Firm Size Distortions and the Productivity Distribution: Evidence from France†

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We show how size-contingent laws can be used to identify the equilibrium and welfare effects of labor regulation. Our framework incorporates such regulations into the Lucas (1978) model and applies it to France where many labor laws start to bind on firms with 50 or more employees. Using population data on firms between 1995 and 2007, we structurally estimate the key parameters of our model to construct counterfactual size, productivity, and welfare distributions. We find that the cost of these regulations is equivalent to that of a 2.3 percent variable tax on labor. In our baseline case with French levels of partial real wage inflexibility, welfare costs of the regulations are 3.4 percent of GDP (falling to 1.3 percent if real wages were perfectly flexible downward). The main losers from the regulation are workers—and to a lesser extent, large firms—and the main winners are small firms. (JEL L11, L51, J8)

A recent literature has documented empirically how distortions can affect aggregate productivity through misallocating resources toward less productive firms. These distortions mean efficient firms produce too little output and employ too few workers (Restuccia and Rogerson 2008). Hsieh and Klenow (2009) argue that such misallocations account for a significant proportion of the difference in aggregate productivity between the United States, China, and India. However, the causes of the distortions remain a black box in these approaches. In this paper, we focus on

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Many empirical papers have shown that deregulation (e.g., Olley and Pakes 1996), higher competition (e.g., Syverson 2004), and trade liberalization (e.g., Pavcnik 2002) have tended to improve reallocation by increasing the correlation between firm size and productivity.
understanding the impact of one specific distortion on the French firm size distribution: regulations that increase labor costs when firms reach 50 workers. This type of size-dependent regulation is relevant to many debates around the world. For example, under the US Affordable Care Act, there are penalties on firms with more than 50 employees that do not offer health care insurance to their employees. Hence, critics have claimed that this will reduce the incentives for efficient firms to grow large whereas supporters are skeptical about the magnitude of any such effect.

The idea that misallocations of resources lie behind aggregate productivity gaps is attractive in understanding some differences in economic performance between the United States and Europe. According to the European Commission (1996), the average production unit in the European Union employed 23 percent less workers than in the United States. Consistent with this, Figure 1 shows that there is a lower proportion of relatively large firms in France compared with the United States. In particular there is a large bulge in the number of firms with employment just below 50 workers in France, but not in the United States. This is also illustrated in Figure 2 which shows the exact number of French manufacturing firms by number of workers.

2To be precise, there are penalties for firms with more than 50 full-time employees who (i) do not offer health coverage and (ii) pay workers too little to buy coverage on their own without using federal subsidies. The penalty is $2,000 for each employee (except for the first 30 employees). If the firm has more than 50 full-time employees and offers some of them coverage but others have to apply for federal subsidies to buy coverage themselves, the firm must pay the lesser of $3,000 for each employee receiving insurance subsidies or $2,000 for each full-time employee (again excluding the first 30 employees). For more details, see: http://www.washingtonpost.com/blogs/wonkblog/wp/2012/11/19/cheer-up-papa-johns-obamacare-gave-you-a-good-deal/.

3Bartelsman, Haltiwanger, and Scarpetta (2013) examine misallocation using microdata across eight OECD countries. They argue that consumption is more than 10 percent below US levels in some European nations because of misallocation. In particular, they find that the Olley-Pakes (1996) covariance term between size and productivity is much smaller in France (0.24 in their Table 1) and other European countries compared to the United States (0.51 in their Table 1). Bloom, Sadun, and Van Reenen (2016) also report a more efficient allocation of employment to better managed firms in the United States than in Europe and developing countries.
There is a sharp fall in the number of firms with exactly 50 employees compared to those who have 49 employees. The burden of French legislation substantially increases when firms employ 50 or more workers. As we explain in detail below, firms of 50 workers or more must create a works council (comité d’entreprise), establish a health and safety committee, report detailed information on all employees to the Labor Ministry, appoint a union representative, and so on. What are the implications for firm size, firm productivity, and aggregate welfare from those laws? Intuitively, some more productive firms that would have been larger without the regulation choose to remain below the legal threshold to avoid these costs. In addition, the higher labor costs make firms above the threshold smaller than they would be in the unregulated economy. In this paper we show how these two sets of changes in the firm size distribution can be exploited to infer the level and distribution of the welfare (and employment) cost of such regulations.

There has been extensive discussion of the importance of labor laws for unemployment and productivity. The OECD and other agencies have developed indices of the importance of these regulations, based on detailed analysis of the laws and expert surveys. It is hard, however, to see how these can be rigorously quantified, since adding up the regulatory provisions has a large arbitrary component. A contribution of our paper is a methodology for quantifying the tax equivalent of a regulation, albeit in the context of a specific model. The calculation is extremely transparent, economically intuitive, and can be applied in many other settings.
We start by setting up a simple model following Lucas’ (1978) observation that the economy-wide observed resource distribution results from allocating productive factors over managers of different ability so as to maximize output. Given limits to managerial time or attention, the better managers are allocated more workers to manage which results in a scale-of-operations effect whereby differences in talent are amplified by the resources allocated. We show that there are four main effects of a size-dependent labor tax with variable and fixed cost components in such a Lucas (1978) style model. (i) Equilibrium wages fall as a result of the reduction in the demand for workers (i.e., some of the tax incidence of regulation falls on workers). (ii) Firm size increases for all firms below the regulatory threshold as a result of the general equilibrium effect on wages. (iii) Firm size decreases to precisely the regulatory threshold for a set of firms that are not productive enough to justify incurring the new regulatory costs. (iv) Firm size decreases proportionally if the variable cost component dominates (as it does empirically) for all firms that are productive enough to incur the additional cost of regulation.

We use the model to guide our estimation of the impact of these regulatory costs. The theory tells us there is a deviation from the correct firm size distribution as a result of the regulation. That is, we expect to see a departure from the usual power law firm size distribution as firms bunch up below the threshold of 50 workers. Given factors such as measurement error, however, the observed empirical departure from the power law is not just at 49 workers but also affects firms of slightly smaller sizes causing a bulge. Similarly, there is not precisely zero mass to the right of the threshold, but rather a valley where there are significantly fewer firms than we would expect from an unbroken power law. Then, at some point the firm size distribution becomes again a power law, with a lower intercept (in log-log space) if the variable cost dominates. The break in the power law from the bulge and valley of firms around the threshold, together with the downward shift in the power law, empirically identify the magnitude of the distortion. We formally allow for measurement error and also develop a dynamic extension to the model with adjustment costs to understand the presence of some firms immediately to the right of the regulatory threshold.

We find that the increase in the per worker variable cost (identified from the downward shift in the power law after 49 workers) is large—about 2.3 percent. What are the welfare losses due to these costs? It is not just the labor regulation, but

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4 In a model of this kind, the source of decreasing returns is on the production size, and is linked to limits to managerial time. For our purposes here, as Hsieh and Klenow (2009) show, this source of decreasing returns is equivalent to having the decreasing returns come from the demand/utility side (as is more common in the recent literature following Melitz 2003).

5 This means the probability density function of firm size is log-linear (e.g., Axtell 2001). There is a large literature on the size and productivity distribution of firms in macro, trade, finance, and industrial organization (IO). Appropriately, the first major study in this area was by Gibrat (1931) who studied French industrial firms, the main focus of the empirical part of our paper.

6 Our descriptive findings are consistent with earlier findings documented in a publication of the French National Statistical Institute (INSEE) by Ceci-Renaud and Chevalier (2010). These authors use the same datasets for employment and thus our descriptive statistics in terms of the firm size distribution are very similar. Their econometric analysis is an implementation of Schivardi and Torrini (2008), which amounts to estimating the distortive impact of the regulations on the firm size distribution by assuming it to be a fixed cost. In contrast to their analysis, we allow the regulation to take a more general form (in particular, we include variable cost components which appear to be the most important component of the cost), and we propose a general equilibrium approach which enables to assess the impact of the regulation on entry and prices (and the firm size distortions they generate).
also the interaction between regulation and downwardly rigid real wages that causes large welfare losses. When wages are fully flexible, the aggregate welfare loss is about 1 percent of GDP. There are also important distributional consequences: aggregate wages and the profits of large firms drop by more than 1 percent, but profits of smaller firms rise by about 7 percent because they enjoy lower equilibrium wages without suffering the regulatory burden. In France, the high minimum wage and powerful unions mean that real wages are partially inflexible. Taking this into account, we find that GDP falls by 3.5 percent mainly through increased unemployment, but also by keeping the most productive firms below their optimal size. Too many employees work for smaller firms, and too few employees work for large firms. In the United States, where wages are more flexible than France, one could speculate that similar size dependent regulations such as the above-mentioned provisions of the Affordable Care Act will have much smaller negative effects.

In terms of the existing literature, one closely related paper is Braguinsky, Branstetter, and Regateiro (2011), which seeks to explain changes in the support of the Portuguese firm size distribution in the context of the Lucas model with labor regulations. Their calibrations also show substantial effects of the regulations on aggregate productivity. Portuguese laws have multiple regulatory thresholds, however, and their firm size data do not show a clear mass point like ours. Thus, their approach cannot exploit the discontinuity to identify the structural parameters of their model. Our paper is also related to the more general literature using tax kinks to identify behavioral parameters (e.g., Saez 2010; Chetty et al. 2011; Kleven and Waseem 2013). Kaplow (2013) discusses issues in the optimal structure of size-related regulations. Finally, our approach of pricing out the cost of regulation as a tax is in the spirit of Posner (1971).

The structure of the paper is as follows. Section I describes our theory, Section II the empirical strategy, Section III the institutional setting and data, and Section IV the main results. We start by showing that the main empirical predictions of the model in terms of the size and productivity distribution are consistent with the data, and we then estimate the parameters of the structural model and use this to show the welfare and distributional impact of the regulation. We present various extensions and robustness tests in Section V. Here we show our results are robust to: considering ways to evade the regulations; allowing for parameter heterogeneity across industries; introducing a second regulatory threshold; allowing for labor-capital substitution; using a Hsieh and Klenow (2009) approach and introducing dynamic considerations. We offer some concluding comments in Section VI. Our extensive online Appendices contain more details on theoretical proofs (A); the econometric estimation (B); various robustness checks (C); capital-labor substitutability (D); dynamics (E); and the regulations themselves (F).

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7 On the quantitative theory side, Guner, Ventura, and Yi (2006, 2008) also consider a Lucas model with size-contingent regulation. They calibrate this to uncover sizable welfare losses. Unlike our paper and Braguinsky, Branstetter, and Regateiro (2011), however, there is no econometric application.
I. Theory

Consider the simplest possible version of Lucas (1978), where there is only one input in production, labor. The primitive of the model is the probability density function (PDF) $\phi(\alpha)$ of managerial ability $\alpha$, $\phi : [\alpha; \alpha_{\text{max}}] \to \mathbb{R}$. We assume that talent is scarce in the sense that $\alpha \phi(\alpha)$ is decreasing in $\alpha$ (i.e., $\phi$ is decreasing in $\alpha$ at a faster rate than $1/\alpha$). A manager who has ability $\alpha$ and is allocated $n$ workers produces $y = \alpha f(n)$, with $f' > 0$, and (given limited managerial time and attention), $f'' < 0$. We study the impact of a regulatory tax on labor which may have both fixed and variable cost components. This tax is only borne by firms after they reach a given size $N$, that is for $n > N$ ($N = 49$ in our main empirical application, although we also study other thresholds).

