Matching and Agglomeration: Theory and Evidence from Japanese Firm-to-Firm Trade∗

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Abstract

Why are economic activities geographically concentrated? In this paper, I argue that increasing returns in firm-to-firm matching is an important source of agglomeration. I open by providing reduced-form evidence of increasing returns in matching using a panel of firm-to-firm trade data covering over a million Japanese firms. Using unexpected supplier bankruptcies as an instrument, I show that the new supplier matching rate upon a supplier loss increases in locations and industries when there are more alternative suppliers selling in the buyer’s location, while this rate remains stable in the presence of other buyers looking for a match. Based on these findings, I develop a new structural trade model that incorporates dynamic firm-to-firm matching across space in a standard Melitz model. In this economy, the presence of more input sellers increases input buyers’ aggregate sales by improving the supplier matching rates and hence reducing their production cost; this, in turn, attracts more suppliers to sell in the location. I calibrate the model to match the reduced-form estimates, and I show that this type of circular causation explains 7% and 16% of spatial inequality of the firm density and the real wages in Japan, respectively. Lastly, I analyze policies to promote economically lagged areas, and I find that (1) subsidies for input suppliers to sell in the target areas are much more effective in improving the welfare of these areas than subsidies to produce in these areas, and (2) improving transportation infrastructure initially decreases and then increases the welfare of the target areas.

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1 Introduction

Economic activities are geographically concentrated. In the case of Japan, 50% of the firms are concentrated in only 3% of the usable area, as observed in Figure 1. There is no shortage of theories that explain why agglomeration may occur. However, there is much less consensus about the empirical and quantitative relevance of the various mechanisms that the literature proposes.

In this paper, I focus on one such mechanism for agglomeration: firms find input suppliers more easily in denser areas. Although this is one of the most classical ideas dating back to Marshall (1890), empirical evidence is limited beyond a cross-sectional correlation (Holmes (1999)). In this paper, I first provide new reduced-form evidence of this agglomeration mechanism. Based on the reduced-form evidence, I develop a new structural trade model to quantify the importance of this mechanism in explaining the spatial distribution of economic activities, as well as understanding its distinctive policy implications specific to this agglomeration mechanism.

The first part of this paper uses a panel of firm-to-firm trade data for over one million Japanese firms to provide reduced-form evidence of increasing returns in firm-to-firm matching. These data allow me to estimate the matching rate with new suppliers upon a supplier loss. If the matching rate improves in the presence of more alternative suppliers, but does not decrease as much by the presence of other buyers, there is evidence of increasing returns in matching. To ensure the exogeneity of a supplier loss, I select plausibly unexpected bankruptcies of suppliers – accidental reasons (CEO death, natural disaster, etc.,) and spillovers from other bankruptcies – that are reported in the data set, and use them as instruments for a supplier loss.

The results are summarized as follows. First, I find that the new supplier matching rate increases in the density of alternative suppliers transporting to sell to the buyer’s location (i.e., the number of suppliers that already serve other firms in the buyer’s location). The magnitude is large: while a loss of one supplier leads to 0.16 new supplier matching per year within the same input sector on average, the matching rate is as high as 0.24 at the 75th percentile and as low as 0.08 at the 25th percentile of the density of alternative suppliers. This heterogeneity of new supplier matching rate is robust by just using within-location-and-buyer-industry variation, i.e., by comparing firms in the same location and industry that loses a supplier in different input industries with a varying supplier density. This alleviates the concern that firm’s selective entry drives the results; i.e., firms which enter in urban areas

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1 Duranton and Puga (2004); Rosenthal and Strange (2004) and Head and Mayer (2004) provide a review.  
2 The data set has been used by several previous papers, including Nakajima et al. (2012); Bernard et al. (2015); Carvalho et al. (2016), and Furusawa et al. (2017).
may be better at supplier matching than those in rural areas.

Second, I find that among various definitions of supplier density, what directly matters for the supplier matching rate is the density of alternative suppliers transporting to sell to the buyer’s location as defined earlier, and once I condition on this, the density of suppliers established and producing in the buyer’s location does not matter. In other words, the first type of supplier density is a sufficient statistic that governs the supplier matching rate. This finding is important, as a typical model in the literature used for spatial policy analysis assumes that agglomeration benefit arises from the latter, often without specifying agglomeration mechanisms. I will investigate the differential policy implications more concretely with a structural model in the latter part of the paper.

Third, I find that conditional on the density of suppliers, the density of other buyers looking for a match does not affect the matching rate. This implies that buyers do not crowd out each other for matching with a supplier. This is in a stark contrast to firm-to-worker matching in the labor market context, where the presence of unemployed workers is often found to decrease other unemployed workers’ reemployment rate. These differences are intuitive. In the context of firm-to-firm matching, suppliers can simultaneously serve multiple buyers without inducing crowding out among buyers, while in the labor market, a vacant job can be filled by only one unemployed worker, necessarily creating crowding out.

Taking all the evidence together, there is evidence of increasing returns in matching: the presence of more suppliers and more buyers improves the firm-to-firm matching rate. Additional findings confirm that such firm-to-firm matching is important for firm production: I find that a supplier bankruptcy leads to a lower sales growth and a higher exit probability of buyer-side firms through an imperfect supplier recovery, and firms do not cope with these supplier bankruptcies by substituting inputs from other existing suppliers.

The second part of the paper develops a structural model to quantify the importance of increasing returns in matching as a source of agglomeration. The model extends a multi-location multi-sector Melitz model to incorporate dynamic firm-to-firm matching in input trade across space. As in a standard Melitz model, potential producers enter in each location by paying a fixed cost and draw an idiosyncratic productivity; upon the realization of this productivity draw, they make a decision to sell into various locations by paying a fixed marketing cost. In addition to these standard assumptions, firms require inputs for production, which they can source from matched suppliers. The matching rate with a supplier increases in the number of suppliers selling in the location but it is unaffected by the number of input buyers in the location; this assumption is in line with the empirical evidence.

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3See, for example, Allen and Arkolakis (2014); Kline and Moretti (2014a); Monte et al. (2015); Ahlfeldt et al. (2015); Faber and Gaubert (2016); Nagy (2017). Redding and Rossi-Hansberg (2016) provide a survey.

4See Petrongolo and Pissarides (2001) for a survey on this literature.
findings of increasing returns in matching in the first part of the paper.

The model exhibits agglomeration through circular causation between the measure of input sellers and downstream market size. In a location with more input sellers, input buyers enjoy a higher supplier matching rate and hence a cost advantage, i.e., a “forward linkage”. This, in turn, creates a larger market for suppliers and encourages more supplier to sell in the location, i.e., a “backward linkage.” The key parameter that governs this circular causation is the elasticity of the supplier matching rate with respect to the number of potential suppliers. I estimate this parameter to match the reduced-form estimates from the first part of the paper.

The first takeaway from the structural estimation is the quantification exercise to understand how important the increasing returns to scale in firm-to-firm matching is in explaining the equilibrium spatial distribution of economic activities. To ask this question, I simulate a counterfactual equilibrium by hypothetically shutting down the increasing returns to scale in matching, i.e., assuming that the elasticity of the supplier matching rate with respect to the supplier density is 0, unlike the estimates of 0.36 from the structural estimation. I find that, under this counterfactual world, the standard deviation of firm density across space would be 7% smaller and that of real wages would be 16% smaller. The remaining geographic variations are induced by the exogenous population distribution and the location and sector-specific productivity heterogeneity, including natural advantages. These results imply that a non-negligible part of the geographic concentration of firms can be explained by increasing returns in firm-to-firm matching.

The second takeaway from the structural estimation is the policy implications. In particular, I study policies that are targeted to promote one of the most economically lagged areas in Japan; Hokkaido area (Figure 1). Large economic disparity of economic activities is an important concern in Japan just as in many countries in the world, and policy makers are facing challenges to promote economically lagged areas. Two prominent examples of such policies are firm subsidy and transportation infrastructure improvement, and I analyze these policies through the lens of my estimated model.

I first analyze firm subsidy. Here, I consider two distinct types of firm subsidy: subsidies for input suppliers to sell in Hokkaido, and subsidies to produce in Hokkaido. Note that the former subsidies can go to firms regardless of their production location, as long as they make input sales in Hokkaido. In reality, the former subsidies are commonly implemented

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5To understand the magnitude, note that Ellison and Glaeser (1999) claim that about 20% of the spatial variation in firm density can be explained by natural advantages (i.e., exogenous location and sector productivity) by correlating the firm density and a proxy for natural advantages. The magnitude at which increasing returns to scale in matching explains the spatial distribution of firm density is quantitatively comparable to their number.
in the form of trade exhibitions or business matching events, and the latter in the form of tax exemptions or subsidies for new business establishment. I find that first type of subsidies is much more effective than the latter to improve the economic welfare of Hokkaido for the same dollar spent. The intuition is simple: because the agglomeration benefit from this mechanism arises from the density of input suppliers selling in the location, the subsidies should be directly targeted to this margin. This result has a strong implication on the recent discussions of place-based policies. Given the past lack of spatial models of agglomeration through firm-to-firm matching in input trade, the literature has only discussed the second type of subsidy, and the first type of subsidy has not attracted an attention despite the prevalence of such policies through business matching events.

Second, I analyze the impacts of transportation infrastructure that improves the access between Hokkaido and the main island of Japan. One may expect that such policy would benefit Hokkaido in a similar way as the subsidies for input sales. In fact, a new bullet train connected Hokkaido and the main island Japan (Tohoku area) in 2016 largely aiming for improving economic welfare in Hokkaido. Surprisingly, I find that the reduction of transportation costs initially decreases and then increases the economic welfare in Hokkaido. The intuition comes from the two counter-forces from transportation infrastructure improvement: On one hand, the reduction of transportation costs benefits firms in Hokkaido through the reduction of unit cost of input goods. On the other hand, reducing transportation costs harms firms by exposing them for more competition. While the first force is initially weak because the matching probability is close to 0, as the transportation cost decreases, more input suppliers sell in Hokkaido, which increases the supplier matching rate and exponentially increases the benefit from a marginal improvement of transportation infrastructure.

Agglomeration is a core issue intersecting in urban economics, economic geography, and international trade, and this paper contributes to these strands of the literature. First, this paper is directly related to the literature of the microfoundation of the agglomeration from increasing returns to scale in matching. In terms of empirics, the closest evidence is limited to a cross-sectional correlation between a fraction of purchased inputs per firm and spatial firm density in the United States (Holmes (1999)). In terms of theory, there are papers that embed increasing returns to scale in matching as a source of agglomeration (i.e., Diamond (1982); Helsley and Strange (1990)), but these models do not incorporate geography and cross-locational matching, which matters policy implications by differentiating the density of sellers and producers as mentioned earlier.

Second, this paper contributes to the literature of economic geography. There is a recent 6See Glaeser and Gottlieb (2008); Kline and Moretti (2014b); Neumark and Simpson (2015) for a review of recent discussions on place-based policies.
wave of quantitative spatial economic models to incorporate realistic geography in theoretical models of the New Economic Geography (NEG) literature as surveyed in Redding and Rossi-Hansberg (2016). This paper’s contribution is to explicitly model a particular micro-foundation of agglomeration and study its distinct policy implications.

Third, this paper is related to several sub-fields of international trade. First, this paper is related to the literature of firm sourcing behavior, with particular emphasis on geographic proximity (Antràs et al. (2014); Bernard et al. (2015, 2016); Blaum et al. (2016); Furusawa et al. (2017)). Second, this paper is related to the literature on firm-to-firm trade network formation (Oberfield (2013); Eaton et al. (2016b); Lim (2016); Tintelnot et al. (2017)). Third, it is related to the literature that studies search and matching frictions in trade relationships (Allen (2014); Startz (2016); Eaton et al. (2016a); Krolikowski and McCallum (2017); Brancaccio et al. (2017)).

The rest of the paper is organized as follows. Section 2 provides the description of the main data set used in this paper, a panel of firm-to-firm trade of over a million Japanese firms. Section 3 provides reduced-form evidence of increasing returns in firm-to-firm matching. Section 4 develops a structural trade model to understand the implications of increasing returns in matching for agglomeration patterns. Section 5 structurally estimates the key parameters of the model and presents the procedures for computing counterfactual equilibrium. Section 6 presents three counterfactual simulations to illustrate the quantitative implication of the agglomeration forces. Section 7 concludes.

2 Data and Descriptive Facts

In this section, I briefly describe this paper’s main data set, a panel of firm-to-firm trade between over one million Japanese firms. I also document a set of descriptive facts that motivates the empirical exercise in Section 3 and the structural model in Section 4.

2.1 Firm-to-Firm Trade Data in Japan

The main data set utilized in this paper comes from a major credit reporting agency, Tokyo Shoko Research (TSR). The data is based on face-to-face and phone interviews, as well as public resources such as financial statements, corporate registrations, and public relations documents. The panel covers three waves, 2006, 2011, and 2014, and it contains basic firm-
level characteristics as well as the precise locations of firm headquarters.

The most important feature of the data set is that it contains dynamic transitions of supplier-buyer relationships. In each period and for each firm, the data reports up to 24 suppliers and buyers for each firm. The information is collected in annual interviews by field surveyors of TSR by asking whether the relationship from previous years continues to exist or whether a new relationship has been initiated. Figure A.1 reports that, while the upper-bound of 24 suppliers is not binding for most firms, there are a non-negligible number of cases with more than 24 suppliers once I include the number of supplier-buyer relationships reported by supplier-side firms (but not by buyer-side firms). Following other recent papers that utilize the same data set (e.g., Carvalho et al. (2016); Bernard et al. (2016)), I define a supplier-buyer relationship to exist if either the supplier-side or the buyer-side firm reports a relationship in each period.

Another important feature of the data set is that it reports the main reasons for bankruptcy in each case. This information is also collected through the interviews by TSR. Table I reports the list of reasons recorded in this data set. In Section 3, I make use of this information as an instrument when estimating matching rate with new suppliers upon a supplier loss.

2.2 Descriptive Patterns of Firm-to-Firm Matching

This section provides a brief overview of the descriptive patterns of the data that motivates the empirical exercise and the structural model in the subsequent sections.

First, an extensive margin of firm-to-firm trade is geographically concentrated, with non-negligible long-distance trade. As is already documented in Nakajima et al. (2013) and Bernard et al. (2016), who use the same data set, and as is replicated in Figure 2, the median of the geodesic distances between buyers and suppliers across all three years is about 25 km, which is much smaller than that of all possible pairs of firms in Japan (464 km). At the same time, the 75th percentile of the geodesic distances of firm-to-firm trade is about 250 km, suggesting that firms are not constrained to trade within locations. These observations are naturally modeled by market penetration decisions à la Melitz (2003) as pursued in Section 4; firms decide their production locations, and based on the production locations, firms engage in input sales in various locations under geographic frictions.

Second, firms in denser areas have more suppliers. Figure 3 shows that it is true, and Table A.1 further confirms that this correlation is robust to various controls, including

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8For the empirical analysis of this paper, I define the location of the firm by its headquarter location. I show robustness by restricting the samples to firms with few establishments to minimize the mismeasurement of firm locations.
industry fixed effects and employment size fixed effects. Together with the strong cross-sectional correlation between number of suppliers and revenue productivity per worker, the results are at least consistent with increasing returns in matching; in denser areas where there are more potential supplier and buyers, the probability that downstream firms match with suppliers for each input sector is high. At the same time, the patterns may be driven by endogenous firm entry resulting from unobserved input demand; i.e., firms who unobservably demand more input goods for production may selectively enter in denser locations.

Third, firms in denser areas have a higher rate of matching with new suppliers, while the separation rate with existing suppliers does not decrease with firm density. From a dynamic point of view, the cross-sectional correlation between the number of suppliers and firm density can be driven either by a higher rate of matching with suppliers or a lower rate of separation with existing suppliers in denser areas. Figure 4 shows that the former is increasing in firm density while the latter is flatter (with a U-shape), indicating that the new matching rate is the key driving force of cross-sectional correlation between the number of suppliers per firm and firm density.

In sum, the descriptive patterns are consistent with agglomeration benefits from increasing returns in firm-to-firm matching; in denser areas there are more buyers and suppliers, giving a better chance of matching with a supplier for each input sector. At the same time, the patterns may be driven by endogenous firm entry resulting from unobserved demand for supplier matching. Testing increasing returns in matching requires, to a first order, identifying a case where firms in different locations are equally in search of suppliers. Section 3 explores this point by studying the matching rate with new suppliers upon an exogenous loss of a supplier.

9A similar finding has been documented in the literature, i.e., externally sourced input share of firms is positively correlated with firm density in the United States (Holmes (1999)).
10Figure A.2 provides a raw correlation; Table A.2 shows that the correlation is not driven by various controls. A similar cross-sectional correlation has been documented by Bernard et al. (2016).
11Firms have suppliers in various input sectors. On average firms have 4.6 suppliers across all three years, and they have suppliers in 2.3 two-digit input sectors and 3.1 four-digit input sectors (Panel (A) of Table 2). There is also a wide dispersion in the number of suppliers and input sectors across firms, as shown in Figure A.3.
12To see this point more formally, note that the following accounting relationship holds at the firm-level:

\[
\text{Number of Suppliers}_{t+1} = \text{Number of Suppliers}_t \times (1 - \text{Probability of Supplier Separation}_t) + \text{Number of New Suppliers}_t.
\]

If a firm has a steady-state number of suppliers, i.e., Number of Suppliers\(_{t+1} = \text{Number of Suppliers}_t\), then it follows that

\[
\text{Number of Suppliers}_t = \frac{\text{Number of New Suppliers}_t}{\text{Probability of Supplier Separation}_t},
\]

hence, the variation in the number of suppliers per firm is decomposed into number of new suppliers and probability of supplier separation.
3 Reduced-Form Evidence of Increasing Returns in Firm-to-Firm Matching

This section provides reduced-form evidence of increasing returns in firm-to-firm matching. Section 3.1 discusses the main empirical strategy; estimating the new supplier matching rate upon a supplier loss instrumented by unexpected supplier bankruptcies. Section 3.2 presents the main results, and Section 3.3 shows additional results on other outcome variables to confirm the importance of matching frictions in this context.

