Product Reviews and the Curse of Easy Information

Kosti Takala*

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Abstract

When there is a new experience good of unknown quality and known price, consumers would like to obtain information on match quality before making their purchase decisions. One way to acquire this information is to search for the experiences of earlier consumers. However, since searching for information is not free, this introduces a problem of sequentially deciding between searching, purchasing, and exiting. By considering a Bayesian learning model with homogeneous consumers, one product and one period, I show that consumer surplus is a non-monotone function of search cost – with an interior maximum. This is due to endogenous pricing, where price may be used to induce more search when search cost is low. The result is shown to extend to heterogeneous consumers. Furthermore, a highly informative signal technology may be as bad for the consumer as a low search cost – they can be seen as two sides of the same coin.

Keywords: Bayesian Learning, Consumer Reviews, Optimal Search

*Department of Economics, Massachusetts Institute of Technology, tkosti@mit.edu. I am thankful to Glenn Ellison and Michael Whinston for their valuable feedback. I have also greatly benefited from my conversations with Alessandro Bonatti and Harry Di Pei.
1 Introduction

Conventional wisdom says more information is better. Consumers should be better off the easier it is for them to search for information on product quality – and the more precise the information is. Product reviews from earlier consumers have become an increasingly important way of acquiring this information, both on the vertical (absolute) and the horizontal (individual match) components, attracting a fair share of recent research (see Chevalier and Mayzlin [2006], among others). Much of earlier search literature has focused on the search for the lowest price (e.g. Diamond [1971] and Stahl [1989]) and obfuscation (see Ellison and Ellison [2009]), but here price is known and search is for the uncertain (match) quality of a product. However, there has not been a satisfactory model of gradual learning about match quality with endogenous pricing to explain how the ease of search and accuracy of information matter in equilibrium. This paper fills that gap.

The aim of the paper is not to merely show that the consumer-optimal search cost is strictly positive, but to derive it analytically and study the complementarity between search cost and signal precision. More importantly, the paper contributes to understanding how each price leads to certain consumer behavior, be it searching three times or not searching at all, and how this implies that the firm’s equilibrium price is a non-monotone function of the search cost. Price acts as a recommendation to search, so the firm may incentivize the consumer to obtain full information, which is related to Lewis and Sappington [1994] who find that sometimes a monopolist should provide its consumers with full information. The difference is that their consumers make no decision to receive the information.

As already evident, the leading application of this paper will be consumer reviews as signals of match quality, although the setup is more general. The ubiquity of reviews on hotels, restaurants, movies, services, and all kinds of other experience goods would suggest that they serve a purpose but the jury is out on whether or not they improve consumer and overall welfare. Chevalier and Mayzlin [2006] have shown empirically that book reviews matter for sales but not whether they improve consumer surplus – or even social surplus for that matter. They also obtain evidence that consumers not only consider the average rating but also read the content of a review. In everything that follows, I will assume that reviews are truthful but will discuss how promotional reviews could be incorporated in my model and what the predictions would be.\footnote{There is a growing literature on promotional reviews, prominent examples of which include Mayzlin et al. [2014] and Luca and Zervas [2016]. These papers argue empirically that promotional reviews exist and are used more when the competition between two firms is intense or when reputational concerns are small.}

It is reasonable to think that technology has made obtaining product information cheaper; reading online reviews seems to be easier than contacting individual friends. Understanding how consumers use reviews when making their decisions is important because it seems to affect observed prices, purchase decisions, and
therefore also welfare. I study how the mere existence of these reviews matters for the pricing strategy of the firm, resulting consumer surplus, and overall welfare— as a function of the search cost. This is slightly in contrast with some of the concurrent research that tries to improve the informational content of an overall rating in terms of vertical or absolute quality without considering the horizontal match component (see Dai et al. [2016]). Furthermore, more precise information may lead to a loss in surplus because searching is costly and sometimes unnecessary from a social point of view. The availability of information practically forces the consumer to use it because she cannot commit to not doing so.

I use a Bayesian learning model to understand how the ease of obtaining reviews (search cost) and the informativeness of these reviews (precision of signal) matter for consumer and social surplus. To do this, we need to explain how a consumer sequentially decides between searching for reviews, purchasing the good, and exiting the market. Second, understanding the consumer’s behavior is important for pinning down the firm’s pricing policy because the firm knows what the consumer will do for each price. In principle, if the firm was better informed on the content of the reviews, its pricing policy could depend on the content. However, I will assume that only the consumer can observe the reviews and will abstract away from what happens when the firm knows which reviews are relevant to the consumer and how.

In the main model, there is one period, one firm and one product with consumer-specific quality. The firm faces a continuum of identical, risk-neutral consumers (or a single representative consumer), who freely observe the price before making their decision. The specificity of quality means that any given consumer may find the product to be a great or a terrible match, independently of one another. Consumption utility is assumed to be binary, with the prior probability of a match being $\pi_0$. This prior corresponds to the share of consumers in the population who would find the product to be of high quality (utility of 1) should they consume. However, no consumer knows whether they will like the product before trying it out. The purpose of searching for reviews is to update one’s beliefs about the population mean.

When a useful review is observed, it is either good or bad: a good product always produces good reviews but a bad product gives good reviews with probability $\mu$. This implies that, when a signal is observed, the belief either moves up or down.

When the firm is in the know, there is a signaling game where the price is a signal of quality, in the spirit of Wolinsky [1983] and Riordan [1986], among others, with one twist: the consumer can (imperfectly) verify the firm’s type by paying the search cost. This game involves multiple equilibria, and is extremely difficult to solve. However, as I will justify in the next section, even if the firm knows the content of the reviews, it may not understand what aspects of those reviews are useful to which type of consumer. Therefore, it is a reasonable assumption to make that the price itself contains no information.

Although not shown here, if the firm perfectly understands how the reviews matter for the consumer’s decision process, then it will be able to extract all ex ante surplus in a semi-separating signaling equilibrium. Basically, higher-type firms price higher on average but low types sometimes imitate them. Full pooling does not work, at least for small values of search cost, because the consumers can then verify the firm’s type by searching.

The consumer may need to search for a while before she gets to a review that matters to her because she is not interested in reviews that seem to come from people unlike her. She may not know how much longer she has to search but she knows the expected time it takes (which is identical for good and bad products).

Note also that we can think of the prior belief, $\pi_0$, as the posterior expectation after observing the average rating of a product, but before studying the actual reviews in more detail and assessing whether they are useful for you.

The assumption that a good product never has any bad reviews is extreme but greatly simplifies the analysis. If we assumed
(but will not reach 1) or falls down to 0. Therefore, there exists a threshold belief, π, such that the consumer will search until this threshold is reached, and only then will she buy. If she ever gets a negative signal, she will exit.

One can think of the prior belief (π0) as the current average rating of a product, assuming binary ratings. It tells the consumer and the firm that a certain share of the reviews are positive but it does not reveal which types of consumers like the product and whether or not some people dislike it because of mere, idiosyncratic, bad luck. In this sense, the purpose of reading reviews is to learn whether types with beliefs similar to yours liked or disliked the product. It is not about learning the absolute quality but where the product is located horizontally.

The firm will set its price knowing that the price determines how many searches the consumer needs to perform to be willing to buy, which in turn determines the consumer’s expected utility. The expected cost of obtaining one useful review is s, which means that the total ex ante expected search costs are fully determined by the firm’s price and the model parameters. The equilibrium pricing and searching policies depend not only on the value of s, but also on the cost of production, c, and the distributional parameters (π0, µ).

The firm can exploit the search behavior by changing its price. When the consumer is willing to search at the current price, increasing the price will lead to her exiting more often, and decreasing the price will lead to a higher likelihood of purchase. The firm typically has to choose between a menu of prices, where each of these prices is optimal for certain consumer behavior: there is one price that is optimal if the firm wants to make the consumer buy without search, there is one price that is optimal if the firm wants to make the consumer search once, and so on. However, under the assumptions we will make, for each value of search cost, there are only two candidates for the optimal price. These candidate prices are functions of the parameters of the model. When the cost of search is low, the firm has an incentive to increase the price because it needs to choose between selling the good at a lower price and a higher probability, or selling it at a higher price and a lower probability. This implies that a low search cost can lead to low consumer surplus because the firm raises its price. The firm is effectively pricing high to turn some consumers into believers who have high beliefs but get little surplus – and some consumers exit with zero beliefs. The firm is more likely to make the consumer search when the production cost is high because then the lost revenue from exiting consumers is not a big concern but the believers can be charged more. In some cases the believers will be charged their full surplus – much like in Lewis and Sappington [1994].

that both types of product can produce both types of reviews, the main results would remain (as long as the signal is informative enough) but the analysis would be more involved because the belief would be able to move up and down. What matters for the consumer (and the firm) is how expensive and unlikely it is for her to reach a belief where purchasing is the optimal decision.

7One can define the search cost as s = \( \bar{s} / \lambda \), where \( \bar{s} \) is the search cost per unit of time, and \( 1 / \lambda \) is the expected time between signals.
In Section 3 we use the main model to understand the consumer’s search behavior and why her utility is non-monotone in the search cost. We not only get the obvious result that consumer surplus is maximized at a strictly positive value of search cost ($s^* > 0$) but are also able to characterize this maximizer. Social surplus, on the other hand, is maximized at $s = 0$ because then the firm gets the first-best surplus. However, as we will see in Section 4, if there is a continuum of consumers with uniformly distributed valuations, and if the production cost $c$ is low enough, both consumer and social surplus will be maximized at the same, strictly positive value of search cost. The intuition for this is that low production costs imply the good should always be sold to as many people as possible but the firm will exclude too many people in equilibrium in order to charge more from the ones that end up purchasing.

In Section 5, we obtain a complementary result concerning the imprecision of the signal ($\mu$): when the consumer gets to choose $\mu$, she will not choose the most informative one ($\mu = 0$) if her search cost is low because then the firm would take advantage and leave her with no utility. The firm might want to set an intermediate price to increase the probability of a sale but the consumer would then utilize her technology, meaning that the firm would be better off setting a high price. Thus, the consumer is often hurt by a better search technology (be it a low search cost or a highly informative signal) because she cannot commit to not using it should the firm set an intermediate price. Hence, the price will be set high when the consumer has access to a good technology (low $s$ and $\mu$). Another important result of Section 5 is that, even though search cost and signal informativeness can be seen as two sides of the same coin, the consumer will always choose a perfectly informative signal if she is also allowed to pick her search cost. This is because both of them are tools for controlling the firm’s pricing decision but the consumer never actually ends up searching in equilibrium. A highly informative signal gives her a good bargaining position because it forces the firm to lower the price, while a high search cost makes sure the firm wants to make the consumer buy. This section complements the more general results obtained by Roesler and Szentes [2017] who show that a certain unbiased signal distribution maximizes consumer surplus (and minimizes the firm’s profit), without any distributional assumptions. However, they assume that the firm has no production cost and there is no cost for the consumer to use the search technology. Therefore, my results on the interaction of search cost and signal precision are valuable in that a higher search cost allows the consumer to use a more precise signal technology, and the firm will still find it profitable to induce no search.

In Appendix C we also consider a bad news model. This means that no news is considered good news. However, for high search costs, the consumer prefers a perfectly informative signal because the firm will not make her use it anyway but it improves her bargaining position relative to the firm (that is, it forces the firm to set a lower price for the consumer not to use the search technology). The issue of no production cost is just a matter of normalization but assuming no search cost is with some loss of generality. If there was a cost of using the technology, the consumer would never use the technology for low prices, which would give the firm an incentive to raise its equilibrium price from $p^*$ to $p^* + s$. It is not entirely clear that the signal distribution they have characterized is optimal under search costs.
because every signal we observe is bad news. This may be seen as a realistic assumption because, while shopping online, we are usually looking for bad news as good signals are more often fabricated. The results obtained in this section mirror those of the main model but are more difficult to interpret.

The paper is structured as follows. Section 1.1 offers a brief overview of the literature. In Section 2, I introduce and justify the main model. Section 3 is devoted to using the model to show how the consumer responds to the firm’s price and what this means for the pricing decision. We also learn why the consumer’s equilibrium utility is non-monotone in the search cost. Section 4 allows for heterogeneous consumers with different valuations, and the results are similar to the model where consumers are identical. Section 5 makes a complementary observation: the informativeness of the signal acts in a way very similar to a low search cost. Section 6 concludes, giving directions for future work.

1.1 Prior Literature

This paper is related to the literatures on persuasion, obfuscation, word-of-mouth communication (WOM), social learning and optimal search, among others. As mentioned already, my paper is not concerned with the dynamics of price setting since the firm makes a one-time decision which then influences how much the consumer will search. However, most of the search literature is concerned with either how search matters for price competition (see Bergemann and Välimäki [1996] for the case where quality is unknown to all parties but can be learned over time), or what the dynamically optimal price is. My finding that there may be an inefficiently high amount of search in equilibrium is similar to what Bergemann and Välimäki [2000] show. In their paper, there is a new product of unknown quality and its performance over time is publicly observable, although search cost is different from mine and corresponds to giving up the safe return, and the future consumers do not have to consider whether or not to acquire information.

More generally, my paper is separated from the bandit literature (to which Bergemann and Välimäki [2000] belongs) in a few ways: while that literature usually assumes a number risky arms with payoffs varying over time and arms, emphasizing the exploration-exploitation tradeoff, I only have one risky arm (the firm) with a constant reward that differs from consumer type to consumer type (match quality) and can only be obtained once. The most important difference is that the reward the consumer may obtain at the end is endogenously chosen by the firm, which determines how long the consumer will search. For a slightly dated survey on this literature, see Bergemann and Valimaki [2006].

A paper closely related to mine is Branco et al. [2012]. Contrary to my discrete setting, they use a continuous-time model of gradual learning to obtain similar results: lower search costs may hurt the consumer because the firm can change its price in response, but this is not true for all beliefs or prior expected utilities.

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10This is not to say bad news cannot be fabricated as well, as mentioned before.
In their model, the consumer searches for information on different product attributes sequentially and her utility is the sum of these characteristics whereas in my model there is only one attribute. Similarly to my paper, these attributes can only take on two values. However, in their paper, the values are the same for every consumer and there is no Bayesian learning aspect (the value of an attribute is learned perfectly), while in my model one consumer may find the product to be of high quality and another may find it low quality.

From the persuasion literature, the most closely related is Kamenica and Gentzkow [2011] where a sender is free to design an experiment to persuade the receiver to take the action the sender prefers. My paper is not directly related but has a similar flavor: the firm (sender) chooses a price to induce an experiment (review search) conducted by the consumer (receiver). If the result of this experiment is bad, there will be no exchange. If the result is good, the consumer will end up with enough consumption utility to at least cover her search cost.

In Lewis and Sappington [1994], the consumers do not know their type but they observe a binary signal, the precision of which is controlled by the seller, and all this is costless. The result is that the seller will either provide no information to sell under the prior beliefs, or he will provide full information and only sell to the consumers who obtain a good signal. The intuition is that for the signal to be profit improving, the seller has to be able to raise his price from the equilibrium with no information (since all consumers are ex-ante identical), but this necessarily excludes all those who obtain a bad signal (expected posterior has to equal the prior). Therefore, if any information is provided, it should be perfect. Which policy is optimal depends on the production cost. My model is closely related to theirs in that the consumers can only be of two types but they are ex-ante identical, and that the seller has the ability to control how much product information the consumers obtain, although he does not have perfect control because signals are random. The most important difference is that the consumers in my model have to pay for the signals, and they make decisions sequentially. Also, my firm cannot choose the signal structure.

It is instructive to make a comparison to the obfuscation literature. Ellison and Ellison [2009] study how sellers use obfuscation to capture a larger slice of the pie. One of the main conclusions is that a better search technology may lead to firms investing in more obfuscation, thereby reducing consumer surplus. In my paper, on the other hand, the seller is not really trying to fool anyone but to give the consumer the incentive to search even though the resulting surplus will largely end up in the seller’s pocket. This is because my seller has only one decision variable, the price, which is perfectly observed by the consumer before she makes any decisions. This is in stark contrast to Ellison and Ellison [2009] because, in their paper (and in most of the literature), the consumers are searching for the lowest price and the sellers know the quality of their products.
My paper is closely related to the literature on social learning but the approach and goals are very different. In the social learning literature, the most common question is whether or not the dynamic arrival of new consumers (who observe something about the actions of previous consumers) leads to the adoption of superior products or informational cascades (see, e.g., Acemoglu et al. [2011] who study which general conditions are needed for asymptotic learning when each agent has a signal and observes some past actions, and Smith and Sørensen [2000] who show that under heterogeneous preferences there may be what they call confounded learning, a type of an informational cascade). My paper, on the other hand, is not concerned with what happens in the limit or whether informational cascades may occur but how one consumer’s search behavior interacts with the firm’s price for a given search cost and what the implications are for welfare.

Ellison and Fudenberg [1995] consider how the structure of WOM communication matters for the choices the agents make, focusing on the dynamic pattern of the market shares of two products. This is the biggest difference between my paper and theirs: I only consider one product and one period. Their most striking result is that, even with naive (that is, non-Bayesian) decision rules, the players may end up adopting the superior product. Contrary to my paper, the agents are not searching optimally but merely take a random sample of $N$ friends in the event they are considering switching from one action to another (in my paper they would pick one friend at a time). Their model is dynamic and stochastic in the sense that the agents make consumption decisions every period, and the product with a higher average payoff may be different from period to period. The model in Ellison and Fudenberg is similar to mine in that payoffs can only be learned from experience although in my model learning is perfect once consumption takes place.

