Smoothingsudden stops

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Abstract

Emerging economies are often exposed to sudden shortages of international financial resources. Yet domestic agents do not seem to take preventive measures against these sudden stops. We highlight the central role played by the limited development of ex ante (insurance) and ex post (spot) domestic financial markets in generating this collective undervaluation of international resources. We study several policies to counteract the external underinsurance. We do this by solving for the optimal mechanism given the constraints imposed by limited financial development, and then considering the main financial policies—in terms of the model and practical relevance—that implement this solution.

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1. Introduction

An emerging economy external crisis can be described as an event in which a country’s international financing needs significantly exceed its international financial resources. Given that such events are a “fact-of-life” in these economies, it is puzzling that domestic agents do not take preventive measures against them. Indeed, quite the contrary: they often increase the likelihood of these events by over-borrowing during capital inflow booms, contracting dollar liabilities, and so on.
A common explanation for this behavior is distortions created by anticipated official interventions, such as crony capitalism, fixed exchange rates, and IFI’s bailouts.¹

We have argued elsewhere that the external underinsurance problem in these economies is more structural in nature than one would conclude from looking only at potentially misguided interventions. Underdeveloped financial markets, a basic feature of emerging economies, lead to a distorted valuation of international resources that, in turn, leads to external underinsurance. In this paper, we take this structure as given, and explore a series of financial policies that solve the underinsurance problem. Since the strategies we discuss are all used in varying form by governments in emerging markets, our main interest is in identifying which strategies work better under certain constraints. We focus on the strategies that work within our model and discuss some of the difficulties they may encounter in implementation.

In our framework, when a country’s international financing needs exceed its international collateral (or liquidity), the domestic price of the latter rises vis-à-vis that of domestic collateral (or liquidity). One manifestation of this phenomenon is a depreciation of the exchange rate, for example.²

However, when domestic financial markets are underdeveloped—in our terminology, when the domestic collateral value of projects is less than their expected revenues—then agents’ external insurance decisions are distorted. Domestic agents in need of external resources cannot transfer the full surplus generated by these resources to other participants in domestic financial markets that do have access to the scarce external funds. Thus, in equilibrium, the scarcity value of external resources is depressed, and private decisions are biased against hoarding international liquidity and thereby insuring against these events. The underinsurance with respect to external shocks takes many forms: excessive external borrowing during booms; a maturity structure of private debt that is distorted toward the short term; dollarization of international liabilities; limited international credit lines; and so on.³

In this paper, we study several of the main financial policies that solve this underinsurance problem, within a unified framework. Section 2 describes the environment that we have used in earlier work, and reproduces the result: collective external underinsurance in the competitive equilibrium with only spot loan markets. One difference in the current model is that we suppress all aggregate shocks. The reason is that our focus is on domestic arrangements to deal with the underinsurance problem. To keep matters simple, we do not discuss international credit lines and other valuable insurance mechanisms that involve foreigners at all.⁴ Alternatively, one can think of our discussion as “net” of these external insurances. A binding aggregate external constraint will be anticipated fully and will still occur. External

¹See, for example, Krugman [12], Burnside et al. [3], or Dooley [9].
²See Caballero and Krishnamurthy [5].
³See Caballero and Krishnamurthy [4, 6].
⁴See Caballero and Krishnamurthy [4, 6] for discussions of insurance arrangements with foreigners.
underinsurance, in its many forms, simply will collapse into excessive international borrowing during capital inflow booms.

While aggregate shocks have no role in the analysis, idiosyncratic ones are central, because they generate the need for domestic financial transactions. Frictions in these transactions are at the root of the external underinsurance problem. In Section 3, we show that if domestic agents are able to write complete insurance contracts with each other, the external underinsurance problem disappears. More domestic insurance increases the distressed firms’ collateral ex post, and hence their capacity to bid for the external resources held by other domestic agents. In equilibrium, this raises the relative price of international to domestic collateral, increasing the incentive to hoard international resources.

However, an important aspect of financial underdevelopment is the absence of these private insurance markets. In Section 4, we assume that idiosyncratic shocks are unobserved, so they cannot be written into insurance arrangements. We solve the mechanism design problem associated with the private information constraint within our structure, and show that the social planner in principle can get around this informational constraint and achieve the competitive equilibrium with complete idiosyncratic insurance markets. We then turn to implementation of the social planner’s mechanism. We begin by analyzing a solution in which private agents form a conglomerate and extend credit lines to each other. While this arrangement is individually incentive compatible, it is not coalition incentive compatible, and hence is not robust to the presence of spot markets, as in Jacklin [11]. Finally, we explore two sets of solutions that require government intervention: capital flows taxation or mandated international liquidity requirements, and sterilization of capital inflows. These solutions can also work, but are subject to other forms of the coalition incentive compatibility problem.

2. A model of external underinsurance

We begin by laying out the model and describing the external underinsurance in the competitive equilibrium with only spot loan markets. In the next sections, we discuss how this result is affected by better domestic insurance arrangements or, in their absence, by centralized arrangements for dealing directly with the external underinsurance problem.

2.1. The environment

Consider a three-date \((t = 0, 1, 2)\) economy with a single consumption good. There are two classes of agents in the economy: of domestic agents and foreigners. Both take as their objective maximizing date 2 expected consumption of the good,

\[
U = E[c_2], \quad c_2 \geq 0.
\]

Each domestic agent is an entrepreneur/manager who owns and operates a production technology within a firm. Investing \(c(k)\) units of the good at date 0
results in capital of $k$ units, where $c(k)$ is strictly increasing, positive, and strictly convex with $c(0) = 0$, $c'(0) = 0$.

As part of the normal ongoing restructuring of an economy, one-half of the firms (randomly chosen) need to re-inject resources into the firm at date 1 to achieve full output. Let $j \in \{i, d\}$ be the type of firm at date 1. Firms that are not hit by this idiosyncratic shock are $i$-types (“intact”) and go on to produce date 2 output of $Ak$. Firms that receive the shock are $d$-types (“distressed”). Their output falls to $ak$, but by reinvesting $I \leq k$ units of good, the $d$-firm can obtain $IA$ additional units of goods at date 2. We normalize $A = A - a$, and assume that $A > 1$. With full reinvestment, $I = k$, a distressed firm obtains the same output as an intact firm, $Ak$. In all cases of interest, $I < k$; thus, we henceforth drop this maximum reinvestment constraint from our discussion (while ensuring that it does not bind in our technical assumptions).

