) - u(\theta - s)\right]}_{\text{liquidity channel, (+)}} \\ & + \underbrace{\beta\left[\int_{\tilde{w}^*_{IBR}}^{\infty}\left(\int_0^w F_{IBR}(x)dx\right)u''(w)dw - \int_{\tilde{w}^*_{FIX}}^{\infty}\left(\int_0^w F_{FIX}(x)dx\right)u''(w)dw\right]}_{\text{risk channel, (+)}} \\ & + \underbrace{\beta\left[u'(\tilde{w}^*_{IBR})\int_0^{\tilde{w}^*_{IBR}} F_{IBR}(w)dw - u'(\tilde{w}^*_{FIX})\int_0^{\tilde{w}^*_{FIX}} F_{FIX}(w)dw\right]}_{\text{optionality channel, (-)}}. \end{aligned} \tag{4.13}$$

Proof. see Appendix A.6. □

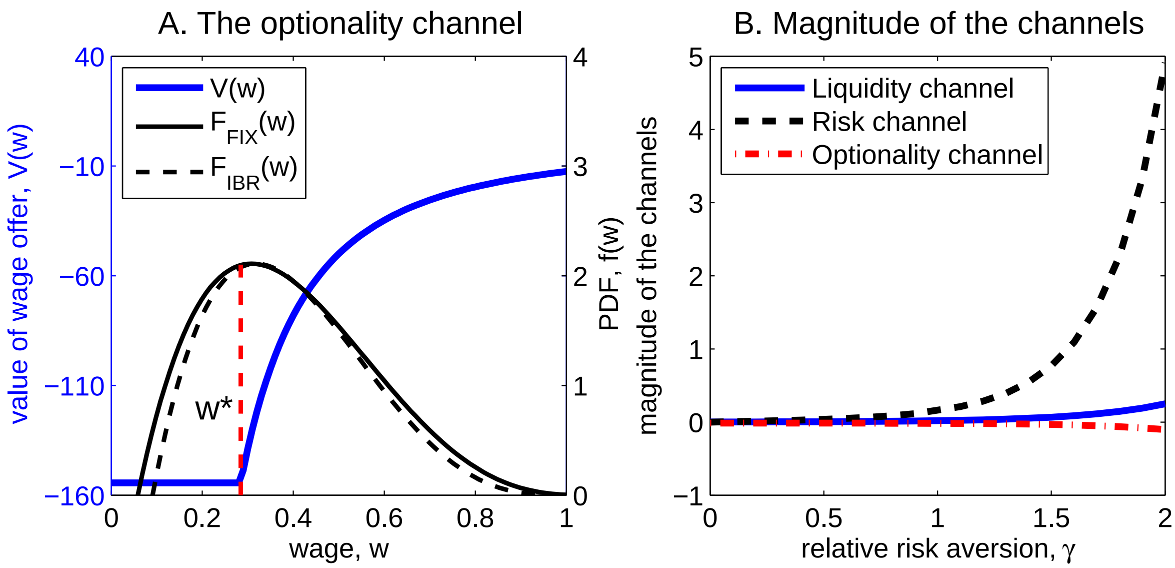
The RHS of equation (4.13) consists of three channels. The first term captures a liquidity channel similar to the one illustrated in Proposition 2. The agent repays less during unemployment under the income-based repayment contract, thus $u((1-\alpha)\theta) > u(\theta - s)$. This implies that the first term is positive, contributing to a higher reservation wage.

The second term captures the risk channel.[24] The income-based repayment contract generates a less risky wage offer distribution according to Lemma 1. Because the agent is risk averse, she would raise the reservation wage to pursue a higher expected return when there are fewer risks in job search. Therefore, the second term is also positive, contributing to a higher reservation wage.

The third term captures the difference in the option value of staying unemployed under the two repayment contracts. Intuitively, the agent has a larger option value of staying unemployed when the wage offer distribution is more dispersed. This is because lower wages would be turned down, and higher wages are more likely to be drawn from a more dispersed wage offer distribution. Essentially, this optionality channel results from the convexity of the value function upon being offered a wage offer:

$$V(w) = \max\{V, W(w)\} = \mathbb{1}_{w \leq w^*}V + \mathbb{1}_{w > w^*}W(w), \tag{4.14}$$

[24] I name the second term as the risk channel in the spirit of Proposition 1. However, it is not exactly the same as the risk channel emphasized in Proposition 1. In Proposition 1, the risk channel refers to the variation in risk aversion, but here the risk channel refers to the variation in the riskiness of the wage offer distribution.



Note: Panel A illustrates the optionality channel. The blue solid line plots the value function upon being offered a wage offer. The black solid line plots the disposable wage offer distribution under the fixed repayment contract, $F_{FIX}(w)$. The black dashed line plots the disposable wage offer distribution under the income-based repayment contract, $F_{IBR}(w)$. The red vertical dashed line represents the reservation wage. The wage offer's upside potential is smaller under the income-based repayment contract due to second-order stochastic dominance. This reduces the option value of staying unemployed and pushes the agent to set a lower reservation wage. Panel B illustrates the magnitude of the three channels. When the agent becomes more risk averse, the magnitude of all three channels increases, but the magnitude of the risk channel increases dramatically and starts to dominate the other two channels. This suggests that the essential feature of the income-based repayment contract is to provide insurance against search risks, making job search more affordable and increasing the reservation wage. The figure is plotted using the CRRA utility, $u(c) = c^{1-\gamma}/(1-\gamma)$ and the beta distribution of wage offers, $Beta(a,b)$, with parameter values: $\gamma = 3$, $a = 2$, $b = 4$, $\theta = 0.1$, $\overline{w} = 1.1$, $\beta = 0.96$, $S = 1$.

Figure 2: A numerical illustration of the optionality channel and the magnitude of the three channels underlying the income-based repayment contract.

where $\mathbb{1}$ refers to the indicator function. The disposable wage offer distribution under the income-based repayment contract, $F_{IBR}(w)$, is less dispersed (see panel A of Figure 2). Thus the optionality channel contributes to a lower reservation wage.

In sum, the liquidity channel and the risk channel push up the disposable reservation wage when the agent switches from the fixed repayment contract to the income-based repayment contract, but the optionality channel has a countervailing effect. When the agent is risk neutral, the risk channel is absent and we can prove that the liquidity channel always dominates the optionality channel.[25] Although I cannot in general prove which channel dominates, the proof of Proposition 4 shows that when the agent has CRRA utility, the net effect is always positive so that the disposable reservation wage under the income-based repayment contract is always higher than that under the fixed repayment contract.

Panel B of Figure 2 provides a numerical example to illustrate the magnitude of the three channels using CRRA utility. When the agent becomes more risk averse, the magnitude of all three channels increases, but the magnitude of the risk channel increases dramatically and starts to dominate the other two channels. This suggests that the essential feature of the income-based repayment contract is to provide insurance against search risks, making job search more affordable and increasing the reservation

[25]To see this, note that a risk-neutral agent has the same reservation wage w^* under both plans. Thus using a proof similar to the proof of Proposition 4, the disposable reservation wage under the income-based repayment contract is higher, suggesting that the liquidity channel dominates.

wage.[26]

4.3.2 Welfare Implication

Because the income-based repayment contract provides insurance, it is not surprising that it increases welfare relative to the fixed repayment contract. I prove this result under CRRA utility:

Proposition 6. *With CRRA utility (and inelastic labor supply), the income-based repayment contract improves the agent's welfare relative to the fixed repayment contract.*

Proof. see Appendix A.6. □

However, this proposition may not hold if labor supply is sufficiently elastic, because the income-based repayment contract also naturally introduces an income-taxish distortion, resulting in an efficiency loss. Moreover, the proof of Proposition 6 also hinges on Proposition 4.[27] This implies that the reservation wage effect of income contingency plays a role in determining the agent's welfare.

In the following, I introduce elastic labor supply to elucidate the tradeoff between the two contracts and the implication of the reservation wage effect on welfare. To this end, I consider a simple mix of the two contracts by assuming that the lender is restricted to using a linear combination of the income-based repayment contract and the fixed repayment contract.[28]

In particular, the lender makes a fraction of debt mS income contingent, and the rest $(1-m)S$ is repaid under the fixed repayment contract, where $m \in [0,1]$. Under this linear contract, in each period the agent repays

$$s = \begin{cases} \alpha\theta + r(1-m)S & \text{if unemployed,} \\ \alpha z + r(1-m)S & \text{if employed with earnings } z = wl(w,\alpha), \end{cases} \tag{4.15}$$

where labor supply $l(w,\alpha)$ is a function of the wage rate w and the repayment ratio α.

For the lender to break even, the repayment ratio α is chosen to satisfy the recoverability constraint[29],

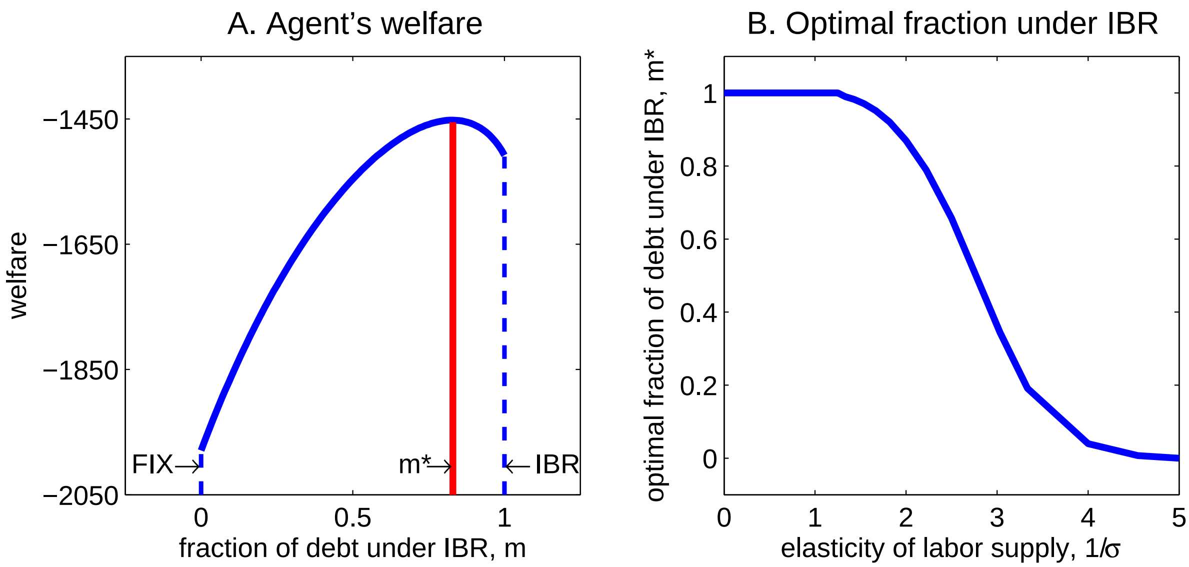
$$\frac{mD}{\beta} = \frac{F(w^*)}{1-\beta F(w^*)}\alpha\theta + \frac{\alpha}{(1-\beta)[1-\beta F(w^*)]}\int_{w^*}^{\overline{w}} wl(w,\alpha)dF(w). \tag{4.16}$$

[26] This result is related to Golosov, Maziero and Menzio (2013)'s insight that insuring search risks would allow agents to search for higher-paid jobs in a directed search model. It is also related to Belhaj, Bourles and Deroian (2014)'s insight that when income risks are endogenous, the agent would be willing to take more risks when there is risk sharing.

[27] For a general DARA utility, the proof is not obtained because it is not clear whether the income-based repayment contract raises the reservation wage.

[28] I consider this linear contract for its simplicity and transparency to illustrate the idea. It is also partially motivated by the numerical examples of Mirrlees (1971) that the optimal schedule is hardly different from an affine function with a constant marginal tax rate. However, numerical simulations from later research show that optimal tax schedules are very sensitive to the utility functions and income distributions.

[29] Because earnings depend on labor supply, which is a function of α, there is a Laffer curve for expected debt repayment, and there may not exist a solution to equation (4.16) when the debt level is high. My following numerical analyses consider the case in which there exist solutions to equation (4.16) and the smaller α is always selected. See Online Appendix B.2 for a related discussion.



Note: This figure illustrates the tradeoff between insurance and the incentive to work. I consider a simple mix of the two contracts by assuming that the lender is restricted to using a linear combination of the income-based repayment contract and the fixed repayment contract. Panel A plots the agent's welfare when the fraction of debt repaid under the income-based repayment contract varies from zero (corresponding to the pure fixed repayment contract) to one (corresponding to the pure income-based repayment contract). It shows that the agent's welfare first increases then decreases due to the benefit from insurance and the distortion on labor supply. The optimal fraction under this parametrization is given by an interior point m^*. Panel B plots the optimal fraction of debt under the income-based repayment contract when the elasticity of labor supply varies. A more elastic labor supply increases the distortion on labor supply, thus making the income-based repayment contract less desirable. The figure is plotted using the GHH utility, $u(c,l) = [c - \phi l^{1+\sigma}/(1+\sigma)]^{1-\gamma}/(1-\gamma)$, and beta distribution of wage offers, $Beta(a,b)$, with parameter values: $a = 2$, $b = 4$, $\gamma = 3$, $\theta = 0.1$, $\overline{w} = 1.1$, $\beta = 0.96$, $S = 1$, $\phi = 1$, $\sigma = 0.47$.

Figure 3: A numerical illustration of the agent's welfare and the optimal fraction of debt repaid under the income-based repayment contract.

I use GHH utility (Greenwood, Hercowitz and Huffman, 1988), $u(c,l) = \frac{1}{1-\gamma}\left(c - \phi \frac{l^{1+\sigma}}{1+\sigma}\right)^{1-\gamma}$, to provide several numerical examples. Panel A of Figure 3 shows that depending on parameter values, increasing m may increase or decrease the agent's welfare due to the the tradeoff in insurance and the incentive to work. The optimal fraction of debt repaid under the income-based repayment contract m^* that maximizes the agent's welfare could be an interior point. Intuitively, there are diminishing returns in providing insurance through the income-based repayment contract due to the decreasing marginal utility of consumption. On the other hand, the distortion on labor supply increases as a higher m increases the repayment ratio α. The optimal value of m^* is achieved when the marginal benefit from providing insurance is equal to the marginal cost of labor supply distortion.

In general, m^* could also be a corner solution, in which case the full income-based repayment contract is strictly better than the fixed repayment contract or vice versa. Panel B of Figure 3 indicates that whether the income-based repayment contract results in a higher welfare crucially depends on the elasticity of labor supply. This is because the elasticity of labor supply determines how responsive labor supply would be when a fraction of income is extracted by the lender. When labor supply is completely inelastic, the income-based repayment contract is strictly better as shown in Proposition 6. However, when labor supply is very elastic, the distortion on labor supply is large; so the fixed repayment contract results in a higher welfare.

In Figure 4, I illustrate that the insurance provided by the income-based repayment contract is more valuable due to the positive response in the reservation wage. In particular, I gradually increase m from 0 to 1 and compare the change in welfare and expected labor supply in two scenarios. In one scenario, I allow the agent to endogenously choose the reservation wage; in the other scenario I fix the reservation wage at the beginning. Because the income-based repayment contract raises the reservation wage, the reservation wage in the first scenario is increasing as m increases (see panel A of Figure 4).

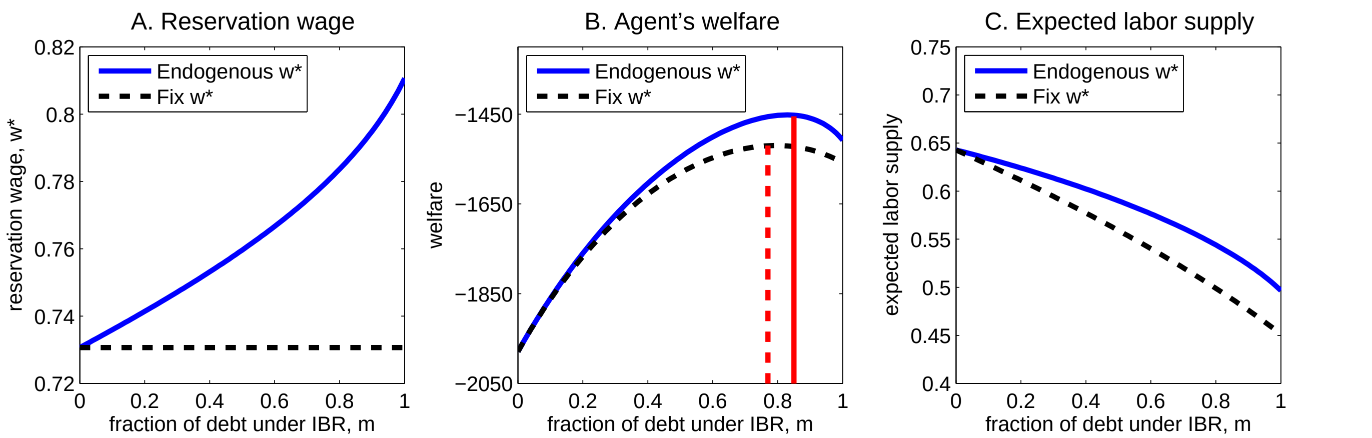
Panel B of Figure 4 indicates that the welfare is significantly higher in the first scenario. This illustrates that the income-based repayment contract increases welfare not only by directly providing insurance, but also by indirectly increasing the reservation wage. In other words, the insurance provided by the income-based repayment contract is more desirable when there are search risks because the agent would choose a higher reservation wage when search risks are partially insured. As a result, the optimal fraction of debt, m^*, is also higher in the first scenario.

Note that this result is not general and would be violated if the elasticity of labor supply is very large. In fact, when labor supply is elastic, the income-based repayment contract raises the reservation wage through an additional channel. This is because repaying debt as a fraction of income disproportionately reduces income more during employment relative to during unemployment because of the negative response in labor supply. This generates a "debt overhang" effect. The "debt overhang" channel not only reduces labor supply, but also further incentivizes the agent to set a higher reservation wage in order to stay unemployed.[30] When the elasticity of labor supply is sufficiently large, the reservation wage could be higher than the efficient one; as a result, fixing the reservation wage at some lower level could be welfare improving. See Appendix B for a detailed discussion.

Panel C compares the expected labor supply in the two scenarios. It shows that the negative effect on labor supply is smaller when the reservation wage is endogenous. This is due to two channels: first, there is a direct positive substitution effect on labor supply as the income-based repayment contract increases the average wage rate by raising the reservation wage. Second, there is an indirect effect due to a lower repayment ratio. This is because a higher reservation wage increases expected repayment conditional on any repayment ratio. Therefore, when the reservation wage increases, the lender would set a lower repayment ratio according to the recoverability constraint (4.7). This in turn alleviates the distortion on labor supply.

These numerical examples suggest that the positive response in the reservation wage under the income-based repayment contract offers a channel that not only increases the agent's welfare but also alleviates the distortionary effect on labor supply. These results highlight that despite the canonical tradeoff between insurance and the incentive to work, the income-based repayment contract is in fact more valuable compared to the fixed repayment contract because of uninsured search risks.

[30] I would like to highlight the distinction between the three channels: the risk channel, the liquidity channel, and the debt overhang channel. Although all three channels raise the reservation wage under the income-based repayment contract, they have divergent welfare implications. The increase in the reservation wage through the risk channel and the liquidity channel is a beneficial response to the correction of the credit and insurance market failures. However, the increase in the reservation wage through the debt overhang channel is a sub-optimal response to the distortion in the relative price of employment and unemployment.



Note: This figure illustrates the reservation wage effect. The blue solid line plots the agent's reservation wage, welfare, and expected labor supply when the reservation wage is allowed to increase as a larger fraction of debt is made income contingent. The black dashed line plots the agent's reservation wage, welfare, and expected labor supply when the reservation wage is fixed at the initial value under the pure fixed repayment contract (see panel A). Panel B shows that the agent's welfare is higher if the reservation wage is allowed to increase; as a result, the optimal fraction of debt repaid under the income-based repayment contract is also larger. Panel C shows that the reduction in labor supply also becomes smaller due to the higher reservation wage. The figure is plotted using the GHH utility, $u(c,l) = [c - \phi l^{1+\sigma}/(1+\sigma)]^{1-\gamma}/(1-\gamma)$, and the beta distribution of wage offers, $Beta(a,b)$, with parameter values: $a = 2$, $b = 4$, $\gamma = 3$, $\theta = 0.1$, $\overline{w} = 1.1$, $\beta = 0.96$, $S = 1$, $\phi = 1$, $\sigma = 0.47$.

Figure 4: A numerical illustration of the reservation wage effect.

4.4 Optimal Repayment Contract

In theory, the income-based repayment contract is not the most efficient way to provide insurance because the repayment ratio is constant regardless of the level of income. As proposed by Dynarski and Kreisman (2013), a progressive system of loan payments that rise with earnings could be welfare improving. In this subsection, I characterize the optimal repayment contract under the assumption that the reservation wage is not contractible. I show that the existence of search risks sets up the optimal contract that also considers the level of reservation wages. The novel implication is that the lender should provide more insurance in an economy with search risks, because this would increase the reservation wage. Therefore, the income-based repayment contract, although not constrained efficient, is designed in the spirit of the optimal repayment contract as it both provides insurance and increases the reservation wage.

To gain some insight, let us begin with the first-best contract. The first-best contract not only provides full insurance against search risks but also sets the reservation wage to $\hat{w}$ to maximize expected income. When labor supply is inelastic, the first-best contract is also incentive compatible because perfect insurance makes the agent indifferent about the reservation wage. This suggests that in contrast to a model without search risks, insurance is more desirable in my model because income risks are controlled by the agent's endogenous job search decisions. The full insurance provided by the first-best contract not only directly increases welfare through consumption smoothing; but also indirectly increases welfare by making a higher reservation wage incentive compatible.

When labor supply is elastic, the first-best contract is not incentive compatible because supplying labor generates disutility. The second-best contract solves the problem in which the lender chooses a nonlinear repayment schedule $\alpha(z)$ conditional on earnings $z = wl$ subject to the recoverability constraint and the agent's incentive compatibility constraints on labor supply and the reservation wage. This problem is more complicated compared to the optimal income taxation problem solved by Mirrlees (1971) as there is an additional incentive compatibility constraint on the reservation wage. As I discuss in Appendix C, if the agent accepts all wage offers arriving in the first period, the problem is mathematically the same as the one solved by Mirrlees (1971) when the government has a utilitarian social welfare function.

Because the closed-form solution of the second-best contract is not attainable, I use the perturbation approach of Saez (2001) to elucidate the economics underlying the second-best contract. In particular, I characterize the second-best contract in terms of the endogenous earnings distribution $H(z)$.[31]

Proposition 7. *Let $g(z)$ be the marginal utility of consumption at earnings z expressed in terms of the shadow cost of debt λ; $\zeta^c(z)$ be the Hicksian (compensated) elasticity of earnings at z; $\eta(z)$ be the income effect on earnings at z; ζ^{z^*} be the elasticity of expected repayment with respect to the reservation earnings z^*; and $u_z(z)$ be the derivative of utility with respect to earnings at z.*

The second-best contract $\alpha^{SB}(z)$ is implicitly determined by:

$$\begin{aligned}\underbrace{\int_z^\infty g(x)dH(x)}_{\text{direct welfare loss}} = \underbrace{1-H(z)}_{\text{mechanical effect, (+)}} & \underbrace{- \frac{z\zeta^c(z)\alpha^{SB}(z)'}{1-\alpha^{SB}(z)'+z\zeta^c(z)\alpha^{SB}(z)''}h(z)}_{\text{elasticity effect, (-)}} \\ & \underbrace{-\int_z^\infty \eta(x)\frac{\alpha^{SB}(x)'}{1-\alpha^{SB}(x)'+x\zeta^c(x)\alpha^{SB}(x)''}dH(x)}_{\text{income effect, (+)}} \\ & \underbrace{+\frac{S(1-\beta)\lambda\zeta^{z^*}}{u_z(z^*)z^*}\int_z^\infty g(x)dH(x)}_{\text{reservation wage effect, (-)}}. \end{aligned} \tag{4.17}$$

The marginal utility of consumption during unemployment is determined by:

$$\underbrace{H(z^*)g(\theta)}_{\text{direct welfare loss}} = \underbrace{H(z^*)}_{\text{mechanical effect, (+)}} + \underbrace{\frac{S(1-\beta)\lambda\zeta^{z^*}}{\beta u_z(z^*)z^*}g(\theta)}_{\text{reservation wage effect, (-)}}. \tag{4.18}$$

Proof. see Appendix C. □

Equation (4.17) characterizes the second-best contract. At the optimum, the direct welfare loss due to a higher marginal repayment rate in the earnings interval $(z, z+dz)$ should be balanced with the benefit of more debt collection, which consists of four effects. First, a higher marginal repayment rate raises debt collection through a mechanical effect as the agent needs to repay more when earnings are above z.

[31] The shape of $H(z)$ depends on the exogenous wage offer distribution $F(w)$ and the repayment schedule $\alpha(z)$. It is difficult to elucidate the economic intuitions in terms of $F(w)$ for a general utility function. But in the problem of Mirrlees (1971), Diamond (1998) provides an intuitive characterization for the optimal income tax using a quasi-linear utility function. I do not adopt this approach because the agent becomes risk neutral when utility is quasi-linear in consumption. As a result, search risks do not matter.

Second, there is a negative elasticity effect as the agent would reduce labor supply when her earnings are within interval $(z, z + dz)$ due to the higher marginal repayment rate. Third, there is a positive income effect as the agent would increase labor supply when earnings are above z due to higher repayment. These three effects are also existent in the formula derived by Saez (2001). The novel effect in equation (4.17) comes from the fourth term, the reservation wage effect.

As I discuss in Appendix B, it is reasonable to assume that ζ^{z^*} is positive, which implies that expected repayment increases with the reservation wage. Given that the shadow value of debt λ is negative, the reservation wage effect should be negative. Therefore, relative to what would be if the reservation wage were non-responsive, a lower optimal marginal tax rate is implemented in the earnings interval in which the reservation wage effect is large. This is because when the lender designs the optimal contract, he also needs to consider the endogenous movement in the reservation wage, and to some extent, try to increase it. Setting a high marginal repayment rate at the earnings where the reservation wage is more responsive would reduce the option value of staying unemployed more and disincentivize the agent from searching for better jobs, which is harmful for debt collection.

The term $\int_z^\infty g(x)dH(x)$ implies that the reservation wage effect is decreasing in z. This is because increasing the marginal repayment rate in $(z, z + dz)$ increases repayment for all earnings $x > z$. Loosely speaking, the decreasing reservation wage effect makes the repayment schedule more progressive, and the lender would have a larger incentive to equalize earnings during employment.[32]

Equation (4.18) determines the marginal utility of consumption during unemployment. Because the lowest earnings are obtained during unemployment, equation (4.18) in fact implicitly determines the intercept of the second-best contract, $\alpha^{SB}(\theta)$. If the agent's reservation wage were non-responsive, then the reservation wage effect is absent in equation (4.18). In this case, the second-best contract subsidizes unemployment so that $g(\theta) = 1$, i.e., the marginal utility of consumption during unemployment is equal to the shadow cost of debt. This is because there is no behavioral response in labor supply during unemployment; thus it is always optimal to equalize the cost of funds to the marginal utility of consumption when the agent is unemployed. However, because the agent's reservation wage is responsive, the negative reservation wage effect incentivizes the lender to set $g(\theta) < 1$, subsidizing the agent more during unemployment, which is financed by higher repayment during employment. Intuitively, this is because providing more liquidity during unemployment increases the agent's reservation wage, which would raise expected repayment.

In sum, the discussion above indicates that in the presence of search risks, the lender has the incentive to provide more insurance both by flattening the income distribution during employment and by subsidizing the income during unemployment. Insurance provision is more valuable because consumption smoothing also indirectly raises the agent's reservation wage. Therefore, the security design in an environment with search risks should take into account both the canonical tradeoff between insurance and the incentive to work, and importantly, the response in the reservation wage. Although the income-based repayment contract is not constrained efficient, it embodies such a concern as it provides insurance and increases the reservation wage.

[32]Note that the argument for higher progressivity is not meant to be rigorous because it also depends on how responsive $\alpha^{SB}(x)$ is for different earnings x.

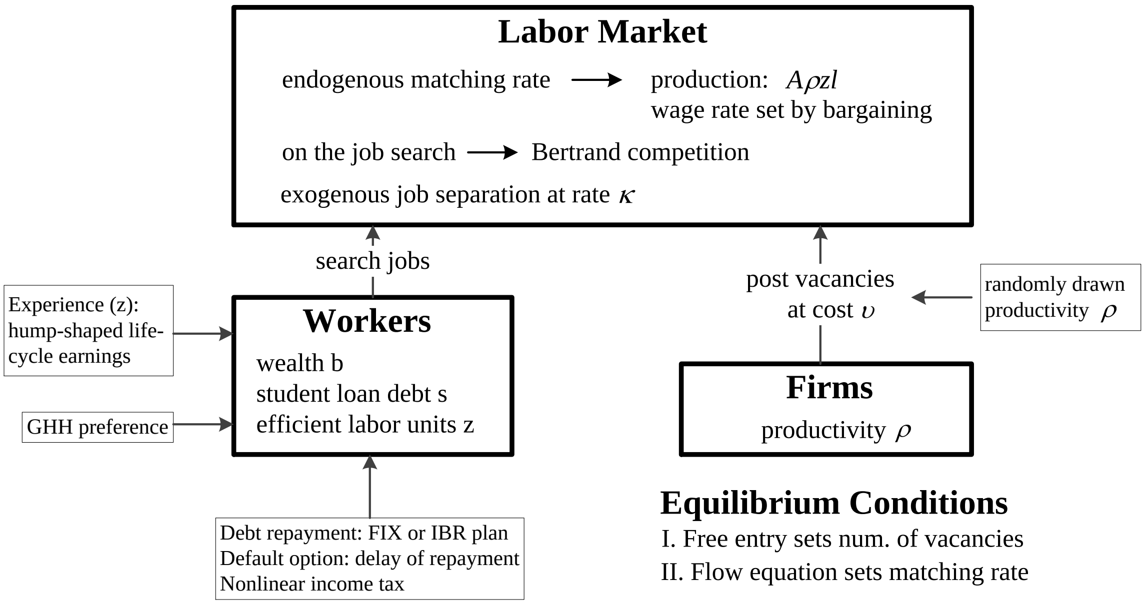


Figure 5: An overview of model ingredients.