Crapis et al. [2016] has a flavor similar to the present paper in that they consider a monopolist and uninformed consumers who can learn from past consumers’ reviews. However, their paper is closer to the social learning literature than mine because the consumers can observe past purchase decisions and the goal of the paper is analyze how the social learning matters for pricing. Their consumers are non-Bayesian and heterogeneous ex ante, while mine are Bayesian and heterogeneous only ex post.

Although I do not consider where the signals come from (which is also different from the social learning literature where signals may be past consumers’ actions), there is a growing literature on incentives to review. For example, Lafky [2014] studies what motivates people to leave reviews, and finds that the reviewers may want to harm the sellers and/or inform future customers. If reviewing is costly, reviews will be more polarized. His model allows the sellers to adjust their quality which my paper abstracts away from for simplicity.

Finally, worth mentioning is Holmstrom [2015], where he argues that transparency is not always good in the money markets because it increases transaction costs (bargaining, asymmetric information), whereas opacity levels the playing field. This idea is closely related to mine because low search costs make information more easily accessible which, due to the pricing policy, increases transaction costs.
2 Model: Costly Binary Signals on Quality

There is a firm selling a product of unknown quality by setting a single, fixed price, and this is the only decision the firm makes. The quality is binary and consumer specific, which means that each consumer will obtain a consumption utility of 1 or 0, independent from one another. In the model, we consider one consumer (representing a continuum of ex-ante identical consumers) who needs to decide whether or not to buy the product. To aid her decision, she can search for existing, truthful consumer reviews (we take no stand on where they come from). That is, the reviews fully reveal the earlier consumers’ experiences, although our consumer may wrongly classify a bad review as a good one. Not all of the reviews are relevant for our consumer because some may come from people whose opinions differ from hers by too much.\textsuperscript{11} The expected cost of obtaining a useful review is \( s \) (called the search cost), which depends on how widespread her preferences are in the population.\textsuperscript{12} The full model is formalized below:

- There is one firm, one consumer, and one product which costs \( c \geq 0 \) to produce.
- The firm’s only decision is to set a price, \( p \), for the product to maximize expected profits.
- Product quality, \( \theta \in \{L, H\} \), is consumer specific and can only take on two values, low or high.
- Consumption utility is binary: \( u(L) = 0, u(H) = 1 \), and the outside option yields \( \bar{u} = 0 \).
- There is no discounting.
- The prior on the product being of high quality is \( \pi_0 \equiv \mathbb{P}(\theta = H) \in (0, 1) \).
- The prior is shared by the consumer and the firm – the firm does not observe the quality or the reviews.
- If the consumer searches, she has to pay an expected cost of \( s \geq 0 \) to obtain a binary signal \( \sigma \in \{G, B\} \).

That is, the signal can be good or bad, with the following conditional probabilities:

\[
\mathbb{P}(\sigma = G|\theta) = \begin{cases} 
1, & \text{if } \theta = H \\
\mu, & \text{if } \theta = L 
\end{cases}
\]

- This signal structure means that there will be a jump in the consumer’s beliefs only when a signal is

\textsuperscript{11}Think about searching for restaurant reviews. If your main criterion is the quality of their spicy food, you should not consider reviews from people who hate spicy food.

\textsuperscript{12}Note: one can think of the expected search cost as a function of a unit search cost and the arrival rate of signals: \( s = \hat{s}/\lambda \). However, this adds unnecessary notation, so it is dropped in the analysis.
observed. Given a prior $\pi_t$, the posteriors can be written as:

$$
\pi_{t+1} := \mathbb{P}(\theta = H|\sigma_0 = \ldots = \sigma_t = G) = \frac{\pi_t}{\pi_t + (1 - \pi_t)\mu} \\
\pi_b := \mathbb{P}(\theta = H|B \in \{\sigma_0, \ldots, \sigma_t\}) = 0.
$$

In particular, this structure leads to a belief that either moves up (but never reaches 1), or falls down to 0. That is, $0 < \pi_t < \pi_{t+1} < 1$ for all $t$, as long as the consumer has observed only good signals. Note also that the arrival rate of the signals is the same for both good and bad quality, meaning that there is no learning without signals – there is no drift in the beliefs.

- The consumer’s value function after $t$ good reviews is:

$$
V(\pi_t) = \max \left\{ 0, \pi_t - p, \left( \pi_t + (1 - \pi_t)\mu \right) V(\pi_{t+1}) - s \right\},
$$

because the consumer can exit (first term), buy (second term), or search for a review (last term), where the expected cost of obtaining a useful review is $s$ and the probability of this review being good is $\pi_t + (1 - \pi_t)\mu$. If the review is bad, continuation value is zero.

- The firm’s profit function at $\pi_0$ can be written as:

$$
\Pi(p, \pi_0) = \mathbb{P}(\text{consumer purchases} | p, \pi_0) (p - c)
$$

- The timing is as follows:

1. Nature draws the consumer-specific qualities according to the true distribution, and earlier consumer reviews are realized.
2. The firm sets the price (without observing the quality or the relevance of reviews).\(^{13}\)
3. The consumer observes the price and sequentially decides whether or not to search for more reviews. If she stops searching after $t$ reviews, she either buys the product ($\pi_t \geq p$) or exits without buying ($\pi_t < p$).
4. Payoffs are realized.

It is in order to comment on the assumption of reviews as trustworthy signals. What is the effect of promotional/fake reviews? Because we are assuming that bad products are thought to be good with

\(^{13}\)If the firm did observe the reviews, the price would act as a signal of quality. This would unnecessarily complicate the model. However, one can also argue that observing the reviews gives the firm no competitive edge because each consumer will find different reviews helpful and will obtain the reviews in a random order.
probability $\mu$ (reviewers do not perfectly understand the quality even though they consume or the receiver of the reviews does not fully understand what the reviewer is saying), it is reasonable to think that positive promotional reviews will, depending on the consumer, have two effects: 1) for some consumers, $\mu$ will increase because good reviews are now more probable and reviewers understand this in equilibrium (slower learning) but cannot observe which reviews are trustworthy, and 2) for other consumers, $\mu$ will stay the same because they can spot the fakes but $s$ will increase because it now costs more to get useful reviews as the proverbial haystack is taller.

3 Non-Monotone Equilibrium Utility

In this section, I will use the above model to show why the consumer’s equilibrium utility is not monotone in her search cost and discuss what forces are driving the result. I will also solve for the consumer-optimal search cost which naturally is strictly positive but depends on the parameters. In a nutshell, the consumer is always hurt by a higher search cost, conditional on behaving the same way. However, if increasing the search cost leads to a change in the consumer’s equilibrium behavior, her utility will jump up, after which it will decrease again. This is due to the interaction between the consumer’s behavior and the firm’s pricing policy. Figures 1 and 2 illustrate what equilibrium payoffs look like, as a function of the search cost. We see that the consumer’s utility is either zero or strictly decreasing, other than at a few points where it jumps up discretely.

Figure 1: Equilibrium utility as a function of search cost when $c = 0$, $\pi_0 = \frac{1}{2}$, and $\mu = \frac{1}{4}$

Utility jumps up at these points because the firm changes its pricing policy to induce one less search from the consumer, which can only be done by setting a lower price. On the other hand, given that the firm does
not change its pricing policy, the consumer’s utility is strictly decreasing in the search cost, while the firm’s profit is strictly increasing as can be seen in Figure 2. Finally, when equilibrium utility is zero, the firm is getting all the social surplus, which means that its profits are decreasing in \( s \).

3.1 Consumer Behavior

Now, to obtain the above equilibrium graphs, we need to proceed step by step. First, the consumer’s strategy can be defined by a threshold belief, \( \bar{\pi} \), such that, for \( \pi \geq \bar{\pi} \), the consumer will buy instantly, and, for beliefs between her prior and \( \bar{\pi} \), the consumer will search until she either crosses the threshold and buys, or obtains a bad review and exits (one of which will surely happen). The firm will never choose a price that makes the consumer exit at the prior belief, as long as production costs are not too high, which we will assume by requiring that \( c \leq \pi_0 \). The threshold \( \bar{\pi} \) will depend on the price \( (p) \) and the search cost \( (s) \).

The probability of observing \( k \in \mathbb{N} \) good reviews in a row is:

\[
P(k \text{ good reviews}) = \pi_0 + (1 - \pi_0)\mu^k,
\]

14 This is implied by the fact that when there are no production costs, it is never socially optimal to search, as the only reason to search would be to avoid incurring the production cost in case the product is bad.

15 One way to show this is as follows: The consumer prefers searching-and-buying over buying instantly if and only if \((\pi_t + (1-\pi_t)\mu)(1 - (\pi_t + (1-\pi_t)\mu)p) - s \geq \pi_t - p \Rightarrow \pi_t \leq \bar{\pi} = 1 - \frac{1}{(1-\mu)p}\). Since \( \pi_t \) is increasing in \( t \), we will reach this threshold at some point and the consumer will prefer searching to buying as long as we are below the threshold. Note also that if she finds searching optimal at \( \pi_0 \), she will surely do so at \( \pi_t \in (\pi_0, \bar{\pi}) \) because the expected cost of reaching \( \bar{\pi} \) is smaller. In equilibrium, exiting is therefore not an option at any point unless a bad signal is observed.

16 Note, however, that the price itself depends on the model parameters \((s, c)\). Everything also depends on the distributional assumptions, here characterized by \((\pi_0, \mu)\).

17 One might think that we should also consider a lower threshold \( \bar{\pi} \) below which the consumer will stop searching and choose to exit. This is true for a more general model where bad news are not conclusive but not for this model because beliefs can only go up or crash down.
which is easy to understand because, if the product is good (which happens with probability $\pi_0$), then the consumer will surely observe $k$ good reviews in a row, but if the product is bad, then the probability of getting these reviews is $\mu^k$.

Now, given the prior $\pi_0$, the purchase threshold $\bar{\pi}$, and the price, the consumer knows the number of good reviews she needs to obtain in a row for her to purchase. Thus, if we assume she needs $n$ good reviews to purchase, her expected value (not taking the search cost into account) is:

$$E[value] = (\pi_0 + (1 - \pi_0)\mu^n) \left(\frac{\pi_0}{\pi_0 + (1 - \pi_0)\mu^n} - p\right) = \pi_0 - (\pi_0 + (1 - \pi_0)\mu^n)p = \pi_0(1 - p) - (1 - \pi_0)\mu^n p.$$ 

Moreover, her expected cost of trying to obtain $n$ good reviews (and exiting when this fails) is:

$$E[\text{cost}] = s(\pi_0n + (1 - \pi_0)(1 + \mu + \mu^2 + \ldots + \mu^{n-1})) = s\left(\pi_0n + (1 - \pi_0)\frac{1 - \mu^n}{1 - \mu}\right).$$

Here, $s$ is the expected cost of obtaining one signal, so the expected cost given that the product is good is $ns$, and the expected cost given that the product is bad is $\frac{1 - \mu^n}{1 - \mu} s$. This means that the consumer's ex ante expected utility from searching $n \geq 0$ times and then buying is:

$$V_n(\pi_0, p) = \pi_0 - (\pi_0 + (1 - \pi_0)\mu^n)p - s\left(n\pi_0 + \frac{(1 - \pi_0)(1 - \pi_0)\mu^n}{1 - \mu}\right).$$

For the consumer to perform exactly $n$ searches, this value function needs to be non-negative and maximized at $n$ (where $n = 0, 1, 2, \ldots$). The following proposition tells us how many times the consumer will search for a given price.

**Proposition 1.** Given a price, $p$, let $n^*$ be the number of searches that satisfies:

$$\frac{s}{(1 - \mu)(1 - \pi_{n^*-1})} < p \leq \frac{s}{(1 - \mu)(1 - \pi_{n^*})}.\quad (2)$$

If $V_{n^*}(\pi_0, p) \geq 0$, the consumer will search exactly $n^*$ times and buy. Otherwise she will exit without search.

Condition (2) guarantees that the value function is maximized at $n^*$ searches but it does not guarantee that this value is non-negative. That is why we also need the non-negativity condition, $V_{n^*}(\pi_0, p) \geq 0$. Note that the consumer will not search at all if $p \leq \frac{s}{(1 - \mu)(1 - \pi_{n^*-1})}$, so that the lower bound does not matter.

**Proof.** Assume that $n^*$ satisfies (2) given some price, $p$. For any $n \in \mathbb{N}$, it is easy to see that $V_{n+1}(\pi_0, p) \leq$
V_n(\pi_0,p) \iff p \leq \frac{s}{(1-\mu)(1-\pi_k)} \quad \text{This is because, assuming the consumer has already seen n reviews, the gain from doing one more search and then buying relative to just buying is }(1-\pi_n)(1-\mu)p-s. The reason is that your expected consumption value stays the same (\pi_n) but you end up saving the price in case the product is bad (1-\pi_n) and you get a bad signal (1-\mu).

On the other hand, because \( p > \frac{s}{(1-\mu)(1-\pi_{k+1})} \), we know that \( V_n^* > V_{n+1} > \ldots > V_0 \). Therefore, \( n^* \) is the number of searches that maximizes the consumer’s search utility. However, if this utility were negative, the consumer should exit immediately. This is why we need \( V_n^*(\pi_0,p) \geq 0 \).

3.2 Pricing Policy

While the previous Section derived the consumer’s optimal search strategy for a given price (and parameters), in this section we show that the choice the firm faces is relatively simple. For Proposition 2 and what will follow, let us define the following (all of which are functions of \( s \)):

- \( p_k := \frac{s}{(1-\mu)(1-\pi_k)} \) , which is the highest price so that the consumer’s optimal number of searches is exactly \( k \) as can be seen in (2), but not requiring that \( V_k(\pi_0,p_k) \geq 0 \).

- \( \Pi_k := (\pi_0 + (1-\pi_0)\mu^k)\left(\frac{s}{(1-\mu)(1-\pi_k)} - c\right) \), which is the profit at \( p_k \).

- \( k^* := \max\{k \mid V_k(\pi_0,p_k) \geq 0\} \), which is the maximum number of searches such that the consumer is willing to participate at \( p_k \).

- \( \hat{p}_{k+1} \), which is the price that satisfies \( V_{k+1}(\pi_0,\hat{p}_{k+1}) = 0 \).

We see the firm’s profit at each price in Figure 3 below. Each line segment ends at \((p_k,\Pi_k)\) for \( k = 0,1,2,\ldots \) but profit drops discretely at \( p_k \) because the consumer searches one more time. Note that each of the lines is less steep than the previous.

Note how increasing the price makes the consumer search more as long as she is willing to participate, as we saw in (2). \( p_k \) is the best the firm can do, given that it wants the consumer to search exactly \( k \) times, while \( p_k^* \) is the highest such price so that the consumer is willing to participate. \( \hat{p}_{k+1} \) is the highest possible price for \( k + 1 \) searches that makes the consumer search \( k + 1 \) times and get no utility. See Figure 3 below for the firm’s profit as a function of its price. We highlight the points where profit drops down because the consumer does one more search.

\[ \text{Note that each of the lines is less steep than the previous.} \]

\[ \text{We see the firm’s profit at each price in Figure 3 below. Each line segment ends at } (p_k,\Pi_k) \text{ for } k = 0,1,2,\ldots \text{ but profit drops discretely at } p_k \text{ because the consumer searches one more time. Note that each of the lines is less steep than the previous.} \]

\[ \text{Note how increasing the price makes the consumer search more as long as she is willing to participate, as we saw in (2). } p_k \text{ is the best the firm can do, given that it wants the consumer to search exactly } k \text{ times, while } p_k^* \text{ is the highest such price so that the consumer is willing to participate. } \hat{p}_{k+1} \text{ is the highest possible price for } k + 1 \text{ searches that makes the consumer search } k + 1 \text{ times and get no utility. See Figure 3 below for the firm’s profit as a function of its price. We highlight the points where profit drops down because the consumer does one more search.} \]

\[ \text{We can write } \hat{p}_{k+1} := \frac{\pi_0(\pi_{k+1}) - \pi_0(\pi_{k+1}) + 1}{\pi_0(\pi_{k+1}) - \pi_0(\pi_{k+1}) + 1} = \frac{\pi_0(\pi_{k+1}) - \pi_0(\pi_{k+1}) + 1}{\pi_0(\pi_{k+1}) - \pi_0(\pi_{k+1}) + 1} = \frac{\pi_0(\pi_{k+1}) - \pi_0(\pi_{k+1}) + 1}{\pi_0(\pi_{k+1}) - \pi_0(\pi_{k+1}) + 1}. \]

\[ \text{We always require that } p_k < \hat{p}_{k+1} \leq p_{k+1}, \text{ because otherwise the firm would not want the consumer to search exactly } k + 1 \text{ times, nor would the consumer be willing to. However, such } \hat{p}_{k+1} \text{ only exists for } k \geq k^* \text{ because the consumer gets a strictly positive utility for all prices below } p_{k+1}. \]
The following Lemma shows that the firm’s profit function $\Pi_k$ is either always increasing or first decreasing and then increasing.