3.1 Empirical Strategy

While no formal test of increasing returns in matching has been conducted in the context of firm-to-firm matching in input trade, that of the labor market has been studied extensively. As reviewed in Petrongolo and Pissarides (2001), early literature uses aggregate relationship between the number of unemployed workers and the job vacancies, while more recent literature uses micro data and estimates reemployment probabilities as a function of the number of job vacancies and unemployed workers in their local areas. For example, Petrongolo (2001) finds that while more job vacancies increase the reemployment rates, more unemployed workers crowd out the reemployment rates at a similar magnitude, supporting the constant returns in matching.

Testing increasing returns in firm-to-firm matching follows the same idea. I estimate the matching rate with new suppliers upon an exogenous supplier loss. If the matching rate improves in the presence of more alternative suppliers, but does not decrease as much by the presence of other buyers, there is evidence of increasing returns in matching. Exogeneity of a supplier loss is important; if the separation is initiated by the buyer-side firm, it may not be in need and search of alternative suppliers, which biases the matching rate estimates. For this purpose, I instrument a supplier loss by unexpected supplier bankruptcies – accidental reasons (CEO death, natural disaster, etc.,) and spillovers from other bankruptcies – from the

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13See, for example, Blanchard and Diamond (1989b,a) for an early attempt to estimate matching functions using aggregate data.

14Other papers that study reemployment probability as a relationship with local market conditions include Bleakley and Lin (2012) and Macaluso (2016). In a similar vein, Jäger (2016) estimates the implication of an unexpected worker death on new hires and demonstrates how it depends on the presence of alternative workers with similar skill sets.
reasons of bankruptcies reported in the data. The validity of these instruments is further confirmed by testing the absence of pre-trends in the outcome variables.

To proxy for the density of alternative suppliers, I count the number of firms for each input industry that supply to some firms in the buyer’s location in the baseline period (2006), which I call “locally-selling suppliers.” Reflecting the nonnegligible presence of long-distance trade as reported in Section 2.2, the proxy includes firms which are established anywhere and does not restrict to local suppliers; in fact, I show that the matching rate is increasing in this proxy, but once it is conditioned, the simple density of suppliers physically established and producing in the buyer’s location does not matter. It should be also stressed that the proxy provides a location and input sector level variation. Such variation allows me to address the concern that firms in a particular location and sector are good at supplier matching in nature, by controlling for the buyer’s location and industry specific heterogeneous effects. Figure 5 provides visual illustrations of this proxy for two industries, the forestry industry and the steel industry; while the forestry industry has higher supplier density in the northern part of Japan, the steel industry has more suppliers in the south.

In turn, I examine various proxies for the density of buyers due to the lack of theoretical guidance on which buyer-side firms are competing each other for supplier matching. The proxies considered include the number of firms in buyer’s industry and location, the number of firms in buyer’s industry and location facing supplier separation in the same input sector, and the number of firms in buyers industry in any locations.

3.2 Main Results

This section presents the evidence of increasing returns in firm-to-firm matching. Section 3.2.1 argues that the new supplier matching rate upon a supplier loss increases in locations and industries when there are more alternative suppliers transporting to sell in the buyer’s location. Section 3.2.2 argues that this particular notion of supplier density is a sufficient statistic for new supplier matching rate; I show that once I condition on it, density of suppliers or firms which are established and producing in the buyer’s location does not matter for the matching rate. Lastly, Section 3.2.3 shows that conditional on the density of suppliers, the density of buyers does not matter for the matching rate.

Table 1 lists the full reasons of the bankruptcies. “Spillover from other bankruptcies” are those caused by management difficulties due to chain reactions such as business partners, subsidiary companies, related bankruptcies, voluntary liquidation, etc. “Accidental reasons” include those with unanticipated accidental problems such as the death of representatives, flood disaster, fire, earthquake, traffic accident, fraud, theft, embezzlement, etc. For “spillovers from other bankruptcies,” I omit the cases in which the buyer-side firms go bankrupt before the suppliers, thereby avoiding cases in which the “spillover” comes from the buyer-side firms in focus.
3.2.1 Matching Rate Increases with the Density of Suppliers

I first show that the new supplier matching rate upon an exogenous separation is higher with more potential suppliers. To show this, I run the following regression:

\[
NewSupplier_{fkt} = \beta_1 Separation_{fkt} + \beta_2 Separation_{fkt} \times \log \text{LocallySellingSuppliers}_{loc(f),k} \\
+ \delta_{loc(f),ind(f),k,t} + \varphi_t(BaselineSupplier_{fkt}) + \epsilon_{fkt},
\]

where \(f\) is the firm, \(k\) is the two-digit input sector of the JSIC industry classification, \(t\) is year (2006 and 2011). \(NewSupplier_{fkt}\) is the number of new suppliers that firm \(f\) matches per year from \(t\) to the next period (2006 to 2011 or 2011 to 2014), and \(Separation_{fkt}\) is the number of suppliers that firm \(f\) loses per year between \(t\) and the next period, including supplier exits and the dissolution of supplier-buyer relationships. \(LocallySellingSuppliers_{loc(f),k}\) is the proxy for the density of alternative suppliers transporting to sell as discussed in Section 3.1, the number of firms in input sector \(k\) that supply to any firms in firm \(f\)’s headquarter location \(loc(f)\) (defined by 0.5 degree grid cell) at the baseline year (2006). \(\delta_{loc(f),ind(f),k,t}\) is the fixed effects at the level of the location and sector of firm \(f\), input sector \(k\), and year \(t\), to make sure that the new supplier matching rate is identified as a comparison between firms in the same location and industry with a same-input-sector supplier with and without a supplier loss. \(\varphi_t(BaselineSupplier_{fkt})\) are the flexible controls for \(BaselineSupplier_{fkt}\), included as fixed effects, to control for the fact that firms with more baseline suppliers mechanically face a higher rate of supplier separation. \(Separation_{fkt}\) and its interaction with \(\log \text{LocallySellingSuppliers}_{loc(f),k}\) are instrumented by \(UnexpectedSupplierBankruptcy_{fkt}\), the number of unexpected supplier bankruptcies that firm \(f\) faces in input sector \(k\) per year from \(t\) to the next period, and its interaction with \(\log \text{LocallySellingSuppliers}_{loc(f),k}\). As reported in Table A.3, the first stage is strongly significant and is not driven by the pretrends. Standard errors are clustered at the firm level, and the regressions are weighted to equalize the weight at the firm and year level. To avoid the case that the results are solely driven by extremely large firms, I take out firms whose baseline number of suppliers is above the 99th percentile for each year.

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16 The industry classification follows the Japan Standard Industrial Classification (JSIC). There are 98 two-digit sectors and 1248 four-digit sectors in JSIC classification. JSIC roughly corresponds to International Standard Industrial Classification (ISIC), although they do not uniquely match. For more detail on JSIC, see http://www.soumu.go.jp/english/dgpp_ss/seido/sangyo/index.htm. Table A.5 reports the robustness checks for different levels of industry classification.

17 Note that the fixed effects \(\delta_{loc(f),ind(f),k,t}\) saturate \(LocallySellingSuppliers_{loc(f),k}\), so \(\beta_2\) is identified only off of the interaction, not by the baseline effects of \(LocallySellingSuppliers_{loc(f),k}\).

18 The reason why the first stage coefficient is less than 1 is because control firms lose suppliers at some rate at the same time as treated firms face a supplier bankruptcy.
Table 3 reports the baseline results. Column (1) starts with the specification with just the average impacts of a supplier loss. The point estimate indicates that a loss of one supplier per year leads to 0.16 more new supplier matching per year in the same two-digit input industry. The fact that it is significantly above 0 implies that firms recover suppliers; the fact that it is significantly below 1 implies that the recovery is far from perfect.\footnote{It should be noted that the estimates of the average effect reported in these tables are underestimated if there is misclassification of input industries. To see this, Table A.4 reports that the re-matching rate is higher than 0.16 when one takes the dependent variable to be the new supplier matching rate in all input sectors. However, the average rematching rate is still significantly below 1, indicating that the supplier rematching is imperfect.}

Columns (2) of Table 3 reports the results with heterogeneous impacts with respect to the number of potential suppliers.\footnote{The log density of locally-selling suppliers is normalized to be mean 0, hence the coefficients on the number of separated suppliers roughly reflect the average impacts of the supplier separation. The same is true for other interactions with the supplier separation.} The inter-quartile range of the log locally-selling suppliers is 2.6, implying that the impact of supplier separation increases by $0.06 \times 2.6 = 0.156$ by going from the 25th percentile to 75th percentile of the log number of potential suppliers. The magnitude is large relative to the average impact of supplier separation; it implies that the yearly recovery rate ranges from 24\% to 8\% by going from the 75th percentile to 25th percentile of the number of potential suppliers.

The rest of the columns show that the positive heterogeneous effects with respect to the locally-selling suppliers are robust. One may worry that these differential matching rates arise because firms in denser locations are better at matching with new suppliers by nature and not because they face more potential suppliers. To deal with these concerns, Column (3) includes the interaction between unexpected supplier bankruptcies (instruments) and firm $j$’s location and year fixed effects. The regression now utilize the within-location variation; two firms in the same location face an unexpected supplier bankruptcy in two different input industries. The results show that the heterogeneous effects with respect to the locally-selling suppliers are robust and still significant.\footnote{Because these added fixed effects saturate the number of unexpected supplier bankruptcy that is used for supplier separation, the average effects of supplier separation are not identified and omitted from the table.} Column (4) further controls for the heterogeneous effects at the level of prefecture and buyer’s industry for each year.\footnote{Region is a geographic administrative unit in Japan, with 8 regions in total. I take regions, rather than the 0.5 degree grid cells, to control for the heterogeneous effects at the level of region, buyer industry and year due to the lack of power by using the latter geographic unit.} The heterogeneous effects with respect to the number of potential suppliers are now identified by the within-location-and-buyer-industry variation, further ensuring that the results are not driven by the selective entry of firms. Furthermore, Column (5) shows that the differential matching rates are not driven by the firm sizes by controlling for the heterogeneous effects with respect to firm’s baseline employment size.
Lastly, I argue that the results are not driven by the correlated shocks over the supply chain, i.e., firms which face unexpected supplier bankruptcy are on a differential trend for supplier matching patterns. For example, if the underlying reason of the supplier bankruptcy is a natural disaster, it is plausible that the natural disaster also hits their buyers directly. To deal with this concern, Column (6) shows that the results are robust to controlling for the firm and year fixed effects to use only the within-firm-across-input-industry variation; i.e., the new supplier matching rate is identified as a comparison within a firm across different input sectors with and without a supplier loss. In addition, Columns (7) and (8) show that firms which face unexpected supplier bankruptcies from 2011 to 2014 do not show differentially higher rates of supplier matching from 2006 to 2011, and this lack of pre-trends hold for the heterogeneous effects. Hence, the results are not driven by the pre-trends.

**Other robustness results.** Table A.5 discusses additional robustness results. In (A) endogeneity of entry, I further deal with the concern related to the endogeneity of supplier density through endogenous firm entry. Of particular concern is a possibility that firms have heterogeneous comparative advantage in supplier matching rate in different input industries, and such unobserved comparative advantage is correlated with the composition of supplier densities across different input industries. To resolve these concerns, I remove the samples where the input sector is the primary sector for the buyer-side firms (Column 1 and 2), as well as only taking buyer-side firms whose CEO’s birth location is the same as the current firm headquarter location. These are the cases where the input sector of concern is not the primary driving force of the entry decision of buyer-side firms.

The rest of the robustness concerns are addressed as follows. In (B) endogeneity of supplier bankruptcy, I show that the results are robust by taking each reason of bankruptcy one-by-one (Columns 4 and 5), as well as controlling for the baseline solvency score of the suppliers (Column 6). (C) heterogeneity of input sectors makes sure whether a particular combination of supplier and buyer industry do not drive the results, by controlling input coefficients of the IO matrix, which proxies the importance of each input industry (Column 7), by controlling the number of suppliers all over Japan (Column 8), and by dividing by manufacturing and non-manufacturing sectors (Columns 9 and 10). The results are further robust to geographic and industry definitions, as well as various sample definitions (Panels D, E, and F).

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23 Another potential concern is the external validity; unexpected supplier bankruptcies are not representative of the supplier separation. To address this concern, Table A.6 shows the results by using supplier exits for any reasons as an instrument for supplier separation, rather than unexpected supplier bankruptcies. Although the presence of pre-trends makes it hard to draw precise causal estimates, the qualitative patterns are consistent with the results using unexpected supplier bankruptcies as an instrument: firms match more with new suppliers upon separation, and they do so more in the presence of more suppliers.
3.2.2 Density of Locally-Selling Suppliers is a Sufficient Statistics of the New Supplier Matching Rate

While the results from Section 3.2.1 confirms that new supplier matching rate upon a supplier loss is higher if the density of locally-selling supplier is higher, it remains to be investigated whether this particular notion of supplier density is the most relevant margin of supplier density among various definitions of supplier density.

To investigate this point, Table 4 scrutinizes what definition of supplier density is actually driving the heterogeneous matching rate. In Column (1) and (2), instead of the density of locally-selling suppliers, I study the heterogeneous rematching rate with respect to the density of firms in any industry whose headquarters are located in the buyer-side firm’s location (“Locally-Established Firms”), as well as the density of suppliers in each input industry established in buyer’s location (“Locally-Established Suppliers”). The results show that, while the rematching rate is higher if these proxies are higher (Columns 1 and 2), they lose significance once I include the interaction of supplier separation and the density of locally-selling suppliers, while that of the “Locally-Selling Suppliers” remains significantly positive (in Columns 3 and 4). This indicates that “Locally-Selling Suppliers” is a sufficient statistics that governs the supplier rematching rate. As mentioned earlier, this finding is important, as a typical model in the literature assumes that agglomeration benefit arises from the latter, often without specifying agglomeration mechanisms. I will investigate the differential policy implications more concretely with a structural model in the latter part of the paper.

While this result provides important policy implications by changing the margin from which agglomeration benefit arises, the results themselves are intuitive: Firms can match with suppliers at a distance, and therefore the relevant margin of supplier density should take such possibility of long-distance matching into account. In fact, in Table A.7 I show that more than half of the newly matched suppliers are outside 0.5 degree longitude times latitude grid cells, and these long-distance matches are what drives the patterns of heterogeneous effects of Table 4.

3.2.3 Matching Rate Does Not Decrease with the Density of Buyers

In this section, I test whether the matching rate decreases with the density of other buyers. The specification is as follows:

\[
\text{NewSupplier}_{fkt} = \beta_1 \text{Separation}_{fkt} + \beta_2 \text{Separation}_{fkt} \times \log \text{LocallySellingSuppliers}_{loc(f),k} \\
+ \beta_3 \text{Separation}_{fkt} \times \log \text{Buyers}_{loc(f),ind(f)} \\
+ \delta_{loc(f),ind(f),k,t} + \varphi_t(\text{BaselineSupplier}_{fkt}) + \epsilon_{fkt},
\]  
(3)
where \( \text{Buyers}_{\text{loc}(f),\text{ind}(f)} \) are various proxies of the number of buyers. One would expect \( \beta_3 < 0 \) if downstream firms crowd out each other for supplier matching. As in regression (2), \( \text{Separation}_{fkt} \) and all of its interactions are instrumented by \( \text{UnexpectedSupplierBankruptcy}_{fkt} \) and its relevant interactions.

The results reported in Table 5 indicate that there is no evidence of crowding out by other buyers. Using the number of firms in the same two-digit industry in firm \( f \)'s location (0.5 degree grid cells) as a proxy for the number of buyers (Column 1), as well as those in the same four-digit industries (Column 2), the coefficients \( \beta_3 \) are precisely estimated and close to 0, while \( \beta_2 \) remains significantly positive. In Column (3), I show that the results remain unchanged by taking the number of firms in firm \( f \)'s industry that also faced an unexpected supplier bankruptcy in the same input sector, exploiting the plausibly exogenous variation in the number of other buyers searching for suppliers. Column (4) shows that the results are also unchanged by counting the buyers in any industry, rather than restricting to those within firm \( f \)'s industry, as long as they face unexpected supplier bankruptcy in the same input sector \( k \). Furthermore, Columns (5) and (6) demonstrate that these results are unchanged when I count firms that faced supplier separation in input sector \( k \), not only those with unexpected supplier bankruptcies. Lastly, one may worry that buyers may be under competition with nonlocal buyers for matching with suppliers. Columns (7) and (8) show that the results are unchanged by defining the buyers as firms in firm \( f \)'s industry located anywhere that also faced an unexpected supplier bankruptcy in the same input sector.

The findings of no crowding out is in stark contrast to worker-to-firm matching in the labor market context, where the presence of unemployed workers is often found to decrease other unemployed workers’ reemployment rate.\(^{24}\) The differences come from the fact that, in firm-to-firm matching, suppliers can simultaneously serve multiple buyers without inducing crowding out among buyers; in the labor market, a vacant job can be filled by only one unemployed worker, necessarily creating crowding out. In other words, the fact that firms can share suppliers limits crowding-out by other buyers.\(^{25}\) In the model, the differences can be expressed that suppliers can simultaneously serve multiple buyers (i.e., many-to-one matching), unlike the one-to-one matching between a vacant job and an unemployed worker.