5 Quantitative Model

My theoretical framework illustrates the mechanism through which debt repayment affects individuals' job search decisions. However, a better assessment of the mechanism and the repayment policy calls for a richer quantitative model. In this section, I develop such a model to quantitatively investigate the implication of debt burden on labor market outcomes.

5.1 Model Overview

The quantitative model is developed based on the theoretical framework with the inclusion of additional ingredients. Figure 5 presents an overview of model structure. I consider a life-cycle economy with overlapping generations, in which there are heterogeneous workers searching for jobs and firms posting vacancies. Workers are different from each other in terms of wealth, student loan debt, and efficient labor units. Firms are ex-ante identical and post vacancies, whose productivity is randomly drawn. Workers and firms meet in the labor market at endogenous matching rates. Employed workers meet other vacancies through on-the-job search and become unemployed after job separation.

Compared to the theoretical framework, the quantitative model has the following additional ingredients, which are introduced not only to match the data on labor market characteristics, but also because they are crucially related to the quantitative importance of the proposed mechanism.

(1). I introduce age-specific efficient labor units to capture the hump-shaped life-cycle earnings profile. Under the fixed repayment plan, borrowers are required to repay debt immediately after college graduation while earnings are low. Thus capturing the life-cycle earnings profile will increase the effect of debt burden on job search decisions through the liquidity channel.

(2). I introduce a realistic default option that allows borrowers to delay repayment. Default provides some sort of insurance, which mitigates the effect of debt burden on job search decisions through the risk channel and the liquidity channel. Thus without introducing default, the model would over-estimate the debt burden.

(3). I introduce on-the-job search and job separation. Intuitively, both features reduce the value of staying unemployed through the optionality channel. Thus without these ingredients, the model would over-estimate the effect of debt burden on job search as the importance of searching for jobs by staying unemployed is exaggerated. In the extreme case, if searching during unemployment is as efficient as searching during employment, then the reservation wage is always equal to UI benefits (Lise, 2013), and the proposed mechanism is absent. Therefore, it is important to introduce these realistic features and ask the data to determine the relative efficiency.

(4). I introduce nonlinear income taxes. Introducing income taxation is important for the quantitative implication of the income-based repayment plan because progressive taxation provides partial insurance, and the distortion on labor supply from income-contingency increases with the income taxes facing indebted agents.[33]

(5). I introduce vacancy posting to endogenize the matching rate and the wage offer distribution. This is to capture the potential equilibrium effect after a large-scale policy change. The model is able to capture two general equilibrium forces: first, borrowers' change in job search decisions would affect non-borrowers' job contact rates by affecting aggregate search effort. Second, workers' change in borrowing decisions would affect firms' vacancy posting decisions by affecting their profit. I use the concept of Nash bargaining and Bertrand competition to determine the wage rate for comparison with the existing literature.

5.2 Workers

There is a continuum of agents of measure one in each cohort who lives for T years. In each year, the oldest cohort of agents dies at age T and a new cohort of agents is born with initial wealth b_0 and student loan debt s_0 randomly drawn from the cumulative distribution function $\Psi(b, s)$, which is estimated using data on college graduates. The assumption that all cohorts of agents are born with the same initial distribution of wealth and student loan debt enables a stationary equilibrium, in which the distribution of agents at the same age is the same across cohorts, although different cohorts reach the same age in different periods.[34] Therefore, in the following, I will describe agents' problem using age index t.

At age $t = 1$, agents enter the labor market as unemployed workers and start job search. Agents' efficient labor units are denoted by z_t. I assume that z_t is determined by

$$\ln z_t = g(t), \tag{5.1}$$

[33]As Stiglitz (2015) points out, the adverse incentive effects from income-based repayment plans are likely to be small, so long as income tax rates and repayment rates combined are not too large.

[34]When evaluating the income-based repayment plan, I also focus on the stationary equilibrium and do comparative statics. The transitional dynamics after the policy change is not very tractable and thus is not analyzed in this paper.

where $g(t)$ is a deterministic trend, which is the same across all agents and only depends on labor market experience t. Following Bagger et al. (2014), I assume the deterministic trend $g(t)$ to be cubic,

$$g(t) = \mu_0 + \mu_1 t + \mu_2 t^2 + \mu_3 t^3. \tag{5.2}$$

Parameters μ_0, μ_1, μ_2, μ_3 are estimated to match the trend in individuals' life-cycle earnings profile that cannot be explained by on-the-job search.

For tractability, I do not consider individual heterogeneity in equation (5.1), a point emphasized by Huggett, Ventura and Yaron (2006) and Bagger et al. (2014). Moreover, the assumption that efficient labor units depend on the number of years in the labor market instead of the number of years in a particular job greatly simplifies the problem as z_t is homogeneous within the same cohort.

Agents have per period utility $u(c, l)$ and discount factor β. I model $u(c, l)$ using the GHH preference (Greenwood, Hercowitz and Huffman, 1988),

$$u(c, l) = \frac{1}{1-\gamma}\left(c - \phi \frac{l^{1+\sigma}}{1+\sigma}\right)^{1-\gamma}, \tag{5.3}$$

where c and l are consumption and labor supply. The benefit of using the GHH preference is that labor supply has a closed-form solution, which greatly simplifies computations. However, the GHH preference does not have an income effect, which suggests that individuals under debt burden would supply more labor and obtain higher earnings.

5.3 Firms

Agents are matched pairwise to jobs, which are created by firms. Following the standard in the literature on search-theoretic models, each firm only creates one job vacancy, thus I do not distinguish between firms and jobs. Jobs are heterogeneous in productivity ρ. There are no productivity shocks, therefore job productivity is constant for a worker-job match until the match breaks up.

Jobs are either vacant or matched with workers and workers are either unemployed or matched with jobs. To simplify notations, I denote $\Omega = (b, s, z)$ as the worker's characteristic. Denote $\phi^u(\Omega)$ as the PDF (i.e., Probability Density Function) of unemployed workers, $\phi^e(\Omega, \rho)$ as the PDF of employed workers matched with jobs whose productivity is ρ, and $v(\rho)$ as the PDF of vacancies. Denote $\Phi^u(\Omega)$, $\Phi^e(\Omega, \rho)$, and $V(\rho)$ as their CDFs (i.e., Cumulative Distribution Function). Denote N_v as the number of vacancies and $\overline{u}$ as the unemployment rate. Because I focus on the stationary equilibrium, all these distributions are time independent.

The number of type-ρ vacancies is

$$N_v(\rho) = N_v v(\rho). \tag{5.4}$$

Because each generation has measure one, and there are T overlapping generations, the number of type ρ jobs in the economy is

$$N(\rho) = (1-\overline{u})T \int \phi^e(\Omega, \rho) d\Omega + N_v(\rho). \tag{5.5}$$

The total number of jobs is

$$N = \int N(\rho) d\rho. \tag{5.6}$$

When a worker Ω is matched with a job ρ, they jointly produce a flow of output using the following production technology:

$$F = Az\rho l, \tag{5.7}$$

where A represents the aggregate productivity that is the same across all firms. Production only uses labor and has constant returns to scale as in Postel-Vinay and Robin (2002). Moreover, match-specific productivity is proportional to job productivity ρ and workers' efficient labor units z. Note that this multiplicative specification implies that job productivity and efficient labor units are complementary, which reflects the specification of Bagger et al. (2014) and is supported by the structural estimation of Lise, Meghir and Robin (2016) using NLSY data on college graduates.

5.4 Labor Market

Matching Job search is a random matching process. Agents contact jobs at endogenous rates that depend on their search effort and the number of vacancies. I allow for on-the-job search and assume that unemployed agents have search efficiency s^u and employed agents have search efficiency s^e.[35] The assumption that search efficiency or job contact rates are different during unemployment and employment is standard in the search literature. For example, Postel-Vinay and Robin (2002) estimate a model with on-the-job search and find that job contact rates are uniformly higher during unemployment across a wide range of occupations.

Denote S as the aggregate level of search effort contributed by both unemployed and employed agents:

$$S = s^u \overline{u} T + s^e (1 - \overline{u}) T. \tag{5.8}$$

The total number of meetings is determined by a Cobb-Douglas matching function,

$$M = \chi S^{\omega} N_v^{1-\omega}, \tag{5.9}$$

where χ and ω are two parameters governing the matching efficiency.

From a vacancy's perspective, the probability of contacting a worker is

$$q = M / N_v. \tag{5.10}$$

[35]In my model, search efficiency is governed by a fixed parameter as in Lise and Robin (2016). A model with endogenous search intensity predicts that indebted unemployed workers would search more and exit unemployment faster, and that indebted employed workers would also search more and change jobs more frequently. The former prediction is already captured by the reservation wage. The quantitative implication of endogenizing search intensity is likely to be small, because the model can roughly replicate the differential unemployment duration and wage income between borrowers and non-borrowers (see subsection 6.3). The latter prediction is not supported by the data, i.e., job-to-job transition rates are similar between borrowers and non-borrowers. Relatedly, introducing search intensity also generates an identification issue. As discussed in Lise (2013), the scale parameter of search effort is not identified without direct observation of search effort. The elasticity parameter can be identified using the variation in unemployment and employment duration across workers with different assets, student loans, and wages. However, these variations are noisy in my sample, indicating that search effort is not likely to be a meaningful decision made by workers. This observation seems to be consistent with the argument of Krusell et al. (2011).

The job contact rates for unemployed workers and employed workers are

$$\lambda^u = s^u M/S; \quad \lambda^e = s^e M/S. \tag{5.11}$$

Denote $W(\Omega, \rho, w)$ as the value of an employed agent Ω in job ρ at wage rate w, $U(\Omega)$ as the value of an unemployed agent Ω, and $J(\Omega, \rho, w)$ as the value of a filled job ρ that pays wage rate w. The value of a vacancy is zero due to the free entry condition. In general, these value functions also depend on age t due to the finite life of agents. This dependence is implicitly captured by efficient labor units z because there is a one-to-one mapping between z and t as specified in equation (5.1).

When an agent and a job meet each other, a match is formed if there exists wage rate w, such that the worker is willing to accept the job and the firm is willing to hire the worker.[36] Thus the participation constraints are

$$W(\Omega, \rho, w) \geq U(\Omega); \tag{5.12}$$

$$J(\Omega, \rho, w) \geq 0. \tag{5.13}$$

Matches break up at an exogenous rate κ. After job separations, workers flow into unemployment as jobs disappear.

Wage Negotiation with Unemployed Workers I consider piece-rate wage contracts as in Bagger et al. (2014). An unemployed worker receives UI benefits θ in every period.[37] The wage income is given by the wage rate w specified in the contract multiplied by the units of labor supply l. Upon forming a worker-firm match, the wage rate is determined through Nash bargaining:

$$w^u(\Omega, \rho) = \underset{w}{\operatorname{argmax}} [W(\Omega, \rho, w) - U(\Omega)]^{\xi} J(\Omega, \rho, w)^{1-\xi}, \tag{5.14}$$

where ξ represents the worker's bargaining power.

Note that the usual linear sharing rule (Pissarides, 1994) is no longer a solution to the Nash Bargaining problem due to the introduction of several features, e.g., risk-averse agents, labor supply, and on-the-job search.[38] Therefore, the wage rate is determined by solving the full maximization problem.

I focus on short-term wage contracts, thus the wage rate is renegotiated in every period, reflecting the change in Ω. The assumption of Nash bargaining links workers' wage rates to their characteristics, implying that wealth, student loan debt, and efficient labor units can influence income. As argued by Krusell, Mukoyama and Sahin (2010), it is logical to assume that workers have the incentive to bargain for higher wages if outside options are strong. Moreover, the results under Nash bargaining are useful

[36] In a search model with risk neutral agents, a necessary and sufficient condition for match formation is that there exists wage rate w, such that the match surplus $W(\Omega, \rho, w) - U(\Omega) + J(\Omega, \rho, w)$ is positive. This condition is no longer sufficient in my model because the linear sharing rule is not a solution to the Nash bargaining problem.

[37] This is an unrealistic assumption I make to simplify computations. In the U.S., the standard time-length of unemployment compensation is six months, although extensions are possible during economic downturns. Incorporating this realistic feature requires an additional state variable to record the time of job separation. To parsimoniously deal with the concern of too much UI benefits, I set a relatively lower value for the parameter θ in my quantitative analysis.

[38] As Shimer (2003) shows, essentially it is because the derivatives of worker's value and job's value with respect to the wage rate are not the same.

for comparison with the existing literature, because it is the most commonly used assumption under risk neutrality.

On-the-Job Search and Poaching I adopt the sequential auction framework of Postel-Vinay and Robin (2002) to model the wage determination during on-the-job search.

The firm's participation constraint (5.13) implies that the highest wage rate that firm ρ can offer to worker Ω is its marginal product of labor, $Az\rho$. Because $W(\Omega, \rho, w)$ is increasing in the wage rate, $W(\Omega, \rho, Az\rho)$ is the highest value that firm ρ can offer to worker Ω. I define this as the the maximal employment value.

Definition 3. *The maximal employment value, denoted by $\overline{W}(\Omega, \rho)$, is the value of worker Ω being employed by firm ρ when the wage rate is set equal to the marginal product of labor $Az\rho$,*

$$\overline{W}(\Omega, \rho) = W(\Omega, \rho, Az\rho). \tag{5.15}$$

The marginal product of labor increases with job productivity ρ, thus more productive firms can offer higher wage rates to workers. This implies that the maximal employment value that a worker can obtain, $\overline{W}(\Omega, \rho)$, increases with job productivity ρ. Because on-the-job search is modeled based on Bertrand competition, the job with higher productivity will keep the worker. Therefore, on-the-job search may trigger job-to-job transitions or wage renegotiations, depending on the relative productivity of the two jobs competing for the worker.

To elaborate, consider a worker Ω working in a job with productivity ρ' and wage w', poached by a new job with productivity ρ. If the maximal employment value of the new job ρ is smaller than the current job's value, i.e., $\overline{W}(\Omega, \rho) < W(\Omega, \rho', w')$, then the worker will discard the new job offer and stay with the current job with the old wage w'.

If the new job can offer a higher job value, then the two jobs will compete to bid up the wage rate. The job with higher productivity is able to overbid the other job and thus keep the worker. There are two cases:

First, if $\rho > \rho'$, the worker currently employed at job ρ' will transfer to job ρ and the old job ρ' will become the negotiation benchmark due to Bertrand competition. This grants the worker an outside option value that is equal to the maximal employment value of ρ'. The new wage rate will be set according to

$$w^e(\Omega, \rho, \rho') = \underset{w}{\operatorname{argmax}}[W(\Omega, \rho, w) - \overline{W}(\Omega, \rho')]^{\xi} J(\Omega, \rho, w)^{1-\xi}, \tag{5.16}$$

where the worker's outside option is captured by the old job's productivity ρ'.

Second, if $\rho \leq \rho'$, the worker will stay with the current employer ρ', but job ρ will be used as the new negotiation benchmark for a wage rise. This grants the worker an outside option value that is equal to the maximal employment value of ρ. The new wage rate will be set to

$$w^e(\Omega, \rho', \rho) = \underset{w}{\operatorname{argmax}}[W(\Omega, \rho', w) - \overline{W}(\Omega, \rho)]^{\xi} J(\Omega, \rho', w)^{1-\xi}. \tag{5.17}$$

Note that in principle it is not clear whether job-to-job transitions increase the wage rate. This is because the wage rate, $w^e(\Omega, \rho, \rho')$, offered by job ρ could be higher or lower than the highest wage rate that could be offered by the negotiation benchmark (i.e., its marginal product of labor, $Az\rho'$) due to the existence of two countervailing forces. On the one hand, job ρ provides the agent a better negotiation benchmark when poached by a more productive job in the future. Therefore, job ρ offers higher potential wage growth resulting from future on-the-job search, which enables a lower current wage rate to keep the worker. On the other hand, job ρ should also directly provide the worker a higher wage rate through Nash bargaining because total surplus increases with job productivity. As a consequence, the relationship between $w^e(\Omega, \rho, \rho')$ and $Az\rho'$ crucially depends on the worker's bargaining power ξ. In the extreme case with $\xi = 0$, firms do not share surplus with workers, and $w^e(\Omega, \rho, \rho')$ is always below $Az\rho'$. At the estimated parameter value, $\xi = 0.45$, it holds that the wage rate offered by the current job is higher than the negotiation benchmark's marginal product of labor, so that job-to-job transitions result in a wage increase.

Reservation Productivity Equation (5.16) nests equation (5.14), if we treat an unemployed agent Ω as being employed in a fictitious job $\rho_u(\Omega)$, such that $\overline{W}(\Omega, \rho_u(\Omega)) = U(\Omega)$. Hence, the negotiation benchmark for an unemployed agent is $\rho_u(\Omega)$ and the wage rate satisfies

$$w^u(\Omega, \rho) = w^e(\Omega, \rho, \rho_u(\Omega)). \tag{5.18}$$

In fact, $\rho_u(\Omega)$ can be considered as the reservation productivity for an unemployed agent Ω, because she is indifferent between being employed at job $\rho_u(\Omega)$ or staying unemployed. On the other hand, job $\rho_u(\Omega)$ is also indifferent about hiring because it is offering the worker the maximal employment value. I define this formally as follows:

Definition 4. *The reservation productivity for an unemployed agent Ω is a fictitious job with productivity $\rho_u(\Omega)$ such that the agent is indifferent between accepting the job or staying unemployed, i.e.,*

$$\overline{W}(\Omega, \rho_u(\Omega)) = U(\Omega). \tag{5.19}$$

For any reservation productivity $\rho_u(\Omega)$, the corresponding marginal product of labor, $Az\rho_u(\Omega)$, can be considered as the reservation wage of an unemployed agent Ω. It is difficult to obtain a formal proof on how the reservation productivity changes with the level of student loan debt, but the intuition is exactly the same as what is discussed in section 4. Therefore, indebted agents set lower reservation productivity and search for a shorter time. Moreover, there are additional forces from Nash bargaining that would affect the reservation wage rate. Because agents are risk averse, a higher level of debt generates two countervailing forces on the wage rate. On the one hand, a higher debt reduces the value of the outside option, which reduces the wage rate for the worker. On the other hand, a higher debt increases the marginal value of liquidity for the worker at the current job, which increases the wage rate for the worker. The second force comes from the fact that with risk-averse agents, the linear sharing rule fails to maximize the Nash bargaining product. Due to the two offsetting forces, the impact of student loan debt on the wage rate through the bargaining channel is small.[39]

5.5 Repayment, Default, and Taxes

Repayment As noted in section 2, federal student loan borrowers can choose from among different repayment plans, but most of them repay under the fixed repayment plan during my sample period. Therefore, I only consider the fixed repayment plan when estimating the model.

I assume that student loan borrowers make fixed payments every period after college graduation until the 10th period. This is consistent with the terms specified in the standard 10-year fixed repayment plan. The interest rate for the fixed repayment plan is variable before July 1, 2006, and fixed thereafter. For simplicity, I consider a fixed interest rate r^s. Hence, the annual payment is given by the standard annuity formula:

$$y_t^{fix} = \frac{r^s}{(1+r^s)\left[1-\frac{1}{(1+r^s)^{10-(t-1)}}\right]} s_t, \quad \text{for } t <= 10. \tag{5.20}$$

Default Unlike other loans, student loans are practically non-dischargeable after default (and bankruptcy). I assume that borrowers incur a cost η if they default on their loans. In the year following the default, borrowers negotiate a new repayment plan that has the same repayment period as the fixed repayment plan.[40] Modeling default option in this way ensures that default time is not a state variable. As a result, in my model default delays the repayment by one period, but the payment in each of the following periods will increase. Moreover, I do not allow repeated default given the complexity of the current setup.[41] If agents default at time t_{def}, the annual payment thereafter is

$$y_t^{def} = \begin{cases} 0, & \text{for } t = t_{def}. \\ \dfrac{r^s}{(1+r^s)\left[1-\frac{1}{(1+r^s)^{10-(t-1)}}\right]} s_t, & \text{for } t_{def} < t <= 10. \end{cases} \tag{5.21}$$

It is also possible that deeply indebted agents may not be able to honor the payment if they had been unemployed for a long time. While this is theoretically possible, it rarely happens in simulations because very few agents take on large debt in the sample. If this involuntary delinquency happens, I assume that agents have to repay all earnings (up to a consumption floor specified below) in every following period until all the past payments required under the fixed repayment plan are repaid.

[39] The impact of the bargaining channel could be large when the level of student loan debt is very high, which is not the case in my estimation sample. This result is also consistent with Krusell, Mukoyama and Sahin (2010)'s finding that wages derived by Nash bargaining are not responsive to the level of wealth unless wealth is very low.

[40] In reality, borrowers can get rehabilitation on their defaulted loans after consequently making several eligible payments. Then borrowers must agree with the U.S. Department of Education on a reasonable and affordable repayment plan. The repayment plans after default are set case by case. Generally, a monthly payment is considered to be reasonable and affordable if it is at least 1.0% of the current loan balance. Volkwein et al. (1998) find that two out of three defaulters reported making payments shortly after the official default first occurred.

[41] In practice, loan rehabilitation is a one-time opportunity, and more severe punishments are imposed on borrowers who default repeatedly.

Income Taxes Agents face progressive income taxes. Following Benabou (2002) and Heathcote, Storesletten and Violante (2014), I model after-tax income y as:

$$y = \varkappa(wl)^{1-\tau}, \tag{5.22}$$

where wl is the pre-tax wage income.

In the U.S., UI benefits are also taxable, thus the formula for unemployed workers is $y = \varkappa\theta^{1-\tau}$. The fiscal parameters $\varkappa$ and τ are set to approximate the U.S. income tax system. The parameter $\varkappa$ determines the overall level of taxation. The parameter τ determines the rate of progressivity because it reflects the elasticity of after-tax income with respect to pre-tax income. When $\tau = 0$, the tax system has a flat marginal tax rate $1 - \varkappa$, and when $\tau > 0$, the tax system is progressive. The values of these two parameters can be inferred indirectly from the data (see section 6).

In the baseline simulation, I assume that the tax revenue is collected to finance the UI benefits and a non-valued public consumption good G:

$$(1-\overline{u})T \iint wl[1-\varkappa(wl)^{-\tau}]\phi^e(\Omega,\rho)d\Omega d\rho = \overline{u}T \int \varkappa\theta^{1-\tau}\phi^u(\Omega)d\Omega + G. \tag{5.23}$$

Because public spending G is non-valued, we can think of this as a residual term being introduced to balance the government budget equation (5.23) (see Heathcote, Storesletten and Violante, 2014).[42]

When conducting the quantitative analyses in section 7, I take the value of G from the baseline as exogenously given. When evaluating the income-based repayment plan, I adjust the parameter $\varkappa$ to balance the budget:

$$(1-\overline{u})T \iint wl[1-(\varkappa-\Delta\varkappa)(wl)^{-\tau}]\phi^e(\Omega,\rho)d\Omega d\rho = \overline{u}T \int (\varkappa-\Delta\varkappa)\theta^{1-\tau}\phi^u(\Omega)d\Omega + G + \text{Forgiveness}. \tag{5.24}$$

The implied value of $\Delta\varkappa$ captures the increase in overall tax level in order to finance the debt forgiveness.

5.6 Value Functions

The timing of events is presented in Figure 6. At the beginning of age t, firms post vacancies at cost ν and existing matched jobs separate at rate κ. Vacancies and agents meet each other at Poisson rates, λ^u, λ^e, and q. Agents then make default decisions (if not yet in default) and repay student loan debt. At the end of age t, unemployed agents receive UI benefits θ, and employed agents supply labor l and negotiate wage rates w with firms based on their negotiation benchmarks' productivity. After receiving income, agents pay income taxes and choose consumption c_t. Following Hubbard, Skinner and Zeldes (1995), I introduce a consumption floor, $\underline{c}$, to model means-tested benefits.[43]

Instead of using the wage rate w as a state variable for an employed worker, the discussions in subsection 5.4 suggest that the negotiation benchmark's productivity is a natural state variable. Therefore,

[42]An alternative way to model the government budget constraint is to assume that the tax revenue net of UI benefits is redistributed equally to all agents. This approach requires us to find a fixed point for the lump-sum rebate, because the

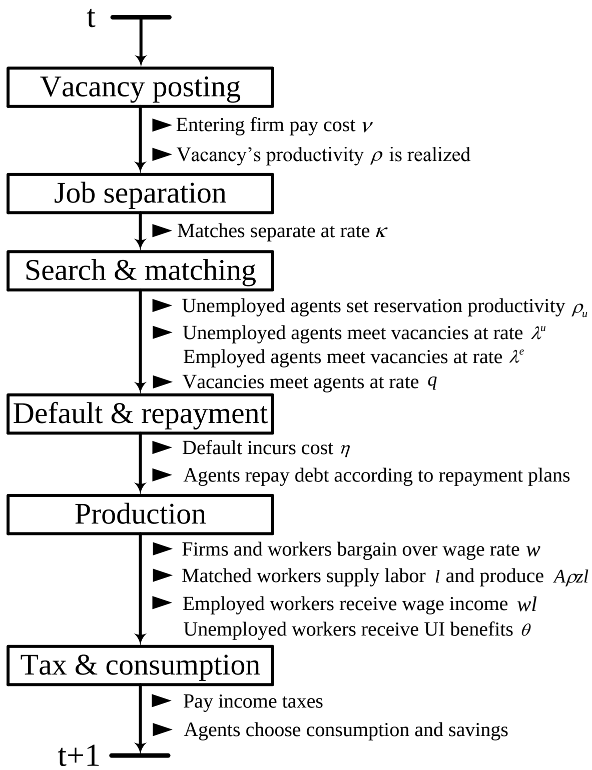


Figure 6: The timing of events.

the state variables are worker characteristic Ω, job productivity ρ, and the negotiation benchmark's productivity ρ'. The value of an employed worker and the value of a job immediately after search and matching can be written as $W(\Omega, \rho, \rho')$ and $J(\Omega, \rho, \rho')$ before default. I add superscript d to represent value functions and variables after default. Below I present the value functions of each participant.

Unemployed Workers An unemployed worker who has defaulted has value

$$U^d(\Omega_t) = \max_{c_t, l_t} \quad u(c_t, l_t) + \beta \Bigg[\underbrace{\lambda^u \int_{x \geq \rho_u^d} W^d(\Omega_{t+1}, x, \rho_u^d) dV(x)}_{\text{accept the job}} + \underbrace{[1 - \lambda^u + \lambda^u V(\rho_u^d)] U^d(\Omega_{t+1})}_{\text{not accept the job}} \Bigg],$$

$$\begin{aligned} \text{subject to} \quad b_{t+1} &= (1+r)(b_t - y_t^{def}) + \varkappa\theta^{1-\tau} - c_t, \\ s_{t+1} &= (1+r^s)(s_t - y_t^{def}), \\ c_t &\geq \underline{c}, \\ b_{t+1} &\geq 0, \end{aligned} \tag{5.25}$$

lump-sum rebate would affect agents' decisions, which would in turn affect total tax revenue.

[43]The impact of student loans on the poor would be exaggerated without this consumption floor. This is because for individuals who need to repay more than what they have, they would be pushed toward zero consumption in that period (and delinquent on part of the repayment). With GHH utility, the local absolute risk aversion is infinite when consumption is zero. As a result, the debt burden will generate an unreasonably large effect on these individuals because they are willing to accept any jobs with wages that enable them to honor the full debt payment. In the simulation, few borrowers hit the consumption floor because of the precautionary savings and UI benefits. The consumption floor is introduced also to initialize the value functions in the final period (see Appendix D.2.2).

where r is the interest rate on deposit and ρ_u^d is the reservation productivity for the unemployed worker Ω_{t+1} who has defaulted. In the objective function, the term $u(c_t, l_t)$ represents the realized utility at age t; the first term in the squared bracket represents the expected value of entering the labor market at age $t+1$; and the second term represents the value of staying unemployed, which could happen when the productivity draw is less than the reservation productivity, $\rho_u^d(\Omega_{t+1})$.

Following Acemoglu and Shimer (2000) and Krusell, Mukoyama and Sahin (2010), I impose the borrowing constraint, $b_{t+1} \geq 0$, so that agents do not have access to other credit apart from student loans. Relaxing this constraint enables the model to parsimoniously capture other types of loans, e.g., consumption loans. I provide a robustness check for the credit limit in section 8. An unemployed worker who has not defaulted yet has the option to default, and her value function $U(\Omega_t)$ can be derived similarly (see Online Appendix C.1).

Employed Workers The value of defaulted employed workers at job ρ, with negotiation benchmark ρ' is given by

$$
\begin{aligned}
W^d(\Omega_t, \rho, \rho') = \max_{c_t, l_t} \quad & u(c_t, l_t) + \beta \Bigg\{ \underbrace{\kappa U^d(\Omega_{t+1})}_{\text{job separation}} + \underbrace{(1-\kappa) \Bigg[[1 - \lambda^e + \lambda^e V(\rho')] W^d(\Omega_{t+1}, \rho, \rho')}_{\text{not poached or poached by a low vacancy}} \\
& + \lambda^e \Bigg(\underbrace{\int_{x \geq \rho} W^d(\Omega_{t+1}, x, \rho) dV(x)}_{\text{transition to a new vacancy}} + \underbrace{\int_{\rho' < x < \rho} W^d(\Omega_{t+1}, \rho, x) dV(x)}_{\text{negotiation for a wage rise}} \Bigg) \Bigg] \Bigg\}, \\
\text{subject to} \quad & b_{t+1} = (1+r)(b_t - y_t^{def}) + \varkappa [w^{e,d}(\Omega_t, \rho, \rho') l_t]^{1-\tau} - c_t, \\
& s_{t+1} = (1+r^s)(s_t - y_t^{def}), \\
& c_t \geq \underline{c}, \\
& b_{t+1} \geq 0,
\end{aligned} \tag{5.26}
$$

In problem (5.26), the first term in the curly bracket captures exogenous job separations at rate κ, in which case the worker becomes unemployed in period $t+1$, and receives $U^d(\Omega_{t+1})$. The job is maintained with probability $1-\kappa$, and the three cases resulting from on-the-job search are captured by the second term. With probability λ^e, the worker gets contacted by a new job x. If the new job's productivity x is larger than the current job ρ, the worker moves to the new job and her current job becomes the new negotiation benchmark. If she samples a job with productivity larger than the current negotiation benchmark but smaller than her current job's productivity, she will stay at the current job with an updated negotiation benchmark. Finally, she may stay with the current job with an unchanged negotiation benchmark either when she is not poached by a new job or the new job's productivity is lower than her current negotiation benchmark. An employed worker who has not defaulted yet has the option to default, and her value function $W(\Omega_t, \rho, \rho')$ can be derived similarly (see Online Appendix C.1).