A. Individual Optimization

Let $\pi(\alpha)$ be the profits obtained by a manager with ability $\alpha$ when she manages a firm at the optimal size:

$$
\pi(\alpha) = \max_n \left\{ \begin{array}{ll}
\alpha f(n) - wn & \text{if } n \leq N \\
\alpha f(n) - w\tau n - F & \text{if } n > N
\end{array} \right.
$$

where $w$ is the worker’s wage, $n$ is the number of workers, $F$ is the fixed cost, and $\tau$ is the variable tax parameter, both of which only apply to firms over a minimum threshold of $N$. Firm size at each side of the threshold is then implicitly determined by the first-order condition:

$$
\alpha f'(n^*) - \tau w = 0, \quad \text{with} \quad \left\{ \begin{array}{ll}
\bar{\tau} = 1 & \text{if } n \leq N \\
\tau = \tau & \text{if } n > N
\end{array} \right.
$$

so that $n^*(\alpha, \tau, w) = f^{-1}(\frac{\tau w}{\alpha})$. To simplify our notation we write this firm size as $n^*(\alpha)$ and omit throughout the explicit reference to the regulatory regime $(N, F, \tau)$ and to wages. Note that firm size is increasing in $\alpha$, and decreasing in $\tau$ and $w$. Managerial ability $\alpha_c$ (subscript $c$ denotes constrained) at the regulatory threshold is given by

$$
\alpha_c = \frac{w}{f'(N)}.
$$

Firms can legally avoid being hit by the regulation by choosing to remain small. The cost of this avoidance strategy is increasing in $\alpha$. The ability level $\alpha_r$ of the marginal manager is defined by the indifference condition between remaining small or jumping to a larger size and paying the regulatory cost:

$$
\alpha_r f(N) - wN = \alpha_r f(n^*(\alpha_r)) - w\tau n^*(\alpha_r) - F,
$$

As in Lucas, this is a one-sector economy. For size-contingent regulations in a multisector Lucas (1978) model, see Garcia-Santana and Pijoan-Mas (2014), who apply it to the Indian textile sector.

Previous studies of this problem, such as particularly Kramarz and Michaud (2010), suggest that the fixed cost element is less important compared to the marginal cost component. Empirically, we also find this result.
where \( n^*(\alpha_r) \) is the optimal firm size for an agent of talent \( \alpha_r \). Subscript \( r \) refers to the marginal firm that is regulated in the sense that they choose to be larger than the regulatory threshold and pay the tax.

**B. Equilibrium**

The most skilled individuals choose to be manager-entrepreneurs, since they benefit from their higher ability in two ways. First, for a given firm size \( n \), they earn more profits. Second, the most skilled individuals hire a larger team, \( n^*(\alpha) \). We denote the ability threshold between managers and workers as \( \alpha_{\text{min}} \); individuals with ability below \( \alpha_{\text{min}} \) are workers. A competitive equilibrium is defined as follows.

**DEFINITION 1:** Given a distribution of managerial talent \( \phi(\alpha) \) over \([\alpha_{\text{min}}, \alpha_{\text{max}}]\), a production function \( y = \alpha f(n) \), a per-worker implicit labor tax \( \tau \), and a fixed cost \( F \) that binds all firms of size \( n > N \), a competitive equilibrium consists of (i) a wage level \( w^* \); (ii) an allocation \( n^*(\alpha) \) that assigns a firm of size \( n^* \) to a manager of skill \( \alpha \); and (iii) a triple of cutoffs \( \{\alpha_{\text{min}} \leq \alpha_c \leq \alpha_r\} \), where \([\alpha_{\text{min}}, \alpha_{\text{min}}] \) is the set of workers, \([\alpha_{\text{min}}, \alpha_c]\) is the set of unconstrained, unregulated managers, \([\alpha_c, \alpha_r]\) is the set of size constrained but unregulated managers, with firm size at \( n^* = N \), and \([\alpha_r, \alpha_{\text{max}}]\) is the set of regulated managers, such that: (E1) no agent wishes to change occupation (worker versus manager); (E2) the choice of \( n^*(\alpha) \) for each manager \( \alpha \) is optimal given their skills, taxes \((\tau, F)\) and wages \( w^* \); (E3) supply of labor equals demand for labor.

Condition (E1) implies continuity of earnings:

\[
\alpha_{\text{min}} f(n^*(\alpha_{\text{min}})) - w^* n^*(\alpha_{\text{min}}) = w^*.
\]

Equilibrium condition (E2), from the first-order condition (2) implies that firm sizes are given by

\[
(6a) \quad n^*(\alpha) = \begin{cases} 
0 & \text{if } \alpha < \alpha_{\text{min}} \\
\frac{\tau w^*}{\alpha} & \text{if } \alpha_{\text{min}} \leq \alpha \leq \alpha_c \\
N & \text{if } \alpha_c \leq \alpha < \alpha_u \\
\frac{w^*}{\alpha} & \text{if } \alpha \geq \alpha_u
\end{cases}
\]

Thus, as Figure 3 shows, we have four categories of agents: workers, small firms, constrained firms, and regulated firms. Finally, condition (E3) requires that supply of workers equals demand for workers:

\[
(7) \quad \int_{\alpha_{\text{min}}}^{\alpha_{\text{min}}} \phi(\alpha) \, d\alpha = \int_{\alpha_{\text{min}}}^{\alpha_{\text{max}}} n^*(\alpha) \phi(\alpha) \, d\alpha,
\]

where \( n^*(\alpha) \) is the continuous and piecewise differentiable function above.
Solving the model involves finding four parameters: the cutoff levels $\alpha_{\text{min}}$, $\alpha_c$, $\alpha_r$, and the equilibrium wage $w^*$. For this we use the four equations (3), (4), (5), and (7). The equilibrium is unique and has the following relevant properties.

**PROPOSITION 1:** The introduction of a tax/variable cost $\tau$ of hiring workers only for firms of size $n > N$: (i) reduces the equilibrium wage $w$ as a result of the reduction in the demand for workers; (ii) increases small firm entry (by reducing $\alpha_{\text{min}}$) and increases firm size for small firms (i.e., $\alpha < \alpha_c$) because of the reduction in $w$; (iii) creates a bulge at $N$ (a spike in the firm size distribution) and valley ($\alpha_r$ increases); and (iv) reduces firm size for all firms that are taxed—i.e., those beyond the threshold $\alpha_r$.

The proof of this proposition is in online Appendix A. Intuitively, the increasing equilibrium schedule that solves the occupational choice equation (5) determining $w$ for each $\alpha_{\text{min}}$ shifts down when the regulatory tax $\tau$ increases (for each cutoff $\alpha_{\text{min}}$ a lower wage $w$ yields indifference), while the decreasing schedule that yields equality between supply and demand (7) also shifts down (left), since all desired team sizes increase and wages must decrease in equilibrium. Thus, the equilibrium $w$ unambiguously decreases as $\tau$ increases (Proposition 1(i)), and this immediately means that all firms which do not pay the regulatory tax are larger (Proposition 1(ii)). Proposition 1(iii) follows since when talent is scarce, the fall in wages is never sufficient to compensate fully the direct impact of the increase in tax.

**PROPOSITION 2:** The introduction of a fixed cost $F$ for sizes $n > N$ (i) reduces the equilibrium wage $w$ and (ii) increases small firm entry (by reducing $\alpha_{\text{min}}$) and increases firm size for small firms (i.e., $\alpha < \alpha_c$); (iii) creates a bulge at $N$ (a spike in the firm size distribution) and valley ($\alpha_r$ increases); and (iv) increases firm size $n$ (through the general equilibrium effect on the wage) for firms beyond the threshold $\alpha_r$.

Again the proofs are in online Appendix A. Intuitively, again the demand for workers is lower, so wages must fall, leading to more entry by small firms. Part (iii) is immediate from equation (4) and similarly part (iv) from the wage reduction.
Example: Consider a power law, \( \phi(\alpha) = \frac{0.6}{\alpha^{1.6}} \) and a returns to scale parameter of \( \theta = 0.5 \). Panel A of Figure 4 shows the firm size distribution for a firm size cutoff at 49 employees, and employment taxes of \( \tau = 0.01 \) (1%) and \( F/w = 0.07 \) (such that \( n_r = 60 \)). As in the distribution in the data, there is a spike at 49 employees that breaks the power law. Panel B of Figure 4 reports the productivity \( \alpha \) as a function of firm size \( n \). It shows that we should expect a spike in the productivity distribution at the point in which the regulation starts to bind. We find strong empirical support for both of these predictions in the data.

C. Empirical Implications

Our econometric work uses the theory as a guide to estimate the welfare losses that result from this regulation. As is well known, the firm size distribution generally follows a power law.\(^{11}\) We follow Lucas (1978) in using the returns to scale function of \( f(n) = n^\theta \). He shows that for it to be consistent with a power law, the managerial ability or productivity distribution must also be a power law, \( \phi(\alpha) = c_\alpha \alpha^{-\beta_\alpha} \), \( \alpha \in [\alpha_\min; \alpha_\max] \) with the constants \( c_\alpha > 0 \) and \( \beta_\alpha > 0 \). In this case, from the first-order conditions in equations (6a) to (6d), firm sizes for the equilibrium wage \( w^* \) are given by \( n^*(\alpha) = \left( \frac{\theta}{w^*} \right)^{1/(1-\theta)} \alpha^{1/(1-\theta)} \) for \( \alpha_\min \leq \alpha < \alpha_c \) and \( n^*(\alpha) = \left( \frac{\theta}{w^*} \right)^{1/(1-\theta)} \tau^{-1/(1-\theta)} \alpha^{1/(1-\theta)} \) for \( \alpha_r \leq \alpha \leq \alpha_\max \).

The distribution of firm sizes \( \chi(n) \) is, by the change of variable formula, \( \chi(n) = \phi(\alpha(n)) \times \frac{1}{p} n^{-\theta} w^* \left( \frac{1-\theta}{\theta} \right) \) (omitting the threshold, and where \( p \) denotes the share of entrepreneurs among all agents in the economy). After some straightforward

\(^{10}\) These values are selected simply to illustrate the graphical patterns.

manipulation, relegated to online Appendix B, the *broken* power law on firm size \( n^* \) is then given by

\[
\chi^*(n) = \begin{cases} 
\left(1 - \frac{\theta}{\theta}ight)^{1-\beta} (\beta - 1) n^{-\beta} & \text{if } \frac{\theta}{1 - \theta} \leq n < N \\
\left(1 - \frac{\theta}{\theta}ight)^{1-\beta} (N^{1-\beta} - Tn_r^{1-\beta}) & \text{if } n = N \\
0 & \text{if } N < n < n_r \\
\left(1 - \frac{\theta}{\theta}ight)^{1-\beta} (\beta - 1) Tn^{-\beta} & \text{if } n_r \leq n
\end{cases}
\]

where \( \beta = \beta_\alpha (1 - \theta) + \theta \) and where \( T = \tau^{\frac{\beta-1}{1-\theta}} \). The upper employment threshold, \( n_r \), is unknown and must be estimated alongside \( \beta \), the power law term, \( \theta \), the return to scale parameter, and \( T \), which is a function of \( \tau \). Note that the shape parameter \( \beta \) in the power law is unaffected by the regulation; instead, in log-log space, the labor regulations generate a parallel shift in the firm size distribution driven by \( T \) (see Figure 4). Thus, the key empirical implication is that the variable part of the regulatory tax can be recovered from the shift \( T \) in the power law.

