Public Debt as Private Liquidity: Optimal Policy

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• **Financial frictions** $\implies$ public debt can be non-neutral

• **Public debt as collateral/buffer stock/outside liquidity** $\implies$ alleviate frictions
  
• **Relevant policy implications** in the aftermath of the Great Recession

  • **Mitigate a financial crisis:** Level vs portfolio composition  
    (QE: Curdia and Woodford, 2011, Gertler and Kiyotaki, 2011)
The Main Issue

- **Relevant policy implications** in the aftermath of the Great Recession
  - Mitigate a financial crisis: Level vs portfolio composition
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  - Relax the ZLB constraint on monetary policy
    (Eggertson and Krugman, 2011, Guerrieri and Lorenzoni, 2011)
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- **Similarities but also differences** with Friedman rule literature
  (Chari, Christiano and Kehoe, 1996, Correia and Teles, 1999)
Missing: Theoretical study of **optimal fiscal policy** is a Ramsey setting in which

- public debt is non-neutral, because it influences the virulence of financial frictions
- but does not generate a free lunch for the government, because taxation is distortionary

**Contribution of this paper:** Fill the gap, offer new lessons for

- optimal long-run quantity of public debt
- desirability of tax smoothing
- optimal policy response to shocks (including financial crises)
How do we do it?

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3. Characterize **analytically** the solution to a class of reduced-form problems that nests the one obtained from our model.
Parenthesis: Absent Financial Frictions

- Without financial friction, the model reduces to a deterministic version of Barro and AMSS
- Optimal policy satisfies
  - **Tax smoothing**: the tax rate (the shadow value of tax revenue) is equated across periods;
  - **Steady-state indeterminacy**: any sustainable level of debt is consistent with steady state.
Main Findings: Deterministic

- When the friction is present, a **tension** emerges between
  1. Easing the friction so as to improve market efficiency/allocation of resources
  2. Exacerbating the friction so as to raise premia and reduce the government’s cost of borrowing
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  - in a benchmark: essentially unique steady state
  
  - more generally: possibly multiple steady states, but each one is locally-determinate
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- Depending on primitives, **2 scenarios** can emerge
  1. Financial distortion vanishes as $t \to \infty \implies$ Friedman rule applies in LR but not SR
  2. Financial distortion preserved as $t \to \infty \implies$ **Friedman rule never applies**
Main Findings: Stochastic

- **Mean reversion**

- Optimal policy response to "wars": less persistent and less volatile

- Optimal policy response to "financial recessions":
  - aforementioned tension $\Rightarrow$ ambiguous effect on planner’s incentives
  - response driven by fiscal considerations, not apparent desire to ease the aggravated friction
  - a financial crisis presents an opportunity for "cheap borrowing"
  - ultimately: **optimal deficit is larger** than in a comparable traditional recession.
The Baseline Model
Micro-Founded Model

  - Infinitely lived agents,
  - competitive markets and flexible prices,
  - Government issues debt and collect taxes by distorting labor supply decisions only.

  - Agents are hit by idiosyncratic shocks \(\implies\) reallocation of goods across agents.
  - The reallocation requires borrowing, borrowing requires collateral.
  - Private supply of collateral is limited as so is the pledgeable income of the private sector.

\[\implies\text{Public debt can serve as collateral and alleviate financial frictions.}\]
Micro-Founded Model

- Helps clarify
  - role of liquidity: risk sharing
  - why debt matters: lack of pledgeable income
- Gives a language to talk about collateral and buffer stock
- Can accommodate effects such as
  - fire sales externalities,
  - inside/outside money,
  - crowding out of capital

**But Importantly:** show that although each of these effects is relevant in its own right, none is key for our results.
Micro-Founded Model: Timing

- Endowment ($e$) + assets ($a$)
- Taste Shock ($\theta$)
- Consumption ($x$) and IOU issuance ($z$)
- Decide labor supply ($h$)
- Receive labor income ($1 - \tau w$)
- Decide savings ($a + 1$) and consumption ($c$)
- IOU repayment
Micro-Founded Model: Timing

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- Receive labor income ($(1-\tau_t)w_t h_{it}$)
- Decide savings ($a_{it+1}$) and consumption ($c_{it}$)
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Micro-Founded Model