\(^{24}\)See Petrongolo and Pissarides (2001) for a survey on this literature.

\(^{25}\)In this sense, the increasing returns in matching documented here is related to “sharing,” in addition to “matching,” among the three classifications of agglomeration mechanisms as introduced in Duranton and Puga (2004).
3.3 Additional Results

This section briefly presents additional results that confirm the importance of matching frictions. Section 3.3.1 demonstrates that a supplier loss leads to lower sales growth and a higher exit probability. Section 3.3.2 illustrates that there is no evidence that firms cope with a loss of supplier by substituting inputs from other existing suppliers.

3.3.1 Sales Growth and Exit Upon Unexpected Supplier Bankruptcy

In this section, I investigate the impacts of unexpected supplier bankruptcy on sales growth and exit. The specification is as follows:

\[
\Delta Y_{ft} = \beta_1 UnexpBankruptcy_{fkt} + \beta_2 UnexpBankruptcy_{fkt} \times \log \text{LocallySellingSuppliers}_{loc(f),k} + \delta_{loc(f),ind(f),k,t} + \phi_t(BaselineSupplier_{fkt}) + \epsilon_{fkt}. \tag{4}
\]

The results are presented in the reduced form rather than the IV form, mainly because the supplier separation cannot be defined when firm \( f \) exits. Compared to regression (2), the outcome variables \( \Delta Y_{ft} \) are defined at the firm and year level, but not at the input sector \( k \) level. Kaido and Miyauchi (in progress) show that even if the outcome variables do not depend on \( k \), above regression consistently estimates average and heterogeneous treatment effects.

Table 6 reports the results. Columns (1) and (2) show the impacts on sales growth, where it is defined as the arc-elasticity (Davis and Haltiwanger (1992)), i.e.,

\[
\Delta Y_{ft} = \frac{1}{2} \frac{Sales_{t}}{Sales_{t} + Sales_{t+1}},
\]

where \( Sales_{t} \) is defined to be 0 if the firm exits in \( t + 1 \). This measure is bounded between 2 and -2, avoids outliers with a large increase in sales from skewing the distribution of percentage changes, and allows inclusion of firms with no sales (i.e., exit). The results show that, while a supplier bankruptcy leads to 3% reduction of sales per year on average relative to control groups, this reduction is mitigated in the presence of more potential suppliers, despite insignificantly. Columns (3) to (6) decompose these impacts in terms of firm \( f \)’s exit and sales growth conditional on survival. While the average effects are driven more by exit margins (Columns 3 and 5), the heterogeneous effects are driven more by the sales growth conditional on survival (Columns 4 and 6). Lastly, Columns (7) and (8) show that these
results are not driven by the pre-trends.\textsuperscript{26}

3.3.2 Evidence of Lack of Substitution by Other Existing Suppliers

While a new supplier matching is one way that firms cope with a supplier loss, they can potentially deal with the shock by sourcing more from other existing suppliers. I show that, in this context, there is no such evidence of substitution by other existing suppliers.

To test this, I run the following specification:

\[
\Delta Y_{fkt} = \beta_1 UnexpBankruptcy_{fkt} + \beta_2 UnexpBankruptcy_{fkt} \times \log LocallySellingSuppliers_{loc(f),k} \\
+ \delta_{loc(f),ind(f),k,t} + \varphi_t(BaselineSupplier_{fkt}) + \epsilon_{fkt}.
\] (5)

The regression specification is the same as regression (4), while now the outcome variables \(\Delta Y_{fkt}\) are various outcomes of other existing suppliers of firm \(f\) in input sector \(k\) at year \(t\). Samples of the regressions are those with more than one baseline suppliers within the input sector.

Table 7 reports the results. Columns (1) and (2) show the impacts on the probability of separation with other existing suppliers. If firms substitute inputs from other existing suppliers, one would expect that the retention rates of other existing suppliers are higher with unexpected supplier bankruptcies. The results show that there is no such evidence. Columns (3) and (4) show that unexpected supplier bankruptcy actually reduces the sales growth of other existing suppliers on average. The results are in fact the opposite of the prediction under the substitution by other existing suppliers; rather, the results imply that other suppliers are complements of the bankrupting suppliers. This evidence of lack of substitution implies that the shock from a supplier loss cannot be mitigated by the presence of other existing suppliers, confirming the importance of new supplier matching.

4 Model of Firm-to-Firm Matching and Agglomeration

This section develops a new structural model building on the reduced-form evidence in Section 3. The model extends a multi-location multi-sector Melitz model (Melitz (2003)) to incorporate dynamic firm-to-firm matching in input trade. As in a standard Melitz model,\textsuperscript{26} while the lack of significance in heterogeneous effects on sales growth and exits are plausibly driven by the noise, another possibility is that the heterogeneity in the importance of matched suppliers offsets the benefit of improved supplier matching rates. More concretely, if suppliers’ productivity is increasing in the proxy of the number of potential suppliers, a loss of these suppliers is more damaging to buyers, which offsets the benefit of a higher recovery rate of suppliers. I show that the structural model in this paper can rationalize this pattern, and I provide suggestive evidence that this is potentially an explanation of the empirical results.
potential producers enter in each location by paying a fixed cost and draw an idiosyncratic productivity; upon the realization of this productivity draw, they make a decision to sell into various locations by paying a fixed marketing cost. In addition to these standard assumptions, firms use input goods for production, which they can source from matched suppliers. The matching rate with a supplier increases with the number of suppliers selling in the location, but it is unaffected by the number of buyers at the location; this assumption is in line with the empirical findings of increasing returns in matching in Section 3.

4.1 Model Set-up

Space is partitioned into a discrete number of locations, denoted by \( i, j, n \in N \). Each location is endowed with \( L_i \) measure of workers who consume final goods. In the baseline, I assume workers are immobile, while I relax this assumption in Appendix C. Time is continuous and denoted by \( t \). In this paper, I only consider a steady-state equilibrium in which aggregate variables (e.g., wages, number of entrants) are constant. Only firm-level variables like supplier matching status vary by \( t \). Without a risk of confusion, the subscript \( t \) is omitted from the aggregate variables.

There is a continuum of potential entrants in each location and sector, where sector is denoted by \( k, m \in K \). I assume that firms do not change production location over their life-cycles. All firms produce both final goods, consumed by final goods consumers, and input goods, used for production by other firms. In this sense, each firm can be simultaneously a buyer and a supplier in input trade. Input trade is only possible when two firms match as a supplier and a buyer. I assume that each buyer-side firm can be matched with at most one supplier in each input sector at a time, though suppliers can be matched with multiple buyers simultaneously.

4.1.1 Technology

Each firm can produce both final goods and input goods with the Cobb-Douglas production function. I assume that the unit cost for both final goods and input goods by firm \( \omega \) in location \( i \) in sector \( m \) is written as follows:

\[
c_{\omega t} = \frac{1}{\varphi_{\omega} A_{i,m}} w_i^{\gamma_{L,m}} \left( \prod_{k \in K} p_{\omega t,k}^{\gamma_{k,m}} \right),
\]

where \( \varphi_{\omega} \) is the exogenous productivity of firm \( \omega \), \( A_{i,m} \) is the exogenous location-sector level productivity, which can be interpreted as natural advantages or other agglomeration forces, \( \gamma_{L,m} \) is the labor share in production for sector \( m \), \( w_i \) is the wage in \( \omega \)’s production location.
\(i, \gamma_{k,m}\) is the input share of sector \(k\)'s input goods in sector \(m\)'s production, and \(p_{\omega_t,k}\) is the unit cost of input goods that firm \(\omega\) has access to in period \(t\). I assume that production function is constant returns to scale, i.e., \(\gamma_{L,m} + \sum_k \gamma_{k,m} = 1\) for all \(m \in K\).

There are two possible ways to source input goods: match with a supplier for customized input goods or purchase in the generalized input goods market. If firm \(\omega\) with production location \(i\) is matched with supplier \(\upsilon\) with production location \(n\) in sector \(k\), firm \(\omega\) can source customized inputs with unit cost at \(c_{\upsilon t}\psi^I_{ni,k}\), where \(c_{\upsilon t}\) is the unit cost of production for firm \(\upsilon\), \(\psi\) is the constant mark-up ratio charged by supplier \(\upsilon\) (\(\psi \geq 1\)), and \(\tau^I_{ni,k}\) is iceberg trade cost of input goods (\(\tau^I_{ni,k} \geq 1\)). The iceberg trade cost captures the combination of shipment cost, transaction cost, and other sources of geographic frictions. If firm \(\omega\) is not matched with a supplier, the firm can source input goods from the generalized input goods market, where these inputs are produced by perfectly competitive fringe suppliers with linear production technology with labor, charging the unit cost of \(\chi_i,k w_i\).

27) Taken together, the cost of input goods is written as

\[
p_{\omega_t,k} = \begin{cases} 
\min \left\{ c_{\upsilon t}\psi^I_{ni,k}, \chi_i,k w_i \right\} & \text{if matched with supplier } \upsilon \text{ producing in } n, \\
\chi_i,k w_i & \text{otherwise.}
\end{cases}
\]  \quad (7)

I consider an equilibrium where \(\chi_{i,k}\) is sufficiently high so that whenever a firm is matched with a supplier, it uses the input goods from the matched supplier rather than sourcing from a generalized input goods market.

4.1.2 Final Goods Demand and Market Structure

As in a standard Melitz model, for firms in sector \(k\) to make final goods sales in location \(j\), they have to pay a fixed marketing cost at a flow rate \(f^F_{j,k}\) regardless of their production location \(i\). For shipping final goods from production location \(n\) to \(j\), the firm incurs an iceberg trade cost \(\tau^F_{nj,k}\). Each seller provides a differentiated variety in a monopolistically competitive manner. Representative final goods consumers have a standard CES utility function:

\[
U = \prod_{k \in K} \left( \int_{\omega \in \Omega_i,k} q_k(\omega) \frac{\sigma - 1}{\sigma} d\omega \right)^{\frac{\sigma}{\sigma - 1} \alpha_k},
\]  \quad (8)

where \(q_k(\omega)\) is the consumption of the goods produced by firm \(\omega\), \(\alpha_k\) is the consumption share of sector \(k\) final goods, \(\sigma > 1\) is the elasticity of substitution, and \(\Omega_i,k\) is the set of varieties available for final goods consumers in location \(i\).

27) Alternatively, one can interpret these generalized input goods as in-house production of input goods by using labor.
4.1.3 Input Goods Demand, Matching, and Market Structure

For firms in sector \( k \) to make input goods sales in location \( j \), they have to pay a fixed marketing cost at flow rate \( f^I_{j,k} \) regardless of their production location \( i \). I assume that these decisions are independent of seller entry for final goods sales described in the previous subsection. All producers in location \( j \) are potential input buyers. I refer to firms in sector \( k \) which pay a fixed marketing cost for input goods sales in location \( j \) as “input sellers” in location \( j \), and its measure by \( S^I_{j,k} \). I refer to firms producing in location \( j \) in sector \( k \) as “input buyers” in location \( j \), and its measure by \( B_{j,k} \). Due to matching frictions, input sellers can be only stochastically matched with input buyers.

In each period \( t \), firm \( \omega \) will randomly match with an input seller in location \( i \) at the Poisson rate \( M_{i,km} \left( S^I_{i,k} \right) \). Following the reduced-form results in Section 3, I assume that the rate that input buyers match with input sellers is increasing in the number of input sellers \( S^I_{i,k} \), but it does not depend on the number of input buyers, i.e., other buyers do not crowd out matching. From the perspective of an input buyer, they match with a supplier at a Poisson rate \( M_{i,km} \left( S^I_{i,k} \right) / S^I_{i,k} \). I also assume that the matching rate with suppliers in input sector \( k \) is independent across all input sectors. Once the relationship is formed, the relationship continues until it is exogenously destroyed at the Poisson rate \( \rho_{i,km} \).

For tractability purposes, I make several additional assumptions, largely in order to simplify the firm-level life-cycle dynamics. First, I assume that firm \( \omega \) makes input goods sales decision in each period based only on its instantaneous unit cost \( c_{\omega t} \). This implies that firms’ input goods sales decisions do not depend on the exogenous productivity of a firm \( \varphi_{\omega} \) as well as the current supplier matching status conditional on the unit cost \( c_{\omega t} \). Second, I assume that, if a supplier stops input sales in location \( j \) or if the supplier dies (explained in Section 4.1.4), the buyers of these exiting suppliers instantaneously recover a supplier from the pool of suppliers which newly start to sell in location \( j \) in the steady state. Similarly, I assume that, if an input buyer dies, the suppliers of exiting buyers instantly match with input buyers which newly enter in location \( j \). These two assumptions essentially shut down the firm-level life-cycle dynamics, and firms instantly reach to the steady-state matching with suppliers and buyers when it is born.

4.1.4 Entry and Exit

Firms in location \( i \) in sector \( m \) exogenously dies at rate \( \xi_{i,m} \). I consider a steady state where the same flow rate of firms enter so that the number of firms, or equivalently input buyers \( B_{i,m} \), is constant over time. Potential entrant \( \omega \) pays a fixed cost \( C_{i,m} \) and draws

\[ \text{Another interpretation of this assumption is that prior to the firm entry and drawing its productivity } \varphi_{\omega}, \text{ each firm determines } f^I_{j,k} \text{ above which all firms pays a fixed marketing cost to make input sales in location } j. \]
a productivity $\varphi$ from a Pareto distribution, where its cumulative distribution function is written as $F(\varphi) = 1 - \left( \frac{\varphi}{\varphi_\omega} \right)^\theta$, where $\varphi$ and $\theta$ are parameters, and $\varphi_\omega \geq \varphi$.

When I analyze the equilibrium, I consider the limit where $\varphi \to 0$. The motivation of this approximation is in line with the assumption that $\varphi$ is sufficiently small in a standard Melitz model; this is to ensure that not all firms make sales (or “export,” in the context of international trade) and the set of sellers is endogenously determined.

4.1.5 Total Expenditure and Trade Balance

Aggregate final goods sales from location $i$ is written as

$$X^F_{i,k} = \sum_{j \in N} \alpha_k w_j L_j \pi^F_{ij,k},$$

where $\pi^F_{ij,k}$ is location $j$’s final goods expenditure share in sector $k$ of goods from location $i$, and $\alpha_k$ is the final goods consumption share of final goods in sector $k$. $\pi^F_{ij,k}$ are endogenously determined in the equilibrium and are derived in the next section.

Aggregate input goods sales from location $i$ is written as

$$X^I_{i,k} = \sum_{m \in K} \sum_{j \in N} \gamma_{k,m} \Psi_{j,km} \left( X^F_{j,m} + X^I_{j,m} \right) \pi^I_{ij,k},$$

where $\Psi_{j,km}$ is the share of final goods sales in location $j$ and sector $m$ sold by firms that are matched with a supplier in input sector $k$, and $\pi^I_{ij,k}$ is location $j$’s input goods expenditure share in sector $k$ of goods from location $i$.

I assume a steady state where each location $i$ has trade deficit $D_i$. Trade balancing condition equates the aggregate sales from location $i$ with the final and input goods purchases in location $i$ net of trade surplus, i.e.,

$$\sum_{k \in K} X^F_{i,k} + \sum_{k \in K} X^I_{i,k} = w_i L_i + \sum_{k,m \in K} \gamma_{k,m} \Psi_{i,km} \left( X^F_{i,m} + X^I_{i,m} \right) + D_i.$$

4.2 Steady-State Equilibrium

In this section, I define and characterize the steady-state equilibrium.

4.2.1 Definition of Equilibrium

To define the equilibrium, I first define the measure of firms in location $i$ in sector $m$ whose unit cost of input goods is below $c$ by $H_{i,m}(c)$. From equations (6), (7), and the Pareto
distribution of $\varphi_\omega$,

$$H_{i,m}(c) = \int_{p_1, \ldots, p_N} B_{i,m} \left\{ 1 - F \left( \frac{1}{c A_{i,m}} w_i^{\gamma_{L,m}} \prod_{k \in K} p_k^{\gamma_{km}} \right) \right\} \prod_{k \in K} dG_{i,k}(p_k)$$  \hspace{1cm} (12)

where $G_{i,k}(\cdot)$ is the distribution of unit cost of input goods $k$. Since $G_{i,k}(\cdot)$ depends on the unit cost of input suppliers selling in location $i$, it in turn depends on $\{H_{j,k}(\cdot)\}$ in all locations and sectors. This constitutes a fixed point problem of unit cost distributions $\{H_{j,k}(\cdot)\}$.  

Using the unit cost distributions of producers $\{H_{i,m}(\cdot)\}$ as defined as above, the equilibrium is defined as follows.

**Definition 1.** The steady-state equilibrium is characterized by unit cost distributions of firms producing in $i$ $\{H_{i,k}(\cdot)\}$, the measure of firms which makes input and final goods sales $\{S^I_{i,k}, S^F_{i,k}\}$, firm entry $\{B_{i,k}\}$ and wages $\{w_i\}$ which satisfy:

1. Stationary stationary distributions of unit costs $\{H_{i,k}()\}$ are determined by (12).

2. Firms optimality conditions are satisfied.

   (a) In each period, firm $\omega$, given its unit cost $\{c_{\omega,t}\}$, determines to make final goods and input goods sales in each sales location $j$ if and only if the expected flow of profit is greater than 0. Per-period expected profits are determined as described in Sections 4.1.2 and 4.1.3. In other words, zero-profit conditions of marginal sellers are satisfied.

   (b) The free entry conditions of potential entrants equate the discounted sum of expected profit with the fixed cost, as described in Section 4.1.4.

3. Goods and labor market clears. The conditions come down to:

   (a) Total expenditure condition of final goods (9) and input goods (10).

