Asset Quality Cycles∗

Masao Fukui†

April 2017

Abstract

Systemic risk builds up endogenously during booms in an economy featuring asymmetric information in asset markets, where investors’ hidden effort choices determine asset quality distribution. Higher asset prices in booms induce more investors to sell their assets, which lowers their incentive to improve quality. This quality deterioration in turn makes the economy vulnerable to future exogenous shocks because market breakdowns become more likely. Private agents do not internalize the fact that their effort choices worsen future adverse selection problems, and thus the planner may improve welfare by taxing trade and thereby lowering asset prices.

Keywords: Adverse Selection, Endogenous Quality, Moral Hazard, Business Cycle

JEL Classification: D82, E32, E44

∗This paper is based on my master’s thesis at the University of Tokyo. I am extremely grateful to my advisor Kosuke Aoki for his guidance and support throughout this project. I am also indebted to Alp Simsek for his detailed advice and suggestions, which substantially improved the paper. I thank Daron Acemoglu, Marios Angeletos, May Bunsupha, Ricardo Cabarello, Nicolas Caramp, Chishio Furukawa, Tomohiro Hirano, Nobuhiro Kiyotaki, Ernest Liu, Philippe Martin, Yuhei Miyauchi, Emi Nakamura, Harry Pei, Sophie Sun, Robert Townsend, Alexis Akira Toda, Kentaro Tomoeda, and Hajime Tomura for helpful comments; and Emily Gallagher for copyediting. Financial supports from the Funai Overseas Scholarship and the Japan Society for the Promotion of Science (Grant 15J11452) are gratefully acknowledged.

†Department of Economics, MIT. Email: fukui@mit.edu.
1 Introduction

Busts always follow booms, and booms are often blamed for being the seed for crises. Existing economic models emphasize “quantity” as a source of fragility, such as the amount of leverage.\(^1\) However the concern that deterioration in the “quality” of assets in boom periods creates fragility in the economy is equally widespread in current policy debates.\(^2\)

Why does the quality of assets deteriorate in booms? How does this create fragility? Is it inefficient? This paper provides a unified framework that speaks to these three questions.

I consider an economy theta features asymmetric information about asset quality in asset markets, where investors’ hidden effort choices endogenously determine quality distribution of assets. Entrepreneurs have a chance to trade projects depending on the realization of idiosyncratic productivity shocks. Productive entrepreneurs naturally become buyers of projects, and unproductive ones sell projects. Each project is either of high or low quality, where quality refers to the efficiency unit. The quality of a project is determined endogenously when entrepreneurs invest. Since improving quality is costly, entrepreneurs who sell projects do not exert effort to maintain quality because they can sell at the same price regardless of the underlying quality. Entrepreneurs who are productive enough to keep their projects in their own hands optimally choose to exert effort.

In my model, quality distribution is endogenously determined by the extensive margin of entrepreneurs’ effort: how many entrepreneurs are sellers. Now consider any shock that leads to higher asset prices. Such a shock induces the marginal entrepreneurs to sell the assets that they otherwise would have kept, and these entrepreneurs stop exerting effort to maintain quality. As a result, the quality of assets in the economy deteriorates. This quality deterioration naturally increases the fragility of the economy because market breakdowns are more likely in the subsequent periods through the standard Akerlof’s (1970) lemon problem. The relationship between asset prices and traders’ endogenous

\(^1\)See, for instance, Lorenzoni (2008), Farhi and Werning (2016), and Korinek and Simsek (2016).
\(^2\)For example, Kindleberger (2015) writes “Minsky emphasized the ‘quality’ of debt to gauge the fragility of the credit structure.”
effort responses is at the heart of this model.

My first main result demonstrates this idea using a deliberately stylized two-period model where entrepreneurs invest and choose effort in the first period, and they trade their projects in the second period. I consider a shock to aggregate productivity to generate movement in asset prices. The analysis further shows that although quality in the economy declines in response to positive shocks, the quality of assets traded in the market increases. An increase in market quality in turn amplifies the response by mitigating adverse selection problems relative to an economy in which the market quality is fixed. It is tempting to think that a deterioration of quality in the economy acts to dampen the booms, but it is actually accompanied by amplification. This happens because those who exert efforts do not sell their projects in the market. Therefore a decline in the aggregate effort level does not translate into a decline in the quality of projects traded in the market. An empirical implication of this result is that looking at the quality of assets that circulating in the market actually tells us little about the quality of assets in the overall economy.

Adverse selection problems are isomorphic to trade taxes, as shown by Kurlat (2013), because buyers cannot avoid paying for lemons. In contrast, the underlying moral hazard implies that trading involves real costs because resources are lost as entrepreneurs decide to sell their projects. I formally show that the previous economy is equivalent to a symmetric information economy with iceberg trade costs and trade taxes. The magnitude of iceberg trade costs is governed by the degree of moral hazard problems. The direct implication of this equivalence is that a planner can improve ex-ante welfare by subsidizing trades. This may at first seem surprising because it means that the planner optimally accepts further deterioration of quality in the economy. This happens because moral hazard is isomorphic to trading technology rather than to distortion, and both private agents and the planner understand that real resources are lost when a trade takes place. Therefore the planner lacks a superior tool over private agents to deal with underlying moral hazard
problems. Instead the planner is able to mitigate adverse selection problems by subsidizing trade because private agents do not internalize the fact that their demand affects the market quality.

By extending the framework above to a dynamic setting, however, I overturn the above conclusion. I assume that projects are also sequentially traded in the next period, when entrepreneurs draw a new productivity. In this dynamic setting, the planner faces a new trade-off: subsidizing trade today worsens future adverse selection problems as a result of quality deterioration. My second main result states that when the planner has little room to improve current market conditions, it is desirable to tax trade and lower asset prices in order to mitigate future adverse selection problems. Contrasting this with the previous result, although the moral hazard problems were part of the technology in the static model, now the underlying moral hazard issues affect future adverse selection problems, which is a distortion. Since private agents do not internalize the fact that their choice of effort affects the future market conditions, the planner plays a role in correcting this externality. The result holds even though I allow ex-post intervention in the future period. This ex-post intervention is inferior to ex-ante intervention because ex-ante intervention improves the quality in the economy, which is the first-best way to deal with adverse selection problems. In general, whether to cool down or boost the asset market depends on the relative benefit of improving market liquidity today or in the future. When the current market liquidity is drying up relative to the future, the planner will find it optimal to boost the current market by sacrificing future market conditions.

After cleanly isolating the main mechanism, I extend the setting into an overlapping generations (OLG) framework to study the accumulation of systemic risk over time. Investment is made in every period, and it accumulates over time in the form of capital stock. Aggregate productivity shocks induce fluctuations in asset prices, which in turn create endogenous movements in the average quality of capital stock as a result of investors’ effort choices. In this environment, systemic risk endogenously and gradually
builds up during long-lasting booms because the quality of the pool of existing capital stock deteriorates during booms. The economy features history dependence: negative shocks of the same size tend to generate severer recessions after long-lasting booms. How fast systemic risk builds up crucially depends on the cyclicality of investment quantity. So far I emphasized that investment quality is counter-cyclical. When the quantity of investment is pro-cyclical, as it is in the data, my mechanism is further amplified. In good times, the quality of investment is low and the quantity is high, while in bad times the quality of investment is high and the quantity is low. Thus systemic risk builds up more rapidly in booms, and the recovery from recessions is slower than in the case where investment quantity is acyclical. This suggests that not only stabilizing asset prices, but also stabilizing investment quantity might be beneficial for economic stability.

Although the model in this paper is abstract, I believe it is useful for understanding numerous boom-bust episodes in the real world. During dot-com bubbles in the United States in the late 1990s, many information technology (IT) startup companies were launched, expecting to be bought out by others. The inability of most of those to sustain themselves as profitable independent companies contributed to the bust of the dot-com bubble. During the real estate bubble in Japan in the late 1980s, investment in real estate became extremely popular, but most of these investments were made just for resale. After prices started to decline, the bankruptcies of real estate companies prolonged the subsequent recession. In the recent Great Recession of the early 2000s, securitization enabled debt to be sold in secondary markets. It is often argued that this created a decline in the quality of debt and was at least partially responsible for the crisis. Indeed, Keys, Mukherjee, Seru, and Vig (2010) provided causal evidence that securitization led to the lax screening of borrowers by banks. More generally, policymakers often try to cool down overheated asset markets, but the rationale behind such interventions has not been theoretically clear. The positive analysis in my model precisely speaks to why the quality of assets declined in those episodes, and why that necessarily results in fragility. My nor-
mative analysis reveals the rationale for why policymakers should attempt to cool down asset markets in booms. These policy implications are distinct from the literature concerning excessive leverage (Lorenzoni (2008); Korinek and Simsek (2016); Farhi and Werning (2016)).

To fix the idea, I will use the dot-com bubble episode as a leading example of my theory. Think of idiosyncratic productivity in the model as the business idea of an entrepreneur. Each entrepreneur starts an internet-related company with his or her idea. Some entrepreneurs’ ideas are good, like Amazon.com and Google, while others are not, such as Pets.com. Each owner is free to sell shares in the company to investors. When the internet was a new technology, market participants found it difficult to accurately assess the value of companies. Moreover, since it was common for internet companies to incur huge initial losses, profits were not a good indicator of quality. Therefore I view asymmetric information as an important friction in the dot-com bubble episode. In the late 1990s, investors looking for “hot” new stocks triggered a period of high demand for tech stocks. This made the market highly liquid. Entrepreneurs with inferior ideas reacted to this by simply creating lemons because they were expecting to sell off the companies to investors eventually. After a while, as investors realized that the majority of these loss-making new tech companies would never become profitable, the market for tech stocks collapsed. Despite its simplicity, I view that my model as one that well describes this story.

1.1 Related Literature

This paper fits into several strands of literature. First, recent studies show that adverse selection problem is potentially crucial to explaining the sudden collapse of the financial market (Bigio (2015, 2016); Guerrieri and Shimer (2014); Kurlat (2013, 2016)). The same motivation has led several papers to explore optimal intervention in the presence of

---

3Even Google and Amazon took several years to show any kind of profits.
adverse selection (Tirole (2012); Philippon and Skreta (2012)). In their models, quality distribution is exogenously given. Now asking where these low-quality assets come from is the natural next step. I offer a model that explains why lemons exist in the first place and show how the creation of lemons interacts with the business cycles.

Several papers also tackle a question similar to mine. Matsuyama (2013) and Martin (2008) provide models of counter-cyclical credit quality. In their theories, the driving force is pro-cyclicality of a borrower’s net worth. Under certain assumptions, higher a borrower’s net worth the more difficult it is for lenders to screen out “bad” projects. In contrast, in my model pro-cyclical asset prices are the driving force for generating counter-cyclical asset quality. I share the same motivation as Gorton and Ordoñez (2014) to explain why busts follow booms. The mechanisms are very different, however. In their theory, a credit boom is accompanied by opacity of information. Moreover, the equilibrium in their model is constrained efficient, while the one here is not.

Eisfeldt (2004), Eisfeldt and Rampini (2006), Cui (2014), and Lanteri (2014) document and explain why capital reallocation is so volatile and pro-cyclical. In my model I take those aspects as given and study the implications for endogenous determination of asset quality.

In independent works, Neuhann (2016) and Caramp (2016) pursue an idea similar to the one in this paper. Neuhann (2016) shares a key insight as with this paper that higher asset prices lead to less effort by investors. In his model, both sellers and buyers are financially constrained. As the net worth of buyers increases, they bid up asset prices. An increase in asset prices reduces sellers’ exposure to risk but at the same time increases their incentive to undertake the investment of low-quality risky projects for sale. As borrow-

---

4 Tirole (2012) provides a brief analysis of an economy where quality distribution is endogenously chosen in adverse selection economy.

