Forward Guidance without Common Knowledge

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How does the economy respond to news about the future?
  - e.g., future interest rates or government spending

Key mechanisms:
  - expectations of choices of others (e.g., of inflation and spending)
  - GE effects (e.g., Keynesian multiplier, $\pi-y$ feedback)
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Standard practice: RE with CK

This paper: RE without CK
  - formalizes frictional coordination
  - structured substitute to relaxing RE
Main Insight and Applications

- Removing CK
  - anchors expectations of the choices of others
  - attenuates GE effects

- Effects increase with horizon
  - as if extra discounting on future outcomes

- Application to ZLB context
  - anchors $\mathbb{E}[\pi]$ and $\mathbb{E}[y]$, for given $\mathbb{E}[R]$ or $\mathbb{E}[g]$
  - lessen forward guidance puzzle
  - offer rationale for the front-loading of fiscal stimuli
  - ...

1. Recast IS and NKPC as Dynamic Beauty Contests
2. Show GE Attenuation and Horizon Effects
3. Application to Forward Guidance and Fiscal Stimuli
4. Related Work
Starting point: textbook NK model
  - key ingredients: forward-looking $c$ and $\pi$

Main departure: remove CK of news about future $R$ or $g$

Auxiliary: enough “noise” to prevent revelation through prices
  - variant: rational inattention

Key friction: uncertainty about how others will respond
  - not uncertainty about the policy per se
  - to understand how it matters → IS and NKPC as beauty contests
\[ c_t = -E_t[r_{t+1}] + E_t[c_{t+1}] \]

- \( \Rightarrow c = f(\text{expected path of } r) \)
The Euler/IS Curve without Common Knowledge

\[ c_t = - \left\{ \sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t[r_{t+k}] \right\} + (1 - \beta) \left\{ \sum_{k=1}^{+\infty} \beta^{k-1} \bar{E}_t[c_{t+k}] \right\} \]

- \( \Rightarrow \) \( c \neq f(\text{expected path of } r) \)
- Key: \( E[\text{behavior of other consumers}] \)
The NK Philips Curve with Common Knowledge

\[ \pi_t = mc_t + \beta E_t[\pi_{t+1}] \]

- \( \Rightarrow \pi = f(\text{expected path of } mc) \)
The NK Philips Curve without Common Knowledge

\[ \pi_t = mc_t + \left\{ \sum_{k=1}^{+\infty} (\beta \theta)^k \bar{E}_t^f [mc_{t+k}] \right\} + \frac{1-\theta}{\theta} \left\{ \sum_{k=1}^{+\infty} (\beta \theta)^k \bar{E}_t^f [\pi_{t+k}] \right\} \]

- \[ \Rightarrow \pi \neq f(\text{expected path of } MC) \]
- Key: \( E[\text{behavior of other firms}] \)
So Far, and What’s Next

- So far:
  - represent IS and NKPC as dynamic beauty contests

- What’s next:
  - consider a more abstract setting
  - develop broader insights
Euler-like condition:

\[ a_{i,t} = \theta_t + \gamma E_{it}[a_{i,t+1}] + \alpha E_{it}[a_{t+1}] \]

- \( \theta_t \) = fundamental, \( a_{it} \) = individual outcome, \( a_t \) = aggregate outcome
- \( \gamma > 0 \) parameterizes PE (e.g., response to own interest rates)
- \( \alpha > 0 \) parameterizes GE (e.g., effect through aggregate income)
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With CK \( \Rightarrow \) representative-agent Euler

\[ a_t = \theta_t + (\gamma + \alpha) E_t[a_{t+1}] \]

\( \Rightarrow \) distinction between PE and GE is irrelevant
An Abstract Dynamic Beauty Contest

- Euler-like condition:

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- Without CK \( \Rightarrow \) dynamic beauty contest

\[ a_t = \theta_t + \gamma \left\{ \sum_{k=1}^{+\infty} \gamma^{k-1} E_t[\theta_{t+k}] \right\} + \alpha \left\{ \sum_{k=1}^{+\infty} \gamma^{k-1} E_t[a_{t+k}] \right\} \]

\( \Rightarrow \) distinction between PE and GE becomes crucial
Question of Interest

- How does $a_t$ respond to news about $\theta_{t+T}$?
  - $c_t$ and $\pi_t$ to news about $R_{t+T}$ or $g_{t+T}$

- Formally:
  - hold $\theta_\tau$ constant (say, at 0) for all $\tau \neq t + T$
  - treat $\theta_{t+T}$ as a random variable (Normally distributed with mean 0)
  - specify information structure about $\theta_{t+T}$
  - study $\phi_T \equiv$ projection coefficient of $a_t$ on $\tilde{E}_t[\theta_{t+T}]$
By iterating, we can express $a_t$ as a linear function of

- 1st-order beliefs: $\bar{E}_t[\theta_{t+T}]$
- 2nd-order beliefs: $\bar{E}_t[\bar{E}_\tau[\theta_{t+T}]] \forall \tau: t < \tau < t+T$
- 3rd-order beliefs: $\bar{E}_t[\bar{E}_\tau[\bar{E}_{\tau'}[\theta_{t+T}]]] \forall \tau, \tau': t < \tau < \tau' < t+T$
- and so on, up to beliefs of order $T$

- Understanding $\bar{E}_t$ of $a_{t+k}$ (e.g., inflation or income)
  \[ \Rightarrow \text{understanding HOB of } \theta_{t+T} \text{ (e.g., interest rate after the ZLB)} \]
Three Basic Insights