D. Welfare Calculations

A manager with talent \( \alpha \) and firm size \( n^*(\alpha) \) produces \( y(\alpha) = \alpha f(n^*(\alpha)) \) and total output in this economy is found by integrating over all agents of different managerial ability:

\[
Y = \int_{\alpha_{\text{min}}}^{\alpha_{\text{c}}} \alpha f(n^*(\alpha)) \phi(\alpha) d\alpha + \int_{\alpha_{\text{c}}}^{\alpha_{\text{r}}} \alpha f(N) \phi(\alpha) d\alpha + \int_{\alpha_{\text{r}}}^{\alpha_{\text{max}}} \alpha f(n^*(\alpha)) \phi(\alpha) d\alpha.
\]

It follows that the welfare change between the regulated and unregulated economy is then given by

\[
\Delta Y = Y(N, \tau, F) - Y(\cdot, 1, 0) = \int_{\alpha_{\text{min}}}^{\alpha_{\text{c}}} \alpha [f(n^*(\alpha)) - f(n_{\text{NR}}^*(\alpha))] \phi(\alpha) d\alpha
\]

where \( n_{\text{NR}}^* \) is firm size given \( w_{\text{NR}}^* \) (where subscript NR denotes “not regulated”), the equilibrium wage rate, and \( \alpha_{\text{min}} \) the cutoff between workers and managers in the unregulated economy. Thus, the welfare losses in our simple framework are the result of adding up three effects.

(i) The top row of equation (9) captures two positive effects on total output from the fall in the equilibrium wage arising from the regulation. First, there
are some additional firms since marginal workers are drawn into becoming entrepreneurs by cheaper labor. Second, the firms which are below the regulatory threshold (and not paying the tax) are able to hire more workers as their wages are lower.

(ii) There is a local output loss, that is the result of the firms which would have been larger but instead are constrained at $N$ workers. This is the second row of equation (9).

(iii) Finally, there is another output loss from the larger firms in the economy, which incur higher labor costs due to the regulatory tax (even after netting off the lower equilibrium wage), and choose too small a size.

Note that $\alpha_{\min}$ unambiguously decreases under regulation. In other words, the measure of firms increases and the measure of workers decreases, so the average firm size will fall in the regulated economy.

E. Partial Inflexibility in Real Wages

We also provide an analysis under the case when real wages are not perfectly flexible downward after the regulation. In France there is a high minimum wage and strong unions (about 90 percent of all workers in France are covered by a collective bargain) which restrict the ability of wages to offset the effect of regulations. More generally, there is likely to be a reservation wage below which individuals will not work, in particular when welfare benefits are generous. Note that these are not the temporary nominal frictions highlighted in the business cycle macro literature, but real rigidities that prevent equilibrium real wages falling by more than a certain amount. Incorporating inflexible real wages requires a small extension of the model. We define the equilibrium with inflexible wages as follows.

**DEFINITION 2:** A competitive equilibrium with inflexible wages consists of: (i) a wage level $w_a^*$ paid to all employed workers indexed by a (for fraction of adjustment), which corresponds to the percentage of adjustment that is feasible, with $w_a^* = w_0^* + a(w^* - w_0^*)$ (thus $a = 1$ means wages are fully flexible); (ii) an allocation $n_a^*(\alpha)$ that assigns a firm of size $n$ to a particular manager of skill $\alpha$; (iii) a triple of cutoffs $\{\alpha_{\min}^a \leq \alpha_c^a \leq \alpha_r^a\}$ as in Definition 1; and (iv) an unemployment rate $u^a$ defined as the number of unemployed workers as a share of the total number of potential workers, such that conditions (E1) and (E2) in Definition 1 hold and (E3) becomes (E3\*): supply of labor is equal to the sum of the demand for labor and unemployment.

The model with partial inflexible wages is solved in the same way as before; the main differences relate to condition (E1) and to the labor market equilibrium. The new version of equation (5) (implied from condition (E1)) now compares the profit

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a small (untaxed) potential entrepreneur with the expected wage of a worker, who earns \( w_a^* \) when she is employed, but zero if she is unemployed:

\[
\alpha_{\min}^a f(n_a^*(\alpha_{\min}^a)) - w_a^* n_a^*(\alpha_{\min}) = (1 - u^a) w_a^*.
\]

The labor market equilibrium condition (E3) is also modified, since now the regulation generates unemployment:

\[
(1 - u^a) \int_{\alpha_{\min}}^{\alpha_{\max}} \phi(\alpha) \, d\alpha = \int_{\alpha_{\min}}^{\alpha_{\max}} n_a^*(\alpha) \phi(\alpha) \, d\alpha.
\]

The remainder of the analysis is otherwise unaltered.

II. Empirical Strategy

A. Empirical Model

We propose a baseline empirical model in which we introduce an error term in our measure of employment so that we can take it to the data. Such empirical model must account for two departures in the data of Figure 2 from the predictions in the theory. First, the departure from the power law does not start precisely at the regulatory threshold \( N \), but slightly earlier: there is a bump in the firm size distribution beginning at around 46 workers. Second, the region immediately to the right of \( N \) does not have zero density, but rather there are some firms with positive employment levels just to the right of the regulatory threshold, \( N \).

In order to account for these empirical departures from the pure theory, we propose a baseline model that allows for measurement error in the employment data. The reasons for this specification are twofold. First, no dataset will perfectly measure the underlying theoretical concept—measurement error is a fact of empirical life. Second, several different regulations start at size 50, and as explained in greater detail in Section IIIA and online Appendix F, they rely on slightly different concepts of employment size, defined respectively in the Code du Travail (labor laws), Code du Commerce (commercial law), Code de la Sécurité Sociale (social security), and in the Code Général des Impôts (fiscal law). The measurement of firm size that we use\(^{13}\) corresponds to the fiscal definition that is a mandatory item reported in the firm’s tax accounts—the arithmetic mean of the number of workers measured at the end of each of the four quarters of the fiscal year. This measure of size is available accurately for essentially all firms. However, it does not exactly correspond to the concept of size that is relevant for all of the regulations. An additional reason for the existence of some firms to the immediate right of the size threshold could be labor adjustment costs. If large changes in employment generate such costs, a firm experiencing a positive shock may spend a period passing through the valley to the right of the 50 employee threshold in order to smooth these adjustment costs. In order

\(^{13}\)Fiscal definition, Article 208-III-3 du Code Général des Impôts. We also test the robustness of our results using alternative datasets and concepts of employment; see online Appendix C.4.
to consider this we build an explicit dynamic model in Section VF. Although more complex, the basic intuitions all go through in this extended model.

Recall that our starting point is the PDF of \( n^* \) as given by equation (8). Employment is measured with error so we assume that rather than observing \( n^*(\alpha) \) we observe \( n(\alpha, \varepsilon) = n^*(\alpha)e^\varepsilon \) where the measurement error \( \varepsilon \) is unobservable. In the data we observe the distribution of \( n \), and thus obtaining the likelihood function requires that we obtain the density function of \( n \). The conditional cumulative distribution function (CDF) is given by (see online Appendix B)

\[
\mathbb{P}(x < n|\varepsilon) = \begin{cases} 
0 & \text{if } \ln(n) - \ln\left(\frac{\theta}{1 - \theta}\right) < \varepsilon \\
1 - \left(\frac{1 - \theta}{\theta}\right)^{1 - \beta}(ne^{-\varepsilon})^{1 - \beta} & \text{if } \ln(n) - \ln(N) < \varepsilon \leq \ln(n) - \ln\left(\frac{\theta}{1 - \theta}\right) \\
1 - \left(\frac{1 - \theta}{\theta}\right)^{1 - \beta}T(n_r)^{1 - \beta} & \text{if } \ln(n) - \ln(n_r) < \varepsilon \leq \ln(n) - \ln(N) \\
1 - \left(\frac{1 - \theta}{\theta}\right)^{1 - \beta}T(ne^{-\varepsilon})^{1 - \beta} & \text{if } \varepsilon \leq \ln(n) - \ln(n_r)
\end{cases}
\]

Let \( \varepsilon \) be normally distributed with mean 0 and variance \( \sigma \). Integrating over \( \varepsilon \) we can compute the unconditional CDF (convolution of the broken power law and Gaussian distributions) simply as

\[
\mathbb{P}(x < n) = \int \mathbb{P}(x < n|\varepsilon) \frac{1}{\sigma} \cdot \varphi\left(\frac{\varepsilon}{\sigma}\right) d\varepsilon,
\]

where \( \varphi \) denotes the gaussian probability density function. In online Appendix B we show that no further constraints on the parameters are required for this object to be a CDF.

**LEMMA 1:** Let \( \varepsilon \) be normally distributed with mean 0 and variance \( \sigma \) so that the measurement error is log normal. Then the function \( \mathbb{P}(x < n) \) is a cumulative distribution function, that is strictly increasing in \( n \), with \( \lim_{n \to 0} \mathbb{P} = 0 \) and \( \lim_{n \to \infty} \mathbb{P} = 1 \) for all feasible values of all parameters, \( \sigma, \theta, T, \beta, \) and \( n_r \).

Thus, taking the derivative of \( \mathbb{P} \) formulated in this way we can obtain the density of the observed \( n \). Given such a density, it is straightforward to estimate the parameters of the model by maximum likelihood (ML). Specifically, ML yields estimates of the parameters \( \hat{\sigma}, \hat{T}, \hat{\beta}, \hat{n}_r \) while \( \hat{\tau} \) and \( \hat{F}_w \) are computed as (see full details in online Appendix B)

\[
\hat{\tau} = \hat{T}^{\frac{1 - \theta}{\beta - 1}}
\]

and

\[
\hat{F}_w = N - \frac{\hat{\tau}}{\hat{\theta}} \cdot \hat{n}_r \cdot \left[ \left( \frac{N}{\hat{n}_r} \right)^{\hat{\theta}} - 1 + \hat{\theta} \right].
\]

\[14\] In online Appendix C.2, we examine the sensitivity of our estimates to this Gaussian assumption and show that the cost of the regulation is robustly estimated.
Figure 5 shows the difference between the pure theory model where employment was measured without error and the true model where there is measurement error. The thickest solid line shows the firm size distribution under the pure model of Section I (same as panel A of Figure 4) whereas the dashed line shows the firm size distribution when we allow for measurement error. The smoothness of the bulge around 50 will depend on the degree of measurement error. If we increase the measurement error to $\sigma = 0.5$ (thin solid line) instead of $\sigma = 0.15$ it is almost impossible to visually identify the effects of the regulation.

