Emerging Market Crises:  
An Asset Markets Perspective

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Abstract

The whole difference between a mild downturn and a deep crisis is the occurrence of sharp fire sales of domestic assets and possibly exchange rates, and the ensuing collapse in the balance sheets of both the financial and non-financial sector. Why and how do such crises materialize? And, most significantly, why doesn’t the private sector take appropriate precautions to avoid the costly consequences of crises? In this paper we argue that a poor contractual environment offers a parsimonious account of these and related phenomena present in emerging markets. We build a stylized model where fire sales follow from the weakening of a country’s international collateral. The lack of precautions, on the other hand, is a consequence of weak domestic collateral, or underdeveloped domestic financial markets, which drive a wedge between the private and social valuation of assets appealing to foreign or uninformed investors. Thus, countries with underdveeled domestic financial markets experience costlier and more frequent crises.

Keywords: Capital flows, fire sales, financial constraints, contractual and corporate governance problems, balance sheets, wasted collateral, domestic and foreign spreads, excessive leverage, amplification, multiple equilibria, currency depreciation, banks.

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1 Introduction

In this paper we provide a model of crises that centers on imperfections in asset markets. We are motivated in part by the recent experience of East Asian countries. By most accounts, the reluctance of international investors to provide resources in order to smooth the impact of a combination of internal and external deficiencies, worsened the situation. As internal and external demand fell, external credit—and capital flows in general—vanished, bringing the real sector, especially nontradeables, to a virtual halt.\footnote{For the first round of crisis-countries—Indonesia, Korea, Malaysia, the Philippines and Thailand—net capital flows declined from 73 billion dollars in 1996 to minus 11 billions in 1997. The actual decline in net inflows was probably higher since “errors and omissions” went from -8.5 billions in 1996 to -19.5 billions in 1997.} As symptoms and consequences, asset deflation and currency depreciations took unexpected proportions, leading many observers to assert that [the declines were] “well beyond what was justified by any reasonable reassessment of economic fundamentals, even in the light of the crisis.” (IMF World Economic Outlook, May 1998, p.4)

The first step we take in this paper is to construct a model to account for these features of real and financial variables during a crisis.\footnote{The literatures on debt crises, currency speculation, and bank-runs, offer important insights on different aspects of assets markets during crises. Most of the effects in these literatures are either amplified or made more likely by the issues we emphasize in this paper. For papers on currency speculation see, e.g., Obstfeld (1994, 1996); on debt crises see, e.g., Calvo (1988), Giavazzi and Pagano (1990), Cole and Kelho (1996); on bank runs see the canonical Diamond and Dybvig (1983) and the applications to open economies by Goldfajn and Valdes (1998) and Chang and Velasco (1998a). Also see Calvo (1998) for an overview and interesting discussion on implications and sources of capital market crises in small open economies.} We build a model in which a country is limited in its external financing by its supply of international collateral. A tightening of this financial constraint can lead to fire sales—i.e., falls in asset prices below fundamentals—and contractions in real activity. An important and realistic aspect of the model is that the response of the economy can be very non-linear and depends on the condition of the aggregate balance sheet of the country.

Agents in our model rationally anticipate crises, and shift resources from good times to these downturns. The central question that this paper asks is why doesn’t the private sector do enough to prevent or dynamically ”arbitrage” them? Analogous to the constraint with respect to external financiers, we introduce a constraint into the domestic financial market whereby financing from another domestic is limited by a firm’s domestic collateral. We show that the private sector in economies with underdeveloped domestic financial markets—represented in our model as a scarcity of domestic collateral—systematically underestimate the social value of their access to international capital markets. As a result, these economies arrive to downturns unprepared, and experience more costly crises. This cross-sectional prediction seems borne out both in East Asia and preceding emerging market crises.
The result arises from the observation that financial constraints create a wedge between internal and external returns. Insufficient domestic collateral creates a spread between the internal rate of return on a good project in the economy and the rate of return that can be credibly promised to outside domestic financiers. This *domestic spread* is just the dual of an inefficient domestic capital allocation process – or poor collateral aggregation in our language. In a dynamic setting, the domestic spread leads to an undervaluation of international collateral that exacerbates crises. The reasoning is straightforward. Since in a crisis there is a shortage of international collateral, abating it requires an ex-ante decision to carry an extra unit of international collateral into the crisis. However, doing so provides the holder less than the full social return on this international collateral – in fact, the shortfall is proportional to the domestic spread. Anticipation of this reduces the private sector’s incentive to accumulate and provision international collateral. The fragility resulting from such under-accumulation, or over-leverage, makes crises more costly.\(^3\) We show that this inefficiency is due to a pecuniary externality. Thus, it is the inefficiency in domestic financial markets, rather than the weakness in international financial links alone, that may justify capital controls and equivalent measures.\(^4\)

In much of our analysis we represent the domestic financial market by a corporate debt market. However, in emerging markets, liquid domestic debt markets are the exception rather than the rule. Invariably, the financial system is bank based. Banks issue claims to foreign investors against balance sheets that are partially backed by international collateral, and lend domestically to those that do not have direct access to international market. This brings a third balance sheet — that of the banking system— into the story, and with it further amplification. As the country faces an international crisis and asset prices collapse, the deterioration in banks’ balance sheets limits their ability to intermediate foreign funds into the domestic system, further deepening the crisis, and so on. If this feedback is strong enough, the size of the crisis depends on the extent of pessimism, as the model exhibits multiple equilibria.

The framework we present gives central roles to credit market constraints arising from

\(^3\)Alternatively, if domestic financial markets are extremely primitive, volatility costs due to external shocks may be replaced by depressed growth and over-exposure to domestic shocks as the economy is unable to aggregate internal resources and use its international collateral. Similarly, if international collateral is very limited to start with, international leverage disappears and with it so does volatility with respect to external shocks. We do not emphasize these cases as our purpose is to emphasize crises in open economies rather than stagnation and traditional domestic business cycles. Also, see Aghion et al (1998), for a model where financial fragility arises from the mechanics of debt accumulation rather than from the undervaluation of collateral provision. Interestingly, they also find that growth volatility is maximized for intermediate levels of financial development.

\(^4\)In Caballero and Krishnamurthy (2000) we argue that this simple observation also sheds light on the choice of exchange rate systems and the conduct of monetary policy in emerging economies.
microeconomic contractual problems in the process of creation and aggregation of international collateral. There are, in principle, many factors behind the incomplete development of both domestic and international financial links in emerging economies, ranging from macroeconomic and political instability to weak corporate governance and contractual environment. Our model emphasizes the latter factors both because they are less well understood within the context of sovereign risk and because their importance has been heightened by the recent Asian crisis.

Relation to the literature. The sovereign debt literature links the aggregate borrowing capacity of a country to its international collateral – represented as the present value of some fraction of exports or exports minus imports (see Eaton and Gersovitz (1981), Bulow and Rogoff (1989)). This is closely related to our identification of international collateral. Since in this literature default and its costs are mostly modeled as a country-wide phenomenon, the question immediately arises on whether the domestic private sector internalizes the effect of their actions on the likelihood of these events. Bardhan (1967), Harberger (1985) and Aizenman (1989) advocated taxing capital flows on the grounds that individual agents do not pay for the increase in a country’s average cost of capital brought about by their international indebtedness. While our results on capital flows and collateral underprovision are related to theirs, our externality stems from imperfections in the domestic financial system rather than from the country’s monopsony power in international financial markets or any sovereign constraint.

We arrive at this result by taking a more microeconomic approach in which collateral is created and aggregated at the individual level. On these aspects, we are more closely related to the new literature on the role of collateral in macroeconomics. Kiyotaki and Moore (1997) place the collateral role of a fixed input —land— and its distribution across agents at the center of their amplification mechanism. Krishnamurthy (1998) shows how a richer set of contracts than that allowed by Kiyotaki and Moore shifts the relevant constraint from that which applies to individual agents to the total availability of collateral in the economy. Holmstrom and Tirole (1998a) argue that the stock market alone may provide insufficient collective collateral if not aggregated properly. They also show that even when wasted corporate collateral (liquidity) is not an issue, public supply of it may be required in the presence of large shocks.

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5 According to the IMF (1998, pp. 63-64), a distinguishing characteristic of the Asian crisis is that purely macroeconomic considerations played a lesser role than in previous crises. Also, see Johnson, et al. (1998) for evidence on the correlation between weak corporate governance and institutional environment (i.e. a weak contractual environment) in an economy and the incidence of the current crisis. As the corporate finance literature has demonstrated, weak corporate governance and, more generally, weak contractual enforcement go hand in hand with credit market constraints.

6 See Bernanke et al (1999) for a thorough survey of the literature on credit constraints and macroeconomics.
In terms of our modeling choices, our paper falls into the family of model's inspired by Diamond and Dybvig (1983). We are closest to Holmstrom and Tirole’s departure from this model in the context of firms. As in that paper, firms in our economy receive a production shock that they must cope with by mortgaging collateral in order to borrow additional funds. The latter can be obtained from other domestic firms or directly from international investors. Analogously, firms in the model of Holmstrom-Tirole can borrow funds either from other firms in the corporate sector or directly from savers. Intermediation renders the first transaction frictionless in Holmstrom-Tirole and their macroeconomic analysis focuses on the implementation and efficiency of the second transaction. In contrast, both transactions encounter frictions in our model. Indeed, and more generally, while there are many differences in terms of substance and modeling aspects with each of the articles on collateral and macroeconomics mentioned above, our central theoretical departure with them as a group is our focus on the presence of two collateral constraints — a domestic and an international one— and their rich static as well as dynamic interactions.

Our underprovision of collateral results are analogous to the free rider problems and underprovision of liquidity studied in the banking literature (see Jacklin (1987), Bhattacharya and Gale (1987), Allen and Gale (1997)). However, the source of the externality behind this inefficiency in our model is the endogenous result of imperfect contract laws and institutions, while theirs is the exogenous divergent ex-post valuation of liquidity by consumers. The literature on crises has focused on models of bank runs and their key sequential service constraint (Diamond and Dybvig (1983), Goldfajn and Valdes (1998) and Chang and Velasco (1998a)). While we do believe that bank failures and runs can aggravate crises, as we discuss in section 4, our model points out that the basic features of crises and their prelude arise even when no agent or institution is committed to supply liquidity. Moreover, the fire sale mechanism we emphasize boosts and feeds from the likelihood of banking crises themselves. Finally our asset pricing implications arise from market segmentation. Merton (1987), Allen and Gale (1994) and Shleifer and Vishny (1993) have noted the impact of segmentation on asset prices.

Outline. Section 2 follows with a description of the model and discusses the aggregate implications of weak connections with international financial markets. Section 3 highlights the implicit transactions in collateral aggregation, and introduces frictions in domestic financial markets. Banks as collateral aggregators and as a source of amplification and multiple equilibria are discussed in section 4. Section 5 concludes, and is followed by an appendix.
2 A Time-to-build model with limited international collateral

In our view, there are two central ingredients behind the recurrent crises affecting emerging economies: weak international financial links and underdeveloped domestic financial markets. We start with a discussion of the first ingredient in isolation, while the next section brings into the analysis frictions in domestic financial markets.

2.1 The Economy

Our goal in this section is to highlight the consequences of suffering insufficient (perceived) international collateral during crises, as well as to understand motives for undertaking precautionary measures in anticipation of more stringent international financial conditions.

*Time.* The world lasts three periods. Time is indexed as \( t = 0, 1, 2 \). Date 0 is the fully flexible period when economic agents design the productive structure, ownership structure, and portfolio allocations. Date 1 is the crisis period, when economic agents struggle to shift resources from the future to cope with shocks in the present. Date 2 represents the unconstrained future, when the economy is (relatively) rich in resources.

*Agents.* Heterogeneity among agents is required to understand the consequences of financial frictions. We have two dimensions of heterogeneity: one between domestic entrepreneurs-consumers and foreign financiers, and another one among domestic entrepreneurs.

The domestic economy is populated by a continuum of unit measure of agents with Cobb-Douglas preferences over date 2 consumption,

\[
  u^d = (c_{t, c_{nt}})^{1/2},
\]

where, \( c_{t} \) is consumption of tradeable (T) goods at date 2 and \( c_{nt} \) is consumption of non-tradeables (N) at date 2. *We denote the date 2 real exchange rate of T-goods in units of N-goods as \( e \).* By specifying preferences over date 2 consumption only, we remove from our analysis standard intertemporal consumption substitution arguments which are not central to our approach.

The rest of the world is represented by foreign investors who have preferences over consumption of tradeable goods at all dates,

\[
  u^f = c_{t,0} + c_{t,1} + c_{t,2}.
\]

The contrast with domestic agents arises because foreign investors may be called on to supply tradeable goods in the date 1 crisis period as well as during the date 0 pre-crisis period. They have large endowments of T-goods at all dates. Additionally, we assume
that they do not discount the future so that the preference structure pins down the gross international interest rate at one.

**Endowments and production.** At date 0, ex-ante homogenous domestic agents are endowed with resources of \( u_0 \) T-goods which can be invested or stored. They can increase these resources by taking on foreign debt of \( d_{0,t} \) - we shall expand on this shortly. Production has a time-to-build aspect. Investments are made at dates 0 and 1, and output is realized at date 2. Let \( k_n \) and \( k_t \) denote the total amount of capital devoted to sectors N and T at the beginning of date 1, inherited from date 0. Then creating capital of \( k_n \) and \( k_t \) requires a date 0 investment totaling \( k_n + k_t \) units of tradeable goods. At date 1, a fraction \( 1 - \rho \) of the firms in the N-sector is spared of further investment and go on to produce \( R_n k_n \) units of N-goods at date 2. The rest experience a productivity fall, \( \Delta_n \equiv R_n - r_n > 0 \), which can be offset by reinvesting a fraction \( \theta_n \leq 1 \) of \( k_n \), in units of the tradeable good, in order to realize output at date 2 of:

\[
\frac{R_n(\theta_n)k_n}{(r_n + \Delta_n \theta_n)k_n} = \frac{k_n}{R_n k_n}
\]

In order to make saving distressed production units sufficiently profitable, we assume that,

\[
\Delta_n \gg R_n(1 - \rho)
\]

Production per unit of capital in the T-sector is simply \( R_t \).\(^7\) The time-line is as follows:

<table>
<thead>
<tr>
<th>Date 0</th>
<th>Date 1</th>
<th>Date 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Borrowing</td>
<td>Crisis</td>
<td>Repayment</td>
</tr>
<tr>
<td>Investment</td>
<td></td>
<td>Consumption</td>
</tr>
</tbody>
</table>

Date 0 Budget Constraint: \( d_{0,t} + w_0 = k_n + k_t \)

T-Production: \( k_t \rightarrow R_t k_t \)

N- Production: Time-to-build

\( k_n \rightarrow (\text{INTACT}) \rightarrow R_n k_n \)

\( \text{Prob = } \rho \) (DISTRESSED)

Need additional investment of

\[
\theta_n k_n \text{ T-goods} = \frac{r_n k_n + \Delta_n \theta_n k_n}{R_n(\theta_n) k_n}
\]

Figure 1: Time Line

\(^7\)Because having shocks in both N and T-sectors does not result in substantial additional insights, we restrict these shocks to the N-sectors. Nonetheless, there are a few interesting peripheral issues that arise once the T-sector is subject to idiosyncratic shocks. For example, a Kiyotaki and Moore (1997) type multiplier may arise.
This time-to-build structure underlines a critical link between financing and production during a crisis. Firms in any economy have ongoing capital needs (working capital, etc.). Starving firms of capital has the effect of shutting down production units in a potentially wasteful manner — to anticipate results, this is our crisis. The dual of this shutting down of production units is that the marginal value of capital during a crisis is very high. This implication of our modeling underlies many of the key results in the paper.

