INTERMEDIATION AND RESALE IN NETWORKS

ONLINE APPENDIX

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Computations for Section 3

To illustrate MPE outcomes for discount factors δ away from 1, consider a bargaining game with $p = 1/2$ in which a seller (player 0) with zero cost bargains with two buyers (players 1 and 2) whose values are $v_1 = 1$ and $v_2 = 0.9$. Using the characterization of MPEs from the proof of Proposition 1, we find that for $\delta > 1/5$, the seller trades with buyer 1 with probability

$$\pi_1 = \frac{\sqrt{404 - 784\delta + 381\delta^2} - 20(1-\delta)}{\delta}$$

and with buyer 2 with probability $1 - \pi_1$. In this case, the payoffs of players $0, 1,$ and 2 are given, in order, by

$$u_0 = \frac{58 - 39\delta - \sqrt{404 - 784\delta + 381\delta^2}}{20(4-3\delta)} \tag{1}$$

$$u_1 = \frac{76\delta - 36 - 39\delta^2 + (2-\delta)\sqrt{404 - 784\delta + 381\delta^2}}{20\delta(4-3\delta)} \tag{2}$$

$$u_2 = \frac{82\delta - 44 - 39\delta^2 + (2-\delta)\sqrt{404 - 784\delta + 381\delta^2}}{20\delta(4-3\delta)}.$$

For $\delta \leq 1/5$, the seller trades exclusively with buyer 1 ($\pi_1 = 1$), which generates the payoffs $u_0 = u_1 = 1/2$ and $u_2 = 0$. The two graphs in Figure 1 plot the probability π_1 and the payoffs (u_0, u_1, u_2) as functions of δ.

Computations for Section 4

Consider the network from Figure 2 formed by the seller (player 0), a single intermediary (player 1), and two buyers (players 2 and 3). Assume that $c_0 = c_1 = 0, v_2 = 1, v_3 = 0.9$, and $p = 1/2$. Let π_1 denote the probability with which the seller trades with player 1. MPE outcomes depend on a discount factor threshold $\underline{\delta} \approx 0.824$.[1] For $\delta \leq \underline{\delta}$, the seller trades exclusively with buyer 2 in the MPE, which generates the equilibrium variables $\pi_1 = 0, u_0^0 = u_2^0 = 1/2, u_1^0 = 0$.

Date: October 9, 2016.

[1]The threshold $\underline{\delta}$ is the middle root of the cubic polynomial $-80 + 256\delta - 253\delta^2 + 73\delta^3$. Its precise formula involves cubic roots of unity (or trigonometric functions).

