On the Misuse of Accounting Rates of Return to Infer Monopoly Profits

By Franklin M. Fisher and John J. McGowan*

Accounting rates of return are frequently used as indices of monopoly power and market performance by economists and lawyers.\(^1\) Such a procedure is valid only to the extent that profits are indeed monopoly profits, accounting profits are in fact economic profits, and the accounting rate of return equals the economic rate of return.

The large volume of research investigating the profits-concentration relationship uniformly relies on accounting rates of return, such as the ratio of reported profits to total assets or to stockholders' equity as the measure of profitability to be related to concentration.\(^2\) Many users of accounting rates of return seem well aware that profits as reported by accountants may not be consistent from firm to firm or industry to industry and may not correspond to economists' definitions of profits. Likewise, they recognize that accountants' statements of assets, hence also stockholders' equity, may fail to correspond to economically acceptable definitions, because accounting practices do not provide for the capitalization of certain activities such as research and development and do not incorporate allowances for inflation. This is to say they are well aware of certain measurement problems which arise in using available accounting information to measure profitability. They seem, however, totally unaware of a much deeper conceptual problem, namely, that accounting rates of return, even if properly and consistently measured, provide almost no information about economic rates of return.\(^3\)

The economic rate of return on an investment is, of course, that discount rate that equates the present value of its expected net revenue stream to its initial outlay. Putting aside the measurement problems referred to above, it is clear that it is the economic rate of return that is equalized within an industry in long-run industry competitive equilibrium and (after adjustment for risk) equalized everywhere in a competitive economy in long-run equilibrium. It is an economic rate of return (after risk adjustment) above the cost of capital that promotes expansion under competition and is produced by output restriction under monopoly. Thus, the economic rate of return is the only correct measure of the profit rate for purposes of economic analysis.\(^4\) Accounting rates of return are useful only insofar as they yield information as to economic rates of return.\(^5\)

\(^*\)Fisher is professor of economics, Massachusetts Institute of Technology. McGowan was Vice-President, Charles River Associates. He died on April 7, 1982. This paper is based on work done for Fisher's testimony as a witness for IBM in *U.S. v. IBM* (69 Civ. 200, U.S. District Court, Southern District of New York). We are indebted to Larry Brownstein, Steven Hendrick, and especially Karen Larson and Leah Hutten for computational and programming assistance. Any errors are our responsibility.

\(^1\)Aside from *U.S. v. IBM*, see, for example, Joseph Cooper, p. 15; the various industry studies in Walter Adams; and the discussion in Philip Areeda and Donald Turner, Vol. II, pp. 331–41.

\(^2\)See the comprehensive reviews of this literature by Leonard Weiss and more recently by F. M. Scherer, pp. 267–95. Additional accounting problems raised by attempting to measure profitability by line of business are discussed extensively in George Benston.

\(^3\)A referee suggests that even the crudest accounting information tells us IBM is more profitable than American Motors (AMC), but we disagree. Surely accounting information tells us IBM generates more dollars of profits per dollar of assets than does AMC but, as the examples below demonstrate, that information alone does not tell us which firm is more profitable in the sense of having a higher economic rate of return.

\(^4\)This is literally true only if the cost of capital is first subtracted. In what follows below, we follow the usual empirical practice of measuring all rates of return before such subtraction.

\(^5\)The existence of a uniquely defined economic rate of return—which we now assume for the theoretical analysis below and which occurs in all the examples—is
Now, it should be obvious that only by the merest happenstance will the accounting rate of return on a given investment, taken as the ratio of net revenue to book value in a particular year, be equal to the economic rate of return that makes the present value of the entire net revenue stream equal to the initial capital cost. Indeed, as we shall see below, accounting rates of return on individual investments generally vary all over the lot. Hence, only if such fluctuations are somehow averaged out by a firm’s investment behavior over time will its accounting rate of return even be roughly constant—let alone approximate the economic rate of return.7

It is easy to show that such averaging requires that the firm grow exponentially, investing in the same mix of investment types each year—an investment type being defined by a time shape of net revenues. Even in such an unrealistically favorable case, the accounting rate of return will generally depend on the rate of growth, equalling the economic rate of return only by accident. Furthermore, the relationship between the accounting and economic rates of return depends on the time shape of net revenues. Hence, only by accident will accounting rates of return be in one-to-one correspondence with economic rates of return. We show by example below that the effects involved cannot be assumed to be small—indeed, they can be large enough to account for the entire interfirm variation in accounting rates of return among the largest firms in the United States.

The plan of the paper is as follows. Section I summarizes the theoretical results which are proved and elucidated in the Appendix. These results establish the relationships among the various rates of return, time shapes, and rates of growth, and demonstrate in principle that accounting rates of return are not informative. The balance of the paper analyzes a series of relatively simple examples to show that the theoretical effects are not so small that they can be neglected in practice. Indeed, they are very large. A ranking of firms by accounting rates of return can easily invert a ranking by economic rates of return.

Before proceeding, we note that some of the theoretical results given below are not new. Ezra Solomon wrote a number of articles culminating in one dealing with the case of exponential growth in 1970. Thomas Stauffer published various theorems a year later (1971) and also attempted to make adjustments to accounting rates of return to correct for alternative cash flow profiles in testimony for the FTC in the Ready to Eat Cereal Litigation.8 J. Leslie Livingston and Gerald Salamon (1971) have also studied and attempted to determine a relationship between the accounting and internal rates of return. Yet, perhaps because Solomon’s focus was on the correct concepts of rate of return and cost of capital for rate regulation, or perhaps because none of the studies cited makes clear just how large the effects involved can be, the importance of these matters for more general industrial organization research appears to have gone largely unnoticed. It is our hope that the self-contained discussion of the present paper and, especially, the mag-

7Throughout this paper we work with accounting rates of return defined as ratios of profits to book values of capital. Similar (but not identical in detail) results apply to accounting rates of return on stockholders’ equity. The precise relations involved can, in principle, be inferred from the results given below. (Such results do apply directly to accounting rates of return on stockholders’ equity even in detail if we consider the firms being analyzed to hold neither debt nor retained earnings.)

7For discussion purposes—and in our examples below—we assume that the firm achieves the same economic rate of return on all its investments, and thus speak of “the” economic rate of return for the firm without worrying about differences between average and marginal rates. This is, of course, the most favorable case for the accounting rate of return for the firm as a whole.

8The proofs given below are different from Stauffer’s proofs, and, we think, more suitable for our present purposes than his where the propositions coincide.
nitudes of the effects exhibited in the examples below will remedy this.

