
Daron Acemoglu

MIT

September 12, 2007
Most process innovations either increase the quality of an existing product or reduce the costs of production.

**Competitive** aspect of innovations: a newly-invented superior computer often replaces existing vintages.

Realm of Schumpeterian *creative destruction*.

Schumpeterian growth raises important issues:

1. Direct price competition between producers with different vintages of quality or different costs of producing
2. Competition between incumbents and entrants: *business stealing effect.*
Continuous time.

Representative household with standard CRRA preferences.

Constant population $L$; labor supplied inelastically.

Resource constraint:

$$C(t) + X(t) + Z(t) \leq Y(t), \quad (1)$$

Normalize the measure of inputs to 1, and denote each machine line by $\nu \in [0, 1]$. 
Preferences and Technology II

- Engine of economic growth: *quality improvement*.
- \( q(ν, t) \) = quality of machine line \( ν \) at time \( t \).
- “Quality ladder” for each machine type:

\[
q(ν, t) = \lambda^{n(ν, t)} q(ν, 0) \text{ for all } ν \text{ and } t, \tag{2}
\]

where:
- \( \lambda > 1 \)
- \( n(ν, t) \) = innovations on this machine line between 0 and \( t \).

- Production function of the final good:

\[
Y(t) = \frac{1}{1 - \beta} \left[ \int_0^1 q(ν, t) x(ν, t \mid q)^{1-\beta} \, dv \right] L^\beta, \tag{3}
\]

where \( x(ν, t \mid q) \) = quantity of machine of type \( ν \) quality \( q \).
Implicit assumption in (3): at any point in time only one quality of any machine is used.

Creative destruction: when a higher-quality machine is invented it will replace (‘‘destroy’’) the previous vintage of machines.
Technology for producing machines and innovation possibilities frontier I

- Cumulative R&D process.
- \( Z(\nu, t) \) units of the final good for research on machine line \( \nu \), quality \( q(\nu, t) \) generate a flow rate

\[ \eta Z(\nu, t) / q(\nu, t) \]

of innovation.
- Note one unit of R&D spending is proportionately less effective when applied to a more advanced machine.
- Free entry into research.
- The firm that makes an innovation has a perpetual patent.
- But other firms can undertake research based on the product invented by this firm.
Once a machine of quality $q(ν, t)$ has been invented, any quantity can be produced at the marginal cost $ψq(ν, t)$.

New entrants undertake the R&D and innovation:

- The incumbent has weaker incentives to innovate, since it would be replacing its own machine, and thus destroying the profits that it is already making (Arrow’s replacement effect).
Equilibrium

- Allocation: time paths of
  - consumption levels, aggregate spending on machines, and aggregate R&D expenditure \([C(t), X(t), Z(t)]_{t=0}\),
  - machine qualities \([q(\nu, t)]_{\nu \in [0,1], t=0}\),
  - prices and quantities of each machine and the net present discounted value of profits from that machine, \([p^X(\nu, t | q), x(\nu, t), V(\nu, t | q)]_{\nu \in [0,1], t=0}\), and
  - interest rates and wage rates, \([r(t), w(t)]_{t=0}\).
Equilibrium: Innovations Regimes

- Demand for machines similar to before:
  \[ x(\nu, t \mid q) = \left( \frac{q(\nu, t)}{p^x(\nu, t \mid q)} \right)^{1/\beta} L \quad \text{for all } \nu \in [0, 1] \text{ and all } t, \quad (4) \]

  where \( p^x(\nu, t \mid q) \) refers to the price of machine type \( \nu \) of quality \( q(\nu, t) \) at time \( t \).

- Two regimes:
  1. Innovation is “drastic” and each firm can charge the unconstrained monopoly price,
  2. Limit prices have to be used.

- Assume drastic innovations regime: \( \lambda \) is sufficiently large

  \[ \lambda \geq \left( \frac{1}{1 - \beta} \right)^{\frac{1-\beta}{\beta}}. \quad (5) \]

- Again normalize \( \psi \equiv 1 - \beta \)
Monopoly Profits

- Profit-maximizing monopoly:

\[ p^x (v, t | q) = q (v, t) . \]  \hspace{1cm} (6)

- Combining with (4)

\[ x (v, t | q) = L. \]  \hspace{1cm} (7)

- Thus, flow profits of monopolist:

\[ \pi (v, t | q) = \beta q (v, t) L. \]
Charaterization of Equilibrium I

- Substituting (7) into (3):

\[ Y(t) = \frac{1}{1 - \beta} Q(t) L, \]  \hspace{1cm} (8)

where

\[ Q(t) = \int_0^1 q(\nu, t) d\nu \]  \hspace{1cm} (9)