5 Relatedly, Kawai (2014) studies a micro model and shows that moral hazard by sellers can destroy gains from trade. My focus is more on business cycles and constrained efficiency, rather than on gains from trade per se. Zryumov (2015) studies a model where entrepreneurs timing decision of entry shapes the time-varying market quality. The quality in the economy is exogenous in his framework, but in my model the quality in the economy is an equilibrium object.
ers’ net worth accumulates over time, systemic risk builds up because low-quality assets are by design more exposed to aggregate shocks. In my model, systemic risk builds up during booms because the quality of the pool of assets worsens over time which exposes the economy to increasingly severe adverse selection problems. The low-quality assets themselves are not more exposed to aggregate risk.

Caramp (2016) studies a model in which agents are exposed to liquidity shocks and because they are financially constrained they instead trade assets to self-insure against shocks. He studies the incentives for ex-ante production of assets and shows that a positive shock today may lead to a collapse of the current market. My model focuses on dynamics. A positive shock is unambiguously good for the current market but increases fragility in the future. Furthermore, while Caramp (2016) focuses on a one-time transitory shock, I demonstrate how long-lasting booms (persistent shocks) contribute to the gradual build up of systemic risk.

More generally, a key difference between the above-mentioned papers and my model is parsimony. All of these papers feature financial friction in addition to adverse selection problems and moral hazard. I focus on a minimal set of ingredients: the interaction between adverse selection and moral hazard. Although financial friction is certainly an important topic when studying financial crises, I believe parsimony is also valuable for the following two reasons. First, the model here is transparent enough to see how each assumption maps to each result. Indeed, my model clarifies that financial friction is not necessary in order to generate endogenous fluctuations in asset quality. Second, although introducing financial friction provides a number of interesting policy implications, such as those for capital requirements or liquidity provision, it also masks the identification of underlying sources of inefficiency that arise from quality deterioration. I demonstrate that lower level of effort is inefficient precisely because private agents do not internalize

---

6For instance, partially destroying buyers’ wealth in Neuhann (2016) and issuing enough government bonds in Caramp (2016) restore the efficiency in their respective frameworks. However these policy implications rely on the presence of financial friction.
the fact that it worsens future adverse selection problems.

Layout. Section 2 presents simple two- and three-period versions of the model to highlight the core mechanism of the paper. Section 3 presents an OLG extension to study the endogenous and gradual accumulation of systemic risk over time. Section 4 concludes.

2 Mechanisms at Play

In this section I present a deliberately stylized model in order to isolate the main mechanism. I start from a two-period model to demonstrate its positive implications. Later I extend to three-period models to study its normative implications.

2.1 Environment

The economy is populated by a mass of ex-ante identical entrepreneurs. Because entrepreneurs live for two periods, the model is a two-period model \( t = 0, 1 \). There are two goods: consumption goods and capital goods. Consumption goods are perishable. There are two types of capital: high- and low-quality capital. They differ in efficiency units when used for production, as will be clear below. I also assume that capital goods are completely irreversible: once consumption goods are transformed into capital, they cannot be consumed.

Entrepreneurs When Young. Entrepreneurs are initially endowed with one unit of consumption good at \( t = 0 \). Their preferences are given by

\[ E_0 c_1. \]

In words, entrepreneurs consume only when they are old and are risk neutral. Young entrepreneurs can save consumption goods via investment technology, which converts a
Entrepreneurs When Old. At the beginning of $t = 1$, entrepreneurs independently draw productivity $z$ from the CDF, $G(z)$, and its PDF is denoted by $g(z)$. The production technology is constant returns to scale in capital and depends on the quality of capital. Specifically,

$$\text{output per unit capital} = \begin{cases} \phi z & \text{from high-quality capital} \\ 0 & \text{from low-quality capital} \end{cases},$$

where $\phi$ is the aggregate productivity component. Therefore, for simplicity, low-quality capital is assumed to be completely useless.

After observing productivity, but before operating any production technology, two sequential events occur. First, entrepreneurs can exert effort to improve the probability of obtaining high-quality capital from their ongoing investments. In particular, fraction $\pi \in \{\pi_h, \pi_l\}$ of an investment is realized as high-quality capital, where $0 < \pi_l < \pi_h \leq 1$. This fraction is $\pi_h$ if entrepreneurs exert effort, and it is $\pi_l$ if they do not. Effort incurs fixed cost $\kappa > 0$ per unit of investment. $\kappa$ can also be interpreted as a maintenance cost. I assume $\kappa$ is arbitrarily small—i.e., $\kappa \rightarrow 0$. I make this assumption for expositional simplicity, and it is not essential. The choice of investment quality is the first key ingredient in the model.

Second, after investment takes place, entrepreneurs can trade capital in a market with asymmetric information where buyers cannot observe the quality of capital. The market for capital is competitive and anonymous. Here, competitiveness implies that entrepreneurs take the price of capital as given. Anonymity ensures that capital is traded at pooling price $p$. An asset market featuring asymmetric information about asset quality is the second key ingredient in the model.
In equilibrium, entrepreneurs with higher productivity become buyers. Buyers observe the quality of the capital after purchase. In order to install high-quality capital, entrepreneurs need to incur quadratic adjustment costs:

$$\Gamma(\lambda^M k^d) = \frac{\gamma}{2} (\lambda^M k^d)^2,$$

where \(\gamma > 0\) is a parameter, \(k^d\) is the amount of capital purchased, and \(\lambda^M\) is the fraction of high-quality capital as a portion of the purchased capital, which will be endogenously determined. The assumption that reallocation of capital incurs quadratic adjustment costs closely follows Eisfeldt and Rampini (2006) and ensures that the demand for capital is bounded. After trading capital, entrepreneurs produce, consume output, and die.

**Supply of Assets.** I analyze the subsequent entrepreneurs’ decisions in a backward manner. Before describing quality-choice decisions, I describe the optimal trading decisions given the type of capital these entrepreneurs own. First, all entrepreneurs find it optimal to sell low-quality capital if price \(p\) is strictly positive because low-quality capital produces nothing and cannot be consumed. Second, entrepreneurs with \(z \leq p/\phi\) choose to sell high-quality capital as well. The reason is clear if we compare two options: either to keep high quality capital to produce \(\phi z\) or to sell it at price \(p\). The latter option will result in a higher amount of consumption if inequality \(z \leq p/\phi\) holds. The same argument makes clear entrepreneurs with \(z > p/\phi\) keep high-quality capital.

Given the decisions above, entrepreneurs optimally choose whether or not to exert effort in their ongoing investment projects. If an entrepreneur is going to sell both low- and high-quality capital, then s/he does not have any incentive to incur strictly positive effort cost \(\kappa\) because whatever its underlying quality, s/he sells at the same price, \(p\). On the contrary, if an entrepreneur’s productivity is high enough to keep high-quality capital, it is optimal to pay the effort cost, \(\kappa\). Thus entrepreneurs exert effort if and only if they plan to keep high-quality capital, which is the case when \(z > p/\phi\). The fraction of high-
quality projects in the economy is defined as

\[ \bar{\lambda}(p/\phi) \equiv \pi_l G(p/\phi) + \pi_h (1 - G(p/\phi)), \quad (2) \]

because those who exert effort yield the fraction \( \pi_h \) of high-quality projects, and those who do not yield the fraction \( \pi_l \) fraction of high-quality projects. The fraction of high-quality projects in the market is defined as

\[ \lambda^M(p/\phi) = \frac{\pi_l G(p/\phi)}{G(p/\phi) + (1 - \pi_h)(1 - G(p/\phi))}. \quad (3) \]

The denominator is the total supply of projects, and the numerator is the supply of high-quality projects because only entrepreneurs with sufficiently low productivity sell high-quality capital, who have only \( \pi_l \) fraction of them.

For now, let us focus on partial equilibrium, where we take \( p \) as exogenous. The following result straightforwardly follows from the above expression and shapes the backbone of the model.

**Proposition 1.** In partial equilibrium, an increase in asset price, \( p \), (i) decreases the fraction of high-quality projects in the economy, \( \bar{\lambda} \); (ii) but increases the fraction of high-quality projects in the market, \( \lambda^M \).

All proofs appear in Appendix A. It should be noted that these results require few structures of the model. Any shocks that lead to higher asset prices will induce marginal entrepreneurs to sell assets they otherwise would have kept, and thus they stop exerting effort to maintain quality. Given that asset prices are highly pro-cyclical in the data, the above proposition predicts counter-cyclicality in average asset quality in the economy. However this does not translate into a decline in the market quality. To understand this, there are two composites of assets sold in the market: (i) the pool of assets sold by entrepreneurs with productivity lower than \( p/\phi \) with a fraction of high-quality capital \( \pi_l \);
(ii) the pool of assets sold by entrepreneurs with productivity higher than $p/\phi$ with a fraction of high-quality capital 0. In response to an increase in asset prices, $p$, the composition shifts from the latter to the former, resulting in an increase in the quality of projects in the market.

**Demand for Assets.** Here I describe the demand side of the assets that move around prices. Each entrepreneur chooses how much capital to purchase, which solves

$$\max_{d \geq 0} \left\{ \phi \lambda^M d - pd - \Gamma(\lambda^M d) \right\}.$$  

Given the specification in equation (1), the demand for capital by an entrepreneur with productivity $z$ is given by

$$d(z; p, \lambda^M) = \max \left\{ \frac{1}{(\lambda^M)^2 \gamma} (\phi z \lambda^M - p), 0 \right\}.$$ (4)

Therefore an entrepreneur with productivity $z > p/(\phi \lambda^M)$ will demand a strictly positive amount of capital.

**2.2 Equilibrium**

As Proposition 1 illustrates, the key is the relationship between asset prices and the quality of assets in the economy. Now I endogenize the asset price as an equilibrium object.

**Aggregation.** It is convenient to normalize prices by the aggregate productivity, $\hat{z} \equiv p/\phi$. The aggregate supply function of capital is given by

$$S(\hat{z}) = G(\hat{z}) + (1 - \pi_h)(1 - G(\hat{z})).$$

The first term represents the fact that entrepreneurs with productivity lower than $\hat{z}$ sell all capital they hold. The second term represents the fact that entrepreneurs with high
enough productivity will sell only low-quality capital which is the fraction $1 - \pi_h$ of their capital holdings because they exert effort.

The aggregate demand for capital can be derived simply by integrating (4):

$$D(\hat{z}) = \int_{\hat{z}/\lambda M(\hat{z})} \frac{\phi}{(\lambda M(\hat{z}))^{2\gamma}} (z\lambda M(\hat{z}) - \hat{z})dG(z),$$

where the market quality, $\lambda M$, is given by (3). Finally, the proportion of high-quality capital in the economy, $\bar{\lambda}$, is given by (2).

**Definition of Equilibrium.** The definition of equilibrium is as follows:

**Definition 1.** An equilibrium consists of the price of capital, $p$, the market proportion of high-quality capital, $\lambda M$, the entrepreneurs’ choice of effort, amount of capital sold, and the amount of capital demanded such that (i) given $p$, the entrepreneurs choose an effort level and trading volume to maximize consumption, and (ii) the market for capital clears: $S(\hat{z}) \geq D(\hat{z})$ with equality whenever $p > 0$, where $\hat{z} \equiv p/\phi$.

