Abstract

An organ transplant can improve a patient’s life while substantially reducing healthcare expenditures. Like many other scarce public resources, organs from deceased donors are rationed to patients on a waitlist via a sequential offer mechanism. The theoretical trade-offs in designing these rationing systems depend on agent preferences. This paper establishes an empirical framework for analyzing waitlist systems and applies it to study the allocation of deceased donor kidneys. We model the decision to accept an organ or wait for a more preferable one as an optimal stopping problem, and develop methods for analyzing counterfactual mechanisms. Our estimates show that while some types of kidneys are desirable for all patients, there is substantial match-specific heterogeneity in values. We then evaluate alternative mechanisms by comparing their effect on patient welfare to an equivalent change in donor supply. Past reforms to the kidney waitlist primarily resulted in redistribution, with similar welfare and organ discard rates as the benchmark first come first served mechanism. These mechanisms and other commonly studied theoretical benchmarks remain far from optimal: we design a mechanism that increases patient welfare by the equivalent to an 18.2 percent increase in donor supply.
1 Introduction

As of January 7, 2019, there were 94,971 patients on the kidney waiting list in the United States, while only 13,483 deceased donor transplants were performed in 2018.\(^1\) Each transplant improves the expected quality and length of a patient’s life while saving hundreds of thousands of dollars in expected dialysis costs (Wolfe et al., 1999; Irwin et al., 2012; Held et al., 2016). Yet, approximately 20 percent of medically suitable organs extracted for transplantation are discarded. Ethical considerations and legal restrictions prevent the use of traditional price-based market-clearing mechanisms to allocate deceased-donor kidneys.\(^2\) It is essential to design mechanisms that efficiently allocate organs, minimize waste, and achieve equitable outcomes while respecting this constraint. Currently, kidneys are allocated through a centralized waitlist. Similar considerations explain the use of waitlist systems to allocate other deceased donor organs, public housing, long-term care, child-care, and child-adoption.

Deceased donor kidneys are allocated without using money because of ethical considerations and legal restrictions make traditional price-based market-clearing mechanisms infeasible. Instead, available kidneys are allocated through a centralized waitlist. Given these constraints, it is essential to find mechanisms that efficiently allocate organs, minimize waste, and achieve equitable outcomes.\(^3\) Similar considerations motivate the use of waitlist systems to allocate other deceased donor organs, public housing, long-term care, child-care, and child-adoption.

Previous research and policy guidance on waitlist design either assumes restrictive forms of preferences or ignores the dynamic incentive for refusing an offer in consideration of future potential offers. Theoretical approaches to designing dynamic assignment mechanisms have found that even qualitative trade-offs are sensitive to whether objects are vertically or horizontally differentiated.\(^4\) Absent clear recommendations from theory, many organ allocation agencies use simulations to predict the effects of alternative allocation rules. The simulations, including those used to design organ allocation rules,\(^5\) do not allow decisions to respond to changes in the system.

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\(^1\)Source: https://optn.transplant.hrsa.gov/data/view-data-reports/national-data/

\(^2\)The National Organ Transplantation Act (NOTA) makes it illegal to obtain human organs for transplantation by compensating donors.

\(^3\)These goals are articulated by the Organ Procurement and Transplantation Network (OPTN), a contractor for the Health Resources & Services Administration (HRSA), in their policy document titled “Concepts for Kidney Allocation” (OPTN, 2011). A committee that was charged with reforming the allocation system adopted a new mechanism in 2014. We discuss these reforms in greater detail below.

\(^4\)Agarwal et al. (2018) compare the results in Su and Zenios (2004), Leshno (2017), Arnosti and Shi (2017), and Bloch and Cantala (2017) and show by example that optimal design depends on the nature of preferences.

\(^5\)For example, Kidney Pancreas Simulated Acceptance Module (KPSAM) used by the kidney allocation committee to evaluate various proposed mechanisms prior to the reforms enacted in 2014 assumes that acceptance decisions on the kidney waitlist do not depend on the mechanism used, thereby ignoring differences in dynamic incentives generated by various mechanisms. Similar methods are used by the organ allocation agencies in the U.K., Scandinavia and France.
This paper develops an empirical framework for analyzing waitlist mechanisms that sequentially assign objects to forward-looking agents, and applies these methods to study the deceased donor kidney allocation system in the U.S. We make several methodological and empirical contributions. First, we develop a method for estimating agent preferences using typical administrative data, and apply it to kidney waitlist data from New York to estimate payoffs from various types of transplants. This step is based on an optimal stopping problem faced by a patient when she is offered a kidney. Second, we define a notion of steady state equilibrium that is amenable to computation and counterfactual analysis of a broad class of mechanisms. Finally, we use these techniques to compare alternative mechanisms on key outcome measures such as efficiency, equity, and organ waste.

Our empirical application uses rich administrative data on the deceased donor kidney allocation system in the New York City area (henceforth, NYRT) between 2010 and 2013. The allocation mechanism used to match deceased donor kidneys with patients relies on a coarse point system and the patient’s waiting time. As soon as an organ becomes available, it is offered to patients on the waitlist in decreasing order of these priority points. The decision of whether or not to accept an offer remains with the patient and the transplant surgeon. Patients that refuse a transplant can remain on the waitlist and may choose to accept the next organ. Even though the timing and quality of future offers are uncertain, it can be optimal to turn down an offer to wait for a more suitable one. There is no penalty in priority points for refusing an offer.

The data suggest that although patients on the NYRT waitlist face extreme scarcity, they strategically wait for offers of desirable organs. While 1,400 patients join the waitlist each year, fewer than 200 deceased donor kidneys are recovered in NYRT. These donors vary widely in quality; some are accepted immediately, while others are rejected by every patient and discarded. The chance of being offered desirable organs increases with waiting time. As a result, patients have an incentive to reject offers and wait for a better kidney. Indeed, Agarwal et al. (2018) document that acceptance rates are higher for patients who are less likely to receive offers in the future. Therefore, consistent with dynamic considerations, patients with a higher option value of waiting are more likely to refuse an offer for an organ.

Motivated by these descriptive facts, we model an agent’s decision to accept an offer as an optimal stopping problem. An agent accepts the current offer if the value from the object is higher than the expected value of continuing to wait. The distribution of potential future offers depends on the mechanism and the strategies of the other agents on the list. Our empirical strategy combines acceptance probabilities with detailed knowledge of the mechanism to recover the value of a transplant as a function of a rich set of patient and donor observ-

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6In what follows, we refer to the decision-maker as the patient because our data do not directly identify cases in which a surgeon makes a decision on behalf of her patient. This approach is reasonable if each surgeon acts in the best interest of each of her patients. Our welfare statements will be based on the revealed preferences of this decision-maker. The effects of various mechanisms on clinical outcomes is left for future research.
ables. Our technique adapts methods for inverting conditional choice probabilities (Hotz and Miller, 1993; Arcidiacono and Miller, 2011; Arcidiacono et al., 2016) to suit dynamic assignment mechanisms.

The estimated values from transplants show that while some donors are systematically more desirable than others, there is substantial match-specific heterogeneity in values. For instance, organs from younger donors are preferred by all patients, but younger patients place a higher value on organs from younger donors. This and other sources of match-specific heterogeneity, such as immunological similarity, create scope for the design of the allocation mechanism to improve match quality by incorporating detailed patient and donor characteristics into the priority system.

Motivated by these facts, we develop methods for predicting equilibrium assignments under alternative mechanisms. This task requires us to solve two technical issues. First, we need to formulate a tractable notion of equilibrium. Computing counterfactuals is challenging because it involves solving a dynamic game with many players. To make progress, we define a notion of a steady-state equilibrium in the spirit of previous approaches to simplifying this exercise (Hopenhayn, 1992; Krusell and Smith, 1998; Weintraub et al., 2008; Fershtman and Pakes, 2012) and develop an algorithm for computing such an equilibrium.

Second, we need to ensure that assignments under counterfactuals of interest are indeed identified. In dynamic models such as ours, counterfactual results may not be invariant to normalizing the payoff of a single action in all states because it restricts payoffs across states (see Aguirregabiria and Suzuki, 2014; Kalouptsidi et al., 2015). We formally show that normalizing the payoff of never receiving an assignment to zero is appropriate for the mechanism design counterfactuals we consider if the value of declining all offers remains fixed. We argue why this assumption is reasonable in our empirical context.

We use these techniques to compare equilibrium assignments under mechanisms used in practice, theoretical benchmarks, and welfare-maximizing mechanisms. The mechanisms used in practice that we consider are the kidney allocation systems used before and after a reform implemented in 2014. To summarize welfare effects of a change in the mechanism on a given patient, we compute the equivalent change in deceased donor supply (arrival rates) under the existing mechanism. We then aggregate these equivalent changes in donor supply across patients as a summary of the welfare effects. This aggregation implicitly makes an interpersonal comparison of utility across patients.

Previously used mechanisms and commonly studied theoretical benchmarks can yield either high average welfare or low discards, but not both. The reform in 2014 resulted primarily in redistribution from older patients to younger patients, with little improvement in the welfare of the average patient. In fact, both the pre- and post-2014 mechanisms yield welfare and organ discard rates within 2.5 percent of the benchmark first come first served mechanism (FCFS) studied in Bloch and Cantala (2017). FCFS induces agents to be selective because
the expected quality of offers weakly increases as time passes. Last come first served (LCFS), which is theoretically studied in Su and Zenios (2004), makes agents less selective because they should expect to receive lower quality of offers in the future if they decline an offer. This mechanism is able to dramatically reduce organ discard rates (25 percent), but at the cost of lowering welfare by 50 percent due to poor match quality.

We find significant scope for using our estimates to improve assignments beyond these solutions. A welfare-maximizing mechanism that tailors offer rates based on agent and object observables can achieve an increase in the average patient’s welfare equivalent to a 18.2 percent increase in donor supply. This mechanism also reduces organ discard rates by more than 7 percent, and as a result, equilibrium queue lengths and waiting times are much shorter than under the pre-2014 mechanism. A concern with the gains we identify is that some types of patients may be substantially worse off relative to pre-2014. Fortunately, we find that there exists a mechanism that increases welfare by an equivalent of 12.2 percent and reduces discards by 4.0 percent while ensuring that no patient type is substantially worse off.

Our solutions are able to reduce discards and increase patient values by simultaneously considering the estimated heterogeneity in patients' transplant values and dynamic incentives. The latter channel is omitted in the KPSAM acceptance module (see SRTR, 2015) used by the Scientific Registry of Transplant Recipients (SRTR) to simulate the effects of various allocation systems. A comparison that ignores dynamic incentives and holds choice probabilities fixed predicts much smaller differences in match quality or discard rates between these mechanisms, understating the effects of changing the mechanism. Moreover, a mechanism that naively prioritizes patients based on predicted transplant value only marginally improves on the pre-2014 mechanism, increasing welfare and reducing discard rates by 3.5 and 0.3 percent, respectively. Therefore, explicitly considering dynamic incentives and equilibrium responses is crucial for identifying these gains.

These results point to the significant scope of using our empirical framework for improving dynamic assignment mechanisms. Previously used approaches for reforming organ allocation systems are unable to identify trade-offs between various mechanisms and potential gains from redesign. Therefore, mechanism design recommendations need to account for equilibrium responses and be specific to the primitives of the market.

Related Literature

Zenios (2004) surveys previous research on organ allocation mechanisms. Unlike our model, the empirical research in this area assumes that acceptance decisions do not depend on the waitlist mechanism. This approach to re-design implicitly assumes that waitlist priorities cannot be used to shape acceptance decisions. Most of the research within economics is on living donor kidney exchange markets (see Roth et al., 2004, 2007, for example), which accounts a small minority of kidney transplants. ⁷

⁷In 2017, organs from deceased donors accounted for 14,038 out of 19,849 kidney transplants. Source:
The most closely related paper to ours is Zhang (2010), who documents that, conditional on a set of donor covariates included in the model, patients lower on the list are more likely to refuse an organ if patients that are higher have refused it. The paper argues that this pattern is most consistent with a model of observational learning. Our approach abstracts away from learning, but allows for donor characteristics observed by agents that are not included in the model to capture this correlation in acceptance behavior. We do this in accordance with our current institutional understanding and to focus on allocation issues and equilibrium responses when simulating changes to the offer system.\footnote{Zhang (2010) uses data from 2002. The information available to patients about donors was dramatically better and standardized during our sample period (2010-2013). This fact significantly reduces the scope for observational learning. Additionally, the tests for observational learning that we are aware of require controlling for all characteristics known to the agents. Absent full controls, both observational learning and unobserved heterogeneity in organ quality result in correlated choices across agents. We leave the development of such tests and methods for incorporating observational learning into our framework to future work.}

The methods in this paper contribute to the growing literature on empirical approaches for analyzing centralized assignment systems (see Agarwal and Somaini, 2019, for a survey). These previous approaches have primarily focused on static assignment mechanisms,\footnote{To our knowledge, the only exceptions are Waldinger (2017), which studies a portfolio choice problem in the context of public housing, and Reeling and Verdier (2018), which studies the decision to enter a repeated lottery in the context of bear hunting licenses. The optimal stopping rule we study differs from these models. Although we estimate our model in the context of organ allocation, a similar stopping rule may also describe behavior in other dynamic matching contexts (Liu et al., 2019).} where the theory is comparatively well-developed and empirical evidence suggests that the difference between different well-coordinated systems is small (Abdulkadiroglu et al., 2017). As we mentioned above, the theory suggests that the most desirable dynamic offer system depends on the nature of preferences, suggesting that estimating these primitives is essential.

Overview

Section 2 describes the institutions, the data, and presents descriptive evidence. Sections 3 and 4 model the optimal stopping problem faced by each agent and details our estimation methods. Section 5 describes our parameter estimates. Section 6 defines a steady-state equilibrium, summarizes results on existence, and presents our approach to welfare comparisons. Section 7 compares predicted outcomes under alternative mechanisms. Section 8 concludes.

## 2 Background, Data and Descriptive Evidence

This section starts with the basics of kidney transplantation before describing the allocation system. Next, we detail our data sources and the information available for the study. Finally, we present key descriptive facts to motivate our model and empirical exercises.

2.1 Basics of Kidney Transplantation

As of December 31, 2016, there were 726,331 cases of End-Stage Renal Disease (ESRD) in the United States (USRDS, 2018). Medicare provides near universal coverage to ESRD patients; it also covers patients under the age of 65. This program cost the federal government $35.4 billion in 2016, accounting for 7.2 percent of overall Medicare paid claims (USRDS, 2018) or approximately 1 percent of the federal budget. Transplantation is the best treatment for ESRD, improving health outcomes while saving an estimated $270,000 over the life of a transplanted patient (Wolfe et al., 1999; Irwin et al., 2012; Held et al., 2016; USRDS, 2018).

A kidney from a deceased donor is considered transplantable to a patient if the patient does not have a pre-existing immune response to proteins on the organ’s cells. A biologically incompatible patient’s immune system will recognize and attack the transplanted organ, resulting in graft failure. Following transplantation, medications allow transplant physicians to limit new immune responses to foreign protein types, but pre-existing immune responses lead to immediate loss of the transplanted organ if not avoided. The specific form of incompatibility is not important for the purposes of this study.

Because of biological compatibility, the transplantation possibilities available to patients differ based on their immune systems. Some patients have immune systems that react to a broader range of organs, even from donors with the same blood type. These patients have fewer transplantation options. A patient’s immune sensitization is commonly measured by Calculated Panel Reactive Antibodies (CPRA), which is the percentage of donors in a representative sample with whom they is tissue-type incompatible. This measure is calculated from blood tests used to predict pre-existing immune responses.

The benefits of a transplant can substantially differ, even conditional on biological compatibility. The circumstances of the donor’s death, kidney function, and the donor’s health prior to death are considered important determinants of organ quality. In some cases, the donor may have an infectious disease that the patient is at risk of getting if she proceeds with a transplant. Size and weight match, and similarity of tissue-protein between the patient and the donor, are also considered important. There are a number of other factors that influence the medical benefits from specific organs for specific patients. We refer the reader to Danovitch (2009) for further details about kidney biology.

The immune system tags foreign objects (antigens) with antigen-specific antibodies so that white blood cells (leukocytes) can defend against them. Each donor has blood-type antigens and specific types of human leukocyte antigen (HLA) proteins that are relevant for kidney transplantation. Some patients have pre-existing antibodies to a subset of these antigens. A transplant recipient’s immune system will immediately attack the donor’s kidney and reject the organ if the recipient has a pre-existing antibody to any one of the donor antigens, making the donor incompatible (Danovitch, 2009). Following transplantation of a compatible kidney, the immune system will attack any donor antigen not present in the patient if such an attack is not attenuated with immunosuppressive medications. Thus, transplant physicians measure pre-existing immune responses and avoid them, whereas they prevent future immune responses by treating a transplanted patient with immunosuppressive medications.

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2.2 The Allocation of Deceased Donor Kidneys

Assignment of a potential donor’s organs begins after brain death is declared or cardiac death is imminent, and necessary consent has been obtained. The Organ Procurement Organization (OPO) in the donor’s area obtains information about the donor from tests and the donor’s medical history. This information is entered into a system, called UNet, that is used to coordinate allocation. This system calculates the order in which patients will be offered the organs, transmits information about the donor to the transplant centers, and records accept/reject decisions. OPO staff usually contact the surgeons for several potential recipients simultaneously. This process can take place while the donor is on life-support and before the potential donor’s organs have been extracted in order to maintain organ viability. Once a kidney has been recovered from the donor, transplant surgeons or patients that indicated initial interest in receiving that kidney receive any new information discovered in the interim and must make final decisions without delay, usually within an hour. A final compatibility blood test is then conducted using samples from the donor and multiple patients that have accepted an organ. The donor’s kidneys are then allocated to the highest priority patients on the waitlist that were willing to accept the organs.

When offering organs, UNet first excludes patients who are not biologically compatible with the donor. This exclusion is based on a detailed patient immunological profile that is submitted when the patient is registered on the waitlist. We mimic this calculation in our analysis and only consider blood and tissue type compatible offers throughout the paper.

Next, UNet screens out patients that have listed pre-set exclusion criteria within the system. These criteria allow patients to automatically exclude kidneys that are transplantable but undesirable because of donor characteristics such as age, health conditions, and kidney function. UNet then orders the remaining set of patients first by priority type, and then within priority type by priority points.

The priority and points system is motivated by both equity and efficiency concerns. Unlike assignment systems for some other organs (for example, livers), the kidney assignment system does not use patient urgency to determine priority. The system used in NYRT prior to 2014 first offers kidneys to patients with a perfect tissue-type match, then to patients from the local OPO in which the organs were recovered, then regionally, and finally nationally. Within each priority group, the points system is based on tissue type similarity, whether or not the patient is pediatric, patient sensitization, and waiting time. The detailed priority system is described in policy section 8 in OPTN (2014).

New kidney allocation rules were implemented on December 4, 2014. This new system gives greater priority to the healthiest patients for the most desirable donors, increases priority for extremely hard to match patients, and reduces emphasis on wait time. Israni et al. (2014) discusses this system and the rationale for the changes. We refer the reader to OPTN (2017) for a detailed description of the priorities and points used.
2.3 Data Sources

This study uses data from the Organ Procurement and Transplantation Network (OPTN). The OPTN data system includes data on all donors, wait-listed candidates, and transplant recipients in the US, submitted by the members of the Organ Procurement and Transplantation Network (OPTN). The Health Resources and Services Administration (HRSA), U.S. Department of Health and Human Services provides oversight to the activities of the OPTN contractor. For tractability, we restrict attention to data on acceptance decisions of patients on the kidney waitlist in the New York Organ Donor Network (NYRT) between January 1st, 2010 and December 31st, 2013. NYRT is the largest donor service area (DSA), in terms of number of patients, that used the standard allocation rules in the United States prior to 2014.

The primary dataset on the waitlist, the Potential Transplant Recipient (PTR) dataset, contains the offers made and accept/reject decisions. This dataset is drawn from the records generated by UNet. In addition, we obtained detailed information on patient and donor characteristics from the Standard Transplantation Analysis and Research (STAR) dataset. This dataset is populated using information gathered in UNet and forms submitted by transplant centers after a transplant is performed.

2.4 Descriptive Analysis

We now describe our sample of patients and donors and document choice patterns. A striking feature of the waitlist is that even though there is extreme scarcity, some donors are rejected by a very large number of patients. Choices suggest that large differences in donor quality combined with substantial priority for waiting time incentivize patients to reject low-quality donors and wait for more attractive offers.

Patients and Donors

Table 1 describes our patient sample. A total of 9,623 patients were registered with NYRT and actively waiting at some point between 2010 and 2013. Panel A shows the state of the waitlist on January 1st of 2010 and 2013, and summarizes a subset of important patient characteristics. Our dataset includes rich information on patient health status, including indicators of patient health (e.g. body mass index, age, total serum albumin), and medical history (e.g. diabetes, years on dialysis). The average patient on the list has waited for a little

\footnote{We end our sample in 2013 to rule out anticipatory effects and to avoid modeling transition dynamics as agents anticipate the new system introduced in December 2014. Reports from the United Network for Organ Sharing (UNOS) that track transplantation rates after the adoption of the new system show the existence of short-term transition dynamics (termed “bolus-effects” in these reports) immediately following the reform (Wilk et al., 2017).}
Table 1: Patient Characteristics

<table>
<thead>
<tr>
<th>Patient Stocks, Arrivals, and Departures</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Panel A: Patient Stocks</strong></td>
</tr>
<tr>
<td><strong>January 1, 2010</strong></td>
</tr>
<tr>
<td>Mean</td>
</tr>
<tr>
<td>Number of Patients</td>
</tr>
<tr>
<td>Years on List</td>
</tr>
<tr>
<td>Years on Dialysis</td>
</tr>
<tr>
<td>Prior Transplant</td>
</tr>
<tr>
<td>Current Age</td>
</tr>
<tr>
<td>Calculated Panel Reactive Antibodies (CPRA)</td>
</tr>
<tr>
<td>Body Mass Index (BMI) at Arrival</td>
</tr>
<tr>
<td>Total Serum Albumin</td>
</tr>
<tr>
<td>Diabetic Patient</td>
</tr>
<tr>
<td>On Dialysis at Arrival</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Panel B: Patient Arrivals and Departures</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td># Patients Arriving</td>
</tr>
<tr>
<td># Patients Departing</td>
</tr>
<tr>
<td># Patients Received Deceased Donor Transplant</td>
</tr>
<tr>
<td># Patients Received Live Donor Transplant</td>
</tr>
<tr>
<td># Patients Died or Too Sick to Transplant</td>
</tr>
<tr>
<td># Patients Departed for Other Reason</td>
</tr>
</tbody>
</table>

Notes: 9,623 patients were active on the NYRT waiting list at some time between January 1st, 2010 and December 31st, 2013. Panel A contains statistics for patients registered in NYRT on January 1st of each calendar year. Panel B contains statistics for patients who joined the NYRT waiting list (arrivals) and who were removed from the waiting list (departures) during each calendar year. Panel C classifies departures by reason. “Departed for Other Reason” includes transfers to non-NYRT transplant centers and miscellaneous departure reasons. Patients who received transplants at a non-NYRT center are included in the Received Deceased Donor Transplant and Received Live Donor Transplant categories.

Over two years, with the average waiting time increasing over time. The average CPRA is about 12 percent, which indicates that there is more than a one-in-ten chance that a patient is tissue-type incompatible with a randomly chosen donor. The allocation mechanism awards priority and points to high CPRA patients because of equity concerns.

Patients in NYRT face extreme scarcity, which results in long waiting times and many patients dying while waiting. The number of new patients joining the list vastly exceeds the number of transplants (Table 1, Panel B). A major contributing factor is that less than 200 donors are recovered from the NYRT area each year (Table 2, Panel A), which is only one-seventh of the number of patients joining the waitlist in NYRT. This scarcity results in the average transplanted patient waiting for over three years before receiving a deceased donor. Patients that do not receive a transplant most commonly leave the list because they either die or become “too sick to transplant” (Table 1, Panel B). A smaller number are fortunate enough to receive a living donor transplant, which is more likely for younger patients and often
occurs within the first year on the waitlist. Finally, some patients leave for other or unknown reasons including a move outside the NYRT area.

Table 2: Donor Characteristics

<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Any Kidney(s) Discarded</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S.D.</td>
</tr>
<tr>
<td><strong>Panel A: Donors Recovered in NYRT, By Number of Organs Allocated</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of Donors Per Year</td>
<td>183.5</td>
<td>44.75</td>
</tr>
<tr>
<td>Median Number of Offers per Donor</td>
<td>26.0</td>
<td>479.0</td>
</tr>
<tr>
<td>Number of Offers per Donor</td>
<td>445.8</td>
<td>1442.6</td>
</tr>
<tr>
<td>Number of Kidneys Transplanted per Donor</td>
<td>1.52</td>
<td>0.80</td>
</tr>
<tr>
<td>Donor Age</td>
<td>43.8</td>
<td>18.0</td>
</tr>
<tr>
<td>Cause of Death -- Head Trauma</td>
<td>25.2%</td>
<td>43.4%</td>
</tr>
<tr>
<td>Cause of Death -- Stroke</td>
<td>43.9%</td>
<td>49.7%</td>
</tr>
<tr>
<td>Diabetic Donor</td>
<td>14.4%</td>
<td>35.2%</td>
</tr>
<tr>
<td>Hypertensive Donor</td>
<td>38.6%</td>
<td>48.7%</td>
</tr>
<tr>
<td>Expanded Criteria Donor (ECD)</td>
<td>31.1%</td>
<td>46.3%</td>
</tr>
<tr>
<td>Non-Heart Beating Donor (DCD)</td>
<td>9.7%</td>
<td>29.6%</td>
</tr>
<tr>
<td>Donor Creatinine</td>
<td>1.3</td>
<td>1.5</td>
</tr>
</tbody>
</table>

**Panel B: All Donors, By Number of Organs Allocated**
<table>
<thead>
<tr>
<th></th>
<th>All</th>
<th>Any Kidney(s) Discarded</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Mean</td>
<td>S.D.</td>
</tr>
<tr>
<td>Number of Donors Per Year</td>
<td>1410.5</td>
<td>896</td>
</tr>
<tr>
<td>Median Number of Offers per Donor</td>
<td>726.0</td>
<td>1183.5</td>
</tr>
<tr>
<td>Number of Offers per Donor</td>
<td>1620.7</td>
<td>2564.3</td>
</tr>
<tr>
<td>Number of Kidneys Transplanted per Donor</td>
<td>0.72</td>
<td>0.89</td>
</tr>
</tbody>
</table>

Notes: Panel A consists of all deceased kidney donors (734) recovered in NYRT and offered to NYRT patients between January 1st, 2010 and December 31st, 2013. Panel B includes all donors (5,642) offered to NYRT patients during the same period, including donors recovered outside NYRT. Offers exclude cases in which the donor did not meet the patient’s pre-determined criteria for acceptable donors, or in which the patient was bypassed by the waitlist system due to operational considerations that did not involve an active choice by the patient or her surgeon.