- Aggregate spending on machines:

\[ X(t) = (1 - \beta) Q(t) L. \]  \hspace{1cm} (10)

- Equilibrium wage rate:

\[ w(t) = \frac{\beta}{1 - \beta} Q(t). \]  \hspace{1cm} (11)
Characterization of Equilibrium II

- Value function for monopolist of variety $\nu$ of quality $q(\nu, t)$ at time $t$:

$$r(t) V(\nu, t \mid q) - \dot{V}(\nu, t \mid q) = \pi(\nu, t \mid q) - z(\nu, t \mid q) V(\nu, t \mid q),$$

where:

- $z(\nu, t \mid q) =$ rate at which new innovations occur in sector $\nu$ at time $t$,
- $\pi(\nu, t \mid q) =$ flow of profits.

- Last term captures the essence of Schumpeterian growth:

  - when innovation occurs, the monopolist loses its monopoly position and is replaced by the producer of the higher-quality machine.
  - From then on, it receives zero profits, and thus has zero value.
  - Because of Arrow’s replacement effect, an entrant undertakes the innovation, thus $z(\nu, t \mid q)$ is the flow rate at which the incumbent will be replaced.
Characterization of Equilibrium III

- Free entry:
  \[ \eta V(\nu, t | q) \leq \lambda^{-1} q(\nu, t) \]
  \[ \text{and } \eta V(\nu, t | q) = \lambda^{-1} q(\nu, t) \text{ if } Z(\nu, t | q) > 0. \] 

- Note: Even though the \( q(\nu, t) \)'s are stochastic as long as the \( Z(\nu, t | q) \)'s, are nonstochastic, average quality \( Q(t) \), and thus total output, \( Y(t) \), and total spending on machines, \( X(t) \), will be nonstochastic.

- Consumer maximization implies the Euler equation,
  \[ \frac{\dot{C}(t)}{C(t)} = \frac{1}{\theta} (r(t) - \rho), \] 
  \[ (14) \]

- Transversality condition:
  \[ \lim_{t \to \infty} \left[ \exp \left( - \int_0^t r(s) \, ds \right) \int_0^1 V(\nu, t | q) \, d\nu \right] = 0 \]
  \[ (15) \]
  for all \( q \).
Definition of Equilibrium

- \( V(\nu, t \mid q) \), is nonstochastic: either \( q \) is not the highest quality in this machine line and \( V(\nu, t \mid q) \) is equal to 0, or it is given by (12).

- An equilibrium can then be represented as time paths of
  - \([C(t), X(t), Z(t)]_{t=0}^{\infty}\) that satisfy (1), (10), (15),
  - \([Q(t)]_{t=0}^{\infty}\) and \([V(\nu, t \mid q)]_{\nu \in [0,1], t=0}^{\infty}\) consistent with (9), (12) and (13),
  - \([p^x(\nu, t \mid q), x(\nu, t)]_{\nu \in [0,1], t=0}^{\infty}\) given by (6) and (7), and
  - \([r(t), w(t)]_{t=0}^{\infty}\) that are consistent with (11) and (14)

- Balanced Growth Path defined similarly to before (constant growth of output, constant interest rate).
Balanced Growth Path I

- In BGP, consumption grows at the constant rate $g^*_C$, that must be the same rate as output growth, $g^*$. 
- From (14), $r(t) = r^*$ for all $t$. 
- If there is positive growth in BGP, there must be research at least in some sectors. 
- Since profits and R&D costs are proportional to quality, whenever the free entry condition (13) holds as equality for one machine type, it will hold as equality for all of them. 
- Thus,
  \[ V(\nu, t | q) = \frac{q(\nu, t)}{\lambda \eta}. \]  
  \[ (16) \]
- Moreover, if it holds between $t$ and $t + \Delta t$, $\dot{V}(\nu, t | q) = 0$, because the right-hand side of equation (16) is constant over time—$q(\nu, t)$ refers to the quality of the machine supplied by the incumbent, which does not change.
Balanced Growth Path II

- Since R&D for each machine type has the same productivity, constant in BGP:

\[ z(\nu, t) = z(t) = z^* \]

- Then (12) implies

\[ V(\nu, t | q) = \frac{\beta q(\nu, t) L}{r^* + z^*}. \tag{17} \]

- Note the effective discount rate is \( r^* + z^* \).