Filled Jobs and Match Surplus The value of a job filled by a worker who has defaulted is,

$$J^d(\Omega_t,\rho,\rho') = \underbrace{[Az_t\rho - w^{e,d}(\Omega_t,\rho,\rho')]l^d(\Omega_t,\rho,\rho')}_{\text{production profit in current period}}$$
$$+\beta(1-\kappa)\Bigg[\underbrace{\lambda^e \int_{\rho'<x<\rho} J^d(\Omega_{t+1},\rho,x)dV(x)}_{\text{negotiation for a wage rise}} + \underbrace{[1-\lambda^e+\lambda^e V(\rho')]J^d(\Omega_{t+1},\rho,\rho')}_{\text{not poached or poached by a low vacancy}}\Bigg], \tag{5.27}$$

where the first term in the squared bracket represents the case in which the poaching job results in a wage increase by raising the negotiation benchmark. The second term represents the case in which the worker does not receive a competitive outside offer.

The match surplus relative to unemployment is given by

$$S^d(\Omega_t,\rho,\rho') = W^d(\Omega_t,\rho,\rho') - U^d(\Omega_t) + J^d(\Omega_t,\rho,\rho'). \tag{5.28}$$

The value of a job filled by a worker who has not defaulted yet is $J(\Omega_t,\rho,\rho')$ and the match surplus is $S(\Omega_t,\rho,\rho')$ (see Online Appendix C.2).

5.7 Stationary Competitive Equilibrium

To close the model, I describe the free entry condition and the flow equations, and define the stationary equilibrium. The free entry condition determines the equilibrium number of vacancies N_v. The flow equation determines the equilibrium unemployment rate $\overline{u}$.

Free Entry Condition The cost of vacancy creation is ν. Following Lise, Meghir and Robin (2016), I assume that once the firm pays the cost, a job is created with productivity ρ being randomly drawn from a CDF $F(\rho)$.[44]

Vacancies last for one period; thus if the created vacancy is not filled by a worker in the current period, the vacancy will be destroyed. This immediately implies that the equilibrium vacancy distribution $V(\rho)$ is the same as $F(\rho)$. In equilibrium, the free entry condition requires that the cost of vacancy creation is

[44] An alternative way to model vacancy creation is to assume that different firms are able to create vacancies of different productivity at convex vacancy creation costs (Lise and Robin, 2016), i.e., $c(N_v(\rho)) = \frac{\nu_0}{1+\nu_1}(N_v(\rho))^{1+\nu_1}$, where $\nu_0, \nu_1 > 0$ are two parameters. The parameters ν_0 and ν_1 govern the response of vacancies to changes in profitability. I do not adopt this specification for two reasons: first, the identification of ν_0 and ν_1 requires times series information on the number of vacancies and its correlation with output. However, my model does not generate time variations in the number of vacancies because I focus on the stationary equilibrium without aggregate shocks. Second, this specification requires estimating all the parameters in general equilibrium, which is not tractable given the complexity of my current setup (see Appendix D for a discussion). The downside of my current setup is that my model can only capture the general equilibrium response in the number of vacancies after a policy change, but not in the composition of vacancies.

equal to its expected value,

$$\frac{v}{q} = \frac{\overline{u}Ts^u}{S}\Bigg[\underbrace{\iint_{\rho>\rho_u^d} J^d(\Omega,\rho,\rho_u^d)\phi^u(\Omega,1)d\Omega dF(\rho)}_{\text{meet defaulted unemployed agents}} + \underbrace{\iint_{\rho>\rho_u} J(\Omega,\rho,\rho_u)\phi^u(\Omega,0)d\Omega dF(\rho)}_{\text{meet non-defaulted unemployed agents}}\Bigg]$$
$$+ \frac{(1-\overline{u})Ts^e}{S}\Bigg[\underbrace{\iiint_{\rho>\rho'} J^d(\Omega,\rho,\rho')\phi^e(\Omega,\rho',1)d\Omega d\rho' dF(\rho)}_{\text{meet defaulted employed agents}} + \underbrace{\iiint_{\rho>\rho'} J(\Omega,\rho,\rho')\phi^e(\Omega,\rho',0)d\Omega d\rho' dF(\rho)}_{\text{meet non-defaulted employed agents}}\Bigg], \tag{5.29}$$

where $\phi^u(\Omega,d)$ represents the PDF conditional on whether unemployed agents have defaulted ($d=1$) or not ($d=0$). Thus, $\phi^u(\Omega) = \phi^u(\Omega,0)+\phi^u(\Omega,1)$. Similarly, for employed agents, $\phi^e(\Omega,\rho) = \phi^e(\Omega,\rho,0)+\phi^e(\Omega,\rho,1)$.

Equation (5.29) states that a new vacancy meets an agent with probability q. Conditional on a meeting, the vacancy meets an unemployed worker with probability $\overline{u}Ts^u/S$ and is filled if the vacancy's productivity is above the reservation productivity, $\rho > \rho_u(\Omega)$. The vacancy meets an employed worker with probability $(1-\overline{u})Ts^e/S$ and is filled if the vacancy's productivity is above the worker's current job's productivity, $\rho > \rho'$.

Flow Equations In the stationary equilibrium, the flows in and out of unemployment balance each other out. The unemployment rate $\overline{u}$ is determined by the following equation:

$$(1-\overline{u})\kappa = \overline{u}\lambda^u\left[\int [1-V(\rho_u^d)]\phi^u(\Omega,1)d\Omega + \int [1-V(\rho_u)]\phi^u(\Omega,0)d\Omega\right], \tag{5.30}$$

where the LHS represents the flow into unemployment due to exogenous separations of employed agents at rate κ, and the RHS represents the flow into employment when unemployed agents contact jobs whose productivity is above their reservation productivity.

Moreover, the flows in and out of employment at every type-ρ job also balance each other out:

$$[\underbrace{\kappa}_{\text{exogenous separation}} + \underbrace{(1-\kappa)\lambda^e(1-V(\rho))}_{\text{transition to more productive jobs}}](1-\overline{u})\int \phi^e(\Omega,\rho)d\Omega \tag{5.31}$$
$$= \underbrace{\lambda^u v(\rho)u\left[\int_{\rho_u^d\le\rho} \phi^u(\Omega,1)d\Omega + \int_{\rho_u\le\rho} \phi^u(\Omega,0)d\Omega\right]}_{\text{unemployed workers find type-}\rho\text{ jobs}} + \underbrace{(1-\kappa)\lambda^e v(\rho)(1-\overline{u})\iint_{x<\rho} \phi^e(\Omega,x)d\Omega dx}_{\text{employed workers transition to type-}\rho\text{ jobs}},$$

where the LHS represents the flow out of employment at the type-ρ job, which happens either at the exogenous separation rate κ, or due to job-to-job transitions, at rate $(1-\kappa)\lambda^e(1-V(\rho))$. The RHS represents the flow into employment at the type-ρ job. The first term captures the case in which unemployed workers meet vacancies with productivity ρ, which is above their reservation productivity.

The second term captures the case in which employed workers at jobs with lower productivity meet type-ρ vacancies and transition to new jobs.

Equilibrium Definition Below I define the stationary competitive equilibrium.

Definition 5. *The stationary competitive equilibrium consists of stationary distributions of unemployed agents, $\phi^u(\Omega)$, employed agents $\phi^e(\Omega, \rho)$, vacancies $V(\rho)$, the number of vacancies N_v, and unemployment rate $\overline{u}$, such that:*

(1). *The job contact rates for agents and firms are determined by the Cobb-Douglas meeting technology according to (5.10-5.11).*

(2). *All unemployed agents Ω make consumption and default decisions by solving problems (5.25) and (C.1) depending on their default status.*

(3). *All employed agents Ω at job ρ with negotiation benchmark ρ' receive wage income and make consumption, labor supply, and default decisions by solving problems (5.26) and (C.5) depending on their default status.*

(4). *Wage rates, $w^e(\Omega, \rho, \rho')$ and $w^{e,d}(\Omega, \rho, \rho')$, are determined by Nash bargaining specified in (5.16) and (5.18).*

(5). *The equilibrium number of vacancies N_v and the vacancy distribution $V(\rho)$ are determined by the free entry condition (5.29).*

(6). *The equilibrium unemployment rate $\overline{u}$ is determined to balance flows in and out of unemployment, as specified in (5.30).*

6 Estimation and Validation Tests

In this section, I present the estimation procedures of my quantitative model. The initial wealth and loan distribution is estimated parametrically using MLE. Based on the estimated distribution, I estimate the model's structural parameters using MSM. Finally, I conduct two validation tests to check the external validity of the model.

6.1 Estimating the Initial Wealth and Loan Distribution

Observing that many students do not borrow at all, I use the parameter p to capture the probability of borrowing. In particular, the initial PDF, $\psi(b, s)$, satisfies

$$\int \psi(b, 0) db = p, \tag{6.1}$$

$$\iint_{s>0} \psi(b, s) dbds = 1 - p. \tag{6.2}$$

The specification here does not consider the fact that borrowing for college study is an endogenous decision, because this paper focuses on how student loan debt affects labor market outcomes. Developing

Table 4: Parameters governing the distribution of wealth and student loan debt.

Parameter	Description	Value	Standard Error
$\underline{b}_0$	Marginal wealth distribution (location, no debt)	0	0
τ_0	Marginal wealth distribution (scale, no debt)	1874.3	158.3
φ_0	Marginal wealth distribution (shape, no debt)	0.6217	0.0754
$\underline{b}_1$	Marginal wealth distribution (location, with debt)	0	0
τ_1	Marginal wealth distribution (scale, with debt)	826.2	50.4
φ_1	Marginal wealth distribution (shape, with debt)	0.5948	0.0533
μ_s	Marginal debt distribution (mean)	8.9823	0.0315
σ_s	Marginal debt distribution (variance)	0.8859	0.0223
p	Percent of youths without debt	0.3875	0.0136
ϑ	Correlation between wealth and debt	-0.1253	0.2332

Note: This table presents the estimated parameter values for the joint distribution of wealth and student loan debt. The marginal wealth distribution for non-borrowers is assumed to be generalized Pareto, captured by parameters β_0, τ_0, and φ_0. The marginal wealth distribution for all borrowers is assumed to be generalized Pareto, captured by parameters β_1, τ_1, and φ_1. The marginal student loan debt distribution for all borrowers is assumed to be log-normal, captured by parameters μ_s and σ_s. The probability mass of non-borrowers is captured by parameter p. The correlation between wealth and debt is estimated using Frank copula, captured by parameter ϑ.

a model with college entry and endogenous borrowing decisions is interesting on its own, for example, to assess the potential interaction between talent and debt burden and to assess the moral hazard issue on borrowing caused by debt forgiveness.[45] In the NLSY97 data used in my estimation, the average AFQT score among borrowers is marginally below that of non-borrowers (see Table 1).[46] Therefore, if the model is developed to account for the difference in talent, the estimated debt burden would be larger as less talented people are less capable to bear risks and face more liquidity needs due to lower wage income. Therefore, my current model in some sense provides a lower bound for the welfare implication of the income-based repayment plan.

To match the distribution observed in the data, I assume that the wealth of agents without debt follows a generalized Pareto distribution with location parameter $\underline{b}_0$, scale parameter τ_0, and shape parameter φ_0:

$$\psi_0(b) = \psi(b, 0) = \frac{1}{\tau_0}\left(1 + \varphi_0 \frac{b - \underline{b}_0}{\tau_0}\right)^{-\frac{1+\varphi_0}{\varphi_0}}. \tag{6.3}$$

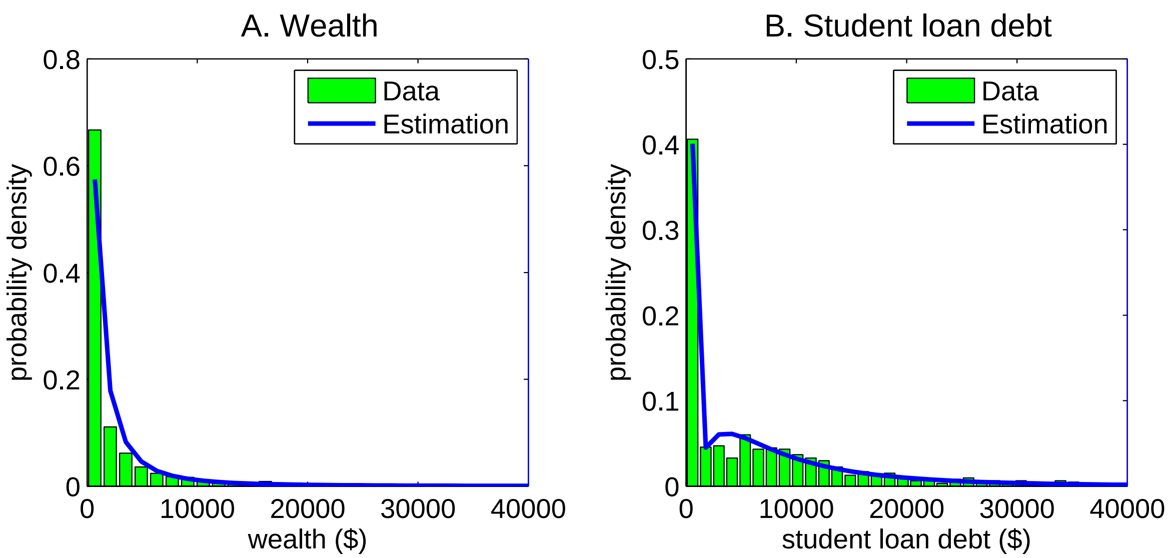
Similarly, for agents with debt, I assume that the marginal distribution of wealth follows a generalized Pareto distribution with parameters $\underline{b}_1$, τ_1, and φ_1,

$$\psi_1(b) = \int\limits_{s>0} \psi(b, s)ds = \frac{1}{\tau_1}\left(1 + \varphi_1 \frac{b - \underline{b}_1}{\tau_1}\right)^{-\frac{1+\varphi_1}{\varphi_1}}. \tag{6.4}$$

The marginal distribution of student loan debt for indebted agents follows a log-normal distribution

[45] The debt forgiveness policy embedded in the income-based repayment plan could potentially generate moral hazard in borrowing as less capable students would enter more expensive private schools, anticipating that the debt will be written-off eventually.

[46] Relatedly, Looney and Yannelis (2015) document the rise in the number of non-traditional student loan borrowers in recent years. These borrowers are usually less talented; they graduate from weak institutions and experience poor labor market outcomes after leaving school.



Note: This figure plots the estimated marginal distribution of wealth and student loan debt. I capture the probability of borrowing using the mass probability p. I assume that the marginal distribution of wealth follows a generalized Pareto distribution with different parameter values for borrowers and non-borrowers. I use the log-normal distribution to capture the marginal distribution of student loan debt for borrowers. The correlation between wealth and debt is estimated using Frank copula.

Figure 7: Estimated and empirical distribution of initial wealth and student loan debt.

with parameters μ_s and σ_s^2,

$$\psi_s(s) = \int \psi(b,s)db = \frac{1}{s\sigma_s\sqrt{2\pi}} e^{-\frac{(\ln s - \mu_s)^2}{2\sigma_s^2}}, \forall s > 0. \tag{6.5}$$

To capture the negative correlation between student loan debt and wealth among borrowers, I use Frank copula, where the single parameter ϑ governs the dependence between the CDF of the marginal distribution of wealth, $\Psi_1(b)$, and the CDF of student loan debt, $\Psi_s(s)$:

$$C(u,v) = \mathbb{P}(\Psi_1(b) \leq u, \Psi_s(s) \leq v) = -\frac{1}{\vartheta} \log\left[1 + \frac{(e^{-\vartheta u} - 1)(e^{-\vartheta v} - 1)}{e^{-\vartheta} - 1}\right]. \tag{6.6}$$

I estimate these parameters using MLE based on the sample consisting of youths with and without student loans separately. The parameter p is estimated to match the percent of youths who did not borrow in the data, and its standard error is estimated by bootstrapping. The estimated parameter values are reported in Table 4. It is shown in Figure 7 that these functional form specifications characterize the empirical distribution of wealth and student loan debt reasonably well.

6.2 Estimating Model Parameters

Following Lise, Meghir and Robin (2016) and Jarosch (2015), I assume that job productivity follows a flexible Beta distribution on support $[0, 1]$ with parameters f_1, f_2.

The are 26 structural parameters to be determined:

$$\Xi = [\underbrace{\varkappa, \tau, \gamma, \sigma, r, \beta, r^s, \omega, \theta, \underline{c}, T}_{\Xi_1}, \underbrace{A, \kappa, s^u, s^e, \chi, \xi, \eta, \nu, \phi, f_1, f_2, \mu_0, \mu_1, \mu_2, \mu_3}_{\Xi_2}]. \tag{6.7}$$

The set of parameters Ξ_1 is determined using external information. The set of parameters Ξ_2 is estimated jointly using MSM to match a set of labor market characteristics. Below I discuss the identification of these parameters.

6.2.1 Externally Determined Parameters

Table 5 presents the values for externally determined parameters.

Table 5: Parameters determined outside the model.

Parameter	Description	Value	Source
$\varkappa$	Overall tax level	2.17	Estimated from March CPS 1997-2008
τ	Rate of tax progressivity	0.11	Estimated from March CPS 1997-2008
γ	Risk aversion	3	Hubbard, Skinner and Zeldes (1995)
σ	Elasticity of labor supply	2.59	Keane (2011), Frisch elasticity=0.33
r	Annual risk-free rate	4.5%	Real interest rate between 1997-2008
β	Discount factor	0.96	Standard practice
r^s	Interest rate on student loans	6.6%	Ionescu (2009), risk premium=2.1%
ω	Meeting technology	0.5	Pissarides and Petrongolo (2001)
θ	UI benefits	\$8,000	40% of average 6-month wage income
$\underline{c}$	Consumption floor	\$900	AFDC, food stamps, and WIC
T	Number of years working	38	Real-life working age of 23 to 60

The fiscal parameters $\varkappa$ and τ are identified using the regression coefficients obtained from regressing log individual after-tax earnings $\tilde{y}_i$ on log individual pre-tax earnings y_i:

$$log(\tilde{y}_i) = log(\varkappa) + (1-\tau)log(y_i) + \varepsilon_i \tag{6.8}$$

The pre-tax earnings data are obtained from March CPS 1997-2008. I use the NBER's TAXSIM program to compute after-tax earnings as earnings minus all federal and state taxes. The estimated values are $\varkappa = 2.17$ and $\tau = 0.11$.

I take advantage of the existing findings to determine the values of γ and σ. The estimated value of risk aversion is highly context-dependent (Barseghyan, Prince and Teitelbaum, 2011; Einav et al., 2012). Therefore, I choose γ according to the literature that is mostly related to this paper. In particular, I set $\gamma = 3$ consistent with the precautionary savings literature (e.g. Hubbard, Skinner and Zeldes, 1995). This value is smaller than the value used by other search and matching models with risk-averse agents (Acemoglu and Shimer, 2000; Krusell, Mukoyama and Sahin, 2010), but is larger than the value used in the literature on macro development (Buera and Shin, 2013; Moll, Townsend and Zhorin, 2016). I take a relatively larger value for the baseline specification also because in dynamic equilibrium models with labor margin, the risk aversion parameter is suggested to have a larger value as agents can adjust labor supply to absorb return shocks (Swanson, 2012). Chetty and Szeidl (2007) also suggest to use a larger value of risk aversion for unemployment shocks. Because the value of γ is the most important parameter that determines the quantitative implication of debt burden, I provide a sensitivity analysis using other values of γ in section 8.

The tax-modified Frisch elasticity of labor supply with respect to pre-tax wage rates is $(1-\tau)/(\sigma+\tau)$. Thus I set $\sigma = 2.59$, which implies that the tax-modified Frisch elasticity is 0.33, broadly consistent with microeconomic evidence (Keane, 2011). Because the GHH utility does not have an income effect, the Frisch elasticity is equal to the Hicksian elasticity. Thus, $\sigma = 2.59$ is also consistent with the estimate of Chetty (2012), who shows that the estimated Hisksian elasticity is 0.33 after taking into account optimization frictions.[47] Because the elasticity of labor supply determines the distortionary effect of the income-based repayment plan, I provide a sensitivity analysis using other values of σ in section 8.

I set the annual risk-free rate to be $r = 4.5\%$, corresponding to the average real interest rate in the U.S. between 1997-2008 (source: World Development Indicators). I set the interest rate on student loans to be $r_s = 6.6\%$, which implies a risk premium consistent with the annualized mark-up over the Treasury bill rate, 2.1%, set by the government for subsidized loans issued before 2006 (Ionescu, 2009).

Following the standard practice, I set the annual discount rate to be $\beta = 0.96$. A robust pattern in many countries is that young people experience faster income growth compared to the old, thus we expect young people tend to be more "impatient", financially, compared to the old. This observation suggests a larger liquidity effect of student loan debt precisely because these loans are required to be paid in early life. In addition to the age-specific heterogeneity in time preference, allowing preference heterogeneity in population is useful to generate high marginal propensities to consume (Carroll, Slacalek and Tokuoka, 2014; Auclert, 2016) and to match the wealth distribution (Krusell, Smith and Jr., 1998). However, due to computational reasons, I only capture the income growth using efficient labor units instead of introducing heterogeneous discount factors in my model.

I set the matching parameter to be $\omega = 0.5$, which lies in the middle of existing estimates using information on the flow of hires and the stock of unemployment and job vacancies (Pissarides and Petrongolo, 2001).

In the U.S., UI benefits generally pay eligible workers between 40-50% of their previous pay. The standard time-length of unemployment compensation is 6 months, although during the recent recession, Congress passed the emergency benefit program to extend that duration to 73 weeks. In my model, unemployed agents receive UI benefits every year. Therefore, I choose a relatively lower value of UI benefits to reflect this discrepancy. I set $\theta = \$8,000$, which amounts to roughly 40% of the average 6-month income in the first five years after college graduation.

Means-tested benefits include Aid to Families with Dependent Children (AFDC), food stamps, and Women, Infants, Children (WIC). In my sample, the percent of youths who had ever received AFDC, food stamps, and WIC by 2013 are 1.3%, 8.4%, and 6.3%. About 11.5% of youths had ever received any means-test benefits during my sample period, with a median monthly benefit level of \$150. Because the

[47] Chetty (2012) also provides a bound on the Frisch elasticity, [0.33 0.47], based on the estimated Hicksian elasticity and empirically reasonable income effect and elasticity of inter-temporal substitution.

take-up rate is far from universal, following Kaplan (2012), the annual consumption floor is set to be $900, half of the median value of means-tested benefits.

Between 2002-2012, the average retirement age is around 60. I set $T = 38$, which corresponds to a real-life working age of 23 to 60.

6.2.2 Internally Estimated Parameters

I now turn to the identification discussion of internally estimated parameters. Parameter A is a scale factor, which is identified from the average wage income, $\$43,933$, during the first five years after college graduation.

Table 6: Parameters estimated jointly using MSM.

Parameter	Description	Value	Standard Error
A	Aggregate productivity	42.0	1.8
κ	Exogenous job separation rate	0.272	0.006
s^u	Search efficiency during unemployment	14.69	1.05
s^e	Search efficiency during employment	1	N/A
χ	Matching efficiency	0.4207	0.0313
ξ	Worker's bargaining power	0.45	0.02
η	Default rate	1.40×10^{-8}	0.16×10^{-8}
ν	Flow cost of vacancy creation	$106,112	$4,684
ϕ	Labor supply scaling factor	5.0×10^{-8}	0.1×10^{-8}
f_1	Vacancy productivity distribution	1.30	0.19
f_2	Vacancy productivity distribution	0.90	0.19
μ_0	Constant term in worker's ability	0.836	0.006
μ_1	Linear term in worker's ability	0.085	0.001
μ_2	Square term in worker's ability	-3.89×10^{-3}	0.02×10^{-3}
μ_3	Cubic term in worker's ability	5.53×10^{-5}	0.06×10^{-5}

Note: This figure presents parameter values estimated jointly using MSM following the two-step estimation procedure detailed in Appendix D. Standard errors are computed by bootstrapping.

The exogenous job separation rate κ is identified from the average duration of employment spells. In the NLSY97 sample, employment spells last for about 2.7 years on average, consistent with the calculations of Shimer (2005) using CPS data.

The search efficiency during employment s^e is normalized to be 1. The search efficiency during unemployment s^u and the parameter governing matching efficiency χ are identified from the average unemployment duration and the average duration of job tenure. In the data, the average unemployment duration is 19 weeks and jobs last for about 2.3 years on average. Because job separations could either result in a transition into unemployment or a transition into another job, the small difference between the average employment duration and the average job tenure implies that on-the-job search is much less efficient compared to searching during unemployment.[48]

[48]In the extreme case where the average employment duration is equal to the average job tenure, there is no job-to-job transitions, which implies the absence of on-the-job search. On the other hand, if the average job tenure is much shorter than the average employment duration, it means most of the job separations are due to job-to-job transitions instead of employment-to-unemployment transitions.

The bargaining parameter ξ is identified from the log wage increases upon job-to-job transitions. In the data, the log hourly wage rate rises by about 18.4% upon job-to-job transitions on average. This estimate is consistent with Lise, Meghir and Robin (2016), who use NLSY79 data.

As argued by Jarosch (2015), the second and third moments of the cross-sectional log wage income distribution provide information useful to pin down the parameters f_1 and f_2 governing the vacancy productivity distribution. During the first five years after college graduation, the cross-sectional log wage income residuals have variance 0.139 and skewness -0.222.

The default cost η is identified from the equilibrium default rate on student loan debt. Using a random 1% sample of National Student Loan Data System (NSLDS), Yannelis (2015) computes that the average two-year cohort default rate for undergrads is 9.26% between 1997-2011.

The flow cost of vacancy creation ν is identified from the vacancy to unemployment ratio. The Job Openings and Labor Turnover Survey (JOLTS) collected job openings information since December 2000 in the United States. I estimate the vacancy to unemployment ratio to be 0.409 using the data between 2001-2013. This estimate is smaller than the estimate of 0.539 provided by Hall (2005), who uses data between 2001-2002.

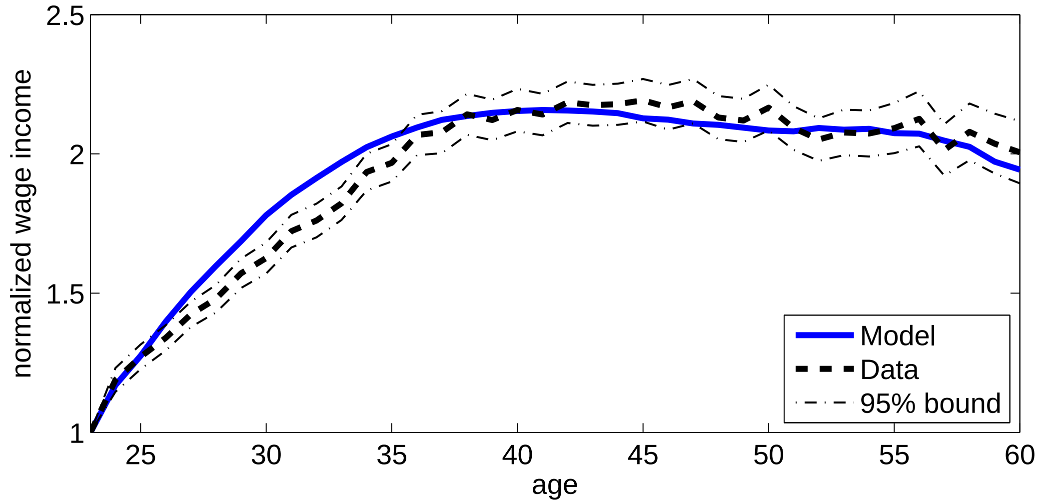
Table 7: Model fit for targeted moments.

Moments	**Model**	**Data**
Average wage income in first 5 years	\$44,012	\$43,933
Average duration of employment spells (year)	2.7	2.7
Average duration of unemployment spells (week)	18.7	19.0
Average job tenure (year)	2.3	2.3
Log wage increase upon job-to-job transitions	0.156	0.184
Variance of log wage income in first 5 years	0.121	0.139
Skewness of log wage income in first 5 years	-0.054	-0.222
Default rate	8.56%	9.26%
Vacancy to unemployment ratio	0.409	0.409
Average hours worked per year	2005	2004
Life-cycle earnings profile	see Figure 8	

Note: This table presents model fit for targeted moments. The life-cycle earnings profile is constructed using March CPS 1997-2008 data from Acemoglu and Autor (2011). The default rate is constructed by Yannelis (2015) using a random 1% sample of NSLDS. The vacancy to unemployment ratio is constructed using JOLTS data between 2001-2013. The remaining moments are constructed using the sample from NLSY97.

Parameter ϕ is a scale factor of labor supply, which is identified from the average number of hours worked in each year. In the data, people with full-time jobs work for roughly 40 hours per week and 2004 hours per year on average.