The domestic economy has no goods at either date 0 or date 1. All investment needs are met by importing goods from abroad, which are paid for with funds raised from loans. We assume that foreigners have large endowments of goods at all dates, and have access to storage with rate of return one.

Firms face significant financial constraints. Neither the plants nor their expected output are valued as collateral by foreigners. Instead, we assume that each domestic agent is endowed with $w$ units of a good that arrive at date 2 and can be pledged as collateral to a foreign lender—i.e. domestic agents can take out loans against $w$ that will be enforced by international courts. Tangibly, we might think of $w$ as revenues from oil exports that reside in foreign bank accounts.

**Assumption 1** (International collateral). Domestic agents may take on loans at either date 0 or date 1 from foreign lenders against the international collateral of $w$, and must satisfy a full collateralization constraint:

$$d_{0,f} + d_{1,f} \leq w.$$  

A domestic agent also can take on a loan from another domestic agent. Unlike foreigners, domestic agents do accept the plants as collateral. However, we assume that these contracts are also imperfect in the sense that not all of the output of $Ak$ is collateral.

**Assumption 2** (Domestic debt and collateral). We assume that domestic courts are additionally able to enforce domestic (local) debt contracts up to an amount of $\lambda ak$ where $\lambda \leq 1$. Thus, the domestic lending constraint is

$$d_{0,l} + d_{1,l} \leq \lambda ak + w - (d_{0,f} + d_{1,f}).$$

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5Throughout the paper we use the word collateral, broadly defined, as the borrowing capacity of agents. For example, $\lambda ak$ can be interpreted as physical collateral, such as land, machinery, or buildings. But $\lambda ak$ can also be interpreted as the verifiable component of output that lenders can seize in bankruptcy proceedings. In this case, $\lambda ak$ is pledgeable output. The distinction between these two forms of borrowing capacity is not crucial for the analysis we conduct.
Assumptions 1 and 2 are the way we define an emerging economy. Thus, we think of an emerging economy as an economy whose pledgeable assets are limited, and where a large share of these assets are part of domestic but not international collateral.

We define an external crisis as a date 1 event in which the financing needs of the economy, $\frac{1}{2}k$, exceeds the international financial resources available to it, $(w - d_{0,f})$. Since they are not central to our concerns and results in this paper, we suppress all aggregate shocks. The external crisis occurs despite being fully anticipated. This simplifies our discussion and means that external underinsurance will only take the form of overborrowing at date 0 (see below). With some abuse of terminology, we will continue referring to overborrowing as external underinsurance. The following assumptions on parameters guarantee that a crisis occurs at date 1 in all the equilibria we study throughout the paper, and that there is external underinsurance in the spot loan market equilibrium:

**Technical Assumption 1.**

1. \[ c'^{-1}\left(\frac{A+a}{2A}\right)a > w - c(c'^{-1}\left(\frac{A+a}{2A}\right)) \]
2. \[ c'^{-1}\left(\frac{A+a}{2A}\right)a < A(w - c(c'^{-1}\left(\frac{A+a}{2A}\right))) \]
3. \[ la < \frac{1}{2}. \]

2.2. Spot loan markets

Let us begin by studying the equilibrium in this economy when agents are restricted to borrowing via a sequence of spot loan contracts. Thus, we rule out domestic insurance arrangements for now.

All of the investment needs of domestic agents (dates 0 and 1) have to be met by importing goods from foreigners. The goods are paid for by issuing date 2 debt claims. Suppose that each firm takes on foreign debt at date 0 of $d_{0,f}$ and invests all of these resources in building a plant of size $k$. Since firms are identical ex ante, without loss of generality we can assume that there is no domestic debt at time 0.

At date 1, a firm finds itself either distressed or intact. If distressed, it borrows up to its maximum international debt capacity in order to take advantage of the high return to rebuilding/restructuring the firm:

\[ d_{1,f} = w - d_{0,f}. \]

These resources then are invested until date 2, yielding $d_{1,f}A$.

After this, it must turn to intact domestic firms for funds. Intact firms have no output at date 1 either, so they must borrow from foreigners if they are to finance the distressed firms. They can do this up to their $w - d_{0,f}$ of financial slack. Unlike foreigners, domestic agents are willing to lend to other domestic against their projects. Since firms can use this collateral to borrow up to $la$, we refer to this quantity as domestic collateral.
Denote the gross interest rate in this domestic loan market as $L_1$. Then the firm takes out the maximum loan as long as $\Delta \geq L_1$:

$$d_{1,f} = \lambda ak.$$  

As a result of domestic borrowing the firm raises $\frac{d_{1,f}}{L_1}$ for investment, to yield $\frac{\lambda ak}{L_1} \Delta$ at date 2.

Combining the above transactions and taking into account that date 0 investment yields $ak$ at date 2, the profits accumulating to this firm at date 2 are

$$V^d = (w - d_{0,f}) \Delta + \frac{\lambda ak}{L_1} \Delta + (1 - \lambda)ak.$$  

Intact firms, on the other hand, have the opportunity to lend to distressed firms at date 1. As long as $L_1 \geq 1$, the intact firm will borrow up to its maximum foreign debt capacity,

$$d_{1,f} = w - d_{0,f},$$  

and invest these resources in the domestic loan market to yield $L_1$. Denote $x_1$ as the face value of date 2 claims that the intact firm purchases. Then the intact firm makes date 2 profits of

$$V^i = x_1 L_1 + Ak + (w - d_{0,f} - d_{1,f}) = (w - d_{0,f})L_1 + Ak.$$  

Finally, at date 0, firms are equally likely to be distressed or intact. Thus, they solve

$$V^{\text{spot}} = \max_{k,d_{0,f}} \frac{1}{2} (w - d_{0,f})(L_1 + \Delta) + \frac{1}{2} (\Delta + (1 - \lambda)ak + \frac{\lambda a}{L_1} \Delta)k$$

s.t. $d_{0,f} \leq w,$

$$c(k) = d_{0,f}.$$  

The only market clearing condition is that the loans issued by distressed firms must equal the loans purchased by intact ones:

$$\frac{1}{2} d_{1,f} = \frac{1}{2} x_1,$$  

where the one-half in front of each microeconomic decision is because each type is of measure one-half in the population.

Definition. Equilibrium in the economy with only sequential spot loan markets consists of decisions, $(k,d_{0,f},d_{1,f},d_{1,f},x_1)$ and the domestic interest rate, $L_1$. Decisions are optimal given $L_1$, and given these decisions, the market clearing condition (1) holds.