Despite this scarcity, undesirable organs have to be offered to many patients in an attempt to allocate them. Table 2 shows that across donors, the mean number of biologically compatible offers that met the pre-set screening criteria is over 400, but the median is much lower, at 26. This skewed distribution arises because undesirable kidneys are rejected by many patients, while desirable kidneys are accepted after only a few offers. Indeed, over 20% of donors have at least one of their viable kidneys discarded.\(^{12}\) Organs from these donors were refused by an average of almost 1,500 patients.

Our observable donor covariates, which should predict organ quality, are correlated with number of offers and discards in the expected ways. Panel A of Table 2 summarizes these

\(^{12}\)The number of transplanted kidneys amongst donors with no discards is less than two because some donors have only one viable kidney for donation.
Table 3: Rates of Receiving and Accepting Offers

<table>
<thead>
<tr>
<th></th>
<th>Number of Patients</th>
<th>Offer &amp; Acceptance Rates</th>
<th></th>
<th></th>
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</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>All Donors</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Annual Rate</td>
<td>% Accepted</td>
<td>Annual Rate</td>
<td>% Accepted</td>
<td>Annual Rate</td>
</tr>
<tr>
<td>All</td>
<td>9623</td>
<td>213.2</td>
<td>0.14%</td>
<td>37.0</td>
<td>0.74%</td>
<td>0.095</td>
</tr>
<tr>
<td>On Dialysis at Regist</td>
<td>6513</td>
<td>211.5</td>
<td>0.15%</td>
<td>38.6</td>
<td>0.77%</td>
<td>0.090</td>
</tr>
<tr>
<td>Not on Dialysis at Re</td>
<td>3110</td>
<td>216.6</td>
<td>0.12%</td>
<td>33.6</td>
<td>0.66%</td>
<td>0.103</td>
</tr>
<tr>
<td>Age 0-49 at Registration</td>
<td>3921</td>
<td>211.8</td>
<td>0.16%</td>
<td>37.0</td>
<td>0.81%</td>
<td>0.087</td>
</tr>
<tr>
<td>Age 50+ at Registration</td>
<td>5702</td>
<td>214.1</td>
<td>0.13%</td>
<td>36.1</td>
<td>0.68%</td>
<td>0.100</td>
</tr>
</tbody>
</table>

Panel B: Offers Within the First 10 Positions that Met Screening Criteria

<table>
<thead>
<tr>
<th></th>
<th>Number of Patients</th>
<th>Offer &amp; Acceptance Rates</th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>All</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Annual Rate</td>
<td>% Accepted</td>
<td>Annual Rate</td>
<td>% Accepted</td>
<td>Annual Rate</td>
</tr>
<tr>
<td>All</td>
<td>9623</td>
<td>0.8</td>
<td>24.53%</td>
<td>0.8</td>
<td>25.81%</td>
<td>0.021</td>
</tr>
<tr>
<td>On Dialysis at Regist</td>
<td>6513</td>
<td>0.8</td>
<td>24.70%</td>
<td>0.8</td>
<td>25.93%</td>
<td>0.017</td>
</tr>
<tr>
<td>Not on Dialysis at Re</td>
<td>3110</td>
<td>0.8</td>
<td>23.99%</td>
<td>0.8</td>
<td>25.40%</td>
<td>0.029</td>
</tr>
<tr>
<td>Age 0-49 at Registration</td>
<td>3921</td>
<td>1.3</td>
<td>25.37%</td>
<td>1.3</td>
<td>26.23%</td>
<td>0.020</td>
</tr>
<tr>
<td>Age 50+ at Registration</td>
<td>5702</td>
<td>0.5</td>
<td>23.34%</td>
<td>0.4</td>
<td>25.17%</td>
<td>0.022</td>
</tr>
</tbody>
</table>

Notes: There were 2,713,172 offers made to NYRT patients between January 1st, 2010 and December 31st, 2013. Panel B restricts to the first 10 NYRT patients in each donor’s offer sequence. An offer Met Screening Criteria if the offer satisfied patient’s pre-determined criteria for acceptable donors. “Annual Rate” columns report annual offer rates computed by patient and then averaged across patients.

characteristics by the allocation outcome for a kidney recovered in NYRT. Donors whose kidney(s) were discarded are older, less likely to die of head trauma, more likely to be diabetic or hypertensive, have a higher creatinine level (an indicator of lower kidney function), and more likely to have donated after cardiac death.

In addition to donors recovered within the local area, NYRT patients are also offered donors from other parts of the country. Panel B shows that a total of 1,410 donors were offered to patients registered with NYRT in the average year. Because most of these donors were recovered elsewhere in the country but offered to NYRT patients after a large number of refusals, these donors are likely to have undesirable organs with a very large number of offers and high discard rates.

**Waitlist, Offers and Acceptance Rates**

We now describe the offer and acceptance rates in the data. We will refer to the decision-maker as the patient for simplicity of exposition, assuming that any decisions made by surgeons are in the best interest of each individual patient.

A patient’s position on the list improves with waiting time, and the mean waiting time falls as we go down the list for a given donor. The average waiting time amongst patients in the top five positions on the list exceeds four years, while this average is approximately 3.5 years for patients in the one hundredth position. Nonetheless, the system is not well approximated
by a first come first served queue. We calculated the fraction of times that two patients who are offered the same donor are ordered identically on the list for the next donor they are both offered. This fraction is 81.5 percent. It would be 100 percent in a first come first served system. Therefore, points and priorities other than for waiting time determine the position on the list.

Patients receive many offers and the overall acceptance rate is extremely low because lower-quality kidneys are offered to a large number of patients. Table 3 describes these overall patterns in offer and acceptance rates. Panel A considers all feasible offers, including biologically compatible offers that did not meet the patient’s pre-set criteria. A typical patient receives over 200 offers per year, but only 0.14 percent of offers are accepted. When interpreting these low acceptance rates, it is important to remember that undesirable organs are offered to a very large number of patients.

Offers from desirable donors are relatively rare and much more likely to be accepted. Kidneys recovered in NYRT are accepted five times more often than those recovered outside NYRT, and 10.8 percent of offers of a perfect tissue-type match are accepted. Panels B further restricts to offers to patients in the first ten positions that met the patients’ screening criteria. These offers are likely to be much more attractive as the related organs have not been refused by many patients. The typical patient can expect to receive less than one such offer each year and is very likely to accept one. And, older patients are less likely to receive one of these desirable offers, in part because they are more likely to depart the waiting list prior to receiving a transplant. This fact puts them at a disadvantage in a mechanism that prioritizes waiting time.

Taken together, these statistics suggest that the supply of desirable donors in NYRT is scarce and it is necessary to wait for several years to gain priority to access these donors. This fact creates a strong incentive for patients to wait.

Evidence on Mismatch

Table 4 provides suggestive evidence of mismatch between donors and transplanted patients. Pediatric patients are very likely to be transplanted, either with a deceased donor kidney or through a living donor. The priority given to these patients is likely an important contributing factor. Adult patients are less fortunate, but interestingly, among adults there is no significant gradient in transplant probability with age. Panel A describes transplanted donors by age for those patients who receive a kidney through the deceased donor waitlist. Pediatric patients are much more likely to receive a transplant from a young donor as compared to older patients. Although there is some assortative matching by age, signs of age mismatch remain.

---

13Our dataset includes information on refusal reasons. Throughout our analysis, we exclude offers that did not result in assignment due to logistical reasons such as surgeon unavailability, OPO operational considerations, or special donor-specific considerations such as expedited organ placement or directed donation.
Table 4: Evidence on Mismatch

<table>
<thead>
<tr>
<th>Patient Age</th>
<th>On Dialysis at Registration</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>0-17</td>
</tr>
<tr>
<td>Patient Population</td>
<td>2.1%</td>
</tr>
<tr>
<td>Deceased Donor Transplants</td>
<td>4.6%</td>
</tr>
</tbody>
</table>

Panel A: Donor Age and Quality

<table>
<thead>
<tr>
<th>Donor Age</th>
<th>0-17</th>
<th>18-35</th>
<th>35-49</th>
<th>50-64</th>
<th>65+</th>
<th>6.8%</th>
<th>8.7%</th>
</tr>
</thead>
<tbody>
<tr>
<td>Donor Age 0-17</td>
<td>25.6%</td>
<td>14.7%</td>
<td>7.5%</td>
<td>4.5%</td>
<td>4.7%</td>
<td>6.8%</td>
<td>8.7%</td>
</tr>
<tr>
<td>Donor Age 18-35</td>
<td>72.2%</td>
<td>36.8%</td>
<td>26.3%</td>
<td>16.4%</td>
<td>12.5%</td>
<td>22.3%</td>
<td>24.6%</td>
</tr>
<tr>
<td>Donor Age 35-49</td>
<td>2.2%</td>
<td>30.5%</td>
<td>34.2%</td>
<td>28.7%</td>
<td>16.6%</td>
<td>27.6%</td>
<td>25.2%</td>
</tr>
<tr>
<td>Donor Age 50-64</td>
<td>0.0%</td>
<td>16.8%</td>
<td>29.9%</td>
<td>42.6%</td>
<td>53.4%</td>
<td>37.3%</td>
<td>34.6%</td>
</tr>
<tr>
<td>Donor Age &gt;= 65</td>
<td>0.0%</td>
<td>1.1%</td>
<td>2.1%</td>
<td>7.9%</td>
<td>12.8%</td>
<td>5.9%</td>
<td>6.9%</td>
</tr>
</tbody>
</table>

Many patients above the age of 65 continue to receive kidneys from young adults and middle-aged donors. Although not reported, these patterns are similar if we focus on a subset of donors with no clear medically undesirable characteristics such as diabetes, cardiac death, high creatinine levels or hepatitis C. Such mismatch motivated the 2014 reforms, which attempted to match healthier patients (typically young adults) to donors whose organs are predicted to last longest.

Another patient characteristic that will be important in the improvements we ultimately identify is whether the patient had already begun dialysis when they joined the waitlist. Some patients with end-stage renal disease who qualify for the waitlist still have marginal kidney function and can avoid dialysis for some time. These patients are relatively healthy compared to patients already on dialysis at registration. The last two columns of Table 4 show that outcomes differ substantially for patients on and off dialysis. While patients not yet on dialysis at registration represent 32 percent of our patient sample, they receive only 27 percent of deceased donor transplants.

Evidence on Response to Dynamic Incentives

A central assumption in our framework is that agents are forward-looking and respond to dynamic incentives. One implication of this assumption is that patients for whom the option value of waiting is lower should be less selective. Agarwal et al. (2018) present descriptive evidence consistent with dynamic incentives using data from all areas of the United States. They find that highly sensitized patients who are immunologically compatible with fewer donors – and who therefore receive fewer offers in the future – are more likely than less sensitized patients to accept an offer. We replicated their research strategy using data from...
patients registered in NYRT and found qualitatively similar patterns.\textsuperscript{14} We refer the reader to their paper for a more detailed discussion of the empirical strategy and potential confounds.

### 3 A Model of Decisions in a Waitlist

This section presents a model of agents’ decisions in a waitlist mechanism that will form the basis of our empirical strategy. It considers a class of sequential assignment mechanisms in which objects, indexed by $j$, are offered to agents, indexed by $i$, waiting on a list who may accept or refuse them.

Objects may be incompatible with some agents. Let $c_{ij} = 1$ if object $j$ is compatible with agent $i$, and 0 otherwise. Incompatibility can arise due to biological reasons in the organ allocation context but they may arise due to other restrictions (e.g. legal) in other contexts.

Throughout the paper, we denote the vector of observed characteristics of agent $i$ with $x_i$. Similarly, $z_j$ denotes observed characteristics of object $j$ that are included in the model; $\eta_j$ denotes a quality index constructed from characteristics that are observed to agents but not included in the model;\textsuperscript{15} and $t_i$ denotes the amount of time the agent has been waiting on the list. The model does not include unobserved agent heterogeneity. We discuss this restriction in section 4.

We begin by defining agent arrival and departure processes, a class of sequential assignment mechanisms, and primitives governing agents’ decisions while on the waitlist. We then provide assumptions on agents’ payoffs and beliefs which will lead to a tractable optimal stopping problem from the agent’s perspective. Though the model is motivated by the structure of our application, it may be useful in other settings in which items are offered sequentially to agents, including other organ allocation settings.

### 3.1 Mechanisms and Primitives

#### 3.1.1 Mechanisms

We consider sequential assignment mechanisms that use a priority score. The mechanism allocates each object as follows:

\textsuperscript{14}Results available in supplementary materials included in the associated replication archive.

\textsuperscript{15}Our dataset includes all the information made available to patients and surgeons at the time of assignment. This includes detailed medical history of the donor, results from dozens of medical tests, and results from physical examination of the organ. While these are recorded in the dataset, there are far too many for us to include in our specifications. Moreover, most of the results are reported as normal for the vast majority of donors. This feature makes it difficult to include these characteristics individually when estimating choice models. The term $\eta_j$ is intended as a quality aggregator of the characteristics that are omitted from the model.
• Step 1 (Ordering): The priority score \( s_{ijt} \equiv s(t; x_i, z_j) \) is calculated for all agents on the waitlist. Ties in the score, if any, are broken using a known tie-breaking rule. For example, ties could be broken either uniformly at random or by waiting time.

• Step 2 (Offers): Each agent may decide to accept or reject the object, with acceptance denoted by \( a_{ij} = 1 \). The mechanism may solicit decisions from multiple agents simultaneously, but may not skip any agents in the priority order. No offers are made to agents that are known to be incompatible with the object.

• Step 3 (Assignment): The objects are assigned to the agents with the highest \( q_j \) priorities for whom \( a_{ij} = 1 \), where \( q_j \) is the number of copies of the object available.

• Step 4 (Arrivals and Departures): An agent is removed from the waitlist once an object has been assigned to her. Other agents may join or leave the list.

Within the set of general offer-based waitlist systems, the primary restriction in our formulation is that an agent’s priority does not depend on either the other agents in the market or their past actions. Such mechanisms are a natural class to consider because they are simple and transparent to implement. Indeed, all deceased-donor organ allocation mechanisms as well as systems considered by the kidney allocation committee during their deliberations prior to the 2014 reform were offer mechanisms based on priority scores.\(^{16}\) In counterfactual analysis, we will compare assignments that result from various mechanisms that obey this structure to benchmark optimal assignments.

Typical administrative datasets from such assignment systems contain information on all characteristics used to determine the priority score. This allows a researcher to calculate the order in which any object would be offered. Our empirical exercises required us to develop computer code for this purpose, and we were able to verify the output of our code using administrative records of the offers that were made during our sample period.

One complication in our setting is that organs must be allocated within a certain time frame that depends on the condition of the organ and various logistical factors. Resource constraints at the Organ Procurement Organization (OPO) can limit the number of patients that can be contacted and offered the organ. While there may be gains in increasing the number of patients that are contacted, we do not have data that can directly speak to the relationship between OPO resources and the number of patients that can be contacted. We therefore treat the maximum number of offers that can be made for each object as exogenous, but assess robustness of our conclusions to removing this limit.

\(^{16}\)Based on an examination of committee reports and public comments downloaded from https://optn.transplant.hrsa.gov/members/committees/kidney-committee/.
### 3.1.2 Payoffs

There are three types of primitive payoffs in the model. The first is the (expected) net present value of agent $i$ being assigned a compatible object $j$ after waiting $t$ periods, denoted $\Gamma_{ij}(t)$. In our application, this term captures the value placed by patients on transplants from various organs. The second is the expected net-present value from from departure without an assignment, $D_i(t)$. In our application, departures occur due to living donor transplants, death, or transfers to other listing centers (see Table 1). We view $D_i(t)$ as incorporating any of those reasons. Finally, agents incur an expected flow payoff while waiting on the list, $d_i(t)$. In our application, $d_i(t)$ is best interpreted as the payoff from living without a functioning kidney, which includes dialysis for most patients.

Two economic implications of the payoffs in our model are worth noting. First, we abstract away from costs of considering an offer. These costs are likely small relative to the value of transplants. Second, we assume that agents only value their own outcomes and not those of others. It may be violated if surgeons value the outcomes of other patients, especially those that they might be treating. NYRT has a total of 10 transplant hospitals staffed with many more kidney transplant surgeons. This limits common agency problems that surgeons might face. We assume that surgeons act in the best interest of each of their patients.

Our empirical framework makes the following assumptions on these payoffs:

**Assumption 1.** (i) The (expected) net present value of an assignment is additively separable in a payoff shock $\varepsilon_{ijt}$:

$$
\Gamma_{ij}(t) \equiv \Gamma(t, x_i, z_j, \eta_j) + \varepsilon_{ijt}.
$$

(ii) The random variables $\varepsilon_{ijt}$ are independent and identically distributed (iid) with a known, non-atomic distribution with cumulative distribution function (cdf) $G$.

(iii) The expected flow payoffs from waiting $d_i(t)$ and the expected payoff from departing without an assignment $D_i(t)$ depend only on $(x_i, t)$.

Restrictions on $\varepsilon_{ijt}$ imposed in Assumptions 1(i) and 1(ii) are common in the dynamic discrete choice literature. They enable an estimation approach based on an inversion technique due to Hotz and Miller (1993). The comparison with other methods and specific functional form and distributional assumptions are discussed in Section 4 below.

The primary restriction in Assumption 1 is that it excludes persistent agent-level unobserved heterogeneity. Section 4 discusses challenges with extending our approach to this case. Instead, our specifications will include a rich set of patient and donor characteristics. This

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\footnote{We can represent the value from a departure as a weighted average over the value from the various events, i.e. $D_i(t) = \sum_k p_{ik}(t) D_{ik}(t)$ where $k$ denotes the type of departure (e.g. obtaining a live donor, death etc.) and $p_{ik}(t)$ is the probability of each type of departure conditional on a departure occurring. The formulation is agnostic about the sources of these payoffs. For example, the net present value of death can include any bequest motives.}
choice is in service of our goal of finding mechanisms that better target offers based on observable characteristics. To do this effectively, it is important to estimate the distribution of payoffs in the model as a function of a rich set of patient and donor characteristics.

### 3.1.3 Arrivals and Departures

Time is continuous. Objects and agents arrive at Poisson rates $\lambda$ and $\gamma$, respectively. The characteristics $x$ of each arriving agent are iid. Similarly, each object’s characteristics $(z, \eta)$ are drawn iid from the cdf $F$ upon arrival. We assume that each object must be assigned before the next object is offered. The Poisson arrival process and continuous time together imply that simultaneous arrivals are zero probability events. Agents may depart the list due to or prior to assignment.

We make the following assumption on the arrival and departure processes:

**Assumption 2.**

(i) Departures prior to assignment and arrivals are governed by independent and exogenous Poisson processes.

(ii) The departure rate of agent $i$ at time $t$ is given by $\delta_i(t) \equiv \delta(t; x_i)$. Further, each agent has a terminal date $T_i < \infty$ at which departure occurs with probability 1.

In our application, we assume that patients die on or before their 100-th birthday. $T_i$ therefore corresponds to the waiting time for a patient on the day they turn 100 years of age.\(^{18}\)

The primary economic restriction in our application is that departures prior to assignment and arrivals do not depend on the design of the kidney waitlist. Table 1 shows that the most common reason for departure without a deceased donor transplant is death or patients becoming “too sick to transplant.” It seems safe to assume that these events are not responsive to the design of the kidney waitlist. The second most common reason is receiving a living donor transplant. Departures due to this reason are exogeneous if the design of the kidney waitlist does not affect the probability of finding a compatible living donor, and if patients always prefer a living donor to staying on the deceased donor waitlist. These conditions are plausible in our setting because living donors are medically preferable to deceased donors because they produce better transplant outcomes in terms of patient and graft survival.\(^{19}\)

\(^{18}\)It is straightforward to extend the framework to allow for agents that could remain on the list forever, $T_i = \infty$. This generalization will primarily change computational techniques and require that the value function for each patient approaches a constant. We restrict our attention to the finite time-horizon case for simplicity of exposition.

\(^{19}\)Living donor superiority is partly driven by the higher medical quality of living donor kidneys, and also by the fact that living donation allows for a better planned transplant. For example, patients receiving a living donor transplant can proactively start immunotherapy. Hart et al. (2017) compare outcomes following living and deceased donation by patient age and primary diagnosis using the chances of graft failure 10 years after transplantation. This statistic for adults transplanted with a deceased donor kidney in 2005 is 52.8%, whereas it is only 37.3% for those that received a living donor.
Similarly, we assume that agent arrivals do not depend on the design of the waitlist. During our sample period, patients could register as soon as the patient starts dialysis or kidney function is sufficiently low (below a glomerular filtration rate of 20mL per minute). Therefore, it is in a patient’s interest to join the waitlist as soon as possible.

3.2 Individual Agent’s Problem

Agents on the waitlist who receive an offer for an object must decide whether to accept it or wait for a future offer. This results in an optimal stopping problem from the perspective of the agent (Pakes, 1986; Rust, 1987). We follow a common estimation strategy in dynamic games by considering an agent’s optimal decision rule taking the distribution of actions of other agents as observed in the data (Pakes et al., 2007; Bajari et al., 2007). Solving for counterfactuals will require a notion of equilibrium, which we discuss in Section 6. This section starts by describing a general formulation of this single-agent problem before making simplifying restrictions.

3.2.1 Beliefs

To make an informed decision about whether to accept an offer, an agent must form beliefs about the organs they may be able to obtain in the future if they declines the current offer. Let $s_j^*$ be the priority score of the pivotal agent that is offered object $j$. That is, an agent with priority at least $s_j^*$ would have been assigned object $j$ if they accepted it. This score is random as it depends on the decisions of all agents on the list when object $j$ is offered, their compatibility, and the number of offers that can be made for the object. Agent $i$ is assigned a compatible object $j$ if they accepts it and their score, $s_{ijt}$, exceeds $s_j^*$. Therefore, it is sufficient for an agent to form beliefs over the probability distribution of $s_j^*$ in order to decide which objects are likely obtainable in the future.

In the kidney allocation context, $s_j^*$ is the lowest priority score amongst the transplanted patients if none of the donor’s kidneys are discarded. If at least one kidney is discarded, then $s_j^*$ is the lowest priority score amongst the patients offered the kidney. This characterization follows because the kidney waitlist offers organs to patients in batches as long as the kidney remains viable for transplantation.

The beliefs about the distribution of the cutoffs priority $s_j^*$ depend on the quality of the organ and the information an agent may have about the competitive environment. In principle, the information set could include the history of offers previously received (and rejected) by the agent as well as identities of the other agents on the waitlist at a given point in time. However, there are several reasons, discussed below, why beliefs are unlikely to be sensitive to such information. We therefore assume that an agent’s beliefs does not depend on such fine information:
Assumption 3. Each agent $i$’s belief that the probability that an object with characteristics $(z_j, \eta_j)$ is compatible and will be available to her after a waiting time of $t$ is

$$\pi (t; z_j, \eta_j, x_i) = H (s_{ijt}; z_j, \eta_j) \times P (c_{ij} = 1 | z_j, x_i),$$

where $s_{ijt} = s (t; z_j, x_i)$ and $H (\cdot; z_j, \eta_j)$ is the cumulative distribution function of the cutoff $s^*_j$ given $(z_j, \eta_j)$.

This assumption embeds three key restrictions. First, it assumes that beliefs are not sensitive to short-term variation in the set of other agents currently on the waitlist. Privacy concerns limit surgeons’ ability to directly obtain such information about other patients. Therefore, the primary threat to this restriction is that some surgeons may be treating multiple patients on the kidney waitlist. This concern is limited by the fact that the NYRT area has many transplant hospitals and surgeons.

Second, it abstracts away from inference about the likelihood of receiving future offers based on past offers. This restriction is supported by both institutional features and empirical observations. The set and order of patients on the waitlist varies significantly across donors due to patient-donor specific compatibility and priority, limiting the ability to predict future offers based on recent experience. However, our data do indicate that recent offers are predictive of future cutoffs. Moreover, we fail to find evidence that a patient’s recent offer history predicts acceptance behavior. Therefore, patients do not seem to be inferring that they face relatively little competition if they receive unexpectedly good or frequent offers.

Third, it assumes that the probability that an organ is compatible depends only on observables and is independent of the cutoff. This restriction is appropriate in our context because surgeons list the blood- and tissue-types that are known to be incompatible with the patient.

Our assumption on beliefs is a reasonable approximation if assessments about which organs are likely to be available are based primarily on a surgeon’s extensive experience treating patients. The main advantage is that the assumption eases analysis relative to more general forms of beliefs because it dramatically reduces the dimension of the information set, and therefore the state space, in the dynamic problem.

\[\text{We cannot reject zero autocorrelation in priority-score cutoffs } s^*_j \text{ across organs ordered by the date on which they arrived, even within donor categories. One would expect non-zero serial correlation across these cutoffs if the offers a patient observed contained information about the likelihood of receiving future offers. Details available on request.}\]

\[\text{We investigated this hypothesis by testing whether, controlling for characteristics of the patient and the current offer, recent offers predict acceptance behavior. Our estimates are precise and suggest that the recent offer rates are not predictive of current acceptance behavior. This result is robust to various measures of recent offer rates and versions that focus on offer rates from desirable donors. Details available on request.}\]

\[\text{Formally, beliefs would be conditioned on a richer information set, denoted } \mathcal{F}_{i,t}, \text{ by replacing } H (s_{ijt}; z_j, \eta_j) \text{ with } H (s_{ijt}; \mathcal{F}_{i,t}, z_j, \eta_j) = P (s^*_j < s_{ijt} | \mathcal{F}_{i,t}, z_j, \eta_j). \text{ In our setting, agents only know the past offers that they have received and refused. Assumption 3 simplifies the problem because including the characteristics of organs offered in the past would lead to a very high-dimensional state space.}\]
The simplification of the state space in Assumption 3 is similar in spirit to equilibrium concepts developed for making the analysis of dynamic games tractable. These concepts either reduce the state space by analyzing a limit with many players (Hopenhayn, 1992; Krusell and Smith, 1998; Weintraub et al., 2008) or model beliefs as being based on past experience (Fershtman and Pakes, 2012). Despite this simplification, the state space continues to be quite rich because, in addition to aspects that influence payoffs, it contains all characteristics that influence priorities or determine whether or not any given object \( j \) is compatible.