I. Summary of Theoretical Results

The main theoretical results, which are proved and elucidated in the Appendix, are as follows:
(a) Unless depreciation schedules are chosen in a particular way, so that the value of the investment is calculated as the present value at the economic rate of return of the stream of benefits remaining in it—a choice which is exceptionally unlikely to be made—the accounting rate of return on a particular investment will differ from year to year, and will not in general equal the economic rate of return on that investment in any year.
(b) The accounting rate of return for the firm as a whole will be an average of the accounting rates of return for individual investments made in the past. The weights in that average will consist of the book value of those different investments which in turn depend on the depreciation schedule adopted, and, particularly, on the amount and timing of such investments.
(c) Unless the proportion of investments with a given time shape remains fixed every year, and unless the firm simply grows exponentially, increasing investments in each and every type of asset by the same proportion for every year, the accounting rate of return to the firm as a whole cannot even be expected to be constant, let alone be equal to the economic rate of return.
(d) Even where the firm does operate in such an unrealistic manner—the case most favorable to the accounting rate of return—the accounting rate of return will vary with the rate of growth of the firm, and will not generally equal the economic rate of return.
(e) The only reliable inferences concerning the economic rate of return that can be drawn (and only in such an unrealistically favorable case) from examination of the accounting rate of return stem from the fact that the accounting rate of return and the economic rate of return will be on the same side of the firm’s exponential growth rate. If the accounting rate of return is higher than the growth rate, then the economic rate of return is also higher than the growth rate. If the accounting rate of return is lower than the growth rate, then the economic rate of return is lower than the growth rate. If the accounting rate of return equals the growth rate, and in this case alone, the economic rate of return is guaranteed to be equal to the accounting rate of return.
(f) Even in the unrealistically favorable exponential growth case, the accounting rate of return depends crucially on the time shape of benefits, and the effect of growth on the accounting rate of return also depends on that time shape. In particular, it is not true that rapidly growing firms tend to understate their profits and slowly growing firms tend to overstate them. The effect can go the other way.
(g) All these results apply both to before- and after-tax rates of return.

II. The Likely Size of the Effects

We now show by example that differences between the accounting and economic rates of return can be quite large indeed. For the sake of economy we examine only differences in after-tax rates of return. We as-

9Such a “natural” depreciation formula—which we shall term “economic depreciation”—was first suggested by Harold Hotelling in 1925. It is somewhat misleading, however, to say that the fundamental conceptual problems discussed in the present paper are basically matters of depreciation accounting. Rather, there exists a particular form of depreciation which will correct those problems which stem from a fundamental difference between the economic and accounting rates of return. These problems arise even where machines never wear out. An example is given in Fisher (1979).

10Two assets are said to be of the same “type” if they yield the same time shape of benefits.

11It is worth pointing out that these results apply to accounting rates of return on total assets, not directly to accounting rates of return on stockholders’ equity. Further, they apply to accounting rates of return on beginning-of-year, not end-of-year or yearly average assets. As the examples below show, the problem of making inferences from accounting rates of return on end-of-year (or average) assets is even worse—if possible—than when beginning-of-year assets are used.

12Compare Cooper, pp. 132–33.
TABLE 1—AFTER-TAX ACCOUNTING RATES OF RETURN
(Percent for the Q-Profile; Six-Year Life; No Delay)

<table>
<thead>
<tr>
<th>Gross Profits (Cash Flow Before-Tax)</th>
<th>Depreciation</th>
<th>After-Tax Profits</th>
<th>Accounting Rate of Return</th>
<th>End-of-Year Assets Accounting Rate of Return</th>
</tr>
</thead>
<tbody>
<tr>
<td>Year</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>23.3</td>
<td>28.6</td>
<td>(5.3)</td>
<td>100.0 (5.3)</td>
</tr>
<tr>
<td>2</td>
<td>44.1</td>
<td>23.8</td>
<td>11.2</td>
<td>71.4 (15.7)</td>
</tr>
<tr>
<td>3</td>
<td>51.9</td>
<td>19.0</td>
<td>18.1</td>
<td>47.6 (38.0)</td>
</tr>
<tr>
<td>4</td>
<td>40.5</td>
<td>14.3</td>
<td>14.4</td>
<td>28.6 (50.3)</td>
</tr>
<tr>
<td>5</td>
<td>20.2</td>
<td>9.5</td>
<td>5.9</td>
<td>14.3 (41.3)</td>
</tr>
<tr>
<td>6</td>
<td>7.8</td>
<td>4.8</td>
<td>1.7</td>
<td>4.8 (35.4)</td>
</tr>
</tbody>
</table>

a Tax rate: 45 percent; After-tax economic rate of return: 15 percent; Sum-of-the-years' digits depreciation.

sume a corporate tax rate of 45 percent, and (for most examples) fix the after-tax economic rate of return at 15 percent while varying growth rates and depreciation methods and the time shape of benefits. Enormous variations in the accounting rates of return are readily generated.

A. The "Q-Profile"

We start with an investment whose benefits begin immediately and last for six years, and follow the time shape exhibited in column 2 of Table 1. For convenience we refer to this shape as the Q-profile. The figures in column 2 are scaled to produce an after-tax economic rate of return of 15 percent on an initial investment of $100 when sum-of-the-years' digits depreciation over a six-year life is used. The remainder of the table shows the calculation of the corresponding accounting rate of return each year.

Plainly, the after-tax accounting rates of return vary substantially. They never equal the after-tax economic rate of return (15 percent), and exceed it in every year with positive net profits. Real-life firms do not generally exhibit such variation in their accounting rates of return because the averaging effects of growth, as it were, attribute profits from past investment to the book value of investments whose profit results are yet to come, rather than to the declining book value of such past investment.

While such an averaging effect tends to stabilize the accounting rate of return, it becomes a hodgepodge devoid of information about the economic rate of return. This point is illustrated by Table 2, which presents asymptotic accounting rates of return assuming constant exponential growth for three different versions of the Q-profile, each with the same tax rate (45 percent) and after-tax economic rate of return (15 percent). The first version (the case of Table 1)

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13 Fifteen percent was roughly the average accounting rate of return in U.S. manufacturing corporations in 1978 (Economic Report of the President, 1979, pp. 279–91). If accounting and economic rates of return tended to coincide, 15 percent would be a reasonable choice for the economic rate of return. Since the rates do not generally coincide, the choice is immaterial. Choosing a lower economic rate of return would reduce the range of accounting rates of return in the results below (for the same examples), but would not affect the conclusions.

With a fixed capital investment, a given time shape of gross profits before depreciation and taxes results in different after-tax economic rates of return for different depreciation methods. To fix the after-tax economic rate of return for a given time shape, therefore, we adjust the height of the gross profit benefit stream proportionally to produce the desired after-tax economic rate of return.

