Forward Guidance without Common Knowledge

George-Marios Angeletos*  Chen Lian**

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*MIT and NBER, **MIT
How does the economy respond to news about the future?
  - e.g., news about future monetary policy

Key mechanism:
  - forward-looking expectations (e.g., of inflation and income)
  - general-equilibrium effects (Keynesian multiplier, $\pi-y$ feedback)
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Standard: Rational Expectations with CK

This paper: Rational Expectations without CK
Main Insight

- Consider a class of “dynamic beauty contests”
  - aggregate outcome today depends on average forecast of aggregate outcome in the future
  - nests NK and more

- Removing CK reduces
  - responsiveness of forward-looking expectations
  - potency of GE effects
  - ability of PM to “manage expectations”

- Effects increase with horizon
  - without CK, it is as if extra discounting of the future

- By imposing CK, we have “biased” our predictions
Origins of lack of CK
- dispersed private information as in Morris-Shin etc
- “rational inattention” (Sims) or “costly contemplation” (Tirole)

Key mechanism
- HOB move less than FOB
- relative role of HOB increases with horizon
  iterating on model’s equations = ascending the hierarchy of beliefs!
Mechanism and Interpretation

- Origins of lack of CK
  - dispersed private information as in Morris-Shin etc
  - “rational inattention” (Sims) or “costly contemplation” (Tirole)

- Key mechanism
  - HOB move less than FOB
  - relative role of HOB increases with horizon
    iterating on model’s equations = ascending the hierarchy of beliefs!

- Remark:
  - the analyst has to think about HOB
  - but the agents inside the model do not:
    in Lucas/REE tradition, they are still statisticians
The NK Model without CK

- Key step: recast IS and NKPC as dynamic beauty contests

- Removing Common Knowledge $\Rightarrow$
  - anchors expectations of $y$ and $\pi$
  - attenuates GE feedback loops (both within and across two blocks)
  - attenuation larger the longer these loops

- Implications:
  - lessen forward guidance puzzle
  - offer rationale for front-loading fiscal stimuli
  - lessen paradox of flexibility
  - ...

1. Framework

2. Recast IS and NKPC as Dynamic Beauty Contests

3. Show GE Attenuation and Horizon Effects

4. Policy Implications

5. Companion Paper and Related Literature
Framework
Framework

- Starting point: textbook NK model
- Main departure: remove CK of fundamental/policy
- Auxiliary: enough “noise” to prevent revelation through prices
  - variant with similar results: rational inattention
Households

- Continuum of households/consumers
- Preferences
  \[ U_i = \sum_{t=0}^{+\infty} \beta^t \left( \log c_{it} - \frac{1}{1+\epsilon} n_{it}^{1+\epsilon} \right), \]
- Budget constraint
  \[ c_{it} + s_{i,t} = \frac{1+R_{t-1}}{1+\pi_t} s_{i,t-1} + w_{it} n_{it} + e_{it} \]

- no risk-sharing, but no liquidity constraints (⇒ OK to log-linearize)
Firms

- Final goods produced by a competitive sector

\[ y_t = \left( \int_0^1 \left( y^j_t \right)^{\frac{s-1}{s}} dj \right)^{\frac{s}{s-1}} \]

- Each variety \( j \) produced by a monopolist

\[ y^j_t = l^j_t \]

- Nominal rigidity a la Calvo
  - fraction \( 1 - \theta \) changes price each period
Important Assumption

- lack of CK (＝dispersed private info) about “fundamentals”
  - path of interest rates [here]
  - path of discount rates [isomorphic]
  - path of government spending [later]
Auxiliary Assumptions

- Appropriate shocks to markups and wages
  - i.i.d over time and unpredictable

- Modeling role: add “noise” in price system
  \[ \Rightarrow \text{limit aggregation of info about “fundamentals”} \]

- Variant: remove the noise in prices, add Rational Inattention
Mapping the IS and the NKPC to Dynamic Beauty Contests
\[ c_t = -E_t[r_{t+1}] + E_t[c_{t+1}] \]

- Key implication: \( c = f(\text{expected path of } r) \)
  - this implication is robust to borrowing constraints
  - even though the aggregate Euler equation itself is different
The Euler/IS Curve without Common Knowledge

- Take formula for optimal consumption (PIH), with $E_{it}$ instead of $E_t$, aggregate across $i$, and impose $y_t = c_t$, to reach the following

$$ c_t = - \left\{ \sum_{k=1}^{+\infty} \beta^{k-1} E_t[r_{t+k}] \right\} + (1 - \beta) \left\{ \sum_{k=1}^{+\infty} \beta^{k-1} E_t[c_{t+k}] \right\} $$