- Combining this with (16):

\[ r^* + z^* = \lambda \eta \beta L. \tag{18} \]

- From the fact that \( g_C^* = g^* \) and (14), \( g^* = (r^* - \rho) / \theta \), or

\[ r^* = \theta g^* + \rho. \tag{19} \]
Balanced Growth Path III

- To solve for the BGP equilibrium, we need a final equation relating \( g^* \) to \( z^* \). From (8)

\[
\frac{\dot{Y}(t)}{Y(t)} = \frac{\dot{Q}(t)}{Q(t)}.
\]

- Note that in an interval of time \( \Delta t \), \( z(t) \Delta t \) sectors experience one innovation, and this will increase their productivity by \( \lambda \).

- The measure of sectors experiencing more than one innovation within this time interval is \( o(\Delta t) \)—i.e., it is second-order in \( \Delta t \), so that

\[
as \Delta t \to 0, \frac{o(\Delta t)}{\Delta t} \to 0.
\]

- Therefore, we have

\[
Q(t + \Delta t) = \lambda Q(t) z(t) \Delta t + (1 - z(t) \Delta t) Q(t) + o(\Delta t).
\]
Balanced Growth Path IV

- Now subtracting $Q(t)$ from both sides, dividing by $\Delta t$ and taking the limit as $\Delta t \to 0$, we obtain

$$
\dot{Q}(t) = (\lambda - 1) z(t) Q(t).
$$

- Therefore,

$$
g^* = (\lambda - 1) z^*.
$$

(20)

- Now combining (18)-(20), we obtain:

$$
g^* = \frac{\lambda \eta \beta L - \rho}{\theta + (\lambda - 1)^{-1}}.
$$

(21)
Proposition Consider the model of Schumpeterian growth described above. Suppose that

\[ \lambda \eta \beta L > \rho > (1 - \theta) \frac{\lambda \eta \beta L - \rho}{\theta + (\lambda - 1)^{-1}}. \]  

Then, there exists a unique balanced growth path in which average quality of machines, output and consumption grow at rate \( g^* \) given by (21). The rate of innovation is \( g^* / (\lambda - 1) \).

- Important: *Scale effects* and implicit *knowledge spillovers* are present.
  - knowledge spillovers arise because innovation is *cumulative.*
Transitional Dynamics

Proposition  In the model of Schumpeterian growth described above, starting with any average quality of machines $Q(0) > 0$, there are no transitional dynamics and the equilibrium path always involves constant growth at the rate $g^*$ given by (21).

- Note only the average quality of machines, $Q(t)$, matters for the allocation of resources.
- Moreover, the incentives to undertake research are identical for two machine types $\nu$ and $\nu'$, with different quality levels $q(\nu, t)$ and $q(\nu', t)$.
This equilibrium is typically Pareto suboptimal.

But now distortions more complex than the expanding varieties model.

- monopolists are not able to capture the entire social gain created by an innovation.
- Business stealing effect.

The equilibrium rate of innovation and growth can be too high or too low.
Quantities of machines used in the final good sector: no markup.

\[ x^S (\nu, t \mid q) = \psi^{-1/\beta} L \]

\[ = (1 - \beta)^{-1/\beta} L. \]

Substituting into (3):

\[ Y^S (t) = (1 - \beta)^{-1/\beta} Q^S (t) L, \]
Maximization problem of the social planner:

\[
\max \int_0^\infty \frac{C_S(t)^{1-\theta} - 1}{1 - \theta} \exp(-\rho t) \, dt
\]

subject to

\[
\dot{Q}_S(t) = \eta (\lambda - 1) (1 - \beta)^{-1/\beta} \beta Q_S(t) L - \eta (\lambda - 1) C_S(t),
\]

where \((1 - \beta)^{-1/\beta} \beta Q_S(t) L\) is net output.
Social Planner’s Problem III

- **Current-value Hamiltonian:**

\[
\hat{H}\left(Q^S, C^S, \mu^S\right) = \frac{C^S(t)^{1-\theta} - 1}{1-\theta} \\
+ \mu^S(t) \left[ \eta (\lambda - 1) (1 - \beta)^{-1/\beta} \beta Q^S(t) L \right. \\
- \left. \eta (\lambda - 1) C^S(t) \right].
\]
Social Planner’s Problem IV

- Necessary conditions:

\[ \hat{H}_C (\cdot) = C^S (t)^{-\theta} - \mu^S (t) \eta (\lambda - 1) \]
\[ = 0 \]

\[ \hat{H}_Q (\cdot) = \mu^S (t) \eta (\lambda - 1) (1 - \beta)^{-1/\beta} \beta L \]
\[ = \rho \mu^S (t) - \dot{\mu}^S (t) \]

\[ \lim_{t \to \infty} \left[ \exp (-\rho t) \mu^S (t) Q^S (t) \right] = 0 \]