   (b) Trade balancing conditions (11).

The equilibrium definition highlights two important distinctions from a standard Melitz model. First, unit cost distributions $\{H_{i,k}(\cdot)\}$ not only depends on exogenous productivity

29More precisely, by noting that this depends on the probability that a firm is matched with a supplier, as well as the distribution of unit cost of suppliers, $G_{i,k}(c)$ is defined as

$$G_{i,k}(c) = \{1 - \Lambda_{i,km}(S^I_{i,k})\} 1[c \leq \chi_{i,km}] + \Lambda_{i,km}(S^I_{i,k}) \frac{\sum H_{i,m}(c/\tau_{i,k})}{\sum H^I_{i,m}(\tau_{i,k}/\tau_{i,k})} 1[c \leq \tau^I_{i,m}].$$
\{A_{i,k}\} \text{ and wages } \{w_i\}, \text{ but also on the steady-state probability of matching with a supplier, } \text{as well as the unit cost distributions of matched suppliers. Second, there are input goods sales } \text{market, where the set of input sellers are determined by zero-profit condition, in addition to the final goods sellers as in a standard Melitz model.}

4.2.2 Characterizing Equilibrium

To further characterize the equilibrium, I make use of an approximation where \(\varphi\) is sufficiently small. It is also a common underlying assumption in a standard Melitz model to ensure that the set of sellers (or “exporters”) is endogenously determined. To see this, if \(\varphi\) is large, all firms may find it beneficial to pay a fixed marketing cost to make sales. Here, to ensure that it is true however low the unit cost of the matched suppliers, I explicitly take an approximation where \(\varphi\) is close to 0. Below, I excerpts the main equilibrium characterization, while the detailed algebraic derivation is left in the appendix.

Unit Cost Distribution of Production

First, I characterize \(H_{i,m}(c)\), the measure of producers whose unit cost of input goods is below \(c\) in location \(i\) in sector \(m\). Appendix B.1.1 shows that

\[
H_{i,m}(c) = (1 - o(\varphi)) \varphi^\theta B_{i,m} A_{i,m} w_i^{-\theta} \prod_{k \in K} \Omega_{i,km} \left( S_{i,k}^I, \overline{c}_{i,k}^I \right) c^\theta,
\]

where \(o(\varphi)\) is the term that goes to 0 as \(\varphi \rightarrow 0\). \(\Omega_{i,km} \left( S_{i,k}^I, \overline{c}_{i,k}^I \right)\) represents the cost advantage from supplier matching, i.e.,

\[
\Omega_{i,km} \left( S_{i,k}^I, \overline{c}_{i,k}^I \right) = 1 - \Lambda_{i,km} \left( S_{i,k}^I \right) + \Lambda_{i,km} \left( S_{i,k}^I \right) \nu_{i,km} \left( \overline{c}_{i,k}^I \right), \tag{13}
\]

where \(\Lambda_{i,km} \left( S_{i,k}^I \right) \equiv \frac{M_{i,km} \left( S_{i,k}^I \right)}{M_{i,km} \left( S_{i,k}^I \right) + \rho_{i,km}}\) is the steady state probability of matching with a supplier, and \(\nu_{i,km} \left( \overline{c}_{i,k}^I \right) \equiv \frac{1}{1 - \gamma_{k,m}} \left( \overline{c}_{i,k}^I / w_i \chi_{i,k} \right)^{-\gamma_{k,m} \theta}\) governs the average cost advantage upon being matched with a supplier.

Gravity Equations

The previous argument shows that \(H_{i,m}(\cdot)\) is approximated by an inverse of Pareto distribution if \(\varphi\) is small. This implies that, just like a standard Melitz model with a Pareto productivity distribution (e.g., Chaney (2008)), the expenditure share of final goods follow a
gravity equation

\[ \pi_{ij,m}^F = \frac{B_{i,k} A_{i,m} \omega_i \left( \tau_{ij,m}^F \right)^{\theta} \prod_{k \in K} \Omega_{i,k,m} \left( S_{ij,k}, \Psi_{ij,k}^F \right)}{\sum_{i' \in N} B_{i',m} A_{i',m} \omega_{i'} \left( \tau_{ij,m}^F \right)^{\theta} \prod_{k \in K} \Omega_{i',k,m} \left( S_{ij,k}, \Psi_{ij,k}^F \right)}. \]  

(14)

as well as the expenditure input goods also follow a gravity equation,

\[ \pi_{ij,m}^I = \frac{B_{i,k} A_{i,m} \omega_i \left( \tau_{ij,m}^I \right)^{\theta} \prod_{k \in K} \Omega_{i,k,m} \left( S_{ij,k}, \Psi_{ij,k}^I \right)}{\sum_{i' \in N} B_{i',m} A_{i',m} \omega_{i'} \left( \tau_{ij,m}^I \right)^{\theta} \prod_{k \in K} \Omega_{i',k,m} \left( S_{ij,k}, \Psi_{ij,k}^I \right)}. \]  

(15)

Furthermore, \( \{\pi_{ij,m}^F, \pi_{ij,m}^I\} \) also correspond to the extensive margin of sellers, i.e. the proportion of firms that sell from location \( i \) to location \( j \) out of all sellers to location \( j \).

**Zero Profit Conditions of Input Goods Sellers**

Zero-profit conditions of input goods sellers yield the number of input sellers \( S_{j,k}^I \) and input seller entry cut-off of unit cost \( \overline{c}_{j,k}^I \). Appendix B.1.3 shows that \( S_{j,k}^I \) is solved as

\[ S_{j,k}^I = \frac{1}{w_{j} X_{j,m}^F} \frac{\psi - 1}{\psi} \sum_{m \in K} \left\{ (1 - \gamma_{k,m}) \gamma_{k,m} \Psi_{j,k,m} \left( X_{j,m}^F + X_{j,m}^I \right) \right\}, \]  

(16)

where \( \Psi_{j,k,m} = \frac{\Lambda_{j,k,m}(S_{j,k}^I)}{1 - \Lambda_{j,k,m}(S_{j,k}^I) + \Lambda_{j,k,m}(S_{j,k}^I)} \) is the share of final goods sales in location \( j \) and sector \( m \) sold by firms that are matched with a supplier in input sector \( k \) (also appearing in total expenditure condition in Equation [10]).

Given the number of sellers \( S_{j,k}^I \), the entry cut-off of sellers is derived as

\[ \overline{c}_{j,k}^I = \left( \frac{S_{j,k}^I}{\sum_{i' \in N} B_{i',k} \omega_{i'} \left( \tau_{ij,m}^I \right)^{\theta} \prod_{l \in K} \Omega_{i',k,l} \left( S_{i',l}^I, \overline{c}_{i',l}^I \right)} \right)^{1/\theta}. \]  

(17)

**Free Entry Conditions**

The expected profit of firms upon an entry depends both on the final goods profits and input goods profits. Appendix B.1.4 shows that the free entry conditions yield the number of firms,

\[ \text{The intuition of the expression of } \Psi_{j,k,m} \text{ is as follows: Firms that are matched with a supplier in input sector } k \text{ enjoy a cost advantage by } \nu_{j,k,m}(\overline{c}_{j,k}^I) \text{ over unmatched firms. Hence, the share sales of firms with a supplier is expanded from } \Lambda_{j,k,m}(S_{j,k}^I), \text{ the steady state probability of matching with a supplier in input sector } k, \text{ by the factor of } \nu_{j,k,m}(\overline{c}_{j,k}^I). \]  

\[ \Psi_{j,k,m} \text{ enters in zero-profit conditions, because the aggregate input sales is written as } \gamma_{k,m} \Psi_{j,k,m} \left( X_{i,m}^F + X_{i,m}^I \right). \]
equivalently as input buyers, as
\[ B_{i,k} = \frac{1}{\xi_{i,k} C_{i,k} w_i} \sum_{j \in N} \left\{ \pi_{ij,k} \frac{\sigma - 1}{\sigma \theta} \alpha_k w_j L_j + \frac{\psi - 1}{\psi} \pi_{ij,k} \sum_{m \in K} \gamma_{k,m}^2 \Psi_{j,km} \left( X_{j,m}^F + X_{j,m}^I \right) \right\}. \] (18)

Together, the equilibrium is characterized as follows:

**Proposition 1.** Under sufficiently small \( \varphi \), the steady-state equilibrium is characterized by \( \{ S_{i,k}^I, B_{i,k}, w_i, X_{i,m}^F, \pi_{ij,k}^F, \pi_{ij,k}^I \} \), which satisfy total expenditure conditions (9, 10), trade balancing conditions (11), gravity equations for input goods (15) and final goods (14), zero profit conditions (16), entry cut-off of input goods sellers (17) and free-entry conditions (18).

### 4.3 Matching and Agglomeration in the Model

In this subsection, I briefly discuss the main agglomeration forces of the model: circular causation between the input seller entry \( S_{i,k}^I \) and the final goods sales of input buyers \( X_{i,m}^F \).

For the sake of the discussion of this section, I will ignore the endogeneity of \( w_i \) and the number of producers \( B_{i,m} \), which constitute a further general equilibrium loop as in the standard Melitz model.

Final goods sales of input buyers in location \( i \) in sector \( m \), \( X_{i,m}^F \), is determined by the total expenditure condition for final goods (Equation [9]). By combining the expenditure share from the gravity equation (15), I have the relationship corresponding to a “forward linkage”:

\[ X_{i,m}^F = \sum_{j \in N} \alpha_k w_j L_j \frac{B_{i,m} \xi_{i,m} w_i^{-\theta \gamma_{i,m}} \left( \pi_{ij,m}^F \right)^{\theta} \prod_{k \in K} \Omega_{i,km} \left( S_{i,k}^I, \pi_{ij,k}^I \right)}{\sum_{i' \in N} B_{i',m} \xi_{i',m} w_i^{-\theta \gamma_{i',m}} \left( \pi_{i',j,m}^F \right)^{\theta} \prod_{k \in K} \Omega_{i',km} \left( S_{i',k}^I, \pi_{i',j,k}^I \right)}. \]

The intuition of these equations is simple: If there are more sellers \( S_{i,k}^I \), producers have a higher chance of matching with a supplier, which gives a cost advantage to producers in location \( i \) (high \( \Omega_{i,km} \left( S_{i,k}^I, \pi_{ij,k}^I \right) \)) and hence increases aggregate final goods sales \( X_{i,m}^F \). The number of sellers, in turn, is again derived by the zero profit condition (16), reproduced here:

\[ S_{i,k}^I = \frac{1}{w_i \pi_{i,k}^F} \frac{\psi - 1}{\psi} \sum_{m \in K} \left\{ (1 - \gamma_{k,m}) \gamma_{k,m} \Psi_{i,km} \left( X_{i,m}^F + X_{i,m}^I \right) \right\}. \]

It shows that \( S_{i,k}^I \) is increasing in the final goods sales of input buyers, \( X_{i,m}^F \). This corresponds to a “backward linkage.” The “forward linkage” and “backward linkage” constitute a positive feedback loop, reinforcing each other to create a force toward agglomeration.

In this circular causation, the elasticity of supplier matching rate with respect to the number of input sellers, \( \lambda \), serve a crucial role. To see this, recall that \( \Omega_{i,k} \left( S_{i,k}^I, \pi_{i,k}^I \right) = \ldots \)
1 − \left\{ \nu_{i,km} \left( \mathcal{E}^I_{i,k} \right) - 1 \right\} \times M_{i,km} \left( S^I_{i,k} \right) / \left\{ M_{i,km} \left( S^I_{i,k} \right) + \rho_{i,km} \right\} by plugging in Equation (13) in the definition of \( \Omega_{i,km} \). Hence, the sensitivity of \( \Omega_{i,km} \) with respect to \( S^I_{i,k} \) is crucially governed by the parameter \( \lambda \), the elasticity of the supplier matching rate with respect to the number of potential suppliers. In Section 5, I estimate \( \lambda \) to replicate the reduced-form estimates in Section 3.

While related, the circular causation through vertical linkages presented here is somewhat distinct from the theoretical models developed by Krugman and Venables (1995) and Venables (1996). In their models, there are no matching frictions, and firms have access to all producers all across the world. There, the circular causation arises from the production location decision of suppliers and from downstream market size. Here, on the other hand, the circular causation arises from supplier market penetration decision à la Melitz (2003) and from downstream market size. The distinction leads to crucial differences in policy implications, as illustrated in the subsidies for input sales in Section 6.2.1.

Just like Krugman and Venables (1995), Venables (1996), and other models of economic geography with agglomeration forces, the circular causation may potentially lead to multiple equilibria. While the presence of multiple equilibria does not cause issues in parameter estimation because the estimation procedure only utilizes the partial equilibrium relationships of the model, it complicates the counterfactual equilibrium simulation. In this paper, I compute counterfactual equilibrium that follow the same bifurcation path as in the observed equilibrium. Section 5 discuss these points further.

5 Equilibrium Computation and Model Estimation

This section discusses the procedure for obtaining counterfactual equilibrium and parameter estimation and calibration.

5.1 Computation of Counterfactual Equilibrium

To compute the counterfactual equilibrium, I follow the approach of “hat algebra” initiated by Dekle et al. (2008). This procedure allows me to limit the set of parameters and baseline variables required to compute the counterfactual equilibrium.

To make the computation simple, I assume that the matching rate is an exponential function of the measure of input sellers, i.e., \( M_{i,km} \left( S^I_{i,k} \right) = \eta_{i,km} \times \left( S^I_{i,k} \right)^{\lambda} \), where \( \{\eta_{i,km}\} \) and \( \lambda \) are parameters. The following proposition describes the conditions of counterfactual equilibrium under two policy counterfactual simulations: Subsidies for input sales and production (Section 6.2.1), and the change in transportation cost (Section 6.2.2).
Proposition 2. Given baseline equilibrium variables \( \{B_{i,k}, w_i, X_{i,k}^F, \pi_{ij,k}^F, \pi_{ij,k}^L, \Lambda_{i,k}, \nu_{i,k}, \Psi_{i,k} \} \), parameters \( \{\lambda, \psi, \theta, \{\gamma_{L,k}, \gamma_{k,m}, \alpha_k\} \} \), counterfactual subsidies for input sales \( \{T_{i,k}^S\} \), subsidies for production \( \{T_{i,k}^P\} \), and changes in transportation costs \( \{\tau_{ij,k}^I, \tau_{ij,k}^F\} \), the counterfactual equilibrium is computed by solving the following set of equations with respect to \( \{\hat{S}_{i,k}^I, \hat{B}_{i,k}, \hat{w}_i, \hat{X}_{i,k}^F, \hat{\tau}_{ij,k}^I, \hat{\tau}_{ij,k}^F, \hat{\pi}_{ij,k}^I, \hat{\pi}_{ij,k}^F\} \):

(i) gravity equation of input goods

\[
\hat{\pi}_{ij,m}^I = \frac{\hat{B}_{i,m} \hat{w}_i^{-\theta} \left( \hat{\tau}_{ij,m}^I \right)^\theta \Pi_{k \in K} \hat{\Omega}_{i,k,m}}{\sum_{i' \in N} \pi_{i'j,m}^I \hat{B}_{i',m} \hat{w}_i^{-\theta} \left( \hat{\tau}_{i'j,m}^I \right)^\theta \Pi_{k \in K} \hat{\Omega}_{i',k,m}},
\]

and that of final goods

\[
\hat{\pi}_{ij,m}^F = \frac{\hat{B}_{i,m} \hat{w}_i^{-\theta} \gamma_{L,m} \left( \hat{\tau}_{ij,m}^F \right)^\theta \Pi_{k \in K} \hat{\Omega}_{i,k,m}}{\sum_{i' \in N} \pi_{i'j,m}^F \hat{B}_{i',m} \hat{w}_i^{-\theta} \gamma_{L,m} \left( \hat{\tau}_{i'j,m}^F \right)^\theta \Pi_{k \in K} \hat{\Omega}_{i',k,m}},
\]

(ii) zero-profit conditions of input goods sellers

\[
\hat{S}_{i,k}^I = \frac{1}{\hat{w}_i} \sum_{m \in K} (1 - \gamma_{k,m}) \gamma_{k,m} \psi_{j,k,m} \hat{\Psi}_{j,k,m} \left( X_{j,m}^F \hat{X}_{j,m}^F + X_{j,m}^l \hat{X}_{j,m}^l \right) + \frac{\theta \psi}{\psi - 1} T_{i,k}^S,
\]

(iii) free entry conditions

\[
\hat{B}_{i,k} = \frac{1}{\hat{w}_i} \sum_{j \in N} \left\{ \pi_{ij,k}^I \hat{\pi}_{ij,k}^F \frac{\alpha_k w_j \hat{L}_j + \psi \theta \tau_{ij,k}^I \sum_{m \in K} \gamma_{k,m} \psi_{j,k,m} \hat{\Psi}_{j,k,m} \left( X_{j,m}^F \hat{X}_{j,m}^F + X_{j,m}^l \hat{X}_{j,m}^l \right)}{\sum_{j \in N} \left\{ \pi_{ij,k}^F \frac{\alpha_k w_j \hat{L}_j + \psi \theta \tau_{ij,k}^I \sum_{m \in K} \gamma_{k,m} \psi_{j,k,m} \hat{\Psi}_{j,k,m} \left( X_{j,m}^F \hat{X}_{j,m}^F + X_{j,m}^l \hat{X}_{j,m}^l \right)}{\sum_{m \in K} (1 - \gamma_{k,m}) \gamma_{k,m} \psi_{j,k,m} \hat{\Psi}_{j,k,m} \left( X_{j,m}^F \hat{X}_{j,m}^F + X_{j,m}^l \hat{X}_{j,m}^l \right)} \right\} \right\} + T_{i,k}^B,
\]

(iv) cost advantages from supplier matching

\[
\hat{\Omega}_{i,k,m} = \frac{1 - \Lambda_{i,k,m} \hat{\lambda}_{i,k,m} + \Lambda_{i,k,m} \psi_{i,k,m} \hat{\lambda}_{i,k,m} \hat{\nu}_{i,k,m}}{1 - \Lambda_{i,k,m} + \Lambda_{i,k,m} \psi_{i,k,m}},
\]

where steady state supplier matching probability is \( \hat{\lambda}_{i,k,m} = 1 / \left\{ 1 - \Lambda_{i,k,m} + \Lambda_{i,k,m} \times (\hat{S}_{i,k}^I)^{-\lambda} \right\} \) and cost advantage upon a match is \( \hat{\nu}_{i,k,m} = \left\{ \hat{S}_{i,k}^I / \left( \sum_{i' \in N} \pi_{i'j,k,m}^I \hat{B}_{i',j,k} \hat{w}_i^{-\theta} \left( \hat{\tau}_{i'j,k,m}^I \right)^\theta \Pi_{k \in K} \hat{\Omega}_{i',j,k,m} \right)^{-\gamma_{k,m}} \right\} \).