**Lemma 1.** $\Pi_k$ is either always increasing or first decreasing and then increasing in $k$.

**Proof.** Note first that $\Pi_{k+1} \geq \Pi_k \iff \frac{c}{s} \geq \frac{\mu - L_k^2}{(1 - \mu)\mu}$. $L_k := \frac{\pi_0}{1 - \pi_k}$ is strictly increasing in $\pi_k$ which is strictly increasing in $k$, so that $\frac{\mu - L_k^2}{(1 - \mu)\mu}$ is strictly decreasing in $k$. In fact, it is decreasing without bound. Therefore, if $\frac{c}{s} \geq \frac{\mu - L_k^2}{(1 - \mu)\mu}$ for some $\tilde{k}$, it also holds for all $k > \tilde{k}$. However, there may be a finite number of periods at the beginning where the profit function is decreasing.

This means that $\Pi_k$ is always increasing if and only if $\frac{c}{s} \geq \frac{\mu - L_0^2}{(1 - \mu)\mu}$ (strictly so if the inequality is strict).

**Proposition 2.** Assume that $s < (1 - \mu)(1 - \pi_0)\pi_0$ and $\pi_0 > c$. If $\Pi_{k^*} \geq \Pi_0$, the firm’s optimal price is either $p_{k^*}$ or $\hat{p}_{k^*+1}$, and the firm prefers $p_{k^*}$ if and only if $s \geq \bar{s}_{k^*}$, where:

$$\bar{s}_{k^*} := \frac{\pi_0 + (1 - \pi_0)(1 - \mu)\mu^{k^*}c}{\pi_0(k + 1) + \frac{(1 + \pi_0)(1 - \mu)}{1 - \mu} + (1 - \pi_0)\mu^{k^*} + \frac{\pi_0^2}{(1 - \mu)(1 - \pi_0)\mu^2}}.$$

If $\Pi_{k^*} < \Pi_0$, firm will set either $p_0$ or $\hat{p}_{k^*+1}$, depending on which one gives higher profits:

1. $\Pi(\pi_0, \hat{p}_{k^*+1}) > \Pi_0 \Rightarrow \hat{p}_{k^*+1}$ is optimal
2. $\Pi(\pi_0, \hat{p}_{k^*+1}) \leq \Pi_0 \Rightarrow p_0$ is optimal.
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Proof. See Appendix A.

What Proposition 2 tells us is that the firm wants to induce either as much search as the consumer is willing to perform (when it is comparing \(p_k\) and \(\hat{p}_{k+1}\)) or no search at all (when setting \(p_0\)). If the firm makes higher profits at \(p_k\) than at \(p_0\), it wants to induce \(k \geq k^*\) searches, but if \(\Pi_{k^*} < \Pi_0\), not inducing any search \((k = 0)\) may be in order. If we make a parametric assumption, we can simplify the firm's pricing policy significantly:

**Corollary 3.1.** Assume that \(\mu \leq L_0^2\). Then firm will always set either \(p_k^*\) or \(\hat{p}_{k^*+1}\), where \(p_k^*\) will be firm-optimal if and only if \(s \geq \hat{s}_k\).

Proof. If the condition holds, \(\xi \geq \frac{\mu \mu^* k^2}{(1-\mu)}\) for all \(c\) and \(s\), so the firm's profit \(\Pi_k\) is always increasing in \(k\). Thus, the firm will want to set the highest \(p_k\) accepted by the consumer, \(p_k^*\), or the price that brings the consumer's utility down to zero, \(\hat{p}_{k^*+1}\). For \(\Pi_{k^*} \geq \Pi(\pi_0, \hat{p}_{k^*+1})\), we need \(s \geq \hat{s}_k\).

### 3.3 Main Theorem

Before going into the main theorem of the section, let us first introduce one more Lemma which makes the proof of the theorem easier, and also helps us understand the problem better:

**Lemma 2.** Let \(s_k\) solve \(V_k(\pi_0, p_k) = 0\). Then \(s_k > s_{k+1} \forall k \in \mathbb{N}_0\), and \(k^* = \max\{k \in \mathbb{N}_0 \mid s_k \geq s\}\).

Proof. First of all, \(V_k(\pi_0, p_k) = \pi_0 - \left[\frac{(\pi_0)(1-\pi_0)\mu^*}{(1-\mu)\mu^*} + \pi_0 + (1-\pi_0)\frac{1-\mu^*}{1-\mu}\right]s\), which is strictly decreasing in \(k\) (since the ex-ante expected consumption value is always \(\pi_0\) but the firm's price \(p_k\) is strictly increasing in \(k\) and the probability of success is strictly decreasing in \(k\)). Thus, for a given pair \((\pi_0, \mu)\), the consumer will never become worse off if the firm-optimal \(k\) decreases. Because \(V_k(\pi_0, p_k)\) is also strictly decreasing in \(s\), we can define a decreasing sequence of search cost thresholds, \(s_0 > s_1 > s_2 > \ldots\), such that if search cost falls strictly between two thresholds, say \(s_k\) and \(s_{k+1}\), then \(V_k(\pi_0, p_k) > 0 > V_{k+1}(\pi_0, p_{k+1})\). This is due to the fact that \(V_k(\pi_0, p_k) > V_{k+1}(\pi_0, p_{k+1})\), which means that we need a strictly higher search cost to bring \(V_k(\pi_0, p_k)\) to zero than for \(V_{k+1}(\pi_0, p_{k+1})\). If \(s > s_0\), even \(V_0(\pi_0, p_0) < 0\), in which case the firm will set \(\hat{p}_0 \equiv \pi_0\), giving the consumer zero utility.

Now, remember that we defined \(k^*\) as the highest number of searches for which \(V_k(\pi_0, p_k) \geq 0\). Therefore, we can also express \(k^*\) as the highest \(k\) such that \(s_k \geq s\), which means that the consumer is (weakly) willing to search \(k\) times at \(p_k\) but she is not willing to search at all at \(p_{k+1}\).

Now we are ready to state the main theorem. It characterizes the consumer-optimal search cost, and provides an analytic solution in some cases.
Theorem 1. Assume that $\pi_0 > c$. Let $L(s) = \frac{\pi_0}{s}$ and $R(\pi_0, \mu) = \frac{\mu - L_0^2}{\mu(1 - \mu)}$. Define $\hat{s}$ as follows:

$$
\hat{s} := \max\{\bar{s}_0, s_1\} = \begin{cases} 
\bar{s}_0 & \text{if } \pi_0 \geq \frac{\sqrt{\mu - c} - \mu}{1 - \mu} \text{ or } c \text{ is high,} \\
(1-\mu)(1-\pi_0) & \text{if } \pi_0 < \frac{\sqrt{\mu - c} - \mu}{1 - \mu} \text{ and } c \text{ is low.}
\end{cases}
$$

If $L(\hat{s}) \geq R(\pi_0, \mu)$, consumer-optimal search cost is $s^* = \hat{s}$. If $L(\hat{s}) < R(\pi_0, \mu)$, optimal search cost is $s^* \leq \hat{s}$.

Proof. See Appendix A.

Note that $c \leq p_0(\hat{s}) \leq \pi_0$, so that both the consumer and the firm are willing to participate at $p_0$. This is because $p_0(\hat{s}) \geq p_0(\bar{s}_0) > c$ and $\pi_0 \geq \max\{p_0(\bar{s}_0), p_0(s_1)\}$.

The condition $L(s) \geq R(\pi_0, \mu)$ corresponds to the case where the firm’s profit function $\Pi_k$ is always increasing. If this holds, the consumer has to have a big enough search cost so that the firm is not willing to induce search. If, however, $L(\hat{s}) < R(\pi_0, \mu)$, the firm’s profit function at the candidate for optimal search cost ($\hat{s}$) is decreasing at first. This means that the consumer’s search cost can be lower than $\bar{s}_0$ and the firm will still find $p_0$ optimal.

Figure 4: Equilibrium utility and profit as functions of the search cost when it is at its consumer-optimal level ($\hat{s} = \bar{s}_0 = \frac{1}{10}$ when $\pi_0 = \mu = \frac{1}{2}$ and $c = 0$). One can see that the optimal search cost equalizes the

[Diagram]

Figure 4 depicts equilibrium utility and profit as functions of the search cost when it is at its consumer-optimal level ($\hat{s} = \bar{s}_0 = \frac{1}{10}$ when $\pi_0 = \mu = \frac{1}{2}$ and $c = 0$). One can see that the optimal search cost equalizes the

\[\text{This is due to the fact that the firm will under no circumstance consider setting } \hat{p}_1 \text{ because that gives lower profits than } p_1 \text{ which is worse than } p_0 \text{ due to V-shaped profits.}\]
firm’s profit from $\hat{p}_1$ to $\Pi_0$ and that $V(\pi_0, \hat{p}_1) = 0$.

When $L(s) \geq R(\pi_0, \mu)$, the Theorem provides an analytic solution to the problem of finding the optimal search cost for the consumer, which can be seen in the following Corollary.

**Corollary 3.2.** Assume that $\pi_0 \geq \sqrt{\frac{\mu}{1-\mu}}$. Then the consumer-optimal search cost is $\bar{s}_0$.

**Proof.** Obvious since the assumption guarantees that $R(\pi_0, \mu) \leq 0$, which in particular implies that $L(s) \geq R(\pi_0, \mu)$ for all $s$ and $c$. Thus, due to the above Theorem, $\bar{s}_0$ is optimal.

We can state another Corollary which makes it easy to compute the consumer’s equilibrium utility when there is no production cost.

**Corollary 3.3.** Assume that $\mu < L^2_0$ and $c = 0$. The consumer’s equilibrium utility can be written as:

$$U^* = \begin{cases} 
V_k(\pi_0, p_k), & \text{if } s \in [\bar{s}_k, s_k] \\
0, & \text{otherwise}
\end{cases}$$

where $k \in \mathbb{N}_0$ is the number of searches in equilibrium. The consumer’s utility jumps up at $s = \bar{s}_k$ and is strictly decreasing in $s$ whenever $s \in [\bar{s}_k, s_k)$ for some $k \in \mathbb{N}_0$. Otherwise it is constant at zero.

**Proof.** The assumption that $\mu < L^2_0$ implies that the firm’s profit function $\Pi_k$ is always increasing in $k$, so that the firm will always set either $p_k^*$ or $\hat{p}_{k+1}$. The firm prefers $p_k^*$ if and only if $s \geq \bar{s}_k^*$, where now:

$$\bar{s}_k = \frac{\pi_0}{(1-\mu)(1-\pi_k)s_k} + \pi_0(k + 1) + (1 - \pi_0) \frac{1 - \mu^{k+1}}{1 - \mu}.$$ 

With the two assumptions we made, for all $k$, $s_{k+1} < \bar{s}_k < s_k$, where $s_k$ is the search cost for which $V_k(\pi_0, p_k) = 0$ (and $s_k$ is decreasing in $k$). This means that if $s \in [\bar{s}_k, s_k]$, the firm’s optimal price will be $p_k$, and if $s \in (s_{k+1}, \bar{s}_k)$, the optimal price will be $\hat{p}_{k+1}$. If $s > s_0$, the optimal price will naturally be equal to the prior since the consumer will never search. The consumer obtains strictly positive utility if and only if $s \in [\bar{s}_k, s_k)$, and this utility is decreasing in $s$ within each interval because the firm is setting $p_k = \frac{s}{(1-\mu)(1-\pi_k)}$ which is increasing in $s$ (and nothing else happens when $s$ increases within the interval). Utility jumps up at $\bar{s}_k$ because there the firm changes its price from the one that makes the consumer search $k+1$ times to the one that makes her search only $k$ times, which surely improves the consumer’s utility.

Note that when there is no production cost and $\mu < L^2_0$, then maximum utility is $\bar{s}_0$. That is, utility equals the optimal search cost.
We saw Corollary 3.3 in action in Figures 1 and 2, where utility always jumps up when the firm decides to induce one less search in equilibrium (from \( k^* + 1 \) down to \( k^* \)). Profit is continuous and alternates between being increasing and decreasing in the search cost. The spikes for large \( k \) (small \( s_k \)) are so small that they do not show up on the scale. We can easily see that the consumer prefers a low search cost conditional on number of searches but a high search cost in that it brings the number of searches down. The optimal search cost is always \( \bar{s}_0 \) which leads to no search.

Next, to understand how the optimal search cost responds to beliefs, consider Figure 5 below. We plot the optimal search cost (\( s^* \)) as a function of the prior for three values of \( \mu \) (the informativeness of the signal), assuming no production cost. We can see that the optimal search cost is non-monotone in the prior and achieves its maximum slightly below \( \pi_0 = 0.6 \). This means that it is more important for the consumer to have a high search cost when the prior is uncertain. The more informative the signal is (the lower \( \mu \) is), the more important it is for the consumer to have a high search cost. This is something we will explore in more detail in Section 5. Finally, note that there are kinks in the optimal search cost when \( \mu > 0 \). This is because \( s_1 \) is optimal for low priors but \( \bar{s}_0 \) becomes optimal for higher ones. When \( \mu = 0 \), \( \bar{s}_0 \) is always optimal.

Figure 5: Optimal search cost as a function of prior and signal precision when \( c = 0 \)

Figure 6 depicts the consumer’s best-possible utility given her prior when there are no production costs for three values of \( \mu \) (assuming she can choose \( s = s^* \)). We see that the consumer is best off when her prior is close to 0.6 and when her signal is really informative, as long as she can choose her search cost. If she is unable to choose her search cost, a low \( \mu \) may not be optimal as we will see in Section 5.
Example

Consider a simple example with $\pi_0 = \mu = \frac{1}{2}$, meaning that the prior is very uncertain and the signal is not very informative because even a bad product gives a good signal half of the time. Assume, as always, that $c < \pi_0 = \frac{1}{2}$. Now, $s_0 = \frac{1}{8}$, $s_1 = \frac{1}{11}$, and $\bar{s}_0 = \frac{1}{10} + \frac{c}{20}$. With these parameter values, $L(\hat{s}) \geq R(\pi_0, \mu) = -2$, so Theorem 1 tells us that the optimal search cost should be $\hat{s}$ (for any $c$). We also have that $\pi_0 \geq \sqrt{\mu - \frac{1}{4}}$ (since $\frac{1}{2} > \sqrt{2} - 1$). Thus, the consumer’s optimal search cost is $\hat{s} = \bar{s}_0 = \frac{1}{10} + \frac{c}{20}$, which is significant relative to the prior of $\frac{1}{2}$, even for zero production costs. The optimal search cost is increasing in the production cost because a higher $c$ makes the firm more likely to induce search. However, it increases very slowly. When the search cost is $s^*$, the firm will set $p^* = p_0 = s^* \frac{s^*}{(1-\mu)(1-\bar{s}_0)} = 4s^* = \frac{2}{5} + \frac{c}{5}$, which will give the consumer an expected utility of $V_0 = \frac{1}{2} - \frac{2}{5} - \frac{c}{5} = \frac{1}{10} - \frac{c}{5}$. The firm, on the other hand, will make a profit of $\Pi_0 = p_0 - c = \frac{2}{5} - \frac{4}{5}c$. Therefore, the product is always sold and the firm and the consumer share the production costs (the consumer pays 20%).

The firm will not pass an increase in the costs directly to the consumer because she would then search. The assumption $c < \pi_0$ guarantees that $p^* > c$ for all $c$. Note that all of the above assumes that the consumer can always choose her search cost for any $c$ so that she will be made to purchase at a low price. Because $\bar{s}_0 > s_1$, it is the firm’s behavior that determines the optimal search cost, not the consumer’s ($s \geq \bar{s}_0$ guarantees that the firm prefers to sell the good without search, whereas $s \geq s_1$ guarantees that the consumer is not willing to search at $p_1$).\(^{24}\)

\(^{24}\)If the consumer was willing to search at $p_1$, the firm would make her do that because the firm’s profit is increasing in $k$, the number of searches.
However, if $\pi_0 = \mu = \frac{1}{4}$ (so that the prior is worse but the signal is more informative), the theorem still tells us that $\hat{s}$ is optimal, and the important values of the search cost are $s_0 = \frac{9}{64}$, $s_1 = \frac{9}{85}$, and $\bar{s}_0 = \frac{9}{100} + \frac{81}{400}c$. We now have that $\pi_0 < \frac{\sqrt{\pi - \mu}}{1 - \mu}$, which means that the consumer-optimal search cost is

$$s^* = \begin{cases} s_1 = \frac{9}{85}, & \text{if } c \leq \frac{4}{51} \\ \bar{s}_0 = \frac{9}{100} + \frac{81}{400}c, & \text{if } c > \frac{4}{51}. \end{cases}$$

The price is now:

$$p^* = \begin{cases} \frac{16}{85}, & \text{if } c \leq \frac{4}{51} \\ \frac{4}{25} + \frac{9}{25}c, & \text{if } c > \frac{4}{51}. \end{cases}$$

Note how the firm bears the whole production cost for low values of $c$ but shares the burden with the consumer for higher values. This is due to the equilibrium nature of the example: we are assuming that the consumer can choose her search cost to be exactly equal to $s^*$. In this case, for low values of $c$, everything stays constant as $c$ increases because it is not the firm’s behavior that matters but the consumer’s (the optimal search cost is high because that makes sure the consumer is not willing to search at $p_1$). However, for higher $c$, it is again the firm’s behavior that determines the optimal search cost, and, therefore, the value of the production cost matters for price and profits.

### 3.4 Welfare Considerations

So far we have only studied why a consumer almost always prefers a strictly positive search cost. However, the following proposition establishes that social surplus (consumer surplus plus firm profit) is always maximized at $s = 0$, but what the firm does is often not socially optimal.