The left panel of Figure 1 illustrates the equilibrium characterization by plotting the supply curve, $S(\hat{z})$, and the demand curve, $D(\hat{z})$. The intersections between the two
curves are equilibria in this economy. As is common in the adverse selection literature, there exist multiple equilibria in general. This comes from the non-monotonicity of the demand curve represented in equation (5). A higher \( \hat{z} \) while holding \( \lambda^M \) constant decreases the demand for capital, but it simultaneously increases the average quality of the capital traded in the market \( \lambda^M \). Since the latter effect acts to increase demand, the demand function is non-monotone in general. For example, there always exists an equilibrium in which the market breaks down—i.e., \( p = 0 \), as long as \( \pi_h < 1 \). Since multiple equilibria are not a focus of my interest here, I follow the literature and focus on the highest price-quantity equilibrium.\(^7\) Wilson (1980) and Stiglitz and Weiss (1981) argue that this might not be a reasonable equilibrium concept when buyers have an incentive to further raise prices to attract a better asset pool. In Appendix B, I show that the sufficient condition such that this issue does not arise is

\[
\frac{d \log \lambda^M(\hat{z})}{d \log \hat{z}} \leq 1, \quad (6)
\]

for all \( \hat{z} > \hat{z}^* \), where \( \hat{z}^* \) is the price in competitive equilibrium. Intuitively, the quality pool of assets should not react too strongly to higher prices.

In a special case with \( \pi_h = 1 \), those entrepreneurs who exert effort are certain to obtain high-quality capital, and they never sell low-quality capital. Therefore market quality is constant at \( \lambda^M(\hat{z}) = \pi_l \), and thus the demand function becomes monotone in price. The right panel of Figure 1 illustrates this case, which leads to the following result:

**Remark 1.** Equilibrium is unique if \( \pi_h = 1 \). Most of my results hold even with \( \pi_h = 1 \).\(^8\)

\(^7\)See Kurlat (2013) or Bigio (2016), for example. All subsequent results here are true at any stable equilibrium.

\(^8\)The only results that do not hold are the ones regarding changes in \( \lambda^M \). If \( \pi_h = 1 \), \( \lambda^M \) is necessarily a constant.
2.3 Equivalence to an Economy with Trade Costs and Trade Taxes

I show that the previous economy is isomorphic to a symmetric information economy with iceberg trade costs and trade taxes. As Kurlat (2013) shows, adverse selection is equivalent to having trade taxes. As it will turn out, moral hazard is equivalent to introducing iceberg trade costs in addition to trade taxes.

Consider a symmetric information economy, where the quality of projects is observable by buyers. In this circumstance all entrepreneurs exert effort to obtain an amount $\pi_h$ of high-quality projects and an amount $1 - \pi_h$ of low-quality projects. Because low-quality projects are useless, they can be omitted from the subsequent analysis. Assume now that the government imposes an *ad-valorem* tax of $\tau_{sym}$ on the purchase of projects. The total revenue collected from the tax, $T = \tau_{sym} p_{sym} S_{sym}$, is rebated equally to all entrepreneurs, where $p_{sym}$ denotes price in this symmetric information economy and $S_{sym}$ is the total supply of assets, which I will describe later. The buyers’ demand for high-quality projects solves

$$
\max_{k_d \geq 0} \left\{ \phi z k_d - p_{sym} (1 + \tau_{sym}) k_d - \Gamma(k_d) \right\}.
$$

Solving the above problem and aggregating over $z$ yields total demand for the projects:

$$
D_{sym}(z_{sym}, \tau_{sym}) = \int_{z_{sym}(1+\tau_{sym})}^{z_{sym}} \phi \left( z - (1 + \tau_{sym}) z_{sym} \right) dG(z),
$$

with $z_{sym} \equiv p_{sym} / \phi$.

On the supply side, in order to deliver a unit of project to other entrepreneurs, sellers must ship $\chi \geq 1$ units of projects. When they use their own projects, they don’t incur these iceberg trade costs. Entrepreneurs with $\phi z \geq p_{sym} \frac{1}{\chi}$ optimally choose to keep their projects, and others with $\phi z < p_{sym} \frac{1}{\chi}$ decide to sell their projects. The total supply of assets is

$$
S_{sym}(z_{sym}) = \pi_h \frac{1}{\chi} G(z_{sym} / \chi),
$$

with $z_{sym} \equiv p_{sym} / \phi$. 
Because there are amounts $\pi_h$ of projects, and each entrepreneur sell $1/\chi$ amount of projects. The following proposition shows that this economy, with a particular choice of $(\tau, \chi)$, delivers an allocation that is equivalent to that in the previous economy.

**Proposition 2.** Suppose $\tau^{\text{sym}} = (1 - \lambda^{M^*} \chi) / (\lambda^{M^*} \chi)$ and $\chi = \pi_h / \pi_l$, where $\lambda^{M^*}$ is the equilibrium value of the asymmetric information economy. Then the allocations (consumption and production for each entrepreneur) of the symmetric information economy with trade costs and trade taxes and the asymmetric information economy are equivalent. Prices are related through $\hat{z}^{\text{sym}} / \chi = \hat{z}^*$, where $\hat{z}^*$ is the equilibrium price in the asymmetric information economy.

Because $\lambda^{M^*} \leq \pi_l$, it follows that $\frac{\pi_h}{\pi_l} \lambda^M \leq 1$, and thus $\tau^{\text{sym}} \geq 0$, which implies that the economy faces taxes. If $\pi_h = \pi_l$ so that there is no moral hazard, then there is no role for trade costs, $\chi = 1$, which is reminiscent of the equivalence result in Kurlat (2013). With moral hazard, $\pi_h > \pi_l$, the fact that entrepreneurs stop maintaining quality when they decide to sell is like having iceberg trade costs because real resources are lost when assets are sold. The direct implication of this fact is that moral hazard changes the technology as opposed to creating distortion. I come back to this issue when discussing the model’s normative implications.

### 2.4 Aggregate Shocks

Now I turn to the comparative statics analysis. To capture business cycle fluctuations in asset prices, I consider a shock to aggregate productivity, $\phi$. A few remarks are in order. I consider a change in $\phi$ while holding $\gamma$ fixed. As is clear from the expression in equation (5), if $\phi$ and $\gamma$ increase proportionally, then there will be no effect on equilibrium. Eisfeldt and Rampini (2006) argue that in order to match the observed data on reallocation, the aggregate productivity (here $\phi$) and the cost of reallocation (here $\gamma$) must negatively co-move along the business cycle. Therefore I view changing $\phi$ while holding $\gamma$ to be the reasonable way to capture business cycle variation in reallocation. Moreover, as Proposi-
tion 1 illustrates, no matter what the underlying shocks are, my main results are robust as long as they produce pro-cyclical fluctuations in asset prices and reallocation. In fact, pro-cyclicality of asset prices and reallocation is one of the most robust empirical regularities. Thus one can view changes in \( \phi \) as a reduced form way to generate pro-cyclical asset prices.

The following proposition shows how a change in aggregate productivity influences reallocation, asset quality in the market, and asset quality in the economy:

**Proposition 3.** An increase in aggregate productivity, \( \phi \), (i) increases the price of capital, \( p/\phi \), and the volume of trade, (ii) reduces the average quality of capital in the economy, \( \bar{\lambda} \), and (iii) increases the average quality of capital traded in the market, \( \lambda^M \).

The first result is straightforward. An increase in \( \phi \) increases the demand, \( D \), without affecting the supply curve \( S \), and thus equilibrium cut-off, \( \hat{z} \), and trading volume must rise to clear the market. The second and third results follow from the combination of the previous result and Proposition 1. Given that the cut-off productivity is higher, \( p/\phi \), fewer entrepreneurs are exerting effort, but more entrepreneurs are selling high-quality projects.

The fact that market quality increases in response to an increase in aggregate productivity plays the role of amplification. Consider two economies, one with asymmetric information and another with symmetric information with trade costs and constant taxes such that, in the absence of shocks, allocations are identical. The asymmetric information economy features endogenous tax rates, where tax rates decrease in response to an increase in market quality, \( \lambda^M \). Combined with the previous proposition, we have

**Corollary 1.** Relative to a symmetric information economy with trade costs and fixed trade taxes, in the economy with asymmetric information, in response to an increase in \( \phi \), asset prices, aggregate output and trading volume increase more.

It is tempting to think that an economy-wide decline in average quality dampens the
response of the economy to a positive productivity shock, but as the above corollary shows, this is not the case. Instead, an increase in market quality mitigates adverse selection problems and thus amplifies the responses of the economy through fostered capital reallocation.\footnote{The amplification mechanism is exactly that described in Kurlat (2013).}

2.5 Normative Implications of a Benchmark Model

What are the normative implications of the model? Before moving to the second-best policy, I briefly explain the first-best allocation. In the first-best allocation (i) all entrepreneurs should exert effort because $\kappa \to 0$,\footnote{$\kappa \to 0$ is not important. This is true as long as $\kappa$ is small enough.} and (ii) entrepreneurs with productivity below $z^{FB}$ should not produce, entrepreneurs with productivity $z \geq z^{FB}$ should produce an amount $z(\tau_h + d(z))$ of output with $d(z) = \frac{1}{\gamma}(\phi z - z^{FB})$, and $z^{FB}$ satisfies $\pi_h G(z^{FB}/\phi) = \int_{z^{FB}} d(z) dG(z/\phi)$. It is clear that the first-best allocation is not attainable in equilibrium. Moreover, the volume of trade is lower than the first best, $\hat{z}^* < \hat{z}^{FB} \equiv z^{FB}/\phi$, where $\hat{z}^*$ is the cut-off of sellers in equilibrium.

In what follows, I ask whether the equilibrium volume of trade is constrained efficient or not. I consider a planner’s problem, where the planner faces the same informational friction as in equilibrium. The planner cannot observe entrepreneurs’ identities, types of capital, and entrepreneurs’ effort choice. I first focus on ex-ante efficiency, where the planner maximizes ex-ante expected welfare. With risk neutrality, this is equivalent to maximizing the total amount of consumption. In this case, Proposition 2 already provides the answer. Recall that Proposition 2 points out that the economy is isomorphic to an economy with trade costs and trade taxes. A natural conjecture from this result is that the planner can undo any distortion coming from trade taxes by subsidizing trade so as to enhance reallocation. Let $\tau$ denote the ad-valorem tax on project purchases in an asymmetric information economy. The following proposition shows that this is indeed
Proposition 4. The equilibrium volume of trade is too low relative to the ex-ante constrained efficient allocation. The planner can maximize ex-ante expected welfare by subsidizing trade, \( \tau = \lambda^M (\hat{z}^{sym}/\chi) - 1 \leq 0 \), where \( \hat{z}^{sym} \) is the asset price in a symmetric information economy without taxes.

Despite the moral hazard concern, the planner always finds it optimal to facilitate trade. In so doing, the planner can improve reallocation, which was distorted by adverse selection, at the cost of lowering average quality in the economy. The economics behind this result is easy to understood if we recall that adverse selection alone is isomorphic to the introduction of trade taxes, but moral hazard is isomorphic to the introduction of trade costs, which is a part of the technology. Both the private agents and the planner understand that some resources will be lost when assets are traded. The planner does not have a tool to deal with moral hazard that is superior to that of the private agents. Therefore the planner optimally deals with the adverse selection problems by subsidizing trade. In a special case where \( \pi_h = 1 \), the equilibrium is constrained efficient (\( \tau = 0 \)). This happens because under \( \pi_h = 1 \) market’s quality does not vary with trading volume, \( \lambda^M = \pi_l \) (no adverse selection). If this is the case, facilitating trade does not improve the market quality.

Previous arguments have considered ex-ante welfare, a criterion before the idiosyncratic productivity shocks have been realized. This may not be an appropriate criterion because a policy may benefit some entrepreneurs by hurting others ex-post. Indeed the subsidization policy described above does not achieve Pareto improvement. Entrepreneurs who neither buy nor sell high-quality assets under trade subsidization are worse off because they must pay a lump-sum tax, which outweighs the benefit from the increased sales price of low-quality projects.