(v) trade balancing condition (no change in trade surplus)

\[
1 = \frac{\sum_{k \in K} X_{i,k}^F \hat{X}_{i,k}^F + \sum_{k \in K} X_{i,k}^l \hat{X}_{i,k}^l - w_i \hat{w}_i \hat{L}_i - \sum_{k,m \in K} \gamma_{k,m} \psi_{i,k,m} \hat{\Psi}_{i,k,m} \left( X_{i,m}^F \hat{X}_{i,m}^F + X_{i,m}^l \hat{X}_{i,m}^l \right)}{\sum_{k \in K} X_{i,k}^F + \sum_{k \in K} X_{i,k}^l - w_i \hat{L}_i - \sum_{k,m \in K} \gamma_{k,m} \psi_{i,k,m} \left( X_{i,m}^F + X_{i,m}^l \right)}},
\]
where input goods sales is 

\[ X_{i,k}^F \hat{\Psi}_{j,k} = \sum_{m \in K} \sum_{j \in N} \gamma_{k,m} \Psi_{j,k,m} \left( X_{i,m}^F \hat{\Psi}_{j,k,m} + X_{i,m}^I \hat{\Psi}_{j,k,m} \right) \tau_{i,j,k}. \]

Proposition 2 states that computing an equilibrium does not require all the parameters in the model; rather, a subset of parameters \{\lambda, \psi, \{\gamma_{L,m}, \gamma_{k,m}, \alpha_k\}, \theta, \sigma\} and baseline variables \{B_{i,k}, w_i, X_{i,k}^F, \pi_{ij,k}, \Lambda_{i,k,m}, \nu_{i,k,m}, \Psi_{i,k,m}\} are sufficient. These parameters and baseline variables are either estimated, calibrated, or directly obtained from the data. Next subsection is devoted to the discussion.

As noted in Section 4.3, there may be multiple equilibria, i.e., there may be multiple set of endogenous variables that solve the system of equations in Proposition 2. For the counterfactual simulations provided later, I obtain the counterfactual equilibrium by gradually changing the subsidies (\{\tau_{S_{i,k}}, \tau_{B_{i,k}}\}) or transportation cost (\{\tau_{F_{i,j,k}}, \tau_{F_{i,j,k}}\}); in this sense, the obtained counterfactual equilibrium follows the same bifurcation paths as in the observed equilibrium. This is an intuitive equilibrium selection rule under the counterfactual policies, particularly when the policies are at a relatively small scale and are unlikely to induce switching to equilibria on a different bifurcation path.

In the counterfactual simulation exercise, I mostly focus on the change in the firm density \(B_{j,k}\) and the real wages \(\frac{w_i}{P_j}\), where \(P_j\) is the consumer price index. The following proposition is useful for characterizing the changes in real wages in each location.

**Proposition 3.** The change in real wages are derived as

\[
\left( \frac{w_i}{P_j} \right) = \prod_{m \in K} \left( \frac{\hat{\tau}_{i,m}^{F} \hat{\omega}_{i,m}^{-\theta \gamma_{L,m}} \left( \hat{\tau}_{i,m}^{F} \hat{\omega}_{i,m}^{\theta} \right)^{\alpha_{m}}}{\prod_{k \in K} \hat{\Omega}_{i,k,m}} \right).
\]

**Proof.** See Appendix B.2.

### 5.2 Parameter Estimation and Calibration

As explained in the previous section, the knowledge of \{\lambda, \psi, \{\gamma_{L,m}, \gamma_{k,m}, \alpha_k\}, \theta, \sigma\} and baseline variables \{B_{i,k}, w_i, X_{i,k}^F, \pi_{ij,k}, \Lambda_{i,k,m}, \nu_{i,k,m}, \Psi_{i,k,m}\} are sufficient for computing counterfactual equilibrium. Below, I discuss the estimation and calibration of these parameters and variables in turn.

#### 5.2.1 Elasticity of Supplier Matching Rate \(\lambda\)

As discussed in Section 4.3, the elasticity of supplier matching rate \(\lambda\) is a particularly important parameter that governs the degree of agglomeration. Below, I illustrate the estimation procedure of \(\lambda\) to match the reduced-form estimates presented in Section 3.
In the model introduced in Section 4, input buyers in location $i$ in sector $m$ who lost a supplier in sector $k$ matches with a new supplier at Poisson rate $\eta_{i,km} \times (\hat{S}_{i,k}^l)^\lambda$. This implies that the probability that the firm matches with an alternative supplier after one year is approximated by $1 - \exp(-\eta_{i,km} \times (\hat{S}_{i,k}^l)^\lambda)$. This object is directly mapped to the data to estimate $\lambda$, i.e.,

$$\lambda = \min_{\lambda} \left\| 1 - \exp(-\eta_{i,km} \times (\hat{S}_{i,k}^l)^\lambda) - \left\{ \hat{\beta}_1 + \hat{\beta}_2 \times \log \text{LocallySellingSuppliers}_{i,k} \right\} \right\|^2$$

where $\hat{\beta}_1$ and $\hat{\beta}_2$ are the reduced-form estimates of Equation (2), where we choose $\hat{\beta}_1 = 0.160$ and $\hat{\beta}_2 = 0.066$ from Column (2) of Table 3. The estimation results suggest the value of $\lambda = 0.36$, implying that 1 percent increase of $\hat{S}_{i,k}^l$ leads to 0.36 percent higher rate of supplier matching.

It should be noted that the multiplicity of equilibria, as discussed in Section 4.3, does not cause issues in the estimation of parameter $\lambda$. This is because the estimation of the parameter is conducted by just using the partial equilibrium relationship of a model without using an entire equilibrium structure. The same argument applies to markup ratio of input sales $\psi_j$, and other baseline parameters explained below.

### 5.2.2 Cost Advantages upon Supplier Matching $\{\nu_{j,km}\}$

Another key set of parameters are the production benefit of supplier matching, $\{\nu_{j,km}\}$. $\{\nu_{j,km}\}$ themselves are not exogenous parameters in the model, and it is partly determined by the productivity of costly generalized input goods that firms use if not matched with a supplier $\{\chi_{j,k}\}$. However, since $\{\nu_{j,km}\}$ are easier to match with data and also because they are sufficient statistics to conduct counterfactual simulation, I directly calibrate $\{\nu_{j,km}\}$ in this paper.

To calibrate $\{\nu_{j,km}\}$, I use the relationship that $\nu_{j,km} = \frac{\Lambda_{j,km}/\Psi_{j,km}}{1-\Lambda_{j,km}/\Psi_{j,km}}$ where $\Lambda_{j,km}$ and $\Psi_{j,km}$ are directly obtained from the data. This relationship is derived from equation (20). Intuitively, $\nu_{j,km}$ is identified from the sales premium of matching with a supplier.

31In order to estimate $\lambda$, further parametric assumptions are required for $\eta_{i,km}$ and $\hat{S}_{i,k}^l$. By noting the relationship $\eta_{i,km} \times (\hat{S}_{i,k}^l)^\lambda = \eta_{i,km} \left( \frac{\psi_{j,km}}{\hat{S}_{i,k}^l} \right)^\lambda \left[ \frac{1}{\hat{S}_{i,k}^l} \sum_{m \in K} \left( \frac{\theta + \gamma_{k,m}(\sigma-1)}{\theta} \gamma_{k,m} \Psi_{j,km} \right)^\lambda \right]$, I parametrize that $\eta_{i,km} \left( \frac{\psi_{j,km}}{\hat{S}_{i,k}^l} \right)^\lambda \sim \log \mathcal{N}(\mu_{\eta,km}, \sigma_{\eta}^2)$ and $\mu_{km} \sim \mathcal{N}(\bar{\mu}_m, \sigma_{\mu}^2)$.
5.2.3 Other Parameters and Baseline Variables

Table 8 provides the remaining list of estimated and calibrated parameters and variables required for computing counterfactual equilibrium following Proposition 2. In terms of parameters, I calibrate \( \{\alpha_k, \gamma_{km}\} \) from the input-output matrix and \( \sigma, \theta \) from the literature (Broda and Weinstein (2006); Head and Mayer (2014)). \( \psi \) are assumed to be equal to \( \sigma/(\sigma - 1) \), implying that the mark-up for final goods sales is the same as that of the input goods sales. For the remaining baseline variables required for the counterfactual simulation, while some variables are directly obtained from the data \( (B_{i,k}, w_i, \Lambda_{i,km}, \Psi_{i,km}) \), others are obtained indirectly from the model relationships.

6 Results of Counterfactual Simulations

This section presents the results from counterfactual simulations. Section 6.1 illustrates how important the increasing returns to scale in matching is in explaining the equilibrium spatial distribution of economic activities. Section 6.2 analyzes two important policies to economically lagged areas; firm subsidies and transportation infrastructure.

6.1 How Much Does Increasing Returns in Matching Explain Geographic Concentration of Economic Activity in Japan?

To understand how important the increasing returns to scale in matching in explaining the equilibrium spatial distribution of economic activities, I hypothetically shut down the increasing returns to scale in matching, i.e., compute the equilibrium under \( \lambda = 0 \) rather than the estimated value of \( \lambda = 0.36 \), and study how the equilibrium changes. More specifically, I assume that the supplier matching rate is \( M_{i,km}(\hat{S}_{i,k}) = \eta_{i,km} \times (\bar{S}_{i,k})^\lambda \), where \( \bar{S}_{i,k} = \frac{1}{N} \sum_{j \in N} S_{j,k} \), i.e., the average number of input sellers in sector \( k \) in the baseline equilibrium. These matching rates are assumed to be unchanged under the counterfactual equilibrium and they do not depend on \( \hat{S}_{i,k} \). Hence, the equilibrium is computed following Proposition 2 except that \( \hat{S}_{j,k} \) are exogenously specified as \( \hat{S}_{j,k} = \bar{S}_{j,k}/S_{j,k} \) rather than zero profit conditions.

Figure 6 shows that counterfactually assuming that \( \lambda = 0 \) significantly reduces the variance of the firm density and real wages across space. Quantitatively, the standard deviation of firm density decreases to 7% of the baseline, while the variance of real wages decreases by 16%. The remaining geographic variations are induced by the exogenous population distribution, \( L_i \), and the heterogeneity of productivity across locations and sectors, \( A_{i,m} \),
which include natural advantages. The results confirm that a significant part of geographic inequality in firm density and real wages are attributed to the increasing returns in matching.

To interpret the magnitude, note that Ellison and Glaeser (1999) document that about 20% of the variation of firm density across cities in the United States is explained by the proxies of natural advantages. Compared to this number, the extent to which the increasing returns to scale in matching explains the spatial dispersion of economic activities is smaller. It should be noted, however, that this magnitude becomes larger if one incorporates other agglomeration forces or labor mobility. To see this, note that while shutting down increasing returns to scale in matching, a big city (e.g., Tokyo) becomes less dense, which reduces other agglomeration forces in Tokyo (e.g., knowledge spillovers), which makes Tokyo less dense.

6.2 Policy Implications to Promote Economically Lagged Areas

In this section, I analyze two policies targeted to promote economically lagged areas. Throughout the world, policies targeting the improvements of disadvantaged locations are common and are often refereed as place-based policies (Kline and Moretti (2014b); Neumark and Simpson (2015)). Here, I study implications of such place-based policies in one of the most economically disadvantaged areas in Japan: Hokkaido (Figure 1). It is one of the most economically lagged areas in Japan; an illustrating event is the bankruptcy of Yubari municipality in 1996 due to the lack of tax revenue. Partly aimed for improving the economic conditions of Hokkaido area, a new bullet train connected Hokkaido island and the northern part of the main island Japan (i.e., Tohoku area) in 2016, and is planned to fully extend over the next 10 to 20 years.

The policies studied in this section are two-folds. First, Section 6.2.1 analyzes the implication of different type of firm subsidies. Second, Section 6.2.2 analyzes the change of iceberg trade cost for input goods between Hokkaido and Tohoku area, motivated by the new bullet train.

6.2.1 Are Input Sales Subsidies More Effective than Production Subsidies?

In this subsection, I analyze firm subsidies. In particular, I compare two types of firm subsidies: subsidies for input suppliers to sell in Hokkaido, and subsidy to produce in Hokkaido. The former are given to suppliers conditional on serving the location as input sellers regardless of their production location, and the latter are given to suppliers who produce in these locations. In the model, these are translated as $T_{i,k}^S$ and $T_{i,k}^B$ in Proposition 2. In reality, the former subsidies are commonly implemented in the form of trade exhibitions or business matching events, and the latter in the form of tax exemptions or subsidies for new business
establishment.

Figure 7 presents the results. While firm density and real wages increase as a function of total dollar spent for both types of subsidies, subsidy for input sales is much more effective than production subsidy for the same dollar spent.\textsuperscript{32} The intuition of these results is simple: Since the agglomeration benefit arises from the density of input suppliers selling in each location, one should directly target this margin. It should be noted, however, that this implication does not hold if the supplier matching elasticity $\lambda$ is not sufficiently large. To see this, if $\lambda \approx 0$, there is no impact on firm density and welfare by subsidizing input sellers. Hence, this results come from the fact that the estimated increasing returns to scale in matching is quantitatively large.

6.2.2 What Are the Welfare Implications of Transportation Infrastructure?

In this subsection, I analyze the impacts of change of iceberg trade cost for input goods between Hokkaido and Tohoku area. Figure 8 shows the results. The x-axis represents the inverse of the change of iceberg trade cost for input goods between Hokkaido and Tohoku areas (i.e., $1/\hat{\tau}_{\text{Hokkaido,Tohoku},k}$ for all $k$). For firm density, improving the transportation infrastructure actually reduces the firm density. As for the real wages in Hokkaido, it exhibits a non-monotonic pattern: it initially decreases and then increases.

The intuition is as follows. There are two counter-forces as a response to the change of input goods trade cost. For one thing, reduction of input goods trade cost increases the unit cost of suppliers selling in Hokkaido, which benefits firms in Hokkaido. On one hand, the reduction of transportation costs benefits firms in Hokkaido through the reduction of unit cost of input goods. On the other hand, reducing transportation costs harms firms by exposing them for more competition. If the latter force is stronger, it reduces the incentive for potential entrants to enter in Hokkaido and reduces firm density in Hokkaido, which results in the welfare reduction. This intuition also explains why there are non-monotonic pattern of real wages. While the first force is initially weak because the matching probability is close to 0, as the transportation cost decreases, more input suppliers sell in Hokkaido, which increases the supplier matching rate and exponentially increases the benefit from a marginal improvement of transportation infrastructure.

\textsuperscript{32}These implications are unchanged even if one considers the impacts of these subsidies in other locations. As shown in Figure A.4, the impacts of subsidies in Hokkaido on locations outside Hokkaido are much smaller in magnitude.
7 Conclusion

This paper investigates the importance of increasing returns in firm-to-firm matching in input trade as a source of agglomeration. I first provide reduced-form evidence of increasing returns to scale in firm-to-firm matching. Using unexpected supplier bankruptcies as an instrument, I show that the new supplier matching rate upon a supplier loss increases in locations and industries when there are more alternative suppliers selling in the buyer’s location, while this rate remains stable in the presence of other downstream firms. Based on these findings, I build a structural trade model to quantify the importance of increasing returns in firm-to-firm matching for the geographic concentration patterns of economic activity in Japan. The structural model highlights distinct policy implications compared to a typical model in the literature that assumes agglomeration benefit arises from local firm density producing in a location. In particular, I find that subsidies for input suppliers to sell in the target location is much more effective than subsidies to produce in these locations. I also find that transportation infrastructure development initially harms and then improves the welfare of the target location.

There are several important directions for future work. First, further understanding of why matching frictions exist and how the policies can reduce such frictions is important. To understand this, Daisuke Miyakawa and I are partnering with Tokyo Shoko Research to conduct a randomized control trial to provide information about potential transaction partners. Such an RCT will reveal how information frictions interact with geographic space and how reducing such frictions improves firm production. Second, dynamic implications of matching frictions and increasing returns in firm-to-firm matching for regional business cycles and long-term growth should be studied both theoretically and empirically. Lastly, other microfoundations of agglomeration should be studied empirically, theoretically and quantitatively. By no means does this paper claim that increasing returns in firm-to-firm matching is the only source of agglomeration. Reflecting Marshall (1890), labor market pooling and knowledge spillovers may be equally important forces that drive agglomeration. With this respect, a general key message of this paper is that both theoretical and empirical understandings of different microfoundations of agglomeration are important for proper understanding of policy consequences, and further research is awaited.
References


Figure 1: Geographic Distribution of Firms in Japan

(A) Map of Japan

(B) Cumulative Distribution of Firms by Local Firm Density

Note: Local firm density is defined by the density of firm headquarters in 2006, evaluated at each grid cell of 0.05 degree latitude by 0.05 degree longitude (approximately 5.5 km by 4.5 km).
Figure 2: Cumulative Distribution of Geodesic Distances between Supplier and Buyers

Note: The graph shows the cumulative distributions of geodesic distance between supplier and buyer’s headquarter locations for the years of 2006, 2011, and 2014.
Figure 3: Number of Suppliers per Firm and Local Firm Density

**Note:** Samples are firms in the data set from 2006. Firm density is defined by the density of firm headquarters in 2006, evaluated at each grid cell of 0.05 degree latitude by 0.05 degree longitude (approximately 5.5 km by 4.5 km).