**Proposition 3.** Social surplus (consumer surplus + profit) is maximized at $s = 0$. For $s > 0$, there may be too much search in equilibrium relative to what would be socially optimal.

**Proof.** For the first part ($s = 0$), note that when there is no search cost, it is optimal to search forever (or as long as one possibly can), which leads to $k^* = \infty$ and $p^* = 1$ (or slightly less if there are only a finite number of signals). Therefore, the probability of success is $\pi_0$, and we will perfectly learn whether the good is of high or low quality. Therefore, social welfare is $W_\infty = V_\infty + \pi_\infty = \pi_0(1 - c)$, which is the theoretical maximum of what we can get in ex-ante terms.

For the second part ($s > 0$), a benevolent planner, who controls everything, would search only when $W_{k+1} - W_k \geq 0$, which is the social gain from one more search. This can be written as $c \geq \frac{s}{(1 - \mu)(1 - \pi_k)}$. If the
search cost is high relative to the production cost, searching will stop early (or not commence at all). This also means that welfare is either always decreasing in the number of searches $k$, or inverse V-shaped: first increasing, then decreasing (see Lemma 4 in Appendix A).

On the other hand, the private gain of the firm can be written as

$$\Pi_{k+1} - \Pi_k = (\pi_k + (1 - \pi_k)\mu)(p_{k+1} - c) - (p_k - c) = (1 - \mu)(1 - \pi_k)c + \frac{(\mu + (1 - \pi_k))^{2 - \mu}}{(1 - \mu)(1 - \pi_k)}s.$$ 

Now, the private gain is bigger than the social gain if and only if $s > 0$ and $(1 - \mu)\pi_k > 0$, both of which hold. Therefore, the firm may induce too much search from a planner’s point of view.

To see this concretely, consider the following example. Take $\mu = \pi_0 = \frac{1}{2}$, $c = \frac{1}{4}$ and $s = \frac{1}{100}$. We can compute that the social gain is positive if and only if $k < \frac{\ln(2/23)}{\ln(1/2)} \approx 3.5$. Thus, the planner would search at $k = 3$ but not at $k = 4$. However, the firm’s profit $\Pi_k$ would be everywhere increasing, so it would find $k^*$ such that $V_{k^* + 1} < 0 \leq V_{k^*}$. In this case, $k^* = 5$. Using Proposition 2, we can now check that the firm prefers $\hat{p}_{k^* + 1}$ over $p_{k^*}$, meaning that it will induce search at $k = 5$ but not at $k = 6$. The firm is not internalizing the loss it imposes on the consumer. We can use the same parameter values but let search cost vary to obtain Figure 7 below. In the figure, we see that the firm often induces strictly more search than would be socially optimal. The two lines overlap only for zero or high search costs. Note that we have plotted starting from a positive $s$ because otherwise the number of searches would be infinite.

Figure 7: Optimal number of searches for the firm and the planner ($\mu = \pi_0 = \frac{1}{2}$, $c = \frac{1}{4}$)

3.5 An application: Searching for Jobs

Consider a job-seeker who is unsure of his options. He values jobs based on the utility ($w_0 \in \{0, 1\}$) they bring him. He knows his options are either great ($w_0 = 1$) or terrible ($w_0 = 0$) but he does not know which.
His expectation is \( \pi_0 = \mathbb{E}(w_0) \). Then a firm comes around and offers him a package worth \( w \). Assume that the job-seeker can spend \( s \) to figure out whether his outside options are good or bad while holding on to the offer. He can also choose to take the outside option without search, which gives \( \pi_0 \). Assume also that the firm always makes an extra profit of 1 if it hires the worker but it costs \( w \) (for simplicity, utility and money are the same – there is risk-neutrality).

The timing is as follows: 1) firm offers \( w \), 2) job-seeker chooses whether to search for outside options, 3) job-seeker rejects or accepts firm’s offer, 4) payoffs are realized.

Assume \( s \leq \pi_0(1-\pi_0) \). If he searches, the job-seeker will get \( U_S = \pi_0 + (1-\pi_0)w - s \), while if he accepts and does not search, he will get \( U_N = w \). He will search if \( U_S > \max\{U_N, \pi_0\} \equiv w_L \equiv \frac{s}{1-\pi_0} < w < \frac{\pi_0 - s}{\pi_0} \equiv w_H \).\(^{25}\)

Now, if the firm sets \( w_H \), it can be sure that the worker will sign the contract, which gives a profit of \( \Pi_H = 1-\pi_0/w_H = s/\pi_0 \), while setting \( w_L \) induces the job-seeker to search and only sign the contract if the search results are bad. That is, \( \Pi_L = (1-\pi_0)(1-w_L) = 1-\pi_0-s \).

We will now find the threshold search cost \( (s^*) \) that makes the firm indifferent between \( w_L \) and \( w_H \):

\[
\Pi_H \geq \Pi_L \iff s \geq s^* \equiv \frac{\pi_0(1-\pi_0)}{1+\pi_0}.
\]

If \( s < s^* \), the firm sets \( w_L \), which encourages the worker to search for other jobs (it equalizes the worker’s expected search utility with the outside option \( \pi_0 \)). This implies the consumer gets \( U^*_S(s) = \pi_0 + (1-\pi_0)\frac{s}{1-\pi_0} - s = \pi_0 \). If \( s \geq s^* \), the firm sets \( w_H \) which will be accepted by the consumer. This gives him \( U^*_N(s) = \frac{\pi_0 - s}{\pi_0} \).

We can check that \( U^*_N(s) \geq U^*_S(s) \iff s \leq \pi_0(1-\pi_0) \) as assumed.

Thus, we have that the consumer searches for \( s \in [0, s^*] \), accepts the firm’s offer for \( s \in [s^*, \pi_0(1-\pi_0)] \), and takes the expected outside option for \( s > \pi_0(1-\pi_0) \).\(^{26}\) This also shows that the consumer’s utility is maximized at \( s = s^* > 0 \) because \( U^*_N(s) > U^*_S(s) = \pi_0 \) for \( s < \pi_0(1-\pi_0) \) and \( U^*_N(s) \) is decreasing in \( s \).

The idea is that when it is easy for the worker to search for other jobs, the firm would have to offer a very high wage to contract with him, so it is best to offer a low wage and let him search. However, if searching is expensive/time consuming, the worker will happily accept an intermediate wage and the firm is happy to pay.

4 Heterogeneous Consumers

Previously, we assumed that there was a single consumer (or a continuum of identical consumers), which meant that the firm was able to extract (nearly) all consumer surplus if it found it optimal to make the

\(^{25}\) The assumption that \( s \leq \pi_0(1-\pi_0) \) guarantees \( w_H \geq \pi_0 \geq w_L \).

\(^{26}\) Taking the expected outside offer corresponds to just picking some job randomly.
consumer search. It also meant that there was little to no deadweight loss.\textsuperscript{27}

In this section, we will add a bit of realism by allowing the consumers’ valuations to be uniformly distributed on \([0, 1]\). That is, \(v \sim U[0, 1]\). This will solve some of the problems mentioned above: the firm will never find it optimal to extract all the consumer surplus because that means excluding too many consumers. Moreover, there may be a real deadweight loss due to the exclusion of low-value consumers whose value is higher than the production cost.\textsuperscript{28}

We will still maintain the assumption that consumption utility is binary: it is always either 0 or \(v\), independently across consumers. This corresponds to saying that a consumer of type \(v\) has consumption utility \(\theta v\), where \(\theta \in \{0, 1\}\), and \(P(\theta = 1) = \pi_0\). This prior is again shared among the consumers. In the rest of the section, we will see what happens in two cases: 1) no information, and 2) one available signal.\textsuperscript{29}

\section{4.1 No information}

Assume first that no additional information is available at any cost. Now, consumer (of type) \(v\) will buy if and only if her utility from buying exceeds that of exiting. Buying gives \(V_B(v) = \pi_0 v - p\), where \(v\) is the consumer’s type and \(p\) the price. Exiting always gives nothing. Therefore, the consumer will buy if and only if \(v \geq \frac{p}{\pi_0}\). The firm will then maximize \(\Pi_B(p) = (1 - \frac{p}{\pi_0})(p - c)\), where the first term is the mass of consumers who prefer to buy. This yields an optimal price of \(p^*_0 = \frac{\pi_0 c}{\pi_0 + \mu}\). The maximized profit is \(\Pi^*_0 = \Pi(p^*_0) = \frac{(\pi_0 - c)^2}{4\pi_0}\).

Consumer \(v\) gets an expected utility of \(V^*_0(v) = \pi_0 v - \frac{\pi_0 c + \mu}{2\pi_0}\) (given that \(v \geq \frac{p^*_0}{\pi_0}\); otherwise nothing). As a whole, the consumers get an average utility of \(V^*_0 = \left(1 - \frac{\pi^*_0}{\pi_0}\right) \frac{V^*_0(1)}{2} = \frac{(\pi_0 - c)^2}{8\pi_0}\). This is because the lowest-value buyer gets 0, the highest type (\(v = 1\)) gets \(V^*_0(1)\), and utility is increasing linearly in \(v\) between the two. The first term represents the mass of the buyers.

\section{4.2 One available signal}

To make things easier, I will assume that, at cost \(s\), each consumer has access to one binary signal \(\sigma\) (the outcome of which is independent across consumers) with the conditional probabilities (as in Section 3):

\[ P(\sigma = G|\theta) = \begin{cases} 1, & \text{if } \theta = H \\ \mu, & \text{if } \theta = L. \end{cases} \]

\textsuperscript{27}There was deadweight loss only when the firm found it optimal to make the consumer search more than a planner would have, as discussed in Corollary 5.4.

\textsuperscript{28}The distributional assumption is for convenience; the results hold for more general distributions.

\textsuperscript{29}Adding more signals would just needlessly complicate the analysis because we could always use arguments like those in Section 3 to show that, all else equal, a consumer prefers not to search.
Every consumer can choose to search once or not at all. Thus, each consumer has three decisions: Search, Buy, or Exit. The expected values of these decisions for consumer \( v \) can be expressed as: \( V_S(v) = \pi_0v - (\pi_0 + (1 - \pi_0)\mu) p - s \), \( V_B(v) = \pi_0v - p \), and \( V_E(v) = 0 \), respectively.

**Proposition 4.** Assume that \( c < \pi_0 \). Then there exist two thresholds, \( \bar{s} \) and \( v^*(s) \), such that, for \( s < \bar{s} \), the firm will set a price \( p^*_S \) to make the consumers with \( v \geq v^*(s) \) search. For \( s \geq \bar{s} \), the consumers with \( v \geq v^*(s) \) will purchase the good at a price \( p^*_B \) that makes them indifferent between buying and searching. All consumers with \( v < v^*(s) \) make the same decision: buy or search. Similarly, everyone with \( v < v^*(s) \) exits.\(^{30}\)

Note that the threshold value \( v^*(s) \) depends on the value of search cost. This is because this is the type of consumer who is exactly indifferent between exiting and participating in the market (be it buying or searching).

**Proof.** See Appendix B.

The search cost threshold \( \bar{s} \) is rather complicated function of the parameters \((\pi_0, \mu, c)\), so we cannot get far analytically unless we make specific parametric assumptions. This is exactly what we will do in the example below.

The threshold for the search cost, \( \bar{s} \), in the previous proposition is next to impossible to write out legibly and to understand but we can use specific parameter values to gain tractability. The following example will consider the case of \( \pi_0 = \mu = \frac{1}{2} \), which we are familiar with from before.

**Example**

Let \( \pi_0 = \mu = \frac{1}{2} \) (and \( c < \pi_0 \), as before). In this case, using Proposition 4, we can calculate: \( A = \frac{65}{16} \), \( B = \frac{9}{16} + \frac{29}{32}c \), and \( C = \frac{1}{16} + \frac{5}{16}c + \frac{9}{256}c^2 \). This gives \( \bar{s} = \frac{1}{26} + \frac{9}{32}c \).

To compute the threshold \( v^*(s) \), we need to know the firm’s equilibrium price as a function of the parameters. Again, using the proof of Proposition 4, we see that the firm’s optimal pricing rule can be written as:

\[
p^* = \begin{cases} 
\frac{1}{3} + \frac{1}{2}c - \frac{2}{3}s, & \text{if } s < \frac{1}{26} + \frac{9}{32}c \\
4s, & \text{if } \frac{1}{26} + \frac{9}{32}c \leq s \leq \frac{1}{16} + \frac{1}{8}c,
\end{cases}
\]

\(^{30}\)We can express the threshold as \( \bar{s} \approx \frac{\mu - \sqrt{\mu^2 - 4\mu c}}{2\mu} \), where \( A = 4(1-\mu)^2(1-\pi_0)^2 \), \( B = 4(1-\mu)(1-\pi_0)(\pi_0 + c) + 2(1-\mu)^2(1-\pi_0)^2(\pi_0 - (\pi_0 + (1-\pi_0)\mu)c) \), and \( C = (1-\mu)^2(1-\pi_0)^2(\pi_0 - (\pi_0 + (1-\pi_0)\mu)c)^2 + 4(1-\mu)^2(1-\pi_0)^2\pi_0c \).
because at \( s \) the firm switches from search pricing to purchase pricing. This allows us to write a single consumer’s utility (knowing that she will search for low \( s \) and purchase for high \( s \), as long as she is willing to participate) as:

\[
V^*(v) = \begin{cases} 
\frac{1}{2}v - \frac{1}{4} - \frac{3}{8}c - \frac{1}{2}s, & \text{if } s < \frac{1}{26} + \frac{9}{52}c \\
\frac{1}{2}v - 4s, & \text{if } \frac{1}{26} + \frac{9}{52}c \leq s \leq \frac{1}{16} + \frac{1}{8}c.
\end{cases}
\]

However, this only applies if \( V^*(v) \) is non-negative, which finally gives us the threshold values for participation:

\[
V^*(s) = \begin{cases} 
\frac{1}{2} + \frac{3}{4}c + s, & \text{if } s < \frac{1}{26} + \frac{9}{52}c \\
8s, & \text{if } \frac{1}{26} + \frac{9}{52}c \leq s \leq \frac{1}{16} + \frac{1}{8}c.
\end{cases}
\]

Therefore, the consumers as a whole get \( V^* \) and the firm gets \( \Pi^* \), where:

\[
V^* = \begin{cases} 
\frac{1}{2}(\frac{1}{2} - \frac{3}{4}c - s)^2, & \text{if } s < \frac{1}{26} + \frac{9}{52}c \\
\frac{1}{2}(1 - 8s)^2, & \text{if } \frac{1}{26} + \frac{9}{52}c \leq s \leq \frac{1}{16} + \frac{1}{8}c
\end{cases}
\]

\[
\Pi^* = \begin{cases} 
\frac{1}{2}(\frac{1}{2} - \frac{3}{4}c - s)^2, & \text{if } s < \frac{1}{26} + \frac{9}{52}c \\
(1 - 8s)(4s - c), & \text{if } \frac{1}{26} + \frac{9}{52}c \leq s \leq \frac{1}{16} + \frac{1}{8}c.
\end{cases}
\]

As you may have noticed, we have omitted what happens when \( s \) is high (higher than \( \frac{1}{16} + \frac{1}{8}c \), to be exact). However, in this case the equilibrium is exactly like the one with no available signals. The intuition is that the search costs are so high the firm is able to charge the no-search monopoly price \( p_0^* \), and no one is willing to search.

The following proposition makes an interesting observation:

**Proposition 5.** Let \( \pi_0 = \mu = \frac{1}{2} \) and \( c < \pi_0 \). The consumers are (strictly) hurt by their having access to the signal if and only if \( s \in \left( \frac{1}{4}c, \frac{1}{26} + \frac{9}{52}c \right) \). In fact, the exact same result applies to social surplus. That is, it may be socially inefficient for the consumers to have access to a signal.

**Proof.** The proof involves simply comparing \( V^*(v) \) to \( V_0^*(v) \) and \( W^* \) to \( W_0^* \), and noting that if \( V^*(v) > V_0^*(v) \), then \( V^* > V_0^* \) (for the consumers as a whole). We can note that for \( s < \frac{1}{4}c \), we always have \( s < \frac{1}{26} + \frac{9}{52}c \) and \( V^*(v) > V_0^*(v) \). Similarly, if \( s \geq \frac{1}{26} + \frac{9}{52}c \), then \( V^*(v) \geq V_0^*(v) \).

**Corollary 4.1.** Assume that \( \pi_0 = \mu = \frac{1}{2} \) and \( c = 0 \). The consumers’ having access to a signal strictly
hurts them and the society as a whole if and only if $s \in (0, \frac{1}{20})$.

To understand the proposition (and the corollary), consider Figures 8 and 9 below. In these figures, we are plotting the firm’s profit and average utility across all consumers in equilibrium. We are fixing the prior and the imprecision of the signal at $\frac{1}{2}$, while Figure 8 fixes the production cost at zero and Figure 9 at 0.1. The colored dotted lines represent what profit and utility would be if there was no possibility of obtaining a signal (as in the no-information case of the previous section). Thus, we see that there are values of $s$ where both profit and utility are lower with than without a signal. Thus, the society as a whole loses as well for some search costs.

Figure 8: Average utility and profit in equilibrium when $c = 0$ and $\pi_0 = \mu = \frac{1}{2}$.