Is Pareto improvement possible?\(^{11}\) In Appendix C, I show that allocation under any

\(^{11}\)Bigelow (1990) derives a sufficient condition for competitive equilibrium to be interim constrained
tax rate in a symmetric information economy can be achieved with some trade taxes in
the asymmetric information economy. This facilitates the analysis because we no longer
need to look at the optimal tax policy in an asymmetric information economy, but in-
stead can focus on a symmetric information economy. Then the standard argument in
the optimal taxation literature implies that a change in the tax rate can bring about Pareto
improvement if the tax rate is beyond the peak of the a Laffer curve. Hence the equilib-
rism in the original economy is ex-interim constrained efficient if the implied tax rate,
\( \tau_{\text{sym}} = (1 - \lambda^M \chi) / (\lambda^M \chi) \), is not too high for the government to collect more revenue
by reducing taxes. For example, when the market is breaking down, the implicit tax rate
is infinite, \( \tau_{\text{sym}} = \infty \). In this case, the government in a symmetric information economy
can increase tax revenue by slightly reducing taxes, which achieves Pareto improvement.
In the asymmetric information economy, this is equivalent to ‘slightly’ subsidizing trade.
These arguments can be made without any reference to moral hazard. How does moral
hazard affect this argument? The moral hazard consideration is likely to bring the econ-
omy to ex-interim constrained efficient allocation. Conditional on the same severity of
adverse selection problems—i.e., the same value of \( \lambda^M \)—the implicit tax rate is lower
when there is moral hazard, \( \chi > 1 \), than when there is no moral hazard, \( \chi = 1 \). This
is because moral hazard implies that trading becomes more technologically costly and
thus trade is less distorted. Indeed, in the extreme case with \( \pi_h = 1 \), the implied tax rate
is zero, so that the economy is both ex-ante and ex-interim constrained efficient. These
results may at first seem surprising because the quality deterioration during booms does
not itself justify the policy intervention. However, I will overturn this conclusion when I
extend the model to a dynamic framework in the following subsection.

efficient in the setup similar to that of Akerlof (1970).
2.6 Extension: Sequential Trade

I extend the previous model by allowing capital to be traded sequentially. I argue that this consideration has novel normative implications. The key idea is that although moral hazard cannot be a rationale for improving the current asset market’s condition, once we take dynamics into account, the creation of low-quality capital will harm the future market’s conditions. This may generate an additional source of inefficiency as well as have different policy implications.

I extend the model to a three-period model \((t = 0, 1, 2)\). The first two periods, \(t = 0, 1\), are the same as before where agents invest, draw idiosyncratic productivity from \(G_1\), choose effort, and trade projects. In the final period, \(t = 2\), agents draw a new idiosyncratic productivity from the distribution function \(G_2\) independently from the past, and trade the capital again. I assume the cost of installing capital at \(t = 2\) is

\[
\Gamma(d_2) = \frac{\gamma}{2}(d_2)^2,
\]

where \(d_2\) is the total capital demanded. Note that unlike at \(t = 1\), entrepreneurs have to incur the cost of installing capital which includes both high- and low-quality capital. The assumption here is that entrepreneurs do not observe quality before installation. If the cost of installing capital at \(t = 2\) involves only high-quality capital, then improvement in market quality at \(t = 2\) has two countervailing effects. On one hand, it increases demand because the purchased assets are more likely to contain high-quality capital (the market quality effect). On the other hand, since there is more high-quality capital circulating, its marginal product is lower (the diminishing marginal product effect). This latter effect complicates the analysis while providing few insights. Thus I abstract away this diminishing marginal product effect so that a lower quality pool at \(t = 2\) unambiguously creates fragility.\(^{12}\) I also assume no depreciation of capital takes place and there is no dis-

\(^{12}\) Readers may feel uncomfortable because the information structure at \(t = 2\) and \(t = 1\) are different. However, I assume \(\pi_1 = 1\) through this section and thus \(\lambda_1^M = \pi_1\), which is a constant. Therefore it is
counting on preferences over time. Each agent’s preferences are given by $E[c_1 + c_2]$. Here I assume that the realization of the aggregate productivity shock at $t = 2$, $\phi_2$, is stochastic and continuously distributed with some distribution function.

**Equilibrium.** I solve for equilibrium using backward induction. At period $t = 2$, given the asset holdings of high- and low-quality capital which are determined at $t = 1$, the agents trade their capital. Entrepreneurs with productivity $z < \hat{z}_2^s$ sell their high-quality capital, and entrepreneurs with $z \geq \hat{z}_2^s$ keep their high-quality capital, where $\hat{z}_2^s \equiv p_2/\phi_2$ is cut-off productivity and $p_t$ is the price of capital at time $t$. These decisions are the same as in the previous model. The aggregate supply of capital is

$$S_2(\hat{z}_2^s; \lambda_2) = (1 - \lambda_2) + \lambda_2 G_2(\hat{z}_2^s),$$

where $\lambda_2$ is the fraction of high-quality capital in the economy at $t = 2$ which is endogenously determined at $t = 1$ (to be described later). The fraction of high-quality capital that circulates in the market at $t = 2$ is given by

$$\lambda_2^M(\hat{z}_2^s; \lambda_2) = \frac{\lambda_2 G_2(\hat{z}_2^s)}{(1 - \lambda_2) + \lambda_2 G_2(\hat{z}_2^s)}.$$

The individual demand for capital purchases solve

$$\max_{d_2 \geq 0} \left\{ \lambda_2^M \phi z d_2 - p_2 k^d - \Gamma(d_2) \right\},$$

and thus the aggregate demand function is

$$D_2(\hat{z}_2; \lambda_2) = \int_{\hat{z}_2^s / \lambda_2^M(\hat{z}_2; \lambda_2)}^{\hat{z}_2^s} \frac{\phi_2}{\gamma}(z \lambda_2^M(\hat{z}_2^s) - \hat{z}_2^s) dG_2(z). \quad (7)$$

equivalent to assuming that entrepreneurs do not observe quality before installing both at $t = 1$ and $t = 2$, but the cost parameter at $t = 1$ is $\gamma_1 \equiv \gamma(\pi_t)^2$. The assumption that the cost involves only high-quality capital in the benchmark model was important only to establish the equivalence result.
The market clearing condition at $t = 2$ is $S_2(\hat{z}_2^s; \lambda_2) \geq D_2(\hat{z}_2^s; \lambda_2)$ with equality whenever $\hat{z}_2^s > 0$. Let $\hat{z}_2^s(\lambda_2)$ denote the solution to this for a given value of $\lambda_2$.

Let us now move on to the characterize the equilibrium at $t = 1$. Each agent takes future quality, $\lambda_2$, as given. The equilibrium is a fixed point in terms of $\lambda_2$, as I describe below. Let $\hat{v}_2^h$ and $\hat{v}_2^l$ denote the values of holding a unit of high-quality capital at the end of $t = 1$, respectively. They are given by

$$
\hat{v}_2^h(\lambda_2) \equiv \mathbb{E}[\phi_2 \max\{z_2, \hat{z}_2^s(\lambda_2)\}], \quad \text{and}
$$

$$
\hat{v}_2^l(\lambda_2) \equiv \mathbb{E}[\phi_2 \hat{z}_2^s(\lambda_2)],
$$

because high-quality capital is sold when productivity falls below the cutoff, and the low-quality capital is always sold at $t = 2$. The expectation is taken over both the aggregate shocks, $\phi_2$, and the idiosyncratic shocks, $z_2$. The productivity cut-off, $\hat{z}_2^s$, at which entrepreneurs are indifferent between selling and holding on to high-quality capital satisfies

$$
\phi_1 \hat{z}_1^s + \hat{v}_2^h(\lambda_2) = p_1, \quad (8)
$$

where the left-hand side is the value of holding on to high-quality capital and the right-hand side is the benefit from selling it. Low-quality capital is always sold in equilibrium because the equilibrium price, $p_1$, will be always higher than $\hat{v}_2^l$ (otherwise there will be infinite demand). The individual demand at $t = 1$ solves

$$
\max_{k^d \geq 0} \left\{ \lambda_1^M (\phi_1 z + \hat{v}_2^h(\lambda_2)) k^d + (1 - \lambda_1^M)\hat{v}_2^l(\lambda_2) k^d - p_1 k^d - \Gamma (\lambda_1^M k^d) \right\},
$$

which can be solved as

$$
d_1(z, \hat{z}_1^s; \lambda_2) = \frac{\phi_1}{\gamma(\lambda_1^M(\hat{z}_1^s))} \left( \lambda_1^M(z) + (1 - \lambda_1^M(\hat{z}_1^s)) (\hat{v}_2^l(\lambda_2) - \hat{v}_2^h(\lambda_2)) \frac{1}{\phi_1} - \hat{z}_1^s \right),
$$

where $\hat{z}_1^s$ is defined in equation (8). The equilibrium at $t = 1$ for a given value of $\lambda_2$ is
characterized by the cut-off productivity $z^s_1$ such that $S_1(z^s_1) \geq D_1(z^s_1)$ holds with equality whenever $z^s_1 > 0$, where

$$S_1(z^s_1) = G_1(z^s_1) + (1 - \pi_h)(1 - G_1(z^s_1)), \quad (9)$$

$$\lambda_1^M(z^s_1) = \frac{\pi_l G_1(z^s_1)}{G_1(z^s_1) + (1 - \pi_h)(1 - G_1(z^s_1))},$$

$$D_1(z^s_1; \lambda_2) = \int d_1(z, z^s_1; \lambda_2)dG_1(z). \quad (10)$$

The resulting fraction of high-quality capital at $t = 2$ is given by

$$\lambda_2 = \pi_l G(z^s_1) + \pi_h(1 - G(z^s_1)). \quad (11)$$

As in the previous section, I deal with the issue of multiplicity by focusing on the highest price-quantity equilibrium in each period. This equilibrium can be characterized as follows. For given $\lambda_2$, we can compute the highest price at $t = 2$ for each $(\lambda_2, \phi_2)$. With this $p_2(\lambda_2, \phi_2)$ in hand, we take $\lambda_2$ as given and can solve for $t = 1$ equilibrium and then compute the highest price and associated cut-off, $z^s_1(\lambda_2)$. Then the entire equilibrium is given by a fixed point $\psi(\lambda_2) = \lambda_2$, where $\psi$ is defined as the following mapping:

$$\psi(\lambda_2) \equiv \pi_l G(z^s_1(\lambda_2)) + \pi_h(1 - G(z^s_1(\lambda_2))).$$

Figure 2 graphically illustrates this mapping. First, $\psi$ is a decreasing function. Intuitively speaking, an increase in future quality improves the future value of low-quality assets more relative to high-quality assets. This is because low-quality assets are always sold at $t = 2$, but high-quality assets are sold only when the productivity at $t = 2$ is sufficiently low. Therefore an expectation of higher $\lambda_2$ lowers the incentive to improve the quality, which implies that $\psi$ is decreasing. However, $\psi$ may not be continuous. The discontinuity comes from the non-monotonicity of the excess demand function at $t = 1$. Therefore an equilibrium in this extended setting may fail to exist. Hereafter I impose the assumption
that \( \pi_h = 1 \), which implies \( \lambda^M(\hat{z}_1) = \pi_I \) for all \( \hat{z}_1 \). This assumption makes the excess demand function at \( t = 1 \) to be monotone, as was illustrated in the right panel of Figure 1. Then I can establish existence of an equilibrium.

**Lemma 1.** Assume \( \pi_h = 1 \). An equilibrium with sequential trading exists. That is, there exists a fixed point, \( \lambda_2 = \psi(\lambda_2) \). Moreover, the fixed point is unique.

Given that \( \psi \) is continuous and decreasing, there exists a unique fixed point. I now turn to comparative statics with respect to \( \phi_1 \). Since an increase in \( \phi_1 \) increases demand at \( t = 1 \) for any given \( \lambda_2 \), the mapping \( \psi \) uniformly shifts down, as illustrated in Figure 2. It follows that the asset equality at \( t = 2 \), \( \lambda_2 \), deteriorates. Defining the fragility of the economy at \( t = 2 \) to be the probability of market breakdown at \( t = 2 \), the following proposition summarizes the result.