Fig. 1 illustrates the market clearing. The horizontal axis shows the quantity of imported goods lent by intact firms/borrowed by distressed firms. The vertical axis is the price of loans $L_1$. The supply is elastic at $L_1 = 1$ up to the point that the intact firms saturate their international collateral constraint of $d_{1,f} = w - d_{0,f}$, at which
point it is completely inelastic. Demand for loans is given by the curve, \( \frac{\lambda k}{L_1} \), which is downward sloping in \( L_1 \).

It is easy to see from the figure that \( D \geq L_1 > 1 \). The figure represents three alternatives for demand: The highest dashed line is the case where there is sufficient domestic collateral that \( L_1 = D \); the middle solid line is the case where \( L_1 \) lies strictly between one and \( D \); the lower dashed line is the case where \( \lambda \) is small and as a result demand is so collateral constrained that intact firms have an excess supply of funds and the interest rate is one.

**Proposition 1.** Under the parameter assumptions in Technical Assumption 1, there is a unique equilibrium in the spot loan market. The interest rate on loans against domestic collateral is

\[
L_1 = \frac{\lambda k}{w - d_{0,f}}.
\]  

\( L_1 \) lies strictly between \( D \) and one (i.e. the equilibrium with the solid demand line in Fig. 1).

**Proof.** See appendix. □

Note that \( L_1 \) lies above the international interest rate of one. This is because the asymmetry between domestic and foreign agents embedded in Assumptions 1 and 2. If foreigners were willing to hold claims against \( \lambda k \), then arbitrage between these and foreign assets would imply that \( L_1 = 1 \). Alternatively, if \( w \) were large, so that on...
the margin some domestic investor was holding claims against both \( w \) and \( \lambda ak \) in their portfolio, then again it must be that \( L_1 = 1 \).

Given this price, the first-order condition for the date 0 program can be written as

\[
c'(k) L_1 + \Delta = \frac{A + (1 - \lambda) a + \frac{\lambda a}{L_1} \Delta}{2},
\]

(3)

where the left-hand side represents the expected opportunity cost of the marginal units of international collateral spent on setting up a plant at date 0, while the right-hand side is the expected marginal revenue associated with the marginal plant.

**Proposition 2.** Consider two economies indexed by \( \lambda \) and \( \lambda' \), where \( \lambda > \lambda' \). Then,

- \( \Delta - L_1(\lambda) < \Delta - L_1(\lambda') \);
- welfare is increasing in \( \lambda \), so that \( V_{\text{spot}}(\lambda) > V_{\text{spot}}(\lambda') \);
- date 0 investment and borrowing are decreasing in \( \lambda \), so that \( k_{\text{spot}}(\lambda) < k_{\text{spot}}(\lambda') \).

**Proof.** Follows after a few steps of algebra, from \( V_{\text{spot}} \), (2), and (3). \( \square \)

The proposition highlights the role of \( \lambda \) on welfare, decisions, and prices. Fixing \( k \), from the market clearing condition we can see that \( L_1 \) is increasing in \( \lambda \). Thus as \( \lambda \) rises, \( L_1 \) rises toward the marginal product at date 1 of \( \Delta \). This has an important effect on date 0 decisions. A firm that decides to borrow less, is essentially “saving” these resources until date 1. At date 1, these resources either are used internally to yield \( \Delta \), or loaned externally, in which case the lender only internalizes \( L_1 \) of this return. Again, this occurs because the borrower is collateral constrained. As \( \lambda \) rises, the spread between \( \Delta \) and \( L_1 \) falls causing firms to save more at date 0. This leads to greater investment at date 1 and increases in welfare. Essentially, as \( \lambda \) rises, prices are less distorted by the credit constraint and the intertemporal savings decision better reflects marginal products.

### 3. Public information of types and date 0 domestic insurance markets

In this section, we show that welfare can be improved through the use of a domestic insurance contract at date 0 that shuffles resources from intact to distressed firms at date 1.\(^6\) At first glance this may seem odd because under our spot market equilibrium, all of the international resources find their way into the hands of the distressed firms. That is, intuition may suggest that the ex post allocation cannot be enhanced by further reallocating domestic collateral to distressed firms, since the scarcity is of international collateral, and this already has been fully transferred. However, in our setup, the welfare gain from domestic insurance comes entirely from affecting the ex post price of international resources, \( L_1 \), and bringing this closer to \( \Delta \)

\(^6\)Recall that our focus is not on the possibility of more or less insurance from foreigners, rather it is on domestic arrangements given the limited access to international financial markets.
so that, ex ante, the borrowing/investment decision of $k$ is less distorted. Moreover, reallocating ex post wealth beyond what is needed to set $L_1 = \Delta$ affects $V^i - V^d$, but not decisions, equilibrium, or ex ante welfare.

**Assumption 3** (Public information). The shock at date 1 is public information, and insurance contracts can be written contingent on type $j \in \{i, d\}$.

Consider the following domestic insurance contract: All firms sign a grand insurance contract at date 0 with repayments in date 2 goods of $x_0, l(i) = -x_{0,l}$ and $x_0, l(d) = x_{0,l} > 0$. Since the types are observable, this contract can be made contingent on type-$j$. Repayments are enforceable as long as

$$x_{0,l} \leq w - d_0, f + \lambda ak.$$ 

Since there is an equal measure of each type, the insurance payments to distressed firms are funded exactly by the receipts from the intact firms.

At date 1, an intact firm sees the domestic interest rate of $L_1 \geq 1$ and has international collateral of $w - d_0, f$, and an insurance liability of $x_{0,l}$. Suppose that the firm lends all of its international collateral at $L_1$, then its total resources are date 2 goods of

$$(w - d_0, f) L_1 + Ak.$$ 

Against this it has the liability of $x_{0,l}$ giving date 2 profits of

$$V^i = (w - d_0, f) L_1 + Ak - x_{0,l}.$$ 

The distressed firm borrows against its international collateral of $w - d_0, f$ and invests the proceeds in production to yield a date 2 return of $\Delta$. As long as $L_1 \leq \Delta$, it borrows $d_{1,l}$ in the domestic debt market, satisfying the constraint that

$$d_{1,l} \leq \lambda ak + x_{0,l}. \tag{4}$$

Thus, it makes date 2 profits of

$$V^d = (w - d_0, f) \Delta + (\Delta - L_1) \frac{d_{1,l}}{L_1} + (1 - \lambda) ak + x_{0,l}.$$ 