Parameters μ_0, μ_1, μ_2, and μ_3 are identified to match the average wage income in each year between ages 23-60. Because NLSY97 does not provide individual labor market histories at this length, I construct the life-cycle earnings profile using March CPS 1997-2008 data downloaded from the website of Acemoglu and Autor (2011).



Note: This figure compares the targeted moments of life-cycle earnings profiles between model and data. The solid line represents the earnings profile generated by the model. The dashed line represents the earnings profile in the data, constructed using March CPS 1997-2008 data from Acemoglu and Autor (2011).

Figure 8: Comparing life-cycle earnings profiles between model and data.

I estimate the set of parameters Ξ_2 using MSM:

$$\hat{\Xi}_2 = \underset{\Xi_2}{\operatorname{argmin}}\, L(\Xi_2) \tag{6.9}$$

The objective function is given by

$$L(\Xi_2) = [\hat{m}_N - \hat{m}_S(\Xi_2)]^T \hat{\Theta}^{-1} [\hat{m}_N - \hat{m}_S(\Xi_2)]. \tag{6.10}$$

where $\hat{m}_N = \frac{1}{N}\sum_{i=1}^{N} m_i$ is the vector of moments computed in the data. $\hat{m}_S(\Xi_2)$ is the vector of moments generated by the model simulation in the stationary equilibrium. $\hat{\Theta}$ is a weighting matrix, constructed from the diagonal of the estimated variance-covariance matrix of $\hat{m}_N$ using bootstrapping. Estimates are not sensitive to alternative choices of weighting matrices because most moments are matched well (see Table 7). The asymptotic variance-covariance matrix for MSM estimators $\hat{\Xi}_2$ is given by:

$$Q(\hat{\Theta}) = (G^T \hat{\Theta} G)^{-1} G^T \hat{\Theta} \hat{S} \hat{\Theta}^T G (G^T \hat{\Theta}^T G)^{-1}, \tag{6.11}$$

where $\hat{S}$ is the variance-covariance matrix; $G = \frac{\partial \hat{m}_S(\Xi_2)}{\partial \Xi_2}|_{\Xi_2=\hat{\Xi}_2}$ is the Jacobian matrix of the simulated moments evaluated at the estimated parameters.[49] The first derivatives are calculated numerically by varying each parameter's value by 1%. The standard errors of $\hat{\Xi}_2$ are given by the square root of the diagonal elements of $Q(\hat{\Theta})$.

Note that the estimation procedure is implemented using a two-step estimation, which allows most

[49]In general, the formula should also incorporate simulation errors, thus the variance-covariance matrix for MSM estimators also depends on the number of simulated agents (Gourieroux and Monfort, 1997). The formula I use does not consider this type of simulation errors because instead of simulating a number of agents, I simulate the distribution of characteristics. Therefore, as long as I focus on the stationary equilibrium, the simulation outcomes are not dependent on randomly drawn shocks.

of the parameters to be estimated in partial equilibrium, without iterating on the equilibrium job contact rates (see Appendix D).

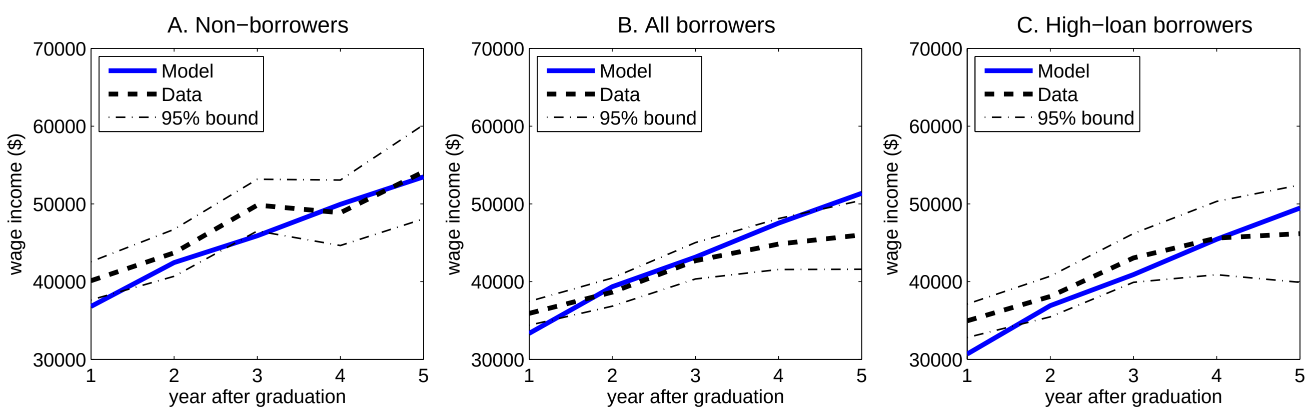
Table 6 presents the internally estimated parameters. The estimated search efficiency for employed workers is about one fifteenth of unemployed workers, which is roughly consistent with the estimate of Gavazza, Mongey and Violante (2016) but is smaller than the structural estimate of Lise, Meghir and Robin (2016) on college graduates from NLSY79. I obtain a relatively small search efficiency due to the small difference between the average length of employment duration and job tenure observed in the sample of college graduates from NLSY97.

6.3 Validation Tests

I conduct two validation tests to provide a type of out-of-sample evaluation of the structure imposed by the quantitative model. First, I check whether the model can replicate several non-targeted moments in the data. The non-targeted moments I choose are those that are informative about the mechanism. Second, I check whether the model can produce several elasticity measures that are consistent with micro estimates in related literature. In particular, the literature on UI benefits and unused credit focus on a mechanism similar to the liquidity channel of student loan debt.

6.3.1 Non-Targeted Moments

The predictions delivered in section 4 suggest that indebted agents are less picky in job search and more likely to end up in lower-paid jobs. The summary statistics listed in Table 1 are consistent with these predictions. Although these only suggest correlations, it is useful to check whether the model-generated discrepancies between non-borrowers and borrowers in unemployment duration and wage income are in line with these summary statistics.



Note: This figure compares the non-targeted moments of annual wage income in the first five years between model and data. The solid line represents the earnings profile generated by the model. The dashed line represents the earnings profile in the data, constructed using the NLSY97 sample. Panel A, B, and C plot wage income for non-borrowers, all borrowers, and high-loan borrowers.

Figure 9: Comparing non-targeted moments: annual wage income in the first five years.

In the data, the average unemployment duration of the first unemployment spell after college graduation is 17.7, 16.4, and 14.1 weeks for non-borrowers, borrowers, and high-loan borrowers. The corresponding values generated by my model are 16.2, 13.8, and 11.3 weeks. In the data, the average hourly wage rate for the first job is $20.0, $16.7, and $16.4 for non-borrowers, borrowers, and high-loan borrowers. The corresponding values implied by the model are $19.9, $18.2, and $17.2. Figure 9 shows that the model also roughly captures the differences among non-borrowers, borrowers, and high-loan borrowers in terms of annual wage income in the first five years after college graduation. The model predicts a smaller discrepancy in wage income between non-borrowers and borrowers relative to the data, but a larger discrepancy between borrowers and high-loan borrowers. This is because in the model, the liquidity channel of student loan debt is increasingly heavier when debt burden increases due to risk aversion.

6.3.2 Comparison to Micro Estimates

I now check whether the model can produce several elasticity measures that are consistent with the micro estimates in related literature. When conducting the following experiments, I focus on partial-equilibrium counterfactual simulations in which the job contact rates and tax rates are fixed, so that the elasticities are estimated in a context consistent with where the micro estimates are obtained. Note that all the elasticities I structurally estimate are based on global elasticities, although some of the micro estimates are local elasticities (see Table 8).

I begin by examining whether the model-implied elasticity of unemployment duration with respect to UI benefits matches the micro estimates using U.S. data. The positive effect of unemployment insurance on unemployment duration is one of the most robust empirical findings. The effect of UI benefits is also delivered from a channel related to job seekers' liquidity constraint, as Chetty (2008) argues that the liquidity effect accounts for 60% of the impact of UI. To estimate the elasticity, I simulate the counterfactual by increasing UI benefits θ by 25%, from $8000 to $10000, corresponding to a 10% increase in UI replacement rate, from 40% of 6-month earnings to 50%. I find that the average unemployment duration increases by about 2.4 weeks, implying that the elasticity of unemployment duration with respect to UI benefits is about 0.50. This elasticity is roughly in line with the estimate of Card et al. (2015), who find that the elasticity is around 0.35 during the pre-recession period (2003-2007) and between 0.65 and 0.9 during the recession and its aftermath.

Next, I check whether the model-generated response in the reservation wage and average wage income are in line with the micro estimates. The estimate of Feldstein and Poterba (1984) indicates that a 10% increase in UI replacement ratio raises the reservation wage by 4% for job losers who are not on layoff. My model generates a larger response in the reservation wage, 6.7%. The empirical evidence on the effect of UI benefits on reemployment wages is mixed.[50] Most existing studies document insignificant effects of UI benefits on reemployment wages (Card, Chetty and Weber, 2007; Lalive, 2007; van Ours

[50]The lack of evidence on reemployment wages could be due to the existence of countervailing forces (Pissarides, 1992; Ljungqvist and Sargent, 1998; Nekoei and Weber, 2016; Schmieder, von Wachter and Bender, 2016) or because unemployed workers do not search more when UI increases or because their established labor market credentials make wage effects less significant. I expect the wage effect of student loans to be more sensible because it affects a much younger population who are entering the labor force with limited liquidity (Card, Chetty and Weber, 2007; Rothstein and Rouse, 2011), but loans are repaid in early careers when wage income is low. This implies that search is less affordable for graduating students who should presumably be highly motivated to search.

Table 8: Comparison to micro estimates.

	Model	Micro Estimates	Source
UI on unemp. dur.	0.50	0.35-0.9	Card et al. (2015)
UI on res. wage	6.7%	4%	Feldstein and Poterba (1984)
UI on reemploy. wage	4.3%	positive	Nekoei and Weber (2016)
		negative	Schmieder, von Wachter and Bender (2016)
		insignificant	Card, Chetty and Weber (2007)
Credit on unemp. dur.	0.5 week	0.15-3 weeks	Herkenhoff, Phillips and Cohen-Cole (2016)
Credit on reemploy. wage	1.3%	0.8%-1.7%	Herkenhoff, Phillips and Cohen-Cole (2016)

Note: This figure compares the model-implied structural estimates with micro estimates. The elasticity of unemployment duration with respect to UI benefits is estimated by simulating the counterfactual with UI benefits θ being increased by 25%, from \$8000 to \$10000, corresponding to a 10% increase in UI replacement rate, from 40% of 6-month earnings to 50%. The effect of UI benefits on the reservation wage and reemployment wage income are also estimated from this counterfactual. The duration and earnings replacement elasticities with respect to unused credit limit are estimated from newly laid off agents due to exogenous job separations in the model. The counterfactual simulation relaxes credit constraints for these agents by 10% of their wage income in previous jobs. The elasticity is estimated using the average difference in unemployment duration and wage income between the baseline economy and the counterfactual economy with relaxed credit constraints.

and Vodopivec, 2008). Schmieder, von Wachter and Bender (2016) find negative effect using U.S. data. Nekoei and Weber (2016) estimate a positive effect of UI benefits on reemployment wages by exploiting an age-based regression discontinuity. My model's simulation results indicate that reemployment wages increase by about 4.3% following a 10% increase in UI replacement rate.

Finally, I use the model to estimate the duration and earnings replacement elasticities with respect to unused credit limit for displaced workers. I then compare these structural estimates with the micro estimates of Herkenhoff, Phillips and Cohen-Cole (2016). Using administrative data from TransUnion and Longitudinal Employment and Household Dynamics (LEHD), Herkenhoff, Phillips and Cohen-Cole (2016) find that increasing credit limits by 10% of prior annual earnings would lead displaced workers to take 0.15 to 3 weeks longer to find a job. Among job finders, the replacement earnings increased by 0.8% to 1.7%.

To evaluate the impact of access to credit on job search and wage income, I isolate newly laid off agents due to exogenous job separations in the model. Denote their prior wage income as $Inc_{-1}(\Omega_{-1}, \rho_{-1}, \rho'_{-1})$ and the set of agents as I_κ. I then simulate these agents' over time until they find the next job, and obtain unemployment duration, $Dur(\Omega)$, and wage income, $Inc(\Omega, \rho, \rho')$. Finally, I run the counterfactual in partial equilibrium to obtain the unemployment duration, $Dur^\Delta(\Omega)$, and wage income, $Inc^\Delta(\Omega, \rho, \rho')$ if these agents were provided with 10% unused credit during unemployment, i.e., the borrowing constraint is relaxed from $b \geq 0$ to $b \geq -0.1Inc_{-1}(\Omega_{-1}, \rho_{-1}, \rho'_{-1})$.

Following Herkenhoff, Phillips and Cohen-Cole (2016), I estimate the duration and earnings elasticity using the following formulas:

$$\epsilon_{dur} = \sum_{I_\kappa} \frac{Dur^\Delta(\Omega) - Dur(\Omega)}{10\%}, \tag{6.12}$$

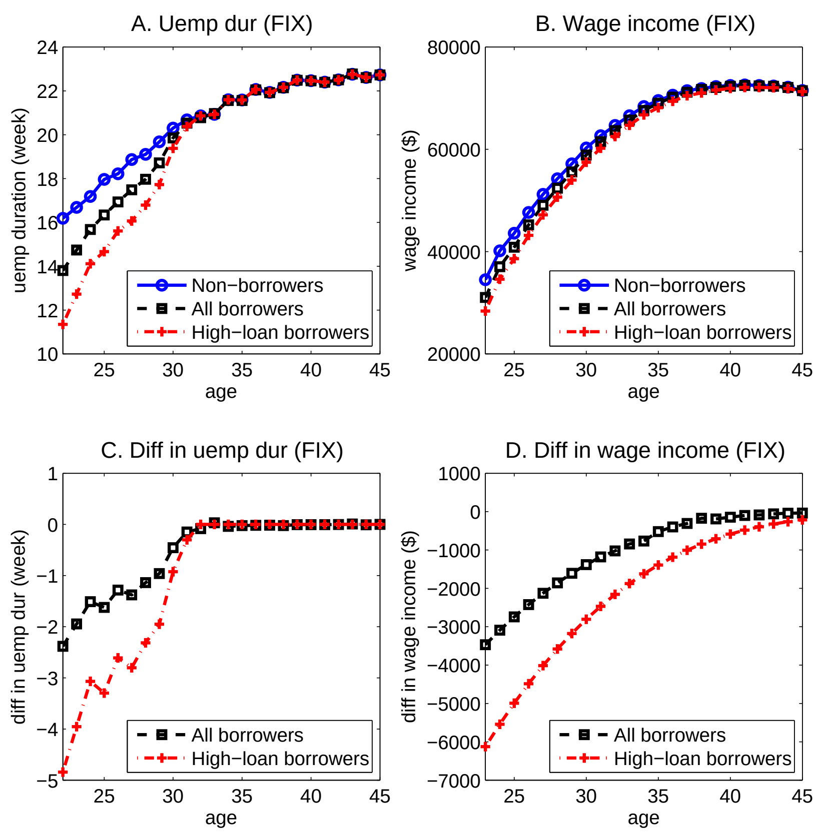
$$\epsilon_{inc} = \sum_{I_\kappa} \frac{[Inc^\Delta(\Omega, \rho, \rho') - Inc(\Omega, \rho, \rho')]/Inc_{-1}(\Omega_{-1}, \rho_{-1}, \rho'_{-1})}{10\%}. \tag{6.13}$$

The structural estimates of ϵ_{dur} and ϵ_{inc} are 0.10 year and 0.13. Therefore, the model predicts that in response to a 10% increase in unused credit, unemployed workers will take 0.5 weeks longer to find a job that on average pays 1.3% more wage income, roughly in line with the micro estimates of Herkenhoff, Phillips and Cohen-Cole (2016).

7 Quantitative Analyses

In this section, I use the estimated model to conduct quantitative analyses. First, I use the model to look at the long-term implications of student loan debt on job search by simulating wage income over the life cycle. Second, I conduct the key counterfactual analysis to evaluate and dissect the effect of the income-based repayment plan.

7.1 Long-Term Effect of Debt Burden



Note: This figure plots the life-cycle unemployment duration and wage income for three groups of agents under the fixed repayment plan. In panels A and B, the blue solid line plots average unemployment duration and wage income for non-borrowers. The black dashed line plots the average unemployment duration and wage income for all borrowers. The red dash-dotted line plots the average unemployment duration and wage income for high-loan borrowers, whose outstanding debt is above the median loan value of borrowers in my sample (i.e., $8,821). Panels C and D plot the difference in unemployment duration and wage income between non-borrowers and borrowers and between non-borrowers and high-loan borrowers.

Figure 10: Life-cycle unemployment duration and wage income under the fixed repayment plan.

To evaluate the long-term effect of debt burden, I simulate the model and track the movement of average unemployment duration and wage income over the life cycle for a single generation. In Figure 10, I plot these aggregate statistics for non-borrowers, all borrowers, and high-loan borrowers between ages 23-45. Panel A and Panel C present that non-borrowers on average spend 2.5 more weeks when searching for their first job compared to all borrowers, and 5 more weeks compared to high-loan borrowers. The difference in unemployment duration decreases over time and is significant in the first 10 years after graduation. This is because in the baseline economy, borrowers are repaying debt under the default 10-year fixed repayment plan, thus their job search decisions would not be affected once the debt is paid off.

However, by contrast, there is a lasting effect of debt burden on wage income beyond the 10th year. Panel C and Panel D indicate that at age 32, even after debt has been paid off, non-borrowers still earn $1,000 more relative to all borrowers, and $2,000 more relative to high-loan borrowers. The difference in wage income is persistent and significant until age 40. This surprising long-term effect of debt is caused by the low job-to-job transition rates. Once debt is paid off, unemployed borrowers would spend roughly the same time on job search as unemployed non-borrowers. However, they are not receiving the same wage income if they are employed, because their current wage offers are outcomes of previous search, which is affected by the debt burden. First jobs matter precisely because of the low search efficiency for employed workers, which is disciplined by the low job-to-job transition rate observed in the data.

7.2 Evaluating the Income-Based Repayment Plan

The 2014 (modified) income-based repayment plan has three main features. First, borrowers are required to repay 10% of their discretionary income if they are new borrowers on or after July 1, 2014. The discretionary income is defined as the difference between pre-tax income and 150% of the poverty guideline. The borrowers who borrowed before July 1, 2014 are eligible for a less generous plan, which requires repaying 15% of the discretionary income.[51] Second, the monthly payment is capped by the amount under the 10-year fixed repayment plan, based on the outstanding loan balance when the borrower initially entered the income-based repayment plan. This implies that the repayment under the income-based repayment plan is never more than the 10-year fixed repayment plan amount.[52] Third, the repayment period is 20 years for the new borrowers and 25 years for the old borrowers. All the remaining balance will be forgiven at the end of the repayment period and the forgiven debt would be considered as taxable income.

I use the model to evaluate the implication of the income-based repayment plan that applies to old borrowers. In particular, I consider a 15% repayment ratio and a 25-year repayment period. I set the poverty guideline based on the average individual poverty level for the 48 contiguous states (excluding Hawaii and Alaska) and the District of Columbia.[53] The inflation-adjusted poverty level is quite stable

[51] As of December 17, 2015, all borrowers with Federal Direct Loans have access to a new repayment plan with monthly payments capped at 10% of the discretionary income. This plan is known as the revised pay as you earn plan (REPAYE).

[52] The repayment cap has an additional implication on labor supply. In any period, if the agent receives income high enough to hit the repayment cap, then the income-based repayment contract has no distortion on labor supply in that period.

[53] The repayment under the income-based repayment plan also depends on family size because the poverty guideline increases with family size. The calibration here does not make this adjustment because my model focuses on individuals.

over time, and the 150% poverty level is set to be $pov = \$15,650$ corresponding to its average value between 1997-2015 measured by 2009 dollars. The interest rate does not depend on repayment plans. Thus the annual payment is given by:

$$y_t^{ibr} = \min\left(0.15 \max(w_t l_t - pov, 0), \quad y_1^{fix}, \quad s_t\right), \quad \text{for } t <= 25, \tag{7.1}$$

where minimizing over the term y_1^{fix} captures the payment cap, and the term s_t ensures that the borrower will never repay more than the amount owed. Note that unemployed workers do not make payments under the income-based repayment plan because UI benefits, $\theta = \$8,000$, are below 150% of the poverty guideline.

In the following, I first simulate the life-cycle outcomes under the income-based repayment plan and compare them to those under the fixed repayment plan. Then, I quantify the aggregate and distributional implications of the income-based repayment plan on various metrics. Finally, I separately quantify the reservation wage effect of the income-based repayment plan.

7.2.1 Life-Cycle Outcomes

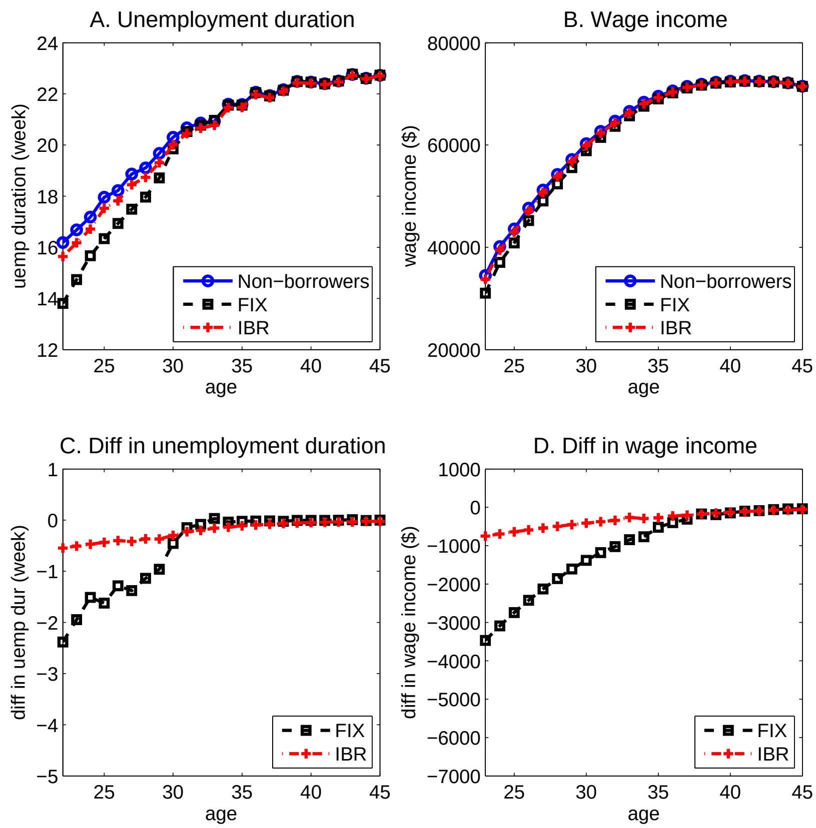
In this subsection, I evaluate the life-cycle outcomes under the income-based repayment plan. Figures 11-12 compare the unemployment duration and wage income for all borrowers and high-loan borrowers under the fixed repayment plan and the income-based repayment plan. Under the income-based repayment plan, borrowers are still spending less time searching for jobs and receiving less wage income on average relative to non-borrowers. However, the difference is much smaller. Immediately after college graduation, all borrowers under the income-based repayment plan on average spend 15.5 weeks searching for their first jobs, which is 0.5 week below the average of non-borrowers. This is a significant improvement relative to the fixed repayment plan, under which borrowers spend 2.5 weeks less in job search relative to non-borrowers. However, because the income-based repayment plan has a longer repayment period, the difference is persistent until age 45.

As a consequence of longer job search, the difference in initial wage income is only about \$900 between non-borrowers and borrowers under the income-based repayment plan, in contrast to \$3,500 under the fixed repayment plan. Moreover, the difference is \$2,000 between non-borrowers and high-loan borrowers under the income-based repayment plan as compared to \$6,000 under the fixed repayment plan.

7.2.2 Distributional Implications on Welfare

In this subsection, I evaluate the distributional implications of the income-based repayment plan on welfare. Following Townsend and Ueda (2010), I proxy the reduction in welfare due to debt burden using wealth compensation. In particular, for any borrower who just graduated from college, the welfare cost of student loan debt under the fixed repayment plan is measured as the amount of wealth that should be transferred to the agent for her to have the same utility as a non-borrower of the same characteristics.[54]

[54] This welfare measure allows me to evaluate the relative ex-post cost of various repayment plans. However, it is not meant to be comprehensive because it does not account for the welfare benefit of student loan debt during college.



Note: This figure plots the life-cycle unemployment duration and wage income for borrowers under the fixed repayment plan and the income-based repayment plan. In panels A and B, the blue solid line plots average unemployment duration and wage income for non-borrowers in the baseline economy (i.e., fixed repayment plan). The black dashed line plots the average unemployment duration and wage income for all borrowers under the fixed repayment plan. The red dash-dotted line plots the average unemployment duration and wage income for all borrowers under the income-based repayment plan. Panels C and D plot the difference in unemployment duration and wage income between non-borrowers and borrowers under the fixed repayment plan and the income-based repayment plan.

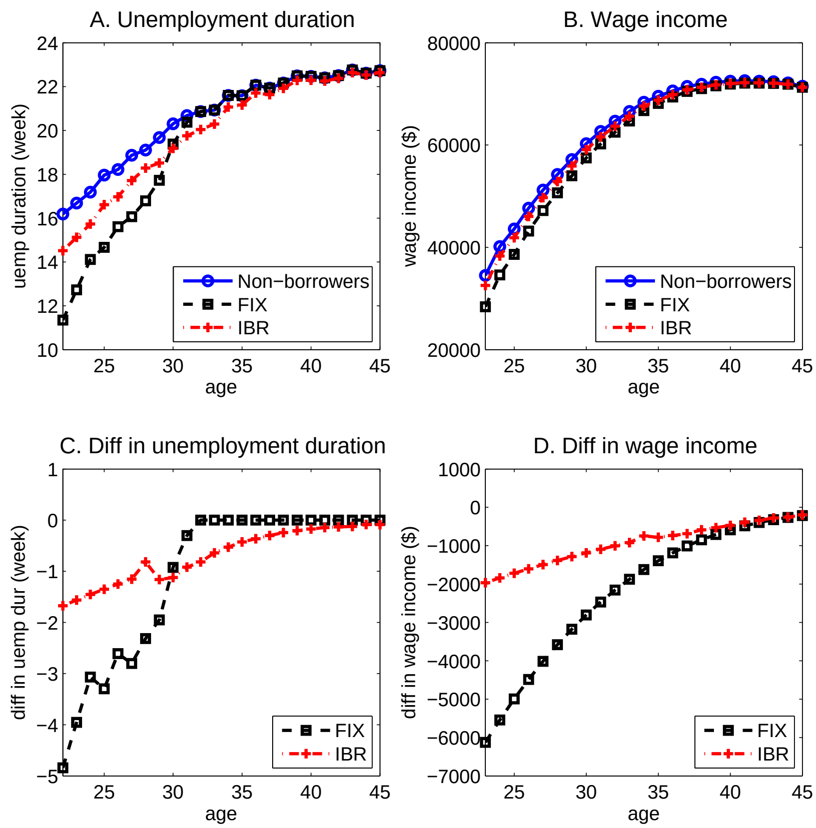
Figure 11: Comparing the life-cycle unemployment duration and wage income for all borrowers under the fixed repayment plan and the income-based repayment plan.

In other words, the non-borrower would be indifferent about accepting the debt and the associated wealth compensation at the same time.

When measuring the welfare cost of student loan debt under the income-based repayment plan, I have to consider the general equilibrium effect caused by the change in equilibrium job contact rates. Therefore, for any agent, I calculate the least amount of wealth transfer that provides her the same utility as a non-borrower of the same characteristic in the baseline economy with the fixed repayment plan.

I measure the improvement on welfare by calculating the difference in wealth compensation between the baseline economy with the fixed repayment plan and the counterfactual economy with the income-based repayment plan.[55] To assess the distributional effect, I do this calculation for borrowers of different levels of wealth and student loan debt. Figure 13 illustrates that adopting the income-based repayment

[55]Therefore, the numbers reported in Figure 13 can be considered as the least amount of wealth compensation that induces the agent to switch from the counterfactual economy with the income-based repayment plan to the baseline economy with the fixed repayment plan.

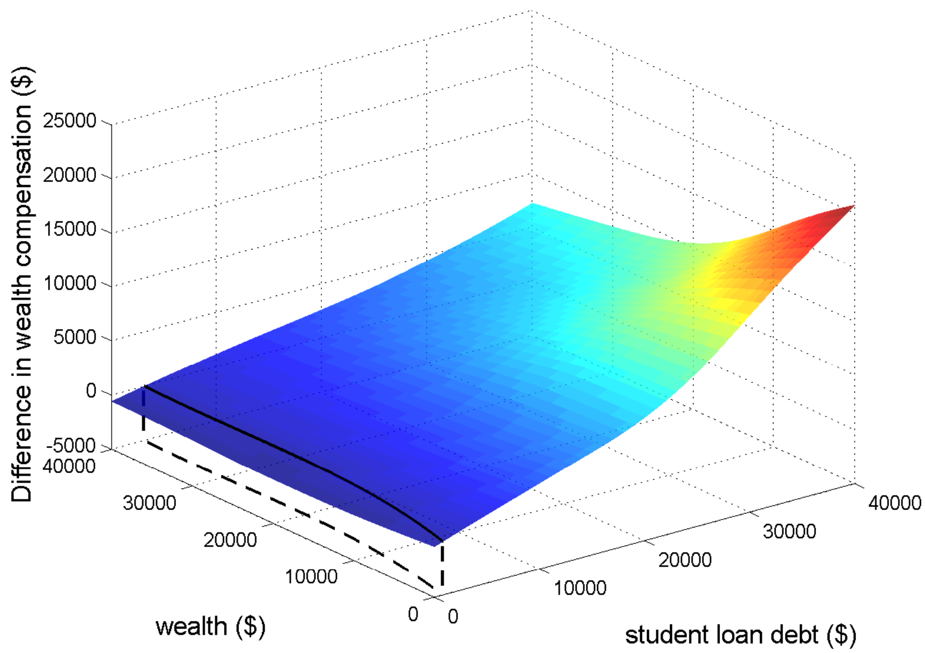


Note: This figure plots the life-cycle unemployment duration and wage income for high-loan borrowers (whose outstanding debt is above the median loan value of borrowers in my sample, i.e., $8,821) under the fixed repayment plan and the income-based repayment plan. In panels A and B, the blue solid line plots average unemployment duration and wage income for non-borrowers in the baseline economy (i.e., fixed repayment plan). The black dashed line plots the average unemployment duration and wage income for high-loan borrowers under the fixed repayment plan. The red dash-dotted line plots the average unemployment duration and wage income for high-loan borrowers under the income-based repayment plan. Panels C and D plot the difference in unemployment duration and wage income between non-borrowers and high-loan borrowers under the fixed repayment plan and the income-based repayment plan.