The domestic economy will have no tradable goods at date 1 to contribute and capital needs can only be accommodated by an increase in imports. In order to finance this increase, domestics must access international capital markets.

*Collateral Constraints.* Foreign investors extend finance only with the security of collateral. Our assumptions on collateral are two-fold. First, a firm in the tradeable sector with capital of $k_t$ expects output at date 2 of $R_t k_t$. We assume that only a fraction $\lambda_t$ of this output can be valued and seized by foreigners in default.\(^8\) Second, we assume that foreigners can only value and seize output of the T-sector, while the N-sector provides no collateral to foreigners. Thus, the quantity $\lambda_t R_t k_t$ constitutes the *international collateral* of the economy.

Almost by definition, an *emerging* economy is one in which project returns are high, however the ability to access international capital is low. These features are captured by assuming that,

$$R_t > 1 \quad \text{but} \quad \lambda_t R_t < 1. \quad (1)$$

While the capital market has weak international links, we assume until the next section that the domestic capital market is frictionless. Residents accept as collateral the full value of output in both N and T-sectors. Thus, for example, a firm in the N-sector with capital of $k_n$ (at date 1) expects output at date 2 of $R_n k_n$. Then, $\lambda_n R_n k_n$ of this output constitutes *domestic collateral*, with $\lambda_n = 1$.

The asymmetric collateral constraint identifies the domestic economy as distinct from the rest of the world. Much of the international economics literature assumes an asymmetric valuation of goods — foreigners only value T-goods, while domestics value both T and N-goods. In contrast, the collateral restriction is that domestic residents extend finance to either tradeable or nontradeable firms, while foreigners accept as collateral only claims on the tradeable sector.\(^9\) It is important to point out that this assumption does not correspond

\(^8\)We appeal to agency conflicts in making this “high-level” assumption. See the appendix for a deeper justification of the assumption based on costly state verification. It is costly for a foreign investor to verify and liquidate the output of $R_t k_t$. This implies that only a fraction of the output can be used as collateral. Also, see La Porta et al. (1998) for worldwide evidence on the high positive correlation between corporate governance problems and ownership concentration (in their interpretation, higher concentration is needed to prevent managers from stealing).

\(^9\)This assumption can be justified from first principles by taking the cost of verifying and liquidating output in the N sector to be infinite for foreigners, while it is finite for domestics.
to the empirical restriction that we should only observe foreign credit extended to firms in the T-sector. In our model, foreigners are willing to hold claims on the N-sector but only as long as their market is “liquid,” in the sense that there are enough claims in the T-sector to allow an instantaneous trade. The restriction implied by the assumption is that at date 1, facing the prospect of having to liquidate N firms, they swap these for claims on the T-sector with domestics who do value N-claims.

The international economics literature has long recognized the importance of international collateral and its relation with a country’s tradeable sector.\(^{10}\) Formal models of sovereign debt renegotiation are built around the question of what international lenders can threaten sovereign countries with in the event of a default (see Eaton and Gersovitz (1981), Bulow and Rogoff (1989)). In this literature, international collateral is typically taken to be some fraction of exports.

While similar in spirit to this identification, our assumption is microeconomic rather than aggregate. Liquidation and seizure of output in the non-tradeable sector is harder for foreigners than it is for domestics, while in the tradeable sector this disadvantage disappears. For instance, it is justified by the fact that cash revenues from exports can be seized before they make it back into the country. This feature was used by Mexico during the 94-95 crisis, when its oil revenues were made part of the collateral backing the liquidity package that it received. In many instances the bias is directly justified by mandates on foreign institutional investors (e.g. limits on real estate investments).\(^{11,12}\)

Alternatively, this assumption is an efficient mechanism to capture the central implications of an expanded model where foreigners’ collateral-bias is only against a fraction of firms—in either sector—and current production of perishable non-tradeable goods (see an earlier draft, Caballero and Krishnamurthy (1998)).\(^{13}\)

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\(^{10}\)See, e.g., Simonsen (1985).

\(^{11}\)Of course there are important exceptions (e.g. “too big to fail” utilities), but see, e.g., Kang and Stulz (1997) for systematic Japanese evidence showing that, for small firms, those that are export oriented are favored by foreign investors. See, e.g. Blommestein (1997), for a discussion of how real estate and other assets considered highly illiquid or exposed to exchange risk are generally avoided (sometimes by mandate) by foreign institutional investors. Interestingly, the very few exceptions to the sovereignty principle, by which the rating on debt issued by a country’s corporate sector is bounded from above by that country’s government debt rating, are for companies which belong to the export sector.

\(^{12}\)The September 1998 report on the Asian crisis by the World Bank, describes firms that borrow in foreign currency as “predominantly large exporting firms with ties to foreign companies, and they have better adjusted to the crisis...” (box 4.3, page 62). The 1997 Industrial Survey in Thailand reflected that of those firms that borrow in foreign currency, 88 percent export, have an average of 818 employees, a debt-equity ratio of 3.12, a relatively high capacity utilization and optimism during the crisis, 70 and 37 percent respectively. The same statistics for firms that do not borrow in foreign currency (75 percent of firms) are: 46 percent, 139 employees, 2.36, 61 percent and 19 percent. See Dollar and Hallward-Dreimeier (1998).

\(^{13}\)There is an extensive literature documenting “home bias” in asset holdings (see, e.g., French and Poterba 1991), matched by asymmetric information explanations (see e.g. Zhou 1998). There are also institutional
Aggregate and idiosyncratic shocks. Shocks affect the economy at date 1 and take two forms. First, as we described above, there is an idiosyncratic shock that demands reinvestment in a fraction $\rho$ of the firms in the $N$ sector. The idiosyncratic shock reflects “normal times.” In the churn of any economy, there are some production units which require more resources than others. Thus, there is a constant need to attract and reallocate resources from some production units to others.

Second, we introduce an external shock at date 1 that affects, in the aggregate, the economy’s ability to attract resources from foreign investors. $\lambda_t R_t k_t$ represents international collateral with respect to foreign investors. There are two states of the world at date 1, $\omega \in \{B, G\}$, differentiated by a low amount of international collateral and a high amount of international collateral. That is,

$$\lambda_t^B R_t^B k_t < \lambda_t^G R_t^G k_t$$

Shocks to international collateral can, in principle, arise either as a shock to $R_t$ or a shock to $\lambda_t$. Terms of trade shocks are declines in $R_t$, while a shortage in the supply of international capital is a decline in $\lambda_t$. To simplify the exposition, we focus only on shocks to $\lambda_t$. In fact, any shock to international collateral is likely to be a combination of these two shocks.

Financial structure. Firms can raise finance at date 0 and date 1 from either domestic or foreign investors. We assume that all finance must be default free and fully collateralized – either by domestic collateral in the case of domestics, or international collateral in the case of foreign investors. Without loss of generality, this allows us to focus on default free debt contracts. Since all firms are homogeneous at date 0 and since debt contracts are default free, we can also restrict attention to date 0 borrowing from foreign investors. Any date 0 borrowing between domestics will involve no net transfers at either date 0 or at a later date and can therefore be ignored.\(^{15}\)

2.2 Balance Sheets and Financial Contracts

Balance sheets. Each entrepreneur has a firm in each sector. While he can issue claims separately on each firm, the entrepreneur consolidates their financial resources in case of distress. We start our characterization of equilibrium by studying the balance sheets of firms at date 1, and by highlighting their dependence on asset prices and exchange rates. Date 0 decisions result in firms arriving at date 1 with installed capital of $k_n$ and $k_t$ and

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\(^{14}\) A “collateral squeeze” versus a “credit crunch.”

\(^{15}\) This assumption is relaxed in section 4 when we introduce domestic banks and therefore lose homogeneity among domestic agents.
foreign debt of \( d_{0,f} \). At date 1, a firm that receives a shock is distressed (S), while a firm that escapes the shock is intact (I). The balance sheet of an intact firm has assets of \( k_t \) and \( k_n \) and foreign debt of \( d_{0,f} \). However, not all of these assets have international collateral value. On one hand, incentive constraints determine that only a fraction \( \lambda_t \) of the shares of \( k_t \) are internationally pledgeable. On the other, shares of \( k_n \) possibly may be converted domestically into international collateral at a discount. Let \( L^\omega \) be the market gross interest rate applied to a -collateralized- claim on one unit of N-goods at date 2, in state of the world \( \omega \). Date 2 output of \( \frac{R_t k_t}{\omega} \) can be exchanged at date 1 for \( \frac{R_n k_n}{\omega} \) units of date 1 T-goods, or equivalently, claims on date 2 T-goods. Then, the international collateral value of the assets of this firm is,

\[
\lambda_t^\omega R_t k_t + \frac{R_n k_n}{L^\omega}. \tag{2}
\]

The debt of this firm with respect to foreigners is \( d_{0,f} \). Hence we can define the equity value of the intact firm as the difference between its assets and liabilities, in terms of T-goods, as

\[
d_t^\omega = \lambda_t^\omega R_t k_t + \frac{R_n k_n}{L^\omega} - d_{0,f}. \tag{3}
\]

The condition of balance sheets play a more central role for distressed firms, since it determines their debt capacity (and from now we use these terms interchangeably). A distressed firm suffers a production shock that requires it to reinvest resources of \( k_n \) T-goods in order to attain \( R_n k_n \) units of N-goods at date 2. Investing \( \theta_n k_n \leq k_n \) means that the firm realizes a lower output of \( \tilde{R}_n(\theta_n) k_n \). The debt capacity of such a firm is:

\[
d_s^\omega = \lambda_t^\omega R_t k_t + \frac{\tilde{R}_n(\theta_n) k_n}{L^\omega} - d_{0,f}. \tag{3}
\]

This expression hints at the sensitivity of balance sheets to asset prices. High interest rates or depreciated exchange rates decrease the value of N collateral and therefore compromise balance sheets. As we show below, this translates directly into constrained reinvestment for the distressed firm at date 1.

**Debt contracts.** Lenders at date 0 do not know if a firm will be distressed or intact. Moreover, the realization of the idiosyncratic shock at date 1 is non-verifiable. Thus lenders are unable to write debt contracts contingent on being distressed or intact. Purely for expositional purposes, we make a simplifying assumption that debt contracts are also not written contingent on the aggregate shock — in fact, most of our results, as we show in the appendix, are preserved even when contracts are fully contingent. Lenders impose the no default constraint that,

\[
d_{0,f} \leq \min_{\omega} \{ \lambda_t^\omega R_t k_t + \frac{R_n k_n}{L^\omega} \}
\]

This ensures that firms always have sufficient resources to repay debts at date 1 even if no reinvestment takes place. Debt levels above this limit introduce considerations of debt
overhang and bankruptcy costs which, while relevant and interesting, are not central to our main concerns.  

The foreign debt constraint requires that (worst-outcome) marketable assets be greater than liabilities. It is important to note that these marketable assets includes both T- and N-assets. This is the sense in which the collateral assumption that we have made does not imply that foreign credit will only be extended to T-firms.

2.3 Microeconomic Decisions and Equilibrium

Date 1 problem. Consider the problem of a distressed firm in raising funds to alleviate its production shock. A choice of \( \theta_n^\omega k_n \) will result in output at date 2 of \( R_t k_t \) T-goods and \( \tilde{R}_n(\theta_n^\omega)k_n \) N-goods. Since \( e^\omega \) is the real exchange rate at date 2, the value of this output in T-goods is,

\[
R_t k_t + \frac{\tilde{R}_n(\theta_n^\omega)k_n}{e^\omega}.
\]

In order to retool a fraction \( \theta_n^\omega \) of the distressed unit, the firm must raise finance and reinvest \( \theta_n^\omega k_n \) T-goods. It can do this in two ways. First, the firm may have some international collateral at date 1 that it can use to borrow directly from foreigners. That is the firm can always raise directly,

\[
d_{t,f}^\omega \leq \lambda^\omega_t R_t k_t - d_{0,f}.
\]

The latter quantity is always positive (see below), and represents the minimum that a firm can raise at date 1. The rest must come from intact firms, which also have access to foreign investors since their capacity to borrow abroad at date 1 is also \( \lambda^\omega_t R_t k_t - d_{0,f} \).

A distressed firm can use its N-collateral to access the international collateral of the intact firms. In equilibrium, the latter discount the N-collateral at a rate of \( L^\omega \geq 1 \) in providing international collateral. Through this “credit chain,” the distressed firm is able to aggregate the international collateral of the economy and pledge this to foreigners to raise resources for date 1 reinvestment. Let \( d_{t,f}^\omega \) and \( d_{t,d}^\omega \) represent the foreign and domestic debt (in units of T-goods) contracted by a distressed firm at date 1. We can then write the problem of a distressed firm as,

\[
(P1) \quad V^\omega_s \equiv \max_{\omega_n^\omega, d_{t,f}^\omega, d_{t,d}^\omega} \quad R_t k_t + \frac{\tilde{R}_n(\theta_n^\omega)k_n}{e^\omega} - d_{0,f} - d_{t,f}^\omega - d_{t,d}^\omega
\]

\[
\text{s.t.} \quad (i) \quad d_{t,f}^\omega = \lambda^\omega_t R_t k_t - d_{0,f} + \frac{\tilde{R}_n(\theta_n^\omega)k_n}{e^\omega} \geq d_{1,f}^\omega + \frac{d_{t,d}^\omega}{L^\omega}
\]

\[
(ii) \quad \theta_n^\omega k_n = d_{t,f}^\omega + \frac{d_{t,d}^\omega}{L^\omega}
\]

\[
(iii) \quad \theta_n^\omega \leq 1.
\]

---

\(^{16}\)This constraint is tighter than it needs to be since the firm in principle could borrow against the pledgeable component of the reinvestment option. Absent renegotiation, this omission is not important since even the tighter constraint is non-binding in equilibrium (see below).
Constraint (i) is a balance sheet constraint (net marketable assets greater than new liabilities), while constraint (ii) reflects that new investment must be fully paid with the resources received by the firm at date 1 in taking on debts of $d_{1,d}^\omega$ and $d_{1,t}^\omega$. Constraint (iii) is purely technological.