Consider (4) more closely. If $x_{0,l} = 0$, we are back in the situation we studied in the previous section and $d_{1,l} = \lambda ak$. Since increasing $x_{0,l}$ from this point only loosens the constraint on $d_{1,l}$, without loss of generality we can set

$$d_{1,l} = \lambda ak + x_{0,l}.$$ 

That is, if the inequality in (4) is strict, then $x_{0,l}$ can be reduced until equality, while only loosening the insurance enforceability constraint and affecting the level of $V^d$ and $V^i$, but not decisions or date 0 welfare (recall that agents are risk neutral). Given
this, the date 0 problem is just
\[
V^{\text{ins}} = \max_{k, d_{0,f}, x_{0,l}} \left[ \frac{1}{2}(w - d_{0,f})(A + L_1) + \frac{1}{2}(A + (1 - \lambda)\mu)k \\
+ \frac{1}{2}(A - L_1)\lambda \mu + x_{0,l} \right] L_1
\]
s.t. \(d_{0,f} \leq w,\)
\(c(k) = d_{0,f},\)
\(x_{0,l} \leq w - d_{0,f} + \lambda \mu.\)

**Lemma 1.** *In the insurance market equilibrium:*

\[L_1 = \Delta.\]

**Proof.** We can see this in two steps. First, from the program, as long as \(\Delta > L_1,\) firms will increase \(x_{0,l}.\) Second, the only limit on \(x_{0,l}\) is the enforceability constraint that \(x_{0,l} \leq w - d_{0,f} + \lambda \mu.\) Suppose that \(x_{0,l} = w - d_{0,f} + \lambda \mu - \delta,\) with \(\delta > 0.\) As \(\delta \to 0,\) the intact firms at date 1 have no international resources, and market clearing in the domestic loan market would require that \(L_1 = \Delta.\) As a comment, there is a large interval within which \(x_{0,l}\) can fall for this to hold. \(\Box\)

Substituting \(L_1 = \Delta\) into the program gives the first-order condition for investment,
\[
c'(k)\Delta = \frac{A + a}{2}. \tag{5}
\]
Contrasting this expression with the first-order condition in the spot market equilibrium, (3), implies:

**Proposition 3.** Let \(k^{\text{ins}}\) be the solution to (5) and \(k^{\text{spot}}\) be the solution to (3). Then:

1. If \(\Delta - L_1^{\text{spot}} > 0,\)
   \[k^{\text{spot}} > k^{\text{ins}} \quad \text{and} \quad V^{\text{ins}} > V^{\text{spot}}.\]

2. If \(\Delta - L_1^{\text{spot}} = 0,\) the two first-order conditions coincide and decisions as well as welfare are the same.

By signing date 0 insurance contracts, firms bid up the price of international collateral at date 1 until it reaches \(\Delta.\) As a result, firms borrow less at date 0 and invest less, thus leading to a better allocation of external resources across dates 0 and 1. Note that the insurance solution leaves no role for \(\lambda.\) Indeed, this is the point: Since the loan market at date 1 is affected by collateral frictions, the date 0 insurance market circumvents these frictions by loosening the domestic collateral constraint.
4. Private information of types and planning solutions

We shall henceforth set \( \lambda \) equal to one, because it plays only a limited role in what follows. More importantly, from now on we shall acknowledge the many difficulties encountered by domestic insurance contracts in emerging economies and assume:

**Assumption 4 (Private information).** The shock at date 1 is private information of the firm.

This assumption means that the insurance contracts in the previous section are not possible since, at face value, all firms will prefer to claim to be distressed and avoid payment. However, the spot loan market is still feasible. We now investigate whether it is still possible to implement the full-insurance solution.

4.1. Mechanism design problem

We take a standard mechanism design approach. The types at date 1 are private information and must be elicited by the mechanism. As usual, we appeal to the revelation principle to focus on direct revelation mechanisms.

Consider the following mechanism. At date 0, agents hand over \( w \) of international collateral to the planner. The mechanism is defined by

\[
\begin{align*}
m & = (k, y_i, y_d, x_i, x_d).
\end{align*}
\]

At date 0, the planner hands resources to create capital of \( k \) to each firm. At date 1, agents send a message of their type, \( j \in \{i, d\} \). They then receive an allocation of international collateral (or imported goods) of \( y_j \) and a claim on date 2 domestically produced goods of \( x_j \).

Thus, the planner solves the following problem:

\[
V^m = \max_m \quad \frac{1}{2}(Ak + y_i + x_i) + \frac{1}{2}(ak + y_dA + x_d)
\]

s.t.  
\[
\begin{align*}
& (RC0) \quad c(k) \leq w, \\
& (RC1) \quad \frac{1}{2}(y_i + y_d) + c(k) \leq w, \\
& (ICC) \quad y_i, y_d \geq 0, \\
& (RCX) \quad x_i + x_d \leq 0, \\
& (DCC) \quad x_i, x_d \geq -ak, \\
& (ICi) \quad Ak + y_i + x_i \geq Ak + y_d + x_d, \\
& (ICd) \quad ak + y_dA + x_d \geq ak + y_iA + x_i.
\end{align*}
\]

The constraints are as follows: RC0 and RC1 are, respectively, date 0 and date 1 resource constraints on importing goods for investment. Since agents hand over all of their international collateral to the planner at date 0, the transfer to them, \( y_j \), must be non-negative. The planner can shuffle claims on date 2 goods—i.e. domestic collateral—at date 1. RCX requires that this shuffling does not create new collateral in the aggregate. DCC states that the most the planner can shuffle away from any of
the agents is given by their domestic collateral constraint, \( ak \). The last two constraints impose incentive compatibility so that each type prefers the bundle intended for it.

The asymmetry between Assumptions 1 and 2 is embedded in RC0, RC1 and DCC. To import goods for investment, only international collateral can be used—hence RC0 and RC1. On the other hand, these goods can be shuffled among domestic agents by transferring claims against domestic collateral—hence DCC. We think this asymmetry is a distinguishing feature of an emerging economy. In a developed economy most assets are both domestic and international collateral; in such a case, we could do away with DCC and rewrite the resource constraints to include the domestic collateral of \( ak \).