Figure 12: Comparing the life-cycle unemployment duration and wage income for high-loan borrowers under the fixed repayment plan and the income-based repayment plan.

plan has significant distributional effect. First, borrowers who are poorer and more indebted would benefit more by switching to the income-based repayment plan because they ask for larger wealth compensation. Second, non-borrowers or borrowers with less debt (≤$3,000) would incur a slight welfare loss when the whole economy adopts the income-based repayment plan. This is because non-borrowers or borrowers with less debt do not benefit much from the income-based repayment plan but they suffer from the reduction in job contact rates caused by fewer vacancy postings and higher aggregate search effort. This distributional effect is arguably helpful in terms of alleviating consumption inequality, because poorer and more indebted agents are also consuming less compared to non-borrowers.[56]

[56]The implication of IBR's distributional benefits coincides with the characteristics of borrowers enrolled in income-driven repayment plans in reality. The Executive Office of the President of the United States (2016) documents that undergraduate-only borrowers in income-driven repayment plans have a median outstanding debt of $25,000 compared with $10,000 in the fixed repayment plan in 2015. Moreover, the average family income based on the first application for federal student aid is $45,000 for those in income-driven repayment plans compared with $57,000 in the fixed repayment plan.



Note: This figure illustrates the distributional effect of the income-based repayment plan. I measure the increase in welfare caused by the income-based repayment plan using wealth compensation. For any borrower who just graduated from college, I calculate the least amount of wealth transfer that induces the borrower to switch from the income-based repayment plan to the fixed repayment plan. The figure plots the wealth compensation for borrowers of different levels of wealth and student loan debt. It is shown that borrowers who are poorer and more indebted benefit more by switching to the income-based repayment plan. In the figure, the area below the black solid line represents welfare losses.

Figure 13: The distributional effect of the income-based repayment plan.

7.2.3 Aggregate Implications

In this subsection, I evaluate the aggregate implication of the fixed repayment plan and the income-based repayment plan on various metrics. Under each repayment plan, I calculate the average unemployment duration, wage income, welfare (measured by wealth compensation), match quality (measured by job productivity), output, and labor supply for two groups of agents, all borrowers and high-loan borrowers between ages 23-32. I then compare the statistics of each group to those of non-borrowers in the baseline economy.

Specifically, all borrowers have $11,873 debt on average and they ask for $7,142 wealth compensation under the fixed repayment plan and $3,703 under the income-based repayment plan. High-loan borrowers have $18,970 debt on average and they ask for $10,788 wealth compensation under the fixed repayment plan and $5,256 under the income-based repayment plan. This suggests that allowing borrowers to have access to the income-based repayment plan would alleviate their debt burden by about half in my sample between 1997-2013. Note that although there is debt forgiveness provided by the income-based repayment plan after 25 years, my simulation results indicate that almost the entire debt in the economy is repaid by most borrowers. This implies that the debt alleviation caused by the income-based repayment plan is almost entirely driven by the insurance channel. There is not much debt forgiveness in my model because the evaluation is based on the sample mostly consisting of borrowers graduated around 2004,

Table 9: Evaluation of the income-based repayment plan.

	Non	**All borrowers**			**High-loan borrowers**		
	-borrowers	FIX	IBR	Difference	FIX	IBR	Difference
Welfare ($)	N/A	7,142	3,703	-3,439	10,788	5,256	-5,532
Unemp. dur. (week)	18.0	16.5	17.4	0.9	14.9	16.3	1.4
		(-8.3)	(-3.3)	(5.0)	(-17.2)	(-9.4)	(7.8)
Match quality	0.711	0.703	0.710	0.007	0.698	0.705	0.007
		(-1.1)	(-0.1)	(1.0)	(-1.8)	(-0.8)	(1.0)
Wage income ($)	51,521	49,382	51,234	1,852	47,542	50,363	2,821
		(-4.2)	(-0.6)	(3.6)	(-7.7)	(-2.2)	(5.5)
Output ($)	67,527	65,936	67,270	1,334	64,788	66,342	1,554
		(-2.4)	(-0.4)	(2.0)	(-4.1)	(-1.8)	(2.3)
Labor supply (hour)	1,889	1,869	1,849	-20	1,853	1,842	-11
		(-1.1)	(-2.1)	(-1.0)	(-1.9)	(-2.5)	(-0.6)

Note: This table compares the aggregate implications of debt burden in the first 10 years after college graduation under the fixed repayment plan and the income-based repayment plan. Column "Non-borrowers" reports outcomes of non-borrowers in the baseline (i.e., fixed repayment plan) simulation. Columns "FIX" report outcomes under the fixed repayment plan for borrowers and high-loan borrowers. Columns "IBR" report outcomes under the income-based repayment plan for borrowers and high-loan borrowers. Columns "Difference" report the difference between "FIX" and "IBR". Statistics in parentheses report the relative percent change using non-borrowers as the benchmark.

with a low average balance of about $11,873. By contrast, in 2014, 70% of students are indebted, and the average balance per borrower is about $27,000. Thus I expect the income-based repayment plan to have more debt forgiveness on 2014 borrowers and be more effective at alleviating the debt burden as the insurance benefits increase with outstanding debt (see Figure 13).[57]

Table 9 also shows that non-borrowers spend 18 weeks on job search on average in their first 10 years. All borrowers and high-loan borrowers on average spend 1.5 weeks and 3.1 weeks less when they are under the fixed repayment plan. The income-based repayment plan raises the reservation wage and increases job search time by 0.9 week for average borrowers.

In terms of match quality, borrowers under the fixed repayment plan are on average matched with jobs that are 1.1% less productive relative to jobs associated with non-borrowers. The income-based repayment plan improves match quality by about 1.0% for average borrowers. The lower match quality translates to lower output and wage income. On average, borrowers under the fixed repayment plan produce 2.4% less and earn 4.2% ($2,139) less compared to non-borrowers in each year of the first 10 years after college graduation. Note that at the estimated parameter values, borrowers already need to repay $1,550 every year on average under the fixed repayment plan. This suggests that debt repayment imposes a double burden on consumption. The indirect reduction in consumption due to inadequate job search is larger than the direct negative effect from debt repayment, which generates even larger consumption inequality between borrowers and non-borrowers.

The income-based repayment plan makes job search much more affordable, and as a result, output and wage income are increased by about 2.0% and 3.6% for average borrowers. Note that output increases precisely because the income-based repayment plan induces a positive reservation wage effect.

[57] I provide a suggestive evaluation for 2014 borrowers in Online Appendix B.3.

In my model, partial insurance provision increases both welfare and output, which is in contrast to an economy without search frictions, where providing insurance increases welfare but reduces output due to the adverse incentive effect on labor supply.

The negative effect on labor supply introduced by the income-based repayment is not large. Borrowers work for 1869 hours on average under the fixed repayment plan, and for 1849 hours under the income-based repayment plan. The difference in hours is small due to the positive substitution effect from having better jobs. The income-based repayment plan improves the job quality of borrowers, which incentivizes them to increase labor supply. This partially offsets the negative substitution effect caused by proportional repayment. In fact, we can think of the difference in unemployment duration as reflecting the adjustment on the extensive margin of labor supply. Table 9 then indicates that the income-based repayment plan reduces labor supply along both the extensive margin and the intensive margin, with the effect on the former being more significant.[58]

There is a small general equilibrium effect. The vacancy to unemployment ratio reduces from 0.409 to 0.405 after the income-based repayment plan is adopted. This is because firms are creating fewer vacancies due to the reduction in profit because more jobs are turned down by borrowers. The higher reservation wage set by borrowers increases aggregate search effort because searching during unemployment is more efficient. Therefore, non-borrowers job contact rates are lower owing to the higher aggregate search effort and the decrease in the number of vacancies. This increases the average unemployment duration of non-borrowers by about 0.5 week.

7.2.4 The Reservation Wage Effect

My theoretical analysis in section 4 indicates that the income-based repayment plan would increase borrowers' reservation wages, and this positive reservation wage effect would further increase borrowers' welfare. In this subsection, I use the model to separately quantify the positive reservation wage effect induced by the income-based repayment plan.

I conduct two experiments. In one experiment, I allow borrowers to adjust reservation wages under the income-based repayment plan as in the previous subsection. In the other experiment, I allow borrowers to make payments according to the income-based repayment plan, but their reservation wages are fixed at the values under the fixed repayment plan.[59] Therefore, in this experiment, the income-based repayment plan provides consumption smoothing but not job search benefits. The simulation outcome would measure the effect of the income-based repayment plan through the pure risk-sharing channel. The difference between the two experiments quantifies the positive reservation wage effect.

Table 10 presents that wealth compensation is \$4,933 and \$7,420 for borrowers and high-loan borrowers on average if reservation wages are fixed. If reservation wages are allowed to adjust, the wealth compensation under the income-based repayment plan would be \$3,703 and \$5,256. Therefore, the adjustment in reservation wages caused by the income-based repayment plan on average contributes

[58]Loosely speaking, this result is consistent with the common view that tax changes have smaller effects on the intensive margin than the extensive margin. For example, Rogerson and Wallenius (2009) find that the adjustment on the extensive margin of labor supply to taxation plays a major role in explaining differences in total hours worked across countries.

[59]For comparison purposes, I control for the general equilibrium effect by fixing the number of vacancies at the value in the first experiment.

Table 10: Quantifying the reservation wage effect of the income-based repayment plan.

	All borrowers			High-loan borrowers		
	(1)	(2)	Difference	(1)	(2)	Difference
Welfare ($)	3,703	4,933	1,230	5,256	7,420	2,164
Unemp. dur. (week)	17.4	16.3	-1.1	16.3	14.8	-1.5
	(-3.3)	(-9.4)	(-6.1)	(-9.4)	(-17.8)	(-8.4)
Match quality	0.710	0.704	-0.006	0.705	0.696	-0.009
	(-0.1)	(-1.0)	(-0.9)	(-0.8)	(-2.1)	(-1.3)
Wage income ($)	51,234	49,394	-1,840	50,363	47,978	-2,385
	(-0.6)	(-4.1)	(-3.5)	(-2.2)	(-6.9)	(-4.7)
Output ($)	67,270	66,164	-1,106	66,342	64,901	-1,441
	(-0.4)	(-2.0)	(-1.6)	(-1.8)	(-3.9)	(-2.1)
Labor supply (hour)	1,849	1,833	-16	1,842	1,808	-34
	(-2.1)	(-3.0)	(-0.9)	(-2.5)	(-4.3)	(-1.8)

Note: Column (1) reports the full effect of the income-based repayment plan when reservation wages are allowed to endogenously increase. Column (2) reports the effect of the income-based repayment plan when reservation wages are fixed at the values under the fixed repayment plan. Therefore, column (2) quantifies the pure risk-sharing channel of the income-based repayment plan. The difference between the two columns quantifies the reservation wage effect. Statistics in parentheses report the relative percent change using non-borrowers as the benchmark.

to a reduction in the wealth compensation by about $1,230 for all borrowers and $2,164 for high-loan borrowers. This implies that about one-third of the difference in wealth compensation between the fixed repayment plan and the income-based repayment plan is attributed to the positive reservation wage effect, and the remaining is due to better consumption smoothing.

Moreover, it is not surprising that almost the entire improvement in match quality, output, and wage income is caused by the positive response in reservation wages. When this channel is shut down, the average values of these variables for all borrowers and high-loan borrowers are similar to those under the fixed repayment plan.

As discussed in the previous subsection, the positive response in reservation wages also enables borrowers to obtain higher-paid jobs, generating a positive substitution effect that mitigates the reduction in labor supply. When reservation wages are fixed, labor supply would be further reduced by 16 hours on average for all borrowers and 34 hours for high-loan borrowers.

8 Robustness Check

I conduct three robustness checks for the quantitative results reported in Tables 9-10. In each robustness check, I reestimate all internally estimated parameters following the procedure in subsection 6.2.2. The simulation results are reported in Online Appendix Tables E.6-E.13.

8.1 Risk Aversion

One important parameter that determines the effect of debt burden on job search is risk aversion γ. In my baseline specification, γ is set to be 3 according to the precautionary savings literature. I now reduce its value to 1.5, according to the macro-development literature on financial frictions. The simulation results indicate that with lower risk aversion, the reduction in wage income is 76.5% of the baseline under the fixed repayment plan. The income-based repayment plan alleviates the debt burden by 30.5% and increases wage income by 1.8%, compared to 47.9% and 3.6% in the baseline. One-fourth of the reduction in debt burden is attributed to the reservation wage effect, as opposed to one-third in the baseline.

8.2 Elasticity of Labor Supply

The elasticity of labor supply determines the behavioral response of borrowers under the income-based repayment plan. In my baseline specification, σ is set to be 2.59 so that the tax-modified Frisch elasticity is 0.33. The micro estimates of intensive margin Hicksian labor supply elasticities range from 0 to 1. Moreover, several papers have noted larger labor market fluctuations at business-cycle frequencies for younger workers (Rios-Rull, 1996; Jaimovich and Siu, 2009). I check the model's implication by setting $\sigma = 0.78$ and $\sigma = 88.89$, corresponding to 1 and 0.01 tax-modified labor supply elasticities. When elasticity is 1, the simulation results indicate that the income-based repayment plan barely alleviates the debt burden or increases wage income due to the large distortion on labor supply. Borrowers' labor supply is on average reduced by 4.7% relative to non-borrowers, compared to 2.1% in the baseline. The reservation wage effect is still positive, as the wealth compensation would increase by $1,302 for borrowers if reservation wages are fixed. When elasticity is 0.01, there is almost no response in labor supply when borrowers switch to the income-based repayment plan. As a result, the income-based repayment plan becomes very effective in alleviating the debt burden. The wealth compensation is reduced by 59.4% on average when all borrowers switch to the income-based repayment plan, compared to 47.9% in the baseline.

8.3 Access to Other Credit

Credit access alleviates the liquidity problem, which would attenuate the effect of debt burden on job search. In the baseline specification, agents cannot borrow; I now relax this assumption. Using data from the Survey of Consumer Finances (SCF), Kaplan and Violante (2014) estimate that the median ratio of credit limit to annual labor income is 18.5% for households aged 22 to 59. Based on this estimate, I allow employed agents to borrow 18.5% of their wage income, and unemployed agents to borrow 18.5% of UI benefits (i.e., $1,500). The simulation results indicate that credit access slightly alleviates the debt burden. The reduction in wage income is 97% of the baseline under the fixed repayment plan. The small difference comes from the fact agents cannot borrow much due to the low income during unemployment. The income-based repayment plan alleviates the debt burden by 44.6% and increases wage income by 3.2%, compared to 47.9% and 3.6% in the baseline. 23.4% of the reduction in debt burden is attributed to the reservation wage effect, as opposed to one-third in the baseline.

9 Conclusion

This paper evaluates the implications of student loan debt on labor market outcomes. My starting point is the observed correlation that indebted college graduates are associated with shorter unemployment duration and lower wage income. Motivated by these facts, I propose a tractable theoretical framework to delineate the economic mechanism through which debt burden drives individuals to be less patient in job search. I argue that the income-based repayment contract would alleviate the negative effect on wage income and improve welfare. I then develop and estimate a rich quantitative model to evaluate the aggregate, distributional, and long-term impact of debt burden and run various counterfactuals to assess the income-based repayment plan.

This paper contributes toward existing literature in three ways. First, this paper presents a novel view on how debt burden affects individuals' job search decisions and labor market outcomes. I illustrate that indebted individuals tend to be less patient in job search, and consequently, they are more likely to end up in lower-paid jobs. The exact effect of debt burden on job search behavior also depends on the repayment schedule due to the existence of the risk channel and the liquidity channel. This view complements the existing view that indebted individuals tend to search for jobs in high-paid sectors and on average have higher wage income (Rothstein and Rouse, 2011).

Second, this paper develops and estimates a quantitative model featuring search frictions to evaluate the implication of student loan debt on labor market outcomes through the proposed mechanism. The simulation results suggest that under the standard fixed repayment plan, there is a lasting effect of debt burden on wage income due to the low job-to-job transition rate. For borrowers, the reduction in consumption caused by inadequate job search is potentially larger than the direct effect from debt repayment. Importantly, the simulation results indicate that the income-based repayment plan is effective in terms of alleviating the debt burden and improving both wage income and welfare.

Third, this paper elucidates and quantifies a novel reservation wage effect of insurance provision in an economy with search risks. Providing insurance not only directly increases welfare through the risk-sharing channel but also indirectly increases it by increasing the reservation wage. I illustrate the reservation wage effect by analytically characterizing the optimal repayment contract and use counterfactual simulations to quantify its importance under the income-based repayment plan. The simulation results imply that one-third of the reduction in debt burden is attributed to the positive response in reservation wages. I argue that this sizable reservation wage effect should be considered when evaluating education financing policies.

In the future, I hope to further the understanding of student loan debt along two directions. First, in work in progress (Ji and Yannelis, 2016), we intend to provide causal evidence for the effect of student loan debt on labor market outcomes using administrative data on federal student loans and de-identified tax records. Second, it is tempting to consider college entry, borrowing, and job search in a unified framework. If students anticipate future debt forgiveness, more students with less talent would go to college and more students would attend expensive private schools. This potentially generates a large ex-ante moral hazard effect that is not addressed in the current model. There could also be moral hazard on the college side. If it is easier for students to obtain loans, private institutions would increase tuition fees due to higher demand for education.

Moreover, the unified framework is also useful to evaluate the current student loan system. From a societal perspective, student loan debt provides a level-playing field for students from low-income families, which arguably helps increase their earnings and reduce inequality. The mechanism proposed in this paper concerns a possible negative effect of student loan debt after graduation, when students are in the labor market. What this paper suggests is that the overall effect of student loans should be reevaluated, because we also need to consider the negative effect of debt burden on labor market outcomes. Importantly, a better designed repayment schedule could be very useful in optimizing the student loan system.

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Appendix

A Proofs

A.1 Proof of Proposition 1

Proof. Rearranging equation (4.3), the reservation wage is implicitly determined by

$$1 = \frac{\beta}{1-\beta} \int_{w^*_{FIX}}^{\overline{w}} \frac{u(w-s) - u(w^*_{FIX} - s)}{u(w^*_{FIX} - s) - u(\theta - s)} dF(w). \tag{A.1}$$

Consider increasing debt by Δs, and denote the reservation wage corresponding to $s + \Delta s$ as $\hat{w}^*_{FIX}$, thus according to (A.1),

$$1 = \frac{\beta}{1-\beta} \int_{\hat{w}^*_{FIX}}^{\overline{w}} \frac{u(w-s-\Delta s) - u(\hat{w}^*_{FIX} - s - \Delta s)}{u(\hat{w}^*_{FIX} - s - \Delta s) - u(\theta - s - \Delta s)} dF(w). \tag{A.2}$$

Define $u_2(x) = u(x - \Delta s)$, we can rewrite (A.2) as

$$1 = \frac{\beta}{1-\beta} \int_{\hat{w}^*_{FIX}}^{\overline{w}} \frac{u_2(w-s) - u_2(\hat{w}^*_{FIX} - s)}{u_2(\hat{w}^*_{FIX} - s) - u_2(\theta - s)} dF(w). \tag{A.3}$$

Let $r(x)$ and $r_2(x)$ be the local absolute risk aversion for $u(x)$ and $u_2(x)$. Thus

$$\begin{aligned} r(x) &> r_2(x) \quad \text{If } u(\cdot) \text{ has IARA;} \\ r(x) &= r_2(x) \quad \text{If } u(\cdot) \text{ has CARA;} \\ r(x) &< r_2(x) \quad \text{If } u(\cdot) \text{ has DARA.} \end{aligned} \tag{A.4}$$

Taking DARA as an example, note that $\theta - s < w^*_{FIX} - s < w - s$ for all $w \in (w^*_{FIX}, \overline{w}]$, thus according to Pratt (1964, Theorem 1),

$$\begin{aligned} 1 &= \frac{\beta}{1-\beta} \int_{w^*_{FIX}}^{\overline{w}} \frac{u(w-s) - u(w^*_{FIX} - s)}{u(w^*_{FIX} - s) - u(\theta - s)} dF(w) \\ &> \frac{\beta}{1-\beta} \int_{w^*_{FIX}}^{\overline{w}} \frac{u_2(w-s) - u_2(w^*_{FIX} - s)}{u_2(w^*_{FIX} - s) - u_2(\theta - s)} dF(w). \end{aligned} \tag{A.5}$$

Then (A.3) and (A.5) imply

$$\int_{\hat{w}^*_{FIX}}^{\overline{w}} \frac{u_2(w-s) - u_2(\hat{w}^*_{FIX} - s)}{u_2(\hat{w}^*_{FIX} - s) - u_2(\theta - s)} dF(w) > \int_{w^*_{FIX}}^{\overline{w}} \frac{u_2(w-s) - u_2(w^*_{FIX} - s)}{u_2(w^*_{FIX} - s) - u_2(\theta - s)} dF(w). \tag{A.6}$$

Because $\int_{w^*_{FIX}}^{\overline{w}} \frac{u_2(w-s)-u_2(w^*_{FIX}-s)}{u_2(w^*_{FIX}-s)-u_2(\theta-s)} dF(w)$ is decreasing in w^*_{FIX}, this implies $\hat{w}^*_{FIX} < w^*_{FIX}$.

Note that Danforth (1974) extends the result of Pratt (1964) to multi-dimensional lotteries. By applying Danforth (1974, Theorem 2), we can obtain a more general result, which indicates that higher debt reduces the agent's reservation wage even in a perfect credit market. ☐

A.2 Proof of Proposition 2

Proof. If the wage offer is accepted at $t = 1$, then the wage income becomes flat in the future. Therefore, the agent would perfectly smooth consumption by saving $s - s_1$ at $t = 1$, and consuming $w - s$ in every period. The value function is

$$W_1(w) = \frac{u(w - s)}{1 - \beta}. \tag{A.7}$$

Under the twisted repayment schedule, suppose that the agent's borrowing constraint is binding when unemployed, i.e., the agent does not save at $t = 1$ if the wage offer is rejected. Then the value function is

$$V_1 = u(\theta - s_1) + \beta \int_{\theta}^{w_2^*} V_2 dF(w) + \beta \int_{w_2^*}^{\overline{w}} W_2(w) dF(w), \tag{A.8}$$

where V_2 and $W_2(w)$ are the value functions of rejecting and accepting the wage offer at $t = 2$ conditional on the wage offer being rejected at $t = 1$. w_2^* is the reservation wage at $t = 2$; It is also the reservation wage for all $t > 2$ because the job search problem is stationary in later periods due to constant debt repayment and zero initial wealth. Therefore, we can write V_2 and $W_2(w)$ as

$$W_2(w) = \frac{u(w - s_2)}{1 - \beta}. \tag{A.9}$$

$$V_2 = \frac{u(\theta - s_2)}{1 - \beta} + \frac{\beta}{1 - \beta} \int_{w_2^*}^{\overline{w}} [W_2(w) - V_2] dF(w). \tag{A.10}$$

The reservation wage at $t = 1$, w_1^*, is determined by

$$V_1 = W_1(w_1^*) \tag{A.11}$$

Substituting equations (A.7) and (A.8) into equation (A.11), we obtain

$$\frac{u(w_1^* - s)}{1 - \beta} = u(\theta - s_1) + \beta \int_{\theta}^{w_2^*} V_2 dF(w) + \beta \int_{w_2^*}^{\overline{w}} W_2(w) dF(w). \tag{A.12}$$

Substituting equation (A.9) and $V_2 = W_2(w_2^*)$ into equation (A.12), we obtain

$$\frac{u(w_1^* - s)}{1 - \beta} = u(\theta - s_1) + \frac{\beta}{1 - \beta} \int_{\theta}^{w_2^*} u(w_2^* - s_2) dF(w) + \frac{\beta}{1 - \beta} \int_{w_2^*}^{\overline{w}} u(w - s_2) dF(w). \tag{A.13}$$

Consider small changes of payments, $\Delta s_1 < 0$, equation (4.4) and assumption $\beta(1 + r) = 1$ imply

$$\Delta s_2 = -r \Delta s_1 = -\frac{1 - \beta}{\beta} \Delta s_1 > 0. \tag{A.14}$$

Differentiating equation (A.13):

$$\Delta w_1^* = -\frac{1}{Q}u'(\theta - s_1)\Delta s_1 + \frac{\beta}{Q(1-\beta)}u'(w_2^* - s_2)F(w_2^*)\Delta w_2^* \\ + \frac{\beta}{Q(1-\beta)}\left[-u'(w_2^* - s_2) + \int_{w_2^*}^{\overline{w}}[u'(w_2^* - s_2) - u'(w - s_2)]dF(w)\right]\Delta s_2, \tag{A.15}$$

where $Q = \frac{u'(w_1^* - s)}{1-\beta} > 0$.

The reservation wage at $t = 2$, w_2^*, is determined by $V_2 = W(w_2^*)$,

$$u(w_2^* - s_2) = u(\theta - s_2) + \frac{\beta}{1-\beta}\int_{w_2^*}^{\overline{w}}[u(w - s_2) - u(w_2^* - s_2)]dF(w). \tag{A.16}$$

Differentiating equation (A.16):

$$\Delta w_2^* = \frac{u'(w_2^* - s_2)\dfrac{1-\beta F(w_2^*)}{1-\beta} - u'(\theta - s_2) - \dfrac{\beta}{1-\beta}\displaystyle\int_{w_2^*}^{\overline{w}} u'(w - s_2)dF(w)}{u'(w_2^* - s_2)\dfrac{1-\beta F(w_2^*)}{1-\beta}}\Delta s_2. \tag{A.17}$$

Substituting (A.14) and (A.17) into (A.15), I obtain

$$\Delta w_1^* = -\frac{1}{Q}[u'(\theta - s_1) - \frac{(1-\beta)F(w_2^*)}{1-\beta F(w_2^*)}u'(\theta - s_2) - \frac{1}{1-\beta F(w_2^*)}\int_{w_2^*}^{\overline{w}} u'(w - s_2)dF(w)]\Delta s_1. \tag{A.18}$$

When the wage offer at $t = 1$ is rejected, the marginal utility of one unit of consumption at $t = 1$ is $u'(\theta - s_1)$, and the marginal utility of one unit of savings is

$$\beta\left[(1+r)u'(\theta - s_2)F(w_2^*) + \frac{r}{1-\beta}\int_{w_2^*}^{\overline{w}} u'(w - s_2)dF(w)\right]. \tag{A.19}$$

In (A.19), the first term captures that the agent would consume $(1 + r)$ at marginal utility $u'(\theta - s_2)$ if the wage offer is below w_2^* at $t = 2$ and rejected. The agent does not save in this case because debt payment is flat during $t \geq 2$ and expected income is higher. The second term captures that the agent would consume r at marginal utility $u'(w - s_2)$ in every future period, $t \geq 2$, if the wage offer w is above w_2^* at $t = 2$ and accepted. This is because both wage income and debt payment are flat in every future period, $t \geq 2$. Thus the agent would only consume the interest of her one unit of wealth to perfectly smooth consumption.

The binding borrowing constraint implies that the marginal utility of one unit of consumption at $t = 1$ is larger than the marginal utility of one unit of savings, i.e.,

$$u'(\theta - s_1) \geq F(w_2^*)u'(\theta - s_2) + \int_{w_2^*}^{\overline{w}} u'(w - s_2)dF(w). \tag{A.20}$$

Substituting (A.20) into (A.18),

$$
\begin{aligned}
\Delta w_1^* &\geq -\frac{1}{Q}\left[\frac{\beta F(w_2^*)[1-F(w_2^*)]}{1-\beta F(w_2^*)}u'(\theta - s_2) - \frac{\beta F(w_2^*)}{1-\beta F(w_2^*)}\int_{w_2^*}^{\overline{w}} u'(w-s_2)dF(w)\right]\Delta s_1 \\
&= -\frac{\beta F(w_2^*)}{Q[1-\beta F(w_2^*)]}\left[[1-F(w_2^*)]u'(\theta - s_2) - \int_{w_2^*}^{\overline{w}} u'(w-s_2)dF(w)\right]\Delta s_1 \\
&= -\frac{\beta F(w_2^*)\Delta s_1}{Q[1-\beta F(w_2^*)]}\int_{w_2^*}^{\overline{w}}[u'(\theta - s_2) - u'(w-s_2)]dF(w) > 0 \qquad \text{(A.21)}
\end{aligned}
$$

□

A.3 Proof of Proposition 3

Proof. Rearranging equation (4.5),

$$
I(w_{FIX}^*) = \frac{\theta F(w_{FIX}^*) + \frac{1}{1-\beta}\int_{w_{FIX}^*}^{\overline{w}} w dF(w)}{1-\beta F(w_{FIX}^*)}. \qquad \text{(A.22)}
$$

Take the first derivative,

$$
I'(w_{FIX}^*) = \frac{f(w_{FIX}^*)}{[1-\beta F(w_{FIX}^*)]^2}\left[\theta - w_{FIX}^* + \frac{\beta}{1-\beta}\int_{w_{FIX}^*}^{\overline{w}}(w - w_{FIX}^*)dF(w)\right]. \qquad \text{(A.23)}
$$

Denote

$$
h(x) = \theta - x + \frac{\beta}{1-\beta}\int_{x}^{\overline{w}}(w-x)dF(w). \qquad \text{(A.24)}
$$

It is straightforward to show that $h(\theta) > 0$, $h(\overline{w}) < 0$, and $h(x)' < 0$. Thus there exists a unique $w_{FIX}^* \in (\theta, \overline{w})$, denoted as $\hat{w}$, such that $I'(\hat{w}) = 0$. When $w^* < \hat{w}$, $I'(w_{FIX}^*) > 0$ and expected income is strictly increasing in w_{FIX}^*; when $w_{FIX}^* > \hat{w}$, $I'(w_{FIX}^*) < 0$ and expected income is strictly decreasing in w_{FIX}^*. Therefore, $\hat{w}$ maximizes expected income and is determined by

$$
\hat{w} - \frac{\beta}{1-\beta}\int_{\hat{w}}^{\overline{w}}(w-\hat{w})dF(w) = \theta. \qquad \text{(A.25)}
$$

Now, I prove that a risk-neutral agent sets her reservation wage to be $\hat{w}$. Because the interest rate is assumed to satisfy $\beta(1+r) = 1$, the risk-neutral agent is indifferent about savings. Without loss of generality, I assume that the risk-neutral agent also behaves hand-to-mouth, like a risk-averse agent. Therefore, her reservation wage is determined by equation (4.3).