An intact firm at date 1 has only one decision: how much finance will it extend to the distressed firm. Suppose that the firm accepts claims at date 1 of $x_{1,d}$ (face value of date 2 T-goods) in return for making a date 1 contribution of $\frac{x_{1,d}}{L^\omega}$, then,

$$ (P2) \quad V_t^\omega \equiv \max_{\omega \in \{G,B\}} \left\{ R_t k_t + \frac{R_{n} k_n}{e^\omega} + x_{1,d}^\omega - \frac{x_{1,d}^\omega}{L^\omega} \right\} \quad \text{s.t.} \quad \lambda_t^\omega R_t k_t - d_{0,f} - \frac{x_{1,d}^\omega}{L^\omega} \geq 0 $$

Date 0 problem. At date 0, a firm looking forward to date 1 can expect to find itself as either distressed or intact, and in either the good or the bad aggregate state of the world. Thus the decision at date 0 is,

$$ (P3) \quad \max_{k_0, k_t, d_{0,f}} \quad \sum_{\omega \in \{G,B\}} \pi^\omega \left( \rho V_{s}^\omega + (1 - \rho)V_t^\omega \right) \quad \text{s.t.} \quad d_{0,f} \leq \min_{\omega} \left\{ \lambda_t^\omega R_t k_t + \frac{R_{n} k_n}{e^\omega} \right\} \quad k_0 + k_t = w_0 + d_{0,f}. $$

where $\pi^\omega$ represents the probability that state $\omega$ occurs.

Equilibrium. Market clearing in the domestic debt market at date 1 (capital letters denote aggregate quantities) requires that the aggregate amount of domestic debt taken on by distressed firms is fully funded by intact firms:

$$ D_{1,d}^\omega = \rho d_{1,d}^\omega $$
$$ X_{1,d}^\omega = (1 - \rho)x_{1,d}^\omega. $$

Therefore, market clearing,

$$ D_{1,d}^\omega = X_{1,d}^\omega, \quad (4) $$

determines the gross discount rate $L^\omega$.

The real exchange rate at date 2, $e^\omega$, is determined by goods-market clearing at date 2. Given the Cobb-Douglas preference structure, the exchange rate only depends on aggregate consumption. The first order condition for a representative consumer yields,

$$ e^\omega = \frac{C_n^\omega}{C_t^\omega}, \quad (5) $$

with

$$ C_n^\omega = (R_n + \rho \Delta_n (\theta_n^\omega - 1)) K_n \quad \text{and} \quad C_t^\omega = R_t K_t - \rho \theta_n^\omega K_n - D_{0,f}. $$

An equilibrium of this economy consists of date 0 and date 1 (contingent) decisions, $(k_n, k_t, d_{0,f})$ and $(\theta_n^\omega, d_{1,f}^\omega, d_{1,d}^\omega, d_{1,t}^\omega)^{\omega \in \{G,B\}}$, respectively, and prices $(e^\omega, L^\omega)^{\omega \in \{G,B\}}$. Decisions are solutions to the firms’ problems (P1), (P2), and (P3) given prices. At these prices, market clearing conditions, (4) and (5), hold.
2.4 Crises and Fire Sales

With the basic setup behind us, we can now characterize crises. Consider the problem of a distressed firm at date 1, as stated in (P1). Halting the productivity decline of a production unit in the N-sector generates a date 2 return of $\Delta_n$ N-goods, but requires a date 1 investment of one T-good. Since foreign lenders charge a gross interest rate of one on collateralized loans, the firm will choose to pledge all of its international collateral to foreign lenders as long as $\frac{\Delta_n}{\epsilon^\omega} \geq 1$ and $\theta_n^\omega \leq 1$. Our assumption that $\Delta_n \gg R_n(1 - \rho)$ allows us to focus on the case where saving distressed units is always profitable and $\frac{\Delta_n}{\epsilon^\omega} > 1$, so that firms borrow as much as possible from foreigners:

$$\frac{\Delta_n}{\epsilon^\omega} > 1 \quad \Rightarrow \quad d^\omega_{1,f} = \min[k_n, \chi^\omega_t R_t k_t - d_{0,f}]. \quad (6)$$

For financial requirements above this level, the firm has to mortgage its domestic collateral. Since foreigners do not accept this collateral directly, distressed firms must borrow from intact firms in order to access foreign resources. In equilibrium, domestic collateral is discounted at the rate of $1/L^\omega$ so that the firm sets its domestic debt as high as possible as long as $\Delta_n/\epsilon^\omega > L^\omega$ and $\theta_n^\omega \leq 1$. The firm is indifferent in its investment scale and debt choice if the marginal product of investing for the firm is equal to the domestic discount rate:

$$d^\omega_{1,d} \leq L^\omega \max[k_n - d^\omega_{1,f}, 0].$$

However if the marginal product of investing is strictly above the domestic discount rate ($\Delta_n/\epsilon^\omega > L^\omega$), this expression holds with equality. The distressed firms’ demand for domestically held international collateral, $d^\omega_{1,f}$, needs to be paired to intact firms’ supply of it. For this, consider the problem of an intact firm at date 1, as portrayed in (P2). The firm lends to the distressed firm at a gross interest rate of $L^\omega$. It accepts debt claims at date 2 with face value of $x^\omega_{1,d}$ and pays for this with T-goods (which it gets from foreigners) of $\frac{x^\omega_{1,d}}{L^\omega}$. The solution to this problem is straightforward:

$$L^\omega > 1 \quad \Rightarrow \quad x^\omega_{1,d} = (\chi^\omega_t R_t k_t - d_{0,f})L^\omega$$

$$L^\omega = 1 \quad \Rightarrow \quad x^\omega_{1,d} \leq \chi^\omega_t R_t k_t - d_{0,f}.$$

The equilibrium value $L^\omega$, is determined by market clearing condition, (4). Distressed firms issue domestic debt claims totaling $D_{1,d} = \rho d_{1,d}$, while intact firms supply all their international collateral as long as $L^\omega \geq 1$. Figure 2 portrays a simple supply and demand diagram. Supply is flat at $L = 1$ and vertical when all international collateral is used up. Demand, on the other hand, is mildly declining and equal to the marginal product $\Delta_n/\epsilon$ when reinvestment is incomplete, and falls sharply as full reinvestment is financed. The mild — and less interesting — decline is due to the depreciation in the long-run exchange
rate induced by increased reinvestment. The sharp decline, on the other hand, reflects the fact that protecting the capital of financially distressed companies is a very high return project.\footnote{Having a menu of reinvestment opportunities - for instance by having more heterogeneity among firms - would certainly smooth the sharpness of the decline. But the essential point captured by our assumption is that there are distinct times when slack rapidly runs out for a large number of companies.}

\begin{figure}
\centering
\includegraphics[width=0.8\textwidth]{figure2}
\caption{Equilibrium and the International Collateral Constraint}
\end{figure}

The figure illustrates two scenarios. In panel (a),

\[ D_{1,t} \leq (1 - \rho)(\lambda^e_t R_t k_t - D_{0,f}), \]

so there is excess supply of international collateral in the domestic market and hence \( L^\omega = 1 \). Since \( d_{1,t} = \max[k_n - (\lambda^e_t R_t k_t - d_{0,f}), 0] \), we can rewrite this condition as,

\[ \rho k_n < \lambda^e_t R_t k_t - D_{0,f} \Rightarrow L^\omega = 1. \quad (7) \]

Notice that the right hand side of this inequality is the aggregate amount of international collateral in the economy once date 0 foreign debts have been paid. We refer to this constraint as the \textit{international collateral constraint}.

Panel (b) illustrates a crisis of international collateral, which occurs when condition (7) ceases to hold. In this case, there is excess demand for funds at date 1 and the domestic interest rate rises above one. Moreover, as intact firms lend as much as they can to distressed firms, the latter compete away all borrower’s surplus:

\[ \rho k_n \geq \lambda^e_t R_t k_t - D_{0,f} \Rightarrow L^\omega = \frac{\Delta_n}{e^\omega}. \quad (8) \]
Let us define the international spread, $s_{if}^\omega$, as

$$s_{if}^\omega \equiv L^\omega - 1.$$  

Figure 3 illustrates that as the date 1 realization of $\lambda_t$ falls, eventually the relevant constraint shifts from (7) to (8) and the international spread jumps from zero to $\frac{\Delta}{\lambda_t} - 1$. For small aggregate shocks and initial leverage, the economy does not suffer at all from declines in $\lambda_t$, while beyond a certain critical level, a contraction in international collateral causes spreads to blow up. While the abrupt jump in spreads is certainly due to the stylized nature of our collateral demand curve, the sharp nonlinearity it illustrates would survive a richer specification. Without exception, the non-linear response of asset prices has occured in all the countries involved in the Tequila, Asian and post-Russian crises.

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Figure 3: Interest Rates and International Collateral

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18 For good realizations of $\lambda_t$ the supply of international collateral is plentiful and borrowers get very attractive terms on financing. This fully protects their returns from re-investment. The situation changes dramatically when there is a shortage of international collateral, as credit is difficult to obtain and all re-investment returns are transferred to domestic liquidity holders.

19 For instance, suppose there was heterogeneity in reinvestment opportunities (i.e. heterogeneous $\Delta_{it}$) such that the demand curve was smoother. In the region where $\lambda_t$ is high and international collateral is plentiful, shifts in $\lambda_t$ will result in no changes in spreads. However, when $\lambda_t$ is low and international collateral is in short supply, shifts will result in much bigger changes in spreads. Funds at date 1 are used to either finance domestic demand or to purchase tradeable goods (or assets) abroad. When there are sufficient funds, it is the foreign interest rates which set domestic interest rates. However, when funds are in short supply all surplus shifts to suppliers and interest rates become very responsive. Our assumption on homogeneous demand, heightens this contrast.

20 Without a surge in nominal interest rates, the fire sale is reflected as a sharp exchange rate depreciation.
As the international spread rises, domestic asset prices collapse. There is a fire sale of domestic assets because the domestic opportunity cost of holding these assets is high when credit is scarce. Foreigners are unable to capture these high returns because at times of crises they only hold and arbitrage claims backed by international collateral. While their arbitrage during normal times keeps the international spread at zero, it is immaterial when the international collateral constraint binds. That is, the interest-parity-condition shifts until domestic equilibrium, rather than international arbitrage, holds.

Fire sales bring about a sharp decline in the balance sheets of distressed firms. In terms of the program (P1), investment at date 1 is no longer determined by technological constraints but by the balance sheet constraint of (i). This shift is the mechanism by which the aggregate shortage of international resources is resolved. When, $\rho K_n < \lambda f R_t K_t - D_{0,f}$, distressed firms — either directly, or through perfect aggregation of the intact firms’ collateral — are able to fund all of their production shock, thus $\theta_n^* = 1$. Conversely, in the fire sales region the rise in $L^\omega$ is simply the mechanism by which limited international collateral is made consistent with full and competitive collateral aggregation, so that

$$\theta_n^* K_n = \frac{1}{\rho} (\lambda f R_t K_t - D_{0,f}) .$$ (9)

Proposition 1 summarizes the main results up to this point:

**Proposition 1 (Regions).** The economy can be in one of two regions at date 1. Region 1 occurs when the international collateral constraint, $\rho K_n \leq \lambda f R_t K_t - D_{0,f}$, does not bind. In this case international spreads are zero and there is full project completion, $\theta_n^* = 1$. Region 2 occurs when the international collateral constraint does bind, in which case international spreads jump to $\frac{\Delta s}{s^0} - 1 > 0$ and distressed projects are downsized: $\theta_n^* K_n = \frac{1}{\rho} (\lambda f R_t K_t - D_{0,f}) < K_n$.

3 Underdeveloped Domestic Financial Markets

In this section we introduce underdeveloped domestic financial markets into the previous setup. We do this to study the date 1 effect of poor domestic collateral on date 0 decisions on international collateral provisioning. That is, we isolate the effect of date 1 demand on the supply decisions at date 0.

Distressed firms use their domestic N-collateral to aggregate the international collateral in the economy. By assuming that all of the output in the N-sector can be collateralized (domestically), we have rendered this transaction frictionless. However an important characteristic of an emerging economy is that it does not allocate capital efficiently within the economy. Weak corporate governance or a weak domestic banking sector are as likely to
hinder domestic collateral aggregation as they are to limit the economy’s access to foreign capital.

The main result of the previous section was to show how a shock to the international supply of funds can result in a sharp rise in interest rates and a real contraction. The rise in domestic interest rates was due to the fact that the marginal product of capital was very high during a crisis. In this section, we study the date 0 response to this anticipation. When there is insufficient domestic collateral, a firm in need of funds is not able to credibly commit the surplus from its high marginal product investment to a supplier of funds. A supplier of funds (intact firms) will purchase a domestic loan that is insufficiently collateralized. As a result, the supplier of funds undervalues the provision of funds to the distressed firms. We show that collateral constraints into the domestic capital market not only amplify and make more costly the scenarios described above, but also create pecuniary externalities which make crises more severe and likely than is socially optimal.

### 3.1 Basic Setup and Domestic Spreads

The environment described in the previous sections needs to be modified only slightly to accommodate weak domestic links. A firm in the N-sector with output of \( R_n k_n \) N-goods at date 2 can pledge a fraction \( \lambda_n R_n k_n \), with \( \lambda_n < 1 \), as collateral to another domestic in order to receive financing. We retain the same bias with regards to foreigners – they only accept claims on the T-sector up to a fraction \( \lambda_T \) of output.

**Balance Sheets.** Return to the balance sheet of a distressed firm, modified now by the domestic debt constraint. Since the firm can only pledge a fraction \( \lambda_n \) of its N-output, its balance sheet, or debt capacity, is reduced to:

\[
q_n^\omega = \lambda_T^\omega R_t k_t + \frac{\lambda_n R_n (\theta_n^\omega) k_n}{e^\omega} - d_{0,f}.
\]

(10)

Its date 1 program must be modified accordingly:

\[
(P1') \quad V_s^\omega \equiv \max_{q_s^\omega} \quad R_t k_t + \frac{\tilde{R}_t (\theta_s^\omega) k_s}{e^\omega} - d_{0,f} - d_{1,f}^\omega - d_{1,d}^\omega
\]

\[
s.t. \quad (i') \quad q_s^\omega = \lambda_T^\omega R_t k_t - d_{0,f} + \frac{\lambda_n R_n (\theta_n^\omega) k_n}{e^\omega} \geq d_{1,f}^\omega + \frac{d_{1,d}^\omega}{e^\omega}
\]

\[
(ii) \quad \theta_n^\omega k_n = d_{1,f}^\omega + \frac{d_{1,d}^\omega}{e^\omega}
\]

\[
(iii) \quad \theta_n^\omega \leq 1
\]

The maximum amount of reinvestment that the distressed firm can finance is given by combining (i') and (ii) with equality, and solving for \( \theta_n^\omega \):

\[
\theta_n^\omega k_n = \frac{1}{1 - \frac{\lambda_n}{e^\omega}} \left( \lambda_T^\omega R_t k_t - d_{0,f} + \frac{\lambda_n R_n (\theta_n^\omega) k_n}{e^\omega} \right)
\]

which is equal to the leverage ratio times the firm’s debt capacity. Thus, as long as,

\[
\lambda_T^\omega R_t k_t - d_{0,f} \geq \left( 1 - \frac{\lambda_n R_n}{e^\omega} \right) k_n,
\]

(11)
holds, the distressed firm is able to finance all of its needs. We refer to this constraint as the *domestic collateral constraint*. Unlike the international collateral constraint, which is purely aggregate and hence affects units through equilibrium prices exclusively, the domestic one affects microeconomic units directly through quantity constraints. It is the latter constraints that are behind the externalities and excess volatility that we discuss below.