- This defines a dynamic beauty contest among the consumers

- Key implication: $c \neq f(\text{expected path of } r)$
  - instead, response of $c$ to news about path of $r$ hinges on HOB
The NK Philips Curve with Common Knowledge

\[ \pi_t = mc_t + \beta E_t[\pi_{t+1}] \]

- Key implication: \( \pi = f(\text{expected path of } mc) \)
The NK Philips Curve without Common Knowledge

- Take formula for optimal rest price, with $E_{it}$ instead of $E_t$, and aggregate across $i$ to reach the following:

\[
\pi_t = mc_t + \left\{ \sum_{k=1}^{+\infty} (\beta \theta)^k \bar{E}_t^f[mc_{t+k}] \right\} + \frac{1-\theta}{\theta} \left\{ \sum_{k=1}^{+\infty} (\beta \theta)^k \bar{E}_t^f[\pi_{t+k}] \right\}
\]

- This defines a dynamic beauty contest among the firms

- Key implication: $\pi \neq f(\text{expected path of } mc)$
  - instead, response of $\pi$ to news about path of $mc$ hinges on HOB
So Far, and What’s Next

- So far:
  - represent the NK model in terms of dynamic beauty contests
  - hint to the role of HOB

- What’s next:
  - consider a more abstract setting
  - nests IS, NKPC, asset-pricing models a la Singleton, and more
  - develop main insights
  - lack of CK = anchored expectations = GE attenuation
  - attenuation increases with horizon: as if myopia
So Far, and What’s Next

- So far:
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- What’s next: the beauty of dynamic beauty contests!
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Consider models in which the following Euler-like condition holds:

\[ a_{i,t} = \theta_t + \gamma E_{it}[a_{i,t+1}] + \alpha E_{it}[a_{t+1}] \]

- \( \theta_t \) = fundamental, \( a_{it} \) = individual outcome, \( a_t \) = aggregate outcome
- \( \gamma > 0 \) parameterizes PE effects, \( \alpha > 0 \) parameterizes GE effects
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aggregate with CK \( \Rightarrow \) representative agent Euler

\[ a_t = \theta_t + (\gamma + \alpha) E_t[a_{t+1}] \]
An Abstract Dynamic Beauty Contest

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- aggregate with CK \( \Rightarrow \) representative agent Euler

\[ a_t = \theta_t + (\gamma + \alpha) E_t[a_{t+1}] \]

- aggregate without CK \( \Rightarrow \) dynamic beauty contest

\[ a_t = \theta_t + \gamma \left\{ \sum_{k=1}^{+\infty} \gamma^{k-1} E_t[\theta_{t+k}] \right\} + \alpha \left\{ \sum_{k=1}^{+\infty} \gamma^{k-1} E_t[a_{t+k}] \right\} \]
Question of Interest

- How does $a_t$ respond to news about $\theta_{t+k}$?
  - consumption response to news about interest rates
  - inflation response to news about marginal costs
  - asset price response to news about dividends
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- Formally:
  - hold $\theta_{\tau}$ constant (say, at 0) for all $\tau \neq t + k$
  - treat $\theta_{t+k}$ as a random variable (Normally distributed with mean 0)
  - specify information structure about $\theta_{t+k}$
  - study $\phi_k = \text{projection coefficient of } a_t \text{ on } \overline{E}_t[\theta_{t+k}]$
The Role of HOB

- By iterating, we can express \( a_t \) as a linear function of
  - 1st-order beliefs: \( \bar{E}_t[\theta_{t+k}] \)
  - 2nd-order beliefs: \( \bar{E}_t[\bar{E}_\tau[\theta_{t+k}]] \quad \forall \tau : t < \tau < t + k \)
  - 3rd-order beliefs: \( \bar{E}_t[\bar{E}_\tau[\bar{E}_{\tau'}[\theta_{t+k}]]] \quad \forall \tau, \tau' : t < \tau < \tau' < t + k \)
  - and so on, up to beliefs of order \( k \)
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- With CK, HOB collapse to FOB and the “usual” predictions apply
  - answer obtained by iterating Euler

\[
\frac{\partial a_t}{\partial \bar{E}_t[\theta_{t+k}]} = \phi^*_k = (\alpha + \gamma)^k
\]
By iterating, we can express $a_t$ as a linear function of

- 1st-order beliefs: $\bar{E}_t[\theta_{t+k}]$
- 2nd-order beliefs: $\bar{E}_t[\bar{E}_\tau[\theta_{t+k}]] \forall \tau : t < \tau < t + k$
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Without CK, we need to understand