### 3.2.2 Value functions

We assume that agents make optimal accept/reject decisions by comparing the net present value of an object to the value of waiting. Since time is continuous and the arrival of objects follows a Poisson process, simultaneous offers are zero probability events and agents consider offers as they arrive.\(^{23}\) Holding the strategies of other agents fixed, agent \( i \) decides to remain on the list instead of accepting object \( j \) if the payoff from an assignment \( \Gamma_{ij}(t) \) is less than the value of continuing to wait conditional on the agent’s type \( x_i \) and current waiting time \( t \), denoted \( V_i(t) \equiv V(t; x_i) \). The Hamilton-Jacobi-Bellman differential equation defining the value of waiting at time \( t \) is:

\[
(\rho + \delta_i(t)) V_i(t) = d_i(t) + \delta_i(t) D_i(t) + \lambda \int \pi_{ij}(t) \int \max \{0, \Gamma_{ij}(t) - V_i(t)\} \ dGdF + \dot{V}_i(t),
\]

where the inner integral computes the expectation over the idiosyncratic payoff shocks \( \varepsilon_{ijt} \) in equation (1), and, with a slight abuse of notation, \( \pi_{ij}(t) = \pi(t; x_i, z_j, \eta_j) \) defined in Assumption 3.

This expression can be derived by considering an agent’s value of waiting at time \( t \) for an infinitesimal duration \( \Delta t \). In the event that no object arrives during this period, the agent incurs flow payoffs from dialysis \( d_i(t) \Delta t \) and may depart exogenously with probability \( \delta_i(t) \Delta t \), incurring a payoff of \( D_i(t) \) if such a departure offers. An object arrives during this period with probability \( \lambda \Delta t \), and its characteristics are drawn from the CDF \( F \). The integral calculates the expected increment in the agent’s value function for each arrival. Specifically, the agent receives an offer for this object with probability \( \pi_{ij}(t) \) and accepts it if \( \Gamma_{ij}(t) > V_i(t) \), yielding an incremental value of \( \int \max \{0, \Gamma_{ij}(t) - V_i(t)\} \ dG \). In the limit as \( \Delta t \to 0 \), the probability that both departures and object arrivals occur within the interval

\(^{23}\)This formulation is consistent with the real-world and provides the most natural interpretation of the arrival of offers and the decisions made in our setting. A discrete-time model would have to define a period length and many offers may arrive during each period. The model would have to explicitly decide the information available about the offers and whether they are considered simultaneously or sequentially. A very short time-interval so that only one offer can arrive results in summations in the calculations below with many terms instead of integrals, adding notational and computational complexity without substantitive differences.
\( \Delta t \) tends to zero, yielding the differential equation above.\(^{24}\)

The differential equation defining \( V_i(t) \) has a unique solution that is determined by the terminal condition \( V_i(T_i) = D_i(T_i) \) because the probability of receiving additional offers in the remaining time vanishes as \( t \to T_i \).

The value of waiting depend on the flow payoffs while waiting on the list, the possibility of and value from exogeneous departures, and the option value of potential offers. As a consequence of the last component, patients may refuse an offer that they may accept if the alternative is never receiving a transplant. Moreover, certain marginal organs may be refused by all patients on the list and discarded even though some patients can benefit from a transplant from the organ relative to no assignment.

### 3.2.3 Normalization and Simplifying the Value Function

A typical dataset from a sequential assignment mechanism such as ours only contains information about accept/reject decisions. As is well understood, data on actions alone do not suffice for identifying all primitives of a dynamic discrete choice model, and the payoff from one action must be normalized in each state (Magnac and Thesmar, 2002). However, Aguirregabiria and Suzuki (2014) and Kalouptsidi et al. (2015) point out that such normalizations may affect counterfactual analysis because they arbitrarily restrict payoffs from specific actions across various states. This fact poses a potentially serious barrier to answering questions that depend on primitives that are not identified from choice data.

Fortunately, the counterfactuals involving changes in the mechanism are identified in our model. Intuitively, in any waitlist mechanism, the trade-offs between accepting an offer and waiting should only depend on payoffs relative to the value of never receiving an assignment. Assumptions 1(iii) and 2(i) together imply that the value of never receiving an assignment does not depend on the mechanism. This discussion suggests normalizing the value of refusing all offers, irrespective of the state.

\(^{24}\)The discretized version of the equation defining the value of waiting at time \( t \) is:

\[
V_i(t) = \frac{1}{1 + \rho \Delta t} \left[ \int d_i(t) \Delta t + \delta_i(t) \Delta t D_i(t) + \lambda \int \pi_{ij}(t) \int \max \{ V_i(t + \Delta t), \Gamma_{ij}(t) \} dG dF \right. \\
+ \left. (1 - (\delta_i(t) + \lambda_i(t)) \Delta t) V_i(t + \Delta t) + o(\Delta t) \right],
\]

where \( \lambda_i(t) = \lambda \int \pi_{ij}(t) dF \) is the rate at which agent \( i \) expects to receive an offer at time \( t \). The leading fraction represents discounting due to time preferences. The first three terms inside the brackets are described in the text. The remainder term includes the payoff in the event that multiple donors or objects arrive, or that a donor arrives and the patient departs, within \( \Delta t \). These events have probability of order \( o(\Delta t) \). Therefore, the remainder is of order \( o(\Delta t) \) as long as all expected payoffs are bounded. Taking the limit as \( \Delta t \to 0 \) under mild continuity conditions yields the differential equation above.
Formally, the value from never being assigned is defined by the differential equation

\[(\rho + \delta_i(t))O_i(t) = d_i(t) + \delta_i(t)D_i(t) + \dot{O}_i(t)\]

and the terminal condition \(O_i(T_i) = D_i(T_i)\). Under Assumptions 1(iii) and 2(i) this value does not depend on the mechanism. Appendix A.1 formally shows that measuring \(V_i(t)\) and \(\Gamma_{ij}(t)\) relative to \(O_i(t)\) suffices for analyzing decisions and differences in welfare under the current and alternative mechanisms.

With this in mind, we normalize \(O_i(t)\) to zero at all \(t\). This normalization implies that \(d_i(t) + \delta_i(t)D_i(t) = 0\) for all \(t\) and that \(D_i(T_i) = 0\). Equation (2) now simplifies to

\[(\rho + \delta_i(t))V_i(t) = \lambda \int \pi_i(t) \max \{0, \Gamma_{ij}(t) - V_i(t)\} \, dGdF + V_i(t)\]

The solution to this differential equation is

\[V(t; x_i) = \int_t^{T_i} \exp(-\rho(\tau - t))p(\tau|t; x_i)\]

\[\left(\lambda \int \pi(\tau; x_i, z, \eta) \max \{0, \Gamma(\tau, x_i, z, \eta) + \varepsilon_{ijt} - V(\tau; x_i)\} \, dGdF_{z,\eta}\right) d\tau,\]

where

\[p(\tau|t; x_i) \equiv \exp\left(-\int_t^\tau \delta(\tau'; x_i) \, d\tau'\right)\]

is the probability that agent \(i\) does not exogenously depart before \(\tau\) conditional on being on the list at \(t\). We have explicitly reintroduced agent and object characteristics into the notation because this equation will form the basis of our empirical strategy. This solution is based on the boundary condition \(\lim_{t \to T_i} V(t; x_i) = D_i(T_i) = O_i(T_i) = 0\) because the probability of receiving an offer after \(t\) vanishes as \(t \to T_i\). A similar result holds for an alternative model that considers the limit case with \(T_i = \infty\).

As can be seen, the advantage of this particular normalization is that we no longer need to estimate the flow payoffs from remaining on the list or the net present value of departing without an assignment. Going forward, we interpret \(\Gamma_{ij}(t)\) and \(V_i(t)\) as values relative to never receiving an assignment.

4 Estimation

The key primitives needed to predict equilibrium allocations and welfare under alternative mechanisms are the transplant values, \(\Gamma(t, x, z, \eta)\). The challenge for estimation is that acceptance decisions in our data depend on both the value of the offered organ and the value of continuing to wait. The two leading techniques for estimating dynamic choice models of
this type are the conditional choice probabilities (CCP) approach (Hotz and Miller, 1993; Aguirregabiria and Mira, 2007; Arcidiacono and Miller, 2011) and the full solution or nested fixed point approach (Miller, 1984; Wolpin, 1984; Pakes, 1986; Rust, 1987). We employ the CCP approach because it affords a computationally tractable estimator that allows us to use detailed knowledge of the mechanism. This section begins by representing the value function and the payoff from assignment in terms of CCPs, and then describes the estimation approach, empirical specification, and alternatives to our approach.

4.1 A CCP Representation

We now develop a representation for our model based on the insights in the literature on CCP estimation of dynamic discrete choice models. This representation will allow us to estimate the model without solving for the integral equation that defines the value function (equation 4).

Let $P_{ijt} \equiv \mathbb{P}(a_{ijt} = 0 | t, x_i, z_j, \eta_j)$ be the conditional choice probability of agent $i$ refusing an offer ($a = 0$) of kidney $j$ at time $t$ given $(t, x_i, z_j, \eta_j)$. Assume for now that $P_{ijt}$ is known—we will estimate this quantity in the next subsection using data on rejection decisions. The CCP approach is based on two key relationships:

**Proposition 1.** Suppose Assumption 1(ii) holds and $P_{ijt}$ is known. Then, (i) the conditional choice probabilities are given by

$$P_{ijt} = G(V(x_i, t) - \Gamma(x_i, z_j, \eta_j, t))$$

and (ii) there exists a known function $\psi$ such that

$$\psi(P_{ijt}) = \int \max\{V(x_i, t), \Gamma(x_i, z_j, \eta_j, t) + \varepsilon_{ijt}\} \, dG - V(x_i, t)$$

$$= \int \max\{0, \Gamma(x_i, z_j, \eta_j, t) - V(x_i, t) + \varepsilon_{ijt}\} \, dG.$$

The first relationship is analogous to the well-known result that CCPs are a known function of the differences between choice-specific conditional value functions. It follows immediately from the fact that $P_{ijt} \equiv \mathbb{P}(V(x_i, t) > \Gamma(x_i, z_j, \eta_j, t) + \varepsilon_{ijt})$ and the definition of $G$ as the CDF of $\varepsilon_{ijt}$. The second relationship expresses the difference between the value of continuing to wait and the expected value from the optimal choice in terms of the CCPs. That is, it expresses the incremental value of making an optimal decision following an offer in terms of the CCPs. This result is shown in proposition 1 of Hotz and Miller (1993).

These two equations can be used to re-write the primitive of interest, $\Gamma(x_i, z_j, \eta_j, t)$. Specifically, substituting part (ii) into the integral equation for $V(x_i, t)$ (equation 4), we get that:
\[ V(t; x_i) = \int_0^{T_t} \exp\left(-\rho(\tau - t)\right) p(\tau|t; x_i) \left( \lambda \int \pi(\tau; x_i, z, \eta) \psi(P_{ij\tau}) \text{d}F_z,\eta \right) \text{d}\tau. \]  

(6)

Using this expression in conjunction with part (i), we obtain the expression

\[ \Gamma(x_i, z_j, \eta_j, t) = -G^{-1}(P_{ijt}) + V(x_i, t). \]

(7)

In our application, we will assume that \( \varepsilon_{ijt} \sim N(0, 1) \). Therefore, \( G = \Phi \) is the CDF of the standard normal, giving us a simple closed-form expression \( \psi(P) = \phi(\Phi^{-1}(P)) - (1 - P) \Phi^{-1}(P) \).

To recover transplant values, we need to estimate the CCPs \( P_{ijt} \), the survival functions \( p \), offer probabilities \( \pi \), the distribution of donor types \( F \), and the parameters \( \rho \) and \( \lambda \). The next subsection turns to these issues.

4.2 Estimation Procedure

We estimate the model in four steps. First, we estimate \( p(\tau|t; x_i) \) and \( \lambda \) using observed patient departures and donor arrivals respectively. Second, we estimate conditional choice probabilities from patient accept/reject decisions. Third, we estimate the integral in equation (6) using the empirical distribution of donor types and offer probabilities to estimate \( F \) and \( \pi \). In the final step, we recover transplant values \( \Gamma(t, x, z, \eta) \) by solving for each patient’s value function at each date by evaluating equation (4).

As is well known, time preferences are not identified from observed choices alone in dynamic discrete choice models (Magnac and Thesmar, 2002). We therefore set the discount rate \( \rho \) to a fixed value of 5 percent per year. Our results are robust to using an annual discount rate of 10 percent. For modest discount rates, most of the discounting of future offers is due to the term \( \delta(t; x) \), which is estimated at approximately 16% per year for the average patient.

Step 1: Estimating Rates of Patient Departure and Donor Arrival

A patient’s continuation value on the waiting list depends on how long she can expect to continue waiting before an exogenous departure. Our dataset contains information on how long each patient is observed on the list without a transplant, and their reason for departure. We can therefore construct a censored measure of the length of time a patient would remain on the list without a transplant. Censoring occurs if the patient is transplanted, or if she is still on the list at the end of the sample period. These censored measures can be used to estimate departure rates independently of payoffs because Assumption 2 implies that, conditional on patient characteristics, departure from the list prior to assignment is exogenous.
We use maximum likelihood to estimate a censored Gompertz proportional hazards model in which the rate of departure takes the form

\[
\delta(t; x_i) = \delta_1 \exp(\delta_2 t) \exp(x_i \beta),
\]

where \(\delta_1 \exp(\delta_2 t)\) is the baseline hazard function for the Gompertz model and \(x_i\) are observed patient covariates. The parametric form for the baseline hazard function has the advantage of allowing for a simple expression for the survival function \(p(\tau|t; x_i) \equiv \exp(-\int_t^\tau \delta(\tau'; x_i) \, d\tau')\). It turns out that the estimated model yields a survival curve similar to the semi-parametric Cox proportional hazards model.

Since donor arrivals are exogeneous, we estimate \(\lambda\) as the empirical average arrival rate. This estimator is the maximum likelihood estimator for a Poisson process.

**Step 2: Estimating the CCPs**

The CCP of refusing an offer in our setting is a function of the characteristics \((t, x_i, z_j, \eta_j)\). The multi-dimensional nature and the continuous variables in this binary response model suggests estimation that uses a flexible basis and a link function (see Arcidiacono and Ellickson, 2011). A particularly convenient choice is to set \(G^{-1}(= \Phi^{-1})\) as the link function. Consequently, we estimate the probit choice model

\[
P_{ijt} = G(\chi(x_i, z_j, t, \theta + \eta_j)),
\]

where \(\chi(\cdot)\) is a flexible set of functions with interactions between its arguments and \(\theta\) is an unknown parameter to be estimated. With this choice of link function, equation (5) implies that the argument of \(G\) in the expression for \(P_{ijt}\) above is equal to \(V(x_i, t) - \Gamma(x_i, z_j, \eta_j, t)\).\(^{25}\) The additive separability of \(\eta_j\) in our empirical specification follows if it is separable in \(\Gamma(\cdot)\).

In our specification of \(\chi(\cdot)\), we include dummies in \(x_i\) and \(z_j\) for categorical variables and piecewise linear splines for their continuous elements, as well as piecewise linear splines in \(t\). The bases in these categorical variables and splines are interacted with each other. The donor unobserved heterogeneity term is parameterized as \(\eta_j \sim N(0, \sigma_\eta)\), with a variance to be estimated. This term captures quality differences across organs due to the detailed medical information available to the agents at the time of assignment, but are not included in the model individually because any given characteristic is relevant for only a small number of donors. It captures correlation in choices for a given organ across patients conditional on the observed characteristics.

\(^{25}\)If the link function \(L^{-1}\) is not equal to the inverse CDF of \(\varepsilon_{ijt}\), then \(V(x_i, t) - \Gamma(x_i, z_j, \eta_j, t) = G^{-1}(L(\chi(x_i, z_j, t, \theta + \eta_j)))\) where the argument of \(G^{-1}\) is equal to \(P_{ijt}\). The remaining estimation steps can proceed exactly as described below, but with minor modifications to the expressions.
Identification of these parameters is based on standard arguments. The parameter $\theta$ is identified by the relationship between the covariates and the probability of acceptance. The variance, $\sigma^2_\eta$, of the donor-specific unobservable is identified because many donors have two kidneys offered to patients. If $\sigma^2_\eta$ is large, then conditional on the observables $x_i$, $z_j$, and $t$, an early acceptance of the first kidney from a donor indicates that the second acceptance should soon follow. In contrast, if $\sigma^2_\eta$ is small, then the position of the first acceptance should have little information about the second. The intuition is similar to those for results on the identification of measurement error models (see Kotlarski’s theorem in Rao, 1992; Hu and Schennach, 2008).

We estimate the parameters $(\theta, \sigma_\eta)$ using a Gibbs’ sampler (McCulloch and Rossi, 1994; Gelman et al., 2014). The resulting estimates are asymptotically equivalent to the maximum likelihood estimator (see van der Vaart, 2000, Theorem 10.1 (Bernstein-von-Mises)).

Step 3: Estimating $V(x_i; t)$ using offer arrivals and CCPs

Next, we use equation (6) to calculate $V(x_i; t)$ by numerically integrating the incremental value generated by offers that a patient can expect to receive in the future. This step requires us to evaluate the integrand at many points, $\tau$. We have estimated the terms $\lambda$ and $p(\tau|t; x_t)$ in Step 1, and have set $\rho$ as discussed earlier. Therefore, the only remaining term is the inner integral in equation (6):

$$W(x_i, \tau; \theta_0) = \int \pi(\tau; x_i, z, \eta) \psi(P_{ij\tau}) dF_{z,\eta} = \mathbb{E}[P(c_{ij} = 1|z_j, x_i) 1\{s_{ijt} > s^*_j\} \psi(P_{ij\tau})|x_i, \tau],$$

where expectations are taken over donor characteristics $(z_j, \eta_j)$, drawn from $F$, and the priority-score cutoff $s^*_j$ drawn from the conditional distribution of cutoffs $H(\cdot|z, \eta_j)$ given $(z_j, \eta_j)$. The second equality is implied by the definition of $\pi_{ij}(t)$ given in Assumption 3. As a notational reminder, $c_{ij} = 1$ if agent $i$ is compatible with object $j$ and $s_{ijt}$ is the priority score of agent $i$ for object $j$ at time $t$.

We estimate this quantity by replacing $P(c_{ij} = 1|z_j, x_i)$ and $\psi(P_{ij\tau})$ with estimated quantities and use the sample analog of the expectation $\mathbb{E}[\cdot|x_i, \tau]$:

$$\hat{W}(x_i, \tau; \hat{\theta}) = \frac{1}{J} \sum_{j=1}^J \hat{P}(c_{ij} = 1) 1\{s_{ij\tau} > s^*_j\} \psi(\hat{P}_{ij\tau}),$$

The Gibbs’ sampler is more convenient than maximum likelihood because our model includes donor unobserved heterogeneity $\eta_j$. The method uses data augmentation on the latent terms $\eta_j$ to solve this problem. We obtain draws of the parameters $\theta$ and $\sigma_\eta$ from a sequence of conditional posterior distributions using a Markov chain given dispersed priors and an initial set of parameters $(\theta^0, \sigma^0_\eta)$. The invariant distribution of the Markov chain is the posterior given the prior and the data. Details on the implementation, including burn-in procedures and convergence diagnostics, are in Appendix A.2.
where \( j \) indexes a donor in our sample. Each element of the summand has three terms. The first term, \( \hat{P} (c_{ij} = 1) \), is an estimate for \( \mathbb{P} (c_{ij} = 1|z_j, x_i) \) constructed using rich information on donor proteins and patient immune system characteristics.\(^{27}\) The second term, \( 1 \{ s_{ijr} > s^*_j \} \), is calculated by comparing the score \( s_{ijt} \) implied by the mechanism’s rules to the observed pivotal priority-score \( s^*_j \) defined in Section 3.2.1. The third term replaces \( \psi (P_{ijr}) \) with the estimate \( \hat{P}_{ijr} = G \left( \chi (x_i, z_j, \tau) \hat{\theta} + \eta_j \right) \), where \( \hat{\theta} \) is the estimated parameter and a draw of \( \eta_j \). Because the distribution of \( s^*_j \) depends on \( \eta_j \), we draw \( \eta_j \) from its distribution given the observed accept/reject decisions of all patients offered donor \( j \).\(^{28}\)

Therefore, our estimate \( \hat{V} (t; x_i) \) is obtained by numerically integrating

\[
\exp (-\rho (\tau - t)) \hat{\rho} (\tau|t; x_i) \hat{W} (x_i, \tau; \hat{\theta})
\]

over the range \( \tau \in [t, T_i] \). Details on the number of points used this procedure for computing \( \hat{V} (t; x_i) \) are provided in Appendix A.2. Substituing \( \hat{W} (x_i, t; \hat{\theta}) \) for \( W (x_i, t; \theta_0) \) is appropriate because the latter is a \( \sqrt{J} \)-consistent estimator of the former for each \( (x_i, t) \) under standard regularity conditions assuming that the dependence of a potential future offer on the organ that has arrived today diminishes with the time-horizon for the future offer.\(^{29}\)

**Step 4: Estimating \( \Gamma \)**

Once \( \hat{V} (t; x_i) \) has been estimated, we recover \( \Gamma (\cdot) \) by inverting \( G : \)

\[
\hat{\Gamma} (t, x_i, z_j, \eta_j) = \hat{V} (t; x_i) - G^{-1} \left( \hat{P}_{ijt} \right) = \hat{V} (t; x_i) - \left( \chi (x_i, z_j, t) \hat{\theta} + \eta_j \right).
\]

This quantity can be calculated for any arbitrary value of \( (t, x_i, z_j, \eta_j) \).

**Discussion**

The main advantage of the CCP estimation approach relative to a full solution/nested fixed point approach is that it can accomodate rich observed heterogeneity while maintaining

\(^{27}\)As mentioned in Section 2.2 a crossmatch is conducted using blood from the donor and patient in case the virtual crossmatch yielded a false negative. We estimate \( \mathbb{P} (c_{ij} = 1|z_j, x_i) \) using the data on instances where a kidney was accepted because of a negative virtual crossmatch, but the transplant did not occur because the final crossmatch was positive.

\(^{28}\)Formally, for each \( j \), we need to draw from a random variable with expectation \( \mathbb{E} \left[ 1 \{ s_{ijr} > s^*_j \} \psi (P_{ijr}) \right] \). For simplicity of exposition, we have dropped the term \( \mathbb{P} (c_{ij} = 1|z_j, x_i) \) as well as explicit conditioning on \( (x_i, z_j, \tau) \). Let \( I_j \) denote all the offers and accept/reject decisions for kidney \( j \). By the law of iterated expectations, this expectation is equal to \( \mathbb{E} \left[ \mathbb{E} \left[ \psi (P_{ijr}) | I_j \right] \right] \) because \( s^*_j \) is measurable with respect to \( I_j \) and \( s_{ijt} \) is a function of \( (x_i, z_j, \tau) \). Therefore, the random variable \( 1 \{ s_{ijr} > s^*_j \} \psi (P_{ijr}) \), where \( \psi (P_{ijr}) = \psi \left( G \left( \chi (x_i, z_j, \tau) \hat{\theta} + \eta_j \right) \right) \) and \( \eta_j \) is drawn from the conditional distribution given \( I_j \) has the desired expectation.

\(^{29}\)Formal conditions and a proof are available upon request.
computational tractability. A full-solution approach would parametrize \( \Gamma (\cdot) \) directly in terms of parameters \( \theta_T \) and compute a likelihood of accepting an offer using the implied value function. Maximizing this likelihood is burdensome because the value function must be computed for many values of \( \theta_T \). This issue is particularly severe if the state space is large as is the case in our application even after the simplification afforded by Assumption 3. Indeed, we compute the compatibility and priority score for each patient and donor using all the variables that enter the assignment mechanism.

Our model abstracts away from patient-level unobserved heterogeneity. An extension that incorporates it would introduce three complications. First, both the departure rates \( \delta_i(t) \) and choice probabilities would ideally depend on unobserved heterogeneity and would need to be simultaneously estimated. Second, a solution to the initial conditions problem would need to be developed for the agents already on the waiting list at the beginning of the sample. Finally, estimating dynamic models with both rich observed and unobserved heterogeneity is demanding on the data. Our goal of targeting offers using observed characteristics weighs in favor of our approach.

We believe that abstracting away from unobserved heterogeneity still yields useful results because our dataset contains a rich set of patient characteristics. Nonetheless, we explored simple specifications in which a limited form of unobserved heterogeneity was included in the choice model only. These specifications yielded qualitatively similar results for the counterfactual analysis (details available on request).

5 Parameter Estimates

This section describes our estimates of patient departure rates, conditional choice probabilities, and the value of transplantation. The estimated models include the rich set of patient and donor observed characteristics summarized in Tables 1 and 2. The baseline characteristics, the linear splines and the interactions among these variables, were chosen by surveying the medical literature. Specifically, we use covariate and spline specifications from the KP-SAM model, which was used by the kidney allocation committee to predict the outcomes of various allocation systems.\(^{30}\) We also include any covariates that were part of the survival models for kidney transplant patients used in Wolfe et al. (2008).

5.1 Estimated CCPs and Departure Rates

Conditional Choice Probabilities: We estimated three specifications for the conditional choice probability of accepting an offer. The first specification includes all of these baseline

\(^{30}\)We obtained the KPSAM module from the Scientific Registry of Transplant Recipients (SRTR). Our dataset contained all but one of the variables used in this model.
variables, but does not include donor unobserved heterogeneity ($\eta$) or the state variable time $t$. The second specification adds donor unobserved heterogeneity, and the third specification adds waiting time interacted with a variety of characteristics. Table 5 presents select parameter estimates (see Table A.4 for the full specification).