**Regions.** Each of the regions in the previous section gives rise to two new regions as $\lambda_n$ is lowered. Region 1 in the previous section, where the international constraint is slack, gives rise to regions I and II. In the former $\lambda_n$ is high so the domestic constraint is slack as well, while in the latter the opposite is true. Similarly, region 2, where the international constraint is binding, yields regions III and IV, sorted again by whether the domestic constraint is slack or binding, respectively.

![Figure 4: Equilibrium when $\lambda_n < 1$](image)

Figure 4 provides examples of equilibria within each of these regions. The solid lines in panel (a) represent the unconstrained demand and supply of $D_{1,t}$. The upper envelope (solid lines) is the domestic demand for funds in the case where there is no friction in the domestic market - it corresponds to the demand curve in the case where $\lambda_n = 1$. As $\lambda_n$ falls there is a point at which the distressed firms’ demand for funds exceeds its domestic collateral. At this point the suppliers of funds face the prospect of making insufficiently collateralized loans and restrict their supply of funds. Equivalently, the supply of fully collateralized debt falls. The long-dashes represent a case in region I, where $\lambda_n$ is high enough that a distressed firm can still purchase all the international collateral it needs. The short dashes portray a situation where $\lambda_n$ is small enough that the firm is unable to fully reinvest since it cannot afford all the international collateral it needs, even when the latter
is available at minimum price.

In the latter case, the internal rate of return on investing at date 1 is high \((\Delta_n/e^\omega >> 1)\). However none of this can be shared with intact firms, as these firms restrict their supply of fully collateralized loans.

Let us define the **domestic spread** as this difference:

\[ s_d^\omega \equiv \frac{\Delta_n}{e^\omega} - L^\omega. \]

This spread reflects the difference between the internal return on investment in a distressed firm at date 1, and the return that the distressed firm is able to promise other domestic investors on a fully collateralized loan.

Panel (b) has a binding international constraint. The long dashes represent a situation in region III, where \(\lambda_n\) is large enough so all surplus can still be transferred to collateral suppliers as in the case with perfect domestic markets. The short dashes, on the other hand, represent a case in region IV, where \(\lambda_n\) is small enough to limit the ability to fully transfer the rents to collateral-suppliers, but still high enough to fully aggregate international collateral (unlike region II).

For a given state of the world in region IV, as \(\lambda_n\) falls, the domestic spread rises. That is, as distressed firms lose domestic collateral the intact firms earn a lower return on their international collateral. This is at the heart of an underdeveloped financial market: firms do not share equally in the surplus generated from investment. A wedge arises between the marginal product of investment and the return to liquidity providers.\(^{21}\) When the domestic market is sufficiently underdeveloped, the domestic spread is positive, as in regions II and IV.\(^{22}\)

**Domestic Interest Rates.**

It is apparent that as \(\lambda_n\) falls, \(L^\omega\) falls as well. Thus we have the counter-intuitive prediction that **weaker domestic financial markets have lower interest rates**. This deserves some examination and explanation.

\(^{21}\)Part of this wedge may be arbitraged by a resource-consuming bank, as in Holmstrom and Tirole (1997). We revisit this issue in section 4.

\(^{22}\)In fact, emerging economies are likely to spend most of the time between these two regions. Region II corresponding to normal times, while region IV represents deep crises. In region II, the domestic spread is \(\frac{\Delta n}{e^\omega} - 1\). In region IV, this spread starts to fall, not because the domestic market is any better, but because as the economy moves into a crisis the international spread rises as the required interest rate on all domestic collateral rises. It is also important to point out that during severe crisis both \(\lambda_n\) and \(\lambda_t\), rather than just the latter, are likely to fall. A shock that reduces international collateral may be one that also reduces domestic collateral. We will capture this feature below by letting the economy fluctuate between regions I and IV rather than II and IV. In region I, no collateral constraint binds, while the opposite is true in region IV. More endogenously, as we develop later in the paper, an external shock to international collateral hinders the economy’s ability to aggregate collateral by compromising the banking sector. This feeds through as a shock to domestic collateral as well.
In our setting all loans are fully collateralized, thus they are default free. If we were to allow risky loans with default, as domestic collateral worsens, the interest rate on these loans would rise to reflect the growing chance of default. In this case the domestic spread would reflect the fact that the increase in risk cannot be fully charged to the borrower.

As an alternative, suppose that borrowers pledged as repayment the full return of \( \frac{\Delta R}{\omega} \), but had insufficient collateral so that there would be default. Then we could still set the domestic interest rate to \( \frac{\Delta R}{\omega} \), as in the previous section, but the partial collateralization of these loans would mean that the expected rate of return received by lenders would be only \( L^\omega \). The latter can be thought of as the "riskless" interest rate (recall the risk neutrality of our lenders at date 1), while the domestic spread can be interpreted as the difference between the marginal product of capital and this riskless rate.\(^{23}\)

By focusing on fully collateralized loans we isolate the effect that lending to firms with poor collateral is bad (or not as good) business, while ignoring the realistic effect that poor collateral induces default premia. For the date 0 decision to supply international liquidity that we focus on below, it is the former effect that is most important.

We summarize the results of this sub-section as follows:

**Proposition 2 (Regions (\( \lambda_n < 1 \))).** When \( \lambda_n < 1 \) the economy can be in one of four regions at date 1.

- **Region I** occurs when both the international collateral constraint (7) and the domestic collateral constraint (11) are slack. In this case, both international and domestic spreads are zero and there is full project completion. \( s_d^{\omega} = s_f^{\omega} = 0, \theta_n^{\omega} = 1 \).

- **Region II** occurs when the international constraint is slack but the domestic constraint is binding. International spreads remain zero, however domestic spreads are positive as the economy fails to aggregate all of its collateral resulting in some projects being downsized. \( s_d^{\omega} > 0, s_f^{\omega} = 0, \theta_n^{\omega} < 1 \).

- **Region III** occurs when the international constraint is binding but the domestic constraint is slack. International spreads jump to \( \frac{\Delta R}{\omega} - 1 \), while domestic spreads remain

\(^{23}\)Our risk-neutral intact firms are only interested in the expected value of loans made. As collateral gets weaker, they restrict the amount of funds lent to equal the expected value of loans made. In region II, when loanable funds are plentiful, the expected return on these loans is simply their opportunity cost of the international interest rate - again, there is no additional charge for default. In region IV, when loanable funds are scarce, the expected return on loans is bid up - but there is no charge for default. In region II, changes in domestic collateral have pure quantity effects. That is, when collateral gets worse, intact firms lend less. In region IV, changes in domestic collateral have purely price effects. Less domestic collateral means the domestic spread rises. What we will be interested in below is the effect of prices on date 0 supply of international collateral. Hence it is a good idea to focus on a model in which changes in domestic collateral have no quantity effects - the vertical supply curve of region IV. Of course in reality, the change in domestic collateral will have a direct effect at date 1 in terms of quantities if the supply curve had some elasticity.
at zero. The economy aggregates all of its collateral, however there is insufficient aggregate resources and some projects are downsized. $s_{d}^{a} = 0, s_{f}^{a} > 0, \theta_{n}^{a} < 1$.

- Region IV occurs when both the international constraint and the domestic constraint are binding. Both, international spreads and domestic spreads are positive and some projects are downsized. $s_{d}^{a} > 0, s_{f}^{a} > 0, \theta_{n}^{a} < 1$.

3.2 The Externality: International Collateral Undervaluation

What are the costs of having an undevoloped domestic financial market? Fixing $k_{n}, k_{t}$ and $d_{0,f}$, the main cost to the economy is that in region II collateral is not aggregated properly, resulting in lower investment and output. But $k_{n}, k_{t}$ and $d_{0,f}$ do not remain fixed as $\lambda$ changes. Focusing on the date 0 problem, we show that the private sector in an economy with underdeveloped financial markets undervalues international collateral. As a result, the economy is over-exposed to the crises brought about by external shocks. We describe the externality in this subsection and its vulnerability consequences in the next one.

The Date 0 Problem. At date 1 the economy can be in either a good or a bad state. To sharpen the focus, we assume that the good state leaves the economy in region I where neither of the collateral constraints bind, while the bad state leaves the economy in region IV where both collateral constraints bind. Although the negative shock is exclusively to international collateral, equilibrium asset prices decline sharply enough so that the domestic collateral constraint becomes binding as well.\footnote{That is, in our scenario a decline in the availability of international financing leads to a decline in domestic “intermediation” as well.}

To arrive at the date 0 problem we work backwards from date 2. In either good or bad state, an entrepreneur/firm at date 2 with wealth (output from production less debt repayments) of $w_{2}$ in units of T-goods solves,

$$\frac{w_{2}}{2}e(\omega)^{1/2} = \max \{ u^{d} \ s.t. \ c_{t} + c_{n}/e(\omega) = w_{2}.\}$$

Since the value function over date 2 wealth is linear in wealth, production and financing decisions at date 1 are taken to maximize the expected value of wealth – which is exactly what (P1) and (P2) reflect.

Good State: $\omega = G$

In the good state, neither collateral constraint binds. Proposition 1 dictates that in this case collateral is fully aggregated and that all N-productions units are saved. The only difference between an intact and a distressed firm, is that the latter must reinvest $k_{n}$. Since the fraction of distressed firms is $\rho$, the expected (as of date 0) date 2 wealth conditional
on state $G$ is:

$$w_2(G) = R_t k_t + \frac{R_n k_n}{e(G)} - \rho k_n.$$  \hspace{1cm} (12)

Market clearing at date 2 (condition (5)), yields a real exchange rate of

$$e(G) = \frac{C_G}{C_t^G} = \frac{R_n K_n}{R_t K_t - D_{0,f} - \rho K_n},$$

while both spreads –domestic and foreign– are equal to zero at date 1.

**Bad State: $\omega = B$**

In the bad state, both collateral constraints bind. Investment is constrained and both the foreign as well as the domestic spread are positive.

The binding domestic collateral constraint implies that in equilibrium:

$$\theta_n^B k_n = \psi \left( \lambda_t^B R_t k_t - D_{0,f} + \frac{\lambda_n r_n k_n}{L^B e(B)} \right) < K_n,$$  \hspace{1cm} (13)

with

$$\psi \equiv \frac{1}{1 - \frac{\lambda_n}{L^B e(B)}}.$$

while the (aggregate) international collateral constraint implies that:

$$\theta_n^B K_n = \frac{\lambda_t^B R_t K_t - D_{0,f}}{\rho}.$$  \hspace{1cm} (14)

Combining the aggregate counterpart of (13) and (14) we obtain the date 1 prices:

$$L^B = \frac{\lambda_n}{1 - \rho} \left( \frac{\Delta_n}{e(B)} + \frac{r_n Z_B}{e(B)} \right),$$

where

$$Z_B \equiv \frac{\rho K_n}{\lambda_t^B R_t K_t - D_{0,f}}.$$

The ratio $Z$ indicates the economy’s exposure to crises. As $Z$ rises, there is less aggregate availability of collateral per unit of distressed capital. Other things equal, this means that the discount of N-claims, $L^B$, rises as well.

The real exchange rate at date 2 is determined as before:

$$e(B) = \frac{C_G}{C_t^B} = \frac{(R_n - \rho \Delta_n(1 - \theta_n^B)) K_n}{R_t K_t - \rho \theta_n^B K_n - D_{0,f}}.$$  \hspace{1cm} (15)

Given these equilibrium prices and availability of reinvestment resources, the expected date 2 wealth conditional on state $B$ is:

$$w_2(B) = R_t k_t + \frac{k_n}{e(B)}(R_n - \rho \Delta_n) - D_{0,f}$$

$$+ \left( \rho \frac{\Delta_n}{e(B)} + (1 - \rho)L^B - 1 \right) (\lambda_t^B R_t k_t - D_{0,f})$$  

$$+ \rho \left( \frac{\Delta_n}{e(B)} - L^B \right) \frac{\lambda_n \tilde{R}_n}{e(B) L^B} k_n.$$

22
The first line of the RHS in (16) reflects the expected wealth, in the bad state, generated by date 0 investment in the N and T sectors, net of foreign debt contracted at date 0. In particular, this line corresponds to the expected wealth assuming that all distressed production units are allowed to fail. The value of saving these production units is captured in the next two lines. The second line reflects the return to having international collateral. With probability \((1 - \rho)\) the firm is intact in which case it borrows against the international collateral and lends to the distressed firm, thereby earning the domestic interest rate of \(L^B\). On the other hand, if the firm is distressed, then it borrows to save its own production units, directly earning the internal return of \(\frac{\Delta P}{e(B)}\) against its international collateral. The last line reflects the return to having domestic collateral. If a firm is distressed, it mortgages its domestic collateral at the domestic interest rate of \(L^B\) and uses these proceeds to save production units, earning \(\frac{\Delta P}{e(B)}\).

Following the logic of the previous paragraph, we can rewrite (16) directly in terms of spreads as,

\[
\begin{align*}
  w_2(B) &= R_t k_t + \frac{k_n}{e(B)}(R_n - \rho \Delta_n) - d_{0,f} \\
  &+ \left( s_f^B + \rho s_d^B \right) (\lambda_t^B R_t k_t - d_{0,f}) \\
  &+ \rho s_d^B \frac{\lambda_n \tilde{R}_n (\theta_n^B)}{e(B) L^B} k_n.
\end{align*}
\]  

It is instructive at this stage to construct two different versions of \(w_2(B)\); one for individual firms and the other for a central planner. To construct the former, we simply substitute in (17) the expression for reinvestment implied by the microeconomic domestic collateral constraint, (13). To construct the central planner’s version, which we denote by \(w_2^*(B)\), we substitute instead the equilibrium collateral constraint, (14). While separately these expressions are obscure, their difference clearly highlights the basic externality:

\[
\begin{align*}
  w_2(B) - w_2^*(B) &= s_d^B \left( \rho \psi \frac{\lambda_n \tilde{R}_n}{L^B e(B)} k_n - (1 - \rho \psi)(\lambda_t^B R_t k_t - d_{0,f}) \right),
\end{align*}
\]  

where \(\rho \psi \leq 1\) follows from equations (13) and (14).25

---

25\(\psi\) is the leverage ratio applied to collateral for an individual distressed firm. That is, it is the multiplier on collateral that determines how much a firm can borrow. A simple way to see that \(\rho \psi < 1\) is to think about the aggregate collateral constraint. If each distressed firm uses a multiple of \(\psi\) and in total the fraction of distressed firms is \(\rho\), then \(\rho \psi \times X\) (Value of Domestic + International Collateral) is the total amount borrowed by distressed firms. In aggregate, however, the country can only borrow against its international collateral. Thus, we must have that,

\[
\rho \psi \times X (\text{Value of Domestic + International Collateral}) \leq \text{Value of International Collateral}
\]

or, \(\rho \psi < 1\).
At a given equilibrium in region IV, \( w_2(B) \) and \( w^*_2(B) \) must be equal. But it is apparent that individuals and the central planner value a marginal unit of international collateral and domestic collateral quite differently. We note three things in particular:

- Individual firms value domestic collateral more than the central planner.
- Individual firms value international collateral less than the central planner.
- The divergence in these valuations are proportional to the domestic spread, \( s^B_d \).