- Households:

$$\max \mathbb{E}_0 \left[ \sum_{t=0}^{\infty} \beta^t (c_{it} + \theta_{it} u(x_{it}) - \nu(h_{it})) \right]$$

s.t.  
$$c_{it} + p_t x_{it} + q_t a_{it+1} = a_{it} + (1 - \tau_t) w_t h_{it} + p_t \bar{e}$$

$$p_t (x_{it} - \bar{e}) \leq \xi w_t h_{it}^{def} + a_{it}$$

$$- a_{it+1} \leq \xi w_{t+1} h_{it+1}^{def}$$
Micro-Founded Model

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\[
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\end{align*}
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- Firms: \( y_t = A h_t \)
Micro-Founded Model

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- **Firms:** \( y_t = A h_t \)

- **Government:** \( q_t b_{t+1} + \tau_t w_t h_t = b_t + g \)
Micro-Founded Model

- **Households:**

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- **Firms:** \( y_t = Ah_t \)

- **Government:** \( q_t b_{t+1} + \tau_t w_t h_t = b_t + g \)

- **Market clearing:** \( y_t = c_t + g, \quad \int x_t(\theta)d\mu(\theta) = \bar{e}, \quad \int a_{t+1}(\theta)d\mu(\theta) = b_{t+1} \).
Proposition

The optimal policy path for the tax rate and the level of public debt solves:

$$\max_{\{s_t, b_{t+1}\}} \sum_{t=0}^{\infty} \beta^t [U(s_t) + V(b_t)] \quad \text{subject to} \quad Q(b_{t+1})b_{t+1} = b_t + g - s_t$$
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1. \(U(s)\): Captures the cost of taxation, \(s \equiv \tau \text{wh}(\tau)\) and \(U'(\cdot) < 0, \ U''(\cdot) < 0\)
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1. $U(s)$: Captures the cost of taxation, $s \equiv \tau \omega h(\tau)$ and $U'(\cdot) < 0, U''(\cdot) < 0$

2. $V(b)$: Value of cross-sectional allocation of asset holdings and morning-good consumption for a given level of public debt, $b$, with $V'(\cdot) > 0$. 

\[\text{Program}\]
A Convenient Reduced-Form Ramsey Problem

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3. $Q(b)$: Price of a bond that supports this allocation, $Q'(\cdot) < 0$. 

   $(\pi(b) \equiv Q(b) - \beta$ is associated liquidity premium)
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   \((\pi(b) \equiv Q(b) - \beta\) is associated liquidity premium\)

4. \(b_{\text{bliss}}\): There exists \(b_{\text{bliss}}\) s.t. \(\forall b \geq b_{\text{bliss}}, V'(b) = 0\) and \(Q(b) = \beta (\pi(b) = 0)\)
The planner chooses a path for \((s, b) \in (0, \bar{s}) \times [\underline{b}, \bar{b}]\) that solves

\[
\max \int_{0}^{+\infty} e^{-\rho t} [U(s) + V(b)] \, dt \\
\text{s.t. } \quad \dot{b} = R(b)b + g - s \\
\quad b(0) = b_0
\]

Dual role of public debt (as in Woodford, Aiyagari-McGrattan or Holmstrom-Tirole):

1. Can improve the allocation of resources (Captured by \(V\)).
2. Can be used to manipulate interest rates (Captured by \(R = \rho - \pi(b)\)).

Key Trade off

Non convex problem due to pecuniary externality.

Key: Dependence of \(V\) and \(R\) (id. \(\pi\)) on \(b\), not the exact reason of this dependence.
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\(\text{Key Trade off}\)
An (even more) Convenient Reduced Form Representation

• The planner chooses a path for \((s, b) \in (0, \bar{s}) \times [\underline{b}, \bar{b}]\) that solves

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Assumptions

Main Assumptions

We consider economies in which the following properties hold:

**A1.** $U$, $V$, and $\pi$ are continuously differentiable.\(^1\)

**A2.** $U$ is concave in $s$, with maximum attained at $s = 0$.

**A3.** There exists a threshold $b_{bliss} \in (0, \bar{b})$ such that $V'(b) > 0$ and $\pi(b) > 0$ for all $b < b_{bliss}$, and $V'(b) = 0$ and $\pi(b) = 0$ for all $b > b_{bliss}$.

**A4.** $\pi(b) \leq \rho$ for all $b$.

\(^1\)To be precise, we allow $V$ and $\pi$ to be non-differentiable at $b = b_{bliss}$. 