Figure 4: New Supplier Matching Rate and Separation Rate by Local Firm Density

**Note:** “New Supplier Matching Rate” is the number of suppliers that firms gain per year (averaged across 2006 to 11 and 2011 to 14), and “Separation Rate” is the probability that firms lose suppliers per year, including both the exits of suppliers and dissolution of trading relationships.
Figure 5: Examples of Geographic Variation of the Density of Locally-Selling Suppliers

(A) Forestry Industry

(B) Steel Industry

Note: The density of locally-selling suppliers is the density the number of firms for each sector that supply to any firms whose headquarter is located in each location (defined by 0.5 degree grid cell) at the baseline year (2006). See Section 3 for more discussion.
Figure 6: Counterfactual Simulation: Shutting Off Increasing Returns in Matching

Note: The graphs show the results from the counterfactual simulations in Section 6.1. Each graph shows the distribution of log firm density and log real wages in the data (in dotted blue lines) and that under the counterfactual equilibrium with $\lambda = 0$ (in solid red lines). Panel (B) shows the same objects for the real wages.
Figure 7: Counterfactual Simulation: Subsidies for Input Sales and Production in Hokkaido

Note: The graphs show the results from the counterfactual simulations in Section 6.2.1. In each graph, blue circle dots represent the impacts of input sales subsidies and red diamond dots represent the impacts of production subsidies. X-axis represents the total subsidies given as a fraction of total input goods expenditure in Hokkaido area in the baseline ($X^I_{Hokkaido}$).
Figure 8: Counterfactual Simulation: Transportation Infrastructure

*Note:* The graphs show the results from the counterfactual simulations in Section 6.2.2. X-axis represents the inverse of the change of iceberg trade cost for input goods between Hokkaido and Tohoku areas (i.e., $1/\hat{\tau}_{Hokkaido,Tohoku,k}$ for all $k$).
Table 1: Main Reasons of Bankruptcies

<table>
<thead>
<tr>
<th>Reasons of Bankruptcies</th>
<th>Number of Bankruptcies 06-14</th>
<th>Percentage</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>reasons used for IV</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>spillover from other bankruptcy</td>
<td>2,310</td>
<td>5.8%</td>
</tr>
<tr>
<td>accidental reasons</td>
<td>615</td>
<td>1.5%</td>
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<tr>
<td><strong>reasons NOT used for IV</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>non-performing sales</td>
<td>26,073</td>
<td>65.4%</td>
</tr>
<tr>
<td>accumulation of debt</td>
<td>5,614</td>
<td>14.1%</td>
</tr>
<tr>
<td>insufficient capital</td>
<td>2,791</td>
<td>7.0%</td>
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<tr>
<td>management failure</td>
<td>1,311</td>
<td>3.3%</td>
</tr>
<tr>
<td>overinvestment in capital</td>
<td>438</td>
<td>1.1%</td>
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<tr>
<td>difficulty in collecting account receivable</td>
<td>249</td>
<td>0.6%</td>
</tr>
<tr>
<td>non-project related failure</td>
<td>232</td>
<td>0.6%</td>
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<tr>
<td>issuance of accommodation debt</td>
<td>155</td>
<td>0.4%</td>
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<tr>
<td>over-accumulation of inventory</td>
<td>51</td>
<td>0.1%</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td>39,839</td>
<td>100.0%</td>
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</table>

*Note:* The frequency of the bankruptcies assigned to each main reason of bankruptcies is reported. “Spillover from other bankruptcy” and “accidental reasons” are the two categories of bankruptcies used for an instrument for a supplier loss in Section 3. “Spillover from other bankruptcies” are those caused by management difficulties due to chain reactions such as business partners, subsidiary companies, related bankruptcies, voluntary liquidation, etc. “Accidental reasons” include those with unanticipated accidental problems such as the death of representatives, flood disaster, fire, earthquake, traffic accident, fraud, theft, embezzlement, etc.
Table 2: Summary Statistics of Japanese Firm-to-Firm Matching in Input Trade

(A) Cross-Sectional Patterns

<table>
<thead>
<tr>
<th>(i) Number of Suppliers per Firm</th>
<th>mean</th>
<th>2006</th>
<th>2011</th>
<th>2014</th>
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</thead>
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<tr>
<td></td>
<td>median</td>
<td>2.65</td>
<td>4.63</td>
<td>4.62</td>
</tr>
<tr>
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<td></td>
<td>99 percentile</td>
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<td></td>
<td>55 percentile</td>
<td>33</td>
<td>35</td>
<td>37</td>
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<table>
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<tr>
<th>(ii) Number of 2-digit Input Sectors with a Supplier per Firm</th>
<th>mean</th>
<th>2006</th>
<th>2011</th>
<th>2014</th>
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<tr>
<td></td>
<td>median</td>
<td>2.39</td>
<td>2.34</td>
<td>2.33</td>
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<table>
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<th>(iii) Number of 4-digit Input Sectors with a Supplier per Firm</th>
<th>mean</th>
<th>3.16</th>
<th>3.11</th>
<th>3.10</th>
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<td></td>
<td>median</td>
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<td>2</td>
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<table>
<thead>
<tr>
<th>(iv) Number of Buyers per Firm</th>
<th>mean</th>
<th>4.64</th>
<th>4.66</th>
<th>4.65</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>median</td>
<td>2</td>
<td>2</td>
<td>2</td>
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</tbody>
</table>

| (v) Number of Firms | 695,418 | 1,030,965 | 1,095,617 |

(B) Dynamic Patterns

<table>
<thead>
<tr>
<th>(i) Probability of Exit</th>
<th>2006 to 11</th>
<th>2011 to 14</th>
<th>per-year average</th>
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<tr>
<td></td>
<td>0.099</td>
<td>0.053</td>
<td>0.019</td>
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<table>
<thead>
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<th>(ii) Change of Suppliers (Conditional on Survival)</th>
<th>2006 to 11</th>
<th>2011 to 14</th>
<th>per-year average</th>
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<tr>
<td>Proportion of Exiting Suppliers</td>
<td>0.10</td>
<td>0.05</td>
<td>0.018</td>
</tr>
<tr>
<td>Proportion of Lost Supplier Relationships</td>
<td>0.37</td>
<td>0.24</td>
<td>0.077</td>
</tr>
<tr>
<td>No. of New Suppliers / No. of Baseline Suppliers</td>
<td>0.54</td>
<td>0.24</td>
<td>0.094</td>
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</table>

<table>
<thead>
<tr>
<th>(iii) Relocation Probability of Firm Headquarters</th>
<th>2006 to 11</th>
<th>2011 to 14</th>
<th>per-year average</th>
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<tr>
<td></td>
<td>0.014</td>
<td>0.008</td>
<td>0.003</td>
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</table>

Note: Proportion of lost suppliers in Row (ii) of Panel (B) includes both the cases of supplier exits and dissolution of trading relationships in the specified time interval. Relocation of firm headquarters in Row (iii) of Panel (B) is defined by the relocation outside the grid cell of 0.5 degree latitude by 0.5 degree longitude. The two-digit and four-digit sectors follow the Japan Standard Industrial Classification (JSIC) with 98 two-digit sectors and 1248 four-digit sectors in JSIC classification. For more detail on JSIC, see [http://www.soumu.go.jp/english/dgpp_ss/seido/sangyo/index.htm](http://www.soumu.go.jp/english/dgpp_ss/seido/sangyo/index.htm).
Table 3: Evidence of New Supplier Matching Rate Increasing in Density of Locally-Selling Suppliers

<table>
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<tr>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separated Suppliers</td>
<td>0.156***</td>
<td>0.160***</td>
<td>0.145</td>
<td>−0.027</td>
<td>−0.027</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.054)</td>
<td>(0.054)</td>
<td></td>
<td>(0.090)</td>
<td>(0.035)</td>
<td>(0.035)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sep. S. x log Locally-Selling Suppliers</td>
<td>0.066**</td>
<td>0.104***</td>
<td>0.096**</td>
<td>0.126***</td>
<td>0.077*</td>
<td>0.005</td>
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</tr>
<tr>
<td></td>
<td>(0.026)</td>
<td>(0.040)</td>
<td>(0.049)</td>
<td>(0.051)</td>
<td>(0.042)</td>
<td>(0.015)</td>
<td></td>
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</tr>
<tr>
<td>Unexp. S. Bankruptcy x Location x Year FE</td>
<td></td>
<td>X</td>
<td>X</td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unexp. S. B. x Buyer Industry x Pref. x Year FE</td>
<td>X</td>
<td></td>
<td>X</td>
<td></td>
<td></td>
<td></td>
<td></td>
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<tr>
<td>Unexp. S. B. x log Employment Size</td>
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<td></td>
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<tr>
<td>Firm x Year FE</td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td>X</td>
<td></td>
</tr>
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</table>

*Note:* Regression specification follows Equation (2). The regression is run at the firm, input sector, and year level. The outcome variable is the number of new suppliers per year, and “Separated Suppliers” indicates the number of lost suppliers per year. “Locally-Selling Suppliers” are defined as the number of firms in each input sector that supply to any firms in the same location (0.5 degree grid cells) in the baseline period (2006). “Separated Suppliers” and its interactions are instrumented by “Unexpected Supplier Bankruptcy” and its relevant interactions. All regressions control for the buyer’s location, buyer’s sector, input sector and year fixed effects, as well as the fixed effects for the baseline number of suppliers. “log Locally-Selling Suppliers” are normalized to be mean 0, and the inter-quartile range is 2.6. Prefecture (Pref.) is a geographic administrative unit in Japan, with 47 prefectures in total. Standard errors are clustered at the firm level, and regressions are weighted to equalize the weight at the firm and year level. *p<0.1; **p<0.05; ***p<0.01.
Table 4: Evidence that Density of *Locally-Selling* Suppliers is a Sufficient Statistics for New Supplier Matching Rate

<table>
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<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Separated Suppliers</td>
<td>0.151***</td>
<td>0.131***</td>
<td>0.124**</td>
<td>0.129***</td>
</tr>
<tr>
<td></td>
<td>(0.053)</td>
<td>(0.050)</td>
<td>(0.055)</td>
<td>(0.049)</td>
</tr>
<tr>
<td>Sep. S. x log Locally-Selling Suppliers</td>
<td>0.077*</td>
<td>0.085*</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.040)</td>
<td>(0.049)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sep. S. x log Locally-Established Firms</td>
<td>0.051*</td>
<td></td>
<td>−0.014</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.030)</td>
<td></td>
<td>(0.045)</td>
<td></td>
</tr>
<tr>
<td>Sep. S. x log Locally-Established Suppliers</td>
<td>0.047**</td>
<td></td>
<td>−0.018</td>
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</tr>
<tr>
<td></td>
<td>(0.020)</td>
<td></td>
<td>(0.034)</td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>2,288,772</td>
<td>2,288,772</td>
<td>2,288,772</td>
<td>2,288,772</td>
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</tbody>
</table>

*Note:* Regression specification follows Equation (3). See the footnote of Table 3 for the definitions of samples, outcome variables and other controls. “Locally-Selling Suppliers” are defined as the number of firms in each input sector that supply to any firms in the same location (0.5 degree grid cells) in the baseline period (2006). “Locally-Established Firms” are the number of firms in *any sectors* whose headquarters are located in the buyer-side firm’s location, and “Locally-Established Suppliers” are the number of firms in *each input sector* whose headquarters are located in the buyer-side firm’s location.
Table 5: Evidence of New Supplier Matching Rate Not Decreasing in Buyers

<table>
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<th>(5)</th>
<th>(6)</th>
<th>(7)</th>
<th>(8)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Supplier Separation</td>
<td>0.179***</td>
<td>0.165***</td>
<td>0.152**</td>
<td>0.120</td>
<td>0.174***</td>
<td>0.124**</td>
<td>0.193***</td>
<td>0.108</td>
</tr>
<tr>
<td></td>
<td>(0.054)</td>
<td>(0.049)</td>
<td>(0.061)</td>
<td>(0.083)</td>
<td>(0.064)</td>
<td>(0.059)</td>
<td>(0.055)</td>
<td>(0.066)</td>
</tr>
<tr>
<td>Sep. S. x log Potential Suppliers</td>
<td>0.091***</td>
<td>0.071***</td>
<td>0.063***</td>
<td>0.060**</td>
<td>0.054**</td>
<td>0.049**</td>
<td>0.066**</td>
<td>0.066**</td>
</tr>
<tr>
<td></td>
<td>(0.032)</td>
<td>(0.027)</td>
<td>(0.023)</td>
<td>(0.024)</td>
<td>(0.025)</td>
<td>(0.025)</td>
<td>(0.026)</td>
<td>(0.026)</td>
</tr>
<tr>
<td>Sep. S. x log Buyers (Same 2-digit Industry)</td>
<td>−0.036</td>
<td></td>
<td></td>
<td></td>
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<tr>
<td></td>
<td>(0.033)</td>
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<tr>
<td>Sep. S. x log Buyers (Same 4-digit Industry)</td>
<td></td>
<td>−0.008</td>
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<td></td>
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<td>(0.020)</td>
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<tr>
<td>Sep. S. x log Buyers (Same 2-digit Industry) with Unexp. S. Bankruptcy</td>
<td></td>
<td></td>
<td>0.012</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td>(0.049)</td>
<td></td>
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<tr>
<td>Sep. S. x log Buyers (all) with Unexp. S. Bankruptcy</td>
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<td>0.027</td>
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<td>(0.046)</td>
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<td>Sep. S. x log Buyers (Same 2-digit Industry) with Supplier Separation</td>
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<td>(0.031)</td>
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<td>Sep. S. x log Buyers (all) with Supplier Separation</td>
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<td>(0.030)</td>
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<tr>
<td>Sep. S. x log Buyers Anywhere (Same 2-digit Industry) with Unexp. S. Bankruptcy</td>
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<td></td>
<td>0.032</td>
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<tr>
<td>Sep. S. x log Buyers Anywhere (all) with Unexp. S. Bankruptcy</td>
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<td>0.035*</td>
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<td>2,288,793</td>
<td>2,288,793</td>
<td>2,288,793</td>
<td>2,288,793</td>
<td>2,288,793</td>
<td>2,288,793</td>
</tr>
</tbody>
</table>

Note: Regression specification follows Equation [3]. See the footnote of Table 3 for the definitions of samples, outcome variables, other controls, and the variable “PotentialLocally-Selling Suppliers”. “log Buyers (Same 2-digit Industry)” indicates the number of firms in the same 2-digit industry in firm f’s location. “log Buyers (Same 2-digit Industry) with Unexp. S. Bankruptcy” indicates the number of firms in the same 2-digit industry in f’s location that faced unexpected supplier bankruptcy in the same 2-digit input sector, while “log Buyers (Same 2-digit Industry) with Supplier Separation” follows the same definition except that firms are counted if they faced any supplier separation (including supplier exits and dissolution of trade relationships) in the same 2-digit sector. “log Buyers (Same 2-digit Industry) Anywhere with Unexp. S. Bankruptcy” follows the same definition as “log Buyers (Same 2-digit Industry) with Unexp. S. Bankruptcy,” except firms in any locations are counted as long as they face unexpected supplier bankruptcy in the same input sector. Standard errors are clustered at the firm level, and regressions are weighted to equalize the weight at the firm and year level. *p<0.1; **p<0.05; ***p<0.01.
Table 6: Impacts of Unexpected Supplier Bankruptcy on Sales Growth and Exit

<table>
<thead>
<tr>
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<th>Sales Growth</th>
<th>Exit</th>
<th>Sales Growth (Surviving)</th>
<th>Sales Growth (Pretrend)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Unexpected Supplier Bankruptcy</td>
<td>−0.031*</td>
<td>−0.036*</td>
<td>0.007*</td>
<td>0.007*</td>
</tr>
<tr>
<td></td>
<td>(0.018)</td>
<td>(0.019)</td>
<td>(0.004)</td>
<td>(0.004)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>−0.005</td>
<td>−0.009</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.012)</td>
<td>(0.012)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>−0.012</td>
<td>−0.013</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.016)</td>
<td>(0.016)</td>
</tr>
<tr>
<td>Unexp. S. Bankruptcy x log Locally-Selling Suppliers</td>
<td>0.013</td>
<td>−0.0005</td>
<td>0.011</td>
<td>0.008</td>
</tr>
<tr>
<td></td>
<td>(0.011)</td>
<td>(0.003)</td>
<td>(0.007)</td>
<td>(0.008)</td>
</tr>
</tbody>
</table>

Observations: 2,496,861 2,496,861 2,496,861 2,496,861 2,309,804 2,309,804 1,017,393 1,017,393

Note: The regression specification follows Equation (4). See the footnote of Table 3 for the definitions of samples, outcome variables, other controls, and the variable “Potential Suppliers”. Sales growth is defined by the arc-elasticity (Davis and Haltiwanger [1992]). Standard errors are clustered at the firm level, and regressions are weighted to equalize the weight at the firm and year level. *p<0.1; **p<0.05; ***p<0.01.
### Table 7: Impacts of Unexpected Supplier Bankruptcy on Other Existing Suppliers