The utility function of the risk-neutral agent has a linear form, i.e., $u(x) = ax + b$. Substituting this into equation (4.3), I obtain

$$
w_{FIX}^* - \frac{\beta}{1-\beta}\int_{w_{FIX}^*}^{\overline{w}}(w - w_{FIX}^*)dF(w) = \theta. \qquad \text{(A.26)}
$$

There is a unique solution to equation (A.26), thus $w_{FIX}^* = \hat{w}$ for the risk-neutral agent. □

A.4 Proof of Proposition 4

Proof. The mileage that CRRA utility buys me is that it is a homogeneous utility function with multiplicative scaling behavior. With CRRA utility, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$, equation (4.8) becomes

$$(w^*_{IBR})^{1-\gamma} = \theta^{1-\gamma} + \frac{\beta}{1-\beta}\int_{w^*_{IBR}}^{\overline{w}} [w^{1-\gamma} - (w^*_{IBR})^{1-\gamma}]dF(w). \tag{A.27}$$

Clearly, w^*_{IBR} does not depend on α. Therefore, under the income-based repayment contract, when the utility has CRRA, the agent's reservation wage is equal to the reservation wage of the agent who has no debt. This suggests that

$$w^*_{IBR} = w^*|_{s=0} > w^*_{FIX}, \tag{A.28}$$

where the last inequality is from Proposition 1 because CRRA utility has decreasing absolute risk aversion. Note that another way to see that the reservation wage does not depend on α when utility has CRRA is to calculate the absolute risk aversion for utility $u((1-\alpha)x)$, which is γ/x, not a function of α. Then, according to the proof of Proposition 1, the reservation wage stays the same because the local absolute risk aversion does not change for any x when α changes.

In fact, we can further show that the disposable reservation wage also satisfies $\overline{w}^*_{IBR} > \overline{w}^*_{FIX}$. This indicates that the liquidity channel plus the risk channel strictly dominates the optionality channel according to Proposition 5. This result is obtained by applying the following lemma to equation (4.10).

Lemma 2. *The reservation wage under the income-based repayment contract satisfies:*

$$w^*_{IBR} < \frac{s}{\alpha}, \tag{A.29}$$

where α solves equation (4.7).

Proof. According to equation (4.8), w^*_{IBR} is determined by

$$u((1-\alpha)w^*_{IBR}) = \frac{(1-\beta)u((1-\alpha)\theta) + \beta\int_{w^*_{IBR}}^{\overline{w}} u((1-\alpha)w)dF(w)}{1-\beta F(w^*_{IBR})}. \tag{A.30}$$

Thus

$$(1-\alpha)w^*_{IBR} = u^{-1}\left[\frac{(1-\beta)u((1-\alpha)\theta) + \beta\int_{w^*_{IBR}}^{\overline{w}} u((1-\alpha)w)dF(w)}{1-\beta F(w^*_{IBR})}\right]. \tag{A.31}$$

Notice that

$$\frac{(1-\beta) + \beta\int_{w^*_{IBR}}^{\overline{w}} dF(w)}{1-\beta F(w^*_{IBR})} = 1, \tag{A.32}$$

thus, we can think of the LHS of equation (A.32) as probability weights, which are imposed on $u((1-\alpha)x)$ to generate the RHS of equation (A.31).

By Jensen's inequality, equation (A.31) can be written as

$$w^*_{IBR} < \frac{(1-\beta)\theta + \beta \int_{w^*_{IBR}}^{\overline{w}} w dF(w)}{1 - \beta F(w^*_{IBR})}. \tag{A.33}$$

According to equation (4.5) and (4.7), α is determined by

$$\frac{s}{\alpha} = \frac{(1-\beta)\theta F(w^*_{IBR}) + \int_{w^*_{IBR}}^{\overline{w}} w dF(w)}{1 - \beta F(w^*_{IBR})}. \tag{A.34}$$

Therefore,

$$\begin{aligned}
\frac{s}{\alpha} - w^*_{IBR} &> \frac{(1-\beta)\int_{w^*_{IBR}}^{\overline{w}} w dF(w) - (1-\beta)\theta[1 - F(w^*_{IBR})]}{1 - \beta F(w^*_{IBR})} \\
&> \frac{(1-\beta)[1 - F(w^*_{IBR})](w^*_{IBR} - \theta)}{1 - \beta F(w^*_{IBR})} \\
&> 0.
\end{aligned} \tag{A.35}$$

□

Using Lemma 2, the disposable reservation wage satisfies

$$\begin{aligned}
\overline{w}^*_{IBR} - \overline{w}^*_{FIX} &= (1-\alpha) w^*_{IBR} - w^*_{FIX} + s \\
&> (1-\alpha) w^*_{IBR} - w^*_{IBR} + s \\
&= \alpha \left(\frac{s}{\alpha} - w^*_{IBR} \right) \\
&> 0.
\end{aligned} \tag{A.36}$$

□

A.5 Proof of Lemma 1

Proof. Proposition 3 indicates that $I(x)$ is increasing in x when $x < \hat{w}$ (note: $\hat{w}$ is the reservation wage chosen by a risk-neutral agent). Therefore, equation (4.7) implies

$$\alpha = \frac{S}{\beta I(w^*_{IBR})} < \frac{S}{\beta I(\theta)} = \frac{s}{\int_{\theta}^{\overline{w}} w dF(w)}. \tag{A.37}$$

The expected disposable wage offer under the two contracts are

$$E_{IBR} = \int_{(1-\alpha)\theta}^{(1-\alpha)\overline{w}} w dF_{IBR}(w) = \int_{\theta}^{\overline{w}} (1-\alpha) w dF(w); \tag{A.38}$$

$$E_{FIX} = \int_{\theta-s}^{\overline{w}-s} w dF_{FIX}(w) = \int_{\theta}^{\overline{w}} (w-s) dF(w). \tag{A.39}$$

Taking the difference,

$$E_{IBR} - E_{FIX} = s - \int_{\theta}^{\overline{w}} \alpha w dF(w) > 0, \tag{A.40}$$

according to equation (A.37). Moreover, because s/α is the unique solution to $F_{IBR}(w) = F_{FIX}(w)$ and $(1-\alpha)\theta > \theta - s$, $F_{IBR}(w)$ single crosses $F_{FIX}(w)$ from below, i.e.,

$$\begin{aligned} F_{IBR}(w) < F_{FIX}(w) \quad & \text{for } w < s/\alpha \\ F_{IBR}(w) > F_{FIX}(w) \quad & \text{for } w > s/\alpha. \end{aligned} \tag{A.41}$$

For $z \in [0, s/\alpha]$, the single-crossing property implies

$$\int_0^z F_{IBR}(w) dw < \int_0^z F_{FIX}(w) dw. \tag{A.42}$$

For $z \in (s/\alpha, \overline{w}]$,

$$\begin{aligned} 0 &< E_{IBR} - E_{FIX} \\ &= \int_0^\infty [1 - F_{IBR}(w)] dw - \int_0^\infty [1 - F_{FIX}(w)] dw \\ &= \int_0^z [1 - F_{IBR}(w)] dw - \int_0^z [1 - F_{FIX}(w)] dw + \int_z^\infty [1 - F_{IBR}(w)] dw - \int_z^\infty [1 - F_{FIX}(w)] dw \\ &< \int_0^z [1 - F_{IBR}(w)] dw - \int_0^z [1 - F_{FIX}(w)] dw \\ &= \int_0^z F_{FIX}(w) dw - \int_0^z F_{IBR}(w) dw. \end{aligned} \tag{A.43}$$

Note that second inequality uses the single-crossing property, and the second equality uses an expectation formula derived below. For a continuous random variable x taking only non-negative values,

$$\begin{aligned} E(x) &= \int_0^\infty x f(x) dx \\ &= \int_0^\infty (-x) d(1 - F(x)) \\ &= [-x(1 - F(x))]_0^\infty + \int_0^\infty [1 - F(x)] dx. \end{aligned} \tag{A.44}$$

The first term in bracket vanishes because

$$1 - F(x) = o(\frac{1}{x}) \quad \text{as } x \to \infty. \tag{A.45}$$

$\square$

A.6 Proof of Proposition 5

Proof. Consider the fixed repayment contract. The disposable reservation wage, $\overline{w}^*_{FIX}$, is determined by

$$
\begin{aligned}
u(\tilde{w}^*_{FIX}) &= u(\theta - s) + \frac{\beta}{1-\beta}\int_{\tilde{w}^*_{FIX}}^{w-s}[u(w) - u(\tilde{w}^*_{FIX})]dF_{FIX}(w) \\
&= u(\theta - s) + \frac{\beta}{1-\beta}\int_{\tilde{w}^*_{FIX}}^{\infty}[u(w) - u(\tilde{w}^*_{FIX})]dF_{FIX}(w) \\
&= u(\theta - s) + \frac{\beta}{1-\beta}\left[M - u(\tilde{w}^*_{FIX}) - \int_{\tilde{w}^*_{FIX}}^{\infty} u'(w)d\left(\int_0^w F_{FIX}(x)dx\right)\right] \\
&= u(\theta - s) + \frac{\beta}{1-\beta}\left[M - u(\tilde{w}^*_{FIX}) - \lim_{x\to\infty} u'(x)\int_0^x F_{FIX}(w)dw + u'(\tilde{w}^*_{FIX})\int_0^{\tilde{w}^*_{FIX}} F_{FIX}(w)dw\right. \\
&\left.+\int_{\tilde{w}^*_{FIX}}^{\infty}\left(\int_0^w F_{FIX}(x)dx\right)u''(w)dw\right], \qquad \text{(A.46)}
\end{aligned}
$$

where $M = \lim_{x\to\infty} u(x)$. The last two equalities are derived by doing integration by parts.

Rearranging the above equation,

$$
\begin{aligned}
u(\tilde{w}^*_{FIX}) = (1-\beta)u(\theta - s) + \beta&\left[M - \lim_{x\to\infty} u'(x)\int_0^x F_{FIX}(w)dw + u'(\tilde{w}^*_{FIX})\int_0^{\tilde{w}^*_{FIX}} F_{FIX}(w)dw\right. \\
&\left.+\int_{\tilde{w}^*_{FIX}}^{\infty}\left(\int_0^w F_{FIX}(x)dx\right)u''(w)dw\right]. \qquad \text{(A.47)}
\end{aligned}
$$

Similarly, for the income-based repayment contract, we have

$$
\begin{aligned}
u(\tilde{w}^*_{IBR}) = (1-\beta)u((1-\alpha)\theta) + \beta&\left[M - \lim_{x\to\infty} u'(x)\int_0^x F_{IBR}(w)dw + u'(\tilde{w}^*_{IBR})\int_0^{\tilde{w}^*_{IBR}} F_{IBR}(w)dw\right. \\
&\left.+\int_{\tilde{w}^*_{IBR}}^{\infty}\left(\int_0^w F_{IBR}(x)dx\right)u''(w)dw\right]. \qquad \text{(A.48)}
\end{aligned}
$$

Taking the difference between (A.47) and (A.48):

$$
\begin{aligned}
u(\tilde{w}^*_{IBR}) - u(\tilde{w}^*_{FIX}) = (1-\beta)\left[u((1-\alpha)\theta) - u(\theta - s)\right] &- \beta\lim_{x\to\infty} u'(x)\int_0^x\left[F_{IBR}(w) - F_{FIX}(w)\right]dw \\
+\beta&\left[\int_{\tilde{w}^*_{IBR}}^{\infty}\left(\int_0^w F_{IBR}(x)dx\right)u''(w)dw - \int_{\tilde{w}^*_{FIX}}^{\infty}\left(\int_0^w F_{FIX}(x)dx\right)u''(w)dw\right]. \\
+\beta&\left[u'(\tilde{w}^*_{IBR})\int_0^{\tilde{w}^*_{IBR}} F_{IBR}(w)dw - u'(\tilde{w}^*_{FIX})\int_0^{\tilde{w}^*_{FIX}} F_{FIX}(w)dw\right] \qquad \text{(A.49)}
\end{aligned}
$$

Because $\lim_{x\to\infty} u'(x) = 0$ and $\lim_{x\to\infty} \int_0^x [F_{FIX}(w) - F_{IBR}(w)]\, dw$ is finite,

$$\lim_{x\to\infty} u'(x) \int_0^x [F_{FIX}(w) - F_{IBR}(w)]\, dw = 0. \tag{A.50}$$

Thus,

$$\begin{aligned} u(\tilde{w}^*_{IBR}) - u(\tilde{w}^*_{FIX}) &= (1-\beta)\,[u((1-\alpha)\theta) - u(\theta - s)] \\ &\quad + \beta \left[\int_{\tilde{w}^*_{IBR}}^{\infty} \left(\int_0^w F_{IBR}(x)dx \right) u''(w)dw - \int_{\tilde{w}^*_{FIX}}^{\infty} \left(\int_0^w F_{FIX}(x)dx \right) u''(w)dw \right] \\ &\quad + \beta \left[u'(\tilde{w}^*_{IBR}) \int_0^{\tilde{w}^*_{IBR}} F_{IBR}(w)dw - u'(\tilde{w}^*_{FIX}) \int_0^{\tilde{w}^*_{FIX}} F_{FIX}(w)dw \right]. \end{aligned} \tag{A.51}$$

In equation (A.51), increasing $\tilde{w}^*_{IBR}$ increases the LHS by $u'(\tilde{w}^*_{IBR})$, more than the increase in the RHS, $\beta F_{IBR}(\tilde{w}^*_{IBR})u'(\tilde{w}^*_{IBR})$. Thus, given $\tilde{w}^*_{FIX}$, there is a unique $\tilde{w}^*_{IBR}$, and whether it is greater or less than $\tilde{w}^*_{FIX}$ depends on the sign of the RHS conditional on $\tilde{w}^*_{FIX} = \tilde{w}^*_{IBR}$.

The first term is positive because $(1-\alpha)\theta > \theta - s$ according to Lemma 1. When $\tilde{w}^*_{FIX} = \tilde{w}^*_{IBR}$, the second term is

$$\beta \left[\int_{\tilde{w}^*_{FIX}}^{\infty} \left(\int_0^w F_{IBR}(x)dx - \int_0^w F_{FIX}(x)dx \right) u''(w)dw \right], \tag{A.52}$$

which is positive because $u''(w) < 0$ and $\int_0^w F_{IBR}(x)dx - \int_0^w F_{FIX}(x)dx < 0$ for all $w > \tilde{w}^*_{FIX}$ according to Lemma 1.

When $\tilde{w}^*_{FIX} = \tilde{w}^*_{IBR}$, the third term is

$$\beta u'(\tilde{w}^*_{FIX}) \left[\int_0^{\tilde{w}^*_{FIX}} F_{IBR}(w)dw - \int_0^{\tilde{w}^*_{FIX}} F_{FIX}(w)dw \right], \tag{A.53}$$

which is negative according to Lemma 1.

□

A.7 Proof of Proposition 6

Proof. The welfare of the agent under the two repayment contracts is given by:

$$\begin{aligned} \text{Welfare}_{IBR} &= \frac{F(w^*_{IBR})u((1-\alpha)\theta)}{1-\beta F(w^*_{IBR})} + \int_{w^*_{IBR}}^{\overline{w}} \frac{u((1-\alpha)w)}{(1-\beta)[1-\beta F(w^*_{IBR})]} dF(w); \\ \text{Welfare}_{FIX} &= \frac{F(w^*_{FIX})u(\theta - s)}{1-\beta F(w^*_{FIX})} + \int_{w^*_{FIX}}^{\overline{w}} \frac{u(w-s)}{(1-\beta)[1-\beta F(w^*_{FIX})]} dF(w). \end{aligned} \tag{A.54}$$

Notice that

$$\frac{(1-\beta)F(w^*) + \int_{w^*}^{\overline{w}} dF(w)}{1-\beta F(w^*)} = 1, \tag{A.55}$$

which allows us to define a CDF $G(w; w^*)$ with parameter w^* as follows:

$$G(w; w^*) = \begin{cases} \dfrac{(1-\beta)F(w^*)}{1-\beta F(w^*)} & \text{if } w \in [\theta, w^*], \\ \dfrac{F(w) - \beta F(w^*)}{1-\beta F(w^*)} & \text{if } w \in (w^*, \overline{w}]. \end{cases} \tag{A.56}$$

Then the welfare equations (A.54) can be written as:

$$\text{Welfare}_{IBR} = \frac{1}{1-\beta} \int_{\theta}^{\overline{w}} u((1-\alpha)w) dG(w; w^*_{IBR}); \tag{A.57}$$

$$\text{Welfare}_{FIX} = \frac{1}{1-\beta} \int_{\theta}^{\overline{w}} u(w-s) dG(w; w^*_{FIX}). \tag{A.58}$$

To prove that $\text{Welfare}_{IBR} > \text{Welfare}_{FIX}$, it is sufficient to show that the lottery with value $(1-\alpha)w$ and CDF $G(w; w^*_{IBR})$ is second-order stochastically dominant over the lottery with value $w - s$ and CDF $G(w; w^*_{FIX})$.

Following the proof of Lemma 1, the single-crossing condition is satisfied because $(1-\alpha)\theta > \theta - s$. Thus I only need to show that the mean of lottery $G(w; w^*_{IBR})$ is larger than the mean of lottery $G(w; w^*_{FIX})$:

$$\int_{\theta}^{\overline{w}} (1-\alpha) w dG(w; w^*_{IBR}) > \int_{\theta}^{\overline{w}} (w-s) dG(w; w^*_{FIX}). \tag{A.59}$$

Define $L(w^*)$ as follows:

$$L(w^*) = \int_{\theta}^{\overline{w}} w dG(w; w^*) = \frac{(1-\beta)\theta F(w^*) + \int_{w^*}^{\overline{w}} w dF(w)}{1-\beta F(w^*)}. \tag{A.60}$$

Taking the first derivative w.r.t. w^*:

$$L(w^*)' = \frac{(1-\beta) f(w^*)}{[1-\beta F(w^*)]^2} \left[\theta - w^* + \frac{\beta}{1-\beta} \int_{w^*}^{\overline{w}} (w - w^*) dF(w) \right]. \tag{A.61}$$

According to the proof of Proposition 3, $L(w^*)' > 0$ as long as $w^* < \hat{w}$, which is always the case because $\hat{w}$ is the reservation wage chosen by a risk-neutral agent.

Proposition 4 shows that with CRRA utility $w^*_{IBR} > w^*_{FIX}$. Then $L(w^*)' > 0$ implies that

$$\int_{\theta}^{\overline{w}} w dG(w; w^*_{IBR}) > \int_{\theta}^{\overline{w}} w dG(w; w^*_{FIX}). \tag{A.62}$$

The repayment ratio α is determined by equations (4.5) and (4.7), thus

$$s = \frac{(1-\beta)F(w^*_{IBR})\alpha\theta + \int_{w^*_{IBR}}^{\overline{w}} \alpha w dF(w)}{1-\beta F(w^*_{IBR})} = \int_{\theta}^{\overline{w}} \alpha w dG(w; w^*_{IBR}). \tag{A.63}$$

This implies

$$\int_{\theta}^{\overline{w}} \alpha w dG(w; w^*_{IBR}) = \int_{\theta}^{\overline{w}} s dG(w; w^*_{FIX}). \tag{A.64}$$

Equations (A.62) and (A.64) lead to (A.59). □

B Understanding the Reservation Wage Effect

In this appendix section, I discuss the conditions under which the reservation wage effect of the income-based repayment contract raises the agent's welfare.

In Figure 4 of the main text, I provided a numerical example showing that the income-based repayment contract also indirectly increases welfare by increasing the reservation wage (i.e., the reservation wage effect). While this result holds for a wide range of empirically reasonable parameter values, it is not generally true.

The goal of this section is to elucidate the economic intuitions. In subsection B.1, I characterize the efficient income-based repayment contract under the assumption that the reservation wage is observable and contractible. I define the reservation wage set by this contract as the efficient reservation wage. In subsection B.2, I show that when labor supply is inelastic, the reservation wage under the income-based repayment contract is below the efficient reservation wage. This explains why the reservation wage effect increases welfare. In subsection B.3, I show that when labor supply is sufficiently elastic, the reservation wage under the income-based repayment contract could be above the efficient reservation wage. This is because there is an additional debt overhang channel under the income-based repayment contract that further increases the reservation wage. The implication of this is that the reservation wage effect could reduce welfare. Finally, in subsection B.4, I provide several numerical examples and discuss that this counter-intuitive result is not likely to happen in reality. Therefore, I argue that the income-based repayment contract indirectly increases welfare by increasing the reservation wage.

B.1 Efficient Reservation Wage

For a certain reservation wage w^*, the agent's welfare under the income-based repayment contract can be expressed recursively:

$$\text{Welfare}_{IBR}(w^*) = F(w^*)\left[u((1-\alpha)\theta, 0) + \beta \text{Welfare}_{IBR}(w^*)\right] + \int_{w^*}^{\overline{w}} \frac{u((1-\alpha)wl, l)}{1-\beta} dF(w). \tag{B.1}$$

Thus, the agent's welfare is

$$\text{Welfare}_{IBR}(w^*) = \frac{F(w^*)u((1-\alpha)\theta, 0)}{1-\beta F(w^*)} + \int_{w^*}^{\overline{w}} \frac{u((1-\alpha)wl, l)}{(1-\beta)[1-\beta F(w^*)]} dF(w). \tag{B.2}$$

The agent determines the reservation wage w^*_{IBR} to maximize welfare under the income-based repayment contract:

$$\begin{aligned}
\max_{w^*} \quad & \text{Welfare}_{IBR}(w^*) && , \\
\text{subject to} \quad & (1-\alpha)wu_1((1-\alpha)wl,l) + u_2((1-\alpha)wl,l) &=\;& 0, \forall w \in [w^*, \overline{w}],
\end{aligned} \tag{B.3}$$

where the constraint is the intra-temporal Euler equation on labor supply, $l(w,\alpha)$. If labor supply is inelastic, the solution to problem (B.3) gives the indifference equation (4.8). Conditional on the reservation wage that solves problem (B.3), the lender sets the repayment ratio α according to the recoverability constraint:

$$\frac{F(w^*_{IBR})\alpha\theta}{1-\beta F(w^*_{IBR})} + \int_{w^*_{IBR}}^{\overline{w}} \frac{\alpha w l(w,\alpha)}{(1-\beta)[1-\beta F(w^*_{IBR})]} dF(w) = \frac{S}{\beta}. \tag{B.4}$$

The reservation wage w^*_{IBR} is inefficient because the agent's reservation wage generates an externality on the lender's revenue. The agent would be better off if she can internalize this effect when choosing the reservation wage. For the discussion of the reservation wage effect, it is useful to introduce the efficient reservation wage as a benchmark.

Definition 6. *The efficient reservation wage, w^*_{EFI}, is the reservation wage that the lender would set under the income-based repayment contract if the reservation wage is observable and contractible, i.e., w^*_{EFI} solves:*

$$\begin{aligned}
\max_{w^*} \quad & \text{Welfare}_{IBR}(w^*) && \\
\text{subject to} \quad & (1-\alpha)wu_1((1-\alpha)wl,l) + u_2((1-\alpha)wl,l) &=\;& 0, \forall w \in [w^*, \overline{w}], \\
\frac{F(w^*)\alpha\theta}{1-\beta F(w^*)} &+ \int_{w^*}^{\overline{w}} \frac{\alpha w l(w,\alpha)}{(1-\beta)[1-\beta F(w^*)]} dF(w) &=\;& \frac{S}{\beta},
\end{aligned} \tag{B.5}$$

where the first constraint is the intra-temporal Euler equation on labor supply, and the second constraint is the lender's recoverability constraint.

Clearly, w^*_{EFI} is different from w^*_{IBR} as the agent takes into account the lender's recoverability constraint when setting the reservation wage.

B.2 Inelastic Labor Supply

To provide some intuitions, I begin by discussing the reservation wage effect when the agent has inelastic labor supply.

Suppose that the agent has CRRA utility, $u(c) = \frac{c^{1-\gamma}}{1-\gamma}$. Denote λ as the Lagrangian multiplier for the recoverability constraint in problem (B.5). The shadow price λ is negative as the agent's welfare decreases when debt S marginally increases. The first order condition that determines the efficient reservation wage is:

$$\begin{aligned}
&\frac{[(1-\alpha)\theta]^{1-\gamma}}{[1-\beta F(w^*_{EFI})](1-\gamma)} + \int_{w^*_{EFI}}^{\overline{w}} \frac{\beta[(1-\alpha)w]^{1-\gamma}}{(1-\beta)[1-\beta F(w^*_{EFI})](1-\gamma)} dF(w) - \frac{[(1-\alpha)w^*_{EFI}]^{1-\gamma}}{(1-\beta)(1-\gamma)} \\
&= \lambda \frac{1-\beta F(w^*_{EFI})}{f(w^*_{EFI})} \alpha I'(w^*_{EFI}),
\end{aligned} \tag{B.6}$$

where $I'(w^*_{EFI})$ is the first derivative of expected income with respect to the reservation wage, characterized by equation (A.23). The RHS of equation (B.6) captures the effect of the reservation wage on expected repayment.

Define

$$g(x) = \frac{[(1-\alpha)\theta]^{1-\gamma}}{1-\gamma} + \int_x^{\overline{w}} \frac{\beta[(1-\alpha)w]^{1-\gamma}}{(1-\beta)(1-\gamma)} dF(w) - \frac{[1-\beta F(x)][(1-\alpha)x]^{1-\gamma}}{(1-\beta)(1-\gamma)}, \tag{B.7}$$

$$h(x) = \lambda \frac{[1-\beta F(x)]^2}{f(x)} \alpha I'(x). \tag{B.8}$$

Equation (B.6) can be rewritten as

$$g(w^*_{EFI}) - h(w^*_{EFI}) = 0. \tag{B.9}$$

In fact, $g(x) = 0$ coincides with the indifference equation (4.8), thus the solution to $g(x) = 0$ gives the reservation wage under the income-based repayment contract, i.e., $g(w^*_{IBR}) = 0$.

The proof of Proposition 4 indicates that with CRRA utility $w^*_{IBR} = w^*|_{s=0}$. When the agent is risk averse, according to Proposition 3, $w^*|_{s=0} < \hat{w}$ and $I'(w^*_{IBR}) > 0$. With $\lambda < 0$, we have $h(w^*_{IBR}) < 0$. Thus

$$g(w^*_{IBR}) - h(w^*_{IBR}) > 0. \tag{B.10}$$

Take the first derivative for $g(x)$ and $h(x)$, we obtain:

$$g'(x) = -\frac{(1-\alpha)[1-\beta F(x)]}{1-\beta}[(1-\alpha)x]^{-\gamma} < 0, \tag{B.11}$$

$$h'(x) = -\frac{\lambda\alpha}{1-\beta}[1-\beta F(x)] > 0. \tag{B.12}$$

Thus

$$g'(x) - h'(x) < 0. \tag{B.13}$$

Equations (B.9-B.13) imply $w^*_{IBR} < w^*_{EFI}$. Therefore, the agent's efficient reservation wage is higher than the reservation wage under the income-based repayment contract when labor supply is inelastic. Intuitively, this is because the efficient reservation wage internalizes the choice of the reservation wage on expected repayment. By increasing the reservation wage, the agent could increase the lender's revenue, motivating the lender to set a smaller repayment ratio α given the recoverability constraint, which in turn increases welfare. The efficient reservation wage is not incentive compatible because facing a lower repayment ratio ex-post, the agent would have the incentive to reduce the reservation wage in order to take fewer risks and increase her utility. As a result, the lender would take a loss.

What this implies is that the income-based repayment contract indirectly raises the agent's welfare by increasing the reservation wage. If we restrict the agent from choosing a higher reservation wage, as in the experiment of Figure 4, the agent's welfare would be lowered because the reservation wage is further away from the efficient one.

B.3 Elastic Labor Supply

Now I turn to the discussion of the reservation wage effect when the agent has elastic labor supply. I show that with elastic labor supply, there is an additional channel that increases the reservation wage under the income-based repayment contract. As a result, the agent could possibly choose a reservation wage higher than the efficient reservation wage.