Given linearity in \( y \)'s and \( x \)'s, we will arrive at corner solutions in them. Since \( c(k) \) is convex, we will have an interior solution in \( k \). In order to see which corners determine the solution, rewrite the two incentive compatibility constraints as

\[
x_i + y_i \geq x_d + y_d
\]
\[
x_d + y_d + (A - 1)y_d \geq x_i + y_i + (A - 1)y_i.
\]

Note that \((x_i + y_i)\) appears as a sum everywhere in the program except in this last incentive compatibility constraint. If we were at an interior point on \( x_i \) and \( y_i \) then the incentive compatibility constraint could be slackened by lowering \( y_i \) and increasing \( x_i \). Thus, consider the solution of \( y_i = 0 \) and \( x_i \) at its highest value. Applying the same argument to \((x_d + y_d)\) dictates a solution of \( y_d \) to be at its highest value and \( x_d \) to be at its lowest. Thus, \( y_d = 2(w - c(k)) \), and \( x_d = -ak \). Combining gives us

\[
m = (k, y_i = 0, y_d = 2(w - c(k)), x_i = ak, x_d = -ak).
\]

and rewriting the optimization problem gives

\[
V^m = \max_k \frac{1}{2}(A + a)k + A(w - c(k)).
\]

The first-order condition to the planning problem is

\[
c'(k)\Delta = \frac{A + a}{2}.
\]

Let \( k^* \) be the solution. The last step is to verify that the solution satisfies the incentive compatibility constraints. That is,

\[
A(y_d - y_i) \geq x_i - x_d \geq y_d - y_i
\]

or

\[
A(w - c(k^*)) \geq ak^* \geq w - c(k^*)
\]

which can be shown to hold under Technical Assumption 1 (see appendix).

**Proposition 4.** The optimal mechanism under private information of types implements the full-insurance public information solution.
This follows directly from comparing the first-order conditions in this solution and
the insurance solution of the previous section.

The mechanism works because it exploits the differential valuation of imported
goods between distressed and intact firms. If a firm claims to be distressed rather
than intact, it receives $2(w - c(k*))$ imported goods, but forgoes $2ak^*$ claims on date
2 goods. The interest rate implicit in this choice is

$$L_1^* = \frac{ak^*}{w - c(k*)},$$

where the technical assumption ensures that $\Delta > L_1^* > 1$. A distressed firm values the
imported goods at $\Delta$, while the intact firm values it at one. Thus, distressed firms
effectively borrow at $L_1^*$ and intact ones lend at $L_1^*$. Both types’ welfare is enhanced,
and the full-insurance solution is achieved.

Finally, note that since $k^* < k$ we have that $L_1^* < L_1$. The mechanism design
solution results in “cheaper” loans to the distressed firms than would prevail in the
spot loan market equilibrium. Of course, the underinsurance problem in the spot
loan market equilibrium is because $L_1$ is low relative to $\Delta$ (Proposition 2).

We now consider three sets of alternative implementations of the planning
solution, noting that each requires the planner to act (and hence be able to monitor)
on a different margin. The tension we noted earlier between the cheap loans of the
planning solution and the underinsurance problem of the spot loan market
equilibrium affects implementation. There is a common problem with each of these
implementations. If a firm can somehow guarantee itself a cheap loan in case it is
distressed, then ex ante it has incentives to increase $k$ even more.

4.2. Domestic credit lines

The first solution we consider is a credit-line/banking arrangement akin to
Diamond and Dybvig’s [8] deposit contracts in the context of consumption
insurance.

Suppose that all firms hand over $w - c(k^*)$ to the bank at date 0. This leaves each
firm with $c(k^*)$ for the purpose of building a plant. The bank then offers each firm
the right to withdraw $w - c(k^*)$ at date 1 as well as a credit line for borrowing an
additional $w - c(k^*)$ at the interest rate of $L_1^*$. Funds not withdrawn at date 1 earn
the interest rate of $L_1^*$ until date 2.\footnote{We do not impose a sequential service constraint as in Diamond and Dybvig [8], which means that $L_1$
is left free to adjust in the out-of-equilibrium event that more than half of the firms decide to withdraw. Thus, there is no “bank-run” equilibrium.}

At date 1, distressed firms return to the bank and withdraw $w - c(k^*)$. In addition,
they choose to take out a further loan against domestic collateral of $ak^*$ at the rate of
$L_1^*$. This gives them imported goods of exactly $2(w - c(k^*))$ which they invest until
date 2 at the private return of $\Delta$. 
Since intact firms’ alternative use of imported goods returns only one, intact firms choose not to withdraw their funds at date 1 and instead wait until date 2, providing them a total return of $L_i^1(w - c(k^*)) = ak^*$.

This structure clearly implements the planner’s solution. However, as was first pointed out by Jacklin [11], it requires the fairly strong restriction that agents not be allowed to make any side trades. That is, all firms must be restricted to trade exclusively with the bank and be barred from trading in a market. If we drop this restriction, the banking arrangement is no longer coalition incentive compatible and the allocation reverts to the competitive equilibrium.\(^8\)

In our context, Jacklin’s critique can be formulated as follows. Suppose that one firm chooses to opt out of the banking arrangement, and privately makes an investment decision of $k$. At date 1, the firm is either distressed or intact. If distressed, suppose that it approaches a firm within the banking arrangement and offers to borrow at the interest rate of $L_i^1$ against domestic collateral of $ak$. Since this return is as good as the return in the banking arrangement, the firm withdraws some of its international collateral and offers it to the rogue firm. The return to the rogue firm is

$$V^d = \Delta(w - c(k)) + \frac{ak}{L_i^1} \Delta,$$

while the firm in the banking arrangement is unaffected. If the firm is intact, it instead offers to lend to a firm in the banking arrangement at the interest rate of $L_i^1$. Once again, the banking firm accepts, and the rogue firm’s profits are

$$V^i = L_i^1(w - c(k)) + Ak.$$

Combining these last two expressions gives us the date 0 program of

$$V^{\text{rogue}}(L_i^1) = \max_k \frac{1}{2}(w - c(k))(\Delta + L_i^1) + \frac{1}{2} \left( \Delta \frac{a}{L_i^1} + A \right) k.$$

The first-order condition for this program is

$$c'(k)(\Delta + L_i^1) = \frac{\Delta a}{L_i^1} + A.$$

Comparing this to the first-order condition of the planning problem, we can see that for $L_i^* < \Delta$ the rogue firm makes a choice of $k > k^*$ and attains strictly higher utility than if it participated in the banking arrangement. Given this, the banking arrangement would unravel.

The problem here is that $L_i^*$ of the planning solution requires cheap loans to be extended to the distressed firms. If a firm can act as in the spot loan market equilibrium, taking this cheap interest rate as given, it will increase investment at date 0 and this exacerbates the underinsurance problem.