Increasing $c$ from 0 to 0.1 means that both the consumers and the firm prefer having a signal to no signal if search cost is low (less than $\frac{1}{4}c$, to be exact). This is why the dotted lines in Figure 9 are below the solid ones for low $s$.

The idea behind the previous two figures is that, for low search costs, the consumers benefit from the signal because searching and utilizing the signal is cheap but the firm cannot set a high price because it does not want to exclude too many consumers (which is in stark contrast to what happens with identical consumers in Section 3). For high search costs, it is too expensive for the firm to make the consumers search but the consumers may still have some bargaining power (depending on how high $s$ is) so that they may gain relative to the no-signal world (for very high search costs everything is the same in both worlds). However, for intermediate search costs, as defined in the proposition, the firm has the incentive to make the
consumers search but they are hurt because they have to use the technology, which is not cheap. In this case the consumers are hurt more than the firm gains, so that social surplus is also lower than with no signal. In short, in some cases having access to a signal may hurt the society because there will be too much search in equilibrium (which in turn is due to a lack of commitment power).

To understand why the lower bound in the proposition is increasing in $c$, we just need to remember that a high production cost leads to a high price and in those cases it is socially optimal to be able to utilize a signal technology (in fact, it is even consumer optimal to do so).

The following lemma establishes that a low search cost may not imply high participation in equilibrium.

**Lemma 3.** Continue to assume that $\pi_0 = \mu = \frac{1}{2}$. If $c < \frac{10}{33}$, the firm’s policy is more exclusive at $s = 0$ than at $s = \bar{s} \equiv \frac{1}{26} + \frac{9}{52}c$. That is, the mass of participating consumers is bigger at $\bar{s}$.

**Proof.** We can simply check what condition has to be satisfied for $v^*(0) > v^*(\bar{s})$. It turns out to be $c < \frac{10}{33}$. \qed

We can now state the main result of the section:

**Proposition 6.** Let $\pi_0 = \mu = \frac{1}{2}$. Assume each consumer can obtain at most one signal at cost $s$. Then:

1. For $c \leq \frac{10}{33}$, each individual consumer’s expected utility is maximized at $\bar{s} \equiv \frac{1}{26} + \frac{9}{52}c$ (given that they purchase). Moreover, total consumer surplus is also maximized at $\bar{s}$.

2. For $c \leq \bar{c} \equiv \frac{\sqrt{51}/13 - 1/2}{2\sqrt{51}/13 - 3/4} \approx 0.142$, total social surplus is maximized at $\bar{s}$. 
3. Socially efficient policy is to set \( p = c \), which yields total social and consumer surplus:

\[
W^*_E = V^*_E = \begin{cases} 
\frac{1}{4} (1 - 2s - \frac{3}{2}c)^2, & \text{if } s < \frac{1}{4}c \\
\frac{1}{4} (1 - 2c)^2, & \text{if } s \geq \frac{1}{4}c,
\end{cases}
\]

and efficient firm profit: \( \Pi^*_E = 0 \).

Note the contrast between this and Theorem 1: social surplus may not be maximized at \( s = 0 \). The intuition is that the firm is excluding too many consumers, whose valuation exceeds the production cost, in order to maximize its (monopoly) profits. This leads to a deadweight loss. Looking at Figures 8 and 9, we see how the firm changes its pricing policy at \( s = \bar{s} \) so that the consumers get a discrete benefit.

**Proof.** To show that each individual consumer benefits from having \( s = \bar{s} > 0 \), note that their utility is piecewise linear in the search cost with one jump and one kink. When \( s < \frac{1}{16} + \frac{1}{8}c \), this utility is decreasing in the search cost everywhere but at \( \bar{s} \) where it jumps up (for low enough \( c \)). Therefore, the consumer’s maximum is either at \( s = 0 \) or at \( s = \bar{s} \). However, when \( c \leq \frac{10}{33} \), the utility is maximized at \( \bar{s} \) – independent of the type, \( v \). Because every consumer’s utility is maximized at \( \bar{s} \), one can expect the total consumer surplus to be maximized there as well. However, it might be that some consumer who was willing to participate at \( s = 0 \) would not do so at \( s = \bar{s} \), but this will not happen with \( c \leq \frac{10}{33} \) as shown in Lemma 3. This proves the first part of the Proposition.

For the second part, we can see that, just like consumer surplus, total surplus \( W^* \) is always decreasing in \( s \) but experiences a discrete jump at \( \bar{s} \). Therefore, we can perform the same steps as above and compare total surplus at two points: \( s = 0 \) and \( s = \bar{s} \). It turns out that total surplus is maximized at \( \bar{s} \) whenever \( c \leq \bar{c} \).

Finally, the last part of the Proposition involves calculating the planner-optimal price (which intuitively turns out to be \( p = c \) because this price excludes the smallest number of consumers while still allowing the firm to make non-negative profits; a lower price would hurt the firm more than it would benefit the consumers even though the marginal consumer’s valuation is still above \( c \)). The surplus calculation comes from noting that, when price equals production cost, \( V'_S(v) > V'_B(v) \iff s < \frac{1}{4}c \) and \( V'_S(v) \geq 0 \iff v \geq 2s + \frac{3}{2}c \). Using these thresholds, we can arrive at the above surplus functions (understanding that for low values of search cost the consumers will search, and for high values they will buy).

The intuition for the above Proposition is that, for low \( s \), the firm is pricing too high, which hurts the consumers in two ways: it forces them to search (extracting more consumer surplus), and it increases \( v \) (the marginal consumer’s valuation) – the firm excludes too many consumers even though all of them have a low search cost. When \( c \leq \bar{c} \), both consumer and social surplus are maximized at a strictly positive search cost.
Figure 10: Mass of participating consumers as a function of search cost when $c = 0$ and $\pi_0 = \mu = \frac{1}{2}$.

because that enables the firm to set a slightly lower price. This can be seen in Figure 10 above, where the mass of participating consumers first decreases in $s$ because fewer of them are willing to search but then jumps up at $\bar{s}$ when the firm finds it optimal to induce no search.\textsuperscript{31} The firm is not alone at fault in setting a high price to induce search because a medium price would still make the consumers search. Only when the search cost is high, can the firm make the consumers buy because they cannot search anymore. Somewhat paradoxically, the consumers’ ability to search for information hurts them because they cannot commit to not searching if the price is intermediate. In fact, the firm is pricing too high even for high values of search cost, $s$, which leads to sub-optimal social surplus (from the point of view of a social planner), as can be seen in Figure 11 below.

In the figure, we plot both the equilibrium social surplus and the planner-efficient social surplus as functions of the search cost. We see that the efficient social surplus is maximized at $s = 0$.\textsuperscript{32} However, in the equilibrium of the game, social surplus is maximized at $s = \bar{s} > 0$. The reason is that the parties (the firm and the consumer) cannot commit to behaving in a certain way. If a consumer, who has a low search cost, sees an intermediate price, she will search for information, which forces the firm to charge a high price (since the consumer is searching even when the price is not as high), which excludes low-value consumers. Only when the consumers have a high search cost can the firm trust them to purchase without search – and only then is an intermediate price optimal.

\textsuperscript{31}We also see that social surplus can be lower or higher than in a model without any signal, depending on the search cost. 
\textsuperscript{32}In fact, social surplus is maximized at every $s$ because a planner would never make the consumers search when $c = 0$. 
Figure 11: Social surplus as a function of search cost in equilibrium when $c = 0$ and $\pi_0 = \mu = \frac{1}{2}$.

To see how the precision of the signal ($\mu$) matters for the equilibrium, consider Figure 12 below where we plot the average equilibrium utility for two values of $\mu$. Note how the two curves start off almost identical. However, the firm changes its pricing policy sooner (lower $s$) when the signal is imprecise (high $\mu$) because the consumer’s beliefs do not increase much after search. On the other hand, the consumer with $\mu = 0$ has to be compensated more for her not to search (she has higher bargaining power in case the firm wants to make her buy). Note, however, that for intermediate $s$, a less precise signal is consumer-optimal. This is due to the fact that the low-$\mu$ consumer cannot commit to not searching which forces the firm to make her search.

5 Consumer Optimal Information Frictions

In this section, it is assumed that the consumer can costlessly choose from a menu of experiments each leading to two possible posteriors: one that perfectly reveals the good to be of low quality (leading to a posterior of 0), and one that improves the consumer’s expectation by an amount of her choice. However, conducting any one of these experiments will cost the consumer $s$. Note that all the experiments the consumer can choose will cost the same, no matter how informative they are in the Blackwell sense, which will only make the results stronger.

The three main results of this section are:

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33They start off exactly the same if there is no production cost and the difference grows in $c$.

34Since we are trying to show that using the best technology (perfectly informative signal) may not be optimal for the consumer, the result will be stronger if the consumer can freely acquire this (or any other) technology.
Figure 12: Utility as a function of search cost and the precision of the signal when $c = 0.1$ and $\pi_0 = \frac{1}{2}$.

1. If the consumer only gets to choose the experiment but not the search cost, and the firm gets to observe the chosen experiment prior to setting its price, the consumer’s optimal signal is not the perfectly informative one but this distortion is decreasing in the search cost, and goes away for high $s$.

   • If the firm has to commit to a price before the consumer chooses the experiment, the consumer will choose the perfectly informative signal. This produces a result similar to Section 3.

2. When the consumer gets to choose both the search cost and the signal, she wants the signal to be perfect and the search cost to be high enough (always strictly positive).

3. When the consumer gets to choose the triple $(q, E_B, E_G)$, where $E_B$ and $E_G$ are the posterior expectations after a bad and a good signal, respectively, and $q$ the probability of a good signal, she will not, in general, choose the perfectly informative experiment $(E_G = 1, E_B = 0)$. She will always choose $E_G = 1$ but the optimal $E_B$ may be strictly positive. However, if search cost $s$ is high enough, the perfectly informative experiment will be chosen.

These results complement the ones obtained in Roesler and Szentes [2017] by adding a search cost and focusing on the relationship between signal precision and search cost.
5 CONSUMER OPTIMAL INFORMATION FRICTIONS

5.1 Model

As in the previous section, there is one consumer (or a continuum of identical consumers) and one firm with a product whose quality can be high or low, yielding a consumption utility of 1 or 0, respectively. Prior probability on the quality being high is $\pi_0$. The firm has a cost of production, $c \geq 0$, and the consumer a search cost, $s \geq 0$. However, there is one twist compared to the previous model: the consumer begins the game by costlessly choosing a search technology which we call $\pi_1$. This technology allows the consumer to conduct an experiment that leads to a posterior of $\pi_1 \geq \pi_0$ with probability $\frac{\pi_0}{\pi_1}$ and a posterior of 0 with the probability $1 - \frac{\pi_0}{\pi_1}$. Conducting the experiment costs $s$ but the consumer can choose any experiment – and the firm observes this choice prior to setting its price. In other words, the timing is as follows: 1) the consumer chooses $\pi_1 \in [\pi_0, 1]$, 2) the firm observes this choice and sets $p \geq 0$, 3) the consumer makes a one-time decision to search or buy, and 4) payoffs are realized.\(^{35}\)

One way to think about the search technology $\pi_1$ is as a search algorithm for reviews. If there was an option to enter your preferences on Amazon and automatically get a summary of all the reviews relevant to your preferences, your posterior would either be high or low. A better technology (higher $\pi_1$) would correspond to a larger range of posterior beliefs for a given prior. Products would be priced higher because firms would know the only ones to buy their products would be those who obtain high posteriors.

Note that this model is still of the same flavor as the baseline model because the consumer’s posterior will always be 0 after a bad signal. In a sense, she can now choose to reach any good posterior $\pi_1$ at cost $s$, while in the baseline model she had to pay more to reach a higher posterior.

5.2 Equilibrium

In this subsection, we will compute the (unique) equilibrium of the game. The takeaway will be that the consumer may choose a less-than-perfect search technology. She will also prefer an interior search cost (as before).

Proposition 7. Assume $s \leq \frac{(\pi_1 - \pi_0)\pi_0}{\pi_1}$, and $c \leq \pi_0$. Let $A(\pi_1, c) \equiv \frac{\pi_0(\pi_1 - \pi_0)}{2\pi_1 - \pi_0} + \frac{(\pi_1 - \pi_0)^2}{\pi_1(2\pi_1 - \pi_0)}c$. Then, if search cost is fixed, the consumer’s optimal choice of search technology is $\pi_1^*(s) = \min\{1, \hat{\pi}_1(s)\}$, where $\hat{\pi}_1(s)$ solves $s = A(\hat{\pi}_1(s), c)$. If both $s$ and $\pi_1$ are for the consumer to choose, she will choose $\pi_1^* = 1$ and $s^* = A(1, c)$.

Proof. To solve this game, let us start from the end. Given a technology $\pi_1$ and a price $p$, the consumer

\(^{35}\)Note that there are no dynamics here. The consumer chooses the posterior she wishes to get by choosing her technology, and by using the technology at cost $s$, she has the chance of reaching that posterior. The fact that there are no dynamics means that the consumer’s posterior does not depend on the firm’s price directly, while in the previous model a higher price implied a higher purchase threshold $\pi.$
will search if and only if \( \frac{s}{\pi_1}(\pi_1 - p) - s \geq \max\{0, \pi_0 - p\} \iff p \in \left( \frac{\pi_1 - \pi_0}{\pi_1 - \pi_0}, \frac{\pi_1 - \pi_0}{\pi_1} \right) \). The assumption that 
\[ s \leq \frac{(\pi_1 - \pi_0)\pi_0}{\pi_1} \]
guarantees that there are some prices that satisfy the above inequalities. \(^{37}\)

Thus, given a search technology \( \pi_1 \), the firm will essentially have the choice between two prices: \( p_S = \frac{\pi_0 - s}{\pi_0} \pi_1 \) ("search price" which makes the consumer search) and \( p_B = \frac{\pi_1}{\pi_1 - \pi_0} s \) ("purchase price" which makes the consumer buy without search). The trade-off from the firm’s perspective is that the search price gives a higher profit conditional on selling but the probability of a sale is less than one, whereas the purchase price is lower but leads to a sale with certainty. Therefore, the firm prefers the search price if and only if:

\[
\frac{\pi_0 - s}{\pi_1} \pi_1(\pi_1 - c) \geq \frac{\pi_1}{\pi_1 - \pi_0} s - c \iff s \leq A(\pi_1, c) \equiv \frac{\pi_0(\pi_1 - \pi_0)}{2\pi_1 - \pi_0} + \frac{(\pi_1 - \pi_0)^2}{\pi_1(2\pi_1 - \pi_0)} c.
\]

This leads to the pricing scheme:

\[
p^* = \begin{cases} 
\frac{\pi_0 - s}{\pi_0} \pi_1, & \text{if } s < A(\pi_1, c) \\
\frac{\pi_1}{\pi_1 - \pi_0} s, & \text{if } s \geq A(\pi_1, c).
\end{cases}
\]

One thing to note is that the assumption \( c \leq \pi_0 \) guarantees that \( A(\pi_1, c) \leq \frac{\pi_0}{\pi_1}(\pi_1 - \pi_0) \), meaning that there are parameter values such that it is best to sell the product without search. Similarly, the assumption that \( s \leq \frac{(\pi_1 - \pi_0)\pi_0}{\pi_1} \) makes it possible for the consumer to search. Now, the consumer’s utility, given \( \pi_1 \), can be written as follows:

\[
V^*(\pi_1) = \begin{cases} 
0, & \text{if } s < A(\pi_1, c) \\
\pi_0 - \frac{\pi_1}{\pi_1 - \pi_0} s, & \text{if } s \geq A(\pi_1, c).
\end{cases}
\]

We see that the consumer always gets zero utility when she is made to search. Note that her best-case utility is \( V_{\max}(\pi_1) = \pi_0 - \frac{\pi_1}{\pi_1 - \pi_0} A(\pi_1, c) = (\pi_0 - c) \frac{\pi_1 - \pi_0}{2\pi_1 - \pi_0} \). This shows that, if the consumer was free to choose both the technology, \( \pi_1 \), and her search cost, \( s \), she would maximize her utility by choosing \( \pi_1^* = 1 \), and \( s^* = A(1, c) = \frac{1 - \pi_0}{2 - \pi_0}(\pi_0 + (1 - \pi_0)c) \). \(^{38}\) This is the lowest possible search cost that makes the firm choose the purchase price (making the consumer search is too expensive because the firm has to pay for it), given that the consumer is using the best possible technology, \( \pi_1^* = 1 \).

What if the consumer cannot choose her search cost but only the technology? Observing that, for \( s > A(\pi_1, c) \), \( V^*(\pi_1) \) is increasing in \( \pi_1 \), and that \( \frac{\partial A(\pi_1, c)}{\partial \pi_1} > 0 \), we obtain the result that the consumer’s optimal search technology is \( \pi_1^* = \min\{\hat{\pi}_1(s), 1\} \), where \( \hat{\pi}_1(s) \) solves: \( s = A(\hat{\pi}_1(s), c) \). The consumer benefits

\(^{36}\)There is a slight abuse of notation here because the inequalities are weak but I assume that the lower bound on price is strict. This is just a matter of convention because we can always make the consumer strictly prefer one action over another by changing the price infinitesimally. Here I have assumed that if the consumer is indifferent between buying and searching, she will buy; and if she is indifferent between searching and exiting, she will search.

\(^{37}\)If this was not the case, even for \( \pi_1 = 1 \), the firm would always set \( p = \pi_0 \) and the consumer would buy without using her search technology, giving her no expected utility.