To illustrate the economic channel, I begin my analysis with risk-neutral agents. Suppose that the agent has quasi-linear utility $u(c,l) = c - \frac{l^{1+\sigma}}{1+\sigma}$. Using equations (4.3) and (4.8), w^*_{FIX} and w^*_{IBR} can be derived from:

$$\frac{\sigma}{1+\sigma}\left(w^*_{FIX}\right)^{\frac{1+\sigma}{\sigma}} = \theta + \frac{\sigma}{1+\sigma}\frac{\beta}{1-\beta}\int_{w^*_{FIX}}^{\overline{w}}\left[w^{\frac{1+\sigma}{\sigma}} - \left(w^*_{FIX}\right)^{\frac{1+\sigma}{\sigma}}\right]dF(w), \tag{B.14}$$

$$\frac{\sigma}{1+\sigma}\left(w^*_{IBR}\right)^{\frac{1+\sigma}{\sigma}} = \underbrace{(1-\alpha)^{-\frac{1}{\sigma}}}_{\text{debt overhang channel}}\theta + \frac{\sigma}{1+\sigma}\frac{\beta}{1-\beta}\int_{w^*_{IBR}}^{\overline{w}}\left[w^{\frac{1+\sigma}{\sigma}} - \left(w^*_{IBR}\right)^{\frac{1+\sigma}{\sigma}}\right]dF(w). \tag{B.15}$$

The only difference between the two equations lies in the term $(1-\alpha)^{-\frac{1}{\sigma}} > 1$, due to the behavioral response in labor supply when the agent is employed and repaying debt under the income-based repayment plan. As a result, the reservation wage under the income-based repayment contract is higher than that under the fixed repayment contract when $\sigma < \infty$. Note that Proposition 3 implies that under the fixed repayment contract, the risk-neutral agent sets the reservation wage equal to $\hat{w}$, which already maximizes expected income. However, the income-based repayment contract further raises the reservation wage, which reduces expected income (before repayment). Intuitively, the agent chooses to set a higher reservation wage to avoid employment because supplying labor is costly. Therefore, elastic labor supply generates an additional force that increases the reservation wage under the income-based repayment contract. This channel is exposed starkly when the agent is risk neutral, because with inelastic labor supply ($\sigma = \infty$), the two reservation wages are equalized, $w^*_{FIX} = w^*_{IBR} = \hat{w}$, due to the absence of the risk channel and the liquidity channel discussed in subsection 4.2.1.

I name the effect on the reservation wage introduced by the elastic labor supply as the *debt overhang channel* of the income-based repayment contract.[60] I would like to highlight the distinction between the three channels: the debt overhang channel, the risk channel, and the liquidity channel. Although all three channels raise the reservation wage under the income-based repayment contract, they have divergent welfare implications. The increase in the reservation wage through the risk channel and the liquidity channel is a beneficial response to the correction of the credit and insurance market failures. However, the increase in the reservation wage through the debt overhang channel is a sub-optimal response to the distortion in the relative price of employment and unemployment.[61] Because the reservation wage controls the extensive participation margin of labor supply, we can interpret this result in an alternative way: the income-based repayment contract generates a moral hazard problem that reduces labor supply

[60] This channel is related to the moral hazard problem in the labor market associated with debt collection policies (Mulligan, 2009).

[61] Due to the behavioral response in labor supply, the income-based repayment contract essentially subsidizes unemployment by reducing income during employment by a proportion, $1-(1-\alpha)^{\frac{1+\sigma}{\sigma}}$, larger than the proportional reduction during unemployment, α.

on the intensive margin. This in turn generates a moral hazard problem that reduces labor supply on the extensive margin, i.e., increasing the reservation wage.

The discussion above suggests that the reservation wage under the income-based repayment contract could be larger than the efficient reservation wage when the risk-neutral agent has elastic labor supply. To see this, I substitute the utility function into equation (B.2) and obtain the agent's welfare:

$$(1-\alpha)\left[\frac{F(w^*)}{1-\beta F(w^*)}\theta+\frac{\sigma}{1+\sigma}\frac{(1-\alpha)^{\frac{1}{\sigma}}}{(1-\beta)[1-\beta F(w^*)]}\int_{w^*}^{\overline{w}} w^{\frac{1+\sigma}{\sigma}}dF(w)\right]. \tag{B.16}$$

By substituting the expression for labor supply, $l=[(1-\alpha)w]^{1/\sigma}$, into equation (B.4), we obtain the recoverability constraint:

$$\alpha\left[\frac{F(w^*)}{1-\beta F(w^*)}\theta+\frac{(1-\alpha)^{\frac{1}{\sigma}}}{(1-\beta)[1-\beta F(w^*)]}\int_{w^*}^{\overline{w}} w^{\frac{1+\sigma}{\sigma}}dF(w)\right]=\frac{S}{\beta}. \tag{B.17}$$

The reservation wage w^*_{IBR} is chosen to maximize the objective function (B.16) with the repayment ratio α set separately according to equation (B.17). The efficient reservation wage w^*_{EFI} is chosen to maximize the objective function (B.16) subject to the constraint (B.17). It is clear that when $\sigma=\infty$, the reservation wage that maximizes the objective function (B.16) also simultaneously maximizes expected repayment, i.e., the LHS of equation (B.17). This implies that the first-order derivative of equation (B.17) with respect to the reservation wage is equal to zero. Therefore, the unconstrained maximization problem yields the same solution as the constrained maximization problem, i.e., $w^*_{IBR}=w^*_{EFI}$. Intuitively, this is saying that the risk-neutral agent would choose the efficient reservation wage that maximizes expected repayment when labor supply is inelastic.

However, when $\sigma<\infty$, the terms inside the bracket of (B.16) differ from those of (B.17) as less weight is given for the value of employment ($\frac{\sigma}{1+\sigma}<1$).[62] This suggests that, compared with the efficient reservation wage w^*_{EFI} that solves the constrained maximization problem, the unconstrained maximization would set a relatively higher reservation wage w^*_{IBR} to avoid employment.

The analysis of a risk-neutral agent presents the stark result that the reservation wage under the income-based repayment contract is always higher than the efficient reservation wage as long as labor supply is elastic. When the agent is risk averse, the risk and liquidity channel of debt repayment would reduce the reservation wage. Therefore, whether the reservation wage under the income-based repayment contract is higher than the efficient one depends on which channel dominates. Intuitively, the strength of the debt overhang channel increases with the elasticity of labor supply. Therefore, when labor supply is sufficiently elastic, the debt-overhang channel would dominate and the reservation wage under the income-based repayment contract would be inefficiently high.[63]

The implication of the debt-overhang channel is that the agent could be better off if the reservation

[62]Intuitively, the agent puts less weight on the value of employment in the objective function because supplying labor generates a dis-utility equaling to $\frac{1}{1+\sigma}$ of the agent's wage income. Mathematically, the efficient reservation wage that solves the constrained maximization problem can be thought of as the average of the reservation wage maximizing (B.16) and the one maximizing (B.16) weighted by the Lagrangian multiplier. Due to the existence of the term $\frac{\sigma}{1+\sigma}$ in (B.16), the reservation wage that maximizes (B.16) is higher than the one maximizing (B.17).

[63]For example, in the extreme case with $\sigma=0$, the second term in equation (B.16) vanishes to zero, and thus $w^*_{IBR}=\overline{w}>w^*_{EFI}$.

wage is restricted at some lower value when being provided with the income-based repayment contract. Therefore, it is not generally true that the income-based repayment contract also indirectly raises welfare by increasing the reservation wage.

B.4 Numerical Examples and Discussions

In this subsection, I provide numerical examples by setting different values for the elasticity of labor supply. The goal of this simple exercise is to show that for empirically reasonable values of risk aversion and the elasticity of labor supply, the income-based repayment contract increases welfare by raising the reservation wage.[64]

In Figure B.1, I report the agent's reservation wage and welfare for different values of parameter σ. In each panel, I vary the fraction of debt under the income-based repayment contract and plot the outcome of interest when the reservation wage is endogenous, fixed at its value under the fixed repayment contract (i.e., $m = 0$), or efficient.

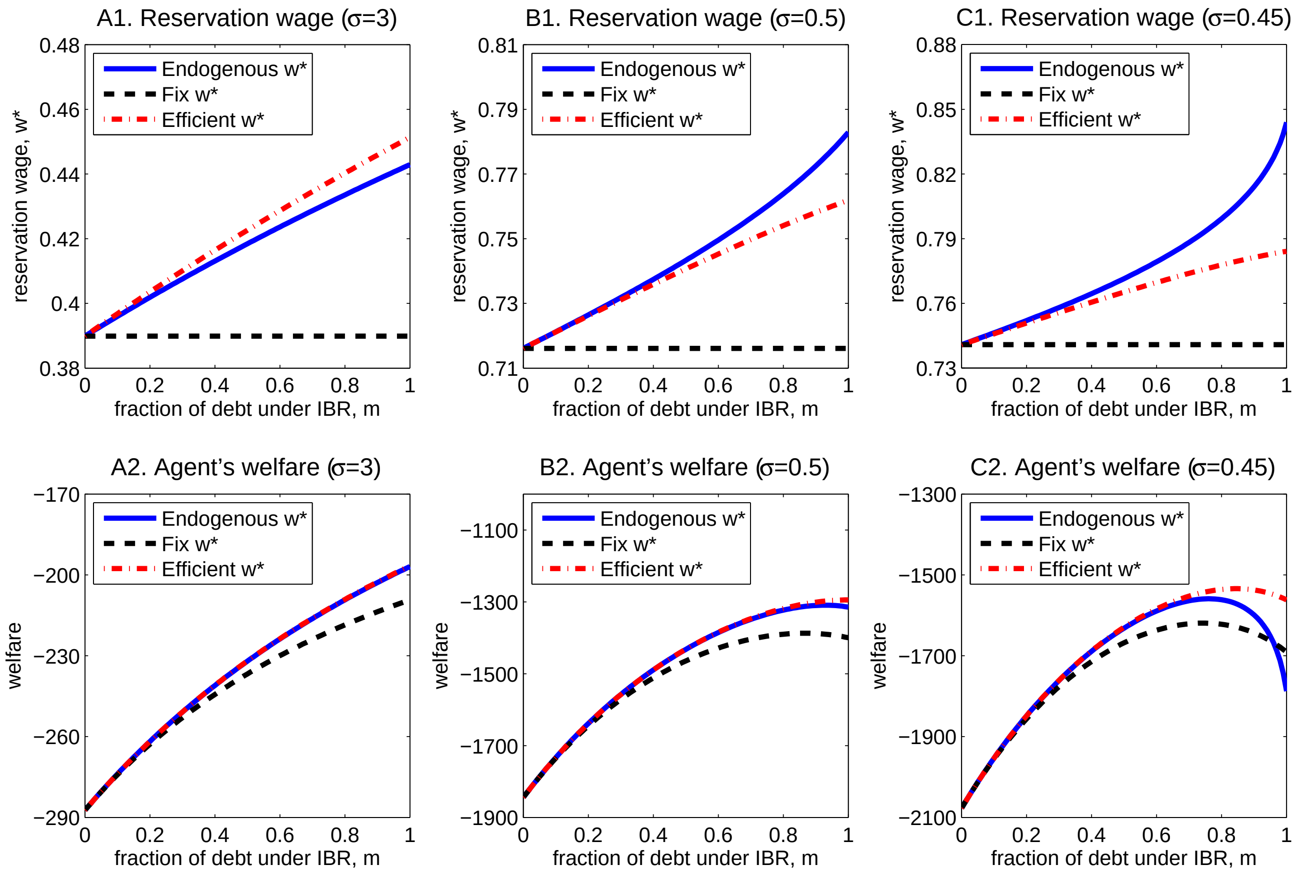
In panels A, I set $\sigma = 3$ to consider an empirically reasonable elasticity of labor supply, 0.33, according to Keane (2011). Panel A2 shows that welfare increases when a larger fraction of debt is made income contingent. It is clear that the inefficiency due to reservation wages is minimal as the welfare with endogenous reservation wages (blue solid line) is almost on top of that under the efficient contract (red dash-dotted line). Importantly, allowing the reservation wage to respond increases the agent's welfare relative to fixing the reservation wage at the beginning (black dashed line). This is because the reservation wage under the fixed repayment contract is too low compared to the efficient reservation wage. Increasing the fraction of income contingency raises the reservation wage, closing the gap to the efficient one (see panel A1) and lowering the repayment ratio.

In panels B, I dramatically increase the elasticity of labor supply to 2 by setting $\sigma = 0.5$. Similar to the result of Figure 4, welfare first increases and then decreases due to the increasing distortion of income contingency on labor supply (see panel B2). The welfare with endogenous reservation wages is still higher than that with fixed reservation wages, but by contrast, the endogenous reservation wage is above the efficient one (see panel B1).

In panels C, I further increase the elasticity of labor supply to 2.22 by setting $\sigma = 0.45$. I obtain the result in which the debt-overhang channel dominates, and increasing the fraction of income contingency indirectly reduces welfare by increasing the reservation wage. Panel C2 shows that the agent's welfare would be higher if the reservation wage is fixed at the beginning. As shown in panel C1, this is essentially caused by the sharp increase in the reservation wage relative to the efficient one when a larger fraction of debt is made income contingent.

In sum, the income-based repayment contract increases the agent's welfare by directly providing insurance. The insurance leads to a higher reservation wage, which may or may not increase the agent's welfare. The key parameters governing whether a higher reservation wage is beneficial are the degree of risk aversion and the elasticity of labor supply. All else equal, a more risk-averse agent sets a lower reservation wage relative to the efficient one under the fixed repayment contract. Thus increasing the

[64] Relatedly, in subsection 7.2.4, I use the quantitative model to evaluate the impact of the higher reservation wage due to the income-based repayment contract, which explains one-third of the reduction in debt burden.



Note: This figure illustrates the reservation wage effect for different elasticities of labor supply. In panel A1, B1, and C1, the blue solid line plots the agent's reservation wage when the reservation wage is allowed to increase as a larger fraction of debt is made income contingent. The black dashed line plots the agent's reservation wage when the reservation wage is fixed at the initial value under the pure fixed repayment contract. The red dash-dotted line plots the agent's efficient reservation wage. The corresponding welfare is plotted in panel A2, B2, and C2. The elasticity of labor supply is 0.33 ($\sigma = 3$), 2 ($\sigma = 0.5$), and 2.22 ($\sigma = 0.45$) in panels A, B, and C. The figure is plotted using the GHH utility, $u(c, l) = [c - \phi l^{1+\sigma}/(1+\sigma)]^{1-\gamma}/(1-\gamma)$ and the beta distribution of wage offers, $Beta(a, b)$, with parameter values: $a = 2$, $b = 4$, $\gamma = 3$, $\theta = 0.1$, $\overline{w} = 1.1$, $\beta = 0.96$, $S = 1$, $\phi = 1$.

Figure B.1: A numerical illustration of the reservation wage effect for different elasticities of labor supply.

reservation wage by providing insurance increases welfare. A larger elasticity of labor supply intensifies the debt overhang channel. Thus when the incentive to work is distorted by the income-based repayment contract, it is more likely to result in a reservation wage too high compared to the efficient one. In this case, by committing to a lower reservation wage, the agent could increase her expected repayment, inducing the lender to set a lower repayment ratio, which consequently increases welfare. However, such commitment is not incentive compatible because ex-post a lower repayment ratio generates a steeper wage offer distribution due to the elastic labor supply. This motivates the agent to stay unemployed longer by setting a higher reservation wage, and the lender would take a loss on debt collection.

Despite the theoretical possibility, in reality, it is plausible that the income-based repayment contract indirectly increases welfare by increasing the reservation wage. This is due to two reasons. First, as suggested by the numerical examples in Figure B.1, a higher reservation wage reduces welfare only when the elasticity of labor supply is about two, while a consensus empirical estimate is usually below one. Second, the theoretical possibility roots from the inefficiency in the income-based repayment contract, which is designed to allow the lender to collect all debt in expectation. In other words, if the repayment ratio is fixed, instead of being varied with the endogenous reservation wage, then the inefficiency would disappear by construction. This is the case in reality, as the government is willing to take a loss by offering debt forgiveness for federal student loans. Using an estimated quantitative model and a more realistic income-based repayment contract, I find a substantial positive reservation wage effect on welfare (see subsection 7.2.4).

C Optimal Repayment Contract

In this appendix section, I solve the optimal repayment contract when labor supply is elastic. I first show that when there is no job search (i.e., the reservation wage is fixed at $w^* = 0$), the mathematical problem is exactly the same as Mirrlees (1971)'s problem with a utilitarian social welfare function. I then show that my problem is different due to the introduction of endogenous job search decisions. I formulate the optimal contracting problem and use the perturbation approach of Saez (2001) to elucidate the economic channels.

C.1 Without Job Search

When the reservation wage w^* is set to be 0, the agent accepts all wage offers drawn from $F(w)$ in the first period. Therefore, the agent's life-time utility conditional on receiving a wage offer w is

$$V(w) = \frac{u(w,l)}{1-\beta}, \tag{C.1}$$

where l is the labor supply that satisfies the first-order condition.

To maximize the agent's expected life-time utility, the lender chooses an optimal nonlinear repayment schedule $\alpha^{SB}(z)$, as a function of the agent's earnings $z = wl$ to collect debt S/β. The nonlinear repayment schedule is not written on wage rates because wage rates are not observable or contractible.[65] The intercept $\alpha^{SB}(0)$ can be thought of as a lump-sum repayment or subsidy that is applied to any realization of earnings. The marginal repayment rate is $\alpha^{SB}(z)'$.

This problem is exactly the same as Mirrlees (1971) if we interpret it in the following way. There is a continuum of agents with different skills w and homogeneous utility functions $\frac{u(c,l)}{1-\beta}$. They work in a static economy and optimally choose their labor supply l in the tax system. The government values a utilitarian social welfare function and optimally designs a nonlinear tax schedule $\alpha^{SB}(z)$ in terms of earnings z to maximize social welfare conditional on collecting S/β revenue.

[65] If wage rates are contractible, then the first-best allocation is attainable because labor supply would not be distorted by repayment contracts. It is reasonable to assume that wage rates are unobservable because if they are observable, then labor supply is also observable from wage income. But this contradicts with the assumption made in the optimal income taxation literature.

The problem is solved by Mirrlees (1971) by applying an optimal control approach on direct truth-telling mechanisms. The advantage of this approach comes from its rigorousness to obtain the technical conditions.[66] However, the derived formula is not useful to elucidate the economic intuitions underlying the optimal contract.

C.2 With Job Search

Now I consider the optimal contracting problem with endogenous job search decisions as specified in section 4. The only departure from the problem of Mirrlees (1971) is that the agent chooses a reservation wage below which the wage offer is rejected. Therefore, in this problem, the types of agents in the problem of Mirrlees (1971) are restricted to a mass point with earnings θ with probability $F(w^*)$ and a continuum of types in $[w^*, \overline{w}]$ with density $\frac{f(w)}{1-F(w^*)}$, where w^* is chosen by the agent to maximize her welfare.

Facing any nonlinear repayment contract $\alpha(z)$ in terms of earnings z, the agent makes two decisions to maximize her welfare. First, the agent chooses a reservation wage w^*. Second, conditional on accepting the wage offer w, the agent chooses her labor supply l. Therefore, the resulting distribution of earnings $H(z)$ depends both on the exogenous wage offer distribution $F(w)$ and the repayment schedule $\alpha(z)$.

Below, I use a perturbation approach inspired by Saez (2001) to characterize the shape of the optimal repayment contract $\alpha^{SB}(z)$. For tractability, I make the following assumptions.

Assumption 1. *Earnings z and utility $u(z - \alpha^{SB}(z), l^{SB}(z))$ weakly increase with wage rates w under the optimal repayment contract $\alpha^{SB}(z)$.*

Assumption 2. *The optimal repayment contract $\alpha^{SB}(z)$ is twice differentiable for all z.*

Assumption 1 is saying that the agent earns more and enjoys higher welfare at jobs with higher wage rates. This is intuitively reasonable given that the monotonicity condition in the mechanism design problem of Mirrlees (1971) requires net earnings $z - \alpha^{SB}(z)$ to be weakly increasing in w. This assumption ensures that there is an injective function under the optimal contract $\alpha^{SB}(z)$, $w \mapsto z = q(w)$. Thus I denote z^* as the earnings corresponding to the reservation wage offer w^*, i.e., $z^* = q(w^*)$.

Assumption 2 comes from Saez (2001). This assumption has additional meaning in the problem I solve because it also restricts the specification of contract off the equilibrium, i.e., for $z \in (\theta, z^*)$ (see Online Appendix B.1 for an illustration). In general, because the agent rejects the wage offer whenever the resulting earnings are below z^*, there exist infinite numbers of optimal repayment contracts in my problem, and some of them could have a discontinuous jump at z^*.[67] This assumption ensures that the reservation wage is derived from a first-order condition instead of being a corner solution. That is, when the reservation wage is slightly changed, the change in the agent's welfare is of second order.

[66] For example, in order to have the local incentive-compatibility constraint being sufficient, the problem is required to satisfy the Spence-Mirrlees single crossing condition and the monotonicity condition.

[67] For example, given the optimal contract $\alpha^{SB}(z)$. We can specify $\tilde{\alpha}^{SB}(z)$ such that $\tilde{\alpha}^{SB}(z) = \alpha^{SB}(z)\ for\ z \geq z^*$ and $z - \tilde{\alpha}^{SB}(z) = \theta - \alpha^{SB}(\theta)\ for\ z < z^*$. Under $\tilde{\alpha}^{SB}(z)$, the net earnings are flat up to the reservation earnings z^*, and there is a discontinuous jump in net earnings at z^*. The contract $\tilde{\alpha}^{SB}(z)$ is incentive compatible because the agent has no incentive to change her reservation earnings z^* as reducing this lowers her utility more than what would be under $\alpha^{SB}(z)$. Moreover, $\tilde{\alpha}^{SB}(z)$ also satisfies the lender's recoverability constraint so it is an optimal contract.

Denote $\lambda < 0$ as the Lagrangian multiplier associated with the lender's recoverability constraint,

$$\frac{H(z^*)}{1-\beta H(z^*)}\alpha^{SB}(\theta) + \frac{1}{(1-\beta)[1-\beta H(z^*)]}\int_{z^*}^{\infty} \alpha^{SB}(z)dH(z) = \frac{S}{\beta}. \tag{C.2}$$

The multiplier λ is also the shadow value measuring the change in the agent's welfare when the amount of debt marginally increases.[68] Denote $g(z) > 0$ as the marginal value of consumption for the agent with earnings z under the optimal repayment contract, expressed in terms of the shadow cost of debt $(-\lambda)$, i.e.,

$$g(z) = \frac{u_1(z - \alpha^{SB}(z), l^{SB}(z))}{-\lambda}, \tag{C.3}$$

where $l^{SB}(z)$ corresponds to the labor supply at earnings z under the optimal contract $\alpha^{SB}(z)$.

I follow Saez (2001) and consider a small perturbation around the optimal repayment schedule $\alpha^{SB}(z)$. Suppose that the marginal repayment rate is increased by $d\alpha$ for earnings between z and $z + dz$, where $z \geq z^*$ (see Online Appendix B.1). This would generate the following effects on expected repayment R, defined as:

$$R = \frac{H(z^*)}{1-\beta H(z^*)}\alpha^{SB}(\theta) + \frac{1}{(1-\beta)[1-\beta H(z^*)]}\int_{z^*}^{\infty} \alpha^{SB}(z)dH(z). \tag{C.4}$$

C.2.1 Various Effects on Expected Repayment

Mechanical effect The agent pays $d\alpha dz$ more when her earnings are above z, with probability $1 - H(z)$. Thus expected repayment increases by

$$M = \frac{1-H(z)}{(1-\beta)[1-\beta H(z^*)]}d\alpha dz. \tag{C.5}$$

Elasticity effect The increase in the marginal repayment rate distorts labor supply when the agent's earnings are between z and $z + dz$, which consequently affects expected repayment. The change in earnings is caused by two effects. First, there is a direct effect due to the increase in $d\alpha$. Second, there is an indirect effect as the agent would face a different marginal repayment rate when her earnings are changed by the direct effect.

As noted by Saez (2001), the direct effect can be decomposed into two parts: an overall uncompensated increase in the marginal rate and an overall increase in virtual income. Therefore, the relevant one that determines the behavioral response is the Hicksian (compensated) elasticity of earnings, which is defined as

$$\zeta^c(z) = \frac{1-\alpha^{SB}(z)'}{z}\left.\frac{\partial z}{\partial(1-\alpha^{SB}(z)')}\right|_u. \tag{C.6}$$

Suppose that the two effects result in an earnings change by Δ, then the direct effect is $-\zeta^c(z)z\frac{d\alpha}{1-\alpha^{SB}(z)'}$,

[68] The negative of λ corresponds to the social value of public funds defined by Saez (2001).

and the indirect effect is $-\zeta^c(z)z\frac{\Delta\alpha^{SB}(z)''}{1-\alpha^{SB}(z)'}$. Hence,

$$\Delta = -\zeta^c(z)z\frac{d\alpha}{1-\alpha^{SB}(z)'} - \zeta^c(z)z\frac{\Delta\alpha^{SB}(z)''}{1-\alpha^{SB}(z)'}. \tag{C.7}$$

This implies that

$$\Delta = -\zeta^c(z)z\frac{d\alpha}{1-\alpha^{SB}(z)' + \zeta^c(z)z\alpha^{SB}(z)''}. \tag{C.8}$$

Following Saez (2001), I assume that $1-\alpha^{SB}(z)' + \zeta^c(z)z\alpha^{SB}(z)'' > 0$ so that bunching of types does not occur. The elasticity effect on expected repayment is

$$\begin{aligned} E &= \frac{\Delta\alpha^{SB}(z)'h(z)dz}{(1-\beta)[1-\beta H(z^*)]} \\ &= -\frac{\zeta^c(z)z\alpha^{SB}(z)'}{1-\alpha^{SB}(z)' + \zeta^c(z)z\alpha^{SB}(z)''}\frac{h(z)}{(1-\beta)[1-\beta H(z^*)]}d\alpha dz. \end{aligned} \tag{C.9}$$

Income effect If the agent accepts a wage offer generating earnings above $z+dz$, her earnings are reduced by $d\alpha dz$ due to the higher marginal rate between z and $z+dz$. This would generate an income effect that induces the agent to work more. As a result, for any $x > z+dz$, earnings increase by $\Delta(x)$, which in turn increases expected repayment. The earnings response $\Delta(x)$ is due to two effects. First, there is a direct effect due to the increase in marginal rate $d\alpha$ between z and $z+dz$. Second, there is an indirect effect due to the change in marginal rates caused by the shift in earnings.

Let $\eta(z) \le 0$ denote the income effect and $\zeta^u(z)$ denote the Marshallian (uncompensated) elasticity of earnings at earnings z, thus the income effect is derived by the Slutsky equation,

$$\zeta^u(z) = \frac{1-\alpha^{SB}(z)'}{z}\frac{\partial z}{\partial(1-\alpha^{SB}(z)')}; \tag{C.10}$$

$$\eta(z) = \zeta^u(z) - \zeta^c(z). \tag{C.11}$$

Therefore, the direct effect is $-\frac{\eta(x)d\alpha dz}{1-\alpha^{SB}(x)'}$ and the indirect effect is $-\zeta^c(x)x\frac{\alpha^{SB}(x)''\Delta(x)}{1-\alpha^{SB}(x)'}$, and the change in earnings is

$$\Delta(x) = -\frac{\eta(x)d\alpha dz}{1-\alpha^{SB}(x)'} - \zeta^c(x)x\frac{\alpha^{SB}(x)''\Delta(x)}{1-\alpha^{SB}(x)'}, \tag{C.12}$$

which implies

$$\Delta(x) = -\eta(x)\frac{d\alpha dz}{1-\alpha^{SB}(x)' + x\zeta^c(x)\alpha^{SB}(x)''}. \tag{C.13}$$

The total income effect on expected repayment is

$$I = -\frac{d\alpha dz}{(1-\beta)[1-\beta H(z^*)]}\int_z^\infty \eta(x)\frac{\alpha^{SB}(x)'}{1-\alpha^{SB}(x)' + x\zeta^c(x)\alpha^{SB}(x)''}h(x)dx. \tag{C.14}$$

Reservation wage effect There is a fourth effect on expected repayment due to the change in reservation earnings, which is not in the problem of Mirrlees (1971). The reservation earnings are determined by the following indifference equation:

$$\frac{u(z^*-\alpha^{SB}(z^*),l^{SB}(z^*))}{1-\beta}=\frac{u(\theta-\alpha^{SB}(\theta),0)}{1-\beta H(z^*)}+\frac{\beta}{(1-\beta)[1-\beta H(z^*)]}\int_{z^*}^{\infty}u(x-\alpha^{SB}(x),l^{SB}(x))dH(x), \tag{C.15}$$

where the LHS of this equation represents the value of being employed at the reservation earnings z^*, and the RHS represents the value of staying unemployed. Assumption 2 ensures that the reservation earnings also satisfy the first-order condition. Rearranging it:

$$1=\frac{\beta}{1-\beta}\int_{z^*}^{\infty}\frac{u(x-\alpha^{SB}(x),l^{SB}(x))-u(z^*-\alpha^{SB}(z^*),l^{SB}(z^*))}{u(z^*-\alpha^{SB}(z^*),l^{SB}(z^*))-u(\theta-\alpha^{SB}(\theta),0)}dH(x). \tag{C.16}$$

Assumption 1 ensures that the integrand is non-negative and decreasing in z^*. The integration is executed from z^* to infinity, thus the RHS of equation (C.16) decreases with z^*. The increase in the marginal repayment rate $d\alpha$ between z and $z+dz$ reduces $u(x-\alpha^{SB}(x),l^{SB}(x))$ for all $x>z$, thus lowering the RHS of equation (C.16). This implies that the reservation earnings z^* would decrease.

For $x>z$, the change $d\alpha$ would change $u(x-\alpha^{SB}(x),l^{SB}(x))$ by

$$\begin{aligned} du(x) &= -u_1(x-\alpha^{SB}(x),l^{SB}(x))d\alpha dz \\ &= g(x)\lambda d\alpha dz. \end{aligned} \tag{C.17}$$

Note that the elasticity effect and the income effect discussed above indicate that labor supply $l^{SB}(x)$ would also change due to the change $d\alpha$, but the Envelope Theorem implies that such a change does not have a first-order effect on utility. Differentiating equation (C.15) and substituting (C.17), we obtain

$$dz^*=d\alpha dz\frac{\beta\lambda}{[1-\beta H(z^*)]u_z(z^*)}\int_{z}^{\infty}g(x)dH(x), \tag{C.18}$$

where $u_z(z)=\frac{du(z-\alpha^{SB}(z),l^{SB}(z))}{dz}$ denotes the marginal change in utility due to a marginal change in earnings at z under the optimal contract $\alpha^{SB}(z)$.