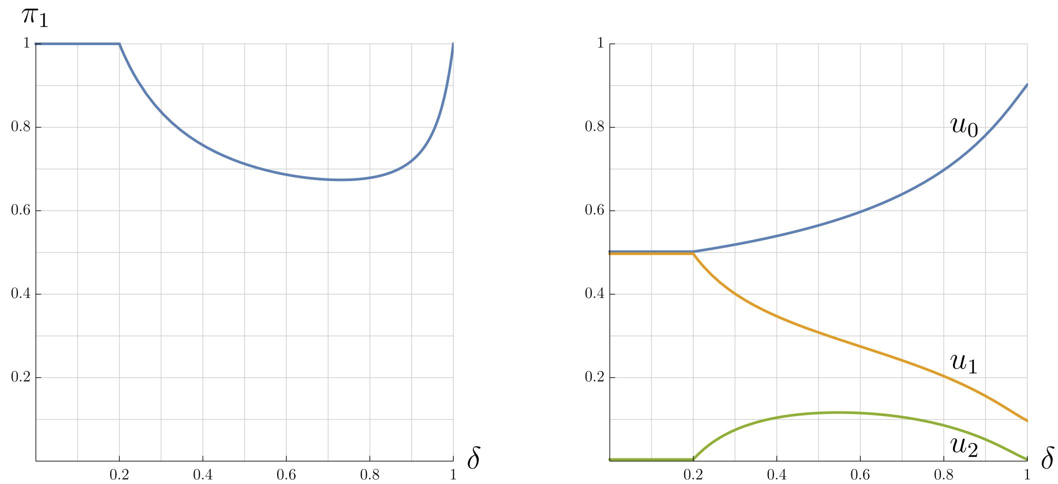

FIGURE 1. Probability of trade with buyer 1 (left graph) and payoffs in the MPE

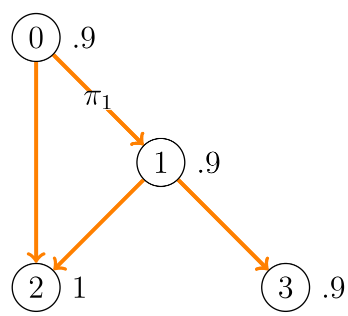

FIGURE 2. The seller trades with positive limit probability with the lowest resale value neighbor, intermediary 1.

If $\delta > \underline{\delta}$, then the initial seller trades with both intermediary 1 and buyer 2 with positive probability in the MPE. In this case, the probability π_1 and the MPE payoffs solve the equations

$$\begin{aligned}
u_0^0 &= \frac{1}{2}(\delta u_1^1 - \delta u_1^0) + \frac{1}{2}\delta u_0^0 = \frac{1}{2}(1 - \delta u_2^0) + \frac{1}{2}\delta u_0^0 \\
u_1^0 &= \pi_1 \left(\frac{1}{2}\delta u_1^0 + \frac{1}{2}(\delta u_1^1 - \delta u_0^0) \right) \\
u_2^0 &= \pi_1 \delta u_2^1 + (1 - \pi_1) \left(\frac{1}{2}\delta u_2^0 + \frac{1}{2}(1 - \delta u_0^0) \right),
\end{aligned}$$

where the continuation payoffs u_1^1 and u_2^1 of players 1 and 2 in the event that the seller trades with player 1 are derived from the bargaining game without intermediation in which player 1 acts as a seller for the two buyers. Hence u_1^1 and u_2^1 are given by the corresponding formulae (1) and (2) from the example in the previous section. We substitute these formulae

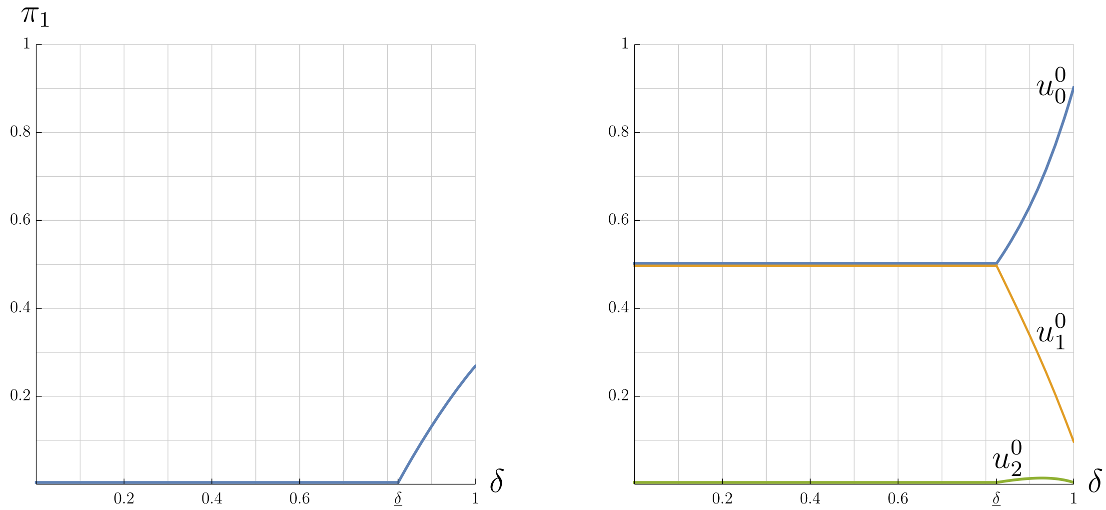

FIGURE 3. Probability of trade with intermediary 1 (left graph) and payoffs of players $0, 1$, and 2 in the MPE

and solve the system of equations above (with the help of Mathematica) to obtain

$$\pi_1 = \frac{1}{4\delta(37-19\delta)}\Big(90 - 19\delta^2 - \sqrt{404 - 784\delta + 381\delta^2} -$$
$$\sqrt{42840 - 80576\delta + 51673\delta^2 - 11704\delta^3 + 361\delta^4 + (412 - 600\delta + 190\delta^2)\sqrt{404 - 784\delta + 381\delta^2}}\Big).$$