**Proposition 5.** An increase in aggregate productivity at \( t = 1 \), \( \phi_1 \), (i) increases the output, trading volume, asset prices, and market quality at \( t = 1 \), but decreases the output, trading volume, asset price, market quality, and average quality in the economy at \( t = 2 \) for all states, and (ii) increases the fragility of the economy at \( t = 2 \). That is, the current boom creates the seed for future crisis.
As we saw in Proposition 3, an increase in current productivity creates the current boom. The current boom, however, lowers the average quality of assets in the economy, and thus negatively affects future economic conditions through by worsening adverse selection problems. The proposition provides a novel perspective on why booms can be seeds for crisis.

I view the above result to be relevant in understanding many real-world boom-bust episodes. The invention of the internet in the late 1990s led to one of the biggest stock market booms in history, which is captured by an increase in \( \phi_1 \) in my model. As early as 1996, Alan Greenspan, the chairman of the Federal Reserve at that time, warned against an overheated market by asking “How do we know when irrational exuberance has unduly escalated asset values, which then become subject to unexpected and prolonged contractions as they have in Japan over the past decade?” Although my model features neither irrationality nor asset price bubbles,\(^{13}\) it captures why overheated asset markets can experience severe contractions thereafter, as a concern of Greenspan.

2.7 Normative Implications under Sequential Trading

What are the policy implications in this extended setting? I again consider a tax on capital purchases for both periods, \( \{\tau_1, \tau_2(\phi_2)\} \) and assume the planner maximizes ex-ante expected welfare, which is equal to the total amount of consumption in \( t = 1 \) and \( t = 2 \). In writing \( \tau_2(\phi_2) \), I assume the planner can also freely intervene at \( t = 2 \) depending on the realization of aggregate productivity shocks. Denote \( \hat{z}^s_1(p_1, \{\hat{z}^s_2\}) \equiv \frac{1}{\phi_1}(p_1 - \tilde{v}^h_2(\{\hat{z}^s_2\})) \) and \( \hat{z}^s_2 \equiv \frac{1}{\phi_2}p_2 \) as the productivity cut-off below which entrepreneurs sell high-quality capital at \( t = 1 \) and \( t = 2 \). I also maintain the assumption that \( \pi_h = 1 \) so that \( \lambda_1^M = \pi_l. \)

\(^{13}\)I conjecture that all my results will be further pronounced when there is an over-valuation of assets.
The demand function at \( t = 1 \) under the tax is given by
\[
d_1(z, p_1, \{\hat{z}_2^{s}\}, \tau_1) = \max \left\{ \frac{\phi_1}{\gamma (\lambda_1^M)^2} \left( \lambda_1^M (z + \frac{1}{\phi_1} \hat{p}_2^{b}) + \frac{1}{\phi_1} (1 - \lambda_1^M) \hat{v}_2 - (1 + \tau_1) p_1 \right), 0 \right\},
\]
and the demand function at \( t = 2 \) is \( d_2(z, \hat{z}_2^{s}, \tau_2, \lambda_2; \phi_2) = \max \left\{ \frac{\phi_1}{\gamma} (z \lambda_2^M - (1 + \tau_2) \hat{z}_2^{s}), 0 \right\} \).

Denote the cut-off productivity above which entrepreneurs become buyers to be
\[
\hat{z}_1^{b}(p_1, \{\hat{z}_2^{s}\}, \tau_1) \equiv \left( (1 + \tau) p_1 - \frac{1}{\phi_1} (\lambda_1^M \hat{d}_2^{b}(\{\hat{z}_2^{s}\}) + (1 - \lambda_1^M) \hat{d}_2^{l}(\{\hat{z}_2^{s}\})) \right) / \lambda_1^M,
\]
\[
\hat{z}_2^{b}(\hat{z}_2^{s}, \tau_2) \equiv \hat{z}_2^{s}(1 + \tau_2) / \lambda_2^M(\hat{z}_2).
\]

Aggregating the individual demand function, we have \( D_1(p_1, \{\hat{z}_2^{s}\}; \tau_1) \equiv \int_{\hat{z}_2^{s}} d_1(z, p_1, \{\hat{z}_2^{s}\}, \tau_1) dG_1(z) \), and \( D_2(\hat{z}_2^{s}, \tau_2, \lambda_2(\hat{z}_2^{s}); \phi_2) \equiv \int_{\hat{z}_2^{s}} d_2(z, \hat{z}_2^{s}, \tau_2, \lambda_2; \phi_2) dG_2(z) \). I assume the planner maximizes ex-ante expected welfare before the realization of productivity at \( t = 1 \). The planner’s problem is
\[
\max_{\tau_1, p_1, (\hat{z}_2(\phi_2), \tau_2(\phi_2))} \int_{\hat{z}_2^{s}(p_1, \{\hat{z}_2^{s}\})} \left( \phi_1 z + \hat{d}_2^{l} \right) \pi_h dG_1(z) + \int_{\hat{z}_2^{s}(p_1, \{\hat{z}_2^{s}\}, \tau_1)} \left[ \phi_1 \lambda_1^M V d_1 - \Gamma (\lambda_1^M d_1) \right] dG_1(z) + \mathbb{E}_{\phi} \left[ -\phi_2 \hat{z}_2 S_2 + \int_{\hat{z}_2^{s}(\hat{z}_2, \tau_2)} \left[ \phi_2 z \lambda_2^M d_2 - \Gamma (d_2) \right] dG_2(z) \right]
\]
s.t. \( S_1(\hat{z}_1^{s}(p_1, \{\hat{z}_2^{s}\})) \geq D_1(p_1, \{\hat{z}_2^{s}\}; \tau_1), \quad S_2(\hat{z}_2^{s}, \lambda_2(\hat{z}_2^{s})) \geq D_2(\hat{z}_2^{s}, \tau_2, \lambda_2(\hat{z}_2^{s}); \phi_2) \)

where \( V = V(z, \{\hat{z}_2^{s}\}) \equiv \frac{1}{\lambda_1^M(\hat{z}_1^{s})} \left( \lambda_1^M (z + \frac{1}{\phi_1} \hat{d}_2^{l}(\{\hat{z}_2^{s}\})) + (1 - \lambda_1^M) \frac{1}{\phi_1} \hat{d}_2^{l}(\{\hat{z}_2^{s}\}) \right) \). I have omitted the arguments to simplify the expression. The first two terms include total production at \( t = 1 \) in addition to the benefit from holding high- and low-quality projects at \( t = 2 \). Since the first two terms include the sales revenue of assets at \( t = 2 \), I am subtracting the payments in the third term because these are merely transfers. The last term is the benefit of reallocation at \( t = 2 \). The two constraints are market clearing conditions for \( t = 1 \) and \( t = 2 \), respectively. The following lemma characterizes the solution:

**Lemma 2.** Assume \( \pi_h = 1 \). The optimal intervention, \( \{\tau_1, \tau_2(\phi_2)\} \), satisfies the following two
conditions.

\[
\tau_1 = \frac{1}{(f_1(z_1^s + a_1^h)g_1(z_1^s)\pi_h)} \frac{\partial \lambda_2}{\partial z_1^s} \mathbb{E}_x \left[ \frac{\partial M}{\partial \lambda_2} \int_{z_2^s} \phi_2 z d_2 dG_2(z) - \phi_2 z_2^s \tau_2(\phi_2)(1 - G_2(z_2)) \right], \quad (12)
\]

\[
\tau_2(\phi_2) = -\frac{1}{\phi_2 z_2 g_2(z_2)\pi_2} \frac{\partial M}{\partial \lambda_2} \int_{z_2^s} \phi_2 z d_2 dG_2(z). \quad (13)
\]

If we look at equation (13), the right-hand side is negative because \(\frac{\partial M}{\partial \lambda_2} \geq 0\). Therefore it follows that \(\tau_2(\phi_2) \leq 0\) for all \(\phi_2\). This result echoes the previous two-period arguments: since the economy at \(t = 2\) is a static adverse selection economy for a given \(\lambda_2\), the planner will find it optimal to subsidize capital purchases to improve the quality of the pool of assets that trade. Looking at the first condition (12), first note that \(\frac{\partial \lambda_2}{\partial z_1^s} = -g_1(z_1^s)(\pi_h - \pi_1) < 0\). In words, the higher asset prices at \(t = 1\) imply lower quality of assets in the next period. Second, \(\frac{\partial M}{\partial \lambda_2} > 0\) because higher quality in the economy translates into higher market quality. Combined with the previous observation that \(\tau_2(\phi_2) \leq 0\), it follows that the right-hand side is positive. This directly implies that \(\tau_1 \geq 0\). These arguments lead to the following proposition:

**Proposition 6.** Assume \(\pi_h = 1\). Then the planner can improve ex-ante welfare by taxing trade at \(t = 1\). That is, the equilibrium volume of trade at \(t = 1\) is too high compared to the ex-ante constrained efficient benchmark.

This result is in sharp contrast to Proposition 4. Proposition 4 states that equilibrium volume of trade was always too low, but now we reach the opposite conclusion. Why is that? In a sequential trading setting, the planner wants to mitigate adverse problems not only at \(t = 1\), but also at \(t = 2\). Although the planner has access to ex-post policy at \(t = 2\), this cannot completely resolve the adverse selection problems. The planner, however, has the first-best way to mitigate the adverse selection problems at \(t = 2\): taxing trade at \(t = 1\) to improve the quality of assets in the economy. Of course, this may worsen the adverse selection problems at \(t = 1\) if \(\pi_h < 1\). Therefore the planner faces a trade-off in choosing tax at \(t = 1\): (i) enhance reallocation to improve current market conditions; (ii) restrict
reallocate to improve future market conditions. In general, the planner strikes a balance between these two effects in choosing whether to tax or subsidize asset purchases. Under \( \pi_h = 1 \), however, the first consideration is necessarily absent because market quality is a constant, \( \lambda^M_1 = \pi_l \). This leads to an unambiguous result, where the planner always finds it optimal to tax asset purchases rather than to subsides them.

The result implies that the sole rationale for policy intervention is to overcome current and future distortions caused by adverse selection problems. In the benchmark model in Section 2.1, the equivalence result states that moral hazard acts as if it is changing technology rather than creating distortion. Here this is no longer the case. Moral hazard creates distortion because it worsens future adverse selection problems. Private agents do not internalize the fact that if they demand fewer assets and lower asset prices, future adverse selection problems will be alleviated. The planner can correct this externality by taxing trades.

The key takeaway here is that when moral hazard interacts with future adverse selection problems, a novel policy implication arises: the economy features excess trading rather than insufficient trading, as predicted by standard adverse selection models. It provides a rationale for why policymakers should cool down over-heated asset markets, such as dot-com bubbles, or mortgage-backed securities (MBS) markets before the recent financial crisis. Faced with higher prices, firms and banks responded by creating low-quality projects or financial assets just for sales. Since private agents do not internalize the fact that these creation of lemons harm future market conditions, the policy leaning against winds is likely to be beneficial for economic stabilization and welfare.

Although I have focused on ex-ante efficiency, the policy described above is quite likely to bring about Pareto improvement even after the realization of idiosyncratic productivity at \( t = 1 \), unlike in the case of the benchmark model. Taxing asset purchases itself harms buyers and sellers at \( t = 1 \). However, they all benefit from improved market conditions through improved quality and subsidization at \( t = 2 \), where the entrepreneurs
draw a new productivity.