Table 5: Conditional Choice Probability of Acceptance (select co-efficients)

<table>
<thead>
<tr>
<th></th>
<th>Base Specification</th>
<th>Unobserved Heterog.</th>
<th>Waiting Time + UH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Calculated Panel Reactive Antibody (CPRA)</td>
<td>0.60 (0.05)</td>
<td>0.68 (0.06)</td>
<td>0.58 (0.09)</td>
</tr>
<tr>
<td>Donor Age &lt; 18</td>
<td>0.27 (0.10)</td>
<td>-0.09 (0.19)</td>
<td>-0.04 (0.20)</td>
</tr>
<tr>
<td>Donor Age 18-35</td>
<td>0.59 (0.12)</td>
<td>-0.06 (0.19)</td>
<td>0.02 (0.19)</td>
</tr>
<tr>
<td>Donor Age 50+</td>
<td>-0.83 (0.16)</td>
<td>-0.77 (0.21)</td>
<td>-0.87 (0.22)</td>
</tr>
<tr>
<td>Expanded Criteria Donor (ECD)</td>
<td>-0.14 (0.02)</td>
<td>-0.53 (0.08)</td>
<td>-0.53 (0.10)</td>
</tr>
<tr>
<td>Donation from Cardiac Death (DCD)</td>
<td>-0.10 (0.02)</td>
<td>-0.51 (0.06)</td>
<td>-0.50 (0.09)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match</td>
<td>2.33 (0.31)</td>
<td>2.92 (0.43)</td>
<td>2.89 (0.44)</td>
</tr>
<tr>
<td>Regional Offer</td>
<td>-1.38 (0.06)</td>
<td>-2.90 (0.19)</td>
<td>-2.92 (0.19)</td>
</tr>
<tr>
<td>National Offer</td>
<td>-1.54 (0.04)</td>
<td>-3.05 (0.12)</td>
<td>-3.11 (0.11)</td>
</tr>
<tr>
<td>Patient on Dialysis at Registration</td>
<td>-0.02 (0.02)</td>
<td>-0.10 (0.02)</td>
<td>-0.09 (0.02)</td>
</tr>
<tr>
<td>Log Waiting Time (years)</td>
<td></td>
<td></td>
<td>0.09 (0.06)</td>
</tr>
<tr>
<td>Log Waiting Time x 1{Over 1 Year}</td>
<td></td>
<td></td>
<td>-0.15 (0.07)</td>
</tr>
<tr>
<td>Log Waiting Time x 1{Over 2 Years}</td>
<td></td>
<td></td>
<td>-0.13 (0.12)</td>
</tr>
<tr>
<td>Log Waiting Time x 1{Over 3 Years}</td>
<td></td>
<td></td>
<td>0.30 (0.11)</td>
</tr>
<tr>
<td>Patient Age x 1{Donor Age &lt; 18}</td>
<td>-0.01 (0.00)</td>
<td>0.00 (0.00)</td>
<td>0.00 (0.00)</td>
</tr>
<tr>
<td>Patient Age x 1{Donor Age 18-35}</td>
<td>-0.02 (0.00)</td>
<td>0.00 (0.01)</td>
<td>0.00 (0.01)</td>
</tr>
<tr>
<td>Patient Age x 1{Donor Age 50+}</td>
<td>0.02 (0.00)</td>
<td>0.02 (0.01)</td>
<td>0.02 (0.01)</td>
</tr>
</tbody>
</table>

Donor Unobservable Std. Dev. 1.02 (0.03) 1.04 (0.04)
Idiosyncratic Shock Std. Dev. 1.00 1.00 1.00

Acceptance Rate 0.140% 0.140% 0.140%
Number of Offers 2713043 2713043 2713043

The estimated coefficients on observed donor characteristics are intuitive and fairly robust across specifications. For example, offers from donors older than 50 years of age are less likely to be accepted than offers from 35 to 50 year old donors, and kidneys from younger donors are even more likely to be accepted.

We also estimate significant patient-level and match-specific heterogeneity in acceptance rates. A perfect tissue type match is very likely to be accepted, much more so than a young donor. Similarly, offers of kidneys with more antigen mismatches (A, B or DR), and regional and national offers are less likely to be accepted. Acceptance rates also depend on patient age. Adult patients of different ages are equally likely to accept a middle-aged donor, but older patients are more likely to accept a donor who is over 50 years old. This pattern is consistent with the idea that it is more important for younger patients to obtain kidneys that are likely to function for a long time.
The third specification shows that acceptance rates fall rapidly with waiting time in the first few years before starting to increase after year three. The increase in acceptance rates is consistent with patients with low waiting time priority being less selective.

Comparing the fit of models suggests that including donor unobserved heterogeneity and a flexible form for the waiting time is important. Although our specification includes the most important donor characteristics, it is not possible to include all characteristics (even if observed in our data) because they are numerous, hard to quantify, or not medically relevant in most cases. As discussed earlier, we interpret the donor unobserved heterogeneity term as a stand-in for these characteristics. Including donor unobserved heterogeneity is important for replicating the trend in acceptance rates across offer number (see Figure A.1(a) in the Appendix). In the data, there is a sharp decline in the average acceptance rate as offer number increases because, as discussed in section 2, undesirable organs are offered to more patients on the list. Only models that include donor unobserved heterogeneity are able to capture this trend. Similarly, it is important to specify a flexible spline in waiting time to fit the average acceptance rates across years waited (see Figure A.1(b)).

Finally, we assessed whether our flexible specification for the CCPs is overfit by measuring out-of-sample fit. To do this, we computed the relative mean-squared error of the model for our estimation sample (which ends in 2013) and the first six months of 2014. The latter period was excluded from estimation in order to avoid anticipatory changes to the mechanism. We assessed the behavior of this statistic as the richness of state space is altered. As expected, the in-sample relative MSE falls with the richness of the specification (see Table A.3). However, the out of sample relative MSE increases substantially if the specification is made richer. This pattern suggests that our specification finds a good in-sample fit without overfitting.

**Departure Rates:** We estimated patient departure models under different parametric assumptions on the baseline hazard of departing from the kidney waitlist prior to transplantation. These models include all of the patient-specific variables included in the CCP model. Table A.2 in the appendix presents these estimates. They reveal significant and robust heterogeneity across patients in their departure rates. For example, departure rates are higher for diabetic patients and for older adults. Patients with blood type A are also more likely to depart, potentially due to better chances of receiving a living donor transplant.

Across specifications, we estimate an increasing baseline hazard of departure, consistent with patients becoming less healthy over time. We chose the Gompertz hazard model because it resulted in a survival curve very similar to the Cox proportional hazards model.

Taken together, we feel comfortable with the fit of the CCPs in the third model and the

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31 The hazard model specifications include the CPRA variable from the CCP model as a linear term, but omits a dummy for CPRA $> 0.8$. Because priority is discontinuous in CPRA at 0.8, it is important to allow acceptance behavior to be discontinuous at this point. Departures without a transplant, however, should not be discontinuous in CPRA. Moreover, specifications of the departures model that included this variable estimated a statistically insignificant coefficient.
Gompertz hazard model. All results that follow use estimates from these specifications.

5.2 Estimated Value of Organ Offers

Below, we develop an interpretable measure for comparing values across agent in terms of an equivalent change in object arrival rates (supply). We then use these units to describe our estimates. Our counterfactual results on patient welfare will also use these units.

Sequential assignment mechanisms can re-assign offers from some agents to others. Motivated by this fact, consider the proportional increase in a patient’s value from a one-time offer for object \( j \) at the time of registration:

\[
EV_{ij} \equiv \int \max \left\{ 0, \Gamma (0, x_i, z_j, \eta_j) + \varepsilon_{ij} - V (0; x_i, \lambda) \right\} dG
\]

where the dependence of \( V \) on the object arrival rate \( \lambda \) is re-introduced for clarity. The numerator is the difference between the value with and without an additional one-time offer for object \( j \) at time 0, and the denominator is the baseline value.

Instead of a one-time offer, the same change in value can be generated by an increase in the object arrival rate. Specifically, let \( \lambda_{ij} \) be defined such that \( V (0; x_i, \lambda_{ij}) - V (0; x_i, \lambda) \) is the numerator of the expression above. Using this definition, we can rewrite \( EV_{ij} \) as

\[
EV_{ij} = \frac{V (0; x_i, \lambda_{ij}) - V (0; x_i, \lambda)}{V (0; x_i, \lambda)} \approx \frac{\lambda_{ij} - \lambda}{\lambda},
\]

where the approximation follows because \( V (t; x_i, \lambda) \) is approximately linear in \( \lambda \) (see equation 4). This approximation is appropriate for small changes in \( \lambda \), so that offer probabilities and acceptance decisions do not change substantially.

The equivalent change in object arrivals is, by definition, invariant to the scale of utility units across agents and will be used to report welfare effects going forward. It equates an additional one-time offer for object \( j \) to the value of an alternative policy that is able to marginally increase the organ supply. This feature makes this quantity similar in spirit to Equivalent Variation at the time of registration. Aggregating this measure yields a notion of social welfare in which the planner equally values the effect of a change in donor supply on all patients.

Figure 1 describes our preferred estimates in these units. The plots show how the value of an organ offer varies across specific patient and donor characteristics, holding all remaining characteristics fixed.

While all patients prefer younger donors, this preference is stronger among healthier patients. Younger patients and patients not on dialysis at registration place a relatively higher value
on younger donors. In contrast, older patients place a high value on offers whether they are from young or from old donors. These patterns are consistent with the CCP estimates and with the differential life expectancy effects of receiving a transplant from a high quality organ on younger and older patients. The higher value placed by older patients on all types of offers is also consistent with descriptive evidence in section 2, which suggests that they receive fewer top 10 offers. Therefore, additional offers represent a higher equivalent increase in donor supply for these patients.

In addition, a perfect tissue type match is especially important for patients. Although not reported, such an offer from a young donor is equivalent to a large increase in overall donor supply for a representative patient. This result is consistent with the fact that organs with a perfect tissue type match are less likely to induce an adverse immune response post-transplant, thereby increasing the life-years afforded by the transplant.

6 Steady State Equilibria and Welfare Comparisons

6.1 Equilibrium Concept

We now define an equilibrium concept for counterfactual analysis. It is intended to capture a large pool of agents waiting for offers. Agents have type $x \in X$, and objects have type $z \in \zeta$, where we henceforth include the unobserved donor characteristic $\eta$ in $z$ for notational simplicity. For computational reasons, we will treat $X$ and $\zeta$ as finite sets.
To simplify notation, albeit with a slight abuse, the rest of the paper replaces subscripts that index individuals $i$ and objects $j$ with types $x$ and $z$, respectively. For instance, we write the value function as $V_x(t)$ instead of $V(t; x_i)$ and the scoring rule as $s_{xz}(t)$ instead of $s(t; x_i, z_j)$. The notation for other quantities such as $\pi$, $\Gamma$ and $\delta$ is adapted analogously.

Agents follow type-symmetric accept/reject strategies, $\sigma_x : \mathbb{R} \times \mathbb{R}_+ \to \{0, 1\}$, indexed by $x \in \mathcal{X}$. The first element of the domain is the payoff of being assigned a particular object, $\Gamma$, and the second element is time waited, $t \in \mathbb{R}_+$. We exclude strategies that depend on richer information because beliefs are restricted to satisfy Assumption 3.

We model the composition of the queue using a single steady state. Specifically, the queue composition will be governed by a probability density function, $m$, defined on the set $\mathcal{X} \times [0, T]$, where $T$ is the maximum wait time. This density governs the distribution of agents of each type and how long they have waited. We write $m_x(t)$ to denote the density evaluated at $(x, t)$. The length of the queue is denoted by $N$.

**Definition 1.** A steady state equilibrium consists of an accept/reject strategy $\sigma^*$, beliefs $\pi^*$, a queue size $N^*$, and a probability measure $m^*$ such that the following conditions hold:

1. Optimality: For each agent of type $x \in \mathcal{X}$ and an offer with net present value $\Gamma$,
   $$\sigma^*_x(\Gamma, t) = 1 \{ \Gamma \geq V_x(t; \pi^*) \},$$
   where $V_x(t; \pi^*)$ is the net present value for type $x$ of declining the object and following the optimal strategy given $\pi^*$ after $t$.

2. Consistent beliefs: For each $(t, x, z)$, the beliefs $\pi^*(t; x, z)$ is consistent with equilibrium offer probabilities. In particular, for mechanisms that uses the scoring rule $s$,
   $$\pi^*_{xz}(t) = H^*_z(s_{xz}(t)) \times P(c = 1 | x, z),$$
   where $H^*_z(s)$ is the probability that the object is available only to agents above the score $s$ if $N^*$ agents are drawn iid from $m^*$, and they follow strategy $\sigma^*$.

3. Steady state detailed balance condition: For each $x \in \mathcal{X}$, $m^*_x(t)$ and $N^*$ satisfy
   $$\dot{m}_x(t) = -m_x(t) \kappa_x(t) \text{ and } m_x(0) = \frac{\gamma_x}{N^*}.$$
where $\gamma_x$ is arrival rate of an agent of type $x$, $\kappa_x(t)$ is the equilibrium departure rate of an agent of type $x$ at waiting time $t$, and $m^*$ is a density: $\sum_{x \in X} \int_0^T m_x(\tau) \, d\tau = 1$.

The first condition states that each agent makes optimal decisions at each point in time given their beliefs. The value from declining an offer is given by the Hamilton-Jacobi-Bellman equation defined in Section 3.2. The second condition imposes that agents have correct beliefs about offer probabilities. In the specific case of a mechanism based on a scoring rule, beliefs depend on the steady state distribution of cutoff scores. The distribution $H^*_z$ governs the cutoffs that arise when agents use strategies $\sigma^*$ and $N^*$ agents are drawn from a distribution governed by $m^*$. The final condition determines the composition of agent characteristics on the list. The left-hand side is the change in the density of agents of type $x$ who have waiting time $t$. The right-hand side term is the rate of departure for those agents. Departures occur for both exogenous reasons and because agents are removed from the waiting list once they are assigned; that is, $\kappa_x(t)$ is the sum of $\delta_x(t)$ and the equilibrium rate at which agents of type $x$ are assigned at time $t$ given the strategy $\sigma^*$ and the offer rates $\pi^*$.

This equilibrium concept abstracts away from transitional dynamics in the size and composition of the queue. The alternative approach of modeling these dynamics is to assume that the queue length and composition follow a Markov process. However, this process is high-dimensional, and would make the counterfactual exercises computationally intractable. As in Assumption 3, this concept approximates the behavior of such a system. For example, part 3(a) sets $N^*$ to be the expected queue length. A law of large numbers can be used to show that, in a long queue, the stationary distribution of the queue length concentrates mass on $N^*$ (details available on request). Hence, we expect that our equilibrium notion will be a good approximation for the behavior of the waitlist.

We compute steady-state equilibria using an algorithm that iterates between computing the value function, optimal decisions, and the steady-state composition of the waitlist. A detailed description with expressions for each step of the procedure and pseudocode is provided in Appendix C. To keep the computational burden manageable, the results we present below are based on a type space given by a random sample of 300 patients and 500 donors drawn from our dataset and a discrete grid for time.34 As we discuss in Section 7, the results reported below are not sensitive to varying these parameters.

We prove the existence of a steady state equilibrium for sequential assignment mechanisms that use a priority score in Theorem 1. The challenge in showing existence arises because the strategies, beliefs, and composition are a function of time, which is a continuous variable. We use the Brower-Schauder-Tychonoff fixed point theorem (Corollary 17.56, Aliprantis and Border, 2006) for general Banach spaces to prove existence. The primary assumptions are

34We discretize time into quarters for the first 15 years after registration, then every 2 years until year 25, and every 25 years thereafter. Finer partitions after the first few years do not affect the results since the probability that a patient survives without a transplant falls dramatically.
technical regularity conditions imposing bounds and Lipschitz continuity of primitive objects. The main substantive condition is that the set of scores used in the mechanism is finite. Our results do not rule out multiplicity of equilibria. However, we did not find multiple equilibria for the set of counterfactuals and specifications that are considered below.

6.2 Welfare Comparisons

Given a mechanism $\mathcal{M}$ and a donor arrival rate $\lambda$, let the **steady-state value** of an agent of type $x$ be the integrated value function over current and future generations:

$$
\bar{V}_x^\mathcal{M} (\lambda) = \frac{\gamma_x}{\rho} \int_0^\infty \exp (-\rho \tau) V_x^\mathcal{M} (0; \lambda) \, d\tau + \int_0^T N^* m_x^* (\tau) V_x^\mathcal{M} (\tau; \lambda) \, d\tau.
$$

(10)

The first term represents the discounted value of agents of type $x$ that are expected to arrive in the future, and the second term represents the value of agents presently on the waiting list. These terms are weighted by the net present value of the total mass of future arrivals $\frac{\gamma_x}{\rho}$ and the equilibrium measure $N^* m_x^* (\tau)$ of patients in the queue, respectively. This value is equal to that of a social planner who considers the net present value of payoffs generated by all future assignments to agents of type $x$.

Let $\mathcal{M}^0$ be the baseline mechanism used during our sample period and $\lambda^0$ be the baseline donor arrival rate. For any mechanism $\mathcal{M}$, define $\lambda_x (\mathcal{M})$, the equivalent donor arrival rate for agents of type $x$, as the solution to the equation $\bar{V}_x^\mathcal{M} (\lambda^0) = \bar{V}_x^{\mathcal{M}^0} (\lambda_x (\mathcal{M}))$. As discussed in Section 5.2, we can express a change in the value function for type $x$ as an equivalent change in the donor arrival rate:

$$
EV_x (\mathcal{M}) = \frac{\bar{V}_x^\mathcal{M} (\lambda^0) - \bar{V}_x^{\mathcal{M}^0} (\lambda^0)}{\bar{V}_x^{\mathcal{M}^0} (\lambda^0)} \approx \frac{\lambda_x (\mathcal{M}) - \lambda^0}{\lambda^0}.
$$

This measure describes the welfare effects for each patient in terms of an alternative policy that keeps the mechanism fixed, but is able to increase (or decrease) organ donation rates. A common challenge when conducting welfare analysis in environments without transfers is the lack of a clear transferable numeraire good that can be used to apply the Kaldor-Hicks criterion. Because the measure $EV_x (\mathcal{M})$ is type-specific, it avoids comparisons of utility across different types of patients. Averages of the measure $EV_x (\mathcal{M})$ across patient types will equally value equivalent changes in donor arrival rates for different patient types. Our results will also report distributional effects on key subgroups so that the reader may independently entertain alternative aggregates.
7 Evaluating Design Trade-Offs

The U.S. Department of Health and Human Services charged the OPTN Kidney Transplantation Committee to reform the system used prior to 2014. The stated goals were to find mechanisms that provided equitable outcomes for patients, efficiently allocated available organs, and minimized organ waste. Motivated by these goals, this section begins with a comparison of previous mechanisms used in practice to benchmark mechanisms, before moving to optimal mechanisms and discussing the sources of gains that we identify.

The qualitative results described below are robust to four variations: first, to the introduction of a limited form of patient unobserved heterogeneity; second, to changing the annual discount factor to 10 percent; third, to variations in the sample and number of patient and donor types; and fourth, to removing the limit on the number of offers that can be made. This last exercise suggests that an increased ability to offer organs to more patients does not obviate the improvements that we identify below.

7.1 The 2014 Reform vs Benchmark Mechanisms

This section compares the mechanisms used prior to 2014 to the re-designed mechanism and two benchmarks from the theoretical literature:

• Post-2014: In December 2014, the kidney allocation mechanism switched to a system that awards greater priority to patients who are extremely difficult to match (high CPRA), and also prioritizes healthier patients for higher quality donors. The rationale for the first change was that high CPRA patients have few opportunities for transplantation and are likely to accept most organs. Therefore, giving them additional priority could reduce organ waste and achieve more equitable outcomes for sensitized patients. The second change intended to offer high-quality kidneys to patients likely to benefit from them most, and in particular to reduce age mismatch.

• First Come First Served (FCFS): One concern of the kidney committee has been to maintain a transparent and procedurally fair offer system. FCFS offers objects to agents in the order they joined the waiting list. It is a procedurally fair and commonly used mechanism. FCFS also has attractive efficiency properties: Bloch and Cantala

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35 These results are available in supplementary materials included in the replication archive associated with this paper.
36 Our calculations only change the allocation mechanism in NYRT. Evaluating a nationwide change would require us to use data on decisions made by all patients in the US, which is burdensome due to the patchwork of variants on points used in approximately half the states. To simplify this task, we keep the system used to prioritize patients from the rest of the United States fixed to the pre-2014 system. We also assume that the policy function of patients from the rest of the US, which governs offers for non-local donors to patients in NYRT, remains fixed.
show that it maximizes agent welfare when values for an object are drawn i.i.d. across agents. This result is driven by the fact that FCFS encourages agents to be selective and choose only objects with high match-specific values. We approximate this system by finely discretizing time on a grid \( t_0, t_1, \ldots, t_L \) and set \( s_{xz}(t) = l \) if \( t \in [t_{l-1}, t_l) \).

- **Last Come First Served (LCFS):** Another goal of the kidney committee has been to minimize organ discards. LCFS provides strong incentives to accept organs because agents that refuse an offer are demoted if other agents arrive in the future. Lower selectivity can reduce waste if the number of patients compatible with a kidney is limited, or if there is a logistical constraint on the number of offers that can be made before the kidney is no longer transplantable. Moreover, Su and Zenios (2004) show that the LCFS system maximizes welfare when there is agreement across agents on the values of various objects, that is, if objects are vertically differentiated in quality and preferences are homogeneous. This is because social welfare depends not on who is assigned the object, but only on the fraction of objects allocated. We approximate this system by finely discretizing time on a grid \( t_0, t_1, \ldots, t_L \) and set \( s_{xz}(t) = L - l \) if \( t \in [t_{l-1}, t_l) \).

Our analysis shows that these theoretical and practical recommendations can either increase welfare or organ discard rates, but not both (Figure 2). Both the pre- and post-2014 mechanisms are very similar to FCFS in terms of average patient welfare and organ discard rates. Although not reported, our results indicate that the 2014 reforms primarily resulted in redistribution towards younger and more highly sensitized patients. In contrast to these three mechanisms, last come first served (LCFS) substantially reduces organ discard rates at the cost of lower welfare.\(^{37}\)

The welfare effects of these mechanisms are consistent with predicted effects on the waitlist and the types of donors transplanted. Table 6, Panel A shows that waiting times, queue lengths, and the quality of the average donor transplanted are similar across the first three mechanisms. In contrast, LCFS results in a dramatic decrease in discard rates and drop in the steady state queue length from 5,113 to 2,961. This decrease in discard rates comes at a significant welfare cost because patients accept organs that are poorly matched to them and of low quality (older and less likely to have died from head trauma). Indeed, a decomposition of the overall welfare changes into the portion predictable using our observable characteristics and the match-specific unobservable characteristics in our model shows that the primary differences are driven by observables.\(^{38}\)

These empirical results can be understood by considering equilibrium incentives in a mechanism to reject an offer and the externality imposed by an agent’s decision to do so. FCFS

\(^{37}\)Our measure of welfare is representative of a patient’s welfare if transplant surgeons help patients make decisions in their best interest. This assumption may not be valid if agency problems between transplant surgeons and the patients they represent significantly skew decisions. In addition, this measure ignores the potential effects of transplantation of healthcare costs that are not internalized by patients and doctors. A
increases selectivity as agents retain their priority when they decline an offer. On the other hand, LCFS reduces selectivity because agents are demoted when other agents join the list. The contrast in selectivity differentially emphasizes two opposing externalities. When agents decline an object, they allow others to receive an assignment earlier, generating a positive externality if the object is desirable. However, by refusing an object and remaining on the list, they also generate a negative externality as they take away future offers.

Which externality dominated depends on preferences and the mechanism. When preferences are highly heterogeneous, the positive externality dominates because agents that reject an offer allow others to receive an offer of average expected value earlier instead of later. Because selective agents only accept offers of high match value, FCFS has desirable efficiency properties when preferences are heterogeneous (Bloch and Cantala, 2017). But, FCFS performs poorly when all agents value each object identically because selective agents pass on only social planner that places weight on these costs may further favor mechanisms that reduce discards.

38Specifically, the observable component includes the portion of the change in welfare effects due to changes in total number of transplants, quality of the average transplanted organ and the observable component of match-specific value. The unobservable part includes the change due to the term $\varepsilon_{ijt}$ in the model.
Table 6: Outcomes in Various Mechanisms

<table>
<thead>
<tr>
<th>Panel A: Steady State Equilibrium, Benchmark Mechanisms</th>
<th>EV&lt;sub&gt;I&lt;/sub&gt;</th>
<th>Waitlist</th>
<th>Transplanted Donors</th>
<th>EV Decomp. &lt;br&gt;ΔV&lt;sub&gt;0&lt;/sub&gt; &lt;br&gt; &gt; -5%</th>
</tr>
</thead>
<tbody>
<tr>
<td>EV&lt;sub&gt;Decomp&lt;/sub&gt;</td>
<td>Queue Length</td>
<td>Reduction in Discard Rate</td>
<td>Years on Waitlist</td>
<td>Age</td>
</tr>
<tr>
<td>Pre-2014 Priorities</td>
<td>--</td>
<td>5113.2</td>
<td>--</td>
<td>2.73</td>
</tr>
<tr>
<td>Post-2014 Priorities</td>
<td>-0.8%</td>
<td>5042.5</td>
<td>0.5%</td>
<td>2.70</td>
</tr>
<tr>
<td>First Come First Served</td>
<td>1.5%</td>
<td>5274.4</td>
<td>-2.1%</td>
<td>2.78</td>
</tr>
<tr>
<td>Last Come First Served</td>
<td>-50.6%</td>
<td>2961.4</td>
<td>24.8%</td>
<td>3.73</td>
</tr>
</tbody>
</table>

Panel B: Steady State Equilibrium, Alternative Systems

| Approx. Opt. Pareto Improving Priorities | 21.4% | 4576.2 | 7.2% | 2.55 | 45.5 | 15.7% | 48.4% | 21.6% | -0.3% | 68.0% |
| Greedy Priorities | 18.2% | 4555.0 | 7.2% | 2.56 | 45.5 | 15.7% | 48.4% | 20.7% | -2.5% | 67.3% |
| Optimal Assignment | 12.2% | 4738.8 | 4.0% | 2.62 | 45.3 | 15.9% | 47.8% | 16.2% | -4.0% | 98.0% |
| 3.5% | 5075.1 | 0.3% | 2.78 | 44.7 | 16.3% | 47.0% | 8.7% | -5.2% | 79.7% |
| Optimal Assignment | 34.3% | 3990.5 | 13.6% | 2.38 | 45.5 | 15.6% | 48.3% | -- | -- | 94.0% |

Panel C: Predictions Assuming no Equilibrium Response

| Pre-2014 Priorities | -- | 5791.2 | -- | 3.39 | 43.8 | 17.2% | 44.6% | -- | -- | -- |
| Post-2014 Priorities | -- | 5753.6 | 0.0% | 3.37 | 43.8 | 17.2% | 44.7% | -- | -- | -- |
| First Come First Served | -- | 5853.7 | -0.1% | 3.33 | 43.8 | 17.3% | 44.5% | -- | -- | -- |
| Last Come First Served | -- | 5668.6 | 2.7% | 5.99 | 44.2 | 16.6% | 45.3% | -- | -- | -- |
| Greedy Priorities | -- | 5708.8 | 1.0% | 3.54 | 43.9 | 17.0% | 44.9% | -- | -- | -- |

lower quality objects onto those lower on the list. In fact, Su and Zenios (2004) show that, when preferences are identical, LCFS forces agents to internalize this negative externality, reducing selectivity and waste, thereby improving upon FCFS.\(^{39}\) The differences in selectivity can induce organ waste, particularly when there is a limit on either the number of patients that are compatible with an organ or the number of offers that can be made before an organ is unviable. But, even in a long list with no limits on the number of offers, the patient in the lowest position is more selective under FCFS than under LCFS because the patient’s position improves over time in the former mechanism, but not the latter.