**B. Identification and Inference**

Our ML estimation over the size distribution allows us to obtain most of the parameters of interest. Intuitively, the slope of the line in Figure 5 (which is the same before and after the cutoff) identifies $\beta$, the power law parameter. The estimates of the two regulatory tax parameters can be recovered from three features of the distribution. First, the downward shift of the power law after 49 employees. Second, the bulge of firms just before the regulatory threshold at 50 employees and third the width of the valley in the size distribution between 49 employees and where the power law recovers at $n_r$. A variable tax $\tau$ without any fixed cost results in a parallel shift of the power law. Instead, if there were only a fixed cost $F$ of the regulation we should only see a bulge, and a valley, but no shift down in the power law after $n_r$. Hence, the existence of a downward shift in the firm size distribution after the regulatory threshold is powerful evidence of a variable cost component of the regulation. Last, the measurement error, $\sigma$, is identified from the size of the random deviations of the size distribution from the broken power law.
Given the empirical estimates of $T$ and $\beta$, we still need an estimate of returns to scale $\theta$ in order to identify the key tax parameter, $\tau$. There are several ways to obtain $\theta$. Our first approach is to calibrate from existing estimates. Since $\theta$ is well recognized to be an important parameter in the macro reallocation literature there are a number of papers to draw on. Basu and Fernald (1997) show a number of estimates and suggest a value of 0.8. We use this as a baseline and consider reasonable variations around this value used in other papers.\(^{15}\) Our second approach is to estimate firm level production functions and sum the coefficients on the elasticities of output with respect to factor inputs to calculate returns to scale.\(^{16}\) Our third method is to use the relationship between size and TFP to back out an estimate of the returns to scale.\(^{17}\)

We implement all these alternative methods to show that our results are robust to plausible values of $\theta$. As previously explained in Section IIA, given an estimate of $\theta$, we have an estimate of the implicit variable tax of regulation as (hats denoting estimated parameters)

\[
\hat{\tau} = \frac{T^{-1 - \hat{\theta}} - 1}{\beta - 1}.
\]

We obtain standard errors for the estimates of the tax using either standard errors clustered at the four-digit industry level and the delta method, or block-bootstrapping at the industry level, with 100 replications.

III. Institutional Setting and Data

A. Institutions: The French Labor Market and Employment Costs

France is renowned for having a highly regulated labor market (see Abowd and Kramarz 2003; Cahuc and Kramarz 2005; Kramarz and Michaud 2010). What is less well known is that most of these laws only bind a firm when it reaches a particular employment size threshold. Although there are some regulations that bind when a firm (or less often, a plant) reaches a lower threshold such as 10 workers, 50 is generally agreed by labor lawyers and business people to be the critical threshold when costs rise significantly. In particular, when firms get to the 50 employee threshold they need to undertake the following duties (see online Appendix F): (i) they must set up a works council (comité d’entreprise) with minimum budget of 0.3 percent of total payroll; (ii) they must establish a committee on health, safety, and working conditions (CHSCT); (iii) a union representative (i.e., not simply a local representative of the firm’s workers) must be appointed if wanted by workers; (iv) they must establish a profit-sharing plan; (v) they incur higher liability in case of a workplace accident; (vi) they must report monthly and in detail all of the labor contracts to the administration; (vii) firing costs increase substantially in the case of collective

\(^{15}\) Guner et al. (2006) use a $\theta = 0.802$ for Japan. Atkeson and Kehoe (2005) use a version of the Lucas model with organizational capital and suggest a value of 0.85. Hsieh and Klenow (2009) use a value of $\theta = 0.5$.

\(^{16}\) Online Appendix C.3.1 details how we do this using a variety of methods such as Levinsohn and Petrin (2003), Olley and Pakes (1996), and the more standard Solow residual approach.

\(^{17}\) Note that in principle, $\theta$ can be recovered from the size distribution itself (see online Appendix C.3). This method relies on rather strong assumptions over the identity of the smallest firm from the indifference condition between being a worker and a manager in equation (5). As discussed in online Appendix C.3, empirically the data are not rich enough to estimate $\theta$ from the size distribution alone (although we can reject very large values of the parameter), so we consider several alternatives in order to examine the empirical robustness of our estimates of $\tau$. 
dismissals of 10 or more workers (Bentolila and Bertola 1990); and (viii) they must undertake to do a formal professional assessment for each worker older than 45.

How important are such provisions for firms? Except in the case of the minimum regulatory budget that is to be allocated to firm councils, it is extremely hard to get a handle on this. For example, what is the opportunity cost of managerial time involved in dealing with works councils, union representatives, health and safety committees, etc.? Our framework is designed to recover the costs of such regulations by examining the revealed preferences of firms. We focus on the 50 employee threshold, but it is straightforward to extend the analysis to other thresholds. Indeed, we implement such an extension to cover the thresholds at both 10 and 50 employees in Section V.

B. Data

Our main dataset is FICUS which is constructed from administrative (fiscal) data covering the universe of French firms between 1995 and 2007. These are based on the mandatory reporting of firms’ income statements to tax authorities for all French tax schemes: the BRN,\textsuperscript{18} RSI, and BNC. The BRN is the standard tax regime; the RSI is a simplified regime that small firms can opt into for a cost; and the BNC covers noncommercial professions such as legal and accounting firms. Although the BRN has been more commonly used by researchers, it misses out on large numbers of small firms. For example, less than 20 percent of single employee firms are in the BRN and even for firms with ten employees the BRN only covers 80 percent of FICUS firms. Hence, for looking at the firm size distribution one needs to use the whole FICUS data.

All firms have to report tax returns (even if they owe no tax in the fiscal year) and there are about 2.2 million firms per year in our data. Our baseline results are on the approximately 200,000 firms active in the manufacturing sector (NACE2 industry classes 15 to 35) as productivity is easier to measure in these industries. But we also present results estimating the model on all the other nonmanufacturing sectors and show the robustness of our results. The laws mainly apply to the administrative unit in FICUS, namely the firm (entreprise), so this dataset is well suited to our purpose.

The employment measure in the FICUS relates to a head-count of the number of workers in the firm averaged over the four quarters of the fiscal year in France. A head-count of employees is taken on the last day of the fiscal year (usually December 31) and on the last day at the end of each of the previous three quarters. Employment is then the simple arithmetic average over these four days. This is close to the concept used for most of the regulations (see online Appendix F), but not all of them and this is one of the motivations for allowing employment to be measured with error with respect to the regulation.\textsuperscript{19} In addition, FICUS contains balance sheet information on capital, investment, the wage bill, materials, four-digit

\textsuperscript{18} See Di Giovanni, Levchenko, and Rancière (2011) and Caliendo, Monte, and Rossi-Hansberg (2015) for other work on these BRN data.

\textsuperscript{19} We attempted to use the differences in the exact regulatory definition of 50 employees to tease out the impact of different regulations (e.g., regulations using head-counts on a single precise date versus those averaging over a longer period). Unfortunately, the data are not sufficiently precise to enable us to uncover such subtle distinctions. This could be an area for future work in other countries where more precise data exist.
industry affiliation, etc. that are important in estimating productivity. We also use the DADS (Déclarations Annuelles de Données Sociales) dataset which contains worker-level information on hours, occupation, gender, age, etc. This dataset provides a variety of proxies to measure firm size in terms of employment and full-time equivalents (see online Appendices F and C.4 for more details). Neither dataset is perfect. FICUS has the advantage that the time period for reporting is common to employment and the accounting measures used to build productivity. The timing of the reporting of DADS employment differs from this accounting information, but it has the advantage of having hours information from which full-time equivalents can be computed, although unfortunately, full-time equivalent information in DADS misses almost one-quarter of jobs. Empirically, we obtain similar results whether we use FICUS or DADS for our econometric estimation (see next section and online Appendix C3). Details of the TFP estimation procedure, which in the baseline specification uses the Levinsohn and Petrin (2003) version of the Olley and Pakes (1996) method are reported in online Appendix C.3.1.

IV. Results

A. Qualitative Analysis of the Data

We first examine some qualitative features of the data to see whether they are consistent with our model. Many commentators have expressed skepticism about the quantitative importance of employment regulations as it is sometimes hard to observe any clear change in the size distribution around important legal thresholds. Figure 6 presents the empirical distribution of firm size around the cutoff of 50 employees for two datasets, FICUS and DADS. The left side panels are the frequencies immediately around 50, whereas the right panels are over a wider support in log-log space. The top left side figure in panel A is the same as Figure 2. As previously discussed, there is a mass point at 49 and a sharp discontinuity in size which is strong evidence for the importance of the regulation. The top right-hand side of Figure 6 shows

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20 In DADS many firms report employment and wage information from December to November rather than from January to December, as is (most frequently) done for value added in the fiscal files.

21 Full-time equivalents are computed from non-annexes jobs only, i.e., those lasting more than 30 days with more than 1.5 hours worked per day, or those that were associated with (total) gross earnings higher than 3 times the monthly minimum wage. As documented in the methodology of the DADS files, in 2002, the non-annexes jobs represent 76.6 percent of all jobs, which means that almost one-quarter of the most short-end, low paid jobs were removed. This is unfortunate as such jobs are in fact covered by labor laws. Note also that the type of contract (full time versus part time) is not observed in the DADS data. It is estimated from the hours paid (taking the seventy-fifth percentile of the distribution of hours per job as a reference), and there is a lot of variation in this due to the heterogeneous implementation of the 35 hours maximum working week.

22 We use a control function approach to deal with unobserved productivity shocks and selection when estimating production functions. Because we have a panel of firms, we can implement this and estimate the production function coefficients. There are several issues with this approach (e.g., Ackerberg et al. 2007; Ackerberg, Caves, and Frazer 2015) to estimating production functions so we also estimate TFP using a variety of other methods (see online Appendix C.3.1 for details).

23 Hsieh and Olken (2014) note this point in their analysis of developing countries like India. See also Schivardi and Torrini (2008) and Boeri and Jimeno (2005) on Italian data; Braguinsky, Branstetter, and Regateiro (2011) on Portuguese data; Abidoye, Orazem, and Vodopivec (2009) on Sri Lanka data; or Martin, Nataraj, and Harrison (2014) on India. These authors find that there is slower growth just under the threshold consistent with the regulation slowing growth (as we also show below), but they find relatively little effect on the cross-sectional distribution. This may be because of the multitude of regulations, variable enforcement or measurement error in the employment data.
Figure 6. The Effect on the Measured Firm Size Distribution Using Alternative Datasets (FICUS and DADS) and Definitions of Employment

Notes: Data sources are from FICUS (corporate tax collection reported to fiscal administration) and DADS (payroll tax collection reported to social administration); see text and online Appendix C4 for more details on datasets. Panels A and B relate to the year 2000, and panel C relates to the year 2002 (as FTE calculation is unavailable in earlier years). All firms in manufacturing. FICUS employment is head-count at end of quarter averaged over four quarters (panel A). Panel B is head-count on December 31. FTE calculated in panel C by the French Statistical Agency INSEE over the available sample of workers (76 percent of all workers). This excludes jobs lasting less than 30 days or less than 1.5 hours worked per day, or those that were associated with (total) gross earnings lower than three times the monthly minimum wage (using as a benchmark, the seventy-fifth percentile of the series of hours per workers in each two-digit industries and size classes, and are bounded above by 1).
this in log-log space clearly indicating the evidence of a broken power law. There
is a downward shift in the intercept of the slope of the power law at 50 employees.

Panel B of Figure 6 uses DADS. We first aggregate employment up to the appro-
priate level for each FICUS firm using head-counts dated on December 31. The
discrete jump at 50 also shows up here, especially in the log-log plot, but less clearly
than in the FICUS data. Panel C of Figure 6 uses full-time equivalents (over one cal-
endar year) which also shows less of a jump than the straight count of employees in
the previous panels, probably because of the way full-time equivalents are estimated
in the files. In online Appendix C.4, we show that our estimates are robust across all
datasets and employment definitions in Figure 6, because the main component of the
estimated tax generates a shift of the distribution of the power law downward rather
than just a mass point at 49.