At the aggregate level, the central planner only cares about creating and protecting international collateral. This is because international collateral is all that can be used to attract foreign investment. However at the microeconomic level, both domestic collateral and international collateral can be used to secure financing. This is the basic tension between the individual and the central planner’s problem.

A positive domestic spread heightens this tension by placing a wedge between the social and private valuation of collateral. A unit of international collateral generates a *social return* of \( \frac{\Delta v}{e(B)} \) at date 1, while a unit of domestic collateral generates a social return of one at date 1. At the individual level, for an intact firm, a unit of domestic collateral generates a return of one. On the other hand, a unit of international collateral only generates a return of \( L^B \). The difference between social and private valuation of \( s^B_d = \frac{\Delta v}{e(B)} - L^B \) results in undervaluation of international collateral. The domestic spread prevents collateral demanders (distressed firms) from transferring the full marginal value of collateral to the collateral providers (intact firms).\(^{26}\)

Underdeveloped financial markets (low values of \( \lambda_n \)) result in high domestic spreads and the systematic mis-valuation of collateral. It is important to realize that the externality we have described is not of the traditional Bardhan-Harberger type, where individual borrowers do not internalize the fact that the country faces an upward slopping supply of foreign loan.\(^{27}\) Indeed, in our setup the economy does face a very steep international funds supply, for the country is rationed after some point, but the externality arises only when *domestic* financial markets are underdeveloped as well. The constrained nature of the domestic demand for collateral limits the price that collateral users can afford. In so doing, a domestic spread arises, which reflects the gap between what an extra unit of collateral is worth for them and

\(^{26}\)Overvaluation of domestic collateral is the necessary counterpart of undervaluation of international collateral. At the individual firm level, for a distressed firm, a unit of international collateral still generates a return of \( \frac{\Delta v}{e(B)} \). On the other hand, a unit of domestic collateral is discounted at the interest rate of \( L^B \) and generates output of \( \frac{\Delta v}{\pi^B} \), providing a return of \( s^B_d > 0 \). At the social level, domestic collateral generates an excess return of zero. At the individual firm level, this mis-pricing causes firms to overvalue the \( \lambda_n, r_n, k_n \) of domestic collateral, resulting in too much domestic collateral relative to international collateral.

\(^{27}\)Which is not to say that such mechanism is not important in reality.
what they can afford. Collateral suppliers anticipate such spread, and therefore see their provision incentives depressed.

Let us now establish this result on the undervaluation of international collateral and its dependence on the underdevelopment of domestic financial markets more formally. We do this in two steps. First, we write down the problem of a central planner who has freedom in making date 0 decisions of \((K_n, K_t, D_{0,f})\) subject to the domestic and international collateral constraints. We arrive at the constrained Pareto efficient decisions of the problem and show, in particular, that these solutions do not depend on \(\lambda_n\). Second, we contrast the central planner’s and the decentralized solution and show that the market equilibrium with \(\lambda_n < 1\) is constrained Pareto inefficient.

**Central planner’s problem.** Consider the problem of a central planner who maximizes the equally weighted sum of utilities of agents in the economy by choosing \(K_n, K_t\) and \(D_{0,f}\) directly at date 0, subject to the respective collateral constraints. In the appendix we show that the solution to this problem is equivalent to the solution of the following program – which resembles that of the individual firm, with the exception of the expression for \(w^*_2(B)\):

\[
\begin{align*}
(P4) \quad & \max_{K_n, K_t, D_{0,f}} \quad \sum_{\omega \in \{G,B\}} \pi^\omega e(\omega) \frac{1}{2} w^{*}_2(\omega) \\
\text{s.t.} & \quad w^*_2(B) = R_t K_t + \frac{(R_n - \rho \Delta_n) K_n}{\epsilon(B)} - D_{0,f} + \left(s^B_f + s^B_d\right) (\lambda^B_t R_t K_t - D_{0,f}) \\
& \quad w^*_2(G) = R_t K_t + \frac{R_n K_n}{\epsilon(B)} - D_{0,f} - \rho K_n \\
& \quad D_{0,f} \leq \min_\omega \left\{ \lambda^\omega_t R_t K_t + \lambda_n \frac{R_n K_n}{\epsilon(B)} \right\} \\
& \quad K_n + K_t = W_0 + D_{0,f}
\end{align*}
\]

**Proposition 3 (Constrained Optimality)** Let \((K^*_n, K^*_t, D^*_{0,f})\) be solutions to \((P4)\). 
(a) These solutions are the constrained Pareto optimal date 0 choices of the economy. (b) \((K^*_n, K^*_t, D^*_{0,f})\) are independent of the strength of domestic financial links, \(\lambda_n\).

Proof: Part (a), see the appendix. Part (b) can be done by simple inspection of program (P4). Once we show that it does not depend on the value of \(\lambda_n\), its solution can’t either.

It is apparent that none of the terms in (P4) depends on \(\lambda_n\) but for the constraint on foreign leverage.\(^{28}\) However, we will show by contradiction that the inequality is always strict:

\[
D_{0,f} < \min_\omega \left\{ \lambda^\omega_t R_t K_t + \lambda_n \frac{\tilde{R}_n(\theta^\omega_n) K_n}{I^\omega e^\omega} \right\}.
\]

Suppose that,

\[
D_{0,f} = \min_\omega \left\{ \lambda^\omega_t R_t K_t + \lambda_n \frac{\tilde{R}_n(\theta^\omega_n) K_n}{I^\omega e^\omega} \right\},
\]

then, the amount of international collateral at date 1 in the bad state is,

\[
\lambda^\omega_t R_t K_t - D_{0,f} < 0.
\]

\(^{28}\)Indeed, note that \((s^B_f + s^B_d) = \Delta_n / \epsilon(B)\).
However if this is the case, then the market for exchanging domestic collateral for international collateral will necessarily break down. Swapping \( \frac{R_0(\theta^\omega_i|K_0)}{e^\omega_i} \) units of domestic collateral must return zero units of international collateral since the supply of international collateral is not positive. \( L^\omega \) rises until such a swap does not take place, or until \( D_{0,f} < \min_\omega \{ \lambda^\omega_t R_t K_t \} \).

**Undervalued international collateral** Let us contrast the central planners problem with the decentralized solution. Taking the case where the good state leaves us in region I and the bad state in region IV, we can write the program of a firm at date 0 as:

\[
(P5) \quad \max_{k_n, k_t, d_{0,f}} \quad \sum_{\omega \in \{G, B\}} \pi^\omega e(\omega) \frac{1}{2} w_2(\omega) \\
\text{s.t.} \quad w_2(G) = R_t k_t + \frac{R_n k_n}{e(G)} - d_{0,f} - \rho k_n \\
\quad w_2(B) = R_t k_t + \frac{(R_n - \rho \Delta_n) k_n}{e(B)} - d_{0,f} \\
\quad \quad + \left( s^B_t + \rho s^B_{d_t} \right) \left( \lambda^B_t R_t k_t - d_{0,f} \right) \\
\quad \quad + \left( \rho s^B_{d_t} \right) \frac{\lambda^B_n R_n k_n}{e(B) e(G)} \\
\quad d_{0,f} \leq \min_\omega \left\{ \lambda^\omega_t R_t k_t + \lambda^\omega_n \frac{R_n k_n}{e(B) e(G)} \right\} \\
\quad k_n + k_t = u_0 + d_{0,f}.
\]

**Proposition 4 (Undervaluation of International Collateral)** Let \((k_n, k_t, d_{0,f})\) be solutions to the program \((P5)\) in the case when \(\lambda_n < 1\) and \(s^B_d > 0\). Then the decisions, \((k_n, k_t, d_{0,f})\), are constrained Pareto inefficient. A central planner can effect a Pareto improvement by perturbing these decisions to \((k'_n, k'_t, d'_{0,f})\), where at least two of the following inequalities are strict,

\[
\begin{align*}
d'_{0,f} &\leq d_{0,f} \\
k'_n &\leq k_n \\
k'_t &\geq k_t
\end{align*}
\]

Proof: see Appendix

The proof follows closely from the logic laid out in the previous section. Note that the only difference between \((P4)\) and \((P5)\) is in the expressions for \(w_2(B)\) and \(w_3(B)\). When \(\lambda_n = 1\) the domestic spread, \(s^B_d = 0\), and there is no difference between the central planner’s and the decentralized solution. In this case, the market equilibrium is constrained Pareto efficient. However, when domestic markets are undeveloped, \(s^B_d > 0\) and there is a divergence in the social and private value of collateral, resulting in the inefficiency.

**3.3 Excess Volatility and Policy**

Proposition 4 has a straightforward but useful corollary:

**Corollary 1** Respectively, let \(Z\) and \(Z'\) denote the indices of exposure corresponding to solutions \((k_n, k_t, d_{0,f})\) and \((k'_n, k'_t, d'_{0,f})\) in proposition 4. Then, \(Z > Z'\).
What is the cost of a higher index of exposure? It is easy to see that as $Z$ rises the economy becomes more volatile — as long as the economy remains fluctuating between regions I and IV, as we have assumed.29

Figure 5 illustrates this. The solid lines represent the supply of international collateral in the bad state and in the good state. In the good state the supply of international collateral is $\lambda' F_R k_t - D_{0,f}$. Given our definition of $Z_G = \frac{\rho K_s}{\lambda' F_R k_t - D_{0,f}}$, the supply curve turns vertical at $\frac{\rho K_s}{Z_G}$. Similarly in the bad state, since there is less international collateral, the supply is constrained at a lower point. The dashed lines represent supply at the Pareto improving decisions, $Z'$.20 While both economies share the equilibrium at $A$ during good states of the world, the less efficient economy falls to $C$ rather than $B$.

![Figure 5: Excess and Costly Volatility](image)

There are two observations we can make from the figure. First, reading off the y-axis, since $L$ varies less across realization of G and B states at $Z'$ than at $Z$, we can see that asset prices are less volatile. Second, reading of the x-axis, since equilibrium reinvestment varies less at $Z'$ than at $Z$, investment and output are also less volatile. When $Z$ is high, there is little surplus of international collateral left even under the best of the scenarios; thus, the economy is nearly defenseless in the event of a bad external shock.

29If the domestic constraints becomes so tight that, e.g., the economy finds itself at all times in the wasted collateral regions, volatility with respect to external shocks may disappear altogether, and be replaced by persistently low reinvestment rates. See our discussion of this issue in the conclusion.

30In this figure, we vary $Z$ by changing $K_t$ and $D_{0,f}$, while keeping $K_s$ constant. Moreover, the figure is only illustrative, as such change in $Z$ also changes the date 2 exchange rate, and hence shifts demand.
While we have stressed the impact of underdeveloped domestic financial markets on investment and output volatility, it also follows that since reinvestment options are curtailed more severely by crisis when $\lambda_n$ is lower, the country’s stock market value suffers more as well.\textsuperscript{31}

4 Endogenous Breakdown in Collateral Aggregation: Amplification and Multiple Equilibria.

Distressed firms access the international collateral of intact firms by borrowing from these firms against their N-collateral. The implicit assumption in the preceding analysis is that this collateral aggregation is done in a perfectly functioning public debt market. Distressed firms issue debt claims in the domestic market secured by N-collateral. Intact firms issue debt claims to foreign investors secured by international collateral. They then use the proceeds from this issue to purchase the debt claims of the distressed firms.

In emerging markets, liquid domestic debt markets are the exception rather than the rule. Invariably, the financial system is bank based – for good reasons: lack of investor protection and the necessity of monitoring to alleviate asymmetric information places banks at the center of the financial system. Banks issue claims to foreign investors against balance sheets that are partially backed by international collateral – both of the distressed and the intact firms. These funds are then channeled to the distressed firms. Implicitly, the intact firm extends credit to the distressed firms against their domestic collateral.

But unlike the immutable debt markets of the previous sections, when asset prices experience sharp falls banks themselves run into trouble. As banks enter distress, the economy’s ability to aggregate its scarce collateral is weakened, triggering further declines in asset prices and real activity. Even worse, if banks are undercapitalized at the outset, this perverse feedback process can lead to multiple equilibria.

This endogenous disintermediation process and its feedback into real activity and asset prices fits conventional wisdom well. Arguably, some form of it plays a role in virtually every emerging market’s external crisis. And in its extreme form of a worst equilibrium among many, it is reminiscent of Indonesia in the recent Asian crisis, Mexico during the tequila crisis, and Chile during the debt crisis of the early 80s, to name a few.\textsuperscript{32}

\textsuperscript{31}That is defining the equity value of all firms as $\rho q^e + (1 - \rho)q^d$, we can show that low $\lambda_n$ lowers this quantity.

\textsuperscript{32}See Gelos and Werner (1999) for a study of lending practices by Mexican banks during the post-liberalization period. Among other things, they document the significant role played by banks in lending to manufacturing firms without access to foreign funds, and their reliance on N-collateral. They argue that this mix contributed significantly to the collapse of the banking system during the 94-95 crisis.
4.1 The Banking System: Amplification and Financial Bottlenecks

In order to highlight the role of banks, in this section we suppress entirely domestic debt markets so that all reallocation must be done via the banking system. Distressed firms in need of funds take loans from banks. To attract these funds, banks offer deposits at a market determined interest rate. Intact firms become the depositors. They mortgage their international collateral to foreign investors and then deposit these funds in the banking system. By intermediating these transactions, the banking system serves to aggregate collateral, which is the only role it has in our model.