1. **Expectations of outcomes = HOB of fundamentals**
   - by iterating, we can express $\bar{E}_t[a_{t+k}]$ in terms of HOB of $\theta_{t+T}$
   - this is true regardless of info structure
   - but CK controls how much $\bar{E}_t[a_{t+k}]$ moves relative to $\bar{E}_t[\theta_{t+T}]$
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2. **HOB vary less than FOB**
   - “unless I am 100% sure that you heard and paid attention to the news, I am likely to think that your beliefs moved less than mine”
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   - “unless I am 100% sure that you heard and paid attention to the news, I am likely to think that your beliefs moved less than mine”

3. **Longer horizons raise the relative importance of HOB**
   - the distant future enters through multiple rounds of GE effects:
     \[ R_{t+T} \rightarrow (c_{t+T}, \pi_{t+T}) \rightarrow (c_{t+T-1}, \pi_{t+T-1}) \rightarrow \cdots \rightarrow (c_t, \pi_t) \]
   - but this is akin to ascending the hierarchy of beliefs!
   - longer horizons therefore raise the load of HOB on outcomes
1. **Attenuation at any horizon**
   - $\phi_T$ bounded between PE effect and CK counterpart:
     $$\gamma^T < \phi_T < \phi_T^* = (\gamma + \alpha)^T$$
   - “CK maximizes GE effect”
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2. **Attenuation effect increases with the horizon**
   - $\phi_T / \phi^*_T$ decreases in $T$
1. Attenuation at any horizon
   - $\phi_T$ bounded between PE effect and CK counterpart:
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2. Attenuation effect increases with the horizon
   - $\phi_T/\phi^*_T$ decreases in $T$

3. Attenuation effect grows without limit
   - $\phi_T/\phi^*_T \rightarrow 0$ as $T \rightarrow \infty$ even if noise is tiny*
Leading example

- Information structure:
  - each agent receives a private Gaussian signal about $\theta_{t+T}$ at $t$
  - no other info arrives up to $t + T$, at which point $\theta_{t+T}$ becomes known

- Implication: a simple exponential structure for HOB

$$\bar{E}_t^h[\theta_{t+T}] = \lambda^{h-1} \cdot \bar{E}_t[\theta_{t+T}]$$

where $\lambda \in (0, 1]$ is decreasing in the amount of noise
Back to our question: How does $a_t$ vary with $\bar{E}_t[\theta_{t+T}]$?

Answer: Same as in a representative-agent model with

$$a_t = \theta_t + (\gamma + \lambda \alpha) E_t[a_{t+1}]$$

- GE effect reduced from $\alpha$ to $\lambda \alpha$
- *as if* myopia / extra discounting
Back to the NK model: Three GE Mechanisms

- Removing CK dulls all these feedback loops
  - as if fewer loops or level-k thinking (but consistent with RE)
ZLB and Forward Guidance

- Let $T$ index length of liquidity trap and horizon of FG
  - $t < T - 1$: ZLB binds and $R_t = 0$ for all
  - $t \geq T + \Delta$: “natural level” and $y_t = \pi_t = 0$
  - let $\Delta = 1$ for simplicity

- Forward guidance
  - policy announcement at $t = 0$ of likely $R_T$
  - modeled as $z = R_T + \text{noise}$

- Our twist: lack of CK about $z$
  - credibility = precision of $z$, or how much $\tilde{E}_0[R_T]$ varies with $z$
  - we bypass this and focus on how $y_0$ varies with $\tilde{E}_0[R_T]$
  - think of this as studying the response of spending and inflation relative to the response of the term structure of interest rates
Leading Example

- Gaussian private signals about $R_T$, no endogenous learning
  - degree of CK indexed by $\lambda \in (0, 1]$ such that $\bar{E}^h[R_T] = \lambda^{h-1}\bar{E}^1[R_T]$
  - consumers vs firms: $\lambda_c$ vs $\lambda_f$

- The power of FG: there exists a function $\phi$ such that

$$y_0 = -\phi(\lambda_c, \lambda_f; T) \cdot \bar{E}_0[R_T]$$

  - measures how much $y$ moves relative to expectations of $R$
  - CK benchmark nested with $\phi^*(T) = \phi(1, 1; T)$
A Numerical Illustration

- Standard parameters as in Gali’s textbook
- Modest friction: 25% prob that others failed to hear announcement
- Large effect: at $T = 5$ years, $\phi$ is less than $1/10$ of $\phi^*$
Remarks

- Three GE effects at work:
  1. inside IS: income-spending feedback
  2. inside NKPC: inflation-inflation feedback
  3. across two blocks: inflation-spending feedback

- All three attenuated when removing CK, but
  - in textbook version of NK, most quantitative bite for (2) and (3)
  - (1) becomes more relevant with short horizons or liquidity constraints
Fiscal Stimuli: Back- vs Front-Loading

- Standard NK prediction:
  - fiscal stimuli work because they trigger inflation
  - better to back-load so as to “pile up” inflation effects

- Our twist:
  - such piling up = iterating HOB
  - not as potent when CK assumption is dropped
  - rationale for front-loading: “minimize coordination friction”
Summary

- Removing CK
  - accommodates frictional coordination
  - attenuates GE effects
  - anchors expectations of inflation and income
  - lessens forward guidance puzzle (and paradox of flexibility too)
  - justifies front loading of fiscal stimuli
  - ...

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Related Work

- Related work that arrests GE by dropping RE
  - cognitive discounting as in Gabaix (2016)
  - level-k as in Garcia & Woodford (2015), Farhi & Werning (2017)

- Our approach has similar implications, but:
  - robust to settings in which GE = strategic substitutability
  - consistent with RE ⇒ immune to Lucas critique, plus no conundrum with what agents do when they see the actual outcomes
  - implies not only discounting but also backward-lookingness
    ⇒ microfoundation of hybrid NKPC, IAC, habit

- Companion papers:
  - “Dampening GE” with Chen Lian
  - “Anchored Expectations” with Zhen Huo
  - ...

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