Since the analytical solution for each of the payoffs u_0^0, u_1^0, and u_2^0 runs over several lines, we offer only a graphical representation. The two graphs in Figure 3 depict π_1 and (u_0^0, u_1^0, u_2^0) as functions of δ. The formula for π_1 implies that $\lim_{\delta\to 1}\pi_1 = (35 - \sqrt{649})/36 \approx 0.26$.

COMPUTATIONS FOR SECTION 6

We use backward induction to find MPEs for the intermediation game played on the network from Figure 4 with $p = 1/2$, zero intermediation costs, and $v_4 = 1$. Subgames 2 and 3 are standard two-player bargaining games. We can easily compute the payoffs, $u_2^2 = u_3^3 = 1/2$.

We next roll back to subgame 1. Since there are no intermediation chains between players 2 and 3, this subgame is strategically equivalent to the bargaining game without intermediaries in which player 1 may resell the good to either "buyer" 2 or 3, who have a common (discounted resale) "value" $\delta/2$. This game has a unique MPE by Proposition 1. By symmetry, player 1 trades with equal probability with intermediaries 2 and 3. Then payoffs in

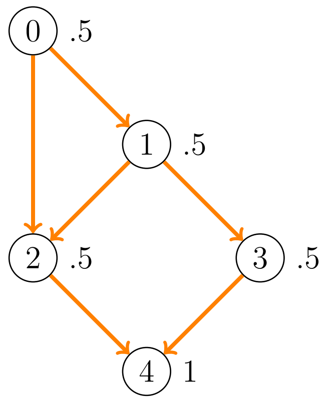

FIGURE 4. Asymmetric trading paths within layers

subgame 1 solve the following system of equations

$$\begin{aligned} u_1^1 &= \frac{1}{2}(\delta u_2^2 - \delta u_2^1) + \frac{1}{2}\delta u_1^1 \\ u_2^1 &= \frac{1}{2}\left(\frac{1}{2}\delta u_2^1 + \frac{1}{2}(\delta u_2^2 - \delta u_1^1)\right) \\ u_3^1 &= u_2^1. \end{aligned}$$

Substituting in $u_2^2 = 1/2$, we immediately find

$$\begin{aligned} u_1^1 &= \frac{\delta(2-\delta)}{8-6\delta} \\ u_2^1 &= \frac{\delta(1-\delta)}{8-6\delta}. \end{aligned}$$

Consider now the bargaining problem faced by the initial seller. An agreement with intermediary 1 generates a lateral intermediation rent of u_2^1 for player 2; this payoff is positive for $\delta \in (0, 1)$, as intermediary 1 resells the good with probability $1/2$ to player 2 in subgame 1. However, in the event of an agreement between the initial seller and intermediary 2, player 1's continuation payoff is 0 since he cannot purchase the good subsequently. Let π_1 denote the probability with which the initial seller selects intermediary 1 for bargaining in an MPE. Then the analysis of Section 4 leads to the following payoff equations

$$\begin{aligned} u_0^0 &= \frac{1}{2}\left(\pi_1(\delta u_1^1 - \delta u_1^0) + (1-\pi_1)(\delta u_2^2 - \delta u_2^0)\right) + \frac{1}{2}\delta u_0^0 \\ u_1^0 &= \pi_1\left(\frac{1}{2}\delta u_1^0 + \frac{1}{2}(\delta u_1^1 - \delta u_0^0)\right) \\ u_2^0 &= \pi_1 \delta u_2^1 + (1-\pi_1)\left(\frac{1}{2}\delta u_2^0 + \frac{1}{2}(\delta u_2^2 - \delta u_0^0)\right), \end{aligned}$$

where u_1^1, u_2^1, and u_2^2 have been computed previously.