**Balance Sheets.** We consider a representative bank in a competitive banking sector (of unit measure). Suppose that at date 0, banks make loans to firms in both N and T sectors and fund these loans by issuing debt to foreigners. In particular, suppose that the assets of the bank are $a_n$ face value of date 2 debt of N-firms and $a_t$ face value of date 2 debt on T-firms. Banks fund this with $d_{0,f}^t$ units of foreign debt (in units of T goods). Then the value of capital (entering date 1) in the bank is the excess of assets over liabilities,

$$d_{0}^t = \frac{a_n}{L^\omega} + a_t - d_{0,f}^t. \tag{19}$$

Foreigners extend credit to banks against the same no-default constraint as before,

$$\frac{a_n}{L^\omega} + a_t - d_{0,f}^t \geq 0.$$

The balance sheets of distressed and intact firms (equations (2) and (3)) are modified slightly:

$$d_{0}^u = \lambda_t^u R_t k_t - a_t + \frac{R_n k_n - a_n}{L^\omega} - d_{0,f}$$

for the intact firm, while for the distressed firm, it is,

$$d_{0}^u = \lambda_t^u R_t k_t - a_t + \frac{\bar{R}_n(\theta_t^u) k_n - a_n}{L^\omega} - d_{0,f}.$$

Both equations are modified to reflect the date 0 lending from the banking sector.

**Capital Constraints.** Notice that we have taken $\lambda_n$ to be one in both balance sheet calculations. That is, in the absence of any further frictions, the banking system is equivalent to the public debt market with $\lambda_n = 1$ and hence collateral aggregation is frictionless. We

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33 At date 0, not all agents in the domestic economy are the same - there are both banks as well as firms. Thus issuance of domestic debt cannot be netted to zero as we have done in previous sections. Additionally, throughout the paper, our firms are conglomerates, each of them with an N-firm and a T-firm in it. We assume here that each of these mini-firms can contract debt independently.

34 The purpose of this section is to highlight the potential date 1 bottleneck brought about by the deterioration in banks' balance sheets. There are several interesting date 0 issues which merit an extensive discussion but which would lengthen this section substantially. For example, one may ask why banks don't sign contingent contracts with borrowers and depositors.
now introduce capital constraints that limit the capacity of the banking system to aggregate collateral.

At date 1, a bank takes in deposits from intact firms of $d^b_{1,d}$ and makes loans of $x^b_{1,d}$ to the distressed firms. Let $L_X$ and $L_D$ represent the loans’ and deposits’ interest rates, respectively. If no other constraint is present, both of these are made at the market determined interest rate of $L$, and the expected date 2 repayment to the bank from this operation is $(x^b_{1,d} - d^b_{1,d})L$ (in units of face value of T-goods at date 2). We assume that capital constraints restrict the size of this operation. In particular,

$$d^b_0 \geq \alpha x^b_{1,d}(\omega),$$

where $0 < \alpha < 1$ is the capital that banks must hold against making loans with present value of $x^b_{1,d}(\omega)$.\(^{35}\) If this constraint binds, $L_X$ is still equal to $L$ but $L_D$ falls to one since there is an excess supply of deposits but a scarcity of loanable funds.

Taking as given the market interest rate (on loans) of $L^\omega$ and date 2 exchange rate of $e^\omega$, the problem of a bank at date 1 is,

\[(P6) \quad \max_{x^b_{1,d},d^b_{1,d}} \quad a_t + \frac{\gamma}{\omega} - d^b_0 - d^b_{1,f}(\omega) + (L^\omega - 1)x^b_{1,d}(\omega) - (L^\omega_D - 1)d^b_{1,d}(\omega) \]

\[\text{s.t.} \quad \begin{align*}
(i) & \quad d^b_0 \geq \alpha x^b_{1,d}(\omega) \\
(ii) & \quad x^b_{1,d}(\omega) \leq d^b_{1,d}(\omega) + d^b_{1,f}(\omega) \\
(iii) & \quad d^b_{1,f} \leq a_t - d^b_0(\omega)
\end{align*}\]

The objective of the bank reflects profits from making loans of $x^b_{1,d}(\omega)$ at the interest rate of $L^\omega$ and taking deposits of $d^b_{1,d}(\omega)$ at the interest rate of $L^\omega_D$. Constraint (i) is simply the capital constraint on banks. Constraints (ii) and (iii) reflect the date 1 resource constraint: making loans of $x^b_{1,d}$ requires raising deposits from intact firms of $d^b_{1,d}$ plus attracting funds against international collateral of $a_t - d^b_{1,d}$ from foreign investors.

Since the formal analysis of equilibrium is very similar to that of the previous section, we relegate it to the appendix. Banks will lend as much as they can whenever $L^\omega > 1$, while in equilibrium $L^\omega = 1$ when demand for funds is less than loanable funds. Firms will behave exactly as in the previous sections. In fact, as long as constraint (i) is not binding, the problem is entirely analogous to that in section 2.4. Panel (a) in figure 6 depicts this scenario. As in that section, there are two regions, (i) and (ii), which are distinguished by whether or not the international collateral constraint binds.

\(^{35}\)In practice, capital adequacy requirements are based on both assets as well as liabilities. For example, BIS standards assign capital requirements to different assets held by banks. The liabilities of the bank are sorted (i.e. common stock, preferred stock) and weighted to determine the amount of capital held by the bank and hence ensure that the capital requirement is met. Capital requirements can be justified from first principles on the basis of moral hazard within the banking sector. See Holmstrom and Tirole (1997), for example.
The interesting new cases arise when the capital requirement \((i)\) does bind. In this case the economy can fall into regions (iii) or (iv) depending on how tightly it binds relative to demand from distressed firms. The dashed line in figure 6 (b) corresponds to a supply of loans which yields a region (iii) equilibrium: loan demand is fully satisfied, interest rates are low and output is high. Conversely, if loan supply is constrained by capital requirements, then interest rates rise to ration the scarce supply (dotted line, region (iv)). The key ingredient to highlight is that now the supply curve for loans bends backwards at the point where the capital requirement binds. This occurs because the value of banks’ capital falls when interest rates rise (see equation (19)).

The backward bending nature of supply plays a significant role in region (iv), and differentiates it from region IV in section 3. While in both cases an aggregate collateral constraint and a microeconomic capital requirement bind, in the banking-intermediated case there is \textit{wasted collateral}, in the sense that not all excess domestic resources are channeled to the distressed firms. Equilibrium is at point A rather than B, where the latter reflects full aggregation. Constrained banks become a financial bottleneck, which bring about simultaneously the worst of each scenario described in the previous sections: sharp declines in asset prices coupled with collateral waste.

\subsection*{4.2 Multiple Equilibria}

The rise in domestic interest rates and fire sale in region (iv) is not only a symptom of the crisis, it is also a cause of the crisis.
We illustrate this in figure 7. As we described above, the contraction in loan supply causes the rise in interest rates. The fall in domestic asset prices amplifies the impact of the crisis by deepening the credit crunch caused by banks’ distressed balance sheets. But it is easy to see that the feedback between asset prices and feasible intermediation brings about the possibility of multiple equilibria.\footnote{This amplification mechanism is similar in spirit to Kiyotaki and Moore (1997) in that a fall in asset prices is reinforced by a tightening financial constraint. However, one key difference between our approach and theirs lies in the identity of the agent with the crucial financial constraint. In Kiyotaki and Moore the key financial constraint is on the demand side - constrained demand reduces the productivity of assets, causing the fall in asset prices, and further constraining demand. However, in our model the key financial constraint is on the supply side - constrained supply causes interest rates to rise, causing bank capital to be compromised, and further constraining supply. In this sense, our model is closer to the amplification mechanism studied in Krishnamurthy (1998).}

![Figure 7: Multiple Equilibria with Banks](image)

Points (A) and (B) in the figure represent two equilibria which are distinguished by low interest rates, non-binding capital requirements (A) and high interest rates, credit crunch (C). Point (B) is also a possibility in which interest rates rise to constrain loan supply to exactly meet demand. While in both (A) and (B) all production units are saved, this is \footnote{A related mechanism is ”the disappearance of markets (and market-makers)” during crises. A concern often expressed by emerging markets’ policy makers in their attempt to place new public debt in international markets during crises.}
not so in (C). The equilibrium in (C) is Pareto inferior to that of (A) and (B). More generally, with $\lambda_n < 1$ or a concave but not rectangular demand function, equilibria can be strictly Pareto ranked. Low interest rate equilibria are more efficient than high rate ones.

5 Final Remarks

Since models are stylized representations of complex realities, it is always unwise to interpret them narrowly when extracting their lessons and relevance. This applies both to assumptions and implications. In the context of our model, there are several implications and assumptions that deserve further discussion in order to facilitate their broader interpretation and empirical assessment.

Our first central implication is the presence of fire sales. That is the emergence of substantial expected excess returns in the midst of a crisis. In the model, fire sales take the form of a sharp rise in domestic interest rates, by which we understand the discount applied to assets that are not good international collateral. There are two related questions that naturally follow this abstract description of fire sales: Do they include the frequently observed collapse in the exchange rate? More generally, which observed rates and prices are the empirical counterparts of the model’s rates and prices?

Our model is designed to highlight collateral and balance sheets effects, and in so doing we have eliminated a series of traditional ingredients which are needed to pin down exchange rates during crises, such as [date 1] consumption decisions involving a choice between tradeables and non-tradeables, or monetary decisions that pin down a nominal exchange rate. Ours is a model of a shift in the interest parity condition brought about by collateral shortages that hinder arbitrage. Indeed, our emphasis is on collateral supply shortages rather than on rising risk premia – although connecting the collateral shortage and risk interpre-

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38 Our model has also disregarded macroeconomic effects of debt-overhang problems - another source of amplification and multiple equilibria that is both empirically plausible and has been pointed out in the literature (see Lamont (1995)). Debt-overhang problems in firms introduce a non-convexity in the investment of firms as a function of the strength of their balance sheets. Given the feedback between investment and the strength of firms’ balance sheets, it is possible to have multiple equilibria. We have simply chosen to emphasize the balance sheet problems in banks where capital constraints are most natural.

39 What is the appropriate policy response in this case? To start, the policy suggestions in the previous section apply here as well. More specifically to banks, however, increasing capital adequacy requirements and liquidity ratios, as Argentina has done after the Tequila crisis, may help avoiding bank runs but it probably exacerbates the problems we have described in this section by raising $\alpha$. At the very least the possibility of making these requirements slightly more pro-cyclical should be pondered (which is often done by valuing assets at outdated prices, as in Japan). The perverse impact this may have on the possibility of bank runs can be offset with improved credit lines, interbank markets, and lender of last resort mechanisms. The latter is more complicated but certainly not impossible in “currency board” scenarios.
tations is possible if a more probabilistic interpretation is given to incentive constraints. But whether the new interest parity condition results from an exchange rate overshooting, or from a sharp rise in domestic rates, or any combination of these two, are all possibilities which are consistent with the model as long as there is a shift in the “collateral/risk premium” term. Internal arbitrage must cause a corresponding decline in stocks and other domestic asset prices. This implication seems highly consistent with the facts. As we argued in the introduction, if there is one robust fact of crises associated to financial factors, it is precisely the sharp decline in “risk appetite,” which is the practitioners’ catch-all term for a higher required return on less than prime assets.

In many situations debt and equity markets are nearly non-existent and most financing occurs through the banking sector. In such cases interest rate surges underestimate the extent of the fire sale, which is partially reflected in rationing and quantity constraints instead. The banks’ reluctance to lend in the midst of a capital flow reversal, even when they have plenty of liquidity at their avail, is also a fairly well established fact. The behavior of Argentine banks in the post-Russian crisis is a good example.

Regardless of the specific mechanism by which the fire sales occur, the dual of this decline in asset prices is that many high marginal product projects are foregone, resulting in the sharp decline in investment and output. This is also a central feature of actual crises.

The second implication, and main one from the point of view of policy, is the undervaluation of international collateral creation and accumulation by the private sector, which leaves the economy unfit to withstand significant shocks to its international collateral. This undervaluation stems not from the traditional Bardhan-Harberger monopsony argument, but from the domestic credit constraint — due to the insufficient development of domestic financial markets — that renders inadequate the compensation received by collateral providers. There are several questions that immediately come to mind with respect to this implication. What are the observable counterparts of the model’s domestic financial underdevelopment? Is there evidence on the connection between crises and such underdevelopment? What is the specific mechanism behind the externality? And, does it have empirical content?

The model’s indicator of the degree of underdevelopment of domestic markets is the domestic spread, by which we mean the gap between the marginal product of distressed firms and the domestic interest rate—in the sense described above. Measuring such gaps directly has always been problematic for supporters of credit rationing models since they involve at least one unobservable. What is reasonably measurable, however, is the size of financial markets and the bulk of the intermediation that takes place in an economy over time. With these measures in mind, the evidence is overwhelmingly in favor of the view that domestic financial markets in emerging economies are extremely underdeveloped compared

\[40\text{Where the probabilistic term must have an exogenous or endogenous aggregate component. Also, see Holmstrom and Tirole (1998b) for a model of asset pricing based on liquidity rather than risk premia.}\]
to those of OECD’s. Interestingly, most of the very few exceptions reside in Asia, a point we return to below.\textsuperscript{41}

More generally, our basic insight is that a limited ability to transfer internal return to outsiders —the genesis and essence of a financial constraint— reduces the appeal of that market to suppliers since it effectively reduces its size. Our model limits these suppliers to domestic producers of T-goods, but in reality international market-makers are also included, and these typically dislike markets that do not support significant volumes. To a first order, this extension follows the same logical steps and conclusions of our stylized analysis.

Although we have emphasized the implications of underdeveloped financial markets for crises, an interesting dimension of our model is that extremely underdeveloped financial markets can lead to lower volatility with respect to external financial shocks. The chief factor when equilibrium falls in the wasted collateral region (region II), is the underdevelopment of domestic financial markets leading to a failure of domestic aggregation of resources rather than to the scarcity of external resources.\textsuperscript{42} This non-monotonic relationship between domestic financial development and vulnerability —low development and volatility, middle development and very high volatility, and high development and low volatility— connects our view with two pieces of supportive evidence.

The first one corresponds to the low to middle development range, where correlation between development and volatility is positive. One of the main empirical predictors of crises is the rapid increase in domestic intermediation. This is often interpreted as evidence of rushed and careless lending, or the result of some moral hazard institution. Some of that is undoubtedly true, but our framework offers a different angle: As financial development rises, the country moves from region II (the wasted collateral region) into region IV, where external factors may lead to crises and fire sales. Moreover, since financial underdevelopment means that private agents undervalue international collateral, they do not take enough precautions against these crises. Within a range, their increased financing and aggregation possibilities do not come hand-on-hand with full incentives to take the adequate precautions required by higher leverage.\textsuperscript{43}

The second one refers to the other end, when development is high. The puzzle of Asian economies is why in countries that could achieve such high levels of leverage –reflecting fairly developed financial markets– were there such dramatic crises? Our perspective is one of “not enough development.” Financial development gave them much growth and long stretches of stability. Still, short of G5 levels of domestic market development, the

\textsuperscript{41}The large spreads for banks in emerging economies between deposit and lending rates may also reflect problems with domestic collateral. These spreads are partly accounted for by the monitoring and legal costs incurred by banks to overcome firms’ contractual problems. Although undoubtedly other factors play a significant role in determining these spreads as well —e.g. inflation, oligopolistic banking structures, etc.

