Confidence and the Propagation of Demand Shocks*

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Abstract

This paper revisits the question of why a negative shock to consumer spending can trigger a recession. Our theory centers on an informational or a behavioral friction that plays a dual role. First, it lets the aggregate supply of today's goods increase with their relative price. Second, it gives rise to a feedback chain between outcomes and beliefs: as output and real returns fall, consumers and firms become pessimistic about the future, which in turn feeds into a further drop in aggregate spending and output, a further drop in confidence, and so on. The first element represents a non-monetary version of Lucas (1972). The second element introduces a novel propagation and amplification mechanism, which can be interpreted as a "confidence multiplier."

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1 Introduction

Fluctuations in aggregate demand, such as those triggered by shocks to the net worth of consumers (Mian and Sufi, 2014) or by swings in their expectations (Beaudry and Portier, 2006; Lorenzoni, 2009), appear to cause business cycles. Yet, the precise mechanism through which a drop in aggregate demand can precipitate a recession, or a fiscal stimulus can offset it, is unclear. This paper adds to the theoretical investigations of this question and, in particular, of the role played by confidence.

Background. To fix ideas, consider the baseline RBC model, abstract from investment, and let a negative shock to the discount rate of the representative consumer proxy for a drop in the aggregate demand for goods today relative to tomorrow.[1] In equilibrium, such a shock does not generate a recession. It only triggers a perfectly-offsetting movement in the relative price of today's goods, the real interest rate. This illustrates the challenge of making sense of demand-driven business cycles.[2]

The baseline New Keynesian model addresses this challenge by adding nominal rigidity and having monetary policy arrest the adjustment in the real interest rate. This effectively equates any demand-driven recession to a monetary contraction (one relative to the flexible-price benchmark). A different tradition equates demand-driven cycles to coordination failures and sunspot fluctuations.[3] This allows "confidence" to play an autonomous role in the business cycle, but *only* in the presence of multiple equilibria; it also begets the question of what triggers the variation in confidence in the first place.[4]

This paper aims at shedding new light on the last question and on the reasons why shifts in aggregate demand can trigger business cycles *even* without the aid of monetary non-neutrality.

Preview. Our theory is built on two key elements. The first one is the confusion, rational or irrational, of inter-temporal and intra-temporal terms of trade: when some agents shift their spending from the future to the present, others perceive a good time to work and produce. The second element is a positive feedback mechanism that lets one's optimism or pessimism reinforce that of others.

The first element builds on Lucas (1972). But whereas that paper focused on breaking monetary neutrality and obtaining a Philips curve, or an aggregate supply for today's goods that increases with the *nominal* price level, we preserve monetary neutrality by letting the numeraire be real future consumption and obtain an aggregate supply for today's goods that only increases with their *relative* price, the real interest rate. This allows us to accommodate the Keynesian narrative of how "demand drives supply," while keeping monetary policy out of the picture.[5]

[1] This proxy is standard in the literature and can be mapped to shocks in consumer credit (Eggertsson and Woodford, 2003; Guerrieri and Lorenzoni, 2017) or shifts in consumer expectations (Beaudry and Portier, 2006; Lorenzoni, 2009).

[2] Adding investment exacerbates the challenge: a negative demand shock causes a *boom* (Barro and King, 1984).

[3] Diamond (1982); Cass and Shell (1983); Cooper and John (1988); Benhabib and Farmer (1994)

[4] Angeletos and La'O (2013) have sought to address the first issue but not the second one. Akerlof and Shiller (2010, ch.1), on the other hand, argue that the role of confidence goes "beyond the rational." They thus advocate a behavioral approach, but, like the literature on multiple equilibria, do not spell out a theory of what drives confidence.

[5] We follow this route, not because we believe that the assumed numeraire is realistic or that monetary policy is neutral (we don't!), but rather because we find the disentangling of business cycles and monetary policy intellectually useful.

The second element hinges on letting the belief friction enter not only aggregate supply but also aggregate demand: relative to a frictionless world, consumers turn excessively optimistic during a boom, and excessively pessimistic during a recession. This introduces a feedback loop between economic activity and beliefs, which is the most novel aspect of our contribution.

To illustrate, consider a negative, aggregate, discount-rate shock. As long as the aggregate supply of today's goods increases with their relative price, whether for the reason articulated above or for some other reason,[6] the shock triggers a drop in employment and output. This justifies a drop in consumer spending even in the absence of any confusion. But because consumers fail to recognize that this was only the product of an *intertemporal* shift in aggregate spending and, instead, interpret it as a negative, persistent, idiosyncratic income shock, aggregate spending falls even more. As this happens, the firms experience an even greater drop in demand, which in turn leads to an even greater reduction in hiring and income, an even greater drop in consumer confidence, and so on.