3 Dynamics

In this section, I study the dynamics of the economy by extending the two-period model in Section 2 in the following two dimensions. First, in order to introduce dynamics in a tractable manner, I assume an OLG structure. Second, I introduce investment and capital accumulation to study the state-dependence of the economy’s response to aggregate shocks. The purpose here is to illustrate how long-lasting booms can contribute to the gradual build-up of systemic risk.

Environment. The economy is populated by overlapping generations of entrepreneurs, where each entrepreneur lives for two periods. As before, there are consumption goods and two types, high- and low-quality capital. Both types of capital depreciate at rate $\delta \in [0, 1)$ every period, and consumption goods are perishable. At $t = 0$, the initial old entrepreneurs are endowed with a unit of capital stock, $i_0 = 1$, with fraction $\lambda^0$ consisting of high-quality capital. The aggregate shock to the economy take the form of the aggregate productivity, $\phi_t$, and $\phi_t$ is revealed one period before, at time $t - 1$. I assume $\phi_t$ follows a Markov process. Let $\bar{\phi}$ denote the unconditional mean of $\phi_t$.

The preferences of entrepreneurs born at $t$ are the same as before, $U_t = \mathbb{E}_t c_{t+1}$. Young entrepreneurs receive all capital stock in the economy from the previous generation (to be described later). In each period, entrepreneurs receive stochastic endowments of consumption goods when they are young. Since entrepreneurs do not value consumption when they are young, they convert all of their endowments in capital goods. The amount of an endowment is a function of the aggregate productivity shocks:

$$i_t = \delta i(\phi_{t+1}),$$
for some function $i(\cdot)$ with $i(\bar{p}) = 1$. This ensures that the aggregate capital stock stays constant in the long-run.

In the next period, the old entrepreneurs first randomly draw idiosyncratic productivity from CDF, $G(z)$, and they then decide whether to exert effort to improve the quality of a new investment. After the investment materializes, the entrepreneurs have a chance to trade both the new and the existing capital. I assume that the vintage of capital is publicly observable, which is arguably a realistic feature of the model. Therefore at time $t$, there are $t$ markets, and the price for capital of vintage $s$ is denoted by $p^s_t$ for $s = 0, 1, \ldots t$.

Old entrepreneurs at time $t$ can produce amount $\phi_t z$ of output from one unit of high-quality capital, and 0 units of output from low-quality capital. Entrepreneurs can purchase capital from the market, but must incur cost to install capital. The quality of this capital is observable after installation. The installation cost is modified as linearly homogenous in the purchasing capital stock and current capital stock holdings for each vintage:

$$\Gamma(k^{s,d}_i) = \gamma \frac{k^{s}_i}{2} \left( \frac{k^{s,d}_i}{k^{s}_i} \right)^2,$$

where $k^{s,d}_i$ is the purchase of capital of vintage $s$, and $k^{s}_i$ denotes the capital holdings of vintage $s$. Given these specifications, the aggregate demand function for capital of vintage $s$ is

$$D(\hat{z}_s^t) = \frac{\phi_t K^s_t}{\gamma} \int_{\hat{z}_s^t / \lambda^M_s}^{\hat{z}_s^t} (\lambda^M z - \hat{z}_s^t) dG(z),$$

where $K^s_t \equiv (1 - \delta)^{t-s} i_s$ is the total capital stock of vintage $s$, $\lambda^M$ is the fraction of high-quality capital traded, which I describe below, and $\hat{z}_s^t \equiv p^s_t / \phi_t$ is the normalized price. The supply side for vintage $s = t$ is

$$S^s(\hat{z}_s^t) = i_s \left[ G(\hat{z}_s^t) + (1 - \pi_h)(1 - G(\hat{z}_s^t)) \right],$$
and the fraction of high-quality capital in circulation is

$$\lambda^M(\hat{z}_t^s) = \frac{\pi_l G(\hat{z}_t^s)}{G(\hat{z}_t^s) + (1 - \pi_h)(1 - G(\hat{z}_t^s))},$$

which is the same as before. The supply side for vintage $s < t$ is given by

$$\hat{S}(\hat{z}_t^s, \lambda^s) = \hat{K}_t^s \left[ G(\hat{z}_t^s) + (1 - \lambda^s)(1 - G(\hat{z}_t^s)) \right],$$

and the fraction of high-quality capital in circulation is

$$\hat{\lambda}^M(\hat{z}_t^s, \lambda^s) = \frac{\lambda^s G(\hat{z}_t^s)}{G(\hat{z}_t^s) + (1 - \lambda^s)(1 - G(\hat{z}_t^s))},$$

where $\lambda^s$ is the fraction of high-quality assets in the economy, which is determined at the time of origination,

$$\lambda^s \equiv \pi_h (1 - G(\hat{z}_t^s)) + \pi_l G(\hat{z}_t^s).$$

The market clearing for each vintage is $S^s(\hat{z}_t^s) \geq D^s(\hat{z}_t^s)$ for all $s$. Note that the embedded constant returns to scale technology imply that the asset prices are independent from the investment amount. Therefore different vintages that have the same value of $\lambda^s$ trade at the same price for $s < t$. I denote $\hat{z}_t(\lambda)$ to be the price of capital with quality $\lambda$, and it is no longer a function of vintage. Let $\Psi_t(\lambda)$ denote the distribution of $\lambda$ across the different vintage of the existing capital stock.

After entrepreneurs produce using capital, each old entrepreneur is randomly matched with a young entrepreneur of the next generation. Then the young entrepreneur makes take-it-or-leave-it offers to purchase capital. Since capital cannot be consumed, the value of capital after its use for production is zero for the old entrepreneurs. As a consequence, all the capital that the old entrepreneurs hold is purchased at a zero price by the young entrepreneurs. These modeling techniques allow one to characterize equilibrium as a repeated static environment with an endogenous state variable $\Psi_t$ and an exogenous state.
variable $\phi_t$. From the analysis in the previous section, it is clear that assets originated in a period with higher $\phi_t$ have lower average quality than assets originated in a period with lower $\phi_t$. Because the average quality is lower, the market for this vintage of assets is more likely to break down. Thus we have the following observation.

**Observation 1.** Assets originated in good times are more fragile than the assets originated in bad times.

Now I turn to an analysis of the aggregate economy. The total amount of existing capital stock (after depreciation) is denoted as $K_t = \sum_{s=1}^{t} K^s_t$. Let us define the average quality of the economy as

$$
\Lambda_t = \frac{K_t}{K_t + i_t} \int \lambda d\Psi_t(\lambda) + \frac{i_t}{K_t + i_t} \lambda^t,
$$

which is the weighted average of the fraction of high-quality capital in the economy. Similarly, the average quality of the capital traded in the market is

$$
\Lambda^M_t = \frac{K_t}{K_t + i_t} \int \lambda^M(\hat{z}^t_t(\lambda)) d\Psi_t(\lambda) + \frac{i_t}{K_t + i_t} \lambda^{M,s}(\hat{z}^t_t),
$$

Also let us define the fragility of the economy, $F_t$, as the expected market size collapsing relative to the entire market size as:

$$
F_t = \mathbb{E}_{\phi} \left[ \frac{K_t}{K_t + i_t} \int \mathbb{1}(\hat{z}_t(\lambda) = 0) d\Psi_t(\lambda) + \frac{i_t}{K_t + i_t} \mathbb{1}(\hat{z}^t_t = 0) \right],
$$

where the expectation is taken over the current aggregate shock.

I define the deterministic steady state of this economy as an equilibrium with the constant aggregate productivity staying constant over time, $\phi_t = \bar{\phi}$ for all $t$, and thus all endogenous variables remain constant over time. For given $\phi_t$, the price of new investment, $\hat{z}^t_t$, and thus the quality of new investment are uniquely pinned down. All the

---

14 A similar assumption is used in Gorton and Ordoñez (2014).
existing capital is of the same quality as the new investment, and thus $\Psi$ has a single mass point. Such a steady state is also stable as long as $\delta > 0$. I also assume $\bar{\phi}$ is large enough so that all markets are active in the steady state. The following proposition shows how long-lasting booms contribute to the buildup of systemic risk.

**Proposition 7.** Starting from the steady state with $\phi_t = \bar{\phi}$, suppose the economy experiences booms accompanied by increased aggregate productivity $\phi_t = \phi^{\text{boom}} > \bar{\phi}$, for $T$ periods. Then the fragility of the economy is increasing in the length of booms, $T$.

This proposition follows from the single observation that as the length of booms increases, the average quality of assets in the economy goes down. In particular, projects originated in boom periods experience deterioration in quality relative to the projects originated earlier. Therefore the markets for this new capital is more likely to suffer from adverse selection problems. As booms become longer, the relative size of these fragile markets grows over time, exposing the entire economy to the risk of more severe recessions. Figure 3 illustrates the result with a numerical example. The economy starts from the steady state then suddenly experiences an increase in aggregate productivity from...
$t = 1$ onward. The left panel indicates that fragility gradually builds up over time, and is driven by a gradual deterioration in average quality in the economy, as shown by the middle panel. The right panel depicts the average quality of assets that are traded. In response to increased asset demand at $t = 1$, market quality jumps up, which further boosts demand for assets. However, as the quality in the economy deteriorates over time, market quality also tends to decline as booms become longer. In the real world, a crisis tend to follow a long-lasting boom period. For example, the Great Recession followed the period of a so-called Great Moderation. These periods were precisely the time when the market for low-quality MBS grew, which in turn triggered the subsequent crisis. The dynamic model allows me to describe these narratives precisely.

So far I have emphasized that investment quality is counter-cyclical. In contrast, data show that the quantity of investment is highly pro-cyclical along the business cycles. How does this observation interact with the counter-cyclical quality? The following proposition compares two economies, one with constant quantity of investment and the other with pro-cyclical quantity of investment.

**Proposition 8.** Starting from the steady state, suppose the economy experiences either booms or recessions, captured by changes in aggregate productivity, $\phi$. Relative to an economy where investment is constant ($\iota(\phi) = 1$ for all $\phi$), in the economy with pro-cyclical quantity of investment ($\iota(\phi)$ is increasing in $\phi$), (i) fragility increases more rapidly during booms; (ii) the fragility of the economy decreases more slowly during recessions.

With pro-cyclical quantity of investment, in booms the amount of investment is high, and the quality of that investments is low. Therefore average quality in the economy rapidly deteriorates in good times. This increases the systemic risk of the economy more relative to a situation in which investment is smooth along the business cycle. The red dashed line in Figure 3 illustrates this. In recessions, with pro-cyclical investment, the quality of investment is high, but the quantity of investment is low. Therefore although the cleansing effect of the recession is present, those forces are weaker with pro-cyclical
investment. This slows down the recovery of the economy. These arguments suggest that policies that attempt to smooth quantity of investment along the business cycle might be beneficial for economic stabilization.

4 Conclusion

In this paper I present a model in which endogenous deterioration of quality during booms creates fragility in the economy. Existing macroeconomic theory emphasizes the role of leverage as a source of systemic risk. I provide here a different but complementary view through the lens of the model. Although many practitioners and policymakers have concerned them-selves with the emergence of inefficient quality deterioration in booms, there has been little work that formalizes these views. My model precisely speaks to why quality deteriorates during booms, why this creates fragility, and why it is inefficient. Although I demonstrate the idea using a fairly abstract framework, the main insights of this paper apply to a wide variety of economic circumstances where adverse selection is a potential source of market failure.

The key prediction of the model concerns the relationship between asset prices and underlying moral hazard problems. This opens up a number of avenues for future research. Empirically, the model delivers a testable prediction of the relation between asset prices and asset quality. Theoretically, introducing asset price bubbles would be an interesting extension of the current model. I conjecture that the mechanism will be further pronounced when asset price bubbles are present, which I believe is a relevant story for understanding many boom-bust episodes.
References


Appendix

A Proofs

A.1 Proof of Proposition 1

It is straightforward to see
\[ \frac{d \lambda(p/\phi)}{dp} = -(\pi_h - \pi_l) \frac{1}{\phi} g(p/\phi) \]

< 0

and
\[ \frac{d \lambda^M(p/\phi)}{dp} = \frac{\frac{1}{\phi} \pi_l g(p/\phi)(1 - \pi_h)}{(G(p/\phi) + (1 - \pi_h)(1 - G(p/\phi)))^2} \geq 0, \]

where the inequalities are strict if \( \pi_h < 1 \).