The change in reservation earnings dz^* does not affect the agent's welfare due to the envelope condition from Assumption 2. However, it affects expected repayment R determined by equation (C.4). Define ζ^{z^*} as the elasticity of expected repayment with respect to the reservation earnings,

$$\zeta^{z^*}=\frac{\partial R/R}{\partial z^*/z^*}. \tag{C.19}$$

Differentiating (C.4), we obtain

$$\zeta^{z^*} = \frac{\beta R + \alpha^{SB}(\theta) - \dfrac{\alpha^{SB}(z^*)}{1-\beta}}{[1-\beta H(z^*)]R} z^* h(z^*)$$
$$= \frac{\beta S + \beta\alpha^{SB}(\theta) - \dfrac{\beta\alpha^{SB}(z^*)}{1-\beta}}{[1-\beta H(z^*)]S} z^* h(z^*), \tag{C.20}$$

where the second equation is obtained by substituting $R = S/\beta$.

In general, ζ^{z^*} could be positive or negative. The discussion in Appendix B suggests that $\zeta^{z^*} > 0$ for empirically reasonable elasticities of labor supply. Therefore, higher reservation earnings increase expected repayment. Using equations (C.18) and (C.20), we obtain the reservation wage effect on expected repayment:

$$RW = \frac{dz^*}{z^*}\zeta^{z^*} R$$
$$= d\alpha dz \frac{S\lambda\zeta^{z^*}}{[1-\beta H(z^*)]u_z(z^*)z^*} \int_z^\infty g(x) dH(x). \tag{C.21}$$

C.2.2 Deriving the Optimal Contract During Employment

The small perturbation around the optimal contract should have no first-order effect on welfare. Therefore, the sum of the four effects, M, E, I, and RW, multiplied by the shadow cost of debt ($-\lambda$) should be equal to the agent's expected welfare loss when earnings are above z. The agent's welfare under $\alpha^{SB}(z)$ is

$$\text{Welfare}_{SB} = \frac{H(z^*)u(\theta - \alpha^{SB}(\theta), 0)}{1-\beta H(z^*)} + \int_{z^*}^\infty \frac{u(x-\alpha^{SB}(x), l^{SB}(x))}{(1-\beta)[1-\beta H(z^*)]} dH(x) \tag{C.22}$$

The expected welfare loss is

$$WL = d\alpha dz \int_z^\infty \frac{u_1(x-\alpha^{SB}(x), l^{SB}(x))}{(1-\beta)[1-\beta H(z^*)]} dH(x)$$
$$= d\alpha dz \int_z^\infty \frac{-\lambda g(x)}{(1-\beta)[1-\beta H(z^*)]} dH(x). \tag{C.23}$$

Again, the Envelope Theorem implies that the change in labor supply has a second-order effect on welfare. At the optimum,

$$WL = -\lambda(M + E + I + RW), \tag{C.24}$$

which implies

$$\underbrace{\int_z^\infty g(x)dH(x)}_{\text{direct welfare loss}} = \underbrace{1-H(z)}_{\text{mechanical effect}} \underbrace{-\frac{z\zeta^c(z)\alpha^{SB}(z)'}{1-\alpha^{SB}(z)'+z\zeta^c(z)\alpha^{SB}(z)''}h(z)}_{\text{elasticity effect}}$$
$$\underbrace{-\int_z^\infty \eta(x)\frac{\alpha^{SB}(x)'}{1-\alpha^{SB}(x)'+x\zeta^c(x)\alpha^{SB}(x)''}dH(x)}_{\text{income effect}}$$
$$+\underbrace{\frac{S(1-\beta)\lambda\zeta^{z^*}}{u_z(z^*)z^*}\int_z^\infty g(x)dH(x)}_{\text{reservation wage effect}}. \tag{C.25}$$

This equation implicitly determines the optimal contract $\alpha^{SB}(z)$. It is different from the one derived by Saez (2001) due to the existence of the reservation wage effect. As a result, it does not admit an explicit solution for $\alpha^{SB}(z)$ because the elasticity of earnings with respect to the reservation earnings, ζ^{z^*}, is a function of $\alpha^{SB}(z)$.

To gain some intuitions, consider the case with inelastic labor supply, which implies that there is no elasticity effect or income effect in equation (C.25). If there are no endogenous search decisions, the reservation wage effect is also absent. Then the optimal contract requires $\int_z^\infty g(x)dH(x) = 1-H(z)$ for all $z > z^*$. This happens only when $g(z) = 1, \forall z > z^*$, suggesting perfect insurance against earnings risks.

When there are search risks, the direct welfare loss is equal to the sum of the mechanical effect and the reservation wage effect. If the agent is provided with perfect insurance, $g(z) = 1$, then the marginal utility does not change when different earnings offers are accepted. This implies that the term $u_z(z^*)$ in the reservation wage effect is equal to zero. In this case, for the reservation wage effect to be well defined, it is required that $\zeta^{z^*} = 0$, which happens when the reservation earnings z^* is set to maximize expected repayment.

Note that the lender can set the reservation wage to maximize expected repayment precisely because the agent with inelastic labor supply is indifferent among different reservation wages when being perfectly insured. Hence, any reservation wage is incentive compatible. This simple discussion with inelastic labor supply highlights the role of reservation wages in optimal contract design: in the context of elastic labor supply, the optimal contract not only cares about the tradeoff between efficiency (incentive to work) and insurance, but also to some extent, uses the reservation wage to increase expected repayment in order to have a smaller distortion on efficiency.

Equation (C.25) characterizes the formula that implicitly determines the optimal marginal repayment rate during employment. In the following, I derive the optimal repayment during unemployment.

C.2.3 Deriving the Optimal Contract During Unemployment

Suppose that repayment is increased by $d\alpha$ during unemployment, which is achieved by smoothly perturbing the repayment schedule below z^* (see Online Appendix B.1) so that Assumption 2 is still satisfied. This is going to have a mechanical effect and a reservation wage effect on expected repayment.

The mechanical effect is given by

$$M = \frac{H(z^*)}{1-\beta H(z^*)} d\alpha, \tag{C.26}$$

which captures the fact that the agent repays more during unemployment. Similar to equation (C.17), for earnings θ, the increase in repayment reduces utility during unemployment by

$$du(\theta) = -u_1(\theta - \alpha^{SB}(\theta), 0) d\alpha = g(\theta)\lambda d\alpha. \tag{C.27}$$

The reservation earnings are determined by equation (C.15). Differentiating this equation and substituting (C.27) yields:

$$dz^* = d\alpha \frac{(1-\beta)\lambda g(\theta)}{[1-\beta H(z^*)]u_z(z^*)}. \tag{C.28}$$

Thus the reservation wage effect is

$$\begin{aligned} RW &= \frac{dz^*}{z^*}\zeta^{z^*} R \\ &= d\alpha \frac{S(1-\beta)\lambda g(\theta)\zeta^{z^*}}{\beta[1-\beta H(z^*)]u_z(z^*)z^*}. \end{aligned} \tag{C.29}$$

According to equation (C.22), this perturbation generates a direct welfare loss:

$$\begin{aligned} WL &= \frac{H(z^*)}{1-\beta H(z^*)} u_1(\theta - \alpha^{SB}(\theta), 0) d\alpha \\ &= -\frac{H(z^*)}{1-\beta H(z^*)} g(\theta)\lambda d\alpha. \end{aligned} \tag{C.30}$$

At the optimum,

$$WL = -\lambda(M + RW), \tag{C.31}$$

which yields

$$\underbrace{H(z^*)g(\theta)}_{\text{direct welfare loss}} = \underbrace{H(z^*)}_{\text{mechanical effect}} + \underbrace{\frac{S(1-\beta)\lambda\zeta^{z^*}}{\beta u_z(z^*)z^*} g(\theta)}_{\text{reservation wage effect}}. \tag{C.32}$$

If the reservation earnings are fixed, then the reservation wage effect is absent in equation (C.32). In this case, the optimal contract subsidizes unemployment such that $g(\theta) = 1$, i.e., to the point where the marginal utility of consumption during unemployment is equal to the shadow cost of debt. This is because there is no behavioral response during unemployment, thus it is always optimal to equalize the cost of fund to the marginal utility of consumption when the agent is unemployed. When there is a negative reservation wage effect, the optimal contract sets $g(\theta) < 1$, indicating that the lender subsidizes the agent more during unemployment. Intuitively, this is because providing more liquidity to unemployment incentivizes the agent to increase her reservation wage and search longer, which would raise expected repayment.

D Estimation and Numerical Methods

In this appendix section, I discuss the estimation and numerical method for the quantitative model in section 5. Different from existing search-theoretic models, the quantitative model is developed to allow most of the parameters being estimated in partial equilibrium without iterating on the equilibrium objects. This largely simplifies the computation and makes the estimation of the full general equilibrium model tractable. Below I first discuss the estimation method and its limitations. Then I discuss the numerical algorithm that solves the model.

D.1 Estimation Method

The standard way to estimate an equilibrium search model is to iterate on the set of parameters Ξ_2 in order to minimize the objective function (6.10). However, this method is not sufficiently tractable due to the large number of parameters in Ξ_2 and the model complexity. The computation burden mainly comes from numerically searching for the equilibrium job contact rates, which are endogenously determined by the firms' job posting decisions and the workers' search decisions. Although searching for the equilibrium objects is not difficult in a standard search model, it is enormously time consuming in my model due to the many features introduced. If there are ways to estimate a subset of parameters without searching for the equilibrium, then the total estimation time would be possibly reduced. This is the logic that underlies my estimation method.

In particular, I estimate the model in two steps: first, I treat the equilibrium job contact rates λ_u and λ_e as parameters and estimate a subset of parameters $\Xi_{2p} = \{A, \kappa, \xi, \eta, \phi, f_1, f_2, \mu_0, \mu_1, \mu_2, \mu_3\}$ along with λ_u and λ_e to match the moments in Table 7 except for the vacancy to unemployment ratio. Second, I fix the values of Ξ_{2p} and estimate the rest parameters $\Xi_{2q} = \Xi_2 / \Xi_{2p} = \{s_e, s_u, \chi, \nu\}$. I normalize s_e to be 1, and the other three parameters are estimated to match the vacancy to unemployment ratio and the job contact rates λ_u and λ_e, which are estimated in the first step. This is straightforward, because equation (5.11) indicates that $s_u = \frac{\lambda_u}{\lambda_e}$. Therefore, the second step only needs to estimate two parameters χ and ν to match two moments, λ_u and the vacancy to unemployment ratio.

Essentially, in the first step, I estimate a partial equilibrium search model with exogenous job contact rates. In the second step, I estimate a general equilibrium search model with only two parameters. This estimation method is much faster because most parameters are estimated in the first step without searching for the equilibrium objects when parameters are optimized. This is because the only equilibrium objects are job contact rates, which are treated as parameters. The estimation in the second step needs to search for the equilibrium objects, but it is much easier now because only two parameters are left to be optimized.

D.1.1 Discussions and Limitations

This two-step estimation method obtains the same result as the standard way of estimating all the parameters together because my quantitative model satisfies three conditions: first, the only equilibrium objects are job contact rates, which are estimated in the first step. Second, all the parameters estimated in the second step affect the model outcomes only through their impacts on job contact rates. Third, all the moments used in the second-step estimation can be exactly matched.

The first condition is satisfied because I assume that the productivity of vacancies is randomly drawn from an exogenous distribution $F(\rho)$. This ensures that the equilibrium vacancy distribution $V(\rho)$ is the same as $F(\rho)$. This condition would be violated if the productivity is not randomly drawn. For example, if different firms can post vacancies of different productivity as in Lise and Robin (2016), then the vacancy distribution is also endogenous. As a result, there is no way to execute the first step due to the unknown vacancy distribution.[69] The limitation of assuming that vacancies' productivity is randomly drawn is that the model cannot capture the potential change in the distribution of vacancies' productivity when repayment policy changes. That is, my model does not capture the possibility that firms would create more productive jobs because the income-based repayment plan motivates borrowers to search for these jobs, the general equilibrium effect proposed by Acemoglu and Shimer (1999, 2000).

The second condition is satisfied because the search efficiency parameters and the vacancy posting cost do not affect either agents' or firms' decisions once the job contact rates are given. It is straightforward to prove that the third condition is also satisfied.[70] If the third condition is not satisfied, this two-step estimation is guaranteed to be inconsistent with the standard way of estimating all parameters together. This is because if we cannot adjust the parameters in the second step to perfectly match the job contact rates, it means that we are over-fitting the model in the first step by selecting those contact rates that could never be achieved in equilibrium. Moreover, if we cannot perfectly match the vacancy to unemployment ratio in the second step, then the estimation result could also be different depending on the weighting matrix. This is because when all the parameters are estimated together, we may want to sacrifice the matched moments in the first step in order to better match the moment in the second step, namely, the vacancy to unemployment ratio.

D.1.2 Estimating Standard Errors

Once the two-step estimation is finished, standard errors of parameters can be constructed in the standard way.

First, I estimate the variance-covariance matrix $\hat{S}$ for all moments. Because the vector of moments in the data can be computed without knowing parameter values, $\hat{S}$ can be computed by bootstrapping the data directly without doing iterated MSM. Specifically, I calculate the moments $N = 200$ times by bootstrapping, then use these N observations of moments to construct the variance-covariance matrix. There are two issues in estimating $\hat{S}$. First, moments are constructed using different data sources. The life-cycle moments are constructed using March CPS, the vacancy to unemployment ratio is constructed using JOLTS, the default rate is constructed using NSLDS, and the remaining moments are constructed

[69] This could be solved if in the first step we treat both the job contact rates and the vacancy distribution as parameters. But then the third condition would be violated because it is almost impossible to fit exactly the distribution of vacancy by selecting the vacancy posting cost.

[70] To see this, note that given λ^u and λ^e, the equilibrium distributions $\phi^u(\Omega)$ and $\phi^e(\Omega, \rho)$ are unique in the stationary equilibrium. The unemployment rate $\overline{u}$ is determined by equation (5.30). Substituting equations (5.10-5.11) into equation (5.29), then N_v is uniquely determined as a function of ν, λ^u, λ^e, and the equilibrium distributions. Thus, there is a unique ν to match the vacancy to unemployment ratio. Because the number of matches M is a function of N_v and χ in equation (5.9), given N_v, χ is uniquely solved to match the job contact rates.

using NLSY97. The covariance between moments constructed in different data sources is set to be zero. Second, the moments in NLSY97 are constructed using different number of observations due to missing values. The covariance between any pair of moments is constructed by bootstrapping non-missing-value observations for both moments. Thus the assumption here is that values are missing randomly, though it is not likely to be true in reality.

In my estimation, I use a diagonal weighting matrix, $\hat{\Theta} = [\text{diag}(\hat{S})]^{-1}$, because covariance is not precisely estimated and may bias the estimated parameter values. The asymptotic variance-covariance matrix for MSM estimators $\hat{\Xi}_2$ is given by:

$$Q(\hat{\Theta}) = (G^T\hat{\Theta}G)^{-1}G^T\hat{\Theta}\hat{S}\hat{\Theta}^T G(G^T\hat{\Theta}^T G)^{-1}, \tag{D.1}$$

where $G = \frac{\partial \hat{m}_S(\Xi_2)}{\partial \Xi_2}|_{\Xi_2=\hat{\Xi}_2}$ is the Jacobian matrix of the simulated moments evaluated at the estimated parameters.[71] The first derivatives are calculated numerically by varying each parameter's value by 1%. The standard errors of $\hat{\Xi}_2$ are given by the square root of the diagonal elements of $Q(\hat{\Theta})$.

D.2 Numerical Method

I solve the model numerically. The computational complexity of this model is extremely large because this is an equilibrium model with five state variables (wealth, debt, efficient labor units, job productivity, and the negotiation benchmark's productivity).[72]

In addition to the complexity introduced by five state variables, the model is hard to solve due to the violation of the linear sharing rule in the Nash bargaining problem. Therefore, for each possible worker-job combination, the algorithm needs to solve a maximization problem whose objective function does not have an analytical solution and is determined endogenously. In the following, I first present the numerical algorithm. Then I describe the initialization of value functions in the final period. Finally, I discuss the implementation of this algorithm.

D.2.1 Algorithm

The model is solved by backward induction using the following algorithm:

(1). Guess the equilibrium job contact rates λ_u for unemployed workers, and $\lambda_e = \frac{s^e}{s^u}\lambda_u$ for employed workers.

(2). Solve the value functions $U(\Omega)$, $W(\Omega, \rho, \rho')$, and $J(\Omega, \rho, \rho')$ in the following steps:

(2.1). Guess wage functions $w(\Omega, \rho, \rho')$ for all Ω, ρ, and ρ'.

[71] In general, the formula should also incorporate simulation errors, thus the variance-covariance matrix for MSM estimators also depends on the number of simulated agents (Gourieroux and Monfort, 1997). The formula I use does not consider simulation errors because instead of simulating a number of agents, I simulate the distribution of characteristics. Therefore, as long as I focus on the stationary equilibrium, the simulation outcomes are not dependent on randomly drawn shocks.

[72] Loosely speaking, solving the model is as difficult as solving the quantitative models of Krusell, Mukoyama and Sahin (2010) and Lise and Robin (2016). Krusell, Mukoyama and Sahin (2010) do not model on-the-job search and Lise and Robin (2016) consider risk-neutral agents. But Krusell, Mukoyama and Sahin (2010) and Lise and Robin (2016) also consider aggregate shocks in their models, which I do not have.

(2.2). Solve problems (5.25-5.27) by backward induction from $t = T$ to $t = 1$ to obtain $U(\Omega)$, $W(\Omega, \rho, \rho')$, $J(\Omega, \rho, \rho')$, and the corresponding policy functions.

(2.3). Solve the Nash bargaining problems (5.16-5.18) to obtain wage $w'(\Omega, \rho, \rho')$.

(2.4). If $w'(\Omega, \rho, \rho') = w(\Omega, \rho, \rho')$ for all Ω, ρ, and ρ', go to step (3); otherwise, go to step (2.1).

(3). Given initial distributions $\Psi(b, s)$ and the policy functions, forward simulate the model from $t = 1$ to $t = T$ to obtain distributions $\phi^u(\Omega)$ and $\phi^e(\Omega, \rho)$.

(4). Compute the equilibrium unemployment rate $\overline{u}$ using equation (5.30) and the aggregate level of search effort S using equation (5.8). Compute the probability of contacting a worker q using the free entry condition (5.29).

(5). Substituting S and q into equations (5.9-5.11) to obtain the number of meetings M, the number of vacancies N_v, and the equilibrium job contact rates λ'_u.

(6). Check if $\lambda'_u = \lambda_u$. If not, go to step (1).

Because I focus on the stationary equilibrium, the value functions and policy functions across different generations are identical. The final period represents age T. When the model is solved in partial equilibrium, the job contact rates λ_u and λ_e are given as parameters. Thus only steps (2) and (3) are executed.

D.2.2 Initialization of Value Functions

The value functions at age T are initialized by assuming that the agent consumes all wealth in the end. In the simulation, all agents should have paid off the outstanding debt before reaching age T. To have a well-defined problem, I also need to specify what happens off the equilibrium, i.e., if there is outstanding debt left at age T. I assume that the agent needs to pay off all the outstanding debt if wealth at age T is sufficient to make the payment. If wealth is not sufficient, I punish the agent to keep the level of consumption at the floor value $\underline{c}$, and the rest wealth is used to repay the debt.

Formally, the value for unemployed workers at age T is:

$$U(\Omega) = \begin{cases} \dfrac{[(1+r)b + \varkappa\theta^{1-\tau} - (1+r_s)s]^{1-\gamma}}{1-\gamma} & \text{if } (1+r)b + \varkappa\theta^{1-\tau} \geq (1+r_s)s + \underline{c} \\ \dfrac{\underline{c}^{1-\gamma}}{1-\gamma} & \text{otherwise} \end{cases} \tag{D.2}$$

The agent dies after age T, and the worker-job match separates as a consequence. Therefore, the value of a filled job at age T is

$$J(\Omega, \rho, \rho') = [Az\rho - w(\Omega, \rho, \rho')]l, \tag{D.3}$$

where $l = \left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1}{\sigma+\tau}} w(\Omega, \rho, \rho')^{\frac{1-\tau}{\sigma+\tau}}$.

To calculate the value for employed workers at age T, I need to solve the Nash bargaining problem to obtain the wage functions at age T. This can be solved directly from a root-finding problem.[73] Depending on whether the agent's wealth is sufficient to repay the debt, there are two cases:

Case 1: Insolvency If the agent is employed at job ρ, the highest wage rate that job ρ can offer is its marginal product of labor $Az\rho$. If the agent could not repay the debt when being offered this wage rate, then she is insolvent and would consume $\underline{c}$. Note that because the current job's productivity is always higher than the negotiation benchmark's productivity, the agent would also be insolvent and consume $\underline{c}$ at the negotiation benchmark. In this scenario, the agent is indifferent between being employed at the current job ρ or at the negotiation benchmark, thus the match surplus for the agent is zero. Moreover, the agent has no incentive to supply labor as this only increases repayment but not consumption. Thus the firm would also obtain zero match surplus. This implies that the value for employed workers at age T is $\underline{c}^{1-\gamma}/(1-\gamma)$ in the case of insolvency.

To pin down the condition for insolvency, consider the highest wage rate $Az\rho$ being offered by job ρ to agent Ω. The after-tax income is $\varkappa(Az\rho)^{1-\tau}$. Substituting the optimal labor supply, $l = \left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1}{\sigma+\tau}}(Az\rho)^{\frac{1-\tau}{\sigma+\tau}}$, the maximum wealth that the agent can obtain is

$$\begin{aligned}\overline{b} &= (1+r)b + \varkappa(Az\rho l)^{1-\tau} \\ &= (1+r)b + \varkappa\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1-\tau}{\sigma+\tau}}(Az\rho)^{\frac{(1+\sigma)(1-\tau)}{\sigma+\tau}}. \end{aligned} \tag{D.4}$$

The agent's utility is

$$\begin{aligned}u(\overline{b}-(1+r_s)s, l) &= \frac{1}{1-\gamma}\left[\overline{b}-(1+r_s)s-\phi\frac{l^{1+\sigma}}{1+\sigma}\right]^{1-\gamma} \\ &= \frac{1}{1-\gamma}\left[\overline{b}-(1+r_s)s-\frac{\phi}{1+\sigma}\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1+\sigma}{\sigma+\tau}}(Az\rho)^{\frac{(1-\tau)(1+\sigma)}{\sigma+\tau}}\right]^{1-\gamma}. \end{aligned} \tag{D.5}$$

For the agent to be insolvent, it should hold that $u(\overline{b}-(1+r_s)s, l) \le u(\underline{c}, 0)$, which requires

$$\overline{b} \le (1+r_s)s + \frac{\phi}{1+\sigma}\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1+\sigma}{\sigma+\tau}}(Az\rho)^{\frac{(1-\tau)(1+\sigma)}{\sigma+\tau}} + \underline{c}. \tag{D.6}$$

Case 2: Solvency When condition (D.6) is not satisfied, the agent is solvent if the highest wage rate is offered by job ρ. Therefore, the actual wage rate $w(\Omega, \rho, \rho')$ that solves the Nash bargaining problem

[73] Note that the wage functions at the final period T can be solved directly from a root-finding problem because the agent consumes all wealth in the final period. In other periods $t < T$, due to the endogenous consumption and savings decisions, multiple iterations are needed to obtain the wage functions as fixed points.

should also satisfy the solvency condition, i.e.,

$$
\begin{aligned}
&(1+r)b+\varkappa\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1-\tau}{\sigma+\tau}} w(\Omega,\rho,\rho')^{\frac{(1+\sigma)(1-\tau)}{\sigma+\tau}} \\
&> (1+r_s)s+\frac{\phi}{1+\sigma}\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1+\sigma}{\sigma+\tau}} w(\Omega,\rho,\rho')^{\frac{(1-\tau)(1+\sigma)}{\sigma+\tau}}+\underline{c}.
\end{aligned}
\tag{D.7}
$$

Otherwise, the agent would obtain a zero match surplus and choose not to supply labor, which also results in a zero match surplus for the firm. Thus both sides could be better off if the firm increases the wage rate to satisfy the solvency condition (D.7). I now derive the wage function $w(\Omega,\rho,\rho')$ under condition (D.7).

The value for employed workers at age T is:

$$
W(\Omega,\rho,\rho')=\frac{1}{1-\gamma}\left[(1+r)b+\varkappa[w(\Omega,\rho,\rho')l]^{1-\tau}-(1+r_s)s-\phi\frac{l^{1+\sigma}}{1+\sigma}\right]^{1-\gamma}, \tag{D.8}
$$

where $l=\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1}{\sigma+\tau}} w(\Omega,\rho,\rho')^{\frac{1-\tau}{\sigma+\tau}}$.

The outside option value for employed workers with negotiation benchmark ρ' at age T is

$$
\overline{W}(\Omega,\rho')=\max\left\{\frac{1}{1-\gamma}\left[(1+r)b+\varkappa(Az\rho'l')^{1-\tau}-(1+r_s)s-\phi\frac{l'^{1+\sigma}}{1+\sigma}\right]^{1-\gamma},\frac{\underline{c}^{1-\gamma}}{1-\gamma}\right\}, \tag{D.9}
$$

where $l'=\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1}{\sigma+\tau}}(Az\rho')^{\frac{1-\tau}{\sigma+\tau}}$. The max operator considers the solvency/insolvency case at the negotiation benchmark ρ'.

The $w(\Omega,\rho,\rho')$ is chosen to maximize the bargaining objective function:

$$
w(\Omega,\rho,\rho')=\underset{w(\Omega,\rho,\rho')}{\operatorname{argmax}}\left[W(\Omega,\rho,\rho')-\overline{W}(\Omega,\rho')\right]^{\xi} J(\Omega,\rho,\rho')^{1-\xi}. \tag{D.10}
$$

Substituting equations (D.3), (D.8) and (D.9) into problem (D.10) and taking the first order condition, we obtain $w(\Omega,\rho,\rho')$ by solving the following root-finding problem:

$$
\begin{aligned}
&\frac{\xi(1-\gamma)\left[B+Kw(\Omega,\rho,\rho')^{\frac{(1+\sigma)(1-\tau)}{\sigma+\tau}}\right]^{-\gamma}\varkappa(1-\tau)\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1-\tau}{\sigma+\tau}} w(\Omega,\rho,\rho')^{\frac{1-2\tau-\sigma\tau}{\sigma+\tau}}}{\left[B+Kw(\Omega,\rho,\rho')^{\frac{(1+\sigma)(1-\tau)}{\sigma+\tau}}\right]^{1-\gamma}-\left[\max\{B+K(Az\rho')^{\frac{(1+\sigma)(1-\tau)}{\sigma+\tau}},\underline{c}\}\right]^{1-\gamma}} \\
&=(1-\xi)\left[\frac{1}{Az\rho-w(\Omega,\rho,\rho')}-\frac{1-\tau}{(\sigma+\tau)w(\Omega,\rho,\rho')}\right],
\end{aligned}
\tag{D.11}
$$

where $B=(1+r)b-(1+r_s)s$ and $K=\varkappa\frac{\sigma+\tau}{1+\sigma}\left[\frac{\varkappa(1-\tau)}{\phi}\right]^{\frac{1-\tau}{\sigma+\tau}}$.

Table D.1: Discretization of state space.

Parameters	Value	Description
n_b	400	Number of wealth grids
Δ_b	\$500	Length of wealth grids
$[\underline{b} \quad \overline{b}]$	[\$0 \$200,000]	Range of wealth
n_s	100	Number of student loan debt grids
Δ_s	\$500	Length of student loan debt grids
$[\underline{s} \quad \overline{s}]$	[\$0 \$50,000]	Range of student loan debt
n_ρ	20	Number of productivity grids
Δ_ρ	0.05	Length of productivity grids
$[\underline{\rho} \quad \overline{\rho}]$	[0 1]	Range of productivity

I use bisection method to solve equation (D.11) with initial lower bound,

$$LB = \left[\frac{1+\sigma}{\varkappa(\sigma+\tau)}\left[\frac{\phi}{\varkappa(1-\tau)}\right]^{\frac{1-\tau}{\sigma+\tau}}\left[[(1-\gamma)\overline{W}(\Omega,\rho')]^{\frac{1}{1-\gamma}} - (1+r)b + (1+r_s)s\right]\right]^{\frac{\sigma+\tau}{(1+\sigma)(1-\tau)}}, \tag{D.12}$$

and upper bound,

$$UB = Az\rho. \tag{D.13}$$

Finally, substituting the solution of $w(\Omega,\rho,\rho')$ into equations (D.3) and (D.8), we obtain $J(\Omega,\rho,\rho')$ and $W(\Omega,\rho,\rho')$.

D.2.3 Implementation

To ensure accuracy, I choose relatively fine grids (see Table D.1). I use grid search to find the optimal consumption when solving agents' problems. The grid search method is computationally time consuming, but is arguably the most robust numerical method for dynamic programming, especially when the model is highly nonlinear. Instead of applying a sequential grid search, I use the golden section search method. The advantage of the golden section search method is that it is robust to the choice of initial values because convergence is guaranteed. However, convergence to the global optimum is not ensured if there are many local optima. Therefore, I further divide the whole decision space into multiple sub-space and select the largest local optimum. I do a robustness check after the estimation using a sequential grid search, and the results are identical. When solving the Nash bargaining problem, I need to invoke the calculation for utility from consumption and utility from the future multiple times. I save the computation time by calculating these values in advance and store them in memory.

The numerical algorithm is implemented using C++. The program is run on the server of MIT Economics Department, *supply.mit.edu*, which is built on Dell PowerEdge R910 running RedHat 6.7 (64-core processor, Intel(R) Xeon(R) CPU E7-4870, 2.4GHz). I use OpenMP for parallelization when iterating value functions and simulating the model. My baseline model requires 40GB of RAM to store the large number of decision rules and value functions.