Our empirical results indicate that horizontal preference heterogeneity is sufficiently strong that the positive externality from rejecting an offer outweighs the negative externality due to possible organ waste. The higher match quality in FCFS relative to LCFS that is driven by higher patient selectivity far outweighs the loss due to additional organ discards. However, as we show in the next section, neither mechanism optimally balances these two forces. Incorporating rich patient and donor characteristics into the design of the mechanism can reduce discards without substantially reducing match quality.

Finally, it is worth noting that our computed steady state queue length for the pre-2014 mechanism is 5,113, which is a little larger than the queue length of 4,508 on January 1,\(^{39}\)Observe that when all agents place an identical value on each object, social surplus is a function only of waste.
2013 (Table 1, panel A). This similarity is remarkable because this moment of the data is not targeted in the CCP approach but matches equilibrium computations from our model and because it suggests that our sample is likely close to the steady state.\footnote{A slightly longer steady state should be expected as the kidney waiting list was growing during our sample period. But, panel A in Table 1 shows that the rate of growth in the length of the waiting list in NYRT declined during our sample period.}

## 7.2 Optimal Mechanisms

We now turn our attention to designing mechanisms that aim to improve welfare, but are subject to varying constraints. We start by giving the designer full flexibility to choose whom to offer a kidney based on the characteristics observed in our data. The designer cannot dictate assignments or condition offers on past actions.\footnote{This restriction is motivated by a need to respect patient and doctor discretion and the concern that forcing assignments or penalizing terminally ill patients may be politically infeasible.}

### • Optimal Offer Rates:

We solve for offer rates $\pi_{xz}(t)$ that maximize $\sum_x \frac{\bar{V}_x^\pi(x_0)}{\bar{V}_x^{x_0}(x_0)}$, where, with a slight abuse of notation, $\bar{V}_x^\pi(x_0)$ is the equilibrium steady-state value for type $x$ under offer rates $\pi$. The offer rates are subject to feasibility constraints, so that the steady-state rate of assignment implied by offer and acceptance rates does not exceed the arrival rate of objects. Ignoring, for the moment, the limit on the maximum number of offers that can be made, for each donor type $z$, we require that

$$\sum_x \int_0^T Nm_x(t) \pi_{xz}(t) P(\Gamma_{xzt} + \varepsilon > V_x(t) | x, z) \, dt \leq q_z.$$ 

The left-hand side is the expected number of objects assigned under offer rates $\pi$ and the right-hand side is the total number of objects available. The mathematical problem we solve includes the limit on the number of offers for each donor and is formally described in Appendix C.2.2.

Because this constraint is placed only on expected quantities, the offer rates may not be implementable for any particular instance of objects of type $z$. The solution provides an upper bound on welfare under any offer mechanism that does not condition on past behavior.

### • Approximately Optimal Priorities:

We can use the solution of the problem above to implement very similar equilibrium offer rates using a scoring rule. Specifically, we set
$s_{xz}(t)$ to the values of $\pi_{xz}(t)$ that solve the problem above. The Kidney Transplantation Committee exclusively focused on mechanisms that used priority rules to offer organs. Such mechanisms are simpler to describe and can be implemented using the existing organ allocation infrastructure. We compute an equilibrium for these priorities using the algorithm outlined in Appendix C.

- **Approximately Optimal Pareto Improving Priorities:** We design a priority system that aims to improve the welfare of the average patient without significantly hurting any patient type. To do this, we use a procedure similar to the one used to derive approximately optimal priorities. Specifically, we first solve for offer rates that are defined identically to the optimal offer rates except that they are constrained to make no agent type worse off than under pre-2014 priorities at the time they join the waitlist. We then solve for the equilibrium allocation under a scoring function $s_{xz}(t)$ that is equal to the values of $\pi_{xz}(t)$ that are the solution to this problem. Although the resulting mechanism may not result in a strict Pareto improvement, we expect that fewer agents will be significantly worse off under these priorities. Our solutions will allow us to quantify these effects.

We find that previously used mechanisms and both FCFS and LCFS are far from optimal; priorities designed using estimated preferences as inputs can substantially increase welfare while also reducing discard rates via a decrease in organ discard rates. Patient welfare under the approximately optimal priorities mechanism increases by 18.2 percent, and discard rates decrease by 7.2 percent (Table 6). Because patients are transplanted at higher rates, they spend less time on the waiting list, and queue lengths fall to below 4,600. In fact, this mechanism achieves most of the gains possible under any offer mechanism. The optimal offer rates, which place an upper bound on welfare from any offer mechanism, performs only marginally better: it increases patient welfare by 21.4 percent. While these gains are large, one drawback of approximately optimal priorities is that some agents are significantly worse off: nearly one-third of patient types are more than 5 percent worse off at registration than they were under the pre-2014 mechanism.

A significant fraction of these gains can be achieved while respecting distributional constraints. The approximately optimal Pareto improving priorities mechanism increases welfare by 12.2 percent and decreases discard rates by 4.0 percent. As under approximately optimal priorities, this mechanism increases welfare through a combination of lower discard rates and improved match quality. Recall that because this mechanism approximates the optimal Pareto improving offer rates with a scoring rule, constraint that no type should be worse off may not be exactly satisfied. Fortunately, 98 percent of patients are no more than 5 percent worse off at registration under our approximation than they were under pre-2014 priorities. Thus, the very strong requirement that no patient type be worse off ex-ante can be approximately satisfied while achieving a substantial improvements in outcomes.
7.3 Incentives, Potential Improvements and Sources of Gains

7.3.1 Importance of Incentives

We now show two ways in which explicitly considering incentives in a waitlist offer mechanism is quantitatively important for arriving at solutions described above. First, a naive mechanism that offers organs in order of predicted value of the transplant does not come close to the solutions described above. To formalize this point, consider the following mechanism:

- **Greedy Priorities:** Patients are prioritized in order of predicted match value. For each donor, it divides patients into 20 equally sized bins based on our predicted welfare gain from assignment, measured in donor supply units ($EV_{xz}^t$). $s_{xz}^t$ is set to 20 for those in the highest bin and to 1 for those in the lowest bin.

The equilibrium under this mechanism only marginally improves patient welfare and discard rates relative to the pre-2014 mechanism (Figure 2). The small difference relative to approximately optimal priorities emphasizes the value of explicitly incorporating incentive constraints into the mechanism design problem: both mechanisms incorporate the rich estimated heterogeneity in preferences, but greedy priorities does not consider incentives.

Second, empirical approaches that ignore incentives mute the differences in predicted equilibrium allocations across mechanisms. Panel C in Table 6 shows that changes in predicted discard rates, queue lengths, and donor characteristics are similar across the various priority mechanisms considered above when acceptance probabilities do not adjust to the new equilibrium. This finding suggests that the KPSAM module (see SRTR, 2015) used to advise the Kidney Transplantation Committee on allocation reform should be modified to incorporate incentives.

7.3.2 Comparison with Maximum Possible Gains

While the mechanisms described above improve both welfare and organ discards, it is unclear whether they achieve most of the possible gains. Specifically, we have only considered indirect mechanisms that do not condition offers on past decisions. To analyze the potential scope for further improvements we calculate an upper bound on possible gains if organs can be assigned by fiat under full information on preferences and rational expectations (but no foresight) about object arrival and agent departure processes:

- **Optimal Assignments:** We maximize the steady-state average welfare by choosing a policy $a(\varepsilon; x, z, t) \in \{0, 1\}$ that assigns objects to agents as a function of the agent type, the object type, time waited, and the preference shocks $\varepsilon$ for all agents that are
currently waiting. The feasibility constraints ensure that the total rate of assignment does not exceed the total arrival rate of objects:

\[ \sum_x \int_0^T N_{m_x}(t) P(c = 1, a = 1|x, z, t) \, dt \leq q_z. \]

The term \( P(c = 1, a = 1|x, z, t) \) is the probability that an object of type \( z \) is compatible \((c = 1)\) and assigned \((a = 1)\) to a randomly chosen agent of type \( x \) that has waited for time \( t \). In addition, we require that the composition of the waitlist be in steady state (Definition 1, part 3). These constraints are described in detail in Appendix C.2.1.

The solution to this problem suggests that approximately optimal priorities achieves over one-half of the possible gains from redesigning the organ allocation system. Optimal assignments yield welfare that is higher than the pre-2014 outcome by an equivalent of a 34.3 percent increase in donor supply (Table 6). At the same time, discard rates for patients in NYRT would decrease by 13.6 percent under this outcome. However, not all patients are better off even in this benchmark case; there are trade-offs between efficiency and distributional objectives. Moreover, the optimal assignment benchmark may be far from achievable by a mechanism because full information and the ability to dictate assignments are extreme assumptions.

### 7.3.3 Sources of Welfare Gains

The gains identified in approximately optimal priorities arise from offering organs to patients who have a high measured match-specific value for them. These patients are also more likely to accept the offers, which has the added benefit of simultaneously reducing discard rates if the limit on the number of possible offers binds.\(^{42}\) Reducing discards can increase expected welfare if match quality is not sacrificed.

This intuition can be illustrated using the assignment of patients by age and dialysis status at the time of registration under various mechanisms. As we discussed in section 5, off-dialysis patients not only value each offer more, but are also more likely to accept an offer. Similarly, we also showed that older patients receive a higher value from each offer.

Approximately optimal priorities increases transplate rates for older patients and reallocates desirable donors from patients on dialysis to patients off dialysis at registration (Table 7). As discussed in section 5.2, older patients prefer organs of all types, and therefore inducing lower

\(^{42}\)Targetting offers based on observed patient and donor characteristics is also likely to result in high values of the transplant inclusive of the idiosyncratic shock. Formally, if the distribution of the idiosyncratic shocks \( \epsilon \) is log-concave and the continuation value is identical for all agents, then both the acceptance probability and the value of transplant conditional on acceptance is increasing in the measured value of a transplant net of the idiosyncratic shock \( \epsilon \).
Table 7: Sources of Welfare Gains

<table>
<thead>
<tr>
<th>Welfare Change</th>
<th>EV, Decomp.</th>
<th>Transplants by Donor Type</th>
<th>Young NYRT Offer</th>
</tr>
</thead>
<tbody>
<tr>
<td>ΔV,0 (0)</td>
<td>EV,</td>
<td>Obs.</td>
<td>Unobs.</td>
</tr>
</tbody>
</table>

Panel A: Patients not on Dialysis at Registration, Age 0-49

<p>| | | | | | | | | | |</p>
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</thead>
<tbody>
<tr>
<td>Pre-2014 Priorities (Baseline)</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>19.4</td>
<td>24.1</td>
<td>49.3</td>
<td>4.0</td>
<td>1.58</td>
<td></td>
</tr>
<tr>
<td>Greedy Priorities</td>
<td>-0.8%</td>
<td>-1.4%</td>
<td>0.4%</td>
<td>1.8%</td>
<td>19.2</td>
<td>11.5</td>
<td>48.5</td>
<td>6.8</td>
<td>3.63</td>
</tr>
<tr>
<td>Approximately Optimal Priorities</td>
<td>9.8%</td>
<td>4.0%</td>
<td>1.5%</td>
<td>2.5%</td>
<td>15.8</td>
<td>26.8</td>
<td>68.1</td>
<td>4.7</td>
<td>1.63</td>
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<td>Approx. Opt. Pareto Improving Priorities</td>
<td>4.6%</td>
<td>2.3%</td>
<td>2.2%</td>
<td>0.1%</td>
<td>16.9</td>
<td>25.3</td>
<td>58.8</td>
<td>3.4</td>
<td>1.75</td>
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</tbody>
</table>

Panel B: Patients not on Dialysis at Registration, Age 50+

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</tr>
</thead>
<tbody>
<tr>
<td>Pre-2014 Priorities (Baseline)</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>8.3</td>
<td>31.9</td>
<td>55.9</td>
<td>1.5</td>
<td>7.78</td>
<td></td>
</tr>
<tr>
<td>Greedy Priorities</td>
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<td>17.1%</td>
<td>37.5%</td>
<td>-20.4%</td>
<td>22.3</td>
<td>24.0</td>
<td>72.6</td>
<td>4.2</td>
<td>4.64</td>
</tr>
<tr>
<td>Approximately Optimal Priorities</td>
<td>105.9%</td>
<td>72.7%</td>
<td>89.2%</td>
<td>-16.5%</td>
<td>25.2</td>
<td>62.1</td>
<td>79.7</td>
<td>10.7</td>
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<tr>
<td>Approx. Opt. Pareto Improving Priorities</td>
<td>69.1%</td>
<td>49.9%</td>
<td>68.0%</td>
<td>-18.1%</td>
<td>15.9</td>
<td>46.6</td>
<td>60.2</td>
<td>3.8</td>
<td>3.17</td>
</tr>
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</table>

Panel C: Patients on Dialysis at Registration, Age 0-49

<p>| | | | | | | | | | |</p>
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<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Pre-2014 Priorities (Baseline)</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>16.4</td>
<td>42.9</td>
<td>81.9</td>
<td>2.0</td>
<td>2.99</td>
<td></td>
</tr>
<tr>
<td>Greedy Priorities</td>
<td>-1.9%</td>
<td>-2.7%</td>
<td>-3.7%</td>
<td>1.0%</td>
<td>23.7</td>
<td>25.6</td>
<td>113.5</td>
<td>7.3</td>
<td>3.90</td>
</tr>
<tr>
<td>Approximately Optimal Priorities</td>
<td>-5.9%</td>
<td>-7.1%</td>
<td>-10.4%</td>
<td>3.3%</td>
<td>2.9</td>
<td>22.3</td>
<td>117.3</td>
<td>0.7</td>
<td>6.69</td>
</tr>
<tr>
<td>Approx. Opt. Pareto Improving Priorities</td>
<td>-0.5%</td>
<td>-2.5%</td>
<td>-3.5%</td>
<td>1.0%</td>
<td>13.1</td>
<td>30.9</td>
<td>105.1</td>
<td>1.9</td>
<td>1.83</td>
</tr>
</tbody>
</table>

Panel D: Patients on Dialysis at Registration, Age 50+

<p>| | | | | | | | | | |</p>
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<thead>
<tr>
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</tr>
</thead>
<tbody>
<tr>
<td>Pre-2014 Priorities (Baseline)</td>
<td>--</td>
<td>--</td>
<td>--</td>
<td>20.4</td>
<td>81.8</td>
<td>130.2</td>
<td>1.2</td>
<td>6.78</td>
<td></td>
</tr>
<tr>
<td>Greedy Priorities</td>
<td>2.1%</td>
<td>1.4%</td>
<td>2.5%</td>
<td>-1.1%</td>
<td>28.0</td>
<td>34.3</td>
<td>188.2</td>
<td>5.7</td>
<td>3.82</td>
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<tr>
<td>Approximately Optimal Priorities</td>
<td>14.9%</td>
<td>7.4%</td>
<td>7.2%</td>
<td>0.2%</td>
<td>18.2</td>
<td>83.2</td>
<td>197.1</td>
<td>2.6</td>
<td>1.68</td>
</tr>
<tr>
<td>Approx. Opt. Pareto Improving Priorities</td>
<td>7.3%</td>
<td>3.0%</td>
<td>3.5%</td>
<td>-0.5%</td>
<td>18.0</td>
<td>89.2</td>
<td>169.8</td>
<td>1.7</td>
<td>2.51</td>
</tr>
</tbody>
</table>

Notes: Transplants by Donor Type is an annual rate. Prob. (%) is the probability that a patient receives an offer from a donor in the relevant group conditional on the donor being offered to some patients. Years Waited is the mean years a patient waited across offers from that donor group. An Old NYRT Donor is over age 35, an Expanded Criteria Donor, or had a cardiac death.

discard rates for them improves welfare. For example, under approximately optimal priorities, transplants from young NYRT donors to older off-dialysis patients is more than three times larger than under the baseline case (Panel B). Meanwhile, on-dialysis patients (Panels C and D) receive fewer kidneys each year from young healthy donors. The distributional goals of the approximately optimal Pareto improving priorities result in less stark, but similar, patterns to those in the unconstrained approximately optimal priorities.

Concerns with using such fine-tuned mechanisms include the possibility that agents may try to game the system by manipulating their characteristics and that such systems may be complicated to understand. While addressing these issue is beyond the scope of this paper, we make a few observations about the former issue. First, all of the patient and donor characteristics we use to determine priority were used in either the pre-2014 mechanism, the post-2014 mechanism, or in a proposal considered by the Kidney Transplantation Committee. Second, if gaming on the use of medical treatments such as delaying dialysis is of concern, then a close substitute such as measured kidney function could be used instead. This limits the potential scope of gaming to submitting fraudulent medical records.
8 Conclusion

Empirical approaches for evaluating dynamic assignment systems are particularly important because theory has not provided sharp guidance on the optimal design of these mechanisms. This paper provides a new equilibrium empirical framework for evaluating dynamic assignment systems that explicitly incorporates dynamic incentives, and applies the framework to the allocation of deceased donor kidneys. Previous reforms of organ allocation systems have been assisted by a simple simulation model that does not account for the dependence of accept/reject rules on agents’ incentives under various mechanisms.

Our findings show that there is significant scope for improving the deceased donor kidney allocation mechanism. Previously used mechanisms and theoretical benchmarks are only able to reduce organ discards at the cost of substantially lowering patient welfare. In contrast, estimates of the value of transplants as a function of observables can be used to identify mechanisms that both reduce discards and improve patient welfare. Moreover, these goals can be achieved while avoiding significant harm to any type of patient. Accounting for changes in agents’ incentives is essential for designing systems that achieve these large gains, and for accurately predicting outcomes under alternative mechanisms. These findings demonstrate the large scope for empirical work to inform the design of dynamic mechanisms more broadly than in the context of deceased donor kidney allocation.

While the planner’s objective function may differ from the ones we consider, our methods can inform design once an objective and a set of acceptable mechanisms has been specified. It requires the analyst to specify a model of behavior and estimate primitive payoffs that are assumed to be stable following the mechanism change.\(^\text{43}\) The approach can be used to both evaluate outcomes from specific proposed mechanisms and to find optimal solutions.

We make several simplifying assumptions that motivate future research. First, we assume that agent beliefs condition on a limited set of variables and we analyze steady state equilibria. Second, we do not allow for patient-level unobserved heterogeneity. Including this feature, potentially with time-varying components, may be important when applying these methods to other empirical contexts or data environments. Third, we assume that arrival rates of patients and objects are exogenous, an assumption that may be particularly important to relax in other applications. Finally, we primarily focus on the outcomes of organ waste and overall welfare. Clinical outcomes, such as life-years gained from transplantation and graft failure rates, are likely important to policy makers.

\(^{43}\)During estimation, we assume that agents do not behave strategically in an attempt to influence a future change in the mechanism. This concern is important only if agents foresee that the designer may use data on decisions to change the mechanism, the number of agents is small and each agent is long-lived. Otherwise, agents should not expect their decision to substantially effect their allocation through a change in the mechanism induced by their influence on estimated parameters. While we cannot directly speak to the first requirement, the later two conditions are not satisfied in the deceased donor kidney allocation context because patients are numerous and have low life-expectancies without a transplant.
References


Nikhil Agarwal, Itai Ashlagi, Michael Rees, Paulo Somaini, Daniel Waldinger.

A Estimation

A.1 Normalization

In this section, we show that the model described in Section 3.2.2 yields the same decision-rules as a model in which \( O_i(t) = 0 \) for all \( t \) (therefore, \( d_i(t) + \delta_i(t) D_i(t) = 0 \) for all \( t \)) and the net present value of a transplant is \( \tilde{\Gamma}_{ij}(t) = \Gamma_{ij}(t) - O_i(t) \). To do this, the next proposition establishes two results which hold for any assignment rule that does not affect the value of being on dialysis or departing without a deceased donor transplant.

Proposition 2. Let \( p_{ij}(t) \) be the conditional probability that \( i \) is assigned object \( j \) given that \( j \) arrives in period \( t \). Let \( V_i(t; p) \) be \( i \)'s value of the assignment rule \( p \). Then,

1. For any assignment rule \( p \), \( \Gamma_{ij}(t) - V_i(t; p) = \tilde{\Gamma}_{ij}(t) - \tilde{V}_i(t; p) \), where \( \tilde{\Gamma}_{ij}(t) = \Gamma_{ij}(t) - O_i(t) \) and\( (\rho + \delta_i(t)) \tilde{V}_i(t; p) = \lambda \int p_{ij}(t) (\tilde{\Gamma}_{ij}(t) - \tilde{V}_i(t; p)) \, dF + \tilde{V}_i(t; p) \)

with boundary condition \( \tilde{V}_i(T_i; p) = 0 \).

2. For any two assignment rules \( p \) and \( p' \), \( V_i(t; p) - V_i(t; p') = \tilde{V}_i(t; p) - \tilde{V}_i(t; p') \)

Proof. Part 1: First, we verify that \( \tilde{V}_i(t; p) = V_i(t; p) - O_i(t) \) satisfies the differential equation above. Note that

\[
(\rho + \delta_i(t)) V_i(t; p) = d_i(t) + \delta_i(t) D_i(t) + \lambda \int p_{ij}(t) (\Gamma_{ij}(t) - V_i(t; p)) \, dF + \tilde{V}_i(t; p) .
\]

Therefore,

\[
(\rho + \delta_i(t)) (V_i(t; p) - O_i(t)) = \lambda \int p_{ij}(t) (\tilde{\Gamma}_{ij}(t) - (V_i(t; p) - O_i(t))) \, dF + \frac{\partial}{\partial t} (V_i(t; p) - O_i(t)) .
\]

Hence, \( \tilde{V}_i(t; p) \) satisfies the necessary differential equation. It is straightforward to check that \( V_i(T_i; p) = O_i(T_i) = D_i(T_i) \) showing that the solution with the boundary condition \( \tilde{V}_i(T_i; p) = 0 \) satisfies the requirements of the proposition.
Part 2: Observe that for any pair of assignment rules, $p$ and $p'$,

$$V_i(t; p) - V_i(t; p') = \lambda \int p_{ij}(t) \left( \Gamma_{ij}(t) - V_i(t; p) \right) dF + \tilde{V}_i(t; p) - \tilde{V}_i(t; p')$$

$$\begin{align*}
&= \lambda \int p_{ij}(t) \left( \left( \tilde{\Gamma}_{ij}(t) - O_i(t) \right) - \left( \tilde{V}_i(t; p) - O_i(t) \right) \right) dF \\
&\quad + \left( \tilde{V}_i(t; p) - \tilde{O}_i(t) \right) - \left( \tilde{V}_i(t; p') - \tilde{O}_i(t) \right) \\
&= \tilde{V}_i(t; p) - \tilde{V}_i(t; p').
\end{align*}$$

\[ \square \]

Refer to the model with $O_i(t) = 0$ for all $t$ and the related value function $\tilde{V}_i(t; p)$ and the payoffs $\tilde{\Gamma}_{ij}(t)$ as the normalized model. Part 1 shows that the normalized model also yields $\Gamma_{ij}(t) - V_i(t; p)$ as the difference in the value of accepting $j$ relative to the value waiting if one expects assignments according to $p$. In particular, the result holds for $p_{ij}(t) = \pi_{ij}(t) 1 \{ \Gamma_{ij}(t) - V_i(t) > 0 \}$. Therefore, the normalized model yields an identical choice rule and value function relative to no assignment.

Part 2 shows that the normalized model yields an identical difference in value functions between any two assignment rules as the original model. To see this, consider any action space $A_i$ and strategy $\sigma_i(t; j) \rightarrow A_i$. As long as the analyst can then evaluate the assignment rule $p_{ij}(t; \sigma)$ as a function of the strategy profile $\sigma = (\sigma_i, \sigma_{-i})$, the result says that the normalized model can be used to determine the difference in values. To solve for equilibria, we can evaluate deviations $(\sigma'_i, \sigma_{-i})$ and compare

$$V_i(t; p(\sigma'_i, \sigma_{-i})) - V_i(t; p(\sigma_i, \sigma_{-i})) = \tilde{V}_i(t; p(\sigma'_i, \sigma_{-i})) - \tilde{V}_i(t; p(\sigma_i, \sigma_{-i})).$$

To identify the value function relative to the current mechanism, we can therefore compute

$$V_i(t; p(\sigma^*)) - V_i(t; \hat{p}) = \tilde{V}_i(t; p(\sigma^*)) - \tilde{V}_i(t; \hat{p}),$$

where $\hat{p}$ denotes the assignment probabilities under the factual mechanism and $p(\sigma^*)$ denotes the equilibrium assignment probabilities in an equilibrium of the counterfactual mechanism.

### A.2 Details on the Estimator

**Gibbs’ Sampler**

Define $y_{ijt} = V(x_i, t) - X(x_i, z_j, \eta_j, t) - \epsilon_{ijt} = \chi(x_i, z_j, \eta_j)\theta - \epsilon_{ijt}$ and $a_{ijt} = 1 \{ y_{ijt} < 0 \}$.

The sampler is initialized at any value of $\theta^0, \sigma^0_i$ and guesses for $\eta_i^0$ and $\eta_i^0$ corresponding to observed decisions such that $y_{ijt}^0 \geq 0$ if and only if agent $i$ rejected object $j$ in period $t$. We
then sample from the conditional posteriors and draws of $y$ given the previous draws. The sampler iterates through the following sequence

\[
\begin{align*}
  y_{ijt}^{s+1} &| \theta^s, \eta^s, a_{ijt} \\
  \eta_j^{s+1} &| y_{ijt}^{s+1}, \theta^s, \sigma^s \eta \\
  \theta^{s+1} &| y^{s+1}, \eta^{s+1} \\
  \sigma^s_{\eta}^{s+1} &| \eta^{s+1}, \sigma^s \\
\end{align*}
\]

(11)

where the conditioning on the priors and the observables is implicit, $y^s$ and $\eta^s$ are vectors with components $y_{ijt}^s$ and $\eta_j^s$, and $y_{ijt}^{s+1}$ is a vector that stacks $y_{ijt}^s$ across all $i, t$. The first two steps involve data augmentation to simplify the sampling problem of the key parameters in the next step. Each of these steps involves draws from a closed-form distribution if the prior distribution on $\sigma^s \eta$ is specified as an inverse-Gamma distribution and the prior for $\theta \sim N(\bar{\theta}, \Sigma_\theta)$. With these priors, the first step involves sampling from a truncated normal, the second and third steps involve sampling from a normal distribution, and the final step involves sampling from an inverse-Gamma.