A.2 Proof of Proposition 2

Set \( \tau_{sym} = (1 - \lambda^M* \chi) / (\lambda^M* \chi) \) and \( \chi = \frac{\pi_h}{\pi_l} \), and guess that \( \hat{z}_{sym} = \chi \hat{z}^* \). I will verify that allocations of symmetric information and asymmetric information are identical. The individual demand function for high-quality capital in a symmetric information economy can be rewritten as
\[ k^{d,sym}(z) = \max \left\{ \frac{\phi}{\gamma} (z - \frac{1}{\lambda^M* \hat{z}^*}), 0 \right\}, \]

which is equivalent to the amount of high-quality capital that entrepreneurs obtain in the asymmetric information economy, \( \lambda^M* \times d(z) = \max \left\{ \frac{\phi}{\gamma} (z - \frac{1}{\lambda^M* \hat{z}^*}), 0 \right\} \). Because the supply function in the symmetric information economy is given by \( S_{sym}(\hat{z}_{sym}) = \pi_l G(\hat{z}^*) \), the supply side of high-quality capital is also same as that for an asymmetric information
economy. The lump-sum transfer is

\[
T = τ^{sym} φ\hat{z}^{sym} S^{sym}(\hat{z}^{sym})
\]

\[
= \frac{(1 - \lambda M^{*} \chi)}{(\lambda M^{*} \chi)} φ\hat{z}^{*} \lambda M^{*} S(\hat{z}^{*})
\]

\[
= (1 - \lambda M^{*} \frac{\tau h}{\pi l}) φ\hat{z}^{*} S(\hat{z}^{*})
\]

\[
= \left(1 - \frac{\pi h G(\hat{z}^{*})}{S(\hat{z}^{*})}\right) φ\hat{z}^{*} S(\hat{z}^{*})
\]

\[
= (1 - \pi h) φ\hat{z}^{*},
\]

which is equivalent to the revenue from sales of lemons for keepers in an asymmetric information economy. Therefore the consumption of keepers \((z \geq \hat{z}^{*})\) in a symmetric information economy and in an asymmetric information economy coincide. The consumption for sellers \((z < \hat{z}^{sym} / \chi)\) in a symmetric information economy is

\[
p^{sym} \pi h \frac{1}{\chi} + T = φ\chi \hat{z}^{*} \pi l + (1 - \pi h) φ\hat{z}^{*}
\]

\[
= φ\hat{z}^{*},
\]

which is the total sales revenue in a symmetric information economy. The above arguments prove that consumption and production for each entrepreneur in the two economies are identical, and prices are related through \(\hat{z}^{sym} / \chi = \hat{z}^{*}\).

### A.3 Proof of Proposition 3

From the market clearing condition, \(\frac{\partial \hat{z}}{\partial \phi} \geq 0\) holds at the highest price equilibrium. Therefore both price and trading volumes increase. The rest follows from Proposition 1.

### A.4 Proof of Proposition 4

This is a special case of Lemma 3.
A.5 Proof of Lemma 1

First, I argue that $\psi$ is continuous in $\lambda_2$. Although the price at $t = 2, \hat{z}_2^s(\phi_2)$, might be discontinuous in $\lambda_2$ for finite numbers of $\phi_2$, once expectations over $\phi_2$ are taken, $v^h_2$ and $v^l_2$ are continuous in $\lambda_2$ because such discontinuous points have measures of zero. Given $\pi_2 = 1, \hat{z}_1^s$ is also continuous in $\lambda_2$ because the excess demand function is monotone. This establishes that $\psi$ is continuous. Next, I argue that $\psi$ is decreasing. Note that the supply function at $t = 1, S_1(\hat{z}_1^s)$, does not depend on $\lambda_2$. The demand function, $D_1(\hat{z}_1^s; \lambda_2)$, is increasing in $\lambda_2$ because

$$
\frac{d}{d\hat{z}_2^s}(v^l_2(\hat{z}_2^s) - v^h_2(\hat{z}_2^h)) = E_\phi \phi_2 \frac{d\hat{z}_2^s}{d\lambda_2} \frac{d}{d\hat{z}_2^s} \left[ \hat{z}_2^s - \int_{\hat{z}_2^s} zdG_2(z) - (1 - G(\hat{z}_2^s))\hat{z}_2^s \right] 
$$

Because the demand function at $t = 2$ is increasing in $\lambda_2$ and the supply function is decreasing in $\lambda_2$, it follows that the price at $t = 2$ is increasing in $\lambda_2$, $\frac{d\hat{z}_2^s}{d\lambda_2} > 0$. Therefore the above expression is positive. This implies that an increase in $\lambda_2$ increases the demand curve at $t = 1$, but keeps the supply curve at $t = 1$ unchanged. As a result, the equilibrium cut-off, $\hat{z}_1^s$, decreases, which in turn lowers average quality in the economy. Thus $\psi$ is decreasing in $\lambda_2$. These arguments show that there exists a unique fixed point $\lambda_2 = \psi(\lambda_2)$.

A.6 Proof of Proposition 5

As argued in the main text, an increase in $\phi_1$ monotonically decreases $\psi$ for all $\lambda_2$. Therefore the equilibrium value of $\lambda_2$ goes down. This not only reduces the production possibility frontier at $t = 2$, but also decreases demand and increases supply at $t = 2$, and thus lowering asset prices and trading volume. Increased supply and decreased demand at $t = 2$ imply that the market at $t = 2$ is more likely to break down.
A.7 Proof of Lemma 2

By substituting the definition of $d_1$ and $d_2$, the planner’s problem can be rewritten as

$$
\max_{\tau_1, \pi_1, \xi_1} \int_{\xi_1} \left( \phi_1 z + v_2^h(\xi_1) \right) \pi_1 dG_1(z) + \int_{\xi_1} \left[ \frac{(\phi_1)^2}{2\gamma(\lambda_1^M)^2} (\lambda_1^M V - (1 + \tau_1) p_1)^2 \right] dG_1(z)
$$

$$
+ \mathbb{E}_\phi \left[ -\phi_2 \pi_2 s_2 (\xi_2, \lambda_2(\xi_1)) + \int_{\xi_1} \left[ \frac{(\phi_2)^2}{2\gamma} ((z \lambda_2^M (\xi_2, \lambda_2))^2 - (1 + \tau_2(\phi))^2) \right] dG_2(z) \right]
$$

s.t. $G_1(\xi_1^s(p_1, \xi_2^s)) + (1 - \pi_1)(1 - G_1(\xi_1^s(p_1, \xi_2^s))) \geq \frac{\phi_1}{\gamma(\lambda_1^M)^2} \int_{\xi_1} \left[ \lambda_1^M V - p_1 (1 + \tau_1) \right] dG_1(z)$,

$$
G_2(\xi_2^s) + (1 - \lambda_2(\xi_1^s))(1 - G_2(\xi_2^s)) \geq \frac{\phi_2}{\gamma} \int_{\xi_2} \left[ z \lambda_2^M (\xi_2^s, \lambda_2) - \xi_2(1 + \tau_2) \right] dG_2(z),
$$

with $V = V(z, \xi_2^s) \equiv \frac{1}{\lambda_1^M(\xi_1^s)} \left( \lambda_1^M(z + \frac{1}{\phi_2} v_2^h(\xi_2^s)) + (1 - \lambda_1^M) \frac{1}{\phi_2} v_2^h(\xi_2^s) \right)$. Note that the assumption $\pi_1 = 1$ implies $\lambda_1^M = \pi_1$. The first-order conditions for $\tau_1$ and $\tau_2$ give

$$
\kappa_1 = \phi_1 p_1 (1 + \tau_1),
$$

$$
\kappa_2 = \phi_2 \xi_2^s (1 + \tau_2),
$$

where $\kappa_1$ and $\kappa_2$ are Lagrangian multipliers on the constraints for market clearing at $t = 1$ and $t = 2$, respectively. The FOC w.r.t. $p_1$ is

$$
\frac{\partial \xi_1^s}{\partial p_1} \tau_1 (\phi_1 \xi_1^s + v_2^h) \xi_1^s \pi_1 + \frac{\partial \lambda_2}{\partial p_1} \mathbb{E}_\phi \left[ \phi_2 \xi_2^s \left( \frac{\phi_2}{\gamma} (z \lambda_2^M - \xi_2(1 + \tau_2)) \right) dG_2(z) \right] = 0,
$$

which can be rearranged as

$$
\tau_1 (\phi_1 \xi_1^s + v_2^h) \xi_1^s \pi_1 + \frac{\partial \lambda_2}{\partial \xi_1^s} \mathbb{E}_\phi \left[ \phi_2 \xi_2^s d_2(z, \xi_2^s) dG_2(z) - \phi_2 \xi_2^s \tau_2 (1 - G_2(\xi_2^s)) \right] = 0,
$$

(A1)
which is (12). The first order condition for $\hat{z}_2(\phi_2)$ is

$$(1 - G(\hat{z}_1)) \tau_1 \frac{\partial v^2_2}{\partial \hat{z}_2} + \frac{(\phi_1)^2}{\gamma(\lambda_1^M)^2} \int_{\hat{z}_1}^{h} \frac{\partial V}{\partial \hat{z}_2} \lambda_1^M (\lambda_1^M V - (1 + \tau_1)p_1) dG_1(z) - \phi_2 S(\hat{z}_2)$$

$$+ g_2(\hat{z}_2) \lambda_2 \phi_2 \hat{z}_2 \tau_2 + \frac{d \lambda_2^{M}(\hat{z}_2)}{d \hat{z}_2} (\phi_2)^2 \gamma \int_{\hat{z}_2}^{h} z(\lambda_2^M - \hat{z}_2(1 + \tau_2)) dG_2(z) = 0.$$ 

By using the market clearing condition, The first three terms can be rewritten as

$$\left(1 - G(\hat{z}_1)\right) \tau_1 \frac{\partial v^2_2}{\partial \hat{z}_2} + \int_{\hat{z}_1}^{h} \frac{\partial (\phi_1 \lambda_1^M V)}{\partial \hat{z}_2} d_1(z, p_1, \lambda_2, \tau) dG_1(z) - \phi_2 S(\hat{z}_2).$$

The total benefit of having higher prices at $t = 2$

$$= \lambda_2 \frac{\partial v^2_2}{\partial \hat{z}_2} + (1 - \lambda_2) \frac{\partial v^2_2}{\partial \hat{z}_2} - \phi_2 S(\hat{z}_2)$$

$$= 0.$$

Intuitively speaking, the direct effect on prices should not affect total welfare because change in prices are just transfers. This direct effect on prices affects welfare only through improving allocation. Thus the condition for $\tau_2(\phi_2)$ reduces to

$$g_2(\hat{z}_2) \lambda_2 \phi_2 \hat{z}_2 \tau_2 + \frac{d \lambda_2^{M}(\hat{z}_2)}{d \hat{z}_2} (\phi_2)^2 \gamma \int_{\hat{z}_2}^{h} z(\lambda_2^M - \hat{z}_2(1 + \tau_2)) dG_2(z) = 0,$$

which can be further rearranged as

$$g_2(\hat{z}_2) \lambda_2 \phi_2 \hat{z}_2 \tau_2 + \frac{d \lambda_2^{M}(\hat{z}_2)}{d \hat{z}_2} \int_{\hat{z}_2}^{h} \phi_2 dz_2(z, \hat{z}_2) dG_2(z) = 0, \quad (A2)$$

This is (13). By plugging (A2) into (12),

$$\tau_1(\phi_1 \hat{z}_1 + v^2_2) g_1(\hat{z}_1) \tau_1 = \frac{\partial \lambda_2^{21}}{\partial \hat{z}_2} E_{\phi} \left[ \phi_2 \hat{z}_2 \tau_2 \left( g_2(\hat{z}_2) \lambda_2 + 1 - G_2(\hat{z}_2) \right) \right],$$

which is (12).
A.8 Proof of Proposition 6

Provided in the main text.