- Combining:

\[ \frac{\dot{C}^S (t)}{C^S (t)} = g^S \equiv \frac{1}{\theta} \left( \eta (\lambda - 1) (1 - \beta)^{-1/\beta} \beta L - \rho \right) . \quad (23) \]
Summary of Social Planner’s Problem

- Total output and average quality will also grow at the rate $g^S$.
- Comparing $g^S$ to $g^*$, either could be greater.
  - When $\lambda$ is very large, $g^S > g^*$. As $\lambda \to \infty$, $g^S / g^* \to (1 - \beta)^{-1/\beta} > 1$.

**Proposition**  In the model of Schumpeterian growth described above, the decentralized equilibrium is generally Pareto suboptimal, and may have a higher or lower rate of innovation and growth than the Pareto optimal allocation.
Policies I

- Creative destruction implies a natural *conflict of interest*, and certain types of policies may have a constituency.
- Suppose there is a tax $\tau$ imposed on R&D spending.
- This has no effect on the profits of existing monopolists, and only influences their net present discounted value via replacement.
- Since taxes on R&D will discourage R&D, there will be replacement at a slower rate, i.e., $z^*$ will fall.
- This increases the steady-state value of all monopolists given by (17):

$$V(q) = \frac{\beta q L}{r^*(\tau) + z^*(\tau)},$$

- The free entry condition becomes

$$V(q) = \frac{(1 + \tau)}{\lambda \eta} q.$$
Policies II

- \( V(q) \) is clearly increasing in the tax rate on R&D, \( \tau \).
- Combining the previous two equations, we see that in response to a positive rate of taxation, \( r^*(\tau) + z^*(\tau) \) must adjust downward.
- Intuitively, when the costs of R&D are raised because of tax policy, the value of a successful innovation, \( V(q) \), must increase to satisfy the free entry condition. This can only happen through a decline the effective discount rate \( r^*(\tau) + z^*(\tau) \).
- A lower effective discount rate, in turn, is achieved by a decline in the equilibrium growth rate of the economy:

\[
g^*(\tau) = \frac{(1 + \tau)^{-1} \lambda \eta \beta \rho - \rho}{\theta + (\lambda - 1)^{-1}}.
\]

- This growth rate is strictly decreasing in \( \tau \), but incumbent monopolists would be in favor of increasing \( \tau \).
Two major differences with previous model:

1. Only one sector experiencing quality improvements rather than a continuum of machine types.
2. The innovation possibilities frontier uses a scarce factor, labor.
Aghion-Howitt Model I

- Consumer side as before, but risk neutral consumers, so:

\[ r^* = \rho \]

- Population constant at \( L \); individuals supply labor inelastically.
- Aggregate production function of final good:

\[
Y(t) = \frac{1}{1 - \beta} x(t | q) (q(t) L_E(t))^{\beta},
\]

(24)

- Market clearing requires:

\[
L_E(t) + L_R(t) \leq L.
\]

- where \( L_E(t) \) is labor used in production, \( L_R(t) \) in the R&D sector.
- Once invented, a machine of quality \( q(t) \) can be produced at the constant marginal cost \( \psi \) in terms of final goods.
Aghion-Howitt Model II

- Normalize $\psi \equiv 1 - \beta$.
- Innovation possibilities frontier: each worker employed in the R&D sector generates a flow rate $\eta$ of a new machine.
- When the current machine used in production has quality $q(t)$, the new machine has quality $\lambda q(t)$.
- Assume that the monopolist can charge the unconstrained monopoly price.
- Then, the demand for the leading-edge machine of quality $q$ is
  \[ x(t | q) = p^x(t)^{-1/\beta} q(t) L_E(t), \]
- Suppose that the monopoly price for highest quality machine is:
  \[ p^x(t | q) = \frac{\psi}{1 - \beta} = 1. \]
- Why is this a “supposition”?
Aghion-Howitt Model III

- Thus demand for the machine of quality \( q \) at time \( t \) is:

\[
x(t \mid q) = q(t) L_E(t),
\]

- Monopoly profits:

\[
\pi(t \mid q) = \beta q(t) L_E(t).
\]

- Aggregate output:

\[
Y(t \mid q) = \frac{1}{1 - \beta} q(t) L_E(t),
\]