<table>
<thead>
<tr>
<th></th>
<th>Pr[Other Supplier Separation]</th>
<th>Other Supplier Sales Growth</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Unexpected Supplier Bankruptcy</td>
<td>0.002</td>
<td>−0.002</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Unexp. S. Bankruptcy x log Locally-Selling Suppliers</td>
<td>0.007</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.004)</td>
<td></td>
</tr>
</tbody>
</table>

| Observations                   | 870,764                       | 870,764                    | 865,615                    | 865,615                    |

*Note:* The regression specification follows Equation (5). See the footnote of Table 3 for the definitions of samples, outcome variables, other controls, and the variable “Potential Suppliers”. Sales growth is defined by the arc-elasticity (Davis and Haltiwanger [1992]). Standard errors are clustered at the firm level, and regressions are weighted to equalize the weight at the firm and year level. *p<0.1; **p<0.05; ***p<0.01.
### Table 8: Estimated and Calibrated Parameters

<table>
<thead>
<tr>
<th>Estimated and Calibrated Parameters</th>
<th>value</th>
<th>source</th>
</tr>
</thead>
<tbody>
<tr>
<td>Elasticity of supplier matching rate $\lambda$</td>
<td>0.36</td>
<td>Section 5.2.1</td>
</tr>
<tr>
<td>Cost advantage upon supplier matching $\nu_{i,km}$</td>
<td></td>
<td>Section 5.2.2</td>
</tr>
<tr>
<td>Input share in production $\gamma_{L,m}, \gamma_{km}$</td>
<td></td>
<td>IO table</td>
</tr>
<tr>
<td>Final goods expenditure share $\alpha_k$</td>
<td></td>
<td>IO table</td>
</tr>
<tr>
<td>Productivity distribution shape parameter $\theta$</td>
<td>5</td>
<td>Head and Mayer (2013)</td>
</tr>
<tr>
<td>Elasticity of substitution $\sigma$</td>
<td>5</td>
<td>Broda and Weinstein (2006)</td>
</tr>
<tr>
<td>Mark-up for input sales $\psi$</td>
<td>1.25</td>
<td>Assume same mark-up as final goods, i.e., $\psi = \sigma / (\sigma - 1)$</td>
</tr>
</tbody>
</table>

### Baseline Variables

<table>
<thead>
<tr>
<th>Baseline Variables</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of producers $B_{i,k}$</td>
<td>data</td>
<td></td>
</tr>
<tr>
<td>Wages $w_i$</td>
<td>data</td>
<td></td>
</tr>
<tr>
<td>Steady state supplier matching probability $\Lambda_{i,km}$</td>
<td>data</td>
<td></td>
</tr>
<tr>
<td>Market share of firms with a supplier $\Psi_{i,km}$</td>
<td>data</td>
<td></td>
</tr>
<tr>
<td>Expenditure share of input goods from $i$ in $j$ $\pi^I_{ij,k}$</td>
<td>Smoothed with gravity equation; See Appendix D</td>
<td></td>
</tr>
<tr>
<td>Expenditure share of final goods from $i$ in $j$ $\pi^F_{ij,k}$</td>
<td>Assume $\pi^F_{ij,k} = \pi^I_{ij,k}$ in the baseline</td>
<td></td>
</tr>
<tr>
<td>Final goods sales $X^F_{i,k}$</td>
<td>Use total expenditure condition (Equation 9)</td>
<td></td>
</tr>
<tr>
<td>Input goods sales $X^I_{i,k}$</td>
<td>Use total expenditure condition (Equation 10)</td>
<td></td>
</tr>
</tbody>
</table>
A Additional Figures and Tables

Figure A.1: CDF of Number of Suppliers

Note: Cumulative distribution of the number of suppliers under two definitions. “Only reported by buyer-side firm” defines the existence of a link if only buyer-side firms report. “Reported by either side” defines the existence of a link if either party reports that the relationship exists.
**Figure A.2: Per-Worker Revenue Productivity and Number of Suppliers per Firm**

*Note:* Samples are based on the data from 2006. The graph shows the average log sales per worker for each number of suppliers, censored at 100.

**Figure A.3: Distribution of Suppliers and Input Sectors per Firm**

*Note:* Samples are based on the data from 2006. The graph shows the CDF of the number of suppliers and number of input sectors with a supplier at the firm level, censored at 20.
Figure A.4: Counterfactual Simulation: Subsidies for Input Sales and Production outside Hokkaido

Note: The graphs show the results from the counterfactual simulations in Section 6.2.1. In each graph, blue circle dots represent the impacts of input sales subsidies and red diamond dots represent the impacts of production subsidies. X-axis represents the total subsidies given as a fraction of total input goods expenditure in Hokkaido area in the baseline ($X^I_{Hokkaido}$).
Figure A.5: Additional Figures for Counterfactual Simulation of Transportation Infrastructure

Note: The graphs show the results from the counterfactual simulations in Section 6.2.2. X-axis represents the inverse of the change of iceberg trade cost for input goods between Hokkaido and Tohoku areas (i.e., $1/\hat{\tau}_{Hokkaido,Tohoku,k}$ for all $k$).
Table A.1: Number of Suppliers per Firm and Firm Density

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
<th>(5)</th>
<th>(6)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log Firm Density</td>
<td>0.784***</td>
<td>0.956***</td>
<td>0.100***</td>
<td>0.268***</td>
<td>0.228***</td>
<td>0.025***</td>
</tr>
<tr>
<td></td>
<td>(0.193)</td>
<td>(0.218)</td>
<td>(0.023)</td>
<td>(0.020)</td>
<td>(0.010)</td>
<td>(0.002)</td>
</tr>
</tbody>
</table>

Sample | excl. Tokyo
4-digit Industry FE | X | X | X | X | X
Employment Size FE | X | X | X | X | X
Prefecture FE | X | X | X | X |
Observations | 696,454 | 696,453 | 670,740 | 670,740 | 591,312 | 670,740 |
R² | 0.002 | 0.034 | 0.928 | 0.929 | 0.895 | 0.447 |

Note: Samples are firms in the data set from 2006. Firm density is defined by the density of firm headquarters in 2006, evaluated at each grid cell of 0.05 degree latitude by 0.05 degree longitude (approximately 5.5 km by 4.5 km). Prefecture is a geographic administrative unit in Japan, with 47 prefectures in total. Standard errors are clustered at the grid cell level. *p<0.1; **p<0.05; ***p<0.01.

Table A.2: Revenue Productivity and Number of Suppliers per Firm

<table>
<thead>
<tr>
<th></th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>log (Supplier Number + 1)</td>
<td>0.373***</td>
<td>0.305***</td>
<td>0.340***</td>
</tr>
<tr>
<td></td>
<td>(0.001)</td>
<td>(0.001)</td>
<td>(0.001)</td>
</tr>
</tbody>
</table>

4-digit Industry FE | X | X
Employment Size FE | X
Observations | 670,092 | 670,091 | 670,091 |
R² | 0.107 | 0.313 | 0.324 |

Note: Samples are firms in the data set from 2006. Firm density is defined by the density of firm headquarters in 2006, evaluated at each grid cell of 0.05 degree latitude by 0.05 degree longitude (approximately 5.5 km by 4.5 km). Robust standard errors are reported. *p<0.1; **p<0.05; ***p<0.01.
Table A.3: First Stages of New Supplier Matching Regression

<table>
<thead>
<tr>
<th></th>
<th>Supplier Separation</th>
<th>Pretrend</th>
<th>(1)</th>
<th>(2)</th>
<th>(3)</th>
<th>(4)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unexpected Supplier Bankruptcy</td>
<td>0.864***</td>
<td>0.861***</td>
<td>0.028</td>
<td>0.034</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.023)</td>
<td>(0.025)</td>
<td>(0.026)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Unexp. Sup. Bankruptcy x log Potential Suppliers</td>
<td>0.007</td>
<td>−0.011</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.012)</td>
<td>(0.011)</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>2,288,793</td>
<td>2,288,793</td>
<td>989,957</td>
<td>989,957</td>
<td></td>
<td></td>
</tr>
<tr>
<td>R²</td>
<td>0.367</td>
<td>0.367</td>
<td>0.243</td>
<td>0.243</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Note: The first stage regression of Equation [3] are reported. See the footnote of Table [3] for the definitions of samples, outcome variables, other controls, and variable definitions. Standard errors are clustered at the firm level, and regressions are weighted to equalize the weight at the firm and year level. *p<0.1; **p<0.05; ***p<0.01.

Table A.4: New Supplier Matching in All Input Sectors

<table>
<thead>
<tr>
<th></th>
<th>New Suppliers (All Industry)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
</tr>
<tr>
<td>Separated Suppliers</td>
<td>0.668***</td>
</tr>
<tr>
<td></td>
<td>(0.188)</td>
</tr>
<tr>
<td>Sep. S. x log Locally-Selling Suppliers</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
</tr>
<tr>
<td>Observation</td>
<td>2.2M</td>
</tr>
</tbody>
</table>

Note: Regression specification follows Equation [2], except that the dependent variables are the new supplier matching rate in any sectors, not only in each input sector. See the footnote of Table [3] for the definitions of samples, outcome variables, other controls, and variable definitions. Standard errors are clustered at the firm level, and regressions are weighted to equalize the weight at the firm and year level. *p<0.1; **p<0.05; ***p<0.01.
Table A.5: Additional Robustness on New Supplier Matching Regression

<table>
<thead>
<tr>
<th></th>
<th>New Suppliers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(A) Endogeneity of Entry (B) Endogeneity of Supplier Bankruptcy (C) Heterogeneity of Input Sectors</td>
</tr>
<tr>
<td></td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9) (10)</td>
</tr>
<tr>
<td>Separated Suppliers</td>
<td>0.070 0.082* 0.122** 0.359** 0.157*** 0.160*** 0.198*** 0.156*** 0.111** 0.296***</td>
</tr>
<tr>
<td>(0.046)</td>
<td>(0.048) (0.050) (0.156) (0.060) (0.056) (0.057) (0.055) (0.044) (0.055)</td>
</tr>
<tr>
<td>Sep. S. x log Locally-Selling Suppliers</td>
<td>0.056** 0.048** 0.054** 0.100* 0.080*** 0.066** 0.087*** 0.097*** 0.042** 0.058**</td>
</tr>
<tr>
<td>(0.022)</td>
<td>(0.023) (0.025) (0.058) (0.028) (0.026) (0.026) (0.032) (0.019) (0.026)</td>
</tr>
<tr>
<td>Observation</td>
<td>1.7M 1.2M 1.6M 2.2M 2.2M 2.2M 2.2M 2.2M 0.6M 1.6M</td>
</tr>
<tr>
<td>Sample</td>
<td>Excl. top 1 Excl. top 3 Local CEO Accidental Only bankruptcy Spillover Ctrl. Ctrl. Input Sector Manufacturing Nonmanufacturing</td>
</tr>
<tr>
<td></td>
<td>Only relationship Only Only bankruptcy Solvency Score Coefficients Total Firm No. Input Sector Input Sector</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th></th>
<th>New Suppliers</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(D) Robustness to Geographic Definition (E) Robustness to Industry Definition (F) Other Misc. Robustness</td>
</tr>
<tr>
<td></td>
<td>(1) (2) (3) (4) (5) (6) (7) (8) (9) (10) (11)</td>
</tr>
<tr>
<td>Separated Suppliers</td>
<td>0.169*** 0.149** 0.145** 0.154*** 0.096* 0.065 0.102* 0.292*** 0.209*** 0.119** 0.128**</td>
</tr>
<tr>
<td>(0.046)</td>
<td>(0.062) (0.069) (0.050) (0.054) (0.071) (0.057) (0.073) (0.048) (0.047) (0.059)</td>
</tr>
<tr>
<td>Sep. S. x log Locally-Selling Suppliers</td>
<td>0.043** 0.048 0.037 0.065*** 0.043 0.050* 0.066* 0.067** 0.085*** 0.050** 0.110***</td>
</tr>
<tr>
<td>(0.021)</td>
<td>(0.029) (0.027) (0.024) (0.028) (0.030) (0.035) (0.026) (0.026) (0.025) (0.022)</td>
</tr>
<tr>
<td>Observation</td>
<td>2.0M 1.7M 2.2M 2.2M 2.6M 2.8M 2.2M 1.2M 2.2M 2.3M 2.4M</td>
</tr>
<tr>
<td>Sample</td>
<td>Excl. Excl. 0.1 Degree 1 Degree 3 digit 4 digit Locally-Selling Relationship Only Incl. Incl.</td>
</tr>
<tr>
<td></td>
<td>Tokyo Est.&gt;5 Grid Cells Grid Cells Sector Sector Suppliers within Same 2-digit Buyer Sector</td>
</tr>
</tbody>
</table>

Note: The specification follows that of Table 3. Column (1) controls for the interaction between unexpected supplier bankruptcy and the input coefficients of the IO matrix. In Column (2), the ranking of industry pair is based on the baseline probability that firms in buyer industries match with suppliers in each input sector. Columns (4) and (5) restrict the instruments for separated suppliers to be unexpected supplier bankruptcy due to accidental reasons and due to other bankruptcy spillovers, respectively. Column (8) and (9) indicate the regressions run at the three digits and four digits industry classification levels, respectively.
Table A.6: New Supplier Matching Regression with All Supplier Exits as an Instrument

<table>
<thead>
<tr>
<th></th>
<th>New Suppliers</th>
<th>Pretrend</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
</tr>
<tr>
<td>Separated Suppliers</td>
<td>0.125***</td>
<td>0.127***</td>
</tr>
<tr>
<td></td>
<td>(0.007)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>Sep. S. x log Locally-Selling Suppliers</td>
<td>0.053***</td>
<td>0.062***</td>
</tr>
<tr>
<td></td>
<td>(0.005)</td>
<td>(0.005)</td>
</tr>
<tr>
<td>Firm x Year x Supplier Exit FE</td>
<td>X</td>
<td>X</td>
</tr>
<tr>
<td>Observations</td>
<td>2,288,793</td>
<td>2,288,793</td>
</tr>
</tbody>
</table>

Note: The specification follows that of Table 3 except that the instruments are supplier exits for any reasons, not unexpected supplier bankruptcies as in Table 3.
Table A.7: New Supplier Matching Within and Outside Locations

<table>
<thead>
<tr>
<th></th>
<th>New Suppliers</th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(Local)</td>
<td>(Nonlocal)</td>
<td>(Local)</td>
<td>(Nonlocal)</td>
</tr>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
<td>(4)</td>
</tr>
<tr>
<td>Supplier Separation</td>
<td>0.063*</td>
<td>0.093**</td>
<td>0.020</td>
<td>0.151***</td>
</tr>
<tr>
<td></td>
<td>(0.037)</td>
<td>(0.037)</td>
<td>(0.037)</td>
<td>(0.046)</td>
</tr>
<tr>
<td>Sep. S. x log Locally-Selling Suppliers</td>
<td>−0.041</td>
<td>0.126***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.034)</td>
<td>(0.038)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sep. S. x log Locally-Established Suppliers</td>
<td>0.077***</td>
<td>−0.095***</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.023)</td>
<td>(0.027)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>2,288,793</td>
<td>2,288,793</td>
<td>2,288,793</td>
<td>2,288,793</td>
</tr>
</tbody>
</table>

Note: The specification follows that of Table 3. “Local” indicates the number of new suppliers within the same 0.5 degree grids, and “Nonlocal” indicates that outside the grids.
Table A.8: Correlation between $\nu_{i,km}$ and Proxy of Potential Suppliers in Reduced-Form Regression

<table>
<thead>
<tr>
<th></th>
<th>log $\nu$</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>log Locally-Selling Suppliers</td>
<td>0.029***</td>
<td>0.031***</td>
<td>0.074***</td>
</tr>
<tr>
<td></td>
<td>(0.002)</td>
<td>(0.003)</td>
<td>(0.009)</td>
</tr>
</tbody>
</table>

|                              |       |       |       |
| Input Sector FE              | X     | X     |       |
| Buyer Sector FE              | X     | X     |       |
| Buyer Location FE            |       | X     |       |
| Observations                 | 89,739| 89,739| 89,739|

Note: See Appendix 5.2.2 for the calibration of $\nu_{i,km}$, i.e., the cost advantage upon supplier matching in the structural model, and Section 3 for the definition of the proxy for the number of potential suppliers in the reduced-form exercise. *p<0.1; **p<0.05; ***p<0.01.
B Proofs and Derivations

B.1 Proof of Proposition 1

This section provides a proof of Proposition 1 following in the order of Section 4.2 by filling the detailed algebraic derivations.