14 This shape was (erroneously) suggested during U.S. v. IBM as being typical of IBM's experience. We use it for convenience.

15 In this context, exponential growth takes place by repeated investment in the same type of project; i.e., all investments have the same time shape of benefits. This is obviously an unrealistic assumption, but one which is more likely to produce equality between accounting and economic rates of return than more realistic assumptions.
Table 2—Asymptotic Accounting Rates of Return (%) on Three Versions of the Q-Profile*  

<table>
<thead>
<tr>
<th>Growth Rate</th>
<th>Six-Year Life (No Delay)</th>
<th>Seven-Year Life (One-Year Delay)</th>
<th>Eight-Year Life (Two-Year Delay)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Beginning-of-Year Assets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>15.2</td>
<td>17.8</td>
<td>18.1</td>
</tr>
<tr>
<td>5</td>
<td>15.2</td>
<td>16.9</td>
<td>17.0</td>
</tr>
<tr>
<td>10</td>
<td>15.1</td>
<td>15.9</td>
<td>15.9</td>
</tr>
<tr>
<td>15</td>
<td>15.0</td>
<td>15.0</td>
<td>15.0</td>
</tr>
<tr>
<td>20</td>
<td>14.8</td>
<td>14.1</td>
<td>14.1</td>
</tr>
<tr>
<td>25</td>
<td>14.7</td>
<td>13.3</td>
<td>13.3</td>
</tr>
<tr>
<td>30</td>
<td>14.5</td>
<td>12.5</td>
<td>12.6</td>
</tr>
<tr>
<td>B. End-of-Year Assets</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>15.2</td>
<td>17.8</td>
<td>18.1</td>
</tr>
<tr>
<td>5</td>
<td>14.5</td>
<td>16.1</td>
<td>16.2</td>
</tr>
<tr>
<td>10</td>
<td>13.7</td>
<td>14.5</td>
<td>14.5</td>
</tr>
<tr>
<td>15</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
</tr>
<tr>
<td>20</td>
<td>12.4</td>
<td>11.8</td>
<td>11.8</td>
</tr>
<tr>
<td>25</td>
<td>11.7</td>
<td>10.6</td>
<td>10.7</td>
</tr>
<tr>
<td>30</td>
<td>11.1</td>
<td>9.6</td>
<td>9.7</td>
</tr>
</tbody>
</table>

*See Table 1.

has no delay between investment and the beginning of the benefit stream, and depreciation is taken over the resulting six-year life. The second version has a seven-year life including a one-year’s delay between investment and initial return. The third has an eight-year life including a two-year delay between investment and initial return. Except for the lag at the beginning and differences in scale, the gross benefit stream is the same in each case. Panel A of the table gives accounting rates of return on beginning-of-year assets; Panel B gives those on end-of-year assets.

Several things are apparent from Table 2. First, the accounting rates of return only equal the economic rate of return of 15 percent when the growth rate is also 15 percent and when the accounting rate of return is measured on beginning-of-year assets. Otherwise, the accounting rates vary from seven points below to almost eleven points above the economic rate of return.

Second, it is not true (as is sometimes stated) that more rapid depreciation, other things equal, tends to understate accounting rates of return. In this example, when the rate of growth is below 15 percent, declining balance and sum-of-the-years’ digits depreciation produces a higher accounting rate of return than straightline depreciation for given growth rates, time profiles, and economic rates of return. The effect is reversed when the growth rate exceeds the economic rate of return of 15 percent. This illustrates a general proposition: more rapid depreciation increases the accounting rate of return (measured on beginning-of-year assets) when the growth is less than the economic rate of return, and decreases the accounting rate of return when the growth rate exceeds the economic rate of return.\(^{16}\) Since this is the only point about depreciation which we wish to demonstrate, we provide only results for sum-of-the-years’ digits depreciation in the rest of this paper.\(^{17}\)

In all the examples in Table 2, firms growing at rates greater than the economic rate of

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\(^{16}\)By Theorem 1, the changeover point is also where the growth rate equals the accounting rate of return on beginning-of-year assets.

\(^{17}\)There is one additional point about depreciation which we shall not bother to exemplify. Since the depreciation method chosen affects the time shape of the after-tax benefit stream, the relationship of after-tax accounting rates to the growth rate is particularly sensitive to the depreciation method. It can even happen that faster growth increases accounting rates of return for one choice of depreciation method and decreases them for another—all for the same pre-tax benefit time shape and the same after-tax economic rate of return. This makes adjustments for growth even harder to make than appears from the examples below.
Table 3—Asymptotic Accounting Rates of Return (%) on Four Versions of the Q-Profile

<table>
<thead>
<tr>
<th>Growth Rate</th>
<th>Ten-Year Life (No Delay, Last Year Spread)</th>
<th>Six-Year Life (No Delay)</th>
<th>Seven-Year Life (One-Year Delay)</th>
<th>Eight-Year Life (Two-Year Delay)</th>
</tr>
</thead>
<tbody>
<tr>
<td>A. Beginning-of-Year Assets</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>13.9</td>
<td>18.1</td>
<td>22.0</td>
<td>25.9</td>
</tr>
<tr>
<td>5</td>
<td>14.5</td>
<td>17.0</td>
<td>19.4</td>
<td>21.7</td>
</tr>
<tr>
<td>10</td>
<td>14.8</td>
<td>15.9</td>
<td>17.1</td>
<td>18.1</td>
</tr>
<tr>
<td>15</td>
<td>15.0</td>
<td>15.0</td>
<td>15.0</td>
<td>15.0</td>
</tr>
<tr>
<td>20</td>
<td>15.1</td>
<td>14.1</td>
<td>13.1</td>
<td>12.3</td>
</tr>
<tr>
<td>25</td>
<td>15.1</td>
<td>13.3</td>
<td>11.4</td>
<td>9.9</td>
</tr>
<tr>
<td>30</td>
<td>15.0</td>
<td>12.6</td>
<td>9.9</td>
<td>7.8</td>
</tr>
<tr>
<td>B. End-of-Year Assets</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>13.9</td>
<td>18.1</td>
<td>22.0</td>
<td>25.9</td>
</tr>
<tr>
<td>5</td>
<td>13.8</td>
<td>16.2</td>
<td>18.5</td>
<td>20.7</td>
</tr>
<tr>
<td>10</td>
<td>13.5</td>
<td>14.5</td>
<td>15.5</td>
<td>16.5</td>
</tr>
<tr>
<td>15</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
<td>13.0</td>
</tr>
<tr>
<td>20</td>
<td>12.6</td>
<td>11.8</td>
<td>10.9</td>
<td>10.2</td>
</tr>
<tr>
<td>25</td>
<td>12.0</td>
<td>10.7</td>
<td>9.2</td>
<td>7.9</td>
</tr>
<tr>
<td>30</td>
<td>11.5</td>
<td>9.7</td>
<td>7.6</td>
<td>6.0</td>
</tr>
</tbody>
</table>

See Table 1.