\(^8\)The result that competitive spot markets may undermine insurance arrangements arises in many settings. See for example, Rothschild and Stiglitz [13], Atkeson and Lucas [1], or Bisin and Rampini [2].
We can take this to its logical end by explicitly accounting for the possibility of side trades in the planning problem. This is done by adding a constraint that

\[ V^m \geq V^{\text{rogue}}(L_1^*) \]

Now the objective in the planning problem is

\[ V^m = (w - c(k^*))A + \frac{1}{2}(a + A). \]

Substituting in \( L_1^* = \frac{ak^*}{w - c(k^*)} \), this can be rewritten as

\[ V^m = \frac{1}{2} (w - c(k^*)) (A + L_1^*) + \frac{1}{2} \left( A \frac{a}{L_1^*} + A \right) k^*. \]

Note that this is the same as the expression for \( V^{\text{rogue}} \) if evaluated at \( k = k^* \). Since both objectives in \( V^m \) and in \( V^{\text{rogue}} \) are strictly concave, they each have a unique maximum, with the maximum in \( V^{\text{rogue}} \) weakly exceeding that of \( V^m \). Given this, we can conclude that the best that the planner can do is to choose \( k^* = k^{\text{rogue}} \) so that

\[ L_1^* = \frac{ak^{\text{rogue}}}{w - c(k^{\text{rogue}})}. \]

These are the same optimality and market clearing conditions that arose in the spot loan markets of Section 2.2. In summary,

**Proposition 5.** (a) The credit-line arrangement implements the full-insurance solution as long as the planner can restrict agents from making side trades. (b) In the absence of this exclusivity restriction, the credit-line arrangement collapses to the competitive equilibrium with spot loan markets.

### 4.3. Capital inflow taxation/liquidity requirement

Let us consider next a tax/transfer scheme based on date 0 borrowing (or investment of \( k \)). Since the primitive problem in the spot loan market equilibrium is that agents overborrow/overinvest at date 0, a tax has the potential of achieving the optimal solution.

The planner taxes all date 0 external borrowing at the rate of \( \tau \) and redistributes the proceeds (\( T \)) in a lumpsum fashion at date 0,

\[ T = \tau d_{0,f} = \tau c(k^*). \]

The program for a firm is

\[ \max_k \frac{1}{2} (w - c(k) - \tau c(k) + T)(L_1 + A) + \frac{1}{2} \left( A + A \frac{a}{L_1} \right) k. \]

This gives the first-order condition

\[ c'(k)(1 + \tau)(L_1 + A) = \left( A + A \frac{a}{L_1} \right). \]
Thus,
\[ 1 + \tau = \frac{2A}{A + L_1} \frac{A + D}{A + a}, \]
where
\[ L_1 = L_1^*. \]

It is straightforward to verify that for \( L_1^* < A \), the optimal tax will be positive.

An alternative, but in spirit similar, implementation of the borrowing tax is an international liquidity requirement. For example, suppose that the planner insisted that a fraction, \( \frac{w - \frac{d_{0,f}}{d_{0,f}}}{d_{0,f}} \), of all foreign borrowings be retained as a liquidity requirement for one-period (i.e. until a crisis arises). Then, since firms choose to borrow \( d_{0,f} \), this arrangement has them saving exactly the right amount until date 1.

Rather than these types of financial policies, a policy which taxes \( k \) (since, \( d_{0,f} = c(k) \)) can achieve the same result. We have chosen to restrict ourselves to studying implementation issues with financial policies, because the frictions we have modeled are financial ones. In practice, a tax on \( k \) likely would encounter other difficulties that our model would overlook.

Unlike the credit-line arrangement, each of these solutions can co-exist with the market for loans, but there is a (less stringent) requirement that the planner observe all external borrowings. If agents could evade the tax/transfer scheme or liquidity requirement, and trade in the loan market at date 1, they would prefer to. Moreover, this incentive rises as more firms fall under the planner’s control, since \( L_1 \) falls.

Finally, it is worth pointing out that this arrangement requires the planner to tax at date 0 and then remove the tax at date 1. If the tax is left active for both periods, the equilibrium would be exactly as in Section 2.2, with the exception that the interest rate on international collateral would rise to \( 1 + \tau \). In general, this will lead to a worse outcome than the case of no-taxation.

**Proposition 6.** If the planner can observe all external borrowing, a borrowing tax or liquidity requirement implements the full-insurance solution.

### 4.4. Capital inflow sterilization

Consider a government that issues \( b \) face value of two period bonds at date 0 in return for international reserves of \( \frac{b}{L_0} \). Thus, the interest rate on these bonds is \( L_0 \), and in order to purchase these bonds, firms increase their external borrowings by \( \frac{b}{L_{0}} \).

At date 1, the government simply buys the bonds plus claims against domestic collateral using its international reserves of \( \frac{b}{L_{0}} \). At date 2, the government finally raises lumpsum taxes of \( T \) in order to balance its budget. Since the investment of reserves at date 1 is done at the interest rate of \( L_1 \), the budget constraint for the
The government is
\[ \frac{b}{L_0} L_1 + T = b, \]
where we note that if \( L_0 = L_1 \), budget balance is achieved without having to raise taxes.

There are two assumptions we make on the government. First, we assume that future tax liabilities are rationally anticipated and constitute a reduction in seizable endowments. Thus, the collateral of each firm is reduced by \( T \), so that the domestic loan capacity becomes
\[ d_{1,1} \leq w + ak - T. \]
Second, we assume that the government bonds that are sold are only domestic collateral. That is, they are like \( ak \) and hence foreigners do not purchase these bonds.\(^9\)

In this context, suppose a firm purchases \( b \) bonds at date \( 0 \). Then its program can be written as
\[
\max_{k, b} \frac{1}{2}(w - c(k) - \frac{b}{L_0})(L_1 + A) + \frac{1}{2}\left(Ak + b - T + \frac{A}{L_1}(ak + b - T)\right)
\]
\[ \text{s.t. } c(k) + \frac{b}{L_0} \leq w. \]

Market clearing is
\[ L_1 = \frac{ak + b - T}{w - c(k) + \frac{b}{L_0}}. \]

There are two cases to consider, depending on whether the international borrowing constraint is slack or not. First, consider the case that, \[ c(k) + \frac{b}{L_0} < w. \] Since firms are at an interior in their purchase of bonds, it must be that \( L_0 = L_1 \), and therefore \( T = 0 \). Substituting this back into the market clearing condition:
\[ L_1 \left( w - c(k) + \frac{b}{L_1} \right) = ak + b, \]
\[ L_1 = \frac{ak}{w - c(k)}. \]
In other words, intervention has no effect in this case.