B.1.1 Unit Cost Distribution

The unit cost of firm \( \omega \) in production location \( i \) in sector \( m \) is reproduced from equation (6) as

\[
c_{\omega t} = \frac{1}{\varphi_{\omega} A_{i,m}} w_{i}^{\gamma_{L,m}} \prod_{k \in K} p_{\omega t,k}^{\gamma_{k,m}},
\]

where

\[
\begin{align*}
p_{\omega t,k} &= \chi_{i,k} w_{i} \quad \text{w.p.} \quad 1 - \Lambda_{i,km} \left( S_{i,k}^{I} \right) \\
p_{\omega t,k} &\sim G_{i,k}^{I} (\cdot) \quad \text{w.p.} \quad \Lambda_{i,km} \left( S_{i,k}^{I} \right)
\end{align*}
\]

hold in the steady state, where \( G_{i,k}^{I} (\cdot) \) is the probability distribution of unit cost of input goods in location \( i \) in sector \( k \). From the argument in Section 4.2, \( G_{i,k}^{I} (\cdot) \) is approximated by the inverse of the Pareto distribution with upper bound \( \tau_{i,k}^{I} \), where \( \tau_{i,k}^{I} \) is the entry cutoff of input goods sellers in sector \( k \) in location \( i \). The measure of firms whose unit cost is below \( c \)
is derived as

\[
H_{i,m}(c) = \int_{p_1,\ldots,p_N} B_{i,m} \left( \frac{1}{\rho A_i} \frac{\gamma_{L,m}^{\gamma_{km} \rho}}{\Pi_{k \in K} \rho_k^{\gamma_{km}}} \right)^{-\theta} \left[ \frac{1}{\rho A_i} \frac{\gamma_{L,m}^{\gamma_{km} \rho}}{\Pi_{k \in K} \rho_k^{\gamma_{km}}} \right] \leq \varphi \right] \prod_{k \in K} dG_{i,k}(p_k) \\
= \int_{p_1,\ldots,p_N} B_{i,m} \left( \frac{1}{\rho A_i} \frac{\gamma_{L,m}^{\gamma_{km} \rho}}{\Pi_{k \in K} \rho_k^{\gamma_{km}}} \right)^{-\theta} (1 - o(\varphi)) \prod_{k \in K} dG_{i,k}(p_k) \\
= (1 - o(\varphi)) \varphi^\theta B_{i,m} \frac{1}{\rho A_i} \frac{\gamma_{L,m}^{\gamma_{km} \rho}}{\Pi_{k \in K} \rho_k^{\gamma_{km}}} c^\theta \times \prod_{k \in K} \left\{ (1 - \Lambda_{i,k,m} (S_{i,k}^I)) + \Lambda_{i,k,m} (S_{i,k}^I) \int_{\rho_k} \left( \frac{\rho_k}{\rho_i \chi_i} \right)^{-\theta_{i,k,m}} \right] dG_{i,k}(p_k) \\
= (1 - o(\varphi)) \varphi^\theta B_{i,m} \frac{1}{\rho A_i} \frac{\gamma_{L,m}^{\gamma_{km} \rho}}{\Pi_{k \in K} \rho_k^{\gamma_{km}}} c^\theta \times \prod_{k \in K} \left\{ (1 - \Lambda_{i,k,m} (S_{i,k}^I)) + \Lambda_{i,k,m} (S_{i,k}^I) \frac{1}{1 - \gamma_{k,m}} \left( \frac{\tau_{i,k}}{\rho_i \chi_i} \right)^{-\theta_{i,k,m}} \right] \\
\equiv (1 - o(\varphi)) \varphi^\theta B_{i,m} \frac{1}{\rho A_i} \frac{\gamma_{L,m}^{\gamma_{km} \rho}}{\Pi_{k \in K} \rho_k^{\gamma_{km}}} \Omega_{i,k,m} (S_{i,k}^I, \tau_{i,k}^I) c^\theta.
\]

\[\Omega_{i,k,m} (S_{i,k}^I, \tau_{i,k}^I)\] represents the cost advantage from supplier matching, i.e.,

\[
\Omega_{i,k,m} (S_{i,k}^I, \tau_{i,k}^I) = 1 - \Lambda_{i,k,m} (S_{i,k}^I) + \Lambda_{i,k,m} (S_{i,k}^I) \nu_{i,k,m} (\tau_{i,k}^I), \tag{19}
\]

where \(\Lambda_{i,k,m} (S_{i,k}^I) \equiv \frac{M_{i,k,m} (S_{i,k}^I)}{M_{i,k,m} (S_{i,k}^I) + p_{i,k,m}}\) is the steady state probability of matching with a supplier, and \(\nu_{i,k,m} (\tau_{i,k}^I) \equiv \frac{1}{1 - \gamma_{k,m}} (\tau_{i,k}^I / \rho_i \chi_i)^{-\gamma_{i,m}}\) governs the average cost advantage upon being matched with a supplier.

### B.1.2 Zero Profit Condition of Final Goods Sellers

As in a standard Melitz model, zero profit conditions of marginal final goods sellers determine the entry cutoff unit cost of final goods sellers as well as the number of final goods sellers. Note that the aggregate final goods sales in sector \(k\) in location \(j\) is \(\alpha_k w_j L_j\), and \(\frac{1}{\sigma - 1}\) fraction of this goes to final goods seller’s profit. Furthermore, the measure of input goods suppliers whose unit cost is below \(c\) is approximated by \(\sum_{i' \in N} B_{i',m} \frac{\gamma_{F,i',m}^{\gamma_{km} \rho}}{\Pi_{k \in K} \rho_k^{\gamma_{km}}} \left( \tau_{i',j,m}^I \right)^{\varphi \theta} \prod_{k \in K} \Omega_{i,k,m} (S_{i,k}^I, \tau_{i,k}^I) \equiv

\]
From these relationships, I solve for the cut-off value of unit cost $c_{j,k}^F$:

\[
\frac{1}{\sigma - 1} \alpha_k w_j L_j = \int_0^{\tau_{j,k}^F} \left\{ \Pi_{j,km}^F(c) + f_{j,k}^F w_j \right\} d\Gamma_{j,k}^F c^\theta
\]

\[
= \int_0^{\tau_{j,k}^F} \left\{ \Pi_{j,km}^F(\tau_{j,k}^F) + f_{j,k}^F w_j \right\} \left( \frac{\tau_{j,k}^F}{c} \right)^{-(1-\sigma)} \Gamma_{j,k}^F \theta c^{\theta-1} dc
\]

\[
= \left\{ \Pi_{j,km}^F(\tau_{j,k}^F) + f_{j,k}^F w_j \right\} \frac{\theta}{\theta + 1 - \Gamma_{j,k}^F \left( \tau_{j,k}^F \right)^\theta}
\]

where the last equation comes from the fact that $\Pi_{j,km}^F(\tau_{j,k}^F) = 0$. This yields the final goods seller entry cutoff as

\[
\tau_{j,k}^F = \left( \frac{\theta + 1 - \alpha_k L_j}{\theta(1-\sigma) f_{j,k}^F \Gamma_{j,k}^F} \right)^{1/\theta}
\]

and the measure of final goods sellers, $S_{j,k}^F \equiv \Gamma_{j,k}^F \left( \tau_{j,k}^F \right)^\theta$, is

\[
S_{j,k}^F = \frac{\theta + 1 - \alpha_k L_j}{\theta(1-\sigma) f_{j,k}^F}
\]

### B.1.3 Zero Profit Condition of Input Goods Sellers

Zero-profit conditions of input goods sellers yield the number of input sellers $S_{j,k}^I$ and input seller entry cut-off of unit cost $\tau_{j,k}^I$. To see this, first observe that aggregate profit from input sales in location $j$ by sector $k$ to sector $m$ is $\psi\gamma_{k,m} \Psi_{j,km}^F(X_{j,m}^I + X_{j,m}^F)$, where $\Psi_{j,km}$ is the share of final goods sales in location $j$ and sector $m$ sold by firms with a supplier in input sector $k$, out of total sales by firms in location $j$ and sector $m$ (introduced in Equation (10)). I first derive $\Psi_{j,km}$ as a function of $\nu_{i,km} \left( \tau_{i,k}^F \right)$ and $\Lambda_{i,km} \left( S_{i,k}^I \right)$. To see this, note that conditional on being matched, the probability of making a sale is $\nu_{i,km} \left( \tau_{i,k}^F \right)$ times the unmatched one. This yields

\[
\Psi_{j,km} \left( S_{j,k}^I, \tau_{j,k}^I \right) = \frac{\Lambda_{j,km} \left( S_{j,k}^I \right) \nu_{j,km} \left( \tau_{j,k}^I \right)}{1 - \Lambda_{j,km} \left( S_{j,k}^I \right) + \Lambda_{j,km} \left( S_{j,k}^I \right) \nu_{j,km} \left( \tau_{j,k}^I \right)}
\]

(20)

Now, the ratio of the profit from input goods of two firms with unit cost $c$ and $c'$ is written as $\frac{\Pi_{j,km}^F(c')}{\Pi_{j,km}^F(c)} = \left( \frac{c'}{c} \right)^{\gamma_{k,m} \theta}$, by noting that the unit cost differences translates to downstream final goods sales by the factor of $\gamma_{k,m}$. Furthermore, the measure of input goods suppliers whose unit cost is below $c$ is approximated by $\sum_{i' \in N} B_{i',m} P_{i',m}^\theta w_{i'} \left( \tau_{i,j,m}^I \right)^\theta c^\theta \Pi_{k \in \Omega_{i,km}} \left( S_{i,k}^I, \tau_{i,k}^I \right) \equiv \text{...} \tag{20}$
\[ \frac{\psi - 1}{\psi} \gamma_{k,m} \Psi_{j,km} \left( X_{j,m}^F + X_{j,m}^I \right) = \int_0^{\tau_{j,k}^I} \Pi_{j,km}(c) d\Gamma_{j,k}^I c^\theta \]

\[ = \int_0^{\tau_{j,k}^I} \Pi_{j,km}(\tau_{j,k}^I) \left( \frac{\tau_{j,k}^I}{c} \right) \gamma_{k,m} c^\theta d\Gamma_{j,k}^I c^\theta - 1 dc \]

\[ = \Pi_{j,km}(\tau_{j,k}^I) \frac{1}{1 - \gamma_{k,m}} \Gamma_{j,k}^I \theta c^\theta \]

\[ = \Pi_{j,km}(\tau_{j,k}^I) \frac{1}{1 - \gamma_{k,m}} S_{j,k}^I. \]

By noting that the zero profit condition is derived as \( f_j^s w_j = \sum_{m \in K} \Pi_{j,km}(\tau_{j,k}^I), \) I have

\[ S_{j,k}^I = \frac{1}{w_j f_j^I} \frac{\psi - 1}{\psi} \sum_{m \in K} \left( (1 - \gamma_{k,m}) \gamma_{k,m} \Psi_{j,km} \left( X_{j,m}^F + X_{j,m}^I \right) \right). \]

The entry cut-off of unit cost is derived as

\[ \tau_{j,k}^I = \left( \frac{S_{j,k}^I}{\sum_{i' \in N} B_{i',k} \left( \tau_{i,j,m}^I \right)^{\theta} \varphi_{i',m} w_{i'}^\theta \prod_{i \in K} \Omega_{i,km} \left( S_{i',j}^I, \tau_{i',j}^I \right)} \right)^{1/\theta}. \]

**B.1.4 Free Entry Conditions**

Total input goods profit of firms in location \( j, \) net of the fixed marketing cost, is written as

\[ \sum_{m \in K} \int_0^{\tau_{j,k}^I} \left( \Pi_{j,km}(c) - w_j f_j^I \right) d\Gamma_{j,k}^I c^\theta \]

\[ = \sum_{m \in K} \frac{\psi - 1}{\psi} \gamma_{k,m} \Psi_{j,km} \left( X_{j,m}^F + X_{j,m}^I \right) - \left( (\gamma_{k,m} + 1) \frac{\psi - 1}{\psi} \gamma_{k,m} \Psi_{j,km} \left( X_{j,m}^F + X_{j,m}^I \right) \right) \]

\[ = \sum_{m \in K} \frac{\psi - 1}{\psi} \gamma_{k,m} \Psi_{j,km} \left( X_{j,m}^F + X_{j,m}^I \right) \]

Total final goods profit of firms in location \( j, \) net of fixed marketing cost, is similarly derived as

\[ \int_0^{\tau_{j,k}^F} \left( \Pi_{j,km}(c) - w_j f_j^F \right) d\Gamma_{j,k}^F c^\theta = \frac{\sigma - 1}{\sigma \theta} \alpha_k w_j L_j. \]

Hence, the free entry condition to enter in location \( i \) is written as

\[ B_{i,k} = \frac{1}{C_{i,k} w_i} \sum_{j \in N} \left( \pi_{ij,k}^F \frac{\sigma - 1}{\sigma \theta} \alpha_k w_j L_j + \frac{\psi - 1}{\psi} \sum_{m \in K} \gamma_{k,m} \Psi_{j,km} \left( X_{j,m}^F + X_{j,m}^I \right) \right), \]
where $\xi_k$ is the bankruptcy rate, and $C_{i,k}w_i$ is the fixed entry cost.

**B.2 Proof of Proposition 3**

From the definition of CES consumer utility and pareto distribution of unit cost of final goods sellers, the consumer price index in location $i$ is written as

$$P_i \propto \prod_{m \in K} \left( S_{i,m}^{F} \right)^{\alpha_m} \left( \frac{S_{i,m}^{F}}{\sum_{m} B_{i,m}^{\theta} \omega_{i,m} w_i^{-\theta \gamma L_i,m} \left( \tau_{i,m}^{F} \right)^{\theta}} \right)^{\frac{\alpha_m}{\theta}}$$

$$= \prod_{m \in K} \left( S_{i,m}^{F} \right)^{\alpha_m \frac{(1-\sigma + \theta)}{(\theta)(1-\sigma)}} \left( \frac{\hat{\pi}_{i,m}^{F}}{\hat{B}_{i,m}^{\omega_{i,m}^{-\theta \gamma L_i,m} \left( \hat{\tau}_{i,m}^{F} \right)^{\theta}}} \right)^{\frac{\alpha_m}{\theta}}$$

Note that under the Pareto distribution $S_{i,k}^{F}$ only depends on $L_i$, hence the values are unchanged under the counterfactual simulations. Therefore, the change of real wages is written as

$$\left( \frac{\hat{w}_i}{\hat{P}_i} \right) = \prod_{m \in K} \left( \frac{\hat{\pi}_{i,m}^{F} \left( \hat{B}_{i,m}^{\omega_{i,m}^{-\theta \gamma L_i,m} \left( \hat{\tau}_{i,m}^{F} \right)^{\theta}} \right)}{\Pi_{k \in K} \hat{\Omega}_{i,km}} \right)^{\frac{\alpha_m}{\theta}}$$

**C Model Extension to Incorporating Labor Mobility**

To incorporate labor mobility, I make a following additional assumption. I assume that workers also consume housing goods in addition to final goods, with Cobb-Douglas utility with share $\beta$. In addition, each worker has heterogeneous preferences for locations, $\epsilon = \{\epsilon_1, \ldots, \epsilon_N\}$. Together, the utility of a worker that draws preference shock $\epsilon$ is written as

$$U_i(\epsilon) = A_i \frac{w_i}{P_i^{1-\beta} R_i^{\beta}} \epsilon_i,$$

where $A_i$ is the exogenous amenity level of the locations and $R_i$ is the rent in location $i$. I assume that housing supply in each location is fixed to 1 at each location. From the land market clearing condition, the rent is determined as $R_i = w_i L_i$ hence the utility function $U_i(\epsilon) = A_i \left( \frac{w_i}{P_i} \right)^{1-\beta} (L_i)^{-\beta} \epsilon_i$. Assuming that $\epsilon_i$ is drawn from Fréchet distribution with scale parameter $\nu$ independently for each worker and location, and normalizing the total population
$\bar{L} = \sum_i L_i = 1$, I have free labor mobility conditions:

$$L_i = \frac{A_i^\nu \left(\frac{w_i}{P_i}\right)^{(1-\beta)\nu}(L_i)^{-\beta\nu}}{\sum_{i'} A_{i'}^\nu \left(\frac{w_{i'}}{P_{i'}}\right)^{(1-\beta)\nu}(L_i)^{-\beta\nu}}. \quad (21)$$

The equilibrium with free labor mobility is simply characterized by just adding free labor mobility conditions in the characterization of Proposition 1 and including $L_i$ as an additional endogenous variables.

**Proposition 4.** Under sufficiently small $\varphi$, the steady-state equilibrium with free labor mobility is characterized by $\{S_i^{l', k}, B_{i,k}, w_i, X_{i,k}^F, \tau_{j,k}^F, \pi_{ij,k}^F, \pi_{i,j,k}^I, L_i\}$ that satisfy total expenditure conditions (9, 10), trade balancing condition (11), gravity equations for input goods (15) and final goods (14), zero profit conditions (16), entry cut-off of input goods sellers (17), free-entry conditions (18), and free labor mobility (21).
D Calibration of Expenditure Share of Input Goods

\( \pi^I_{ij,k} \)

By noting that \( \pi^I_{ij,k} \) also corresponds to the extensive margin of trade, empirical frequency of the probability of sourcing input goods from location \( i \) in sector \( k \), \( \pi^I_{ij,k} \), can be in principle directly obtained from the data set by counting the fraction of sector \( k \) suppliers producing in location \( j \) from location \( i \). However, this measure is noisy due to the sparseness of the data and involves many zeros. To deal with this, I first estimate the parametrized gravity equation (15) to obtain the smoothed predictor of \( \pi^I_{ij,k} \). To implement this, I estimate the following model

\[
\bar{\pi}^I_{ij,k} = \frac{\delta_{i,k} \tau^\theta_{ij,k}}{\sum_{i' \in N} \delta_{i',k} \tau^\theta_{ij,k}}
\]

where I parametrize \( \tau^\theta_{ij,k} = (D_{ij})^{\kappa_k} \) and \( D_{ij} \) is the straight line distance between \( i \) and \( j \). \( \delta_{i,k} \) is the location fixed effects, defined as \( \delta_{i,k} \equiv B_{i,k} w_i^{-\theta} \). The model is estimated by a Poisson regression as suggested by Santos Silva and Tenreyro (2006). I use the predictor of the model

\[
\bar{\pi}^I_{ij,k} = \frac{\hat{\delta}_{i,k} \hat{\tau}^\theta_{ij,k}}{\sum_{i' \in N} \hat{\delta}_{i',k} \hat{\tau}^\theta_{ij,k}}
\]

for the baseline \( \pi^I_{ij,k} \).