Characterizing Optimal Debt Provision
Necessary Conditions for Optimality

- Set of necessary conditions

\[\begin{align*}
\dot{\lambda} &= V'(b) - \lambda \pi(b) (\sigma(b) - 1) \\
\dot{b} &= g + (\rho - \pi(b)) b - s(\lambda)
\end{align*}\]

+ transversality condition: \(\lim_{t \to \infty} e^{-\tau t} \lambda(t) b(t) = 0\).
Euler Equation: \[ \dot{\lambda} = V'(b) - \lambda \pi(b) (\sigma(b) - 1) \]

- Think of a static problem: max social value + seigniorage revenue for a given \( \lambda \)

\[
\max_b \Omega(b, \lambda) \equiv V(b) + \lambda \pi(b) b
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- FOC

\[ V'(b) - \lambda \pi(b)(\sigma(b) - 1) = 0 \]

Marginal social gain from easing the financial friction (Allocation Efficiency)

Marginal cost in terms of seigniorage revenue (Interest Rate Manipulation)
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Marginal social gain from easing the financial friction (Allocation Efficiency)

Marginal cost in terms of seigniorage revenue (Interest Rate Manipulation)

- **Tempting**: interpret the above condition as the optimal steady state debt provision decision \( \Rightarrow \text{Misleading!} \)

- Tax Smoothing acts as an adjustment cost.
Optimality Conditions

• Set of necessary conditions

\[
\begin{align*}
\dot{\lambda} &= V'(b) - \lambda \pi(b) (\sigma(b) - 1) \\
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\end{align*}
\]

+ transversality condition: \( \lim_{t \to \infty} e^{-\tau t} \lambda(t) b(t) = 0. \)

• **Non-convex problem** \( \implies \) **not sufficient** \( \implies \) Skiba (1978), Brock-Dechert (1983)

• Study the global dynamics.
A Useful Benchmark

**Benchmark**

(i) the elasticity $\sigma$ is monotone;

(ii) the ratio $V'/\pi$ is constant.
A Useful Benchmark

**Benchmark**

(i) *the elasticity* $\sigma$ *is monotone*;

(ii) *the ratio* $V'/\pi$ *is constant*.

(i) Guarantees that $\pi(b)b$ is single-peaked (Laffer curve for seigniorage);

(ii) Any social gain following an increase in liquidity is exactly compensated by an increase in borrowing cost.
### Result

*Under the previous assumptions, there exists a unique pair $(b^*, \lambda^*)$, such that for any $b_0 \leq b_{\text{bliss}}$ the economy converges to $(b^*, \lambda^*)$.**
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Result

There exists a threshold level, \(\hat{g}\), for government spending such that

- For any \(g \leq \hat{g}\), \(b^* = b_{\text{bliss}}\): An analogue to the Friedman rule holds in the long run;
- For any \(g > \hat{g}\), \(b^* < b_{\text{bliss}}\): it is optimal for the government to squeeze liquidity to increase seigniorage revenue to finance expenditures.
Phase Diagram: Benchmark I: $g \leq \hat{g}$.
Phase Diagram: Benchmark II: $g > \hat{g}$.

\[ \dot{\lambda} = 0 \quad \gamma(b) \quad \dot{\lambda} = 0 \quad \dot{\lambda} = 0 \]

\[ \lambda_{\text{skiba}} \quad \lambda^* \quad \psi(b) \quad b = 0 \]

Useful Lemma 22
• Relax monotonicity of $\sigma$ and the invariance of $V'(b)/\pi(b)$ to $b$.

• Things get more intricate.

• Many situations can occur $\implies$ Only illustrate a typical one
Beyond the Benchmark: A Typical Situation
Beyond the Benchmark: A Typical Situation

\[ \dot{\lambda} = 0 \]

\[ \dot{b} = 0 \]

\[ \lambda = 0 \]

\[ \tilde{b}, \tilde{b}, \tilde{b}, \tilde{b} \]

\[ b_L^\#, b_M^\#, b_H^\# \]

\[ b_{bliss}, b_{skiba}, \bar{b} \]
Theorem

Let $B^\# \equiv \{ b \in (b, b_{\text{bliss}}) : \gamma(b) = \psi(b) \text{ and } \gamma'(b) \leq \psi'(b) \}$ be the set of the points at which $\gamma$ intersects $\psi$ from above. In every economy, there exists a threshold $b_{\text{skiba}} \in [b, b]$ and a set $B^* \subseteq B^\#$ such that the following are true along the optimal policy:

(i) If either $b_0 \in B^*$ or $b_0 > \max\{b_{\text{bliss}}, b_{\text{skiba}}\}$, debt stays constant at $b_0$ for ever.