In the other case, the international constraint binds. Suppose that the government sells enough bonds that \( c(k^*) + \frac{b}{L_0} = w \). The first-order condition for the private sector is
\[ c'(k) \frac{L_0}{L_1} L_1 + A = \frac{1}{2} \left( A + a \frac{A}{L_1} \right). \]

\(^9\)See the appendix in Caballero and Krishnamurthy [5] for a model justifying this assumption in terms of a risk of suspension of convertibility.
As before, the right-hand side is the return from an extra unit of \( k \). The left-hand side is the opportunity cost of these resources. \( c'(k) \) could otherwise be invested in the government bonds at \( L_0 \), sold at \( L_1 \) at date 1, and the proceeds reinvested at either \( L_1 \) or \( \Delta \). Given the intervention, optimality for the private sector requires that the interest rate on these bonds be

\[
L_0 = \frac{A + \frac{A}{L_1^*} a}{c'(k^*)(L_1^* + \Delta)} L_1^*.
\]

Since \( c'(k^*) = \frac{A + a}{2A} \), we arrive to

\[
L_0 = \frac{A + \frac{A}{L_1^*} a}{A + a} \frac{2A}{A + L_1^*} L_1^*.
\]

For \( L_1^* < \Delta \), we have that \( L_0 > L_1^* \). Since, after purchasing these bonds, the private sector has exactly \( c(k^*) \) left, firms invest the optimal amount of \( k^* \), and the full-information solution is achieved.

Essentially, the implementation has the government “subsidizing” savings by offering a bond with an interest rate exceeding \( L_1 \). It requires no knowledge of date 0 borrowing or investment. However, it does require that the government be able to tax and issue bonds.

On the one hand, since we have assumed that taxes come out of otherwise privately seizable endowments, this tax power is not any stronger than what we gave the private sector.\(^{10}\) On the other hand, it does come with a buried assumption. As in the banking arrangement we first discussed, if agents had the option to not pay taxes and not buy government bonds, but be allowed to trade with the firms who are paying taxes, they would prefer this option. As in the banking arrangement, the sterilization policy is not coalition incentive compatible. However, it seems reasonable to believe that coalition incentive compatibility with respect to taxes is easier to achieve than that of ruling out side trades in a private banking arrangement.

We label this policy as sterilization because, in practice, emerging markets that sterilize do accumulate international reserves on the one hand and issue government bonds on the other. However, our bond policy is “real” and may seem closer to fiscal than to monetary policy. In Caballero and Krishnamurthy [5], we have argued that emphasizing this “real” side of a sterilization policy sheds light on observed outcomes that are puzzling when only the standard, purely monetary, side is considered.

**Proposition 7.** Sterilizing capital inflows at date 0 by issuing two-period government bonds, and consequently reversing the transaction at date 1, achieves the full-insurance solution as long as the planner has the power to tax endowments and bonds are not viewed as international collateral by foreign investors.

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\(^{10}\) There are interesting issues to explore if we allow the government to seize more than the private sector. Holmstrom and Tirole [10] and Woodford [14] have studied the real effects of government bond policy in this case.
5. Final remarks

As in our previous papers, we have synthesized emerging markets’ volatility in terms of two basic ingredients: weak links with international financial markets and underdeveloped domestic financial markets. The need for external insurance stems from the former insufficiency, while the latter is behind the external underinsurance problem.

The contribution of this paper is twofold: First, we have explicitly modeled the informational constraint on domestic insurance markets and thus have been able to discuss the feasibility of contractual arrangements to solve the underinsurance problem. Second, we have explored in a unified setting several of the main international liquidity management strategies available to these countries.

If domestic agents can write complete insurance contracts with each other, then external underinsurance disappears. However, the mechanism behind this result is not a standard insurance channel. The main problem for the economy during a sudden stop is not in the domestic allocation of its limited international collateral but on the aggregate amount of the latter. Domestic insurance improves efficiency by aligning the price of international collateral with its marginal product. In this sense, domestic insurance relates to our discussion in Caballero and Krishnamurthy [7] of the incentive—as opposed to the standard aggregate demand—benefit of a countercyclical monetary policy in economies subject to sudden stops. In fact, we argue further that such policy could in some instances substitute for the absence of domestic insurance.

If domestic insurance is not possible—i.e. when types are unobservable—we show that it is possible to design mechanisms that could attain the same aggregate outcomes and welfare as the full-information case. We also highlight a common Achilles’ heel of these solutions, which is their failure to meet a coalition incentive compatibility constraint. In practice, this means that implementation will be complicated by the existence of secondary markets or ways to opt out of the mechanism. Among the solutions of these type that we study, bond policy is probably more robust than the others, we argue, but it also can have potentially large drawbacks if the intervention is not large enough and if public bonds have illiquid secondary markets during crises (see [5]).

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Appendix A

A.1. Proof of Proposition 1

The equilibrium in the text is derived assuming that in the firm’s program:

- (A) The maximum reinvestment constraint for the distressed firm at date 1 is not binding:
  \[ k > w - c(k) + \frac{\lambda a k}{L_1}; \]  
  \[ \text{(A.1)} \]
- (B) \( L_1 < \Delta \);
- (C) \( L_1 > 1 \).

The text stated that the parameter assumptions in Technical Assumption 1 ensured that these conditions will be met in equilibrium, and that under those parameter assumptions, the equilibrium is unique. We explain this in detail in this appendix.

A.2. Detailed characterization of the spot loan market equilibrium

Assuming (A) holds, the distressed firm solves

\[ V^d(k) = \max_k \left\{ \frac{1}{2} (w - c(k))(L_1 + \Delta) + \frac{1}{2} \left( A + (1 - \lambda) a + \frac{\lambda a}{L_1} \Delta \right) k \right\}. \]

The FOC gives

\[ \frac{1}{2} c'(k)(L_1 + \Delta) = \frac{1}{2} \left( A + (1 - \lambda) a + \frac{\lambda a}{L_1} \Delta \right). \]  
  \[ \text{(A.2)} \]

This equation implicitly defines a function \( k(L_1) \) which is decreasing in \( L_1 \).

If (A) does not hold, then the distressed firm reinvests \( k \). It borrows the resources at interest rate of \( L_1 \). Thus,\(^{11}\)

\[ V^d_0(k) = \max_k \left\{ (w - c(k))L_1 + \left( A - \frac{L_1}{2} \right) k \right\}. \]

Define \( k_0(L_1) \) as the solution to \( k_0 = w - c(k_0) + \frac{\lambda a k_0}{L_1} \) (i.e. the boundary of the maximum reinvestment region). We will return to the latter expression, (A.3), shortly.