1. how HOB co-move with $\bar{E}_t[\theta_{t+k}]$
2. how HOB load in $a_t$
• For every $h$, multiple kinds of $h$-order beliefs $\Rightarrow$ great complexity

• To make progress, assume constant info between “now” and “then”
  - $\bar{E}_\tau = \bar{E}_t$ for all $\tau \in \{t, \ldots, t + k - 1\}$

• Not strictly need, but useful starting point
  - completely irrelevant under CK
  - without CK, reduces “across period” HOB to “within period” HOB
  - distinguishes our results from prior work that focuses on learning
1. **Attenuation at any horizon**
   - \( \gamma^{k-1} < \phi < \phi^* \) (\( \phi \) bounded between PE effect and CK counterpart)
   - “CK maximizes GE effect”
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2. Attenuation effect increases with horizon
   - ratio $\phi/\phi^*$ decreases in $k$
     - longer horizons = iterating on beliefs = more weight on HOB
     - but HOB are more anchored than LOB
     - the more we iterate, the more potent this anchoring
   - it is as if the agents discount the future more heavily
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3. **Attenuation effect grows without limit as $k \to \infty$**
   - provided $\lim_{h \to \infty} Var(\bar{E}_t^h) = 0$ (which is “generically” true)
   - $\phi/\phi^* \to 0$ as $k \to \infty$ even if noise is “tiny”
Leading example

- Information structure:
  - each agent receives a private signal about $\theta_{t+k}$ at $t$
  - given by $x_{it} = \theta_{t+k} + \epsilon_{it}$, where $\epsilon_{it}$ is Normal idiosyncratic noise
  - no other info arrives up to $t+k$, at which point $\theta_{t+k}$ becomes known

- Implication: a simple exponential structure for HOB

$$\bar{E}_t^h[\theta_{t+k}] = \lambda^{h-1} \bar{E}_t[\theta_{t+k}]$$

where $\lambda \in (0, 1]$ is decreasing in the amount of noise

- Key observation (robust to richer info structures):
  - HOB are anchored relative to FOB
  - CK obtained as $\lambda \to 1$ and “maximizes” the responsiveness of HOB
• Back to our question: How does $a_t$ vary with $\bar{E}_t[\theta_{t+k}]$?

• Answer: Same as in representative agent model with

$$a_t = \theta_t + (\gamma + \lambda \alpha)E_t[a_{t+1}]$$

- as if myopia / extra discounting of future outcomes
- contrast with prior work that tries to reduce $\gamma$
- similar effect in Gabaix and Farhi-Werning, but here RE
• Results robust to various forms of learning

• Recast friction as product of rational inattention
Robustness, Implications, and What’s Next

- Results robust to various forms of learning
- Recast friction as product of rational inattention
- Direct implications:
  - IS: attenuate $c$ response to news about distant real $r$
  - NKPC: attenuate $\pi$ response to news about distant $mc$
  - AP: attenuate $p$ response to news about distant dividends

Next: application to liquidity trap

Caveats: endogeneity of $r$ and $mc$, GE feedback between IS and NKPC

Deal with these caveats

Obtain lessons for forward guidance, fiscal stimuli, etc.
Robustness, Implications, and What’s Next

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  - deal with these caveats
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Application: Forward Guidance
ZLB and Forward Guidance

- Go back to NK model, consider ZLB context
- Let $T$ index length of liquidity trap and horizon of FG
  - $t < T - 1$: ZLB binds and $R_t = 0$ for all
  - $t \geq T + \Delta$: “natural level” and $y_t = \pi_t = 0$
  - let $\Delta = 1$ for simplicity

- Forward guidance
  - policy announcement at $t = 0$ of likely $R_T$
  - modeled as $z = R_T + \text{noise}$

- Remark
  - credibility has to do with how much $\bar{E}_0[R_T]$ varies with $z$
  - we focus on how $y_0$ varies with $\bar{E}_0[R_T]$
Leading Example

- Information structure
  - initial private signal

\[ x_i = z + \epsilon_i, \quad \epsilon_i \sim \mathcal{N}(0, \sigma^2_\epsilon) \]

- \( \epsilon_i \) can be interpreted as the product of rational inattention
- limit with no endogenous learning (large markup and wage shocks)
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- Degree of CK indexed by \( \lambda \in (0, 1] \)
  \[ \bar{\mathbb{E}}^h[R_T] = \lambda^{h-1} \bar{\mathbb{E}}^1[R_T] \]
  - consumers vs firms: \( \lambda_c \) vs \( \lambda_f \)
  - CK benchmark nested with \( \lambda_c = \lambda_f = 1 \)
**The Power of Forward Guidance**

- **Question:** How does $y_0$ vary with $\bar{E}_0[R_T]$?