A.9 Proof of Proposition 7

Because investment quality is decreasing in \( \phi \), the quality of new investments made during booms declines. Therefore these markets are more likely to collapse. As \( T \) increases, the relative size of these fragile markets increases and thus the \( F_t \) increases over time.

A.10 Proof of Proposition 8

When investment is pro-cyclical, the investment quantity during booms is larger than an economy that features constant investment. This implies that the relative size of the fragile markets grows more quickly. Thus the fragility, \( F_t \), also increases more rapidly. In recessions, the quality of investment is higher than in steady state. With pro-cyclical investment, the quality improves more slowly, as does the fragility.

B  Walrasian Equilibrium and Buyers’ Equilibrium

The goal is to derive Equation (6). The buyer’s problem for a given price \( \hat{z} \equiv p/\phi \) is

\[
V^b(\hat{z}) \equiv \max_{k^d \geq 0} \left\{ z\lambda^M(\hat{z})k^d - \hat{z}k^d - \frac{1}{\phi} \Gamma(\lambda^M(\hat{z})k^d) \right\}.
\]

Buyers do not have any incentive to raise prices beyond competitive equilibrium price, \( \hat{z}^* \), if

\[
\frac{dV^b(\hat{z})}{d\hat{z}} \leq 0
\]

(B3)
for $\hat{z} > \hat{z}^*$. The envelope theorem implies

$$
\frac{dV^b(\hat{z})}{d\hat{z}} = z \frac{d\lambda^M(\hat{z})}{d\hat{z}} k^{d^*} - k^{d^*} - \frac{\gamma}{\phi} \frac{d\lambda^M(\hat{z})}{d\hat{z}} (k^{d^*})^2.
$$

Noting that $k^{d^*} = \frac{\phi}{(\lambda M)^{\lambda M}} (z \lambda M - \hat{z})$, condition (B3) can be rewritten as

$$
\frac{d\lambda^M(\hat{z})}{d\hat{z}} \frac{1}{\lambda M} \hat{z} - 1 \leq 0,
$$

which is equivalent to equation (6). For example, when $G$ is given by standard log-normal and $\pi_h = 0.9$ and $\pi_l = 0.1$, this condition holds as long as $\hat{z}$ is greater than the 9th percentile.

In the case where buyers do not observe quality before installation of capital, the buyers’ value for a given price is

$$
V^b(\hat{z}) \equiv \max_{k^d \geq 0} \left\{ z \lambda^M(\hat{z}) k^d - \hat{z} k^d - \frac{1}{\phi} \Gamma(k^d) \right\}.
$$

Applying the same argument as above, the buyers do not have any incentive to raise prices if

$$
z_{\text{max}} \frac{d\lambda^M(\hat{z})}{d\hat{z}} \leq 1,
$$

where $z_{\text{max}}$ is the upper bound of $z$.

### C Ex-interim Efficiency in Benchmark Model

To facilitate the analysis, instead of looking at polices in an economy with informational asymmetry, I look for a tax policy in an economy with symmetric information. The following lemma shows that policy in one economy can be mapped to the policy in another economy.
Lemma 3. For any tax policy $\tau$ in an economy with asymmetric information, there exists a tax rate in an economy with symmetric information that delivers an equivalent allocation. The tax rates are related through $\tau_{\text{sym}} = \frac{1 + \tau - \lambda M^* \chi}{\lambda M^* \chi}$, where $\lambda M^*$ is the equilibrium value in an asymmetric information economy. Conversely, for any tax policy $\tau_{\text{sym}}$ in an economy with asymmetric information, there exists a tax rate in an economy with asymmetric information that delivers an equivalent allocation. The tax rates are related through $\tau = \lambda M(\hat{z}_{\text{sym}} / \chi) \chi \tau_{\text{sym}} + \lambda M(\hat{z}_{\text{sym}} / \chi) \chi - 1$, where $\hat{z}_{\text{sym}}$ is the equilibrium price in a symmetric information economy.

Proof. The proof consists of mimicking the same procedure as was applied in Proposition 2. Start from an asymmetric information economy with tax rate $\tau$. Set $\tau_{\text{sym}} = \frac{1 + \tau - \lambda M^* \chi}{\lambda M^* \chi}$ and $\chi = \pi_h / \pi_l$, and guess that $\hat{z}^* = \hat{z}_{\text{sym}} / \chi$ holds. Then the individual demand function for high-quality capital in a symmetric information economy can be rewritten as

$$k^{d,\text{sym}}(z) = \max \left\{ \frac{\phi}{\gamma} (z - \frac{1}{\lambda M^*} (1 + \tau) \hat{z}^*), 0 \right\},$$

which is equivalent to the amount of high-quality capital that entrepreneurs obtain in the asymmetric information economy, $\lambda M^* \times d(z) = \max \left\{ \frac{\phi}{\gamma} (z - \frac{1}{\lambda M^*} (1 + \tau) \hat{z}^*), 0 \right\}$. The supply function in the asymmetric information economy is given by $S(\hat{z}_{\text{sym}}) = \pi_l G(\hat{z}^*)$, which is again the same as the supply of high-quality capital in the asymmetric information economy. The lump-sum transfer is

$$T = \tau_{\text{sym}} p_{\text{sym}} S(p_{\text{sym}} / \phi)$$

$$= \frac{(1 + \tau - \lambda M^* \chi)}{(\lambda M^* \chi)} \phi \chi \hat{z}^* \lambda M^* S(\hat{z}^*)$$

$$= (1 - \lambda M^* \frac{\pi_h}{\pi_l}) \phi \hat{z}^* S(\hat{z}^*)$$

$$= (1 - \pi_h) \phi \hat{z}^* + \tau \phi \hat{z}^* S(\hat{z}^*),$$

which is the amount that keepers of high-quality capital receive from the sales of low-quality assets and lump-sum transfers. The total consumption for sellers ($z < \hat{z}_{\text{sym}} / \chi$)
is

\[ p^{sym} \pi_h \frac{1}{\chi} + T = \phi \chi \hat{z}^* \pi_l + (1 - \pi_h) \phi \hat{z}^* + \tau \phi \hat{z}^* S(\hat{z}^*), \]

\[ = \phi \hat{z}^* + \tau \phi \hat{z}^* S(\hat{z}^*), \]

which includes the sales of all the projects and lump-sum transfers in the asymmetric information economy. Therefore allocations are equivalent.

I will prove the converse. Start from a symmetric information economy with tax rate \( \tau^{sym} \) and \( \chi = \pi_h / \pi_l \). Guess that \( \hat{z}^* = \hat{z}^{sym} / \chi \). Set \( \tau = \lambda^M (\hat{z}^{sym} / \chi) \chi^{sym} + \lambda^M (\hat{z}^{sym} / \chi) \chi = 1 \) in the asymmetric information economy. Then the individual demand function for capital in an asymmetric information economy can be written as

\[ k^d(z) = \max \left\{ \frac{\phi}{\gamma (\lambda^{M*})^2} (z \lambda^{M*} - \lambda^{M*} \chi (1 + \tau^{sym}) \hat{z}^*), 0 \right\} \]

\[ = \max \left\{ \frac{\phi}{\gamma \lambda^{M*}} (z - (1 + \tau^{sym}) \hat{z}^{sym}), 0 \right\}. \]

Thus the effective demand for high-quality capital is \( \lambda^{M*} \times k^d(z) = \max \left\{ \frac{\phi}{\gamma} (z - (1 + \tau^{sym}) \hat{z}^{sym}), 0 \right\}, \) which is the same as the demand for high-quality capital in the symmetric information economy. The sum of the sale of low-quality capital and lump-sum transfers in the asymmetric information economy is

\[ \phi \hat{z}^* (1 - \pi_h) + \tau \phi \hat{z}^* S(\hat{z}^*) = \phi \hat{z}^{sym} / \chi \left[ (1 - \pi_h) + \tau S(\hat{z}^*) \right] \]

\[ = \phi \hat{z}^{sym} / \chi \left[ (1 - \pi_h) + (\lambda^M / \chi (1 + \tau^{sym}) - 1) S(\hat{z}^*) \right] \]

\[ = \phi \hat{z}^{sym} / \chi \left[ (1 - \pi_h) - S(\hat{z}^*) + \pi_h G(\hat{z}^*) (1 + \tau^{sym}) \right] \]

\[ = \phi \hat{z}^{sym} \pi_l G(\hat{z}^*) \tau^{sym} \]

\[ = \phi \hat{z}^{sym} S^{sym} (\hat{z}^{sym}) \tau^{sym}, \]

which is same as the lump-sum transfer each agent receives in an asymmetric information economy.
economy. The sales revenue from high-quality assets is also same because \( \phi \hat{z}^* = \phi \hat{z}^{sym} / \chi \). Therefore we confirm that the allocations are identical.

The above Lemma shows that it is sufficient to look for \( \tau^{sym} \) instead of looking at tax policy in an economy with asymmetric information. Therefore I ask that starting from a symmetric information economy with tax rate \( \tau^{sym*} = \frac{1 + \tau - \lambda M^* \chi}{\lambda M^* \chi} \), whether one can achieve Pareto improvement by changing the tax rate. Obviously, increasing the tax rate does not achieve this goal because it increases the distortion in the economy. In order to bring about Pareto improvement by reducing taxes, it is necessary and sufficient to improve welfare of the entrepreneurs who neither sell nor buy because both buyers and sellers will gain additional benefit. This implies that the equilibrium with asymmetric information is constrained efficient if it is not possible to increase tax revenue by reducing taxes. Letting \( T \equiv \tau^{sym} \phi \hat{z}^{sym} \pi_1 G(\hat{z}^{sym} / \chi) \) be the total revenue in a symmetric information economy, the sufficient condition for this to be true is

\[
\frac{dT}{d\tau^{sym}} \geq 0
\]

for all \( \tau^{sym} \in [0, \tau^{sym*}] \). Note that this is the exactly the same as the argument behind the Laffer curve in the optimal taxation literature. The condition can be rewritten as

\[
(1 + g(\hat{z}^{sym} / \chi))(\hat{z}^{sym} / \chi) \frac{d \log \hat{z}^{sym}}{d \log \tau^{sym}} \geq -1,
\]

where \( \frac{d \log \hat{z}^{sym}}{d \log \tau^{sym}} \leq 0 \) is the price elasticity with respect to taxes.\(^{15}\) Conditional on \( \frac{d \log \hat{z}^{sym}}{d \log \tau^{sym}} \), the condition is likely to hold when there are smaller masses of entrepreneurs around the cut-off to sell (\( g(\hat{z}^{sym} / \chi) \) low), or there is a greater volume of trade (high \( G(\hat{z}^{sym} / \chi) \)).

\[^{15}\]The expression for \( \frac{d \log \hat{z}^{sym}}{d \log \tau^{sym}} \) is

\[
\frac{d \log \hat{z}^{sym}}{d \log \tau^{sym}} = \frac{-(1 - G(\hat{z}^{sym}(1 + \tau^{sym})) \phi \hat{z}^{sym})}{(1 - G(\hat{z}^{sym}(1 + \tau))) \phi (1 + \tau^{sym}) + \pi_h \frac{1}{\chi \pi} g(\hat{z}^{sym} / \chi)}
\]