Return of 15 percent have accounting rates of return on beginning-of-year assets less than the economic rate of return, while those growing at rates less than the economic rate of return all have accounting rates of return on beginning-of-year assets greater than the economic rate of return. Contrary to what might be expected, this qualitative relationship provides no practical basis for adjusting accounting rates of return so that they will accurately reflect economic rates of return.

Table 2, for example, shows that firms which use sum-of-the-years digits depreciation and grow at 5 percent have accounting rates of return on beginning-of-year assets which range from 17.0 to 21.7 percent. Thus, even for firms with the same growth rate and depreciation method, the required adjustment varies from 2 to 6.7 percentage points depending upon the time profile. Clearly, the time profile, depreciation method, and growth rate must all be known before accounting rates of return can be adjusted to reflect economic rates of return.

In the foregoing examples, for a given time shape, faster-growing firms have lower accounting rates of return than slower-growing ones with the same economic rate of return. We have seen that even if this were a universal phenomenon, it would not provide a way to adjust accounting rates of return to reflect economic rates of return, since different firms will generally have different time shapes and therefore require different adjustments. The difficulties are even worse in practice, because the accounting rate of return can actually rise with the growth rate, causing slower-growing firms to have their economic rates of return understated. Thus, even the strong assumption that firms have the same time profile is insufficient to permit adjustment of accounting rates of return; the specific profile must also be known in order to make inferences about the ranking of economic rates of return.

We demonstrate this phenomenon by taking the original Q-profile (six-year life and no delay) and spreading the last year’s gross profits out evenly over five years (years 6–10) instead of having them all in year 6. Table 3 shows that this small change in the profile produces an increasing relationship between the growth rate and the accounting rate of return. The original results for sum-of-the-year’s digits depreciation are reproduced for ease of comparison.

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18So simple a relationship does not hold if the accounting rate of return is based on end-of-year assets.
Focusing on the first column (10-Year Life), we see that the accounting rate of return on beginning-of-year assets actually begins by rising with the growth rate, reaching the value of the economic rate of return (as it must) at a 15 percent growth rate, and then going slightly above it before falling back again. (It is a special feature of this particular example that these values are all close to the economic rate of return of 15 percent.) The behavior of the accounting rate of return on end-of-year assets is different. This magnitude falls with the growth rate (in this example), but it exhibits still another phenomenon. As opposed to the previous example, where the accounting rates of return on both beginning- and end-of-year assets were above the economic rate of return of 15 percent for low growth rates and below it for large ones, here the accounting rate of return on end-of-year assets starts and finishes below the economic rate of return of 15 percent. There is no rate of growth for which the accounting rate of return on end-of-year assets equals the economic rate of 15 percent.

The impossibility of making inferences about relative profit rates should be obvious even within the confines of these examples, all of which represent only relatively slight variations on the same profile. Every one of the firms exhibited in Table 3 has the same underlying after-tax economic rate of return. Yet their after-tax accounting rates of return on end-of-year assets vary from 6.0 to 25.9 percent.19

Further, it is impossible to infer anything about relative profitability by attempting to adjust for growth rates. For example, each row of Table 3 involves firms with the same growth rate, so that there is nothing to adjust for in comparing them; yet, except for the special row corresponding to the point where the growth rate is equal to the true after-tax economic rate of return, the after-tax accounting rates of return continue to vary. For the row corresponding to 5 percent growth, for example, after-tax accounting rates of return vary between 13.8 and 20.7 percent. For the row corresponding to 25 percent, they vary between 7.9 and 12.0 percent. Further, it is not correct to say that slow-growing firms have accounting rates of return that overstate their economic rate, while fast-growing firms have accounting rates of return that underestimate them. Continuing to use accounting rates of return on end-of-year assets, the firm just introduced (10-Year Life) has an accounting rate of return which understates its economic rate of return at all levels of growth. If one uses beginning-of-year assets, it has accounting rates of return which tend to understate its economic rate of return at low rates of growth and (slightly) overstate it at higher ones.

Moreover, the phenomenon of accounting rates of return increasing with the growth rate can be considerably more marked if we use other profiles. Table 4 shows the before-tax benefit stream (corresponding to an initial investment of $100, an economic rate of return of 15 percent, and sum-of-the-years' digits depreciation over a six-year life) for two other profiles (X firm and Y firm). Table 5 shows the after-tax accounting rates of return for these firms when they grow exponentially at various rates. The after-tax accounting rates of return on beginning-of-year assets rise rather rapidly with the growth rate. The after-tax accounting rate of return on end-of-year assets also rises with the growth rate. However, as was also the case for the variation on the Q-profile examined earlier, it does not rise by enough to get to the economic rate of return of 15 percent.

19Here and later, the results for beginning-of-year assets are similar.

<table>
<thead>
<tr>
<th>Year</th>
<th>$X$ Firm ($)</th>
<th>$Y$ Firm ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>90.2</td>
<td>107.0</td>
</tr>
<tr>
<td>2</td>
<td>27.1</td>
<td>10.7</td>
</tr>
<tr>
<td>3</td>
<td>18.0</td>
<td>10.7</td>
</tr>
<tr>
<td>4</td>
<td>9.0</td>
<td>10.7</td>
</tr>
<tr>
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<td>9.0</td>
<td>10.7</td>
</tr>
<tr>
<td>6</td>
<td>9.0</td>
<td>10.7</td>
</tr>
</tbody>
</table>

*See Table 1.
### Table 5—Asymptotic Accounting Rates of Return (%)
For X-Firms and Y-Firms*

<table>
<thead>
<tr>
<th>Growth Rate</th>
<th>Beginning-of-Year Assets</th>
<th>End-of-Year Assets</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>X Firm</td>
<td>Y Firm</td>
</tr>
<tr>
<td>0</td>
<td>12.9</td>
<td>12.5</td>
</tr>
<tr>
<td>5</td>
<td>13.6</td>
<td>13.3</td>
</tr>
<tr>
<td>10</td>
<td>14.3</td>
<td>14.2</td>
</tr>
<tr>
<td>15</td>
<td>15.0</td>
<td>15.0</td>
</tr>
<tr>
<td>20</td>
<td>15.7</td>
<td>15.8</td>
</tr>
<tr>
<td>25</td>
<td>16.3</td>
<td>16.6</td>
</tr>
<tr>
<td>30</td>
<td>16.9</td>
<td>17.3</td>
</tr>
</tbody>
</table>

*See Table 1.