**Computing the Value Function**

Given $t$, for each patient $i$, the value of continuing is given by equation (6). Using equation (9), the sample analog of the value of continuing is given by

\[
\hat{V}_i(t) = \lambda \int_t^{T_i} \exp(-\rho(\tau - t)) p(\tau|t; x_i) \hat{W}(x_i, \tau; \hat{\theta}) d\tau.
\]

We numerically approximate this integral. First, we re-write $\hat{V}_i(t)$ as follows:

\[
\begin{align*}
  \hat{V}_i(t) &= \lambda \int_t^{T_i} \exp(-\rho(\tau - t)) p(\tau|t; x_i) \frac{1}{J} \sum_{j=1}^J 1 \left\{ c_{ij} = 1 \right\} 1 \left\{ s(\tau; x_i, z_j) > s^*_j \right\} \psi(\hat{P}_{ij\tau}) d\tau \\
  &= \lambda \frac{1}{J} \sum_{j=1}^J 1 \left\{ c_{ij} = 1 \right\} \int_t^{T_i} \exp(-\rho(\tau - t)) p(\tau|t; x_i) 1 \left\{ s(\tau; x_i, z_j) > s^*_j \right\} \psi(\hat{P}_{ij\tau}) d\tau \\
  &= \lambda \frac{1}{J} \sum_{j=1}^J 1 \left\{ c_{ij} = 1 \right\} \int_{\tau_{ijt}}^{T_i} \exp(-\rho(\tau - t)) p(\tau|t; x_i) \psi(\hat{P}_{ij\tau}) d\tau,
\end{align*}
\]

where $\tau_{ijt} = \inf \left\{ \tau > t : s(\tau; x_i, z_j) > s^*_j \right\}$, with $\tau_{ijt} = T_i$ if $s(\tau; x_i, z_j) < s^*_j$ for all $\tau \leq T_i$.

For each $i$ and $j$, we approximate the integral above using $B = 40$ equally spaced points $q^b = \frac{b}{B+1}$ for $b = 1, \ldots, B$ on the unit interval. Let $\tau_{ijt}^b = F^{-1}(q^b; \rho, \tau_{ijt}, T_i)$ where $F(\cdot; \rho, \tau_{ijt}, T_i)$ is the cumulative distribution function of an exponential random variable with parameter $\rho$ that is truncated between $\tau_{ijt}$ and $T_i$. We therefore compute the value
function as

\[ \hat{V}_i(t) = \frac{\lambda}{\rho} \sum_{j=1}^{J} 1\{c_{ij} = 1\} \sum_{b=1}^{B} p(\tau_{ij}^b | t; x_i) \psi(\hat{P}_{ij}^b). \]

This procedure ensures that there are \( B \) points of evaluation for each possible donor and patient-time pair. The numerical performance is superior to an alternative that approximates the integral in equation (6) as a sum over a fixed set of draws because some patient, donor, time combinations may have a very small window of availability, \( [\tau_{ijt}, T_i] \).

### A.3 Auxiliary Models

#### Positive Crossmatch Probability

Table A.1: Positive Crossmatch Model

<table>
<thead>
<tr>
<th>Dependent Variable: Positive Crossmatch</th>
<th>1.025 (0.152)</th>
</tr>
</thead>
<tbody>
<tr>
<td>CPRA</td>
<td>-1.374 (0.474)</td>
</tr>
<tr>
<td>0 or 1 HLA Mismatches</td>
<td>0.199 (0.0856)</td>
</tr>
<tr>
<td>2 or 3 HLA Mismatches</td>
<td>-0.449 (0.0930)</td>
</tr>
<tr>
<td>0 DR Mismatches</td>
<td>-0.590 (0.684)</td>
</tr>
<tr>
<td>CPRA x 1{0 or 1 HLA Mismatches}</td>
<td>-0.477 (0.169)</td>
</tr>
<tr>
<td>CPRA x 1{2 or 3 HLA Mismatches}</td>
<td>-0.587 (0.0827)</td>
</tr>
<tr>
<td>CPRA 0</td>
<td>-3.389 (0.811)</td>
</tr>
<tr>
<td>CPRA - 0.8 if CPRA &gt; 0.8</td>
<td>-0.0325 (0.00846)</td>
</tr>
<tr>
<td>Log Dialysis Time at Registration (Years)</td>
<td>1.035 (0.0812)</td>
</tr>
<tr>
<td>Log Dialysis Time at Registration x 1{Over 5 Years}</td>
<td>0.0108 (0.00490)</td>
</tr>
<tr>
<td>Patient Age at Registration (Years)</td>
<td>-0.0272 (0.00628)</td>
</tr>
<tr>
<td>Age at Registration - 35 if Age &gt; 35</td>
<td>-0.254 (0.170)</td>
</tr>
<tr>
<td>Constant</td>
<td>3876</td>
</tr>
</tbody>
</table>

Notes: Coefficient estimates from a probit regression of positive crossmatch on patient CPRA, the number of tissue type mismatches, patient age, and years on dialysis at registration. The sample is all offers accepted by NYRT patients between 2010 and 2013. Positive crossmatches are identified by the appropriate refusal code in the PTR data. CPRA is measured on a [0,1] scale.

Not all accepted offers result in transplantation because additional testing may yield a positive crossmatch indicating that the patient is likely to develop an immune response to the donor’s kidney. These transplants are not carried out, and if possible the organ is placed with another patient. To account for positive crossmatches when computing value functions and conducting counterfactual simulations, we estimate a probit model to predict the probability that a patient has a positive crossmatch with an organ they have accepted. The specification
includes interactions between the patient’s CPRA and the number of HLA mismatches with
the donor, in addition to controls for patient age and number of years on dialysis. We use a
subset of the variables included in the CCP model to avoid overfitting. Coefficient estimates
and standard errors are displayed in Table A.1. The results are intuitive and consistent
with medical knowledge. For example, higher CPRA is associated with a higher positive
crossmatch probability, as are more tissue-type dissimilarities (as measured by DR or HLA
mismatches). This is consistent with the view that patients with more sensitized immune
systems may be more likely to test positive against foreign antibodies, even if they have not
tested positive in the past.

Maximum Number of Offers and Discards

In our setting, some organs may not be offered to all compatible patients in NYRT. This
usually occurs for two reasons. First, the organ may become unsuitable for transplantation if
it remains outside donor’s body for too long. Second, the organ may be accepted by a patient
in another OPO. We call these events “timeouts.” Timeouts are driven by a combination of
factors, including whether the organ remained in the donor’s body during the offer process;
the rate at which offers were made, which depends on patient/surgeon response times and
the number of patients simultaneously contacted; and decisions of patients outside NYRT.

We model the maximum number of offers that can be made for a given organ using a censored
exponential hazards model. Duration is the number of observed offers. Censoring occurs if
the organ is placed, or if it is discarded after being offered to all compatible patients. The
hazard function is given by

$$
\lambda_o (z) = \lambda_o \exp (z \beta)
$$

where $z$ are characteristics of the donor, $\beta$ is a vector of coefficients, and $\lambda_0$ is the constant
baseline hazard rate. We allow the timeout hazard to depend on geography and indicators
of donor quality. Specifically, we control for whether the donor is an expanded criteria donor
(ECD); the donor’s cause of death (DCD); and whether the donor was recovered in NYRT,
as well as interactions among these variables. The estimated timeout hazards are inputs in
the counterfactual exercises.

In addition, we model the probability that a donor’s unallocated kidneys are discarded after
the maximum number of offers has been reached using a probit model. The model is estimated
by tracking whether the kidneys were ultimately transplanted or not. With the remaining
probability, the donor’s kidneys are allocated to a patient not registered in NYRT. This
probit model includes the identical set of covariates used to model the maximum number of
offers that can be made. This part of the model does not influence allocation and incentives
for patients in NYRT. It is used to properly account for changes in discards for kidneys not
allocated to patients in NYRT.
### Table A.2: Survival Model Estimates

<table>
<thead>
<tr>
<th></th>
<th>Gompertz (1)</th>
<th>Weibull (2)</th>
<th>Cox (3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Diabetic Patient</td>
<td>0.0812</td>
<td>0.0739</td>
<td>0.0850</td>
</tr>
<tr>
<td></td>
<td>(0.0336)</td>
<td>(0.0336)</td>
<td>(0.0336)</td>
</tr>
<tr>
<td>Bloodtype A Patient</td>
<td>0.159</td>
<td>0.127</td>
<td>0.165</td>
</tr>
<tr>
<td></td>
<td>(0.0437)</td>
<td>(0.0436)</td>
<td>(0.0438)</td>
</tr>
<tr>
<td>Bloodtype O Patient</td>
<td>0.00394</td>
<td>0.00400</td>
<td>0.00385</td>
</tr>
<tr>
<td></td>
<td>(0.0392)</td>
<td>(0.0392)</td>
<td>(0.0392)</td>
</tr>
<tr>
<td>Calculated Panel Reactive Antibodies (CPRA)</td>
<td>-0.000126</td>
<td>-0.000211</td>
<td>-0.000275</td>
</tr>
<tr>
<td></td>
<td>(0.00150)</td>
<td>(0.00150)</td>
<td>(0.00150)</td>
</tr>
<tr>
<td>CPRA = 0</td>
<td>0.190</td>
<td>0.179</td>
<td>0.181</td>
</tr>
<tr>
<td></td>
<td>(0.0738)</td>
<td>(0.0738)</td>
<td>(0.0739)</td>
</tr>
<tr>
<td>CPRA - 80 if CPRA &gt;= 80</td>
<td>-0.0230</td>
<td>-0.0204</td>
<td>-0.0225</td>
</tr>
<tr>
<td></td>
<td>(0.00650)</td>
<td>(0.00650)</td>
<td>(0.00650)</td>
</tr>
<tr>
<td>Age (at Registration)</td>
<td>-0.0418</td>
<td>-0.0363</td>
<td>-0.0361</td>
</tr>
<tr>
<td></td>
<td>(0.0150)</td>
<td>(0.0151)</td>
<td>(0.0151)</td>
</tr>
<tr>
<td>Age - 18 if Age &gt;= 18</td>
<td>0.0399</td>
<td>0.0356</td>
<td>0.0348</td>
</tr>
<tr>
<td></td>
<td>(0.0184)</td>
<td>(0.0186)</td>
<td>(0.0186)</td>
</tr>
<tr>
<td>Age - 35 if Age &gt;= 35</td>
<td>-0.00988</td>
<td>-0.0121</td>
<td>-0.0104</td>
</tr>
<tr>
<td></td>
<td>(0.00966)</td>
<td>(0.00966)</td>
<td>(0.00966)</td>
</tr>
<tr>
<td>Age - 50 if Age &gt;= 50</td>
<td>0.0236</td>
<td>0.0231</td>
<td>0.0242</td>
</tr>
<tr>
<td></td>
<td>(0.00729)</td>
<td>(0.00728)</td>
<td>(0.00729)</td>
</tr>
<tr>
<td>Age - 65 if Age &gt;= 65</td>
<td>0.0241</td>
<td>0.0233</td>
<td>0.0238</td>
</tr>
<tr>
<td></td>
<td>(0.00927)</td>
<td>(0.00926)</td>
<td>(0.00929)</td>
</tr>
<tr>
<td>Prior Transplant</td>
<td>0.0513</td>
<td>0.0590</td>
<td>0.0546</td>
</tr>
<tr>
<td></td>
<td>(0.0552)</td>
<td>(0.0550)</td>
<td>(0.0552)</td>
</tr>
<tr>
<td>Body Mass Index (BMI)</td>
<td>-0.0155</td>
<td>-0.0145</td>
<td>-0.0156</td>
</tr>
<tr>
<td></td>
<td>(0.00639)</td>
<td>(0.00639)</td>
<td>(0.00640)</td>
</tr>
<tr>
<td>Missing BMI</td>
<td>-0.0680</td>
<td>0.0736</td>
<td>-0.104</td>
</tr>
<tr>
<td></td>
<td>(0.199)</td>
<td>(0.199)</td>
<td>(0.200)</td>
</tr>
<tr>
<td>BMI &gt;= 18.5</td>
<td>-0.0382</td>
<td>-0.0450</td>
<td>-0.0356</td>
</tr>
<tr>
<td></td>
<td>(0.106)</td>
<td>(0.106)</td>
<td>(0.106)</td>
</tr>
<tr>
<td>BMI &gt;= 25</td>
<td>0.00882</td>
<td>0.00346</td>
<td>0.00918</td>
</tr>
<tr>
<td></td>
<td>(0.0492)</td>
<td>(0.0492)</td>
<td>(0.0492)</td>
</tr>
<tr>
<td>BMI &gt;= 30</td>
<td>0.0509</td>
<td>0.0429</td>
<td>0.0513</td>
</tr>
<tr>
<td></td>
<td>(0.0595)</td>
<td>(0.0595)</td>
<td>(0.0595)</td>
</tr>
<tr>
<td>Total Serum Albumin</td>
<td>-0.163</td>
<td>-0.160</td>
<td>-0.156</td>
</tr>
<tr>
<td></td>
<td>(0.0549)</td>
<td>(0.0550)</td>
<td>(0.0548)</td>
</tr>
<tr>
<td>Missing Total Serum Albumin</td>
<td>-0.533</td>
<td>-0.461</td>
<td>-0.490</td>
</tr>
<tr>
<td></td>
<td>(0.189)</td>
<td>(0.189)</td>
<td>(0.189)</td>
</tr>
<tr>
<td>Total Serum Albumin &gt;= 3.7</td>
<td>-0.0645</td>
<td>-0.0630</td>
<td>-0.0681</td>
</tr>
<tr>
<td></td>
<td>(0.0591)</td>
<td>(0.0592)</td>
<td>(0.0591)</td>
</tr>
<tr>
<td>Total Serum Albumin &gt;= 4.4</td>
<td>0.0512</td>
<td>0.0405</td>
<td>0.0505</td>
</tr>
<tr>
<td></td>
<td>(0.0510)</td>
<td>(0.0509)</td>
<td>(0.0510)</td>
</tr>
<tr>
<td>On Dialysis at Registration</td>
<td>-0.149</td>
<td>-0.169</td>
<td>-0.142</td>
</tr>
<tr>
<td></td>
<td>(0.113)</td>
<td>(0.113)</td>
<td>(0.113)</td>
</tr>
<tr>
<td>Log Years on Dialysis at Registration</td>
<td>-0.00139</td>
<td>0.00451</td>
<td>-0.00291</td>
</tr>
<tr>
<td></td>
<td>(0.0185)</td>
<td>(0.0185)</td>
<td>(0.0185)</td>
</tr>
<tr>
<td>Log Years on Dialysis at Registration x 1[Over 5 Years]</td>
<td>0.187</td>
<td>0.181</td>
<td>0.181</td>
</tr>
<tr>
<td></td>
<td>(0.110)</td>
<td>(0.110)</td>
<td>(0.110)</td>
</tr>
<tr>
<td>Constant</td>
<td>-5.870</td>
<td>-5.308</td>
<td>-5.566</td>
</tr>
<tr>
<td></td>
<td>(0.342)</td>
<td>(0.352)</td>
<td>(0.362)</td>
</tr>
<tr>
<td>Constant</td>
<td>0.00000922</td>
<td>0.0000210</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(0.0000210)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observations</td>
<td>9623</td>
<td>9623</td>
<td>9623</td>
</tr>
</tbody>
</table>
Figure A.1: Model Fit

(a) By Position

(b) By Waiting Time

Table A.3: Out-of-sample Model Validation

<table>
<thead>
<tr>
<th>Specification</th>
<th>Estimation Sample</th>
<th>Validation Sample</th>
</tr>
</thead>
<tbody>
<tr>
<td>Sparse Specification</td>
<td>87%</td>
<td>85%</td>
</tr>
<tr>
<td>Baseline Specification</td>
<td>81%</td>
<td>84%</td>
</tr>
<tr>
<td>Richer Specification</td>
<td>77%</td>
<td>91%</td>
</tr>
<tr>
<td>Richest Specification</td>
<td>73%</td>
<td>152%</td>
</tr>
</tbody>
</table>

Note: Validation sample includes offers made between January 1, 2014 and June 30, 2014. The relative mean squared error normalizes the MSE relative to a baseline estimator that predicts a constant CCP in each period. The sparse specification reduces the interactions and knots in the piecewise linear terms included in $\chi(\cdot)$ from our baseline specification so that we estimate about one fourth of the co-efficients. The richer specification increases the number of interactions and knots in the piecewise linear terms by a factor of four from the baseline, and the last specification further increases the number of terms by another factor of three.
## Table A.4: Conditional Choice Probability of Acceptance (Detailed)

<table>
<thead>
<tr>
<th></th>
<th>Base Specification</th>
<th>Unobserved Heterog.</th>
<th>Waiting Time + UH</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>(1)</td>
<td>(2)</td>
<td>(3)</td>
</tr>
<tr>
<td>Constant</td>
<td>-3.70 (0.02)</td>
<td>-4.47 (0.03)</td>
<td>-4.49 (0.05)</td>
</tr>
<tr>
<td>Patient Diabetic</td>
<td>-0.06 (0.01)</td>
<td>-0.05 (0.02)</td>
<td>-0.03 (0.02)</td>
</tr>
<tr>
<td>Calculated Panel Reactive Antibody (CPRA)</td>
<td>0.60 (0.05)</td>
<td>0.68 (0.06)</td>
<td>0.58 (0.09)</td>
</tr>
<tr>
<td>CPRA &gt; 0.8</td>
<td>0.27 (0.05)</td>
<td>0.10 (0.06)</td>
<td>0.12 (0.08)</td>
</tr>
<tr>
<td>CPRA = 0</td>
<td>-0.10 (0.02)</td>
<td>-0.02 (0.03)</td>
<td>-0.02 (0.03)</td>
</tr>
<tr>
<td>CPRA - 0.8 if CPRA &gt; 0.8</td>
<td>0.01 (0.03)</td>
<td>0.00 (0.11)</td>
<td>-0.04 (0.10)</td>
</tr>
<tr>
<td>Donor had Prior Transplant</td>
<td>0.38 (0.02)</td>
<td>0.36 (0.02)</td>
<td>0.14 (0.03)</td>
</tr>
<tr>
<td>Donor Age &lt; 18</td>
<td>0.27 (0.10)</td>
<td>-0.09 (0.19)</td>
<td>-0.04 (0.20)</td>
</tr>
<tr>
<td>Donor Age 18-35</td>
<td>0.59 (0.12)</td>
<td>-0.06 (0.19)</td>
<td>0.02 (0.19)</td>
</tr>
<tr>
<td>Donor Age 50+</td>
<td>0.01 (0.02)</td>
<td>0.02 (0.06)</td>
<td>0.04 (0.07)</td>
</tr>
<tr>
<td>Patient History of Hypertension</td>
<td>0.17 (0.09)</td>
<td>-0.16 (0.32)</td>
<td>-0.16 (0.36)</td>
</tr>
<tr>
<td>Donor Creatinine 0.5-1.0</td>
<td>0.06 (0.03)</td>
<td>0.02 (0.11)</td>
<td>-0.01 (0.11)</td>
</tr>
<tr>
<td>Donor Creatinine 1.0-1.5</td>
<td>0.01 (0.03)</td>
<td>0.00 (0.11)</td>
<td>-0.04 (0.10)</td>
</tr>
<tr>
<td>Donor Creatinine &gt;= 1.5</td>
<td>0.13 (0.03)</td>
<td>-0.21 (0.11)</td>
<td>-0.23 (0.11)</td>
</tr>
<tr>
<td>Donor Pancreas Offered</td>
<td>0.36 (0.03)</td>
<td>0.54 (0.09)</td>
<td>0.56 (0.09)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match</td>
<td>0.13 (0.02)</td>
<td>-0.53 (0.08)</td>
<td>-0.53 (0.10)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match Run</td>
<td>0.01 (0.01)</td>
<td>0.05 (0.05)</td>
<td>0.06 (0.04)</td>
</tr>
<tr>
<td>Patient Blood Type A</td>
<td>0.23 (0.31)</td>
<td>2.92 (0.43)</td>
<td>2.89 (0.44)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match</td>
<td>0.07 (0.01)</td>
<td>-0.28 (0.07)</td>
<td>-0.28 (0.07)</td>
</tr>
<tr>
<td>Perfect Tissue Type O</td>
<td>0.04 (0.01)</td>
<td>0.10 (0.01)</td>
<td>0.10 (0.01)</td>
</tr>
<tr>
<td>Donor Serum Albumin</td>
<td>0.05 (0.01)</td>
<td>-0.11 (0.01)</td>
<td>-0.01 (0.01)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match</td>
<td>0.01 (0.00)</td>
<td>0.02 (0.01)</td>
<td>0.02 (0.01)</td>
</tr>
<tr>
<td>Patient Age 18-35</td>
<td>0.00 (0.00)</td>
<td>0.00 (0.00)</td>
<td>0.00 (0.00)</td>
</tr>
<tr>
<td>Patient Age 65 if Age &gt;= 65</td>
<td>-0.01 (0.00)</td>
<td>0.00 (0.01)</td>
<td>-0.01 (0.01)</td>
</tr>
<tr>
<td>Log Waiting Time (years)</td>
<td>-0.15 (0.07)</td>
<td>-0.13 (0.12)</td>
<td>0.30 (0.11)</td>
</tr>
<tr>
<td>Log Waiting Time x 1(Over 1 Year)</td>
<td>-0.07 (0.03)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Patient BMI at Departure</td>
<td>0.07 (0.03)</td>
<td>0.06 (0.04)</td>
<td>0.06 (0.04)</td>
</tr>
<tr>
<td>Patient BMI - 25 if BMI &gt;= 30</td>
<td>0.02 (0.01)</td>
<td>0.02 (0.01)</td>
<td>0.02 (0.01)</td>
</tr>
<tr>
<td>Patient BMI - 30 if BMI &gt;= 30</td>
<td>-0.01 (0.01)</td>
<td>-0.02 (0.01)</td>
<td>-0.02 (0.01)</td>
</tr>
<tr>
<td>Patient Serum Albumin</td>
<td>-0.02 (0.03)</td>
<td>-0.01 (0.03)</td>
<td>-0.01 (0.03)</td>
</tr>
<tr>
<td>Serum Albumin - 3.7 if &gt;= 3.7</td>
<td>-0.04 (0.05)</td>
<td>-0.07 (0.06)</td>
<td>-0.06 (0.06)</td>
</tr>
<tr>
<td>Serum Albumin - 4.4 if &gt;= 4.4</td>
<td>0.12 (0.05)</td>
<td>0.16 (0.06)</td>
<td>0.16 (0.06)</td>
</tr>
<tr>
<td>Log Dialysis Time at Registration (Years)</td>
<td>0.04 (0.00)</td>
<td>0.05 (0.01)</td>
<td>0.05 (0.01)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match x Diabetic Patient</td>
<td>-0.44 (0.19)</td>
<td>-0.39 (0.27)</td>
<td>-0.29 (0.27)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match x Patient Age</td>
<td>0.03 (0.16)</td>
<td>0.06 (0.23)</td>
<td>0.06 (0.23)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match x CPRA</td>
<td>0.85 (0.35)</td>
<td>1.35 (0.48)</td>
<td>1.53 (0.48)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match x ECD Donor</td>
<td>-0.63 (0.16)</td>
<td>-0.72 (0.23)</td>
<td>-0.72 (0.23)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match x DCD Donor</td>
<td>-0.46 (0.33)</td>
<td>-1.03 (0.47)</td>
<td>-1.05 (0.47)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match x NYRT Donor</td>
<td>0.44 (0.18)</td>
<td>-0.02 (0.26)</td>
<td>-0.02 (0.26)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match x NYRT Donor x ABO Compatible</td>
<td>0.02 (0.17)</td>
<td>0.09 (0.24)</td>
<td>0.08 (0.24)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match x AB matching</td>
<td>0.16 (0.03)</td>
<td>0.06 (0.04)</td>
<td>0.05 (0.04)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match x AB matching</td>
<td>-0.02 (0.03)</td>
<td>-0.05 (0.04)</td>
<td>-0.05 (0.04)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match x AB matching</td>
<td>-0.03 (0.03)</td>
<td>-0.01 (0.04)</td>
<td>-0.01 (0.03)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match x AB matching</td>
<td>-0.05 (0.04)</td>
<td>0.18 (0.22)</td>
<td>0.19 (0.25)</td>
</tr>
<tr>
<td>Perfect Tissue Type Match x AB matching</td>
<td>0.13 (0.04)</td>
<td>0.24 (0.15)</td>
<td>0.25 (0.15)</td>
</tr>
<tr>
<td>Term</td>
<td>Coefficient</td>
<td>Std. Error</td>
<td>Coefficient</td>
</tr>
<tr>
<td>------</td>
<td>-------------</td>
<td>------------</td>
<td>-------------</td>
</tr>
<tr>
<td>NYRT Donor x 1{Donor Age 50+}</td>
<td>-0.45</td>
<td>(0.03)</td>
<td>-0.69</td>
</tr>
<tr>
<td>Patient Age x 1{Donor Age &lt; 18}</td>
<td>-0.01</td>
<td>(0.00)</td>
<td>0.00</td>
</tr>
<tr>
<td>Patient Age x 1{Donor Age 18-35}</td>
<td>-0.02</td>
<td>(0.00)</td>
<td>0.00</td>
</tr>
<tr>
<td>Patient Age x 1{Donor Age 50+}</td>
<td>0.02</td>
<td>(0.00)</td>
<td>0.02</td>
</tr>
<tr>
<td>Patient Age - 35 if Age &gt;= 35 x 1{Donor Age 18-35}</td>
<td>0.02</td>
<td>(0.01)</td>
<td>0.00</td>
</tr>
<tr>
<td>Patient Age - 35 if Age &gt;= 35 x 1{Donor Age 50+}</td>
<td>-0.01</td>
<td>(0.01)</td>
<td>0.00</td>
</tr>
<tr>
<td>Log Waiting Time x Prior Transplant</td>
<td>0.23</td>
<td>(0.02)</td>
<td>0.00</td>
</tr>
<tr>
<td>Log Waiting Time x Patient Diabetic</td>
<td>-0.03</td>
<td>(0.02)</td>
<td>0.00</td>
</tr>
<tr>
<td>Log Waiting Time x CPRA</td>
<td>0.08</td>
<td>(0.05)</td>
<td>0.00</td>
</tr>
<tr>
<td>Log Waiting Time x 1{CPRA &gt;= 80}</td>
<td>0.00</td>
<td>(0.05)</td>
<td>0.00</td>
</tr>
<tr>
<td>Log Waiting Time x Patient Serum Albumin</td>
<td>-0.01</td>
<td>(0.01)</td>
<td>0.00</td>
</tr>
<tr>
<td>Log Waiting Time x Patient BMI at Departure</td>
<td>0.00</td>
<td>(0.00)</td>
<td>0.00</td>
</tr>
<tr>
<td>Log Waiting Time x 1{Patient Blood Type A}</td>
<td>0.01</td>
<td>(0.03)</td>
<td>0.00</td>
</tr>
<tr>
<td>Log Waiting Time x 1{Patient Blood Type O}</td>
<td>-0.01</td>
<td>(0.03)</td>
<td>0.00</td>
</tr>
<tr>
<td>Patient BMI Missing</td>
<td>-1.27</td>
<td>(0.61)</td>
<td>0.00</td>
</tr>
<tr>
<td>Patient Serum Albumin Missing</td>
<td>-0.05</td>
<td>(0.12)</td>
<td>0.00</td>
</tr>
<tr>
<td>Donor Unobservable Std. Dev.</td>
<td>1.02</td>
<td>(0.03)</td>
<td>1.04</td>
</tr>
<tr>
<td>Idiosyncratic Shock Std. Dev.</td>
<td>1.00</td>
<td>1.00</td>
<td>1.00</td>
</tr>
</tbody>
</table>

Acceptance Rate | 0.140% | 0.140% | 0.140% | 0.140% | 0.140% | 0.140% |
Number of Offers | 2713043 | 2713043 | 2713043 | 2713043 | 2713043 | 2713043 |
Number of Donors | 5642 | 5642 | 5642 | 5642 | 5642 | 5642 |
Number of Patients | 9494 | 9494 | 9494 | 9494 | 9494 | 9494 |
B  Equilibria: Existence and Approximations

B.1  Approximating Offer Probabilities

This section derives a computationally tractable approximation to offer probabilities given a scoring rule \( s \), a large waitlist \( N^\ast \) and an acceptance policy function.