We refer to this feedback mechanism as the "confidence multiplier." It is connected to the Keynesian income-spending multiplier, but differs from it in three respects. First, it does not rest on nominal rigidity. Second, it requires a certain departure from full information and/or full rationality, one that lets consumers be overly optimistic during a boom and overly pessimistic during a recession. Third, it allows the optimism or pessimism of one consumer to feed into that of others.

PE vs GE, and a macroeconomic complementarity. Another way to understand our results is in terms of the PE and GE effects of aggregate demand shocks. In the frictionless RBC benchmark that serves as our starting point, the PE effect is perfectly offset by the GE adjustment in the real interest rate. The first element of our paper arrests this countervailing GE effect by letting aggregate supply be positively sloped. The second element adds a *new* GE effect, in the form of the confidence multiplier.

In contrast to the neoclassical GE effect, the GE effect added here reinforces the PE effect. In other words, the economy is effectively transformed from a game featuring strategic substitutability to one featuring strategic complementarity. This echoes Angeletos, Lorenzoni, and Pavan (2010), Benhabib, Wang, and Wen (2015), Gaballo (2017), and Chahrour and Gaballo (2018), who also argue that informational frictions can be the source of macroeconomic complementarity—even of multiple equilibria—but do not contain the particular mechanism identified here.

Blending the rational and the irrational, and empirical backdrop. Our baseline model maintains rationality but allows an informational friction in the traditions of Lucas (1972) and Sims (2003): workers and consumers alike are confused because they do not have access or do not pay enough attention to all relevant information. Empirical evidence in support of such a friction abounds.[7] In our setting,

[6] For example, because of heterogeneity and incomplete markets, as in the flexible-price models contained in Beaudry and Portier (2013) and Guerrieri and Lorenzoni (2017).

[7] For example, Coibion, Gorodnichenko, and Ropele (2018) show, using a survey by the Bank of Italy, that firms ignore relevant, public information about the state of the economy: they change both their expectations and their behavior when such information is made salient to them. Cavallo, Cruces, and Perez-Truglia (2017), on the other hand, use an experiment

such a friction helps rationalize both an upward-sloping aggregate supply and the aforementioned feedback loop between "confidence" and market outcomes.

The same feedback loop emerges in a variant in which agents are *irrational* in a specific way, forming extrapolative beliefs as in Greenwood and Shleifer (2014) and Gennaioli, Ma, and Shleifer (2016). This allows for a behavioral re-interpretation of our theory, which we welcome. It also underscores that the key for our purposes is, not the distinction between irrationality and informational friction, but rather the over-reaction of certain beliefs about future to recent experiences.

At first glance, this may appear to contradict the evidence that forecasts of aggregate outcomes such as inflation and GDP tend to under-react to aggregate shocks. However, this is *not* the case: in our setting, the expectations of aggregate outcomes display under-reaction, simply because agents have incomplete information about the underlying aggregate shock. Overreaction is only present in the expectations that consumers form about their *own* income and firms about their *own* earnings.

The relevant evidence is therefore that contained in Rozsypal and Schlafmann (2018) and Gennaioli, Ma, and Shleifer (2016) about, respectively, consumer expectations of income and CFO expectations of firm earnings. Although these papers do not directly address the feedback mechanism we study here, they do offer evidence of overreaction in the relevant kind of beliefs.

Fiscal Policy. During the Great Recession, Robert Shiller blogged that a fiscal stimulus could help boost consumer confidence,[8] prompting the following response by N. Gregory Mankiw:[9]

> "Yale's Bob Shiller argues that confidence is the key to getting the economy back on track. I think a lot of economists would agree with that. The question is what it would take to make people more confident. ... Until we figure it out, it is best to be suspicious of any policy whose benefits are supposed to work through the amorphous channel of 'confidence.'"

Our theory puts a specific structure on this amorphous channel, in the manner described already. In so doing, it also helps accommodate the idea that fiscal stimuli can boost confidence. Furthermore, our theory predicts that fiscal stimuli are more effective when they are front-loaded. This contrasts the prediction of the New Keynesian model that back-loading is more effective, at least as long as the nominal interest rate is not adjusted enough. In that model, back-loading stimulates demand by piling up inflationary expectations. In our setting, front-loading triggers a virtuous feedback loop between economic outcomes and consumer beliefs.

Layout. Section 2 expands on the relation of our paper to the literature. Section 3 introduces our baseline model and studies our frictionless benchmark, which embeds the lessons of the standard RBC model and represents our point of departure. Sections 4 and 5 contain our main results. Section 6 considers a few extensions and discusses the broader insights. Section 7 concludes.

to show that consumers do not process information optimally even when such information is readily available to them.