\textsuperscript{42}See Aghion et al. (1999) for similar implication in a model of endogenous credit cycles.

\textsuperscript{43}Eventually, however, incentives catch up with financing opportunities as financial development continues.
economies left themselves fragile and exposed to crises — albeit unlikely ex-ante — that proved very damaging.\textsuperscript{44}

The latter discussion takes us to the externality and its relevance beyond the confines of the model. In the latter, and probably in reality as well, domestic microeconomic agents do not worry about the nature — domestic or international — of collateral per-se, once the relative prices and market liquidity of these assets is taken into consideration.\textsuperscript{45} But the central planner does, since during crises it is the international collateral constraint that binds first.\textsuperscript{46} Individual entrepreneurs do their right microeconomic precautionary savings calculations, it is just not in the instrument-mix that the central planner needs.

At the right prices, which is what happens in our model when domestic financial markets are well functioning, entrepreneurs align their portfolio and leverage selections with those of the central planner. But when microeconomic units are credit constrained in equilibrium, prices and interest rates do not reflect fully the marginal value of resources. This is simply the dual of a credit constrained equilibrium, and as such it appears as a robust feature of emerging economies as well.

Finally, the third implication worth expanding on is that the above factors can generate a serious financial bottleneck, with potential multiple equilibria consequences, due to the collapse of banks’ balance sheets. Is this channel empirically relevant? And, does the underlying amplification mechanism extend beyond banks?

In our model, rather than emphasizing the depositors-lenders maturity mismatches and the potential runs that come with it, we have chosen to emphasize the role of domestic banks in transforming domestic international collateral. Banks borrow from foreigners to lend to local firms that foreigners would not lend to directly. Some of these firms are in the N-sector. Regardless of the nominal denomination of the domestic loans, banks are naturally mismatched in their effective denominations. A collapse in asset prices and real-exchange rates reduces the banks’ ability to intermediate foreign resources into the economy at the worst of times. In extreme circumstances, the possibility of multiple equilibria arises to determine not so much whether there is a crisis or not, but its magnitude once embroiled in the crisis.

\textsuperscript{44}Having said this, it is probably the case that their levels of leverage was beyond their international collateral. A big component of the crisis, especially right after Thailand, probably had to do with the realization of this mismatch by international investors. Traditional moral hazard arguments may have played a role here as well, although we continue to believe that the central problem of emerging markets is too little — and too volatile — not too much capital inflows.

\textsuperscript{45}Note that this claim is not inconsistent with our assumption that foreigners do not accept certain class of assets as collateral. Their aversion to these assets comes from their perception that they have an informational and incentive disadvantage with respect to certain assets, and hence are priced out of those markets given their uncertainties.

\textsuperscript{46}In a sense, during normal times our model operates with two “currencies,” while during crises it is only one that matters for total reinvestment and activity, while the other one only decides internal transfers.
This mechanism, as we have argued, plays a role in virtually all emerging markets’ external crises. And in its extreme form of a worst equilibrium among many, it reminds us of Indonesia in the recent Asian crisis, Mexico during the tequila crisis and Chile during the debt crisis of the early 80s, to name a few. Moreover, it complements naturally and feeds into the bank run type of multiple equilibria, as a reduced balance sheet means less residual resources left to those depositors that run late. This interaction probably had something to do with the Argentine bank run of the mid-90s.

But the mechanism we capture goes well beyond the banks themselves. In our model, banks are distinct from corporations in that the latter can separate incentive problem in their T-component and their N-component. But this is clearly a modeling simplification not a feature of reality. Once such sharp separation is blurred, a collapse in the value of the N-component of a corporation affects its international collateral as well, and the feedback and amplification mechanism of our banks is extended directly to the corporate sector. Similarly, internal contagion may also arise when sectors are complementary in “production,” as is the case, e.g., with a government whose claims lose their rating, and via the sovereignty principle, causes a depreciation in the rating of domestic corporations as well.47

All of our asset price and real implications arise from an abstract liquidity “shock.” How should these shocks be interpreted in reality? They can arise as pure financial shocks, as often occurs in the so called “contagion” scenarios. Are these totally unrelated to fundamentals? Most likely not, but by now most countries know quite well whether they are on the short list for a contagion induced shock. As such, taking our liquidity-shocks-abstraction as given and thinking about its implications – the domestic factors that amplify them, and the generic type of policy responses that are most appropriate – is a fruitful exercise.

But shocks need not be purely financial, as the latter may be the reflection of a real shock to terms of trades, productivity, domestic wages, loss of competitiveness and a myriad factors that weaken domestic corporations and a government’s credit-quality perceptions. Each of these shocks has its own idiosyncrasies, but our point of view is that absent the financial impact of these shocks, none of them should have the real consequences that recent history has offered. The recent bout of uncertainty around Argentina’s prospects is a case in point. Argentina’s loss of competitiveness vis-à-vis Brazil seems a second order issue when considered in isolation – at worst just a 2 or 3 percent output equivalent shock for a country that has growth prospects well above OECD’s. However, it becomes a first order issue when one considers the uncertainty that this engenders: Will interest rates soar, and

47 One of the main justifications for the sovereignty principle is that as the government finds itself in payments trouble, it is likely to implement measures — e.g. suspension of convertibility — that compromise the private sector’s repayments as well. Indeed, the very few broad exceptions allowed by S&P to the sovereignty principle is reserved for highly dollarized economies, where convertibility is difficult to suspend.
in so doing compromise fiscal stability, the convertibility law, and with it the private sector at large? It seems safe to argue that absent imperfect financial markets considerations, much of this uncertainty would be abated.

We have omitted many factors that play central roles in specific crises; currency attacks and unsustainable budget deficits, to name a few. Our goal has not been to explain a specific episode, but rather, to offer a useful framework within which the economics of emerging markets crises can be discussed. The standard intertemporal substitution driven, deep markets, neoclassical model, which while useful as a benchmark for macroeconomic analysis of developed economies, is too distant from emerging markets’ reality to be of much use. The alternative we offer is a stylized framework that brings to the fore what we view as the backbone of emerging markets’ structure: underdeveloped domestic asset markets coupled with weak international financial links. Standard crises ingredients can be built on this structure as, for example, we are doing in Caballero and Krishnamurthy (2000) where we discuss the liquidity management and monetary policy for emerging economies within the context of our “asset markets perspective.”
6 Appendix

6.1 Incentive Constraints

We have appealed to agency conflicts in order to justify the assumption that entrepreneurs be required to retain a fraction of their shares. One possible formulation to justify this illiquidity assumption is as follows.

Consider a contract between a firm and a lender at date 0. Suppose that a firm investing one unit at date 0 yields date 2 flows of $\hat{R}_t$ where $E[\hat{R}_t] = R_t$, and the support of $\hat{R}_t$ is $[0, \infty)$. Now assume that the realization of $\hat{R}_t$ is private information of the firm. A lender can observe this payment only by paying a cost of $c$ (in units of T-good). Then, it is fairly standard to show that an optimal contract will be a debt contract (see Gale and Hellwig (1985)). The contract will specify a face of $f$; if $\hat{R}_t > f$ the firm makes the repayment of $f$; otherwise, the firm defaults and lenders pay the audit cost and receive $\hat{R}_t - c$.

Assume that each firm has a continuum of such projects each with i.i.d. return of $\hat{R}_t$. Each project is individually financed via a debt contract. Then define,

$$\theta_t = \frac{E[\hat{R}_t - f|\hat{R}_t > f] \text{Prob}[\hat{R}_t > f]}{R_t} < 1$$

This is the share of each firm that the entrepreneur necessarily holds. Lenders receive expected flows of,

$$E[\hat{R}_t - c|\hat{R}_t < f] \text{Prob}[\hat{R}_t < f] + f \text{Prob}[\hat{R}_t > f]$$

scaled by $R_t$ this is,

$$\frac{E[\hat{R}_t - f|\hat{R}_t < f] \text{Prob}[\hat{R}_t < f]}{R_t} - c \frac{\text{Prob}[\hat{R}_t < f]}{R_t} + f \frac{\text{Prob}[\hat{R}_t > f]}{R_t} = 1 - \theta_t - c \frac{\text{Prob}[\hat{R}_t < f]}{R_t}$$

This is the component of firms that can be held by outsiders. In our formulation we have suppressed the cost $c$, so that this component is simply the complement of what the firm holds itself. However, this makes no qualitative difference to the problem.

6.2 Derivation of the Central Planner’s Program: Proposition 3

In this section we show how to arrive at the central planner’s program, (P5) in the text.

Consider a central planner that maximizes the weighted sum of utilities of agents in the economy.

$$\max \sum \pi^w \left[ \rho \left( \frac{c_{-i}^{w}}{c_{-i}^{w}} \right)^{\frac{1}{2}} + (1 - \rho) \left( \frac{c_{i}^{w}}{c_{i}^{w}} \right)^{\frac{1}{2}} \right]$$

It is easy to show that this can be rewritten in terms of aggregate consumption. The first order condition for individual consumption gives $c_{-i}^{w} = c_{i}^{w}$. This implies that aggregate
consumption satisfies,

\[ \frac{c_{n,s}^\omega}{c_n^\omega} = \frac{\alpha}{\rho} \quad 0 < \alpha < 1 \]

\[ \frac{c_{t,s}^\omega}{c_t^\omega} = \frac{\alpha}{\rho} \]

\[ \frac{c_{n,i}^\omega}{c_n^\omega} = \frac{1 - \alpha}{1 - \rho} \]

\[ \frac{c_{t,i}^\omega}{c_t^\omega} = \frac{1 - \alpha}{1 - \rho} \]

The central planners objective can then be rewritten as,

\[ \rho\left(\frac{c_{n,s}^\omega c_{t,s}^\omega}{c_n^\omega c_t^\omega}\right)^\frac{1}{\gamma} + (1 - \rho)\left(\frac{c_{n,i}^\omega c_{t,i}^\omega}{c_n^\omega c_t^\omega}\right)^\frac{1}{\gamma} = \left(\frac{C_n(\omega)C_t(\omega)}{C_n^\omega C_t^\omega}\right)^\frac{1}{\gamma} \]

or,

\[ \max \sum \pi^\omega \left[ \rho\left(\frac{c_{n,s}^\omega c_{t,s}^\omega}{c_n^\omega c_t^\omega}\right)^\frac{1}{\gamma} + (1 - \rho)\left(\frac{c_{n,i}^\omega c_{t,i}^\omega}{c_n^\omega c_t^\omega}\right)^\frac{1}{\gamma} \right] = \max \sum \pi^\omega \left(\frac{C_n(\omega)C_t(\omega)}{C_n^\omega C_t^\omega}\right)^\frac{1}{\gamma}. \]

Consider next the aggregate consumption relations. In the good state there is full project completion, so that,

\[ C_n(G) = R_nK_n \]

\[ C_t(G) = R_tK_t - \rho K_n \]

In the bad state only \( \theta_nK_n \) is completed,

\[ C_n(B) = k_n(R_n - \rho\Delta_n(1 - \theta_n^B)) \]

\[ C_t(B) = k_tR_t - \rho\theta_n^B k_n - d_{0,f} \]

where,

\[ \theta_n^B = \frac{\lambda_n^B R_t K_t - D_{0,f}}{\rho} \]

Substituting in these relations for aggregate constraints, the central planners problem is,

\[ (PA1) \max_{k_n, \theta_n, d_{0,f}} \sum_{\omega \in \{G, B\}} \pi^\omega \left(\frac{C_n(\omega)C_t(\omega)}{C_n^\omega C_t^\omega}\right)^\frac{1}{\gamma} \]

s.t. \[ C_n^G = R_nk_n \]

\[ C_t^G = R_tk_n - d_{0,f} - \rho k_n \]

\[ C_n^B = k_n(R_n - \rho\Delta_n(1 - \theta_n^B)) \]

\[ C_t^B = k_tR_t - \rho\theta_n^B k_n - d_{0,f} \]

\[ \theta_n^B = \frac{\lambda_n^B R_t K_t - D_{0,f}}{\rho} \]

\[ D_{0,f} < \min\left\{ \frac{\lambda_t^\omega R_t K_t}{1 - \rho^\omega}, \frac{R_n}{1 - \rho^\omega} \theta_n^B K_n \right\} \]

\[ K_n + K_t = W_0 + D_{0,f} \]

A few remarks first. (PA1) incorporates both the domestic and international collateral constraints. Thus, the central planner is operating under the same constraints as agents
in the decentralized equilibrium. Prices do not appear in either objective or constraints - except for the no-default debt constraint. Moreover, this constraint is the only one in which \( \lambda_n \) appears. As we showed earlier, this constraint will never bind.

Let us next rewrite this program so that it can be compared to the decentralized solution, (P5), more easily. In doing this it is easier to rewrite (PA1) and reintroduce prices, thereby getting (P4):

\[
\begin{align*}
(P4) \quad \max_{K_n, K_t, D_{0,f}} & \quad \sum_{\omega \in \{G,B\}} \pi^\omega e(\omega) \frac{3}{2} w_2^\omega(\omega) \\
\text{s.t.} & \quad w_2^B(B) = R_t K_t + \left( \frac{R_n - \rho \Delta_n}{\epsilon(B)} \right) K_n - D_{0,f} + \left( s_B^B + s_B \right) \left( \lambda_t^B R_t K_t - D_{0,f} \right) \\
& \quad w_2^G(G) = R_t K_t + \frac{R_n K_n}{\epsilon(e)} - D_{0,f} - \rho K_n \\
& \quad D_{0,f} \leq \min_\omega \{ \chi^\omega_n R_t K_t + \lambda_n \frac{\tilde{R}_n(\theta_n^e)}{1 - \epsilon^e} K_n \} \\
& \quad K_n + K_t = W_0 + D_{0,f}
\end{align*}
\]

(P4) arrives from substituting back in expressions for the exchange rate into the FOC for (PA1).