(ii) If $b_0 < b_{\text{skiba}}$ and $b_0 \notin B^*$, then debt converges monotonically to a point inside $B^*$.

(iii) If $b_{\text{skiba}} < b_{\text{bliss}}$ and $b_0 \in (b_{\text{skiba}}, b_{\text{bliss}})$, debt converges monotonically to $b_{\text{bliss}}$. 
Optimal Policy
Optimal Policy

\[ \psi(b) \]

\[ \gamma(b) \]

\[ \lambda^* \]

\[ \lambda \]

\[ b^* \]

\[ b_{skiba} \]

\[ b_{bliss} \]

\[ b_0 \]

\[ \bar{b} \]
Optimal Policy

\[ \lambda \]

\[ \psi(b) \]

\[ \gamma(b) \]

\[ b_0 \quad b^* \quad b_{skiba} \quad b_{bliss} \quad b_{skiba} \]

\[ \lambda^* \]
Optimal Policy

- \( \psi(b) \)
- \( \gamma(b) \)
- \( \lambda^* \)
- \( b^* \)
- \( b_0 \)
- \( b_{\text{skiba}} \)
- \( b_{\text{bliss}} \)
- \( b_{\text{skiba}} \)
Optimal Policy

\[ \psi(b) \]

\[ \lambda \]

\[ b_{skiba} \quad b_0 \quad b_{bliss} \quad \overline{b} \]

\[ \lambda(b) \]
**Theorem**

Any economy belongs to one of the following three non-empty classes:

(i) **Economies in which** \( B^* = \emptyset \) **and** \( b_{skiba} = b \).

(ii) **Economies in which** \( B^* \neq \emptyset \) **and** \( b_{skiba} \in (b, b_{bliss}) \).

(iii) **Economies in which** \( B^* \neq \emptyset \) **and** \( b_{skiba} \geq b_{bliss} \).

Furthermore, \( \psi_{bliss} > \gamma_{bliss} \) is sufficient for an economy to belong to the last class.
Optimality of Steady State Public Debt

Proposition

Let $\Omega(b, \lambda) = V(b) + \lambda \pi(b)b$ be the liquidity plus seignorage. Consider an economy in which the set $B^* \neq \emptyset$, then take any $b^* \in B^*$ and let $\lambda^* = \psi(b^*)$.

- If $\gamma'(b^*) < 0$, $b^*$ attains a local maximum of $\Omega(b, \lambda^*)$.
- If instead $\gamma'(b^*) > 0$, $b^*$ attains a local minimum of $\Omega(b, \lambda^*)$.

Economies do not necessarily converge towards the optimal level of debt.

- There exist economies in which $B^*$ is a singleton and, nevertheless, $b^*$ attains a local minimum of $\Omega(b, \lambda^*)$.
Optimality of Public Debt

Local Maximum

Non-Optimal Debt

\[ \psi(b) \quad \gamma(b) \]

\[ \lambda^* \]

\[ b^* \quad b_{bliss} \quad \bar{b} \]

\[ \lambda \]

\[ b \]
Additional Insights
• **Key difference from FR literature:** All government liabilities offer liquidity services.
On the Friedman Rule

- **Key difference from FR literature:** All government liabilities offer liquidity services.

- Assume the Government can issue money like assets (Bonds), $m$, and hold a position in non-money asset, $n \implies b = m + n$

$$\dot{m} + \dot{n} = [\rho - \pi(m)]m + \rho n + g - s \iff \dot{b} + s = \rho b - \pi(m)m + g$$

- For a given level of **total** liabilities ($\dot{m} + \dot{n} = 0$), the government can change liquidity ($m$) w/o affecting neither its fiscal position ($b$) nor the interest rate ($\rho$) just by varying the composition of liabilities ($m/n$).

---

**Complete separation between liquidity provision and fiscal position**
On the Friedman Rule

- Formally:

\[
\max \int_0^{+\infty} e^{-\rho t} [U(s) + V(m)]dt \quad \text{s.t.} \quad b_0 = \int_0^{+\infty} e^{-\rho t} [\pi(m)m + s - g]dt
\]

- \(s\) and \(m\) are constant over time \(\implies\) back to tax smoothing.
- \(m^*\) may or may not coincide with the Friedman rule
- If it does, unlike in our model, it does in each and every period.
On the Friedman Rule

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- $s$ and $m$ are constant over time $\implies$ back to tax smoothing.
- $m^*$ may or may not coincide with the Friedman rule
- If it does, unlike in our model, it does in each and every period.