For sufficiently high δ, it is impossible that $\pi_1 \in \{0,1\}$. For instance, $\pi_1 = 1$ implies that $u_0^0 = u_1^0 = \delta u_1^1/2$ and $u_2^0 = \delta u_2^1$. In particular, the MPE payoffs of players 1 and 2 converge to 1/4 and 0, respectively, as $\delta \to 1$. Then for high δ, we have $\delta u_2^2 - \delta u_2^0 > \delta u_1^1 - \delta u_1^0$, so the initial seller prefers to bargain with intermediary 2 instead of 1. A similar contradiction obtains assuming that $\pi_1 = 0$ for high δ.

Hence we need $\pi_1 \in (0,1)$ for high δ. Then the seller's indifference between the two intermediaries requires that $\delta u_1^1 - \delta u_1^0 = \delta u_2^2 - \delta u_2^0$. Appending this constraint to the system of equations displayed above, we find that for δ sufficiently close to 1 there is a unique solution,

$$
\begin{aligned}
u_0^0 &= \frac{\delta(12 - 13\delta + 2\delta^2 + \delta^3) - (1-\delta)\delta\sqrt{144 - 280\delta + 185\delta^2 - 42\delta^3 + \delta^4}}{4(4-3\delta)^2} \\
u_1^0 &= \frac{(2-\delta)(1-\delta)(-12 + 9\delta + \delta^2 + \sqrt{144 - 280\delta + 185\delta^2 - 42\delta^3 + \delta^4})}{4(4-3\delta)^2} \\
u_2^0 &= \frac{(1-\delta)\left(8 - 2\delta - \delta^2 - \delta^3 + (2-\delta)\sqrt{144 - 280\delta + 185\delta^2 - 42\delta^3 + \delta^4}\right)}{4(4-3\delta)^2} \\
\pi_1 &= \frac{4 + \delta - \delta^2 - \sqrt{144 - 280\delta + 185\delta^2 - 42\delta^3 + \delta^4}}{4\delta(2-\delta)}.
\end{aligned}
$$

In particular, $\lim_{\delta \to 1} \pi_1 = 1 - 1/\sqrt{2}$.

MPEs have the agreement structure and the payoffs described above if and only if $\delta > 4/5$. For $\delta \leq 4/5$, the seller trades exclusively with buyer 2 in the MPE and payoffs are given by $u_0^0 = u_2^0 = \delta/4$ and $u_1^0 = u_3^0 = 0$.

Proof of Proposition 9

Let G' denote the network resulting from the vertical integration of a pair of intermediaries $i < j$ linked in an arborescence G. Then G' is also an arborescence. All lateral intermediation rents in arborescences are 0, so each subgame in either G or G' is strategically equivalent to a bargaining game without intermediation and resale values solve the corresponding payoff equations from the proof of Proposition 1. In particular, all MPEs are outcome equivalent for every δ in either network. The statements in this proof apply to MPEs for every δ, and we omit this quantifier for the rest of the argument.

The proof of Proposition 1 implies that increasing a buyer's value or adding new buyers boosts seller profits. Moreover, increasing a buyer's value leads to higher probability of trade with that buyer. The arguments below apply these observations to several subgames.

Since i inherits j's downstream neighbors after the merger and at least one of these neighbors has a higher resale value than j (Lemma 1), while the resale values of i's other neighbors are identical in G and G', it must be that i's resale value increases following the merger. It then follows by induction that the resale value of every seller along the unique path from node 0 to i and the probability that each such seller acquires the good in equilibrium increase after the merger and that other resale values do not change. This implies that the profits of the initial seller and player i and the probability that i receives the good increase following the merger.

We are left to prove that consolidated intermediary profits increase following the vertical integration of i and j. Since the probability that i receives the good increases with the merger, we only have to show that the sum of payoffs of i and j in subgame i in network G does not exceed i's resale value in G'. If i does not trade with j in subgame i in network G, then j's payoff in subgame i is 0 and the previous claim is true because i's resale value increases after the merger. If i trades with j in subgame i with positive probability in G, then the incentive constraint for trade requires that the sum of payoffs of i and j in subgame i does not exceed j's resale value in G. Since i inherits all of j's downstream neighbors in G', j's resale value in G does not exceed i's resale value in G'. We established that the sum of payoffs of i and j in subgame i in G does not exceed i's resale value in G', as claimed.