\(^{38}\)It can be easily checked that \( s^* \leq \frac{(\pi_1 - \pi_0)\pi_0}{\pi_1} \) as long as \( c \leq \pi_0 \).
from increasing $\pi_1$ as long as the firm is still willing to sell the good without search, leaving the consumer with positive utility.

In particular, if the search cost is fixed, the proposition is saying that the consumer will often be better off not obtaining the best possible search technology even though that would be socially optimal. This is because the firm would then exploit this technology and make her search (it would know the technology exists when setting the price). When the consumer gets to choose her technology as well as the cost of using it ($s$), she always prefers the socially optimal technology. This is the most important insight of this section: even though $s$ and $\pi_1$ are complementary tools in that both a high $s$ and a low $\pi_1$ make the firm set the purchase price ($p_B$), the consumer prefers using $s$ instead of $\pi_1$. Why? Because in equilibrium she will not end up searching, it does not directly matter what the value of $s$ is. We know that in any equilibrium where the condition binds ($s = A(\pi_1, c)$) and $\pi_1 < 1$, the consumer prefers to raise both $\pi_1$ and $s$ so that the condition still binds because that leads to a lower purchase price $p_B$.

To see how this works, consider the following corollary:

**Corollary 5.1.** Assume $c = 0$. Then the consumer-optimal posterior as a function of search cost is

$$\pi^*_1(s) = \begin{cases} \frac{\pi_0^2 - \pi_0 s}{\pi_0 - 2s} & \text{if } 0 < s \leq \frac{\pi_0(1 - \pi_0)}{2 - \pi_0} \\ 1 & \text{if } \frac{\pi_0(1 - \pi_0)}{2 - \pi_0} \leq s < \pi_0(1 - \pi_0). \end{cases}$$

**Proof.** We need $s = A(\pi^*_1, 0) \iff \pi_1 = \frac{\pi_0^2 - \pi_0 s}{\pi_0 - 2s}$ but this is less than one only if $s \leq \frac{\pi_0(1 - \pi_0)}{2 - \pi_0}$. Otherwise the consumer should obtain the perfectly precise signal.

To understand the corollary, consider Figure 13 below. We plot the consumer’s optimal (good) posterior as a function of the search cost for three priors when there are no production costs. We see that the consumer will not choose the perfectly informative experiment unless her search cost is high enough. The threshold search cost for choosing the perfect experiment is increasing in the prior until $\pi_0 = 2 - \sqrt{2}$ (the red line), after which it starts to decrease (not seen in the figure). An interesting observation is that there are values of $s$ for which the optimal posterior is higher under $\pi_0 = 0.4$ than under $\pi_0 = 0.59$. Thus, it is not the case that the optimal posterior increases monotonically in the prior for all $s$. If the consumer becomes more convinced of the products high quality ex ante, she may need to use a less informative experiment (meaning a lower $\pi_1$) to keep the firm from setting a high price.
The corollary means that the consumer’s equilibrium utility is

\[ V^*(\pi_1^*(s)) = \begin{cases} 
  s, & \text{if } s \leq \frac{\pi_0(1-\pi_0)}{2-\pi_0} \\
  \pi_0 - \frac{s}{1-\pi_0}, & \text{if } \frac{\pi_0(1-\pi_0)}{2-\pi_0} \leq s \leq \pi_0(1-\pi_0).
\end{cases} \]

If she could choose both the cost of search and the technology, she would go with \( s = \frac{\pi_0(1-\pi_0)}{2-\pi_0} \) and \( \pi_1 = 1 \). One way to understand the consumer’s optimal policy and her value function, is to remember that she is the one who chooses the search technology, and the firm observes this choice when setting its price. The consumer will not choose the socially optimal search technology (\( \pi_1 = 1 \)) when her search cost is low because if she did, the firm would make her use it and get no surplus. By choosing an intermediate search technology, the consumer forces the firm to set a low price that induces no search. In effect, the consumer is shooting herself in the leg to avoid having to conduct expensive search. Note also that the distortion is decreasing in the search cost, which is intuitive, because the higher the search cost is, the less of an incentive the firm has to make the consumer search. Thus, the consumer does not have to tie her hands as much when her search cost is high. All this leads to a value function that is increasing in the search cost at first but then decreases after the search cost crosses the critical threshold, \( \hat{s} = \frac{\pi_0(1-\pi_0)}{2-\pi_0} \). After the threshold, increasing the search cost does not allow the consumer to improve her search technology because it is already set at its maximum value of 1. Therefore, the good will always be sold and higher search costs are detrimental to the consumer.
(because they reduce her bargaining power, as her ability to search is reduced).

Because the good is always sold without search, social surplus is always exactly \( \pi_0 \) (assuming no production cost, of course). However, the firm’s profit, just like the consumer’s value, is not monotone in the search cost:

\[
\Pi^*(\pi_1(s)) = \begin{cases} 
\pi_0 - s, & \text{if } s < \frac{\pi_0(1-\pi_0)}{2-\pi_0} \\
\frac{s}{1-\pi_0}, & \text{if } s \geq \frac{\pi_0(1-\pi_0)}{2-\pi_0}.
\end{cases}
\]

Thus, the firm’s profit is V-shaped, whereas the consumer’s utility is the inverse of that. The consumer’s optimum is the firm’s minimum, and vice versa.

So far we have been assuming that the consumer chooses her search technology before the firm sets its price. However, in some settings the opposite might be true. Therefore, the following proposition briefly explores what happens when the firm gets to commit to a price before the consumer chooses her technology.

**Proposition 8.** Assume that \( s \leq \pi_0(1-\pi_0) \) is fixed. Assume that the timing is slightly different from the previous propositions: 1) the firm sets \( p \), 2) the consumer chooses \( \pi_1 \), 3) the consumer buys, searches, or exits. Then the consumer’s optimal policy is to always use the best search technology (\( \pi_1^* = 1 \)). The consumer’s using the optimal search technology will be reflected in the price, giving her the utility:

\[
V^* = \begin{cases} 
0, & \text{if } s < A(1,c) \\
\pi_0 - \frac{1}{1-\pi_0}s, & \text{if } s \geq A(1,c),
\end{cases}
\]

where \( A(1,c) = \frac{\pi_0(1-\pi_0)}{2-\pi_0} + \frac{(1-\pi_0)^2}{2-\pi_0}c \) is the threshold where the firm changes its price.

Note that this result is essentially the same as the results in Section 3, the only difference being that now the consumer can choose her technology. She would again want to have an intermediate search cost.

**Proof.** The consumer now chooses \( \pi_1^* \) to maximize \( \frac{\pi_0}{\pi_1}(\pi_1 - p) - s \), after which she chooses to search (giving her a payoff of \( \frac{\pi_0}{\pi_1}(\pi_1 - p) - s \)), purchase \( (\pi_0 - p) \), or exit (0). We see immediately that the optimal choice of search technology is \( \pi_1^* = 1 \) because it does not impact the price in any way. Then we see that the consumer’s search region is defined by \( p \in \left( \frac{s}{1-\pi_0}, 1 - \frac{s}{\pi_0} \right) \). This means that the firm will be choosing between two prices: \( p_B \equiv \frac{s}{1-\pi_0} \) and \( p_S \equiv 1 - \frac{s}{\pi_0} \), leading to the cut-off search cost \( A(1,c) = \frac{\pi_0(1-\pi_0)}{2-\pi_0} + \frac{(1-\pi_0)^2}{2-\pi_0}c \) (where the firm sets...
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Thus, the consumer’s utility function can be written as:

\[ V^* = \begin{cases} 
0, & \text{if } s < A(1, c) \\
\pi_0 - \frac{1}{1 - \pi} s, & \text{if } s \geq A(1, c) 
\end{cases} \]

This shows that the firm’s issue in the previous model is one of commitment. There, the firm cannot commit to a price, which means that the consumer can affect the firm’s decision by choosing an intermediate search technology. Here, however, the firm is able to commit, meaning that it can force the consumer to search and get zero utility when the search cost is low (which is essentially just Theorem 1 in disguise).

5.3 A More General Model

So far, we have assumed that the consumer can choose her technology such that she will either perfectly learn that the product is of low quality, or she will obtain a posterior expectation of \( \pi_1 \) with probability \( \frac{\pi_0}{\pi_1} \). Now, to be more general, and to allow the quality to be any real number between 0 and 1, assume that the consumer has the freedom to choose any experiment with two possible signals: a bad and a good signal. The only restriction is that the expected posterior has to be equal to the prior.

To fix notation, assume that the prior distribution of quality, \( \theta \), on \([0, 1]\) is \( F \), and that there are two signals which lead to posteriors \( F_B \) and \( F_G \), respectively. The only requirement here is that the expected posterior distribution be equal to the prior. In other words, \( F = qF_G + (1 - q)F_B \), where \( q \) is the probability of a good signal. Assume also that \( E_G \equiv \mathbb{E}_G[\theta] \geq \mathbb{E}[\theta] \equiv E \geq \mathbb{E}_B[\theta] \equiv E_B \), so that the good signal always (weakly) improves the expected quality relative to the prior (and similarly, the bad signal lowers the expected quality). The actual distributions do not matter, and we only need to know their expected values (as long as they satisfy the requirement that expected posterior be equal to the prior).\(^{39}\)

**Proposition 9.** Assume that \( s < 1 - E \) and \( c < E \). The consumer’s optimal search technology characterized by the good posterior, \( E_G^* = 1 \), and the probability of a good signal, \( q^* = \min\{E, \hat{q}\} \), where \( \hat{q} \) solves

\[ s = A(\hat{q}, 1, E, c) \equiv q - \frac{E}{2-q} + \frac{(1-q)^2}{2-q} c. \]

Here, \( A(q, E_G, E, c) \) is a search cost threshold such that the firm switches from making the consumer search to making her buy at the threshold. Thus, the consumer wants to be exactly at the threshold.

\(^{39}\)Note that assuming only two possible signals is without loss of generality when the consumer can choose the posterior beliefs because the only two things that matter are 1) the probability that the posterior exceeds the price, and 2) expected posterior given that it exceeds the price. The first one is the only thing that matters for the firm when it is setting its price while the second matters for the consumer’s utility. However, conditional on the firm setting a given price, the actual number of signals is inconsequential because the consumer can choose which price the firm will set.
Proof. Now, given the search technology \((q, E_G)\), the consumer will search if:

\[
q(E_G - p) - s > \max\{0, E - p\} \iff p \in \left(\frac{E - qE_G}{1 - q} + \frac{s}{1 - q}, \frac{E}{1 - q} - \frac{s}{q}\right).
\]

Thus, the firm has to choose between two prices: \(p_S = E_G - \frac{s}{q}\), and \(p_B = \frac{E - qE_G}{1 - q} + \frac{s}{1 - q}\), where the former makes the consumer search and the latter makes her buy. The firm will choose \(p_S\) if and only if:

\[
\frac{E - qE_G}{1 - q} + \frac{s}{1 - q} - c < q(E_G - \frac{s}{q} - c) \iff s < qE_G - \frac{E}{2 - q} + \frac{(1 - q)^2}{2 - q} - c, \quad \text{if} \quad s \geq A(q, E_G, E, c).
\]

This allows us to write the consumer’s utility, given the parameters \((E, c)\) and the chosen technology \((q, E_G)\):

\[
V(q, E_G, E, c) = \begin{cases} 0, & \text{if } s < A(q, E_G, E, c) \\ \frac{q}{1 - q}(E_G - E) - \frac{s}{1 - q}, & \text{if } s \geq A(q, E_G, E, c), \end{cases}
\]

where the function \(A(\cdot)\) is as defined in (6). This shows, again, how the firm gets all the surplus when the consumer is made to search, and how the firm sometimes finds it optimal to give the consumer some surplus in exchange for her not searching. We see that the consumer’s utility is strictly positive if and only if \(A(q, E_G, E, c) \leq s < q(E_G - E)\), which is possible only when \(c < E\).

Because both \(A(q, E_G, E, c)\) and \(V(q, E, G, E, c)\) are strictly increasing in \(E_G\), we want to set it as high as possible without violating \(s \geq A\) (for a given \(q\), that is). Thus, we want to set \(E_G\) so that \(s = A(q, E_G, E, c)\), which yields: \(E_G^*(q, s) = \min\left\{\frac{E - (1-q)^2}{q(2-q)} + \frac{s}{q}, 1\right\}\).

Take then any \(q < E\).\(^{40}\) If the optimal posterior \(E_G^*(q, s) < 1\), then we can show that \(V(q, E_G^*(q, s), E, c) = \frac{1-s}{2-q}(E - c)\), which is decreasing in \(q\). In particular, this implies that we should reduce \(q\), which decreases both \(E_G^*(q, s)\) and \(V(q, E_G^*(q, s), E, c)\).\(^{41}\) If, on the other hand, \(E_G^*(q, s) = 1\), we have \(s \geq A(q, 1, E, c)\). However, now \(\partial V(q, 1, E, c)/\partial q > 0\) and \(\partial A(q, 1, E, c)/\partial q > 0\) (these follow from the assumptions \(s < 1 - E\) and \(c < E\), respectively), which imply that we should increase \(q\) as much as we can, as long as \(s \geq A(q, 1, E, c)\). Putting everything together implies that the consumer maximizes her utility by choosing \(E_G^* = 1\) and \(q^* = \min\{E, \hat{q}\}\)

\(^{40}\)Because if \(q > E\), we need either \(E_B < 0\) or \(E_G > 1\), which is impossible. Furthermore, if \(q = E\), then the only option is \(E_G = 1\) and \(E_B = 0\).

\(^{41}\)\(\partial E_G^*/\partial q = \frac{2(1-q)(c-E)}{q^2(2-q)^2} - \frac{s}{q^2} < 0\) because \(c < E\).
such that \( s = A(q, 1, E, c) \). We can even write \( \hat{q} \) analytically:

\[
\hat{q} = 1 - \frac{\sqrt{s^2 + 4(1-c)(1-E-s)} - s}{2(1-c)}
\]

This concludes the proof.

Figure 14: Optimal \( q^* \), bad posterior (\( E^*_B \)) and maximized utility (\( V^* \)), assuming \( \pi_0 = \frac{1}{2} \) and \( c = 0 \).

In Figure 14, we see how the optimal probability of a high posterior (\( q^* \)) and the resulting bad posterior (\( E^*_B \)) depend on the search cost. Note that if the consumer was forced to choose \( q = E \) (perfect experiment because \( E^*_G = 1 \)), her utility would remain at zero until it jumped up at the point where \( q^*(s) = 1 \).

The intuition for the proposition is that the consumer wants to get the highest possible utility when a good signal is observed (\( E^*_G = 1 \)) but she may want to make sure not to observe the good signal too often (if \( q^* < E \)). One can verify that, if it is easy for the consumer to search, she wants to have \( q^* < E \), which also implies that \( E_B > 0 \).\(^{42}\) This is because the firm does not care about the consumer’s posterior conditional on it being above \( p \), while the consumer wants the highest posterior because, combined with a low enough \( q \), it gives her more bargaining power and forces the firm to charge a lower price. However, when searching is expensive (high \( s \)), the consumer wants the best possible technology because it improves her bargaining power and forces the firm to charge a lower price (\( q^* = E \), \( E_B = 0 \), and \( E_G = 1 \)). We can also state this as a corollary:

\(^{42}\)Simply because \( E^*_G = 1 \), \( E_B = \frac{E - q}{1 - q} > 0 \Leftrightarrow q < E \).
Corollary 5.2. For \( s \geq \frac{1}{2-E} [1 - (1-c)(1-E)] \), the consumer will choose the perfect technology: \( E_\text{G}^* = 1 \) and \( q^* = E \).

Proof. \( \hat{q} \geq E \iff 2(1-E)(1-c) + s \geq \sqrt{s^2 + 4(1-c)(1-E-s)} \iff s \geq \frac{1}{2-E} [1 - (1-c)(1-E)] \). \( \square \)

Corollary 5.3. Assume that \( c < E \) and that \( s \) is small enough for \( q^* < E \). Then \( q^* \) is increasing in \( s \) and decreasing in \( c \). Furthermore, this means that \( E_B^* \) is decreasing in \( s \) and increasing in \( c \).

Note how \( E_B^* \) is positive whenever \( q^* < E \) (which happens when \( s \) is low), which makes sense because the consumer wants to limit the informativeness of her experiment when search cost is low. Similarly, a high production cost will make the firm more willing to induce search, which means that the consumer needs a worse technology – this manifests in the form of a lower \( q^* \) and a higher \( E_B^* \). However, the posterior after a good signal always remains at 1 because that way the consumer can be sure that she is buying a good product after a good signal.

Proof. That the inequalities \( \partial q^*/\partial s > 0 \) and \( \partial q^*/\partial c < 0 \) hold can be seen by simply differentiating \( q^* \) (we need \( q^* < E \), because otherwise there is no change). To obtain the results regarding \( E_B^* \), we can just note that \( E_B^* = \frac{E^* - q^* E_G^*}{1-q^*} = \frac{E^* - q^*}{1-q^*} \), which is decreasing in \( q^* \). Thus, \( E_B^* \) moves in the opposite direction from \( q^* \). \( \square \)

Now, as an example, consider the case with \( s = c = 0 \). In this case, \( q^* = 1 - \sqrt{1-E} < E \), and \( E_B^* = \frac{E^* - q^*}{1-q^*} = 1 - \sqrt{1-E} \). Thus, the consumer will set a high \( E_B^* \) to get a better position vis-à-vis the firm. This is socially inefficient because we are not using the most effective technology but it is optimal for the consumer.