- **Why?** The supply of liquidity is isolated from fiscal policy
- Assume instead that $n$ has (even a tiny) role as liquidity then fiscal and liquidity considerations are intertwined and we are back to our trade-off,
Crowding Out Effect of Debt

- Aiyagari-McGrattan (1998): Public debt crowds out capital because debt is a substitute for capital as a buffer stock.
- Model la Holmstöm-Tirole (1998): firms have to borrow to finance their capital need.
- Agents can relax future financial constraints by saving in the form of capital and debt $\Rightarrow$ Possible crowding out.
Crowding Out Effect of Debt

- Aiyagari-McGrattan (1998): Public debt crowds out capital because debt is a substitute for capital as a buffer stock.

- Model la Holmstöm-Tirole (1998): firms have to borrow to finance their capital need.

- Agents can relax future financial constraints by saving in the form of capital and debt → Possible crowding out.

- But another conflicting effect:
  - More debt that serves as collateral alleviate the financial friction.
  - Improves the allocation of capital and raises the ex-ante return to capital → Crowding in.

- Overall effect depends on micro details and calibration.
**Borrowing Cheap?**

- **Recent great recession**: low interest rates = signal that it is cheap for the government to borrow.
- Krugman and DeLong: US government should have run an expansionary fiscal policy
  - for Keynesian-stimulus reasons
  - **but also** because interest rates were extraordinarily low.
Borrowing Cheap?

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  - for Keynesian-stimulus reasons
  - **but also** because interest rates were extraordinarily low.

- **Misleading claim!**

- **Barro/AMSS**: Adding deterministic variations in the discount rate does not justify changes in the optimal fiscal mix!
- **Why?** The interest rate captures the cost of borrowing **but also** what a society desires (No wedge between the interest rate and the CP’s discount rate)
Borrowing Cheap?

- **Our framework**: More borrowing may be optimal during a crisis

- **But** not simply because of low interest rates!
Our framework: More borrowing may be optimal during a crisis

But not simply because of low interest rates!

Because low interest rate is a manifestation of
  • the aggravated financial friction
  • the associated increase in the liquidity premium the Govt can extract.

Optimal response to adverse financial shocks?
Effect of shocks

- War shocks: Model departs from standard tax smoothing (both across time and states) and exhibits mean reversion.

- Here look at a financial shock that causes
  - income to fall
  - tax basis to shrink
  - private and social value of liquidity to increase
Effect of shocks

- Both the $\dot{b} = 0$ and $\dot{\lambda} = 0$ shift.
- 3 effects drive movements in the debt:
  - Tax Smoothing (shock is temporary)
  - Increase in the marginal value of providing liquidity (Increase liquidity)
  - Increase in the opportunity cost of liquidity (Squeeze liquidity)
- Net effect is ambiguous and depends on the calibration and the micro details.
Effect of shocks

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• Consider the case: $\frac{V'(b)}{\pi(b)}$ and $\sigma(b)$ remain constant $\implies \dot{\lambda} = 0$ locus does not shift
  $\implies$ Response of $b$ is dictated by fiscal considerations only (increase in tax burden)
Financial Shock: No change in $\dot{\lambda} = 0$
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$\Delta T | \Delta \pi(b) b = 0$
Financial Shock: No change in $\dot{\lambda} = 0$
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\[ \Delta T \mid \Delta \pi(b) = 0 \]

\[ \Delta \pi(b) b \]
Financial Shock: No change in $\dot{\lambda} = 0$
**Effect of shocks**

**Case 1:** No direct interest rate effect $\implies$ traditional recession (raise debt and taxes)

**Case 2:** Change in interest rate compensates for decrease in tax $\implies$ no change in $b$ and $\tau$. The drop in tax revenue is debt financed ($\dot{b} = 0$ and $\dot{\lambda} = 0$ unaffected)

**IRF to a Financial Shock**

- **Interest Rate**
- **Primary Deficit**
- **Public Debt**
- **Optimal Tax Rate**

$\Delta \pi(b)b = 0; \quad \Delta \pi(b)b = -\Delta T$
• **Question:** Is it always optimal to supply debt to alleviate financial frictions?

• **Not always!** The government may wish to exploit its collateral producing capacity in order to earn rents from the private sector and thus reduce its reliance on distortionary tax revenue sources.