In all panels of Figure 6, firm size seems to approximate a power law in the
employment size distribution prior to the bulge around 50. After 50, there is
a sharp fall in the number of firms and the line becomes flatter before resuming
another power law with a similar slope. Broadly, outside a distorted region around
50 employees, one could describe this pattern a broken power law with the break at
50.\textsuperscript{24} The finding of the power-law for firm size in France is similar to that for many
other countries and has been noted by other authors (e.g., Di Giovanni, Levchenko,
and Rancière 2011), but the finding of the break in the law precisely around the main
labor market regulation is a specific feature of French data. There does appear to be
some break in the power law at firm size 10 which corresponds to the size thresholds
from other pieces of labor and accounting regulations (see online Appendix F). In
order to avoid conflating these issues we initially focus our analysis first on firms
with 10 or more employees in the main analysis, and therefore on the additional
costs generated by regulations at threshold 50 relative to average labor cost for firms
having 10 to 49 employees. We extend later (Section VC) our analysis to the case
with two thresholds (10 and 50) and show that our main specification does indeed
pick up the main costs of the regulation.

Our basic model has the implication that more talented managers leverage their
ability over a greater number of workers. Figure 7 examines this idea by plotting
mean TFP levels by firm size.\textsuperscript{25} Productivity appears to rise monotonically with
size, although there is more heteroskedasticity for the larger firms as we would
expect because there are fewer firms in each bin. The relationship between TFP and
size is broadly log-linear. What is particularly interesting for our purposes, however,
is the bulge in productivity around the 50 employee threshold. We mark these points
in darker shading. This is consistent with our model (see panel B of Figure 4) where
some of the more productive firms who would have been just over 50 employees in
the counterfactual unregulated economy, choose to be below 50 employees to avoid
the cost of the regulation. Firms just below the cutoff are a mixture of firms who

\textsuperscript{24} See Howell (2002) for examples of how to estimate these types of distributions. More generally, see Bauke
(2007) for ways of consistently estimating power laws.

\textsuperscript{25} TFP is difficult to measure accurately (see online Appendix C). We do not have firm-specific information on
prices so high measured TFP may reflect higher mark-ups as well as higher productivity. Additionally, adjustment
costs will drive a wedge between the true and estimated production function parameters (see Section VI). Such
measurement issues may explain why some of the firms to the right of the threshold have lower measured TFP.
would have had a similar employment level without the implicit tax and those firms whose size is distorted by the size-related regulation.

We also examined dynamic patterns in the data. Firms are much more likely to stay at size 49 and not grow. Twelve percent of firms were exactly of size 49 employees for two years running compared to only 4 percent of firms who stayed exactly at size 47 and only 2 percent of firms who stayed at exactly size 52 employees (online Appendix Figure A1). Only 0.32 percent of firms with 49 employees or less crossed over the threshold to 50 (or more) workers in an average year (about 550 firms), compared to 0.40 percent of firms with 47 employees or less who crossed the 48 employee threshold (about 700 firms). Another way of seeing this is that the average duration of firms at 49 employees was 0.30 of a year, compared to 0.23 of a year for firms with 50 employees (online Appendix Figure A2). These dynamic patterns are consistent with firms being reluctant to grow beyond the threshold at 49 employees. We will explore this more in the context of an explicit dynamic model in Section VI.

B. Econometric Results

The key parameters are estimated from the size distribution of firms using the ML procedure described in Section II. Table 1 shows a set of baseline results using different values of $\theta$ for French manufacturing firms in 2000.\footnote{We use a sample of firms with between 10 and 1,000 employees correcting our estimates for censoring at these lower and upper known thresholds.} We begin by using
a calibrated value of $\theta = 0.8$ from Basu and Fernald (1997) in column 1. The slope of the power law, $\beta$, is about 1.8 and significant at conventional levels. The upper employment threshold, $n_r$, is estimated to be an employment level of 59 and we obtain a standard deviation of the measurement error of 0.12. Turning to the estimates of the tax equivalent costs of the regulation, we estimate the implicit variable labor tax to be $\tau - 1 = 0.023$ (2.3 percent) and highly significant.

One surprise in Table 1 is that the fixed cost estimate, $F/w$, is negative and equal to about 94 percent of a single worker’s wages. The implications of this for the implied regulatory cost is shown in panel A of Figure 8. The bold line is our main estimates of the total regulatory costs from column 1 of Table 1 (as a proportion of the average wage in the economy). These are zero until the firm employs the fiftieth worker where they jump discontinuously to 20.9 percent $(= 50 \times 0.023 - 0.941)$ before rising linearly with the estimated variable cost of 2.3 percent. If the fixed cost were zero then the total cost function would be given by the dashed line. The negative fixed cost implies that there is a sudden jump in costs at the threshold, but that it is much less than what we would expect from just the variable cost.\footnote{A cross check on the plausibility of the results is to see how much extra profits would be gained from switching from 49 to 59 workers. Using the estimate in column 1 of Table 1 this is €13,863 = $[F/w + (\tau - 1) \times n_r] \times w$ = $[-0.941 + 0.023 \times 59.27] \times €31,912$. If we take a cost of capital of 15 percent to reflect the greater riskiness of small firms, the difference in entrepreneurial rents for firms between the two size classes using accounting data is similar at €13,521. This suggests our results are consistent with basic accounting data.}

So no firm ever benefits from crossing the regulatory threshold as the total variable costs exceed the fixed costs.
This is also reflected in the fact that the bulge in the firm size distribution just below the regulatory threshold is smaller than it would be without the estimated negative fixed cost. One economic explanation for the negative fixed cost is the presence of a small within-firm positive spillover from the effort in information collection that firms must undertake to comply with the costly regulatory reporting requirements (such as monthly reports on every worker’s pay and conditions to the Labor Ministry). Having to collect such information may also enable a firm to make efficiency savings in organizing labor or other parts of their business. Online Appendix C.1 shows that in general small spillovers that are concave can lead to an estimate of a negative fixed costs of the sort we find.

Column 2 of Table 1 considers an alternative calibration of $\theta = 0.85$ from Atkeson and Kehoe (2005). The obtained results are stable, although the estimate of the variable cost of regulation falls from 2.3 percent to 1.7 percent. This is because the importance of the distortions of the tax depend on returns to scale. When returns to scale are close to unity the most efficient firms produce a large share of output, so it only takes a small distortionary tax to have a large effect on the size distribution. As decreasing returns set in it takes a much larger estimate of $\tau$ to rationalize any given distorted distribution (the downward shift, bulge, and valley in firm size). To further illustrate this effect, column 3 considers $\theta = 0.50$ (as in Hsieh and Klenow 2009). This case implies a large tax of 5.9 percent. Column 4 goes to the other extreme with a $\theta = 0.9$ generating a smaller tax rate of 0.7 percent. Column 5 of Table 1 uses the

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28 These could be through adopting information technologies specifically designed to keep track of inputs like labor in the firm such as Enterprise Source Planning (ERP) software. Bloom et al. (2014) discuss more details of the economics and organizational consequences of ERP introduction. It could also mean adopting the kind of management practices discussed by Bloom, Sadun, and Van Reenen (2016).
TFP estimates from the production function to estimate the TFP-size relationship as in Figure 7. This generates an estimate of the variable tax of 2.4 percent. Column 6 uses the returns to scale parameter directly estimated from a production function, giving a value of $\theta = 0.860$. Other parameter estimates remain stable and we obtain an estimate of the variable tax of 1.6 percent.

Our results use the year 2000 cross section but are robust to pooling across all years between 1995 and 2007 data or choosing individual years (see online Appendix Table A1). These experiments show that the upper employment threshold lies between 58 and 60 workers, the variable tax $(\tau - 1)$ between 1.7 percent and 2.3 percent.

Panel B of Figure 8 shows how the fit of the model compares with the raw data using the estimated parameters in column 1 of Table 1. Although imperfect, we seem to do a reasonable job at mimicking the size distribution even around the regulatory threshold. Another way to assess the model’s performance is to recall that we are using the distribution only of firm size and employment to estimate the parameters of our model. Thus, we can also assess the model’s performance by using it to predict the distribution of output, a moment we are not targeting in the baseline estimation. If alternative margins of adjustment were important, then the observed distribution of output would be very different (and total output larger) than what is predicted by the model. In particular, we would expect to underestimate the share of output produced by large firms, and to overestimate the share of output produced by small firms, as larger firms can reduce their regulatory cost by increasing hours, skills, or capital intensity (see below for evidence of this).

Panels A and B of Table 2 compare the actual and predicted distributions of firms and employment. Unsurprisingly we match these moments closely as these are the data that we are using to fit our parameters. Panel C has the output estimation. The proportion around the bulge of 49–59 workers predicted by our model is close (at 4.1 percent) to the actual data (3.5 percent). As expected, we underestimate the output of larger firms, but not by too much. Our parameters suggest that 68.6 percent of output should be in firms with over 58 employees, whereas the number is 71.6 percent in the data. Symmetrically, we overestimate the output share of the small firms (27.2 percent versus 24.9 percent). These mispredictions are exactly as basic economics suggests as the baseline results do not account for other margins of substitution. Nevertheless, as shown by the standard errors, our estimates are within the 95 percent confidence intervals which is a good performance for such a simple model. In Section V we will explicitly model some substitution margins (e.g., over capital) and come to a similar conclusions over the robustness of our model.

C. Changes in the Level and Distribution of Welfare

We can fully calculate the impact of the regulation on the firm size distribution, output, and welfare. As shown in Section I, the slope of the power law does not change as a result of the implicit tax. According to the theory, the impact of the tax is a parallel move upward of the firm size distribution at sizes $n < 49$, a spike at $n = 49$, and a parallel move down for $n > 58$. Thus, the counterfactual firm size distribution is a power law with the exponent $\beta$ we calculated in our analysis, and $\tau = 1, F = 0$. The position of the intercept is pinned down by the labor market condition, which requires that the total number of agents in the economy is constant,
and by the minimum firm size which in our specification is pinned down by the returns to scale parameter and also stays constant.29

As discussed in Section IE, an important factor influencing the welfare effects of the regulation is the degree of real wage flexibility, \( a \). We begin by assuming that real wages are perfectly flexible \( (a = 1) \). But given that France has a high minimum wage and strong unions we calibrated the degree of inflexibility using three alternative methods (see online Appendix B.5.3 for details). In our baseline model we used the average difference in the structural unemployment rates between France and the United States between 2002 and 2007 from OECD (2015). The United States serves as the benchmark of the least regulated labor market in the OECD. We then use our parameter estimates to fit the value consistent with the difference in unemployment rates. This generated \( a = 0.70 \), i.e., real wages fall by 70 percent of what we would expect in a world of fully flexible real wages. Second, we used the estimates in Aeberhardt, Givord, and Marbot (2012) of the impact of changes in the value of the minimum wage over the entire wage distribution. Their quantile regression estimates imply \( a = 0.67 \), a very similar degree of flexibility to the first method. Thirdly, we use the degree of partial adjustment estimated by the official midterm macro-model used by the French Finance Ministry (e.g., de Loubens and Thornary 2010). This has a higher degree of rigidity \( a = 0.62 \) after two years. We will take \( a = 0.70 \) as our baseline, but the similarity of the calibration of the wage rigidity parameter across several different methods is reassuring.