Corollary 5.4. If the consumer is not free to choose \( q \), and she can only pick \( E_G \), then the optimal technology may have \( E_G^* < 1 \).

Proof. This is obvious because \( E_G^*(q,s) = \min \left\{ \frac{E-(1-q)^2}{q(2-q)} + \frac{q}{q}, 1 \right\} \) but we cannot adjust \( q \), so it may be that \( s = A(q,E_G^*,E,c) \) at \( E_G^* < 1 \). \( \square \)

For example, if we assume that \( E = \frac{1}{2} \) and restrict the consumer to consider only those experiments that have \( q = \frac{1}{2} \), the condition for \( E_G^* < 1 \) becomes \( s < \frac{1}{2} - \frac{3}{4}E \). This means that for low values of the search cost \( (s < \frac{1}{6}) \), the consumer’s optimal experiment has \( E_G^* < 1 \) and \( E_B^* > 0 \). More specifically, \( E_G^* = \frac{1/2}{3/4} + 2s = \frac{2}{3} + 2s \), and \( E_B^* = \frac{1}{3} - 2s \). The consumer could easily (and for free) obtain a better technology but this would end up hurting her through the pricing of the firm.

6 Concluding Remarks

In this paper, we have seen how the consumer may benefit from having an imperfect search technology and/or a high search cost because then the firm will find it optimal to set a lower price. However, the
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The consumer always prefers to have a perfect technology if she can choose a high enough search cost so as to make the firm willing to sell without search. If the consumer cannot choose her search cost but she has more flexibility in choosing her technology, she may control the firm’s behavior by choosing a technology that is not socially optimal (an experiment that is not perfectly informative).

The reason for non-monotone utility (and profits) is that the consumer has two actions that the firm can induce by pricing high or low. For low search costs, the consumer has a lot of bargaining power when she buys because the firm has to offer a low price to prevent her from searching. However, a low search cost means that the firm can set a very high price and still make the consumer search. This is why the firm prefers to induce search when search is cheap. On the other hand, when search costs are high, the consumer is willing to purchase the good at a higher price because she does not have as much bargaining power in case the firm wants to sell. The maximum price the firm can set and make the consumer search is low when search costs are high, so the firm prefers selling the good outright.

Social surplus in equilibrium is generally lower than what would be optimal if one party had all the bargaining power. If search cost was zero or so high that the consumer had no bargaining power, social surplus would be at the planner-optimal level. If there is an ex-ante contracting stage, we can always reach the socially efficient course of action by giving one party the full bargaining power and requiring that this party make a lump-sum transfer to the other party. The size of this transfer can be anything as long as both players are weakly better off than in the original equilibrium.

When we introduce a continuum of consumers with different valuations for a good match, we see that the main result holds: a non-zero search cost is still optimal for low production costs – both for the consumers and the society as a whole. However, the equilibrium of the game is never efficient in the first-best sense; a social planner would like to include more consumers but the firm is excluding them to extract more from those who participate.

To get back to the issue of positive promotional reviews, let us remind ourselves how they might affect the consumers: 1) for some consumers, \( \mu \) will increase because good reviews are now more probable and reviewers understand this in equilibrium (slower learning) but cannot observe which reviews are trustworthy, and 2) for other consumers, \( \mu \) will stay the same because they can spot the fakes but \( s \) will increase because it now costs more to get useful reviews as the proverbial haystack is taller. This means that if the firm is already setting \( p_0 = \frac{s}{(1-\mu)(1-\pi_0)} \), increasing \( \mu \) will help it increase the price (since increasing \( \mu \) also leads to lower \( \hat{s} \)) until \( p_0 = \pi_0 \). If, however, \( s_1 < s < \hat{s}_0 = \hat{s} \), then increasing \( \mu \) may lead to the firm having to switch from \( \hat{p}_1 \) to \( p_0 \), which will surely improve consumer surplus. On the other hand, if the consumers are sophisticated enough so that they can spot the fakes, there is no effect on \( \mu \) but the increased \( s \) will work the same way: if \( s > \hat{s}_0 \) to start with, the consumers will be worse off but if \( s_1 < s < \hat{s}_0 \), the consumers may...
It is left for future research to determine what happens in this game when there is competition and (ex-ante) product differentiation, or when the consumers differ in their search cost. It would also be interesting to learn what the effect of promotional reviews is: how do they matter for profits and utility?

References


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Appendices

A General Results for Section 3

Let $k_{fb} := \min \{ k \mid c < \frac{s}{(1-\mu)(1-\pi_0)} \}$, which is the socially optimal (first best) number of searches. It is derived in the Lemma below:

Lemma 4. $k_{fb}$ is the socially optimal number of searches.
Proof. Note that social surplus can be written as $W_k = (\pi_0 + (1 - \pi_0)\mu^k)(\pi_k - c) - \mathbb{E}[\text{cost}]_k = \pi_0 - (\pi_0 + (1 - \pi_0)\mu^k)c - \mathbb{E}[\text{cost}]_k$. Therefore, $W_{k+1} - W_k = (1 - \mu)(1 - \pi_0)c - s \geq 0 \iff c \geq \frac{s}{(1 - \mu)(1 - \pi_0)}$. Because the RHS of this inequality is strictly increasing in $k$ and without bound, there has to be some $k$ for which the inequality is reversed, meaning that there is some $k_{fb}$ such that $\frac{s}{(1 - \mu)(1 - \pi_{k_{fb} + 1})} < c < \frac{s}{(1 - \mu)(1 - \pi_{k_{fb}})}$. This means that $W_0 < W_1 < \ldots W_{k_{fb} - 1} \leq W_{k_{fb}}$ and $W_{k_{fb}} > W_{k_{fb} + 1} > \ldots$. □

The following Lemma shows that the firm will not set the socially optimal price $p_{k_{fb}}$ because it will always have an option that is at least as good.

Lemma 5. Assume that $\pi_0 > c$. Then $k_{fb} \leq k^* + 1$, which means that the firm will never set $p > \hat{p}_{k^*+1}$.

Proof. If, contrary to the claim, $k_{fb} > k^* + 1$, we would know that $c \geq \frac{s}{(1 - \mu)(1 - \pi_{k_{fb} + 1})} > \hat{p}_{k^*+1} > p_k^*$. However, this would mean that the firm makes negative profits at $\hat{p}_{k^*+1}$ and the consumer gets zero at that price. Thus, social surplus would be negative at $\hat{p}_{k^*+1}$. However, the only way social surplus can be negative is if $\pi_0 < c$ (since social surplus is increasing in this range and $W_0 = \pi_0 - c$). But this is a contradiction because we assumed that $\pi_0 > c$. □

Proposition 2 Assume that $s \leq (1 - \mu)(1 - \pi_0)\pi_0$ and $\pi_0 > c$. If $\Pi_k^* \geq \Pi_0$, the firm’s optimal price is either $p_k^*$ or $\hat{p}_{k^*+1}$, and the firm prefers $p_k^*$ if and only if $s \geq \tilde{s}_k^*$, where:

$$\tilde{s}_k^* = \frac{\pi_0 + (1 - \pi_0)(1 - \mu)\mu^k c}{\pi_0 (k^* + 1) + \frac{(1 + \pi_0)}{1 - \mu} + (1 - \pi_0)\mu^{k^*} + \frac{\pi_0^2}{(1 - \mu)(1 - \pi_0)\mu^{k^*}}}.$$

If $\Pi_k^* < \Pi_0$, firm will set either $p_0$ or $\hat{p}_{k^*+1}$, depending on which one gives higher profits:

1. $\Pi(\pi_0, \hat{p}_{k^*+1}) > \Pi_0 \Rightarrow \hat{p}_{k^*+1}$ is optimal
2. $\Pi(\pi_0, \hat{p}_{k^*+1}) \leq \Pi_0 \Rightarrow p_0$ is optimal.

Proof. The assumptions guarantee that the consumer will get a non-negative payoff in equilibrium and that the firm will not consider $\hat{p}_{k_{fb}}$ (by Lemma 5). The firm is now choosing its price knowing that it will induce a number of searches from the consumer. If $\Pi_k^* \geq \Pi_0$, the firm should set either $p_k^*$ or $p_{k^*+1}$ because the consumer is willing to search at these prices and the firm’s profit is at least as high as at $p_0$. Because $\Pi_k^*$ is either always increasing or V-shaped (Lemma 1), the firm should not consider any price between $p_0$ and $p_k^*$. The firm will choose $p_k^*$ over $p_{k^*+1}$ if and only if $\Pi_k^* \geq \Pi(\pi_0, \hat{p}_{k^*+1}) \iff s \geq \tilde{s}_k^*$. Note that the firm does not want to set a price higher than $\hat{p}_{k^*+1}$ because it will be the residual claimant to the full social surplus but social surplus is decreasing at that point due to Lemma 5.

\footnote{If search cost is higher than $(1 - \mu)(1 - \pi_0)\pi_0$, the equilibrium will feature $p^* = \pi_0$.}
If, however, $\Pi_k < \Pi_0$ so that $\Pi_0$ is V-shaped and the firm’s profit at $p_k$, is less than that at $p_0$, equilibrium price will be either $p_0$ or $\hat{p}_{k+1}$. It cannot be anything higher than $\hat{p}_{k+1}$ because the firm again gets the full social surplus which is decreasing by Lemma 5. It also cannot be anything between $p_0$ and $p_k$, because those prices give profits that are lower than $\Pi_0$. Which one of the two prices is optimal depends on the profits as given in the Proposition.

Let us remind ourselves of the Lemma introduced before Theorem 1:

**Lemma 2** Let $s_k$ solve $V_k(\pi_0, p_k) = 0$. Then $s_k > s_{k+1} \forall k \in \mathbb{N}_0$, and $k^* = \max\{k \in \mathbb{N}_0 \mid s_k \geq s\}$.

Now we can restate and prove the Theorem.

**Theorem 1** Assume that $\pi_0 > c$. Let $L(s) = \frac{c}{s}$ and $R(\pi_0, \mu) = \frac{\mu - L_0^2}{\mu(1-\mu)}$. Define $\hat{s}$ as follows:

$$\hat{s} := \max\{\hat{s}_0, s_1\} = \begin{cases} 
\hat{s}_0 = \frac{(1-\mu)(1-\pi_0)}{(1-\mu)(1-\pi_0)}(\pi_0 + (1-\pi_0)(1-\mu)c) , & \text{if } \pi_0 \geq \frac{\sqrt{\mu-\mu}^2}{1-\mu} \text{ or } c \text{ is high,} \\
 s_1 = \frac{(1-\mu)(1-\pi_0)\pi_0}{(1-\mu)(1-\pi_0)+\pi_0(1-\pi_0)} , & \text{if } \pi_0 < \frac{\sqrt{\mu-\mu}^2}{1-\mu} \text{ and } c \text{ is low.}
\end{cases}$$

If $L(\hat{s}) \geq R(\pi_0, \mu)$, consumer-optimal search cost is $s^* = \hat{s}$. If $L(\hat{s}) < R(\pi_0, \mu)$, optimal search cost is $s^* \leq \hat{s}$.

**Proof.** When $L(s) \geq R(\pi_0, \mu)$, the firm’s profit function $\Pi_k$ is strictly increasing in $k$, meaning that the firm always wants to induce the highest possible number of searches $k$. The lower $s$ is, the more the consumer is willing to search, which implies a higher equilibrium number of searches, $k^*$. Using (1), we can compute the consumer’s value function at the relevant prices:

$$V_k(\pi_0, p_k) = \pi_0 - \left[ \left( \pi_0 + (1-\pi_0)(1-\pi_0) \mu^k \left( 1 - \mu \right)^k \right)^2 \left( 1 - \mu \right)^k \right] s,$$

$$V_{k+1}(\pi_0, \hat{p}_{k+1}) = 0.$$

Due to Lemma 2, we can construct a decreasing sequence of threshold values for the search cost, $s_0 > s_1 > s_2 > \ldots$, such that if the search cost falls between two thresholds, say $s_k$ and $s_{k+1}$, then $V_k(\pi_0, p_k) > 0 > V_{k+1}(\pi_0, p_{k+1})$. If $s > s_0$, even $V_0(\pi_0, p_0) < 0$, in which case the firm will set $\hat{p}_0 \equiv \pi_0$, giving the consumer zero utility.

The consumer will (typically) get a strictly positive utility if and only if her search cost $s \geq \bar{s}_k$, as long as $s < s_0 = (1-\mu)(1-\pi_0)\pi_0$ because otherwise the consumer will always just buy at a price that makes her indifferent between exiting and buying.\(^{44}\) $\bar{s}_k$ is a threshold such that the firm will choose $p_k$ over $\hat{p}_{k+1}$ for $s \geq \bar{s}_k$, and this was introduced in Proposition 2.

\(^{44}\)Here, we have broken the tie in the firm’s decision so that it will make the consumer buy if it is indifferent between the two strategies.
To obtain the value of the search cost that maximizes the value function, one needs to find the lowest value of the search cost that still makes the firm want to set \( p = p_0 \). The consumer’s equilibrium value when the firm prices at \( p_0 \) is \( V_0(\pi_0, p_0) = \pi_0 - \frac{\hat{s}}{(1-\mu)(1-\pi_0)} \), and the value at \( p_1 \) is \( V_1(\pi_0, p_1) = \pi_0 - \frac{(1-\mu)(\mu(\pi_0 + (1-\pi_0)\mu))}{(1-\mu)(1-\pi_0)} \), because these prices make the consumer search 0 and 1 times, respectively. Thus, \( s_0 = (1-\mu)(1-\pi_0)\pi_0 \) and \( s_1 = \frac{(1-\mu)(1-\pi_0)\mu\pi_0}{(1-\mu)(1-\pi_0)(\mu+\pi_0(1-\pi_0)\mu)} \), where \( s_0 > s_1 \). Furthermore, \( \bar{s}_0 = \frac{(1-\mu)(1-\pi_0)}{(1-\mu)(1-\pi_0)+\pi_0 + (1-\pi_0)(1-\mu)c} (\pi_0 + (1-\pi_0)(1-\mu)c) \), which makes the firm indifferent between \( p_0 \) and \( \bar{p}_1 \). Note also that \( \bar{s}_0 > (1-\mu)(1-\pi_0)c \) because \( \pi_0 > c \), which means that if \( s \geq \bar{s}_0 \), the firm will never have to consider \( \bar{p}_{k,1} \).

Now, \( L(\hat{s}) \geq R(\pi_0, \mu) \) guarantees that the firm will set \( p = p_0 \). We can verify that \( s_0 > \bar{s}_0 \) as long as \( \pi_0 > c \) (this is assumed), which means that \( \hat{s} := \max\{\bar{s}_0, s_1\} < s_0 \). One can check that \( \bar{s}_0 \geq s_1 \) for all \( c \geq 0 \) if and only if \( \pi_0 \geq \frac{\sqrt{\pi-\mu}}{1-\mu} \). However, if \( \pi_0 < \frac{\sqrt{\pi-\mu}}{1-\mu} \), \( s_1 > \bar{s}_0 \) for low values of production cost.

If \( L(\hat{s}) < R(\pi_0, \mu) \), we can do better than \( \hat{s} \) because the firm’s profit function is first decreasing, meaning that the consumer does not have to constrain the firm as much by choosing a high search cost. Since \( \Pi_1 < \Pi_0 \), there exists \( s^* \leq s_1 \leq \hat{s} \) such that, using Proposition 2, the firm will set \( p_0 = \frac{s}{(1-\mu)(1-\pi_0)} \) for any \( s \geq s^* \). Thus, the consumer-optimal choice is the smallest such search cost, \( s^* \).

\[ \square \]

B Proof of Proposition 4

**Proposition 4.** Assume that \( c < \pi_0 \). Then there exist two thresholds, \( \hat{s} \) and \( \underline{v}^*(s) \), such that, for \( s \geq \hat{s} \), the firm will make the consumers with \( v \geq \underline{v}^*(s) \) search. For \( s \geq \hat{s} \), the consumers with \( v \geq \underline{v}^*(s) \) will purchase the good at a price that makes them indifferent between buying and searching. All consumers with \( v \geq \underline{v}^*(s) \) make the same decision: buy or search. Similarly, everyone with \( v < \underline{v}^*(s) \) exits.

**Proof.** Now, given a price \( p \), a consumer with valuation \( v \) will search if and only if \( V_S(v) \geq \max\{V_B(v), V_E(v)\} \). The value of search is higher than the value of buying if and only if \( p > \bar{p}_B = \frac{s}{(1-\mu)(1-\pi_0)} \), where \( \bar{p}_B \) is the highest price that still makes the consumer buy the good without search. Note how this does not depend on the consumer’s type as the expected consumption value is the same whether or not you search or buy. Consumer \( v \) prefers searching to exiting if and only if \( v \geq \underline{v}(p) = \frac{\pi_0 + (1-\pi_0)\mu + s}{\pi_0} \). In other words, if one consumer finds searching optimal, then every consumer whose value is high enough finds searching optimal and no one finds buying optimal.

---

\(^{45}\)The explanation is that when \( s > (1-\mu)(1-\pi_0)c \), it is not socially optimal to search at \( k = 0 \), which means that it is also not optimal for the firm to induce search more than once (since it is the residual claimant to the full social surplus at \( \bar{p}_{k,1} \) for any \( k \)).