In what follows, we fix a particular agent \( i \) with priority score \( s \). Ties are broken randomly, and it is therefore without loss of generality to consider each agent’s tiebreaker to be drawn from a uniform distribution on the unit interval. Let \( 1 - \alpha \) be the tie-breaker for agent \( i \).

There are two reasons why an offer for a kidney may be the last offer that can be made. The first reason is that the agent who received the offer accepts it. The second one is that the kidney expires after the offer is made. The first of these is governed by the acceptance behavior of agents and the priority rule. The second is governed by the probability that the agent’s position on the list exceeds the maximum number of possible offers for the kidney. This model, specified in equation (12), yields a probability, \( p_0 = \lambda_o (z) \) for an object of type \( z \), of failure before the next offer is made. For simplicity, we fix \( z \) drop if from the notation.

An agent receives an offer if the total number of acceptances and expirations after offers to agents with a higher priority score than agent \( i \) is strictly less than the number of copies of the object available. Consider the probability of receiving an offer by considering waitlists that are composed of \( N \) agents randomly drawn with distribution governed by \( m \). For each agent drawn from \( m \), the probability that this agent is ordered above \( i \) and that the kidney is either accepted by the agent or expires is:

\[
p(s, \alpha) = m_H (s) p_H (s) + m_E (s) \alpha p_E (s).
\]

The first term represents the case when an agent with a higher priority (group \( H \)) is drawn. The probability of the kidney becoming unavailable conditional on an agent drawn from a higher priority group is:

\[
p_H (s) = p_0 + (1 - p_0) \frac{1}{m_H (s)} \sum_{t, x} m(t; x) 1 \{ s(t; x) > s \} \mathbb{P} (\Gamma (t; x) + \varepsilon > V_x (t)).
\]

The second term represents the probability that an agent with priority score \( s \) is drawn. The term \( p_E (s) \) is defined analogously as \( p_H (s) \).

The number of times that an kidney would become unavailable after being offered to an agent ordered above \( i \) is a binomial random variable \( X \) with parameters \( N \) and \( p(s, \alpha) \). In this notation, an object is available to agent \( i \) if \( X < q \), where \( q \) is the total number of copies of the object. Hence, the probability that \( i \) receives an offer is given by

\[
\int_0^1 \mathbb{P} (X < q|s, \alpha) d\alpha,
\]

(13)
where we have integrated over the tie-breaker $\alpha$, and explicit conditioning on $N$ is subsumed for simplicity.

For large $N$ and small $p(s, \alpha)$, the distribution of $X$ approaches the distribution of a Poisson random variable with parameter $Np(s, \alpha)$. Therefore, the expression in equation (13) yields the following expression for $\pi_x(t)$:

$$
\pi_x(t) = \int_0^1 \sum_{q' < q} e^{-Np(s, \alpha)} \frac{(Np(s, \alpha))^{q'}}{q'!} \, d\alpha,
$$

where we use the Poisson approximation to re-write $P(X < q|s, \alpha)$. As a reminder, the object type $z$ is dropped from the notation for simplicity as it is fixed, although the offer probabilities depend on it. This integral can be solved for in closed form. If $q \in \{1, 2\}$, that is

$$
\pi_x(t) = \frac{e^{-Np(s, 0)} - e^{-Np(s, 1)}}{N(p(s, 1) - p(s, 0))} + 1 \{q = 2\} \frac{(1 + Np(s, 0))e^{-Np(s, 0)} - (1 + Np(s, 1))e^{-Np(s, 1)}}{N(p(s, 1) - p(s, 0))}.
$$

(14)

### B.2 Existence of Steady-State Equilibria

This section proves that a steady-state equilibrium exists for sequential offer mechanisms that use a scoring rule. Throughout, we assume that $\chi$ and $\zeta$ are finite sets, and that $T$ is finite. This assumption greatly simplifies the technical argument. Moreover, they are also used when computing steady-state equilibria.

We make the following regularly assumptions on the primitive objects:

**Assumption 4.** (i) The exogenous arrival and departure rates $\lambda$ and $\gamma_x$ are finite

(ii) The exogenous departure rate $\delta(t; x)$ is bounded below by $\delta > 0$ and bounded above, uniformly for $t \in [0, T)$ and all $x \in \chi$

(iii) The conditional probability density function $f_{\Gamma|t,x,z}$ exists, and is uniformly bounded

(iv) The conditional moment $E[|\Gamma| | \tau, x, z] = \int |\Gamma| \, dF_{\Gamma|\tau,x,z}$ is uniformly bounded in $t, x, z$

(v) The family of functions $g(t; x, z, \Gamma) = F_{\Gamma|t,x,z}(\Gamma)$ indexed by $\Gamma, x, z$ is Lipschitz continuous in $t$ with a common constant

(vi) The object arrival rate $\lambda$ is strictly less than the total agent arrival rate $\sum \gamma_x$

(vii) The set of scores $S = \{s(t; x, z) : (t, x, z) \in [0, T] \times \chi \times \zeta\}$ is finite.

Most empirical models will satisfy the continuity and boundedness assumptions above. The two substantive assumptions that are worth discussing are parts (vi) and (vii). Part (vi)
assumes that the objects that need to be assigned are scarce. This assumption ensures that
the queue is unlikely to be empty. Part (vii) imposes a restriction on the mechanisms for
which we prove existence. The assumption is used to ensure that the set of all functions
\( \pi_{xz}(t) \) is sufficiently small (more precisely, compact). Other assumptions that yield this
conclusion would also suffice.

Our main result proves existence of a steady-state equilibrium.

**Theorem 1.** Suppose Assumption 4 is satisfied. Then a steady-state equilibrium for a se-
quential offer mechanism with a scoring rule exists.

**Proof.** The proof proceeds by applying the Brower-Schauder-Tychonoff Fixed Point Theorem
(Corollary 17.56, Aliprantis and Border, 2006). The proof proceeds in three parts. First, we
define a set \( \Omega \) to which the equilibrium objects belong. Second, we define a map \( A : \Omega \to \Omega \).
Finally, we show that \( A \) has fixed points, and that the fixed points of \( A \) yield steady-state
equilibria.

**Part 1, Definition of \( \Omega \):** The equilibrium objects are defined by five types of functions:

1. The conditional choice probability, given \( t \) and the agent and object characteristics \( x \)
   and \( z \). We consider these choice probabilities as a function \( p_\sigma : [0, T] \times \chi \times \zeta \to [0, 1] \).

2. The value function \( V : \chi \times [0, T] \to \mathbb{R}_+ \). It is convenient to define this function,
   although it is somewhat redundant with the choice probabilities above.

3. The offer probabilities \( \pi : [0, T] \times \chi \times \zeta \to [0, 1] \) where \( \pi(t; x, z) = H_z(s_{xz}(t)) \times
   \mathbb{P}(c_{ij} = 1|x, z) \).

4. The distribution of agent types \( m : \chi \times [0, T] \to \mathbb{R}_+ \).

5. The queue length \( N \in \mathbb{R} \).

We denote the tuple of these objects with \( \omega = (p_\sigma, V, \pi, m, N) \). We endow each of the
functions in the first four objects with the supremum norm over their respective domains.
The norm for \( \omega \) is denoted \( ||\omega|| = ||p_\sigma|| + ||V|| + ||\pi|| + ||m|| + |N| \). Therefore, \( \omega \) is an element
of a Banach space.

We further restrict \( \omega \) to belong to a subset \( \Omega \) of this Banach space. Specifically, we restrict
these components as follows:

1. The functions \( V_x(t) \) are uniformly bounded by \( \lambda T \sup_{\tau,x,z} \int |\Gamma| \, dF_{\Gamma|\tau,x,z} \), and are Lipschitz
   continuous with a common constant \( \lambda \sup_{\tau,x,z} \int |\Gamma| \, dF_{\Gamma|\tau,x,z} \).
2. The functions \( p_x (t; x, z) \) that are uniformly bounded by 1, and Lipschitz continuous with a common constant \( K \), where

\[
K = \lambda \sup_{\tau, x, z} \left( \int |\Gamma| \frac{dF_{\Gamma|\tau,x,z}}{d\Gamma} \int \sup_{\Gamma \geq \tau, x, z} f_{\Gamma|\tau,x,z} (\Gamma) \right) + \sup_{\Gamma \geq \tau, x, z} \left| F_{\Gamma|\tau,x,z} (\Gamma) - F_{\Gamma'|\tau,x,z} (\Gamma') \right| / |t - t'|.
\]

Note that Assumption 4 implies that \( K \) is finite.

3. The functions \( \pi_{x,z} (t) \) such that \( \pi_{x,z} (t) = \pi_{x,z} (t') \) if \( s_{xz} (t) = s_{xz} (t') \) with range \([0, 1]\),

4. The functions \( m_{x} (t) \) are uniformly bounded by \( T \sup_{x} \gamma_{x} \) and are Lipschitz continuous with a common constant \( \sup_{x, \tau} \gamma_{x} \delta (\tau; x) \).

5. The term \( N \in [\bar{N}, \bar{N}] \), where \( \bar{N} = (\sum_{x} \gamma_{x}) \inf_{x} \int_{0}^{T} \exp (- \int_{0}^{T} \delta (\tau; x) d\tau) / \lambda \) and \( \bar{N} = \sum_{x} \gamma_{x} / \bar{\delta} \). Note that \( \bar{N} > 0 \) because Assumption 4 requires that \( \sum_{x} \gamma_{x} < \lambda \) and \( \delta (\tau; x) \) is uniformly bounded above.

**Part 2, definition of** \( A : \Omega \rightarrow \Omega \): Denote \( A_{V} [\omega] \) as the \( V \) component of \( A [\omega] \), where \( \omega \in \Omega \). Likewise, define \( A_{\pi}, A_{p_x}, A_{m} \) and \( A_{N} \). This map is defined as follows:

\[
A_{V} [\omega] (x, t) = \int_{t}^{T} \exp (- \rho (\tau - t)) \frac{p(\tau|t; x)}{\lambda} \int \frac{\pi (\tau; x, Z)}{\max \{0, \Gamma - V (\tau; x)\}} dF_{\Gamma|x,z} dF_{Z} d\tau
\]

\[
A_{p_x} [\omega] (x, z, t) = \int 1 \{ \Gamma \geq A_{V} [\omega] (x, t) \} dF_{\Gamma|x,z,t}
\]

\[
A_{m} [\omega] (x, t) = \gamma_{x} \exp \left( - \int_{0}^{t} \delta (\tau; x) + \lambda \int \pi (\tau; x, Z) p_{x} (\tau; x, z) dF_{\Gamma|x,z} d\tau \right) / N
\]

\[
A_{N} [\omega] = \max \left\{ \bar{N}, \min \left\{ \frac{T \sum_{x} \gamma_{x}}{\sum_{x} \int_{0}^{T} \kappa_{x} (t) dt}, \bar{N} \right\} \right\}
\]

\[
A_{\pi} [\omega] (x, z, t) = H_{x} (s_{xz} (t); A_{p_x} [\omega], A_{m} [\omega], A_{N} [\omega]) \times \mathbb{P} (c_{ij} = 1|x, z)
\]

where

\[
p(\tau|t; x) = \exp \left( - \int_{t}^{\tau} \delta (\tau'; x) d\tau' \right)
\]

is the probability that agent of type \( x \) departs the list prior to \( \tau \) conditional on being on the list at \( t \). It is clear that \( A \) is well defined. To ensure that the image is a subset of \( \Omega \), we need to show that \( A [\omega] \in \Omega \) for all \( \omega \in \Omega \). We do this for each of the components separately:

1. \( A_{V} \): Since \( \exp (- \rho (\tau - t)), p(\tau|t; x) \) and \( \pi (\tau; x, Z) \) are in \([0, 1]\), and

\[
\int \max \{0, \Gamma - V (\tau; x)\} dF_{\Gamma|x,z} \leq \int \frac{|\Gamma|}{dF_{\Gamma|x,z}}.
\]

we have that \( A_{V} [\omega] \) is uniformly bounded by \( \lambda T \sup_{\tau, x, z} |\Gamma| dF_{\Gamma|x,z} \). Further, for any
\( A \), \( t, t' \in [0, T] \), with \( t < t' \), we have that

\[
|A_V [\omega] (t) - A_V [\omega] (t')| \\
= \left| \int_t^{t'} \exp (-\rho (\tau - t)) p (\tau | t; x) \left( \lambda \int \pi (\tau; x, Z) \int \max \{0, \Gamma - V (\tau; x)\} dF_{\Gamma | \tau, x, Z} dF_{Z} \right) d\tau \right| \\
\leq \lambda |t' - t| \sup_{\tau, x, z} \int |\Gamma| dF_{\Gamma | \tau, x, Z}. 
\]

Therefore, \( A_V [\omega] \) satisfies the necessary restrictions.

2. \( A_{p_\sigma} \): Note that is \( A_{p_\sigma} [\omega] \) uniformly bounded by 1. Moreover, for any \( x \) and \( z \), and \( t, t' \in [0, T] \), we have that

\[
|A_{p_\sigma} [\omega] (t, x, z) - A_{p_\sigma} [\omega] (t', x, z)| \\
= \left| \int \{ \Gamma \geq A_V [\omega] (x, t) \} dF_{\Gamma | x, z, t} - \int \{ \Gamma \geq A_V [\omega] (x, t') \} dF_{\Gamma | x, z, t'} \right| \\
= \left| \int (1 \{ \Gamma \geq A_V [\omega] (x, t) \} - 1 \{ \Gamma \geq A_V [\omega] (x, t') \}) dF_{\Gamma | x, z, t} \right| \\
\quad + \left| \int 1 \{ \Gamma \geq A_V [\omega] (x, t') \} d \left( F_{\Gamma | x, z, t} - F_{\Gamma | x, z, t'} \right) \right| \\
\leq \left| \int \max \{A_V [\omega] (x, t), A_V [\omega] (x, t')\} \right| \\
\quad + \left| F_{\Gamma | x, z, t} (A_V [\omega] (x, t')) - F_{\Gamma | x, z, t} (A_V [\omega] (x, t)) \right| \\
\leq \lambda |t' - t| \sup_{\tau, x, z} \left( \int |\Gamma| dF_{\Gamma | \tau, x, z} \sup_{\Gamma} f_{\Gamma | \tau, x, z} (\Gamma) \right) + \sup_{\Gamma, x, z} \left| \left| F_{\Gamma | t, x, z} (\Gamma) - F_{\Gamma | t', x, z} (\Gamma) \right| \right| |t - t'| \\
\leq \left| \lambda \sup_{\tau, x, z} \left( \int |\Gamma| dF_{\Gamma | \tau, x, z} \sup_{\Gamma} f_{\Gamma | \tau, x, z} (\Gamma) \right) + \sup_{\Gamma, x, z, t, t'} \left( \left| \left| F_{\Gamma | t, x, z} (\Gamma) - F_{\Gamma | t', x, z} (\Gamma) \right| \right| \right) \right| |t - t'|. 
\]

Therefore, \( A_{p_\sigma} [\omega] \) satisfies the necessary restrictions.

3. \( A_{\pi} \): Observe that \( A_{\pi} [\omega] (x, z, t) \in [0, 1] \) and \( A_{\pi} [\omega] (x, z, t) = A_{\pi} [\omega] (x, z, t') \) if \( s_{xz} (t) = s_{xz} (t') \) by construction.

4. \( A_m \): Since \( \exp \left(-f_0^t \delta (\tau; x) + \lambda \int \pi (\tau; x, Z) p_\sigma (\tau; x, Z) dF_{\Gamma | \tau, x, Z} d\tau \right) \leq 1 \) and \( N \geq 1 \), we have that \( A_m [\omega] \) is uniformly bounded by \( \sup_{x} \gamma_x \). Further, for any \( t, t' \in [0, T] \), with \( t < t' \), we have that

\[
|A_m [\omega] (t) - A_m [\omega] (t')| \leq \gamma_x \exp \left(-f_0^t \delta (\tau; x) + \lambda \int \pi (\tau; x, Z) p_\sigma (\tau; x, Z) dF_{Z} d\tau \right) \\
\leq \gamma_x \exp \left(-f_0^t \delta (\tau; x) d\tau \right) \leq |t' - t| \sup_{x, \tau} \gamma_x \delta (\tau; x). 
\]
Therefore, $A_m [\omega]$ satisfies the necessary restrictions.

5. $A_N$: By construction, $A_N [\omega]$ belongs to $\left[1, \frac{T \sum \gamma_x}{\delta}\right]$, satisfying the necessary restrictions.

Part 3, existence of equilibria: It is straightforward to verify that $\Omega$ is convex. Lemma 1 implies that the components $\Omega_V$, $\Omega_m$ and $\Omega_{p_x}$ are compact sets. Lemma 2 shows that $\Omega_{\pi}$ is compact. Assumption 4 (i), (ii) and (vi) imply that $N > 0$ and $\bar{N}$ is finite, implying that $\Omega_N$ is compact. Therefore, $\Omega$ is compact because it is the cartesian product of compact sets. Lemma 3 shows that $A$ is a continuous map. Therefore, by the Brouwer-Schauder-Tychonoff Theorem (Corollary 17.56, Aliprantis and Border, 2006) implies that there exists $\omega^* \in \Omega$ such that $A [\omega^*] = \omega^*$.

To complete the proof, we show that any fixed point $\omega^* = (p^*_o, V^*, \pi^*, m^*, N^*)$ corresponds to a steady-state equilibrium. Observe that for each $x$,

$$V^* (t; x) = \int_t^T \exp (-\rho (\tau - t)) p (\tau | t; x) \left( \lambda \int \pi^* (\tau; x, Z) \max \{0, \Gamma - V^* (\tau; x)\} dF_{\Gamma | \tau, x, Z} dF_Z \right) d\tau.$$

Therefore, $V^* (t; x)$ is the value of declining an offer and following the optimal strategy given the offer rate $\pi^*$. Therefore,

$$p^*_o (x, z, t) = A_{p_o} [\omega^*] (x, z, t) = \int 1 \{\Gamma \geq V^* (t; x)\} dF_{\Gamma | x, z, t}.$$

For each $x, z, t$, the quantity $F_{\Gamma | x, z, t}^{-1} (p^*_o (x, z, t)) = V^* (t; x)$. Therefore, $\sigma^* (\Gamma, t) = 1 \{\Gamma \geq F_{\Gamma | x, z, t}^{-1} (p^*_o (x, z, t))\}$ is an optimal strategy, satisfying requirement 1 in Definition 1.

By construction, $\pi^* (x, z, t) = A_{\pi} [\omega^*] (x, z, t) = H_z (s_{xz} (t); p^*_o, m^*, N^*) \times \mathbb{P} (c_{ij} = 1 | x, z)$ satisfies requirement 2 of Definition 1 because $p^*_o$ equals the acceptance probability of a type $z$ object by an agent of type $x$ at time $t$.

Finally, $m^* = A_m [\omega^*]$ and $N^* = A_N [\omega^*]$ together satisfy requirement 3 in Definition 1. The restriction of $A_N [\omega^*]$ to $[\underline{N}, \bar{N}]$ cannot strictly bind because $\underline{N}$ and $\bar{N}$ denote the smallest and largest possible queue lengths given the exogenous arrival and departure rates.

\[\square\]

**B.3 Lemmata**

**Lemma 1.** Suppose $X \subset L_{\infty} ([a,b])$ is the set of all functions on the bounded interval $[a,b]$ that are uniformly bounded by $K_1$ and have a common Lipschitz constant $K_2$. Then $X$ is compact.
Proof. Note that the set of functions $X$ is uniformly equicontinuous. By the Arzela-Ascoli theorem, any sequence of functions $x_n \in X$ has a uniformly convergent subsequence $x_{n_k}$. Let the limit of this sequence by $x^*$, i.e. for each $t$, $x^*(t) = \lim_{k \to \infty} x_{n_k}(t)$. Therefore, 
\[ \sup_t |x^*(t)| = \lim_{k \to \infty} \sup_t \left| x_{n_k}(t) \right| \leq K_1. \]
Similarly, 
\[ |x^*(t) - x^*(t')| = \lim_{k \to \infty} |x_{n_k}(t) - x_{n_k}(t')| \leq K_2 |t - t'|. \]
Hence, $x^* \in X$. Consequently, we have that $X$ is sequentially compact, which is equivalent to $X$ being compact. 

\[ \square \]

Lemma 2. Assumption 4(vii) implies that the set $\Omega_\pi$ consisting of functions $\pi : [0, T] \times \chi \times \zeta \to [0, 1]$ endowed with the supremum norm such that $\pi_{xz}(t) = \pi_{xz}(t')$ if $s_{xz}(t) = s_{xz}(t')$ is compact.

Proof. Assumption 4(vii) and finiteness of $\chi$ and $\zeta$ imply that the set of scores $s_{xz}(t)$ over all $\chi$, $\zeta$ and $[0, T]$ is finite. Therefore, $\Omega_\pi$ is an element of a finite dimensional Euclidean space. Further, $\Omega_\pi$ is closed and bounded by definition. By the Heine-Borel theorem, $\Omega_\pi$ is compact.

\[ \square \]

Lemma 3. Suppose Assumption 4 is satisfied. Then the map $A : \Omega \to \Omega$ is continuous.

Proof. We do this for each component of $A$ separately.

$A_V$: Let $\Omega_0$ be an arbitrary subset of $\Omega$. Consider $\omega \in \bar{\Omega}_0$, where $\bar{\Omega}_0$ is the closure of $\Omega_0$. Since $\omega \in \bar{\Omega}_0$, there exists a sequence $\omega_n \in \Omega_0$ such that $\| \omega_n - \omega \| = \varepsilon_n \to 0$. Denote $\bar{V}_n = A_V [\omega_n]$ and drop $x$ from the notation as it belongs to a finite set. Now, consider
\[
\left| \bar{V}_n(t) - \bar{V}(t) \right| 
\leq T \lambda \sup_{t,z} \left| \pi_n(t; z) \int \max \{0, V_n(t)\} - \pi(t; z) \int \max \{0, V(t)\} dF_{\Gamma|^t,z} \right| 
+ T \lambda \sup_{t,z} \left| \pi_n(t; z) \int \max \{0, V_n(t)\} - \pi(t; z) \int \max \{0, V(t)\} dF_{\Gamma|^t,z} \right| 
+ T \lambda \left( 1 + \sup_{t,z} \int |\Gamma| dF_{\Gamma|^t,z} \right) \varepsilon_n.
\]
Since $\varepsilon_n \to 0$, Assumption 4(i) and (iv) imply that the right hand side converges to zero. Therefore, $A_V \left[ \bar{\Omega}_0 \right] \subset A_V [\Omega_0]$, implying that $A_V$ is continuous (Theorem 2.27, Aliprantis and Border, 2006).
$A_{p_x}$: Continuity follows by noting that $A_N$ is continuous in the sup-norm, and $F_{t,x,z}$ is absolutely continuous with respect to Lebesgue measure for each $t, x, z$ (Assumption 4(iii)).

$A_m$: It is sufficient to fix $x$ because $\chi$ is a finite set. Lemma 4 implies that the map defined by $A_\kappa [\omega] (t) = \delta (t; x) + \lambda \int \pi (t; x, Z) p_{\sigma} (t; x) \, dF_Z$ is continuous. Moreover, $\sup_t A_\kappa [\omega] (t)$ is bounded above (Assumption 4(i)). Therefore, $A_\kappa [\omega] (t) = - \int_0^t \delta (\tau; x) + \lambda \int \pi (\tau; x, Z) \sigma (\Gamma, t) \, dF_Z \, d\tau$ defines a continuous map from $\Omega$ to $C ([0, T])$. Because a composition of continuous functions is continuous, and $g (a) = \gamma_x \exp (a) / N$ is continuous for all $N > 0$, we have that $A_m$ is continuous.

$A_N$: First we show that $A_N [\omega_n] = (A_{p_x} [\omega], A_m [\omega], A_N [\omega])$. We have shown that $\bar{A}$ is continuous and compact. Note that for any sequence $\omega_n$,

$$\sup_{x,z,t} | A_\pi [\omega_n] (x, z, t) | \leq \sup_{x,z,t} | H_z (s_{xz} (t); \bar{A} [\omega_n]) | \leq \sup_{z,s} | H_z (s; \bar{A} [\omega_n]) |,$$

where the first inequality follows from the fact that $\mathbb{P} (c_{ij} = 1 | x, z) \in [0, 1]$ and the second inequality follows from set inclusion. Therefore, Lemma 5 and continuity of $\bar{A}$ imply that for each $\bar{z}$, $\sup_{s,z} | H_z (s; \bar{A} [\omega_n]) | - H_z (s; \bar{A} [\omega]) | \to 0$ if $\omega_n$ converges to $\omega$. Since $z$ belongs to a finite set, we therefore have that $\sup_{x,z,t} | A_\pi [\omega_n] (x, z, t) - A_\pi [\omega] (x, z, t) | \to 0$. Hence, $A_\pi$ is a continuous map. Finally, Lemma 5 and the fact that $\bar{A} [\Omega]$ is compact imply that the image of $A_\pi$ is compact.