29 Note that this is not the case when the regulation binds for firms of all sizes (i.e., \( N = 0 \) and \( \tau > 1 \)), or when wages are rigid. See Section I and online Appendix B for the details of the derivation.
Panel A of Figure 9 presents the change in the firm size distribution in the world with regulation (bold line) and without regulation (dashed line) using the estimated parameters from our model (the lower figure zooms in around the threshold to make the changes more visible). As the theory led us to expect, in the counterfactual unregulated economy there are fewer firms under 49 employees. This is because in the regulated economy (i) there is a spike at 49 employees for those firms who are optimally avoiding the regulation, and (ii) since equilibrium wages have fallen there is an expansion in the number of small firms. Compared to the unregulated economy, the regulated world has fewer large firms since although wages have fallen there is an additional regulatory tax. Panel B presents the same information in terms of the output distribution which has a similar pattern.

We illustrate the effects of different degrees of real wages inflexibility in Figure 10 for jobs (left-hand-side figures) and welfare (right-hand-side figures). Panel A has full flexibility $a = 1$, panel B is our baseline case of $a = 0.7$. Turning first to the top-left figure, we examine the changes in employment in the regulated economy compared to an unregulated economy for small firms, medium firms, large firms and the overall economy. With fully flexible wages there is basically no aggregate
loss of jobs, but we can see that there is an increase in employment in small firms of nearly 200,000, and decreases in employment in medium sized and large firms. In our baseline scenario in panel B, there are overall job losses of around 140,000 as unemployment emerges. The same pattern emerges of the largest job losses coming from the biggest firms and actually some gains in the smaller firms.

The welfare losses in terms of income (wages or profits) are given in the right-hand side of Figure 10 with the exact numbers in Table 3. With fully flexible wages (panel A) there is an overall loss of about 1.3 percent of GDP. Large firms and
Wage adjustment

1. Degree of flexibility, \( a \), scale parameter \( 0.8, 0.8, 0.8, 0.8 \)

3. Percentage of firms paying tax

2. Percentage of firms avoiding the regulation, \( \delta \)

1. Unemployment rate

4. Change in labor costs (wage reduction) for small firms below 49

5. Change in labor costs (wage reduction but tax increase), firms above 49

6. Excess entry by small firms (percent increase in number of firms)

7. Increase in size of small firms

8. Increase in size of large firms

9. Annual welfare loss (as a percentage of GDP)
   a. Implicit tax
   b. Output loss
   c. Total (Implicit tax + Output loss)

10. Winners and losers
   a. Change in expected wage for those who remain in labor force
   b. Average gain by entering entrepreneurs of small firms
   c. Average profit gain by small unconstrained firms
   d. Average profit gain by firms constrained at 49
   e. Change in profit for large firms

Notes: Based on the baseline estimates in column 1 of Table 1 under the assumption that the maximum firm size is 10,000. Column 1 assumes real wages fully adjust. Columns 2 to 4 assume partial adjustment. Percentage firms avoiding the regulation (row 2) is mass who move to the spike at 49. Percentage firms paying tax (row 3) is mass of agents with productivity greater than \( \alpha_u \) relative to agents with productivity greater than \( \alpha_{\text{max}} \). Change in labor costs for small firms (row 4) is equilibrium wage effect. Change in labor costs for large firms (row 5) is row 4 plus the estimated implicit tax (7−1). Excess entry (row 6) is the difference in ln/mass of agents having productivity greater than \( \alpha_{\text{min}} \) minus ln/mass of agents having productivity greater than \( \alpha_{\text{max}} \), where \( \alpha_{\text{min}} \) is the threshold between workers and entrepreneurs in the counterfactual economy without regulation. Increase in size of small firms (row 7) corresponds to \( \ln(n^*) - \ln(n_0) \) for firms having productivity smaller than \( \alpha_c \). Increase in size of large firms (row 8) corresponds to \( n^* - n_0^* \) for firms having productivity greater than \( \alpha_c \). Implicit tax (row 9a) corresponds to the total amount of implicit tax \( \int_{\alpha_c}^{\alpha_{\text{min}}} \left[ (1 - \tau) \cdot w^* \cdot n^*(\alpha) + k|\sigma(\alpha)| d\alpha \right] \) as a share of total output, \( Y^* \). Output loss (row 9b) corresponds to \( \ln(Y^*) - \ln(Y_0^*) \). For winners and losers (row 10), we compute the average (percentage point) changes in expected wages or profits for agents in each of the following bins: 10a labor force \([\alpha; \alpha_{\text{min}}] \); 10b new entrepreneurs \([\alpha_{\text{min}}; \alpha_{\text{max}}] \); 10c small firms \([\alpha_{\text{min}}; \alpha_c] \); 10d. constrained firms \([\alpha_c; \alpha_{\text{max}}] \); 10e. large firms \([\alpha_c; \alpha_{\text{max}}] \). In the partially rigid wages case in columns 2 to 4, expected wages are computed as \( (1 - u_{\text{ID}}^*) \cdot w^*_k \) where \( u \) denotes the unemployment rate.

Table 3—Welfare and Distributional Analysis

<table>
<thead>
<tr>
<th>Description of benchmark:</th>
<th>Perfectly flexible real wages (1)</th>
<th>Difference in US/French structural unemployment (2)</th>
<th>Diffusion of min. wage indexation (3)</th>
<th>Standard midterm assumption adjustment (4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wage adjustment (degree of flexibility, ( a ))</td>
<td>1.0 0.7 0.67 0.62</td>
<td>0.8 0.8 0.8 0.8</td>
<td>1.0 0.7 0.67 0.62</td>
<td>1.0 0.7 0.67 0.62</td>
</tr>
<tr>
<td>Unemployment rate</td>
<td>0 2.3 2.5 2.6</td>
<td>2.3 2.5 2.6 2.7</td>
<td>2.3 2.5 2.6 2.7</td>
<td>2.3 2.5 2.6 2.7</td>
</tr>
<tr>
<td>Change in labor costs (wage reduction) for small firms (below 49)</td>
<td>–1.792 –1.258 –1.120 –1.109</td>
<td>0.502 1.036 1.095 1.185</td>
<td>0.502 1.036 1.095 1.185</td>
<td>0.502 1.036 1.095 1.185</td>
</tr>
<tr>
<td>Excess entry by small firms (percent increase in number of firms)</td>
<td>8.958 6.292 5.996 5.548</td>
<td>2.667 0.539 0.303 0.055</td>
<td>2.667 0.539 0.303 0.055</td>
<td>2.667 0.539 0.303 0.055</td>
</tr>
<tr>
<td>Increase in size of large firms</td>
<td>8.958 6.292 5.996 5.548</td>
<td>2.667 0.539 0.303 0.055</td>
<td>2.667 0.539 0.303 0.055</td>
<td>2.667 0.539 0.303 0.055</td>
</tr>
<tr>
<td>Annual welfare loss (as a percentage of GDP)</td>
<td>1.304 1.302 1.302 1.302</td>
<td>0.022 2.148 2.384 2.744</td>
<td>0.022 2.148 2.384 2.744</td>
<td>0.022 2.148 2.384 2.744</td>
</tr>
<tr>
<td>Implicit tax</td>
<td>1.304 1.302 1.302 1.302</td>
<td>0.022 2.148 2.384 2.744</td>
<td>0.022 2.148 2.384 2.744</td>
<td>0.022 2.148 2.384 2.744</td>
</tr>
<tr>
<td>Output loss</td>
<td>1.304 1.302 1.302 1.302</td>
<td>0.022 2.148 2.384 2.744</td>
<td>0.022 2.148 2.384 2.744</td>
<td>0.022 2.148 2.384 2.744</td>
</tr>
<tr>
<td>Total (Implicit tax + Output loss)</td>
<td>1.304 1.302 1.302 1.302</td>
<td>0.022 2.148 2.384 2.744</td>
<td>0.022 2.148 2.384 2.744</td>
<td>0.022 2.148 2.384 2.744</td>
</tr>
</tbody>
</table>

Workers are losers whereas smaller firms are the winners. Labor costs rise by 0.5 percent for large firms and their size is reduced by 2.5 percent (rows 5 and 8 of Table 3). Small firms’ costs fall by 1.8 percent and these firms are 9.0 percent larger (rows 4 and 7 of Table 3). The pure deadweight output loss is quite small (0.02 percent of GDP, row 9b) as the wage adjustment is able to undo most of the cost, but the overall welfare loss depends on how one regards the implicit tax revenues, which are 1.3 percent of GDP (row 9a). These are not actual revenue streams to the government but rather are the costs of reporting and enforcing the regulations.30

30The modest magnitude of the welfare cost in the case of flexible wages is perhaps unsurprising as the regulation does not cause the rank order of firm size to change with the ability distribution. Hopenhayn (2014) shows in a general context that first-order welfare losses from misallocation require some rank reversals between ability and size.
that firms bear—diverted employee time, the costs of employing union officials, in-house lawyers, etc. One view is that these also simply represent waste so the welfare cost are in the total sum in 9c (1.326 percent). An alternative view is that these amenities are valued by workers. But if this was the case we would expect to see workers over the 50 employee threshold receiving lower wages as a compensating differential for these amenities (e.g., Lazear 1990). We can examine this by looking at wages around the regulatory threshold (see Figure 11). As expected the wage is upward sloping in size, but there does not appear to be a fall in wages after the regulatory threshold, which suggests that workers do not place much benefit on the extra regulations. A caveat to this is that there may be positive externalities from the regulations, for example if the health and safety committee reduces accidents and so saves public healthcare costs. On the other hand, there may be negative externalities, for example through more industrial disputes.

Returning to Figure 10, it is clear that welfare losses are larger if real wages are more inflexible. In panel B welfare falls by 3.45 percent in our baseline scenario where wages adjust by 70 percent. Since equilibrium wages are not falling as much as in the fully flexible case, larger firms lose from not being able to offset the regulatory cost and low ability agents are losing because there are higher levels of unemployment.31 Small firms increase their size by about 6 percent, but large firms

31 In the basic model, workers are homogeneous so they make an ex ante decision to enter based on their expected wage (relative to their expected profits from being an entrepreneur) and there is a random draw ex post to determine who will be unemployed. In our risk-neutral setup, low ability individuals lose out to a similar degree regardless of the degree of wage inflexibility. We consider heterogeneous production skill among employees in one of the extensions below.
drop in size by 5 percent. The welfare losses increase in the degree of downward real wage rigidity. Looking across columns 3 and 4 of Table 3, welfare losses increase (to 3.7 percent and 4.0 percent) as unemployment rises.

Table 4 examines the robustness of the welfare results to returns to scale. We use a much lower value of $\theta = 0.5$ (as in column 3 Table 1 and Hsieh and Klenow 2009) and repeat the analysis. Welfare losses rise in the perfectly flexible wage case of column 1 to 1.93 percent (from 1.33 percent) but fall a bit to 3.29 percent (from 3.45 percent) in our baseline case of column 2 that has a degree of partial wage rigidity calibrated to US-French structural wage differences. The robustness of our baseline welfare results to a large change in the scale parameter is reassuring.

In summary, we have three main quantitative results. First, aggregate welfare losses from the regulation are around 3.5 percent of GDP with employment losses of 140,000 for realistic levels of partially inflexible real wages. Second, welfare and job losses are smaller as wages become more flexible. Third, the regulation

\[32\] Since the model parameters are changed by using a lower returns to scale parameter ($\theta = 0.5$ instead of 0.8) the calibrated wage inflexibility parameter that is consistent with this model also changes a bit (from $a = 0.70$ to $a = 0.68$).

\[33\] The implied elasticity of labor demand is 2 in Table 4. This is consistent with French evidence in Kramarz and Philippin (2001) and Abowd et al. (2009), who found elasticities of about 2 for men and 1.5 for women. These are more reasonable than the higher implied elasticity (of 5) in Table 3. We note, however, that a recent study by Cahuc, Carcillo, and Le Barbanchon (2016) on France does find elasticities in that range (4 to be precise).
redistributes income away from workers and larger firms and toward small firms (i.e., those with mediocre managerial ability) across all scenarios.