### III. Conclusions

That the accounting rate of return—after tax as well as before tax—is a misleading measure of the economic rate of return is evident from examining cases of single projects such as in Table 1. The cases shown in later tables are unduly favorable to the accounting rate of return in that they mask its behavior by averaging. That averaging effect is achieved by the quite unrealistic assumption that investment by the firm always brings in the same time shape of returns, and that the firm grows each year by increasing its investments at the same percentage rate. Even on such favorable terms, it is impossible to infer either the magnitude or direction of differences in economic rates of return from differences in accounting rates of return. This is because such inferences require not only correction for growth rates, but also knowledge of the time shapes of returns.

The level and behavior of the accounting rate of return are both sensitive to the type of time shape used. Even within the Q-profile example, the rates vary depending on when the time shape begins and how the last few years are spread out. There is every reason to suppose that firms differ in the time shapes of their investments, and that a particular firm's investments will also differ among themselves. Thus, comparisons of accounting rates of return to make inferences about monopoly profits is a baseless procedure.

This conclusion can be most dramatically demonstrated by juxtaposing accounting rates of return for firms with different time shapes and different economic rates of return. When this is done, it is easy to see that firms with higher accounting rates of return can have lower economic rates of return. Table 6 gives after-tax economic rates of return and after-tax accounting rates of return on end-of-year assets for three growth rates (0, 5, and 10 percent), and for each of the six time shapes already discussed as well as two other "one-hoss shay" time shapes. For each growth rate, the examples are chosen so that the eight firms represented are ranked in ascending order of economic rates of return and in descending order of accounting rates of return—a complete reversal even with growth rates constant.

Examination of Table 6 shows again that no inference about relative after-tax economic rates of return is possible from after-tax accounting rates of return. For example, the lowest after-tax economic rate of return in the table is that for the Q-profile with an eight-year life at a zero growth rate. For that firm, the after-tax economic rate of return is 13 percent. Yet, its after-tax accounting rate

---

20 The one-hoss shay time shapes have a constant return (no lag) for four and six years, respectively, and zero returns thereafter.
Table 6—After-Tax Economic Rates of Return (E) and Asymptotic Accounting Rates of Return on End-of-Year Assets (A) For Eight Time Shapes

<table>
<thead>
<tr>
<th>Growth Rate</th>
<th>0 Percent</th>
<th>5 Percent</th>
<th>10 Percent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>E</td>
<td>A</td>
<td>E</td>
</tr>
<tr>
<td>Q-Profile 8-Year Life (2-year delay)</td>
<td>13.0</td>
<td>21.6</td>
<td>16.0</td>
</tr>
<tr>
<td>7-Year Life (1-year delay)</td>
<td>14.0</td>
<td>20.2</td>
<td>17.0</td>
</tr>
<tr>
<td>One-Hoss Shay 6-Year Life (no delay)</td>
<td>15.0</td>
<td>20.0</td>
<td>18.1</td>
</tr>
<tr>
<td>4-Year Life (no delay)</td>
<td>16.0</td>
<td>19.8</td>
<td>19.0</td>
</tr>
<tr>
<td>Q-Profile 6-Year Life (no delay)</td>
<td>16.1</td>
<td>19.6</td>
<td>19.05</td>
</tr>
<tr>
<td>10-Year Life (no delay; last year spread)</td>
<td>18.0</td>
<td>16.9</td>
<td>20.0</td>
</tr>
<tr>
<td>X Firm</td>
<td>19.0</td>
<td>16.2</td>
<td>21.0</td>
</tr>
<tr>
<td>Y Firm</td>
<td>19.2</td>
<td>15.8</td>
<td>21.2</td>
</tr>
</tbody>
</table>

*Tax rate: 45 percent; Sum-of-the-years’ digits depreciation.

The economic rate of return on end-of-year assets is 21.6 percent, the second highest accounting rate of return in the table, and a value well above that of 15.8 percent for the Y firm at zero growth, corresponding to a 19.2 percent economic rate of return. The 21.6 percent accounting rate of return so encountered is even above the 18.9 percent figure obtained for the Y firm at 10 percent growth—a figure which corresponds to an economic rate of return of 23.2 percent, the highest in the table, and more than 10 percentage points above the economic rate of return of 13 percent for the Q-profile with an eight-year life at zero growth. Similar examples of reversals occur throughout the table.

Moreover, it is not true that faster-growing firms should have their accounting rates of return adjusted upwards relative to slower growing ones. Consider the comparison between the Q-profile with a ten-year life at zero growth and the Q-profile with an eight-year life at 5 percent growth. The faster-growing firm has an accounting rate of return (22.6 percent) already greater than that of the slower-growing firm (16.9 percent), but its economic rate of return (16.0 percent) is below that of the slower-growing firm (18.0 percent). Adjusting the faster-growing firm's accounting rate of return upwards relative to that of the slower-growing one will make things worse, not better.

As all of this makes clear, there is no way in which one can look at accounting rates of return and infer anything about relative economic profitability or, a fortiori, about the presence or absence of monopoly profits. The economic rate of return is difficult—perhaps impossible—to compute for entire firms. Doing so requires information about both the past and the future which
outside observers do not have, if it exists at all. Yet it is the economic rate of return which is the magnitude of interest for economic propositions. Economists (and others) who believe that analysis of accounting rates of return will tell them much (if they can only overcome the various definitional problems which separate economists and accountants) are deluding themselves. The literature which supposedly relates concentration and economic profit rates does no such thing, and examination of absolute or relative accounting rates of return to draw conclusions about monopoly profits is a totally misleading enterprise.

APPENDIX

I: BEFORE-TAX ANALYSIS

A. The Accounting Rate of Return on Individual Investments

We begin our analysis of the problem by considering the before-tax accounting and economic rates of return on a single investment. Later we shall consider the firm as being made up of a series of such investments which may be (but need not always be) of the same type. The after-tax case is treated below and shown to be isomorphic, although more complex.

An investment may be thought of for heuristic purposes as a "machine" costing one dollar. If this is invested at time 0, the firm experiences a stream of net benefits as a result. Such benefits include all changes in revenues and costs (other than the initial capital cost) which accrue to the firm as a result of making the investment. The flow of such benefits at time \( \theta \) is denoted by \( f(\theta) \).