**Lemma 4.** Fix $x$. The map $A_\kappa : \Omega \to L_{\infty} ([0, T])$, where $A_\kappa [\omega] (t) = \delta (t; x) + \lambda \int \pi (t; x, Z) p_{\sigma} (t; x, Z) \, dF_Z$ is continuous if $\lambda$ is finite, and $\pi$ and $p_{\sigma}$ are uniformly bounded by 1.

**Proof.** Let $\Omega_0$ be an arbitrary subset of $\Omega$. Consider $\omega \in \bar{\Omega}_0$. Since $\omega \in \bar{\Omega}_0$, there exists a sequence $\omega_n \in \Omega_0$ such that $\| \omega_n - \omega \| = \varepsilon_n \to 0$. Now, consider $A_\kappa [\omega_n] (t) = ...
\[ \lambda \int \pi_n (t; x, Z) p_{\sigma_n} (\tau; x, Z) dF_{\Gamma|\tau,x,Z} , \] where we endow the range with the supremum norm.

\[ \| A_\kappa [\omega_n] - A_\kappa [\omega] \| = \lambda \left\| \int \pi_n (t; x, Z) p_{\sigma_n} (t; x, Z) dF_Z - \int \pi (t; x, Z) p_\sigma (t; x, Z) dF_Z \right\| \]

\[ \leq \lambda \sup_{z,t} |\pi_n (t; x, z) p_{\sigma_n} (t; x, z) - \pi (t; x, z) p_\sigma (t; x, z)| \]

\[ \leq \lambda \sup_{z,t} |\pi_n (t; x, z) (p_{\sigma_n} (t; x, z) - p_\sigma (t; x, z))| \]

\[ + \lambda \sup_{z,t} |(\pi_n (t; x, z) - \pi (t; x, z)) p_\sigma (t; x, z)| \]

\[ \leq \lambda \sup_{z,t} |p_{\sigma_n} (t; x, z) - p_\sigma (t; x, z)| + \lambda \sup_{z,t} |\pi_n (t; x, z) - \pi (t; x, z)| \leq 2\lambda \varepsilon_n. \]

Therefore, \( A_\kappa [\hat{\Omega}_0] \subset \overline{A_\kappa [\Omega_0]} \), implying that \( A_\kappa \) is continuous (Theorem 2.27, Aliprantis and Border, 2006).

\[ \square \]

**Lemma 5.** Fix \( z \). The map \( A_H : \Omega \rightarrow L_\infty (\mathbb{R}) \) defined by \( A_H [\omega] (s) = H_z (s; p_\sigma, m, N) \) is continuous.

**Proof.** We omit \( z \) from the notation for simplicity as it is fixed. Equation (14) derives the following expression for \( A_H \):

\[ A_H [\omega] (t, x, z) = \int_0^1 \sum_{q' < q} \frac{e^{-np(s,\alpha)} (np(s,\alpha))^{q'}}{q!} \, d\alpha, \]

where \( p(s, \alpha) = m_H (s) p_H (s) + m_E (s) \alpha p_E (s) \), \( p_H (s) = p_0 + (1 - p_0) \frac{1}{m_H (s)} \sum_{t,x} m (t; x) \{ s(t; x) > s \} p_\sigma \)

\( p_E (s) \) is defined analagously as \( p_H (s) \), and \( p_0 \) is the probability of a failure occurring. Here, we have \( \mathbb{P} (\Gamma (t; x, z) + \varepsilon > V_z (t)) \) with the acceptance probabilities \( p_\sigma (t; x, z) \). Recall that \( m_H (s) = \sum_{t,x} m (t; x) \{ s(t; x) > s \} \) and \( m_E (s) = \sum_{t,x} m (t; x) \{ s(t; x) = s \} \). We are now ready to prove continuity of \( A_H \). We do this by first proving continuity of the components \( m_H, m_E, p_H \) and \( p_E \).

Continuity of \( m_H \) and \( m_E \): Consider a sequence \( m_n \) that converges in sup norm on \( x, t \) to \( m \) we have that

\[ |m_{\kappa, H} (s) - m_H (s)| = \sum_{x} \int_0^T |m_n (t; x) - m (t; x)| \{ s(t; x) > s \} \, dt \]

\[ \leq |\chi| T \sup_{x,t} |m_n (t; x) - m (t; x)|. \]

Because this bound is independent of \( s \), we have that \( \sup_s |m_{\kappa, H} (s) - m_H (s)| \) converges to zero. Therefore, \( A_{m_H} : \Omega \rightarrow L_\infty (\mathbb{R}) \) defined by \( A_{m_H} [\omega] (s) = m_H (s) \) is a continuous map because \( A_{m_H} (\hat{\Omega}_0) = \overline{A_{m_H} (\Omega_0)} \) for any \( \Omega_0 \subseteq \Omega \) (Theorem 2.27, Aliprantis and Border, 2006).
An identical argument yields that \( A_{m_E} : \Omega \to L_{\infty} (\mathbb{R}) \) defined by \( A_{m_E} [\omega] (s) = m_E (s) \) is a continuous map. We do this by first proving continuity of the various components.

**Continuity of \( p_H \) and \( p_E \):** We show the argument only for \( p_H \) because the argument for \( p_E \) is identical. Consider a sequence of \( \omega_n \) that converges to \( \omega \), and the map \( A_{p_H} : \Omega \to L_{\infty} (\mathbb{R}) \) defined by \( A_{p_H} [\omega] (s) = p_0 + (1 - p_0) \frac{1}{m_H (s)} \sum_{t,x} m (t; x) 1 \{ s (t; x) > s \} p_\sigma (t; x) \). Since \( p_0 \) is fixed, we only need to show continuity of the map from \( \omega \) to \( \frac{1}{m_H (s)} \sum_{t,x} m (t; x) 1 \{ s (t; x) > s \} p_\sigma (t; x) \).

For each \( s \), we have that

\[
\left| \frac{1}{m_{n,H} (s)} \sum_{t,x} m_n (t; x) 1 \{ s (t; x) > s \} p_{n,\sigma} (t; x, z) - \frac{1}{m_H (s)} \sum_{t,x} m (t; x) 1 \{ s (t; x) > s \} p_{\sigma} (t; x) \right|
\]

\[
\leq \left| \frac{1}{m_{n,H} (s)} \sum_{t,x} m_n (t; x) 1 \{ s (t; x) > s \} p_{n,\sigma} (t; x) - \frac{1}{m_H (s)} \sum_{t,x} m (t; x) 1 \{ s (t; x) > s \} p_{\sigma} (t; x) \right|
\]

\[
+ \left| \frac{1}{m_H (s)} \sum_{t,x} m (t; x) 1 \{ s (t; x) > s \} p_{\sigma} (t; x) - \frac{1}{m_H (s)} \sum_{t,x} m (t; x) 1 \{ s (t; x) > s \} p_{\sigma} (t; x) \right|
\]

\[
\leq \left| \frac{1}{m_{n,H} (s)} \sum_{t,x} m_n (t; x) 1 \{ s (t; x) > s \} - \frac{1}{m_H (s)} \sum_{t,x} m (t; x) 1 \{ s (t; x) > s \} \right| |p_{n,\sigma} (t; x)|
\]

\[
+ \left| \frac{1}{m_H (s)} \sum_{t,x} m (t; x) 1 \{ s (t; x) > s \} \right| |p_{n,\sigma} (t; x) - p_{\sigma} (t; x)|
\]

\[
\leq \left| \frac{1}{m_{n,H} (s)} \sum_{t,x} m_n (t; x) 1 \{ s (t; x) > s \} - \frac{1}{m_H (s)} \sum_{t,x} m (t; x) 1 \{ s (t; x) > s \} \right| + |p_{n,\sigma} (t; x) - p_{\sigma} (t; x)|
\]

\[
= |p_{n,\sigma} (t; x) - p_{\sigma} (t; x)|
\]

The first inequality follows from the triangle inequality. The third inequality follows from the fact that \( |p_{n,\sigma} (t; x) - p_{\sigma} (t; x)| \) is bounded by 1 and \( m_{n,H} (s) = \sum_{t,x} m (t; x) 1 \{ s (t; x) > s \} \) by definition. The final inequality follows from the definition that \( m_{n,H} (s) = \sum_{t,x} m_n (t; x) 1 \{ s (t; x) > s \} \) and \( m_H (s) = \sum_{t,x} m (t; x) 1 \{ s (t; x) > s \} \) for all \( s \). If \( \omega_n \) converges to \( \omega \), we have that \( \sup_{t,x} |p_{n,\sigma} (t; x) - p_{\sigma} (t; x)| \) converges to zero. Therefore, we have that \( \sup_{s} |A_{p_H} [\omega_n] (s) - A_{p_H} [\omega] (s)| \) also converges to zero. Hence, \( A_{p_H} \) is continuous because \( A_{p_H} (\Omega_0) = \frac{A_{p_H} (\Omega_0)}{A_{p_H} (\Omega_0)} \) for any \( \Omega_0 \subseteq \Omega \) (Theorem 2.27, Aliprantis and Border, 2006).

**Continuity of \( p (s, \alpha) \):** The map \( A_{p_H} : \Omega \to L_{\infty} (\mathbb{R} \times [0, 1]) \) defined by \( A_p [\omega] (s, \alpha) = m_H (s) p_H (s) + m_E (s) \alpha p_E (s) \) is continuous because \( \alpha \) is bounded by 1, the maps from \( \omega \) to \( m_H (s) \), \( p_H (s) \), \( m_E (s) \), \( p_E (s) \) are continuous, and multiplication and addition are continuous.

**Continuity of \( A \):** The map from \( \Omega \) to \( \sum_{q' < q} e^{-Np(s, \alpha)(Np(s, \alpha))^{q'}} \frac{e^{-Np(s, \alpha)(Np(s, \alpha))^{q'}}}{q!} \) is continuous because the components are continuous, and the composition and multiplication operators are continuous. This term is bounded by 1. Therefore, the integral \( \int_0^1 \sum_{q' < q} e^{-Np(s, \alpha)(Np(s, \alpha))^{q'}} \frac{e^{-Np(s, \alpha)(Np(s, \alpha))^{q'}}}{q!} \, \text{d} \alpha \) defines a
continous map from $\Omega$ to the $L_\infty([0,T])$ for each $x$. 

\section{Computational Details}

\subsection{Counterfactual Scoring Mechanisms}

We compute steady-state equilibria for counterfactual scoring mechanisms using the algorithm described below. The algorithm uses a discrete time grid $t = t_0, \ldots, t_l, t_{l+1}, \ldots, T$, arbitrary initial beliefs $\pi^0$, and a sample of patients and donors as inputs. The baseline results we present below are based on a type space given by a random sample of 300 patients and 500 donors drawn from our dataset. Further, we discretize time into quarters for the first 15 years after registration, then every 2 years until year 25, and every 25 years thereafter. These results are not sensitive to a larger set of types used to calculate equilibria of scoring mechanisms and to finer partitions after the first few years since the probability that a patient survives without a transplant falls dramatically. Details are available in supplementary materials included in the replication archive associated with this paper.

Given this grid and the patient and donor types, an equilibrium is computed by iterating through the following steps three key steps until convergence:

1. Compute the value function $V^k_x(t_l)$, given beliefs $\pi^{k-1}$, via backwards induction from the value of waiting in the next period $V^k_x(t_{l+1})$. This calculation also yields patient strategies $\sigma^k_x(\Gamma, t) = 1\{\Gamma \geq V^k_x(t)\}$ and departure rates $\kappa^k_x(t)$.

2. Compute the queue composition $m^k$ via forward simulation.

3. Compute $\pi^k(t; x, z)$, which is the probability that an agent of type $x$ is offered an object of type $z$ using the queue composition and the accept/reject strategies $\sigma^k_x(\Gamma, t)$.

4. For step $k > 1$: Terminate if the change in value functions and queue length/compositions between iterations -- $\sup_x, t [V^k_x(t_l) - V^{k-1}_x(t_l)], \sup_x, t [m^k_x(t_l; x) - m^{k-1}_x(t_l)], N^k - N^{k-1}$ -- are uniformly below a chosen tolerance level. If these conditions are not satisfied, repeat steps 1-4.

If this algorithm terminates, the resulting accept/reject rules yield an equilibrium (up to the threshold tolerance). Because the equilibrium we compute may not be unique, we tried different starting values for $\pi^0$. Our experiments at the estimated parameters do not indicate that multiplicity is a concern in our setting. We describe each of these steps and detail the pseudocode below.

\textbf{Value Function Computation (Backwards Induction):}
Algorithm 1 Steady State Equilibrium

1: Inputs: Patient and donor characteristics, scoring rule \( s \), parameters \( \Gamma, \delta, \rho \), and patient age grid \( \{t_0, \ldots, t_L = T\} \). Let \( t_{0x} \) be the arrival time for patient of type \( x \).

2: Outputs: \( V^*, \pi^*, m^*, N^* \)

3: Initialize \( k = 0 \) and beliefs \( \pi^k(t) \) for all \( x \) and \( t \in \{t_0, \ldots, t_L\} \)

4: repeat
5: \( V^k \leftarrow \) Backwards Induction(\( \pi^k \))
6: \( \kappa^k_x(t_i) \leftarrow \delta_x(t_i) + \lambda \sum_{z} \pi^k_{x,z}(t_i) \mathbb{P}(\Gamma(t_i; x, z) + \varepsilon > V^k_x(t_i)) \)
7: \( m^k, N^k \leftarrow \) Forward Simulation(\( \kappa^k \)) \quad \triangleright \text{Waitlist Composition}
8: \( \pi^k \leftarrow \) Compute Offer Probabilities(\( V^k, m^k, N^k \)) \quad \triangleright \text{Offer Probabilities}
9: \( k \leftarrow k + 1 \)
10: until \( k > 1, \|V^k - V^{k-1}\|_\infty < \varepsilon, \|m^k - m^{k-1}\|_\infty < \varepsilon \), and \( N^k = N^{k-1} \) \quad \triangleright \text{Convergence}

11: \( V^* \leftarrow V^k, m^* \leftarrow m^k, N^* \leftarrow N^k, \pi^* \leftarrow \pi^k \)

12: function Backwards Induction(\( \pi \))
13: for all \( x \) do
14: \( \text{Set } V_x(T) = 0 \)
15: \text{for all } \( x \) and \( t_i = t_{L-1} \) to \( t_{0x} \) do
16: \( \text{Compute } V_x(t_i) \text{ by solving for } v \text{ in equation (15)} \)
17: \text{end for}
18: \text{end for}
19: return \( V_x(t_i) \) for all \( x \) and \( t_i \in \{t_{0x}, \ldots, T\} \)
20: end function

21: function Forward Simulation(\( \kappa \))
22: for all \( x \) do
23: \( m_x(t_0) \leftarrow \lambda_x \)
24: \text{for all } \( t_i = t_{0x+1} \) to \( T \) do
25: \( m_x(t_{i+1}) \leftarrow m_x(t_i) \exp(-\kappa_x(t_i)(t_{i+1} - t_i)) \)
26: \text{end for}
27: \text{end for}
28: \( N^k \leftarrow \sum_{x,t} m^k_x(t_i) \kappa^k_x(t_i) \) \quad \triangleright \text{Waitlist Size: Definition 1, part 3(b)}
29: \( m_x(t_i) \leftarrow m_x(t_i)/N^k \) for all \( x \) and \( t_i \)
30: return \( m_x(t_i) \) for \( t_i \in \{t_{0x}, \ldots, T\} \) and \( N^k \)
31: end function

32: function Compute Offer Probabilities(\( m, V, N \))
33: \( p^s(t_i; x, z) \leftarrow \mathbb{P}(\Gamma(t_i; x, z) + \varepsilon > V_x(t_i)) \) for all \( x, t_i \)
34: for all \( s = \max s(t_i; x, z) \) to \( \min s(t_i; x, z) \) do
35: \( \text{Compute } \pi \text{ using equation (14)} \)
36: \text{end for}
37: return \( \pi^k \)
38: end function
For a small \( h \), the value function derived in equation (3) can be approximated as

\[(\rho + \delta_x (t)) V_x (t) \approx \lambda \int \pi_x (t; z) \int \max \{0, \Gamma (t; x, z) + \varepsilon - V_x (t)\} \, dGdF + \frac{V_x (t + h) - V_x (t)}{h} \]

Because the right-hand side is monotonically decreasing in \( V_x (t) \), there is a unique value of \( V_x (t) \) that satisfies the equation. We will use this expression to obtain the value function by backwards induction. At iteration \( k \), given \( V^k_x (t_{l+1}) \) we use the bisection method to calculate the value of \( v \) that solves:

\[(\rho + \delta_x (t_l)) v = \lambda \int \pi^k_x (t_l; z) \int \max \{0, \Gamma (t_l; x, z) + \varepsilon - v\} \, dGdF + \frac{V^k_x (t_{l+1}) - v}{t_{l+1} - t_l} \tag{15} \]

Because this problem can be written as finding \( v = f (v) \) where \( f (\cdot) \) is strictly decreasing, we can take any initial guess \( v_0 \) and set the lower bound to \( \min (v_0, f (v_0)) \) and the upper bound to \( \max (v_0, f (v_0)) \). We use the initial guess \( v_0 = V^k_x (t_{l+1}) \).

**Offer Probabilities, \( \pi_{x,z} (t) \):**

The expression in equation (14) can be simplified and solved for analytically. We use that solution in our algorithm.

**Waitlist Size/Composition (Forward Simulation), \( m, N \):**

We use \( \kappa_x (t) \) and \( \gamma_x \) to update the queue composition. Solving the ODE in Definition 1, part 3(a), we get that for any \( h > 0 \),

\[m_x (t + h) = m_x (t) \exp \left( - \int_0^h \kappa_x (t + \tau) \, d\tau \right),\]

where \( m_x (0) = \lambda_x \). Approximating \( \kappa_x (t + \tau) = \kappa_x (t + h) \) for all \( \tau \in (0, h) \), we have that

\[m_x (t_{l+1}) = m_x (t_l) \exp \left( - \kappa_x (t_{l+1}) (t_{l+1} - t_l) \right). \tag{16} \]

Finally, we scale the output so that \( m_x (t_l) \) is a probability measure.

The size of the waitlist, \( N \), is determined by part 3(b) of Definition 1.

**C.2 Optimal Assignments and Optimal Offer Rates**

The objective functions for these two problems are identical. It is given by:

\[\sum \frac{1}{V^M_0 (\lambda_0)} \left[ \frac{\gamma_x}{\rho} V_x (0) + \sum_l N m_x (t_l) (t_{l+1} - t_l) V_x (t_l) \right],\]
where $\tilde{V}_x^{M_0} (\lambda_0)$ is defined in equation (10), $\tilde{m} \equiv N m$ and $V$ are choice variables with interpretations as in the rest of the paper. The variable $\tilde{m}$ differs from $m$ in only that it integrates across $\chi \times [0, T]$ to the total queue length, instead of being a probability density function that integrates to 1. The constraints on the two problems differ and each has a separate, third choice variable. For the optimal assignment mechanism, we choose assignment policies parametrized by $\mu$. For the optimal offer mechanism, we choose offer rates parametrized by $\pi$. We describe these variables and constraints below. The nonlinear problem is solved using KNITRO optimizer interfaced with MATLAB.

C.2.1 Optimal Assignments

The allocation maximizes the objective function above by assigning an object of type $z$ to agents currently on the list. The social planner knows the payoffs $\Gamma_{xzt}$ as well as the idiosyncratic shocks $\varepsilon$. The planner also knows the steady-state distribution of agents waiting for an assignment but not the future arrivals of objects or agents. The choice variable is the probability $\mu_{zxt}$ with which a compatible object of type $z$ is allocated to an agent of type $x$ who has waited for $t$ periods. Given $\mu$, the assignment is made to compatible agents of type $x$ that have waited for $t$ periods and have the highest draws of $\varepsilon$. Choosing $\mu$ is equivalent to choosing a cutoff $\bar{\varepsilon}_{xzt}$ such that $\mu_{xzt} = P (a (\varepsilon; x, z, t) = 1) = \int 1 \{ \varepsilon > \bar{\varepsilon}_{xzt} \} \ dG$, where the integral is taken with respect to $\varepsilon$.

There are three constraints:

1. Value Function: Finally, we chose value to equal the agent’s net present value from the expected stream of assignments under the policy $\mu_{zxt}$:

$$
\left( 1 + \left( \rho + \delta_x (t_l) + \lambda \sum_z f_z \mu_{zxt} c_{xz} \right) (t_{l+1} - t_l) \right) V_x (t_l) = (t_{l+1} - t_l) \lambda w_x (t_l) + V_x (t_{l+1}),
$$

where

$$
w_x (t) = \sum_z f_z c_{xz} \int (\Gamma_{xzt} + \varepsilon) 1 \{ \varepsilon > \bar{\varepsilon}_{xzt} \} \ dG
$$

$f_z$ is the probability that the object type is $z$, and integrals are taken with respect to $\varepsilon$. This expression of $V$ and $w$ is obtained from solving the value function from following the policy of accepting offers with $\varepsilon$ above $\bar{\varepsilon}_{xzt}$, with offers made whenever an object arrives. The term $w_x (t)$ denotes the expected value to an agent of type $x$ conditional on an object arriving.

2. Feasibility: The total mass of type $z$ objects that are assigned upon arrival must not exceed the mass of objects that arrive. Specifically, for each $z$, we impose the constraint:

$$
\sum_{x,l} \tilde{m}_x (t_l) (t_{l+1} - t_l) c_{xz} \mu_{zxt_l} \leq q_z,
$$
where $c_{xz}$ is the known (estimated) compatibility probability. The left hand side is the cumulative product of the (discretized) masses of each type of agent on the waitlist, $\tilde{m}_x(t_l)(t_{l+1} - t_l)$, multiplied by the assignment probabilities $c_{xz}\mu_{xzt_l}$ for each agent. This quantity cannot exceed the mass of objects that arrive, $q_z$.

3. Steady-State Composition: The measure of agents of type $x$ that have waited for $t$ periods is in steady state. This constraint is analogous to equation (16) above. Specifically, for each $x$ and $l > 0$, we have that

$$\tilde{m}_x(t_{l+1}) = \tilde{m}_x(t_l) \exp\left(-\left(\delta_x(t_l) + \lambda \sum_z f_z c_{xz}\mu_{xzt_l}\right)(t_{l+1} - t_l)\right)$$

$$\tilde{m}_x(t_0) = \gamma_x.$$

The term $\lambda \sum_z f_z c_{xz}\mu_{xzt_l}$ is the cumulative assignment rate across object for an agent of type $x$ at time $t_l$. This, when added to $\delta_x(t_{l+1})$, yields the total departure rate.

In addition, we impose that each $\mu_{xzt}$ belongs to unit interval.

### C.2.2 Optimal Offer Rates

The problem maximizes the objective function above by choosing a probability of offering an object of type $z$ to agents currently on the list. The social planner has full information about the payoffs $\Gamma_{xzt}$, but does not know the idiosyncratic shocks $\varepsilon$. She knows the steady-state distribution of agents waiting for an assignment but not the future arrivals of objects or agents. The choice variable in this problem is the probability $\pi_{xzt}$ with which an object of type $z$ is offered to an agent of type $x$ who has waited for $t$ periods upon arrival. Agents make optimal choices, given $\pi$, on which offers to accept.

As before, there are three constraints:

1. Value Function: Finally, we chosen value to equal the agent’s net present value from the expected stream of assignments under the policy $\pi_{xzt}$:

$$(1 + (\rho + \delta_x(t_l))(t_{l+1} - t_l)) V_x(t_l) = (t_{l+1} - t_l) \lambda w_x(t_l) + V_x(t_{l+1}),$$

where

$$w_x(t) = \sum_z f_z \pi_{xzt} c_{xz} \int \max \{0, \Gamma_{xzt} + \varepsilon - V_x(t)\} dG,$$

$f_z$ is the probability that the object type is $z$, and integrals are taken with respect to $\varepsilon$. As for the optimal assignment problem, $w_x(t)$ is the expected value to an agent of type $x$ conditional on an object arriving. However, in this problem, the agent makes optimal decisions and offers do not depend on the payoff shocks. Therefore, an assignment
occurs only if the agent is offered the object and the agent accepts. Acceptance occurs if the payoff shock exceeds $V_x(t) - \Gamma_{xt}$.

2. Feasibility: The total mass of type $z$ objects that are assigned upon arrival must not exceed the mass of objects that arrive. Specifically, for each $z$, we impose the constraint:

$$\sum_{x,l} \tilde{m}_x(t_l)(t_{l+1} - t_l) \pi_{xtl} \left[ c_{xz} \int 1 \{ \Gamma_{xtl} + \varepsilon > V_x(t_l) \} \, dG + p_{0,z} \right] \leq q_z,$$

where the integral is taken with respect to $\varepsilon$. This constraint is also analogous to the feasibility constraint in the optimal assignment problem. The difference is that the assignment rate $c_{xz}\mu_{xt}$ is replaced by the term

$$\pi_{xtl} \left[ c_{xz} \int 1 \{ \Gamma_{xtl} + \varepsilon > V_x(t_l) \} \, dG + p_{0,z} \right].$$

The term $\pi_{xtl}$ denotes the probability that an agent of type $x$ receives an offer for an object of type $z$ after she has waited for $t_l$ periods. The term in brackets is the probability that any such offer is the last offer for the object that can be made. It is the sum of the probability that object is compatible and transplanted

$$c_{xz} \int 1 \{ \Gamma_{xtl} + \varepsilon > V_x(t_l) \} \, dG$$

and the probability that no more offers can be made after this one. This term arises from the technological constraint on the number of offers that can be made for an object. The model used to determine $p_{0,z}$ is described in Appendix A.3.

This constraint only places a restriction on the expected number of assignments. Therefore, the offer rates $\pi_{xt}$ may not be implementable for specific donor arrivals.

3. Steady-State Composition: The measure of agents of type $x$ that have waited for $t$ periods is in steady state. Specifically, for each $x$ and $l > 0$, we have that

$$\tilde{m}_x(t_{l+1}) = \tilde{m}_x(t_l) \exp \left( -\left( \delta_x(t_{l+1}) + \lambda \sum_z f_z \mu_{xtl} \right)(t_{l+1} - t_l) \right)$$

$$\tilde{m}_x(t_l) = \gamma_x,$$

where

$$\mu_{xt} = \pi_{xt} c_{xz} \int 1 \{ \Gamma_{xt} + \varepsilon > V_x(t) \} \, dG.$$

The constraint differs from the optimal assignment problem in only that the assignment probability $\mu_{xt}$ depends on agents’ acceptance decision.

In addition, we impose that each $\pi_{xt}$ belongs to unit interval.