[8]See also Akerlof and Shiller (2010, ch. 1).

[9]Greg Mankiw's blog, January 27, 2009: http://gregmankiw.blogspot.com/2009/01/.

2 Related literature

Our paper's theme that informational frictions help capture the role of "confidence" or "sentiment" in business cycles echoes Angeletos and La'O (2013), Benhabib, Wang, and Wen (2015), and Huo and Takayama (2015). These works attribute the variation in confidence to extrinsic shocks and do not feature the feedback mechanism at the core of our paper. By contrast, the business-cycle trigger in our setting is intrinsic and "confidence" is modeled as a propagation and amplification mechanism.

Related in this respect are Chahrour and Gaballo (2018) and Ilut and Saijo (2018). These works focus on different economics—the former on the confusion of supply shocks for demand shocks, the latter on ambiguity, uncertainty, and learning—but share the broader idea of letting intrinsic shocks drive consumer or producer beliefs away from the full-information, full-rationality benchmark.

Chahrour and Gaballo (2018) also share the idea that signal-extraction problems can be the source of macroeconomic complementarity in a business-cycle context. But they do not study the particular kind of confusion we consider here and therefore do not share our "confidence multiplier." They also highlight the possibilities of multiple equilibria and of discontinuities with respect to the level of noise, which are interesting but not the focus of our paper. Similar points apply to Benhabib, Wang, and Wen (2015) and Gaballo (2017). At the same time, the informational origin of the strategic complementarity is the common thread that distinguishes these papers and ours from a literature on "beauty contests" that studies settings in which the strategic complementarity exists *without* the informational friction (Morris and Shin, 2002; Woodford, 2003; Angeletos and Pavan, 2007; Myatt and Wallace, 2012).

By letting aggregate supply respond to the real interest rate *without* the aid of nominal rigidity and monetary policy, our paper connects to Beaudry and Portier (2013, 2018) and Guerrieri and Lorenzoni (2017). The flexible-price models in these works achieve the same objective with a different method, by adding heterogeneity and credit frictions. What distinguishes our contribution is not only the emphasis on beliefs but also the feedback mechanism on the demand side.

An additional difference between our paper and that of Beaudry and Portier (2018) is the following. Their flexible-price model features an aggregate supply that *decreases* with the real interest rate. This is because a higher rate in that model maps to a higher financial distortion. For our purposes, instead, the real interest rate should be interpreted as the relative scarcity of today's goods: the supply side of our model abstracts from financial distortions and instead seeks to match the partial-equilibrium logic that the supply of a good *increases* with its relative price.

Last but not least, our focus on the combination of informational frictions and demand shocks is reminiscent of Lorenzoni (2009). That paper proposes a new interpretation of demand shocks, in terms of noisy expectations about future productivity and income, but maintains the propagation mechanism of the New Keynesian model. Our paper instead revisits the propagation mechanism itself, adding a new, belief-based multiplier. The two contributions are complements, not substitutes.

3 The Model

In this section, we spell out the micro-foundations of our setting, study a benchmark that epitomizes the RBC framework, and explain the departure made from it.

3.1 Micro-foundations

The building block of our model is a neoclassical, production economy, as in the flexible-price, or RBC, core of the modern business-cycle paradigm. To simplify the analysis, we abstract from investment, shut down wealth effects on labor supply, allow only two periods, denoted by $t \in \{1, 2\}$ and loosely interpreted as, respectively, the "short run" and the "long run." We let trading be centralized in the second period, at which point everything is common knowledge. The trading is nevertheless geographically segmented in the first period, so as to accommodate an informational friction. We allow for monopoly power but abstract from monetary non-neutrality by letting prices be flexible and transactions be quoted in real terms, relative a second-period composite good.

Islands and agents. There is a continuum of islands, indexed by $i \in [0, 1]$. On each island, there is a continuum of firms, each being a monopolistically competitive producer of a differentiated good. We use (i, j) to identify both the j-th variety produced on island i and the firm producing it. On each island, there is also a representative household, indexed by $h = i$ and consisting by a worker and a consumer. In both periods, a worker is employed in the island the household resides in. In the first period, a consumer "visits," and can purchase the goods of, only a non-representative sample of the islands. In the second period, the consumer trades and consumes all the goods.