First write (PA1) as follows,

\[
\begin{align*}
\max_{K_n, K_t, D_{0,f}, c_{n,t}, c_{t,t}, c_{n,s}, c_{t,s}} & \quad \sum \pi^\omega \left[ \rho^\omega_n (c_{n,t}^\omega, c_{t,t}^\omega) \right] + (1 - \rho) (c_{n,s}^\omega, c_{t,s}^\omega) \\
\text{s.t.} & \quad \rho c_{n,t}^\omega + (1 - \rho) c_{t,t}^\omega = C_t^\omega \\
& \quad \rho c_{n,s}^\omega + (1 - \rho) c_{t,s}^\omega = C_n^\omega \\
& \quad C_n^G = R_n k_n \\
& \quad C_t^G = R_t k_t - d_{0,f} - \rho k_n \\
& \quad C_n^B = k_n (R_n - \rho \Delta_n (1 - \theta_n^B)) \\
& \quad C_t^B = k_t R_t - \rho \theta_n^B k_n - d_{0,f} \\
& \quad \theta_n^B = \lambda_t^B R_t K_t - D_{0,f} \\
& \quad D_{0,f} \leq \min_\omega \{ \chi^\omega_n R_t K_t + \lambda_n \frac{\tilde{R}_n(\theta_n^e)}{1 - \epsilon^e} K_n \} \\
& \quad K_n + K_t = W_0 + D_{0,f}
\end{align*}
\]

Let \( \alpha = (K_n, K_t, D_{0,f}) \) be the vector of date 0 choices, and rewrite the problem as,

\[
\begin{align*}
\max_{\alpha, c_{n,t}, c_{t,t}, c_{n,s}, c_{t,s}} & \quad \sum \pi^\omega \left[ \rho^\omega_n (c_{n,t}^\omega, c_{t,t}^\omega) \right] + (1 - \rho) (c_{n,s}^\omega, c_{t,s}^\omega) \\
\text{s.t.} & \quad \rho c_{n,t}^\omega + (1 - \rho) c_{t,t}^\omega = C_t^\omega(\alpha) \\
& \quad \rho c_{n,s}^\omega + (1 - \rho) c_{t,s}^\omega = C_n^\omega(\alpha) \\
& \quad F(\alpha) = 0
\end{align*}
\]

where \( F(\alpha) \) reflects the date 0 constraints. Forming the Lagrangian,

\[
\mathcal{L} = \sum \pi^\omega \left[ \rho^\omega_n (c_{n,t}^\omega, c_{t,t}^\omega) \right] + (1 - \rho) (c_{n,s}^\omega, c_{t,s}^\omega) - \mu_t^\omega (\rho c_{n,t}^\omega + (1 - \rho) c_{t,t}^\omega - C_t^\omega(\alpha)) - \mu_n^\omega (\rho c_{n,s}^\omega + (1 - \rho) c_{t,s}^\omega - C_n^\omega(\alpha)) - \mu_F F(\alpha)
\]

which gives the F.O.C.’s,

\[
\frac{1}{2} \left( \frac{c_{n, t}^\omega}{c_{t, t}^\omega} \right) = \mu_t^\omega
\]

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\[
\frac{1}{2} \left( \frac{c_{\omega, i}}{c_{i, i}} \right)^{1/2} = \mu_{i}^{\omega}
\]
\[
\frac{1}{2} \left( \frac{c_{\omega, s}}{c_{i, s}} \right)^{1/2} = \frac{1}{\mu_{n}^{\omega}}
\]
\[
\frac{1}{2} \left( \frac{c_{\omega, i}}{c_{i, i}} \right)^{1/2} = \frac{1}{\mu_{n}^{\omega}}
\]
\[
\mu_{F}^{\omega} \frac{\partial C_{n}^{\omega} (\alpha)}{\partial \alpha} + \mu_{n}^{\omega} \frac{C_{n}^{\omega} (\alpha)}{\partial \alpha} = \mu_{F}^{\omega} \frac{\partial F(\omega)}{\partial \alpha}
\]

Notice that \( \mu_{i}^{\omega} = \frac{1}{\mu_{n}^{\omega}} = e(\omega)^{1/2} \). Substituting this into the last constraint,
\[
\frac{1}{2} e(\omega)^{1/2} \left( \frac{\partial C_{n}^{\omega} (\alpha)}{\partial \alpha} + \frac{1}{e(\omega)} \frac{C_{n}^{\omega} (\alpha)}{\partial \alpha} \right) = \mu_{F}^{\omega} \frac{\partial F(\omega)}{\partial \alpha}
\]

Which is just,
\[
\frac{1}{2} e(\omega)^{1/2} \frac{\partial w_{s}^{\omega} (\omega)}{\partial \alpha} = \mu_{F}^{\omega} \frac{\partial F(\omega)}{\partial \alpha}
\]

This is the same F.O.C. as (P4) gives.

### 6.3 Proof of Proposition 4

Consider the Lagrangian of (P5),
\[
\mathcal{L} = \sum_{\omega \in \{G, B\}} \pi^{\omega} e(\omega)^{1/2} w_{2}(\omega) - \mu (k_{n} + k_{t} - w_{0} - d_{0,f})
\]

Which gives F.O.C.’s,
\[
\frac{\partial \mathcal{L}}{\partial k_{n}} = \pi^{G} e(G)^{1/2} \frac{\partial w_{2}^{G} (G)}{\partial k_{n}} + \pi^{B} e(B)^{1/2} \frac{\partial w_{2}^{B} (B)}{\partial k_{n}} - \mu = 0
\]
\[
\frac{\partial \mathcal{L}}{\partial k_{t}} = \pi^{G} e(G)^{1/2} \frac{\partial w_{2}^{G} (G)}{\partial k_{t}} + \pi^{B} e(B)^{1/2} \frac{\partial w_{2}^{B} (B)}{\partial k_{t}} - \mu = 0
\]
\[
\frac{\partial \mathcal{L}}{\partial d_{0,f}} = \pi^{G} e(G)^{1/2} \frac{\partial w_{2}^{G} (G)}{\partial d_{0,f}} + \pi^{B} e(B)^{1/2} \frac{\partial w_{2}^{B} (B)}{\partial d_{0,f}} + \mu = 0
\]

Denote the solutions to these equations as \((k_{n}, k_{t}, d_{0,f})\). Next consider the F.O.C.’s to the Lagrangian of (P4),
\[
\frac{\partial \mathcal{L}^{*}}{\partial k_{n}} = \pi^{G} e(G)^{1/2} \frac{\partial w_{s}^{G} (G)}{\partial k_{n}} + \pi^{B} e(B)^{1/2} \frac{\partial w_{s}^{B} (B)}{\partial k_{n}} - \mu = 0
\]
\[
\frac{\partial \mathcal{L}^{*}}{\partial k_{t}} = \pi^{G} e(G)^{1/2} \frac{\partial w_{s}^{G} (G)}{\partial k_{t}} + \pi^{B} e(B)^{1/2} \frac{\partial w_{s}^{B} (B)}{\partial k_{t}} - \mu = 0
\]
\[
\frac{\partial \mathcal{L}^{*}}{\partial d_{0,f}} = \pi^{G} e(G)^{1/2} \frac{\partial w_{s}^{G} (G)}{\partial d_{0,f}} + \pi^{B} e(B)^{1/2} \frac{\partial w_{s}^{B} (B)}{\partial d_{0,f}} + \mu = 0
\]
We can evaluate the partial derivatives in $\mathcal{L}^*$ at the choices $(k_n, k_t, d_{0,f})$.

\[
\begin{align*}
\frac{\partial \mathcal{L}^*}{\partial k_n} &= \frac{\partial \mathcal{L}}{\partial k_n} - \beta e(B) \gamma \lambda_n k_n < 0 \\
\frac{\partial \mathcal{L}^*}{\partial k_t} &= \frac{\partial \mathcal{L}}{\partial k_t} + \beta e(B) \gamma s_d (1 - \gamma) k_t^B > 0 \\
\frac{\partial \mathcal{L}^*}{\partial d_{0,f}} &= \frac{\partial \mathcal{L}}{\partial d_{0,f}} - \beta e(B) \gamma s_d (1 - \gamma) < 0
\end{align*}
\]

Since, $s_d^B > 0$ and $0 < \gamma < 1$, we can conclude that welfare can be improved by choosing at least one of,

\[
\begin{align*}
d'_{0,f} &\leq d_{0,f} \\
k'_n &\leq k_n \\
k'_t &\geq k_t
\end{align*}
\]

But by the budget constraint, if one of these holds strictly, at least one of the others must do as well.

### 6.4 Contingent Debt Contracts

In this section we rewrite the problem of sections 3 and 4 to include debt contracts that are written contingent on the state of the world $\omega \in \{G, B\}$. The no-default constraint is now written as,

\[
d'_{0,f} \leq \lambda_t^\omega R_t k_t + \frac{r_n k_n}{L^\omega e^\omega}
\]

The problem at date 1 is only altered by the fact that date 0 decisions result in a different amount of date 1 debt international collateral than would be the case if debt contracts were not contingent.

We can rewrite the date 0 program of (P5) as,

\[
(P5C) \quad \max_{k_n, k_t, d'_{0,f}} \quad \sum_{\omega \in \{G, B\}} \pi^\omega e(\omega) \frac{1}{2} w_2(\omega)
\]

\[
s.t. \quad w_2(G) = R_t k_t + \frac{r_n k_n}{e^G} - d'_{0,f} - \rho k_n \\
w_2(B) = R_t k_t + \frac{(r_n - \rho \Delta_n) k_n}{e^B} - d'_{0,f} \\
\quad \quad \quad + \left( s_d^B + \rho \psi s_d^B \right) (\lambda_t^B R_t k_t - d'_{0,f}) \\
\quad \quad \quad \quad + \left( \rho \psi s_d^B \right) \frac{r_n k_n}{e^B [L^\omega e^\omega]}
\]

\[
d'_{0,f} \leq \lambda_t^\omega R_t k_t + \lambda_n \frac{r_n k_n}{L^\omega e^\omega} \\
k_n + k_t = w_0 + E^\omega[d'_{0,f}]
\]

Allowing for contingency does increase the amount of insurance possible by the domestic economy by offsetting some of the fall of $\lambda_t^\omega$. However there will still be under-insurance
relative the central planners problem as the basic externality is unaffected. We can rewrite
the central planners problem for a comparison as,

$$(P4C) \quad \max_{K_0, K_t, D_{0,f}} \sum_{\omega \in \{G, B\}} \pi^\omega e(\omega) \frac{1}{T} \pi^\omega w^\omega(B)$$

s.t. \quad w^\omega(B) = R_t K_t + \frac{R_s K_s}{e(B)} - D_{0,f}^B + \left(s^B + s_f^B\right) (\lambda^R_t R_t K_t - D_{0,f}^R)

w^\omega(G) = R_t K_t + \frac{R_s K_s}{e(G)} - D_{0,f}^G - \rho K_n

D_{0,f}^\omega < \lambda^R_t R_t K_t + \lambda_n \frac{\theta_n}{L^\omega} K_n

K_n + K_t = W_0 + E_\omega[D_0,f]

It is apparent that the expressions for $w^\omega(B)$ and $w^P_2$ are different - again reflected in the
domestic spread. For the same reasons as in the text, this will result in undervaluation of
international collateral.

6.5 Supporting Formulae for Banking Section

The solution to program $(P6)$ is determined by the binding constraints. Constraint (i)
requires that loans satisfy,

$$\left[ x^1_{1,d}(\omega) \leq \frac{a_t}{\alpha} \right].$$

Given an amount of loans, the demand for deposits and foreign borrowing by banks will be
determined by the rate of interest,

$$L^\omega_D > 1 \quad \Rightarrow \quad x^b_{1,d}(\omega) = d^1_{1,d}(\omega) + a_t - \phi_f^b$$

$$L^\omega_D = 1 \quad \Rightarrow \quad x^b_{1,d}(\omega) \leq d^1_{1,d}(\omega) + a_t - \phi_f^b.$$

Turning next to the decisions of firms, demand for loans by distressed firms and deposits
from intact firms follow from the same logic as before (see $P1$ and $P2$). For ease of exposition,
we make one notational change with respect to sections 2 and 3. There, we had denominated
the quantity of debt (supplied by distressed firms and demanded by intact firms) in units
of face value of date 2 T-goods. In what follows, the quantity of debt is in terms of the date
1 present value of debt. Intact firms extend deposits to banks of $x_{1,d}(\omega)$,

$$L^\omega_D > 1 \quad \Rightarrow \quad x_{1,d}(\omega) = \lambda^R t R_t k_t - d_{0,f} - a_t$$

$$L^\omega_D = 1 \quad \Rightarrow \quad x_{1,d}(\omega) \leq \lambda^R t R_t k_t - d_{0,f} - a_t.$$

Distressed firms take out bank loans of $d_{1,d}(\omega)$ to save production units,

$$\frac{\Delta_n}{e^\omega} > L^\omega \quad \Rightarrow \quad d_{1,d}(\omega) = \max[k_n - d^\omega_{1,f}, 0],$$

$$\frac{\Delta_n}{e^\omega} = L^\omega \quad \Rightarrow \quad d_{1,d}(\omega) \leq \max[k_n - d^\omega_{1,f}, 0].$$

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Vis-a-vis the previous analysis, market clearing conditions need to be modified slightly to take into account banks. The aggregate amount of deposits in banks must equal the aggregate deposits of the intact firms,

\[ d_{1,d}^1(\omega) \equiv D_{1,d}^0(\omega) = X_{1,d}(\omega) = (1 - \rho)x_{1,d}(\omega) \]

Likewise, the total loan supply from banks must equal the new domestic debt issued by distressed firms,

\[ d_{1,d}^b(\omega) \equiv X_{1,d}^b(\omega) = D_{1,d}(\omega) = \rho d_{1,d}^0 \]

Taken together and combined with the capital constraint on banks this implies that,

\[ \lambda_t^\omega R_t k_t - d_{0,f}^b - d_{0,f}^b \leq \frac{d_{0,f}^b}{\alpha} \Rightarrow d_{1,d}(\omega) = (1 - \rho)x_{1,d}(\omega) \]

\[ \lambda_t^\omega R_t k_t - d_{0,f}^b - d_{0,f}^b > \frac{d_{0,f}^b}{\alpha} \Rightarrow d_{1,d}(\omega) = (1 - \rho)x_{1,d}^\omega = \frac{d_{0,f}^b}{\alpha} \]

Consider the first inequality. When the capital requirement on banks does not bind, banks aggregate domestic collateral perfectly – they channel all of the excess debt capacity of the intact firms to the distressed firms. The economy behaves exactly as if \( \lambda_n = 1 \) in a public debt market (section 2.4). Asset prices and investment depend on the international collateral constraint,

\[ \rho k_n < \lambda_t^\omega R_t k_t - d_{0,f} - d_{0,f}^b \Rightarrow L^\omega = L_D^\omega = 1 \quad \theta_n^\omega = 1 \]

\[ \rho k_n \geq \lambda_t^\omega R_t k_t - d_{0,f} - d_{0,f}^b \Rightarrow L^\omega = L_D^\omega = \frac{\Delta_n}{\varepsilon^\omega} \quad \theta_n^\omega < 1. \]

When the capital requirement \((i)\) binds, the economy can fall into regions \((iii)\) or \((iv)\) depending on how tightly it binds relative to demand from distressed firms. In this case, \( L_D^\omega = 1 \) and:

\[ \rho(k_n - (\lambda_t^\omega R_t k_t - d_{0,f} - A_t)) < \frac{d_{0,f}^b}{\alpha} \Rightarrow L^\omega = 1 \quad \theta_n^\omega = 1 \]

\[ \rho(k_n - (\lambda_t^\omega R_t k_t - d_{0,f} - A_t)) \geq \frac{d_{0,f}^b}{\alpha} \Rightarrow L^\omega = \frac{\Delta_n}{\varepsilon^\omega} \quad \theta_n^\omega < 1. \]
References