- **Answer:** There exists a function $\phi$ such that

$$y_0 = -\phi(\lambda_c, \lambda_r, T, \kappa) \cdot \bar{E}_0[R_T]$$

- **standard:** $\phi^*$ increases with $T$ and explodes as $T \to \infty$
- **here:** $\phi$ vs $\phi^*$
Main Results

- Attenuation for any horizon
  - three GE effects at work:
    1. inside IS: income-spending feedback
    2. inside NKPC: inflation-inflation feedback
    3. across two blocs: inflation-spending feedback
  - all three attenuated; but quantitative bite only for (2) and (3)
Main Results

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- **Attenuation effect increases with horizon**
  - $\phi/\phi^*$ decreases in $T$
  - $\phi/\phi^* \to 0$ as $T \to \infty$, even if $\lambda \approx 1$
  - for $\lambda_c$ small enough, $\phi \to 0$ in absolute, not only relative to $\phi^*$
A Numerical Illustration

- Modest friction: $\lambda_c = \lambda_f = 0.75$
  - 25% prob that *others* have failed to hear announcement
  - all other parameters standard, as in McKay et al (2015)
Additional Results
Fiscal Stimuli: Back- vs Front-Loading

- Standard NK prediction:
  - fiscal stimuli work because they trigger inflation
  - better to back-load so as to “pile up” inflation effects

- Our twist:
  - such piling up = iterating HOB
  - not as potent when CK assumption is dropped
  - rationale for front-loading: “minimize coordination friction”
Paradox of Flexibility

- In standard NK model, $\phi^*$ increases with $1 - \theta$
  - paradox: price flexibility makes MP more powerful
  - related: price flex exacerbates ZLB constraint/deflationary spiral

- Our twist: $\phi/\phi^*$ decreases with $1 - \theta$
  - price flexibility increases attenuation effect
  - lack of CK lessens paradox of flexibility

- Intuition: same causes (GE effects), same resolution
As if representative agent discounts more heavily the future

\[ y_t = \Lambda_t E_t [y_{t+1}] - \{R_t - \lambda_t E_t [\pi_{t+1}]\} \]
\[ \pi_t = M_t \beta E_t [\pi_{t+1}] + m_t \kappa y_t \]

where \( \Lambda_t, \lambda_t, M_t, m_t \in (0, 1) \).

Reminiscent of Gabaix (2016), but:
- preserve Euler conditions at individual level
- preserve Rational Expectations

lack of CK = as if “cognitive discounting”
Companion Paper and Related Literature
- Angeletos and Lian (2016)
- Gabaix (2016), Farhi and Werning (2017)
Goal: formalize notion “GE Adjustment is Weak / Takes Times”
- within a more abstract (flexible) demand+supply setting

Define and characterize an appropriate “frictionless benchmark”
- assumes REE+CK, replicates Arrow-Debreu

Depart from this benchmark in two possible ways
- either by replacing REE with certain kinds of “bounded rationality”
- or by maintaining REE and, instead, removing CK
Key Findings

- **Lack of CK = GE attenuation**
  - REE alone restricts GE effect in an interval
  - standard practice picks the upper bound!
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- **Level-k Thinking = GE attenuation when GE amplifies PE**
  - but $\neq$ when GE offsets PE
  - problem due to “overshooting” in conjectures about others

- Reflective Eq as in Garcia-Schmidt and Woodford (2015)
  - similar to Level-k Thinking, but “smooth”
  - bypasses aforementioned problem, attenuates GE

- Cognitive discount as in Gabaix (2016)
  - by assumption, agents discount reaction of others
  - can capture GE attenuation
  - but free to capture opposite property too

- Bridge to older tradition on T attonement and Cobweb dynamics
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Take-Home Lessons

- GE attenuation = robust prediction of relaxing either REE or CK

- In this regard, lack of CK = substitute of bounded rationality

- But, some differences: lack of CK =
  1. immune to Lucas critique
  2. no conundrum with what agents do when they see that the actual outcomes are inconsistent with their beliefs
  3. no ad hoc “default” point, no “freedom” to obtain the opposite result
  4. easier to adapt to stationary settings with recurring shocks
Conclusion
Conclusion

- Relaxing CK ⇒
  - attenuates or slows down GE mechanisms
  - sheds new light on “managing expectations”
  - lessens NK puzzles, favors front-loading

- Related ongoing work
  - revisit effects of $G$, $T$, deficits in both RBC and NK
  - demand-driven cycles and keynesian multipliers w/o nominal rigidity
  - ...