• Provide a full characterization of
  ✓ long term properties and
  ✓ the global dynamics
  of optimal policy in a model where public debt can alleviate the financial frictions created by lack of sufficient collateral for asset trades.
2 Topical Insights

• Do low interest rates during recessions make it “cheap” for the Government to borrow?
  • Argument has no place in the standard Ramsey framework.
  • **Why?** no wedge between interest rate and discount rate of planner
  • May make sense in our framework (if the low interest rate reflects the financial friction.)
2 Topical Insights

• Do low interest rates during recessions make it “cheap” for the Government to borrow?
  • Argument has no place in the standard Ramsey framework.
  • **Why?** no wedge between interest rate and discount rate of planner
  • May make sense in our framework (if the low interest rate reflects the financial friction.)

• **Optimal policy response to a financial crisis**
  • crisis raises the marginal value of providing liquidity $\implies$ Increase debt
  • but it also raises the opportunity cost of doing so $\implies$ Squeezing liquidity
  Not clear!

  • In a benchmark (one where these 2 effects cancel each other), optimal policy response
    dictated by budgetary considerations $(\lambda)$ rather than by the apparent increase in the social
    value of easing the friction
THANK YOU!
Social Value of Debt

- $V(b)$ is the value of the following problem:

$$
\max_{(p,q)\in \mathbb{R}_+^2 \& (x,a):[\theta,\bar{\theta}] \to \mathbb{R}_+ \times [\phi, +\infty)} \int \theta u(x(\theta))\varphi(\theta)d\theta
$$

subject to

\[
\int x(\theta)\varphi(\theta)d\theta = \bar{\theta}
\]

\[
\int a(\theta_-)\varphi(\theta_-)d\theta_- = b
\]

\[
\phi + a(\theta_-) - p(x(\theta) - \bar{\theta}) \geq 0 \quad \forall (\theta, \theta_-)
\]

\[
\theta u'(x(\theta)) \geq p \quad \forall \theta
\]

\[
[\theta u'(x(\theta)) - p] [\phi + a(\theta_-) - p(x(\theta) - \bar{\theta})] = 0 \quad \forall (\theta, \theta_-)
\]

\[
a(\theta_-) + \phi \geq 0 \quad \forall \theta_- \]

\[
\beta + \mathcal{U}_a(a(\theta_-), \theta_-, p) \leq q \quad \forall \theta_- \]

\[
[\mathcal{U}_a(a(\theta_-), \theta_-, p) - \pi] [a(\theta_-) + \phi] = 0 \quad \forall \theta_- \]
Define $\mathcal{H}(b, \lambda) = \max_s H(s, b, \lambda) \equiv U(s) + V(b) + \lambda(s - [\rho - \pi(b)]b - g)$, we have

**Lemma (Skiba, 1978, Brock and Dechert, 1983)**

*For any $b_0$ and any $\lambda_0 \in \Lambda(b_0)$, the path in $\mathcal{P}(b_0)$ that starts from initial point $(b_0, \lambda_0)$ yields a value that is equal to $\mathcal{H}(b_0, \lambda_0)/\rho$.***

**Lemma**

$\mathcal{H}(b, \lambda)$ is convex in $\lambda$ (upper envelop of linear functions of $\lambda$).
Aiyagari-McGrattan use a shortcut to avoid the computational challenge of studying optimal debt in incomplete market economies.

\[ \Rightarrow \text{Restrict debt and taxes to be constant, abstract from transition and study Steady State welfare} \]

Using our notations, amounts to maximize \( U(s) + V(b) \) s.t. \( r(b)b = g - s \)

\[ \Rightarrow b^{AMG} = \arg\max_b \left[ V(b) - \lambda^{AMG}(\rho - \pi(b))b \right] = \arg\max_b \left[ \Omega(b, \lambda^{AMG}) - \lambda^{AMG}\rho b \right] \]

FOC: \( \Omega_b = \lambda^{AMG}\rho > 0 \) while we get \( \Omega_b = 0 \Rightarrow b^{AMG} < b^\star \).
• Aiyagari-McGrattan’s exercise
  • underestimates the optimal long-run level of debt
  • results in a debt level below $b_{\text{bliss}}$ even when long-run satiation would be optimal

• Why? Because this exercise treats the entire payments on debt, $r(b)b$, as a cost, while the social planner should view debt issuance as a profit generating exercise (seignorage) to the tune of $\pi(b)b$. 

i.i.d. Case

Debt and Taxes in our Model; Debt and Taxes in AMSS; Government Spending.

Persistent Case

Public Debt

Optimal Tax Rate

Debt and Taxes in our Model; Debt and Taxes in AMSS; Government Spending.

Our Model; Lucas-Stokey; Government Spending Shock