The economic rate of return on a machine, \( r \), is that discount rate which makes the discounted value of the benefit stream equal to the capital costs of the investment. In other words, \( r \) satisfies

\[
(\text{A1}) \quad \int_0^\infty f(\theta) \exp(-r\theta) \, d\theta = 1.
\]

We assume that the integral in (A1) is monotonically decreasing in \( r \) so that (A1) has a unique positive solution. This will be true if the negative portion of the net benefit stream (if any) precedes the positive portion. This is the usual case.

Now the firm adopts a depreciation schedule for this machine. Let \( V(\theta) \) denote the book value of the machine as of time \( \theta \). Then \( -V'(\theta) \) is the rate of depreciation at \( \theta \), where the prime denotes differentiation. Plainly, \( V(0) = 1 \), and it makes sense to suppose that \( V(\infty) = 0 \), although this latter condition is not really needed.

Accounting profits attributable to this machine at time \( \theta \) will be equal to net benefits less depreciation. We can think of the accounting rate of return for this machine as the accounting rate of return which the firm would have if this were its only asset. Denoting that rate by \( b(\theta) \),

\[
(\text{A2}) \quad b(\theta) = (f(\theta) + V'(\theta))/V(\theta).
\]

The first question which comes immediately to mind is that of when \( b(\theta) = r \) for all \( \theta \) within the life of the machine. We prove this will occur if and only if the depreciation schedule adopted by the firm always values the machine as the discounted value of the future benefit stream, discounting at the economic rate of return, \( r \) (see Hotelling).

THEOREM 1: \( b(\theta) = r \) if and only if

\[
(\text{A3}) \quad V(\theta) = \int_{\theta}^{\infty} f(u) \exp(-r(u-\theta)) \, du.
\]

If (A1) has more than one solution, then the economic rate of return is ill-defined and there is even less point in considering whether the accounting rate of return yields information about it.
PROOF:
(a) Suppose (A3) holds. Differentiating with respect to $\theta$, we obtain

$$V'(\theta) = -f(\theta) + rV(\theta),$$

which when substituted in equation (A2) yields $b(\theta) = r$.

(b) Suppose $b(\theta) = r$. Then, from (A2),

$$V'(\theta) = rV(\theta) - f(\theta).$$

This is a linear differential equation with an additive forcing function $(-f(\theta))$. Its solution is therefore in the form

$$V(\theta) = C \exp(r\theta) + z(\theta),$$

where $z(\theta)$ is any particular solution of (A5) and $C$ is a constant to be determined by the initial conditions. However, by part (a) of the proof, the integral on the right-hand side of (A3) is a particular solution of (A5). Hence $z(\theta)$ can be taken as that integral. Do this and note that $z(0) = 1$ by (A1), the definition of the economic rate of return. Since we have $V(0) = 1$, setting $\theta = 0$ in (A6) yields $C = 0$, and the theorem is proved.

Thus even where the firm has a single simple investment with no ambiguity about marginal vs. economic rates of return, the accounting rate of return will not equal the economic rate of return except for a particular choice of a depreciation schedule—which choice we may term "economic depreciation."

The reason for this is not hard to find. The book value of the firm's assets reflects the investment expenditures made in the past less the depreciation already taken on them. The benefits for which such investments were made are at least partly in the future. Yet the accounting rate of return takes gross profits before depreciation as the benefit flow which happens to be currently occurring. Unless depreciation is chosen so as to reflect the change in future benefits in the appropriate way, there is no reason to suppose that such a calculation should equal the economic rate of return, and Theorem 1 shows that the two will generally not be equal.

Will firms tend to adopt an "economic depreciation" schedule yielding book value as in equation (A3)? This is pathologically unlikely. Except in the simple "Santa Claus" case of $f(\theta) = k \exp(-\lambda \theta)$ which corresponds to exponential depreciation or other similarly special cases corresponding to straightline or other standard depreciation methods, the benefit stream from investment when plugged into (A3) will not yield depreciation schedules anything like those used by real-life firms to optimize after-tax profits given IRS rules or those schedules used for nontax purposes. Real investments will almost invariably have complicated time shapes for their benefit streams. Further, even relatively simple shapes yield economic depreciation schedules which are quite far from actual ones. To see this, one need only observe that if $V(\theta)$ satisfies equation (A3), there is no reason that $V'(\theta)$ must always be negative. Indeed, if the time stream of benefits starts low and then has a hump a few years out, taking economic depreciation would require writing up the value of assets for the first few years. Yet there is nothing bizarre about such an example.

We must, therefore, with pathologically unlikely exceptions, expect that the accounting rate of return on a particular machine, $a(\theta)$, will generally not equal the economic rate of return $r$. (How far off it can be is demonstrated by examples.) This should make us suspect that the same thing will generally be true of the firm as a whole, and we now go on to explore that question.

B. The Accounting Rate of Return for the Firm as an Average

It is fairly plain that the best hope for an accounting rate of return equal to the economic rate of return will occur if all investments made by the firm are exactly alike, since otherwise (as shown below), changes in the mix of investment types will change the accounting rate. So we begin by considering the case in which all machines are like the machine above.

It now becomes necessary to distinguish calendar time, denoted by $t$, from the age of a machine, denoted by $\theta$. We let $I(t)$ be the
value of investment made at $t$ (equals the number of machines purchased). Let $K(t)$ denote the book value of the firm's assets at $t$ and $\pi(t)$ the value of its accounting profits at $t$. Then,

\begin{equation}
\begin{aligned}
K(t) &= \int_{-\infty}^{t} I(u) V(t-u) \, du \\
&= \int_{0}^{\infty} (t-\theta) V(\theta) \, d\theta,
\end{aligned}
\end{equation}

where $\theta = t - u$. Similarly,

\begin{equation}
\begin{aligned}
\pi(t) &= \int_{-\infty}^{t} I(u)(f(t-u)+V'(t-u)) \, du \\
&= \int_{0}^{\infty} (t-\theta)(f(\theta)+V'(\theta)) \, d\theta \\
&= \int_{0}^{\infty} (t-\theta)V(\theta)b(\theta) \, d\theta,
\end{aligned}
\end{equation}

using (A2).

Hence, letting $a(t)$ be the firm's accounting rate of return at $t$:

\begin{equation}
\begin{aligned}
a(t) &= \frac{\pi(t)}{K(t)} \\
&= \left[ \int_{0}^{\infty} (t-\theta)V(\theta)b(\theta) \, d\theta \right] \\
&\quad \bigg/ \left[ \int_{0}^{\infty} (t-\theta)V(\theta) \, d\theta \right];
\end{aligned}
\end{equation}

so that we have proved

\textbf{Lemma 1:} At any time $t$, the accounting rate of return for the firm as a whole is a weighted average of the individual accounting rates for its individual past investments, the weights being the book values of those past investments.