- Equilibrium wage:

\[
w(t \mid q) = \frac{\beta}{1 - \beta} q(t).
\]

- Focus on a “steady-state equilibrium” with constant flow rate of innovation \( z^* \).
Aghion-Howitt Model IV

- Even with constant $z$, consumption and output growth will not be constant because of the stochastic nature of innovation.
- A constant number (and thus fraction) of workers, $L^*_R$, must be working in research. Since $r^* = \rho$, this implies that the steady-state value of a monopolist is:

$$V(q) = \frac{\beta q (L - L^*_R)}{\rho + z^*},$$

- Free entry:

$$w(q) = \eta V(\lambda q).$$

- In addition, given the R&D technology, we must have

$$z^* = \eta L^*_R.$$
Combining the last four equations:

\[
\frac{\lambda (1 - \beta) \eta (L - L^*_R)}{\rho + \eta L^*_R} = 1,
\]

which uniquely determines the steady-state number of workers in research as

\[
L^*_R = \frac{\lambda (1 - \beta) \eta L - \rho}{\eta + \lambda (1 - \beta) \eta},
\]  

(25)

as long as this expression is positive.

Since there is only one sector undergoing technological change and this sector experiences growth only at finite intervals, the growth rate of the economy will have an\textit{ uneven} nature.
Proposition  Consider the one-sector Schumpeterian growth model presented in this section and suppose that

\[
0 < \lambda (1 - \beta) \eta L - \rho < \frac{1 + \lambda (1 - \beta) \rho}{\ln \lambda}. \tag{26}
\]

Then there exists a unique steady-state equilibrium in which \( L^*_R \) workers work in the research sector, where \( L^*_R \) is given in equation (25). The economy has an average growth rate of \( g^* = \eta L^*_R \ln \lambda \). Equilibrium growth is “uneven,” in the sense that the economy has constant output for a while and then grows by a discrete amount when an innovation takes place.
The uneven pattern of economic growth in the previous model is driven by the discrete nature of innovations in continuous time.

Another source of uneven growth more closely related to creative destruction is that future growth reduces the value of current innovations, because it causes more rapid replacement.

We focus on an equilibrium path with endogenous growth cycles.

Now assume that $L_R$ workers in research leads to innovation at the rate

$$\eta(L_R) L_R,$$

where $\eta(\cdot)$ is a strictly decreasing function, representing an externality in the research process.
Uneven Growth II

- Free entry condition:
  \[ \eta (L_R (q)) V (\lambda q) = w (q) \]

- Look for an equilibrium with the cyclical property that the rate of innovation differs in an odd-numbered innovation versus an even-numbered innovation.
- Possible when all agents in the economy expect there to be such an equilibrium (i.e., it is a “self-fulfilling” equilibrium).
- Denote the number of workers in R&D for odd and even-numbered innovations by \( L^1_R \) and \( L^2_R \).
- Then, in any equilibrium with such cyclical pattern:
  \[
  V^2 (\lambda q) = \frac{\beta q (L - L^2_R)}{\rho + \eta (L^2_R) L^2_R} \quad \text{and} \quad V^1 (\lambda q) = \frac{\beta q (L - L^1_R)}{\rho + \eta (L^1_R) L^1_R}.
  \] (27)
And the free entry conditions is:

\[ \eta (L_R^1) V^2 (\lambda q) = w (q) \] and \[ \eta (L_R^2) V^1 (\lambda q) = w (q) , \]

Therefore, equilibrium conditions:

\[
\eta (L_R^1) \frac{\lambda (1 - \beta) q (L - L_R^2)}{\rho + \eta (L_R^2) L_R^2} = 1 \text{ and } \eta (L_R^2) \frac{\lambda (1 - \beta) q (L - L_R^1)}{\rho + \eta (L_R^1) L_R^1} = 1. \] \hspace{1cm} (28)

These two equations can have solutions \( L_R^1 \) and \( L_R^2 \neq L_R^1 \), which would correspond to the possibility of a two-period endogenous cycle.
Labor Market Implications

- So far creative destruction only destroyed the monopoly rents of incumbent producers. In more realistic settings, it may:
  - Dislocate previously employed workers.
  - Destroy firm-specific skills: workers (and firms) may be less willing to make specific human capital and other investments.
Conclusions

- Forward-looking incentives driving growth in this Schumpeterian model as well.
- But the “industrial organization” of growth is richer than in the basic Romer or Grossman-Helpman model.
- Most important ideas:
  - Creative destruction
  - Business stealing effect and conflict of interest
  - Growth and planning horizons of incumbents related
  - Possible uneven growth