V. Extensions and Robustness

In this section we consider several extensions to our framework and robustness tests of the results. We allow the parameters to be different by industry (Section VA); examine whether firms respond by splitting up their operation (Section VB) and/or substituting into other factors (Section VD); allowing for another threshold (Section VC) and in Section VE comparing our results to Hsieh and Klenow (2009).

A. Industry Heterogeneity

We examine the other main sectors outside manufacturing in our baseline case where $\theta = 0.80$ see (online Appendix Table A2). We first estimated the model separately for each of four other sectors: transport, construction, wholesale/distribution, and business services industries. The tax $\tau$ is significant in all sectors, highest in Transport (3.5 percent) and smallest in Business Services (0.8 percent). We also estimated production functions separately by three-digit sectors and implemented column 6 of Table 1 allowing the scale ($\theta$) and all other parameters to be freely estimated. There is substantial degree of heterogeneity with some sectors having estimates of the regulatory tax from near zero to over 3 percent and this is related to industry characteristics in an intuitive way. For example, when labor costs are a smaller share of total value added, the estimated regulatory taxes tend to be bigger and when the capital-labor ratio of the sector is high they tend to be smaller. The distortion associated with the regulation is less damaging in sectors when labor is a less important factor.

B. Compliance, Misreporting, and Changing Corporate Structure

A business group could respond to the regulation by outright misreporting employee numbers. The authorities and unions are well aware of this incentive and threaten hefty fines and prison sentences for employers who lie to the fiscal or social security authorities. In some countries there is certainly evidence of cheating from tax returns (e.g., Almunia and Lopez-Rodriguez 2015), although generally researchers are surprised at how low are these rates of noncompliance given the incentives (e.g., Kleven, Kreiner, and Saez 2011). We doubt these evasion strategies can be a major factor behind our results. First, we show below that there is some adjustment on a number of other margins that is consistent with a real effect of the regulation such as investment and hours of work (see below). There is no incentive for the firm to report these other items in a systematically

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34 Some of the industries have insufficient number of firms to perform this estimation but we are still able to do this for a large number of sectors. The full results are available on request and are plotted in online Appendix Figure A3.

35 Note also that the standard, economy wide regulations described in online Appendix F interact with industry-specific regulations and agreements, which are sometimes also size dependent. This generates some industry-level variation in the strength of regulations which is reflected in our estimates.
misleading way. Second, hiding taxable revenues is much easier than hiding workers who have a very physical presence with a legal contract, health, and pension rights. Third, in 2009 alone the French state employed 2,190 *agents de contrôle* to monitor firms compliance. That is about 1 agent per 100 firms with 10 or more employees, a rather high degree of observation. Fourth, hidden workers could be considered an additional factor of production. As we discussed earlier, our model performs reasonably well in predicting output even abstracting away from such considerations, and Section VD below shows more formally that our results are robust to this extension.

A more subtle way of avoiding the regulation for business is by splitting a company into smaller subsidiaries. For example, a firm which wished to grow to 50 employees could split itself into two 25-employee firms controlled by the group owner. There are costs to such a strategy: the firm must then file separate fiscal and legal accounts, demonstrate that the affiliates are operating autonomously, and suffer from greater problems of loss of control. One way to check for this issue is to split the sample into those firms that are stand-alone businesses and those that are subsidiaries/affiliates of larger groups. Panel A of online Appendix Figure A4 compares the power law for these two types of firms. For both stand-alone firms and affiliates we can observe the broken power law at 50. The fact that the discontinuity exists for stand-alone firms implies that our results are not being driven solely by corporate restructuring.

### C. A Model with Two Thresholds: Additional Threshold at 10 Employees

We have focused on the most obvious of the regulatory threshold at size 50, but there is another important threshold at 10 employees where employers must pay monthly (rather than quarterly) social security obligations, transport aid, and a higher training tax. Online Appendix Table A3 reports results for this analysis for manufacturing and the other main sectors. The new threshold is estimated to involve near zero variable regulatory cost in all cases (the highest is in transport which is only 0.5 percent compared to 3 percent at 50). The fixed cost at 10 is most often insignificant, except in the construction and trade industries, where the point estimate is positive but small (around 2 percent of the cost of one worker). Importantly, adding this new threshold does not impact materially our estimates of the regulatory costs at 50 which remains at about 2 percent in most sectors.

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37 A more extreme reaction of the firm would be to engage in franchising. This has some further costs as the CEO no longer has claims over the residual profits of the franchisee and loses much control. In any case, franchising is rare in manufacturing.
38 Panel B repeats the distribution for the stand-alones, but now considers affiliates aggregated up to the group level. Although the power law is still broken at 50, it is less pronounced than at the subsidiary level in panel A which could indicate some degree of corporate restructuring in response to the regulation. One might ask why there should be any discontinuity at 50 employees at the group level when aggregating across multiple subsidiaries? The reason is that a small number of regulations do bind at the group level, mainly through case law.
39 Another way to look at this issue is to examine patterns of entry and exit. If firms where splitting up we would expect to see a mass of new entry around the bump of 45 to 49 employees. Online Appendix Figure A5 shows that this is not the case: entrants tend to be small (panel C) and the fraction of entrants in each size class is rather uniform for firms between 35 and 75 employees (panel A). There is somewhat of an exit spike at 50 employees (panel B), but this is to be expected if some firms have mistakenly crossed the threshold and been hit by a large regulatory cost shock.
including manufacturing. Online Appendix Table A4 uses these two threshold estimates to redo our welfare and distributional analysis. Reassuringly, the welfare costs are concentrated on the 50 employee threshold, and both the flexible and partially rigid wages cases are virtually the same as in the main analysis with only one threshold. Our analysis thus suggests that the costs introduced after size 50 are the important regulatory costs.

D. Allowing for Capital-Labor Substitution and Other Margins of Adjustment

Firms may react to the regulation by substituting labor for capital or other inputs. Figure 12 shows that investment per worker increases on average with firm size, but spikes up (like employment) around the threshold of 50 employees, consistent with capital-labor substitution. To analyze the robustness of our baseline estimation to such substitutability, we add capital as an input into a CES production function (see online Appendix D). A manager who has ability $\alpha$ and is allocated $n$ workers and $k$ units of capital produces $\alpha f(n, k) = \alpha (\lambda n^\rho + (1 - \lambda) k^{\rho})^{\theta}$ with $\theta \in (0, 1)$ and $\rho \in \mathbb{R}$. The production function is increasing and strictly concave in $n$ and $k$. As before, let $\pi(\alpha)$ be the profits obtained by a manager with ability $\alpha$ when she manages a firm at the optimal size. These profits are given by

$$
\pi(\alpha) = \max_{n,k} \begin{cases} 
\alpha f(n, k) - wn - rk & \text{if } n \leq N \\
\alpha f(n, k) - wt n - rk - F & \text{if } n > N
\end{cases},
$$
where \( w \) is the worker’s wage, \( n \) is the number of workers, \( r \) is the marginal cost of capital, and \( k \) is the number of units of capital. Firm size at each side of the threshold (but not at the threshold) is then determined by the first-order condition:

\[
\begin{align*}
\text{(16)} & \hspace{1cm} \alpha \theta (\lambda n^\rho + (1 - \lambda) k^\rho)^\theta - 1 \lambda n^{\rho - 1} - w \tau = 0 \quad \text{with} \quad \begin{cases} \\
\frac{\tau}{\bar{\tau}} = 1 & \text{if } n < N \\
\frac{\tau}{\bar{\tau}} = \tau & \text{if } n > N
\end{cases} \\
\text{(17)} & \hspace{1cm} \alpha \theta (\lambda n^\rho + (1 - \lambda) k^\rho)^\theta - 1 (1 - \lambda) k^{\rho - 1} - r = 0.
\end{align*}
\]

Dividing equation (16) by equation (17), we obtain the labor-capital ratio:

\[
\text{(18)} \quad \frac{n}{k} = \left[ \frac{r \lambda}{w \tau (1 - \lambda)} \right]^\eta = \gamma^\eta \quad \text{with} \quad \begin{cases} \\
\frac{\tau}{\bar{\tau}} = 1 & \text{if } n < N \\
\frac{\tau}{\bar{\tau}} = \tau & \text{if } n > N
\end{cases}
\]

where \( \eta = \frac{1}{1 - \rho} \) is the elasticity of substitution.

We reestimate the extended model as before using ML (as in Table 1 column 1). Figure 13 reports the sensitivity of the estimated variable costs of regulation to capital-labor substitution. As the elasticity of substitution increases, the estimated variable tax \( \tau - 1 \) decreases from the baseline no substitution case \((\eta = 0)\) of around 3.1 percent to about 2.2 percent in our highest substitution case (an
unrealistically high $\eta = 6$). Thus, it is stable to reasonable assumptions over substitutability.

Given these parameter estimates, we can calculate in Figure 14 the sensitivity of our welfare losses to the elasticity of substitution. In panel A for fully flexible wages, welfare losses for no substitutability ($\eta = 0$) are essentially the same (1.3 percent) as our baseline in column 1 of Table 3. As expected, greater degrees of substitutability reduce the welfare loss, but not tremendously (about 1.1 percent at $\eta = 3$ for example). Panel B presents partial wage inflexibility as calibrated to the French/US unemployment difference (column 2 of Table 3). Again, welfare changes are quite stable across a wide range of values of substitutability. The level of welfare loss of 2.5 percent is somewhat lower than in the baseline of 3.4 percent. This reflects the ability of employers to avoid reducing their output so much in the face of higher labor costs by substituting into capital.

Although welfare losses are somewhat smaller than in the baseline case, the ability of firms to substitute labor for capital does not warrant a major change in our conclusions. The main message remains that the extent to which the wage adjustment can transfer the cost of the regulation to the workers determines the cost of the regulation. If wages were fully flexible, the welfare losses would be about one-half of the level they are given the current level of French wage rigidity.

Apart from capital, the most obvious other way firms could adjust is by increasing hours per worker (e.g., through longer overtime) rather than expanding the number

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**Figure 14. Allowing for Capital/Labor Substitutability (CES): How Welfare Losses Change with Assumptions over Substitutability**

Notes: Parameters estimated by ML. The figure shows how welfare changes when we alter the assumptions over the elasticity of substitution (ranging between 0 and 6). Panel A looks at the case of fully flexible wages and panel B looks at the case of partially rigid wages, where the adjustment parameter $a$ is calibrated to match the difference in the structural employment rates between France and the United States ($u = 2.622$ percentage points). The price of capital is assumed to be completely flexible in both panels.

---

40 See online Appendix D for details of how our baseline model and CES model differ. This is true even at $\eta = 0$ (Leontief), which is why the estimates of $\tau$ at this boundary case are not identical. Most empirical estimates have $\eta \leq 1$ (Hamermesh 1996). More recently, Karabarbounis and Neiman (2013) argue for a higher $\eta$, using $\eta = 1.3$ as their preferred estimate.