The program for the intact firm is as described earlier. For any \( L_1 \geq 1 \), the intact firm will be willing to saturate its foreign debt capacity and lend \( w - c(k) \) in the domestic loan market.

\(^{11}\)In this range, the distressed firm borrows against its domestic collateral at the rate of \( L_1 \). If the firm is distressed its profits are \( ak + k(A - L_1) + (w - c(k))L_1 \). If the firm is intact its profits are \( Ak + (w - c(k))L_1 \).
Assuming that the maximum reinvestment constraint does not bind, the market clearing condition in the domestic loans market at date 1 is

$$L_1(k) = \min \left[ A, \max \left[ 1, \frac{\lambda ak}{w - c(k)} \right] \right].$$

The reason for the min and max operator is as follows. If $\frac{\lambda ak}{w - c(k)} > A$, then it does not pay for the distressed firm to saturate its domestic borrowing capacity. In this case, it borrows less, so that $d_1L = A(w - c(k)) < \lambda ak$, and the equilibrium interest rate is $L_1 = A$. If $\frac{\lambda ak}{w - c(k)} < 1$, it does not pay for the intact firm to lend any funds in the domestic loan market. In this case, it lends less until the point where $x_1 = \lambda ak < w - c(k)$ and $L_1 = 1$.

Under conditions (B) and (C), we can drop the min and max operators and simply write

$$L_1(k) = \frac{\lambda ak}{w - c(k)}. \quad (A.4)$$

Note that $L_1(k)$ is an increasing function of $k$.

Jointly, Eqs. (A.2) and (A.4) define unique equilibrium values of $L_1$ and $k$. Since these equations were derived assuming that conditions (A)–(C) held, we need to ensure that conditions (A)–(C) indeed do hold.

First consider $k(L_1)$ defined from the first-order condition earlier. The largest value this attains over the interval $L_1 \in [1, A]$ is at $L_1 = 1$. The smallest value that $k$ reaches is at $L_1 = A$. Thus, define

$$\tilde{k} = c^{-1} \left( \frac{A + (1 - \lambda) a + \lambda a A}{1 + A} \right)$$

and

$$\bar{k} = c^{-1} \left( \frac{A + a}{2A} \right).$$

As long as $L_1(\tilde{k}) > 1$ and $L_1(\bar{k}) < A$, then there must be an equilibrium point $L_1$ and $k$ such that the solutions to Eqs. (A.2) and (A.4) produce an $L_1$ that lies strictly between one and $A$.

The conditions for such an equilibrium are

$$\lambda ak > w - c(\bar{k})$$

and

$$\lambda ak > A(w - c(\bar{k})).$$

Note that Technical Assumptions 1 and 2 guarantee that these conditions will be met. While Technical Assumption 2 corresponds exactly to the condition we have derived, Technical Assumption 1 is actually stronger than required. That is, $\lambda ak > \lambda ac^{-1} \left( \frac{A + a}{2A} \right) > w - c \left( c^{-1} \left( \frac{A + a}{2A} \right) \right) > w - c(\bar{k})$.

We will explain why we impose the stronger assumption shortly.
Finally, we need to check that the maximum reinvestment constraint does not bind. This constraint can be written as
\[ k(1 - \lambda a/L_1) \geq k(1 - \lambda a) > w - c(k). \]

We know (since \( L_1 > 1 \)) that
\[ \lambda ak > w - c(k). \]

Thus, as long as
\[ k(1 - \lambda a) > \lambda ak, \]

the maximum reinvestment constraint will not bind. For \( \lambda a < \frac{1}{2} \) (Technical Assumption 3), this is satisfied.

### A.3. Maximum reinvestment constraint

Focusing on the case where the maximum reinvestment constraint did not bind, we have shown that the technical assumption gives a single equilibrium satisfying conditions (A)–(C). To show that this equilibrium is unique, we show that there does not exist another equilibrium in which the maximum reinvestment constraint does bind.

Return to expression (A.3). We will show that for any equilibrium value of \( 1 < L_1 < A \), the marginal benefit of increasing \( k \) is strictly positive in the region where the maximum reinvestment constraint binds.

Since \( c(\cdot) \) is convex, \( V_0^d(k) > V_0^d(k_0) \) for all \( k < k_0 \). Consider the derivatives around the boundary point of \( k_0 \). The derivative of the value function from the left as \( k \to k_0 \) is

\[ V_0^- = -c'(k_1)L_1 + A - \frac{L_1}{2}. \]

From the right, in the region where the maximum reinvestment constraint binds, the derivative is

\[ V_0^+ = -c'(k_1)\frac{A + L_1}{2} + \frac{1}{2}(A + \lambda a A/L_1 + (1 - \lambda)a). \]

Now,
\[
V_0^- - V_0^+ = (A - L_1/2) + c'(k)\left(\frac{A + L_1}{2} - L_1\right) - \frac{1}{2}(A + \lambda a A/L_1 + (1 - \lambda)a)
\]
\[ = \frac{1}{2}\left(A - L_1 - \frac{\lambda a}{L_1}(A - L_1)\right) + \frac{1}{2}c'(k_1)(A - L_1)
\]
\[ > \frac{1}{2}\left(A - L_1 - \frac{1}{2}(A - L_1)\right) + \frac{1}{2}c'(k_1)(A - L_1)
\]
\[ > 0. \]

The last step follows from Technical Assumption 3 (\( \lambda a < 1/2 \)) and the fact that \( A > L_1 > 1 \).
Finally, $V_0^+$ is greater than zero since we have an interior solution in the region where the maximum reinvestment constraint does not bind.

Thus $V_0^{id}(k) > V_0^{id}(k_0) > 0$ for all $k < k_0$. This implies that the maximum reinvestment constraint will never bind. Essentially, investment at date 0 is sufficiently profitable that the distressed firm will always choose a $k$ sufficiently high that the maximum reinvestment constraint will not bind at date 1.

A.4. Incentive compatibility constraints

In the mechanism design solution, 
\[
c'(k) = \frac{A + a}{2A}.
\]

The incentive compatibility conditions required that 
\[
A(w - c(k)) \geq ak \geq w - c(k).
\]

This is certainly satisfied if 
\[
ac^{-1}\left(\frac{A + a}{2A}\right) < A\left(w - c\left(c^{-1}\left(\frac{A + a}{2A}\right)\right)\right)
\]

and 
\[
ac^{-1}\left(\frac{A + a}{2A}\right) > w - c\left(c^{-1}\left(\frac{A + a}{2A}\right)\right).
\]

These parameter assumptions correspond to Technical Assumptions 1 and 2 (when $\lambda = 1$).

References