It should be obvious that this result would also be true if machines were not always of one type.

We now ask whether such an average will equal the economic rate of return. First consider whether the average can even be independent of $t$. This can happen in two ways. First, $b(\theta)$ might be independent of $\theta$. We know from Theorem 1 that this will happen for $b(\theta) \equiv r$ only for the cases of economic depreciation already discussed which we rule out. It is easy to show that $b(\theta) \equiv q \neq r$ is impossible.\textsuperscript{24}

The other way in which $a(t)$ might be independent of $t$ would be if the relative weights in the average did not change over time.\textsuperscript{25}

\begin{equation}
\frac{I(t_1 - \theta)V(\theta)}{I(t_2 - \theta)V(\theta)} = k,
\end{equation}

whence

\begin{equation}
\frac{I'(t_1 - \theta)}{I(t_1 - \theta)} = \frac{I'(t_2 - \theta)}{I(t_2 - \theta)},
\end{equation}

for all $(t_1, t_2)$. Evidently it must then be the case that

\begin{equation}
I(t) = M \exp(gt)
\end{equation}

for some constant growth rate $g$.

The remainder of our investigation will concern the case of exponential growth with the scale factor, $M$, set equal to unity. This case is the most favorable to accounting rates of return approximating economic rates of return, since in its absence accounting rates of return will not even be constant, even though the economic rate of return is well defined and constant.

C. The Effect of the Growth Rate in Exponential Growth

We are now dealing with a case in which the accounting rate of return is (at least

\textsuperscript{24}To see this, observe that a proof essentially the same as that of Theorem 1 would show that $b(\theta) \equiv q$ if and only if

\begin{equation}
V(\theta) = C \exp(q\theta) + z(\theta),
\end{equation}

where

\begin{equation}
z(\theta) = \int_{0}^{\infty} f(u) \exp(-q(u-\theta)) \, du,
\end{equation}

but $V(0) = 1$ so that $C = 1 - z(0) = 0$ if $q \neq r$. Then (A4) yields $V(\infty) = \pm \exp$ depending on $q \geq r$ and this is not possible.

\textsuperscript{25}For a given distribution of $b(\theta)$ there might be other possibilities, but these would be even more special than the case of economic depreciation already discussed. The statement in the text is true if $a(i)$ is to be constant despite unknown variations in $b(\theta)$ with $\theta$.}
asymptotically) constant and given as

\begin{equation}
\pi^*(t) = \int_{-\infty}^{t} \exp(\mu) f(t-u) \, du
\end{equation}

where \(a\) denotes the (asymptotic) constant value. This is still a weighted average of the accounting rates of return on individual investments. Plainly, the growth rate \(g\) affects the weights. Since the accounting rates of return on individual investments will almost always not be constant in view of Theorem 1, changes in the weights will usually affect the average.

The present section studies such effects and asks, in particular, what inferences can be drawn concerning the economic rate of return \(r\), from knowledge of the accounting rate of return \(a\), and the growth rate \(g\), without information on the time shape of benefits, \(f(\cdot)\), since the latter information is plainly never available from the books of the firm—even assuming it is known in detail to the firm’s forecasters.

The first thing to say in this regard is that while (as we shall show) there exist values of \(g\) for which \(a = r\), these values will be the exception. One cannot expect accounting and economic rates of return to coincide even in the most favorable case of exponential growth and a single investment type except by the merest accident. What information can be gleaned from the accounting rate of return is analyzed in this section.

It will be convenient to set up the problem a little differently from the analyses above. Let \(\pi^*(t)\) denote the gross profits of the firm before depreciation. Let \(\delta(t)\) denote total depreciation taken at time \(t\). Let \(K^*(t)\) denote the undepreciated value of the firm’s capital stock. Let \(D(t)\) denote the total depreciation already taken on that stock. Finally, let \(a^* = \pi^*(t)/K^*(t)\), so that \(a^*\) is the accounting rate of return which would be observed if there were no depreciation. The following relationships hold:

\begin{equation}
a = \frac{\pi^*(t) - \delta(t)}{K^*(t) - D(t)},
\end{equation}

\begin{equation}
\pi^*(t) = \int_{-\infty}^{t} \exp(\mu) f(t-u) \, du
\end{equation}

\begin{equation}
= \int_{0}^{\infty} \exp(g(t-\theta)) f(\theta) \, d\theta
\end{equation}

\begin{equation}
= \exp(g t) \int_{0}^{\infty} \exp(-g\theta) f(\theta) \, d\theta
\end{equation}

\begin{equation}
= \exp(g t) \pi^*(0),
\end{equation}

\begin{equation}
K^*(t) = \int_{-\infty}^{t} \exp(g u) \, du
\end{equation}

\begin{equation}
= \exp(g t)/g
\end{equation}

\begin{equation}
\delta(t) = \int_{-\infty}^{t} \exp(g u) V'(t-u) \, du
\end{equation}

\begin{equation}
= \int_{0}^{\infty} \exp(g(t-\theta)) V'(\theta) \, d\theta
\end{equation}

\begin{equation}
= \exp(g t) \delta(0),
\end{equation}

\begin{equation}
D(t) = \int_{-\infty}^{t} \delta(u) \, du
\end{equation}

\begin{equation}
= \int_{-\infty}^{t} \exp(g u) \delta(0) \, du
\end{equation}

\begin{equation}
= \delta(0) \exp(g t)/g.
\end{equation}

Evidently, we have proved:

**Lemma 2**: \(\delta(t)/D(t) = g\).

We now study the effects of \(g\) on \(a^*\).

**Lemma 3**: (a) If \(g = r\), then \(a^* = r = g\).

(b) \(d \log a^*/d \log g < 1\).

(c) \(a^*\) and \(r\) are always on the same side of \(g\). That is, \(a^* < g \iff r < g\); \(a^* = g \iff r = g\); \(a^* > g \iff r > g\).

**Proof**:

(a) Using equations (A15) and (A16),

\begin{equation}
a^* = g \pi^*(0)
\end{equation}

\begin{equation}
= g \int_{0}^{\infty} \exp(-g\theta) f(\theta) \, d\theta.
\end{equation}
If \( g = r \), then \( \pi^*(0) = 1 \) by the definition of the economic rate of return (A1), whence \( a^* = g = r \).

(b) From (A19),

\[
(A20) \quad \log a^* = \log g + \log \pi^*(0),
\]

but examination of \( \pi^*(0) \) shows that it is necessarily decreasing in \( g \) since it is the discounted integral of future benefits from a single machine discounted at the rate \( g \). Thus \( d \log a^*/d \log g < 1 \).

(c) These statements follow directly from (a) and (b).