\(^{46}\)Note that this condition guarantees that \( \bar{s}_0 \geq s_1 \) for all \( c \).

\(^{47}\)We can express the threshold as \( \hat{s} = \frac{\sqrt{\pi-\mu - \frac{A}{C}}}{B} \), where \( A = 4 + (1-\mu)^2(1-\pi_0)^2 \), \( B = 4(1-\mu)(1-\pi_0)(\pi_0 + c) + 2(1-\mu)^2(1-\pi_0)^2(\pi_0 - (\pi_0 + (1-\pi_0)\mu)c) \), and \( C = (1-\mu)^2(1-\pi_0)^2(\pi_0 - (\pi_0 + (1-\pi_0)\mu)c)^2 + 4(1-\mu)^2(1-\pi_0)^2\pi_0c \).
The firm cannot just maximize the profits assuming that everyone will buy the good because the consumers have the option to search. Therefore, the firm has to choose between making the consumers search and making them buy. Above we found that, as long as \( p \leq \hat{p}_B \), the consumers will indeed buy the good without search. Therefore, the firm’s optimal purchase price is:

\[
p_B^* = \begin{cases} 
\hat{p}_B \equiv \frac{s}{(1-\mu)(1-\pi_0)}, & \text{if } \frac{s}{(1-\mu)(1-\pi_0)} \leq \frac{\pi_0+c}{2} \\
p_0^* \equiv \frac{\pi_0+c}{2}, & \text{if } \frac{s}{(1-\mu)(1-\pi_0)} \geq \frac{\pi_0+c}{2}.
\end{cases}
\]

In other words, should the firm decide to sell the good without search, it will always set the maximum price that makes the consumer buy instead of searching (\( \hat{p}_B \)) unless this price is higher than the optimal price with no information (\( p_0^* = \frac{\pi_0+c}{2} \) from the case with no signals). This leads to the purchase profit function:

\[
\Pi_B = \begin{cases} 
\frac{\pi_0+c}{2} - \frac{s^2}{(1-\mu)(1-\pi_0)^2\pi_0} - c, & \text{if } \frac{s}{(1-\mu)(1-\pi_0)} \leq \frac{\pi_0+c}{2} \\
\frac{s}{(1-\mu)(1-\pi_0)^2\pi_0}, & \text{if } \frac{s}{(1-\mu)(1-\pi_0)} \geq \frac{\pi_0+c}{2}.
\end{cases}
\]

Therefore, type \( v \) consumer will get (given that she is willing to buy):

\[
V_B^*(v) = \begin{cases} 
\frac{\pi_0 v}{2} - \frac{s}{(1-\mu)(1-\pi_0)}, & \text{if } \frac{s}{(1-\mu)(1-\pi_0)} \leq \frac{\pi_0+c}{2} \\
\frac{\pi_0 v}{2} - \frac{\pi_0+c}{2}, & \text{if } \frac{s}{(1-\mu)(1-\pi_0)} \geq \frac{\pi_0+c}{2}.
\end{cases}
\]

This translates into an average utility of:

\[
V_B^* = \begin{cases} 
\frac{\pi_0}{2} + \frac{s^2}{2(1-\mu)(1-\pi_0)^2\pi_0} - \frac{s}{(1-\mu)(1-\pi_0)}, & \text{if } \frac{s}{(1-\mu)(1-\pi_0)} \leq \frac{\pi_0+c}{2} \\
\frac{(\pi_0-c)^2}{8\pi_0}, & \text{if } \frac{s}{(1-\mu)(1-\pi_0)} \geq \frac{\pi_0+c}{2}.
\end{cases}
\]

And, finally, we can compute the social surplus:

\[
W_B^* = \begin{cases} 
\frac{\pi_0}{2} + \frac{s^2}{2(1-\mu)(1-\pi_0)^2\pi_0} - \left(1 - \frac{s}{(1-\mu)(1-\pi_0)}\right) c, & \text{if } \frac{s}{(1-\mu)(1-\pi_0)} \leq \frac{\pi_0+c}{2} \\
\frac{3(\pi_0-c)^2}{8\pi_0}, & \text{if } \frac{s}{(1-\mu)(1-\pi_0)} \geq \frac{\pi_0+c}{2}.
\end{cases}
\]

Now, if the firm decides to make the consumers search, its profit function will be: \( \Pi_S = \pi_0 + (1 - \pi_0)\mu(1 - v(p))(p - c) \), because only the consumers who expect a positive utility will search (those above \( v(p) = \frac{(\pi_0+(1-\pi_0)\mu)p+c}{\pi_0} \)) and only a fraction of them will obtain a good signal \( \pi_0 + (1 - \pi_0)\mu \). Maximizing
this yields the optimal search price:\footnote{As long as $\pi_0 \geq s + (\pi_0 + (1 - \pi_0)\mu)c$, because otherwise optimal price is less than production cost.}

\[ p_S^* = \frac{\pi_0 - s + (\pi_0 + (1 - \pi_0)\mu)c}{2(\pi_0 + (1 - \pi_0)\mu)}. \]

This means that the equilibrium threshold value for those who search is:

\[ v_S^* = \frac{\pi_0 + s + (\pi_0 + (1 - \pi_0)\mu)c}{2\pi_0}, \]

and those with $v < v_S^*$ will exit. The firm’s maximized search profit is:

\[ \Pi_S^* = \frac{(\pi_0 - s - (\pi_0 + (1 - \pi_0)\mu)c)^2}{4\pi_0}. \]

Finally, this means that consumer $v$ ($v \geq v_S^*$) gets:

\[ V_S^*(v) = \pi_0 v - \frac{\pi_0 + s + (\pi_0 + (1 - \pi_0)\mu)c}{2}, \]

and the consumers as a whole get:

\[ V_S^* = \frac{(\pi_0 - s - (\pi_0 + (1 - \pi_0)\mu)c)^2}{8\pi_0}, \]

which means that the social surplus is:

\[ W_S^* = \frac{3(\pi_0 - s - (\pi_0 + (1 - \pi_0)\mu)c)^2}{8\pi_0}. \]

Whether or not the consumers will be made to search depends on the firm’s profits in the two cases ($\Pi_S^*$ versus $\Pi_B^*$). However, because we have assumed that $\pi_0 \geq c$, $\Pi_B \geq \Pi_S$ always holds when

\[ \frac{s}{(1-\mu)(1-\pi_0)} \geq \frac{\pi_0 + c}{2}, \]

that is, for high values of the search cost (and not too high production costs), the firm prefers selling the good without search. So, if we then assume that

\[ \frac{s}{(1-\mu)(1-\pi_0)} \leq \frac{\pi_0 + c}{2}, \]

this implies $\Pi_S^* > \Pi_B$ if and only if the condition of the Proposition holds (if and only if $s < \bar{s}$, where $\bar{s}$ is a function of $A$, $B$, and $C$). \hfill $\Box$

\section{A Bad News Model}

In this section, I will briefly go over a slightly different model and obtain results similar to the models covered earlier. In particular, the consumer’s utility is shown to be non-monotone in her search cost.
Let the setup be similar to the previous section in that news arrive exponentially. To be more specific, the product can be of high or low quality (yielding utilities 1 and 0, respectively). The consumer has a prior $\pi_0$ on the quality being high, but she can improve this belief by searching for information at a unit cost of $\tilde{s} \geq 0$. If the consumer searches for information, bad news arrive at rate $\lambda > 0$ only when the quality is low.\(^{49}\) If the product is good, nothing is observed.

**Proposition 10.** Given a price $p$, search cost $s \equiv \frac{\tilde{s}}{\lambda}$, and an initial belief $\pi_0$ such that $\pi_0 > \frac{s}{1 - 2\tilde{s}}$, the consumer will search until her belief exceeds $\hat{\pi} \equiv 1 - \frac{s}{p}$. Her initial value of search can be written as:

$$V_S(p) = \pi_0(1 - p) - \left(1 + \pi_0 \frac{p - 2s}{p - s} \ln \left(\frac{p - s}{L_0 s}\right)\right) s.$$  

If $s < \pi_0(1 - \pi_0)$, there exists a threshold $\hat{s}$ such that the firm will make the consumer buy the product for $s \geq \hat{s}$, giving her weakly positive utility, and the firm will make her search for information for $s < \hat{s}$, giving her no utility. The maximal utility is obtained at $s = \hat{s} > 0$.

**Proof.** Assuming that the probability the consumer assigns on the product being of high quality at time $t \geq 0$ is $\pi_t \equiv \mathbb{P}(H)$, the belief can be shown to evolve as follows: $\pi_t + d\pi_t = \frac{\pi_t}{\pi_t + (1 - \pi_t)(1 - \lambda dt)}$. Taking the limit $(dt \to 0)$, one obtains:

$$\dot{\pi}_t \equiv d\pi_t/dt = \lambda \pi_t(1 - \pi_t)$$

$$\Rightarrow \pi_t = \frac{L_0 e^{\lambda t}}{1 + L_0 e^{\lambda t}},$$

where $L_0 = \frac{\pi_0}{1 - \pi_0}$ is the (given) prior likelihood ratio. Note that, in a time interval of length $dt$, the searching consumer will obtain a bad signal and her belief will fall all the way down to 0 with probability $(1 - \pi_t)\lambda dt$.

This allows us to determine the threshold belief, $\hat{\pi}$, such that for beliefs higher than the threshold, the consumer prefers to buy the good (instead of searching for more information). This threshold is obtained simply by equating the expected utility from buying now to the expected utility from searching for a "small amount of time" $(dt)$ and then buying:

$$\hat{\pi} - p = -\hat{s}dt + (1 - \hat{\pi})\lambda dt \cdot 0 + (1 - (1 - \hat{\pi})\lambda dt)(\hat{\pi} + d\hat{\pi} - p)$$

$$\Leftrightarrow \hat{\pi} = 1 - \frac{s}{p},$$

where $s \equiv \tilde{s}/\lambda$ is the "normalized search cost". This buying threshold works as long as $\hat{\pi} > p$, which requires that $p \in (\frac{1}{2} - \sqrt{\frac{1}{4} - s}, \frac{1}{2} + \sqrt{\frac{1}{4} - s})$ and $s \leq \frac{1}{2}$. Otherwise, exiting will yield a higher utility. Setting $\pi_0 = \hat{\pi}$ and

\(^{49}\)Here it is informative to use not only the expected search cost $s = \tilde{s}/\lambda$, but also the arrival rate, $\lambda$. 

solving for the price, gives us $p_0 = \frac{s}{1 - \pi_0}$. It is easy to see that for prices $p \leq p_0$, the purchase threshold is lower than the consumer’s prior, implying that the consumer should purchase instantaneously (assuming that $\pi_0 > p_0$).

If we let $\hat{t}(\hat{\pi}) = \min\{t \mid \pi_t \geq \hat{\pi}, t \geq 0\}$, this will be the first time the consumer’s belief crosses the threshold (it is unique, and exists for all $\hat{\pi} \in (0,1)$). Using (8), we can, in fact, write $\hat{t}(\hat{\pi}) = \frac{1}{\lambda} \ln(\frac{p-s}{L_0 s})$, or $e^{\hat{t}(\hat{\pi})} = \frac{p-s}{L_0 s}$.

Thus, the time spent searching is increasing in the price and decreasing in the search cost (holding the price constant).

We are now ready to compute the consumer’s value function at $\pi_0$, knowing that she will buy at the prior if $p \leq \min\{p_0, \pi_0\} = \min\{\frac{s}{1-\pi_0}, \pi_0\}$, which can be written as:

$$p \leq \begin{cases} \frac{s}{1-\pi_0}, & \text{if } s \leq \pi_0(1-\pi_0) \\ \pi_0, & \text{else.} \end{cases} \quad (10)$$

To compute the value of searching until belief hits $\hat{\pi}$ or until there is a bad signal, one needs to compute the costs and benefits of search. To this end, expected costs can be written as:

$$E[\text{costs}] = \left(\pi_0 \hat{t} + (1-\pi_0) \int_0^{\hat{t}} t e^{-\lambda t} dt\right) \hat{s} \quad \text{integrate by parts}$$

$$= \left(\pi_0 \hat{t} + (1-\pi_0) - (1-\pi_0)(1+\hat{t})e^{-\hat{t}}\right) s \quad \text{or } e^{\hat{t}} = \frac{p-s}{L_0 s}$$

$$= \left(1 + \pi_0 \frac{p-2s}{p-s} \ln(\frac{p-s}{L_0 s}) - \pi_0 \frac{p}{p-s}\right) s,$$

and expected benefit as:

$$E[\text{benefit}] = \mathbb{P}(\text{purchase})(\hat{\pi} - p) \quad \hat{\pi} = 1 - s/p$$

$$= \left(\pi_0 + (1-\pi_0)e^{-\hat{t}}\right) \left(1 - \frac{s}{p} - p\right)$$

$$= \pi_0 \left(1 - \frac{p^2}{p-s}\right).$$

Combining the benefits and the costs, and simplifying, we can write the consumer’s search value as:

$$V_S(p) = \pi_0(1-p) - \left(1 + \pi_0 \frac{p-2s}{p-s} \ln(\frac{p-s}{L_0 s})\right) s, \quad \text{for } p \geq p_0 = \frac{s}{1-\pi_0} \quad (11)$$

Remember that the above assumes that the price is such that the consumer will find it beneficial to start
searching (and keep doing so until her belief hits $\hat{\pi}$ or 0).\footnote{Note that if the consumer decides to search until $\hat{\pi}$ at $\pi_0$, then she will choose to continue searching until $\hat{\pi}$ at every $\pi \in (\pi_0, \hat{\pi})$, as long as she obtains no negative signals.}

The firm will then choose its price, knowing that the consumer will search until her belief hits the threshold, $\hat{\pi}$. If the firm does not want the consumer to search at all, it can set its price at $p = p_0 \equiv \frac{s}{1 - \pi_0}$ (assuming this is less than the prior belief). If the firm makes the consumer search, note first that the value of searching until $\hat{\pi} > \pi_0$ is strictly decreasing in $p$, and that it is positive at $p = 2s$ under the condition given in the proposition ($\pi_0 > \frac{s}{1 - 2s}$). This implies that there exists a price, $p^*$, such that $V_S(p^*) = 0$. Let us then find the firm’s optimal policy:

$$
\max_p \Pi(p) = \max_p \left( \pi_0 + (1 - \pi_0)e^{-\lambda t}(p - c) \right) = \max_p \pi_0 \frac{p(p - c)}{p - s} \quad (12)
$$

$$
\Rightarrow \Pi'(p) = \pi_0 \frac{p(p - 2s) + cs}{(p - s)^2} > 0 \quad \forall p > 2s. \quad (13)
$$

This means that if the firm makes the consumer search, it will always set $p = p^*$ because its profit function is strictly increasing in the price. Thus, the firm needs to compare two prices: $p_0$ and $p^*$. Purchase price, $p_0$, will be optimal if and only if:

$$
\frac{s}{1 - \pi_0} - c \geq \pi_0 \frac{p^*(p^* - c)}{p^* - s}. \quad (14)
$$

Otherwise the search price, $p^*$, will maximize profits. One can show that $p^*$ is decreasing in $s$ (essentially because $V_S(p)$ is decreasing in both $p$ and $s$, meaning that $p^*$ has to decrease if $s$ increases to keep $V_S(p^*) = 0$). Furthermore, the firm’s search profits $\Pi(p^*)$ are decreasing in $s$ because a higher $s$ forces the firm to charge a lower price even though it would like to set $p = 1$.\footnote{Technically, $p = 1$ never leads to a sale because the consumer’s posterior never reaches 1, but we can assume that $p = 1$ corresponds to $p = 1 - \epsilon$ for small $\epsilon > 0$.} Therefore, there exists a value of the search cost, say $\hat{s}$, such that for $s \geq \hat{s}$, the left-hand side of (14) is at least as high as the right-hand side (because LHS is increasing in $s$, and RHS is decreasing in $s$). However, it can be that $\hat{s} \geq \pi_0(1 - \pi_0)$ which leads the firm to set $p_0$, giving the consumer no utility.

The consumer obtains her maximal, strictly positive utility at $s = \hat{s}$ whenever $\hat{s} < \pi_0(1 - \pi_0)$. Her utility is zero for $s < \hat{s}$ and positive but decreasing for $s \in [\hat{s}, \pi_0(1 - \pi_0))$. We can write this out:

$$
V^* = \begin{cases} 
0, & \text{if } s < \hat{s} \\
\pi_0 - \frac{s}{1 - \pi_0}, & \text{if } s \geq \hat{s}.
\end{cases} \quad (15)
$$

This concludes the proof.
**Corollary C.1.** Social welfare is a non-monotone function of the search cost. It is maximized at \( s = 0 \) but it is not monotonically decreasing because it jumps up at \( s = \hat{s} \).

*Proof.* The proof of the corollary is simple. Social welfare is maximized at \( s = 0 \) because then the firm will make the consumer search for so long that we can be absolutely sure that the product is of high quality, giving expected welfare \( S = \pi_0(1 - c) \). Welfare is decreasing in \( s \) for \( s < \hat{s} \) because we cannot make absolutely sure that the product is of high quality and we also incur a positive search cost. However, at \( s = \hat{s} \), the firm is indifferent between making the consumer search and not but the consumer experiences a discrete jump in her utility, implying that the social welfare jumps up. \( \Box \)