41 See online Appendix D.7 for a comparison of the CES, two input case with the single input specification.

42 Note that the implied value for of $a$ is different for every value of $\eta$ in Figure 14 to be consistent with (calibrated) structural French/US unemployment difference.
of employees. The number of annual hours does increase just before the threshold of 50 employees (online Appendix Figure A6), from the expected around 1,650 to 1,710, that is by around 3.6 percent. Firms are limited in their ability to increase hours not just because of imperfect substitutability between overtime and regular hours but also because of France’s 35 maximum hours per week regulation. There also is evidence for adjustment along other margins such as using more intermediate inputs to mitigate the costs of the regulation. Allowing for these other margins does not substantially affect our main findings, as is also suggested by the accuracy of our output predictions in Table 3.

E. A Comparison with the Hsieh-Klenow (2009) Approach

Hsieh and Klenow (2009) take a related approach to examining regulatory distortions by examining the distributions of size and revenue-based productivity measures in the United States, India, and China. Like us, they found that such distortions could have substantial macroeconomic effects. Their approach, however, focuses on the variation in marginal revenue productivity (MRP) as an indicator for distortions because, when factor prices are the same, MRPs should be equalized across firms even when underlying managerial ability is heterogeneous. In our context, we can follow Hsieh and Klenow (2009) and estimate the distortion (τ) from the change in the MRP of labor. The relation can be seen in online Appendix Figure A7. Naïvely using data around the threshold to look at the change in MRP would be incorrect, however, as this local distortion in the MRP is due to the decision of firms in the 50–58 range to optimally choose to be at 49.\footnote{Note that the treatment effect in the regression discontinuity design would reflect the local distortion and not the global distortion and would lead to misleading estimates of the implicit tax. Instead, our approach would suggest comparing the MRP of labor for firms away from the threshold.} We rather implement the Hsieh-Klenow method of value added per worker (relative to the industry average) as an index of the MRP of labor for firms having 20 to 42 or 60 to 150 employees and obtain an estimate of the tax of 2.86 percent (panel A of online Appendix Figure A8).\footnote{This is using the average for firms between 20 to 42 workers compared to firms with 57 to 200 workers. Reasonable changes of the exact thresholds make little difference. Another approach is to use our own estimates of the MRP of labor from the model \(\alpha \theta (n^*)^{\theta - 1} \). When doing this we obtain a value of the implicit tax of 3.3 percent (Panel B of online Appendix Figure A8).} Although we prefer our more structural approach, we note that these estimates are similar to our own estimate of the implicit tax of 2.3 percent.

F. A Dynamic Extension to the Model

Our baseline model as well as all extensions presented in the remainder of Section V are essentially static. In this subsection, we consider a dynamic extension of our framework to investigate whether the existence of adjustment costs (rather than simply measurement error) can explain the positive mass of firms to the right of the threshold. The dynamic approach will also be helpful in interpreting some of the dynamic features of the data such as growth rates around the regulatory threshold.
This experiment however comes at the cost of increased complexity: in particular, we only investigate here the partial equilibrium properties of the dynamic framework.\footnote{Results in this section are based on numerical simulations of the steady-state distributions of firm size and growth in this new framework (e.g., Judd 1998). Factor prices are treated as exogenous, common to all firms and time invariant.}

The full details of our dynamic model are in online Appendix E, the main features are as follows. Upon entry firms draw their management ability (TFP) from a known distribution $\phi(\alpha)$ that is distributed as a power law. As in, e.g., Melitz (2003), there is an endogenous profit threshold such that some firms will immediately exit after observing their ability as they cannot cover their fixed costs. The dynamic nature of the model is due to the fact that TFP is allowed to change over time in this setting according to an AR(1) process: after a firm is founded it is subject each period to a shock which will usually cause it to want to change size. This is the only source of stochastic variation shifting the environment. We allow for quadratic adjustment costs in labor (and capital), such that a firm will not always adjust immediately to its long-run frictionless optimal size.\footnote{An alternative way to dynamically model the regulatory tax would be to consider it as a purely sunk cost (or sunk cost plus variable cost) as in Gourio and Roys (2014). This has similar predictions on the firm size distribution and simplifies the dynamics compared to the model presented here (as there are no convex adjustment costs). However, the raw data on employment dynamics do not seem compatible with this approach. Panel A of online Appendix Figure A1 shows how employment adjusts around the threshold at 49 for all firms and then the subset of firms who had (in the past five years) already had 50 or more employees (panel B). If the regulatory cost was sunk the latter group, who presumably have already paid the sunk cost, should not bunch much around 49 employees. However, whether we look at firms who make no change in employment in panel A, we observe a big spike at 49 employees for both groups. This does not seem consistent with a simple sunk cost story.}

Free entry implies that agents choose to become entrepreneurs and enter until the expected value of entry is equal to the sunk entry costs. Last, as in the baseline static model, we assume that in the regulated economy there is a variable cost ($\tau$) and fixed cost ($F$) that has to be paid when firms cross the discrete employment threshold at 50 employees.\footnote{We complement this set of assumptions with an exogenous death rate (in addition to the endogenous exit decision) that is modeled as an i.i.d. shock that causes some (small) margin of firms to go out of business. This is not a necessary feature of the model as the TFP process is AR(1) rather than random walk, hence even a firm with a very high TFP draw will tend to revert to the average over the long run.}

The steady-state equilibrium of this model ensures that the expected cost of entry equals the expected value of entry given optimal factor demand decisions. This equilibrium is characterized by a distribution of firms in terms of their state values $\alpha, k, n$ (plus their optimized choice of materials $m$). We simulated the model numerically for 20,000 firms over 100 periods (years), and used the last 25 years to describe the obtained steady state in online Appendix Figures A11 to A14. All structural parameters are either set to their estimated values or calibrated to values that have been proposed in the literature (online Appendix Table A6), while we provide comparative statics for different values of the adjustment costs and of the regulatory tax parameters.

This exercise shows essentially that the intuitions underlying the static model and key empirical features of the data are preserved in such a setting. The regulatory tax distorts the steady state firm size distribution and generates a bulge just to the left of the regulatory threshold, a valley to the right of the threshold, and then a continuation of the power law of the firm size distribution, which are increasing in the magnitude of the regulatory tax. The important difference with the static approach
is that firms in the valley are fully optimizing, and not just affected by measurement error. To see this assume that current managerial ability (or productivity) would put an incumbent firm to the left of the regulatory threshold and consider what would happen if such a firm received a large positive productivity shock. The firm would like to move to a larger size and earn more profits by hiring more workers. However, since there are convex adjustment costs, spending a period in the valley that is suboptimal from a static point of view may be optimal from a dynamic point of view. Large jumps in employment are extremely costly because of convex adjustment costs and so landing for a period or two in the valley may be less costly than paying these large adjustment costs to make a large immediate expansion. When adjustment costs increase, these considerations become more important and the mass of firms in the valley rises.

To conclude, we see this dynamic extension as an important exercise, since we believe in reality a mixture of adjustment costs and measurement error helps explain the valley (i.e., the existence of a positive mass of firms in the dominated area to the right of the regulatory threshold). Importantly, we checked it does not delivery fundamentally new insights compared to the static model, but leave its full estimation and thorough welfare analysis for future research. At this stage, we believe the considerably simpler approach of our basic model has much to recommend it.

VI. Conclusions

The costliness of labor market regulation is a long-debated subject in policy circles and economics. We have tried to shed light on this issue by introducing a structural methodology that uses a simple theoretical general equilibrium approach based on the Lucas (1978) model of the firm size and productivity distribution. We introduce size-specific regulations into this model, exploiting the fact that in most countries labor regulation only bites when firms cross specific size thresholds. We show how such a model generates predictions over the equilibrium size and productivity distribution and moreover, can be used to generate an estimate of the implicit regulatory tax of the regulation. Intuitively, firms will optimally choose to remain small to avoid the regulation, so the size distribution becomes distorted with too many firms just below the size threshold and too few firms just above it. We show how the regulation creates welfare losses by (i) allocating too little employment to more productive firms who choose to be just below the regulatory threshold; (ii) allocating too little employment to more productive firms who bear the implicit labor tax (whereas small firms do not); and (iii) through reducing equilibrium wages (due to some tax incidence falling on workers) which encourages too many agents with low managerial ability to become small entrepreneurs rather than working as employees for more productive entrepreneurs.

We implement this model for France where many labor laws bite when a firm has 50 employees or more. We find that the qualitative predictions of the model fit very well. First, there is a sharp fall off in the firm size distribution precisely at 50 employees, resembling a broken power law. Second, there is a bulge in productivity just to

48 The loss of resources generated by adjustment costs might have a nontrivial impact on total welfare.
the left of the size threshold. Third, there is a shift downward in the power law consistent with an increase in variable labor costs after passing the regulatory threshold.

We then estimate the key parameters of the theoretical model from the firm size distribution. Our approach delivers quite a stable and robust cost of the employment regulation which seems to place an additional cost on labor of about 2.3 percent of the wage. We show that we expect this cost to translate into relatively small output losses when real wages are flexible (about 1.3 percent of GDP) but large losses of 3.4 percent of GDP in the more realistic case when real wages are (partially) downwardly rigid. Furthermore, there are large distributional effects regardless of wage flexibility with workers losing substantively and small firms benefiting from the regulation. This is unlikely to be an intended consequence of the laws. Our welfare calculations are subject to the caveat that there may be some other externalities to society from the regulations—e.g., works councils may have wider benefits than any individual worker would take into consideration.

This is just the start of our research program of opening up the black box of firm distortions used in many macro models. Size-contingent regulations are ubiquitous and our methodology can be used for other regulations, other parts of the size distribution, other industries and other countries. We showed in the paper that all the intuitions of our baseline static model are preserved in a dynamic extension allowing for shocks to productivity (or managerial ability) and adjustment costs in terms of labor or capital. Estimating the parameters of such a dynamic model is however left for future research at this stage. Going further we could allow firms to invest in order to improve their managerial capabilities. Such investments enable small firms to grow and since size-contingent regulations tax this growth over the threshold, they may well discourage investment and therefore inhibit the dynamics of growth in the economy. On the other hand, the regulations could conceivably encourage firms to “strike for the fence” and go for radical innovations rather than incremental innovations which would put them only marginally to the right of the threshold.

Finally, we have assumed for simplicity that workers are homogeneous. One can extend our baseline model to allow for heterogeneous workers in the framework of Garicano and Rossi-Hansberg (2006). In this model managers’ ability allows them to solve those problems that workers could not solve. More skilled production workers endogenously match with more talented managers, because managers can leverage their ability over a larger mass of output in this case (as in our basic model, span of control costs are in the number of workers managed, not the skill of these workers). In this setup the regulation introduces an additional matching friction which reduces aggregate output. As before, aggregate real wages will fall, and if there is a minimum wage unskilled workers will tend to become unemployed. The qualitative implications of this extension are therefore the same as our baseline model. Structurally estimating this model is several orders of magnitude more difficult however due to the matching model of skill that would be introduced. We leave consideration of such a model for future work (see Lise, Meghir, and Robin 2016 for an attempt to structurally model heterogeneous firms and workers in a matching model).

49 For example, the retail sector has a large number of size-contingent regulations with big boxes being actively discouraged in many countries and US cities (e.g., Bertrand and Kramarz 2002, or Baily and Solow 2001).
Despite these caveats, we believe that our approach is a simple, powerful, and potentially fruitful way to tackle the vexed problem of the impact of regulation on modern economies.

REFERENCES