Using Lemmas 2 and 3, we can now proceed to the main result of this section for the magnitude of interest, the accounting rate of return \( a \), itself.

THEOREM 2: \( a \) and \( r \) are always on the same side of \( g \). That is, \( a < g \leftrightarrow r < g; \quad a = g \leftrightarrow r = g; \quad a > g \leftrightarrow r > g \).

PROOF:

By definition, \( \pi^*(t) = a^*K^*(t) \). By Lemma 2, \( \delta(t) = gD(t) \). Substituting in (A14)

\[
(A21) \quad a = \frac{a^*K^*(t) - gD(t)}{K^*(t) - D(t)} \geq g,
\]

accordingly as \( a^* \geq g \). The desired result now follows from Lemma 3.

A diagram may be illuminating here. In Figure 1, the growth rate is measured on the horizontal axis and rates of return on the vertical axis. The 45° line indicates where growth rates and rates of return are equal. Theorem 2 states that the accounting rate of return must be above the 45° line to the left of the dashed line at \( g = r \); it must pass through \( H \), the point of intersection of the dashed line and the 45° line; and it must be below the 45° line to the right of the dashed line.

Can we say more than this? The answer is in the negative without information on the time shape of benefits \( f(\cdot) \). In particular, it is not the case that the direction of change of \( a \) with respect to \( g \) is signed. Nor is it true that \( r \) must lie between \( a \) and \( g \). These facts are exemplified in the text.

II. AFTER-TAX ANALYSIS

These same results apply to the analysis of the relationship between the after-tax economic rate of return and the after-tax accounting rate of return. This is obvious if the depreciation schedule used is not that used for tax purposes; in that case, the effect of taxes is just to change the benefit profile with the analysis the same as before, given the new benefit profile \( f(\cdot) \). Moreover, the same thing is true if tax depreciation is used. To see this, let \( \alpha \) be the tax rate \( 0 < \alpha < 1 \) (assumed constant for simplicity). Let \( r' \) denote the after-tax economic rate of return. Then \( r' \) satisfies

\[
(A22) \quad \int_0^\infty (1 - \alpha)f(\theta) + \alpha d(\theta) \times \exp(-r'\theta) d\theta = 1,
\]

where \( d(\theta) \) denotes depreciation on an asset of age \( \theta \) and \( f(\theta) \) denotes its before-tax benefits, as before.

This reflects the fact that the choice of a depreciation schedule, \( d(\cdot) \), affects after-tax
returns. Define

\[ f^*(\theta) = ((1 - \alpha)f(\theta) + \alpha d(\theta)). \]

We now show that the analysis of the before-tax case applies directly to the after-tax case with \( f^*(\cdot) \), the after-tax benefit schedule, replacing \( f(\cdot) \), the before-tax benefit schedule. 26

To see this, observe that the denominator of the accounting rate-of-return (whether total capitalization or stockholder’s equity) will be the same before and after taxes. The numerator in the after-tax case, after-tax profits less depreciation, will be:

\[ \int_{-\infty}^{t} (1 - \alpha)(f(t - \theta) - d(t - \theta))I(\theta) \, d\theta, \]

\[ = \int_{-\infty}^{t} ((1 - \alpha)f(t - \theta) + \alpha d(t - \theta))I(\theta) \, d\theta \]

\[ - \int_{-\infty}^{t} d(t - \theta)I(\theta) \, d\theta, \]

\[ = \int_{-\infty}^{t} f^*(t - \theta)I(\theta) \, d\theta - \int_{-\infty}^{t} d(\theta)I(\theta) \, d\theta. \]

But this is the same numerator as would be encountered in the before-tax analysis for a firm with the same depreciation schedule, but before-tax benefits \( f^*(\cdot) \). For such a firm, \( r' \) would be the before-tax economic rate of return. Hence analysis of the after-tax case is identical to that of the before-tax case with an appropriate adjusted definition of the benefit schedule. All previous results apply to it. 27

27 It is interesting (and revealing of the full unity of the before- and after-tax analyses) to note what happens in the case of "economic depreciation" examined above. In that case, it turns out that the (pathologically unlikely) choice of an economic depreciation schedule involves the same depreciation schedule whether economic depreciation is chosen before or after tax. Assets are valued at the present value of all remaining benefits either before or after tax; it makes no difference. Further, that choice of depreciation schedule makes the after-tax economic rate of return \( r' \) relate to the before-tax economic rate of return \( r \), in the natural (but—except with this depreciation schedule—not inevitable) way: \( r' = r(1 - \alpha) \). To show these things, return to the differential equation (equation (A5)) from which we derived the formula for economic depreciation in the before-tax case.

\[ (a) \quad V''(\theta) = rV(\theta) - f(\theta). \]

Consideration of the before-tax analysis shows that if and only if \( V(\cdot) \) satisfies this and \( V(0) = 1 \), then

\[ (b) \quad V(\theta) = \int_{\theta}^{\infty} f(u) \exp(-r(u - \theta)) \, du \]

the present value of future benefits. Now, choose \( V(\cdot) \) and hence \( d(\cdot) \) to satisfy (b) and therefore (a). Then

\[ (c) \quad (1 - \alpha)V''(\theta) = r(1 - \alpha)V(\theta) - (1 - \alpha)f(\theta), \]

\[ (d) \quad V''(\theta) = r(1 - \alpha)V(\theta) \]

\[ - ((1 - \alpha)f(\theta) + \alpha d(\theta)) = r(1 - \alpha)V(\theta) - f^*(\theta), \]

since \( d(\theta) = -V'(\theta) \). But this is in the same form as (a). Hence, as in (A6):

\[ (e) \quad V(\theta) = \int_{\theta}^{\infty} f^*(u) \exp(-r'(u - \theta)) \, du + C \exp(r'\theta), \]

with \( r' = r(1 - \alpha) \). Here, \( C \) is a constant of integration; however \( C = 0 \), since (b) shows that \( V(\infty) \) is finite. Since \( V(0) = 1 \), we have

\[ (f) \quad 1 = \int_{0}^{\infty} f^*(u) \exp(-r'u) \, du, \]

which shows that \( r' \) is the after-tax economic rate of return. From (b) and (g) with \( C = 0 \), \( V(\theta) \) is both the before- and the after-tax present value of the remaining benefit stream at \( \theta \), whence economic depreciation is the same in both cases. See Paul Samuelson.
Thus, in after-tax analysis as in before-tax analysis, there is no reason to believe that differences in the accounting rate of return correspond to differences in economic rates of return. Our computer examples show the effects can be very large; the belief that they are small enough in practice to make accounting rates useful for analytic purposes rests on nothing but wishful thinking.

REFERENCES