Trading and numeraire. There is no fiat money ("dollar bills"), no nominal rigidity, and no monetary authority controlling the quantity of money (as in Lucas, 1972), the real interest rate (as in the New Keynesian model), or anything else of consequence. Instead, all payments take place in real, privately-issued IOUs, where one IOU equals the promise to deliver, in the second period, a basket containing one unit of every commodity. All prices are denominated relative to the value of this basket. And the real interest rate is determined in a Walrasian fashion, letting demand meet supply. As anticipated in the Introduction, this modeling approach is useful, even if hard to map to the real world, because it helps disentangle our theory from the role of monetary policy.[10]

Preferences and Demand Shocks. The preference of household $h \in [0, 1]$ is represented by

$$\mathcal{U}^h = U\left(c_1^h, n_1^h\right) + \beta^h U\left(c_2^h, n_2^h\right), \tag{1}$$

where n_t^h is the household's labor supply in period $t \in \{1, 2\}$, c_t^h is its effective consumption in period

[10] It is possible to recast the analysis in a monetary variant in which prices are quoted in dollars and the aforementioned real IOUs are replaced by dollar-denominated balances kept in the central bank. This dilutes the aforementioned disentangling by letting monetary policy be non-neutral, but does not affect the essence of our results regarding the confidence multiplier.

$t \in \{1, 2\}$ (defined below), β^h is its discount factor between period 1 and period 2, and U is the per-period utility function. The latter takes a GHH form:

$$U(c, n) = \frac{(c - v(n))^{1-\frac{1}{\psi}}}{1 - \frac{1}{\psi}} \qquad \text{and} \qquad v(n) = \frac{n^{1+\kappa}}{1+\kappa}, \tag{2}$$

for some scalars $\psi, \kappa > 0$. The GHH specification shuts down wealth effects on labor supply. The scalar κ is the inverse of the Frisch elasticity of labor supply. The scalar ψ controls the elasticity of intertemporal substitution.

Because the discount factor β^h determines the household's demand for all goods (inclusive of leisure) in period 1 relative to that in period 2, allowing for random variation in it serves as a proxy for shifts in inter-temporal spending attitudes. In particular, we let

$$\beta^h = \bar{\beta}\delta^h,$$

where $\bar{\beta} \sim \log\mathcal{N}(-\frac{1}{2}\sigma_\beta^2, \sigma_\beta^2)$ is an aggregate shock and $\delta^h \sim \log\mathcal{N}(-\frac{1}{2}\sigma_\delta^2, \sigma_\delta^2)$ is an idiosyncratic shock, i.i.d. across h.[11] The friction introduced shortly removes common knowledge of $\bar{\beta}$, and of the associated aggregate fluctuations, but allows each household h to know δ^h, its own discount factor.

We next let the effective consumption in each period be given by a nested CES aggregator of the consumption of the differentiated goods consumed by the household. Specifically,

$$c_t^h = F\left(\left\{c_{i,t}^h, \xi_i\right\}_{i \in \mathcal{C}_t^h}\right) \qquad \text{and} \qquad c_{i,t}^h = H\left(\left\{c_{i,j,t}^h\right\}_{j \in [0,1]}\right), \tag{3}$$

where the functions F and H are CES aggregators, $c_{i,j,t}^h$ is the consumption of variety j from island i in period t, $c_{i,t}^h$ is a consumption index for all the varieties consumed from island i in period t, $\mathcal{C}_t^h$ is the set of islands which household h "visits" (i.e., consumes the products of) in period t, and $\xi_i \sim \log\mathcal{N}(-\frac{1}{2}\sigma_\xi^2, \sigma_\xi^2)$ is an island-specific taste shock, i.i.d. across islands. The modeling role of this shock is to limit the information that the firms, or the households, of an island can extract from local demand about the underlying aggregate shock.

One can think of the islands as different categories of expenditure (e.g., food vs entertainment) and the different goods within each island as different varieties of the same category. To simplify the exposition, we fix the elasticity of substitution across islands to 1. On the other hand, to make sure that the monopolist's problem is well defined and the markup is finite, we let the elasticity of substitution across the different varieties of the same island be $\epsilon > 1$.[12]

[11] The mean of $\log\bar{\beta}$ is set to $-\frac{1}{2}\sigma_\beta^2$ so that that $\mathbb{E}\left[\bar{\beta}\right] = 1$, and similarly for δ^h. We also assume that a law of large number holds for the continuum of agents so that $\int_{h\in[0,1]} \delta^h dh = \mathbb{E}\left[\delta^h\right]$. The same principles apply throughout the paper.

[12] That is, $F\left(\{c_{i,t}^h, \xi_i\}_{i\in\mathcal{C}_t^h}\right) = \exp\left(\frac{1}{\int_{i\in\mathcal{C}_t^h} \xi_i di} \int_{i\in\mathcal{C}_t^h} \xi_i \ln(c_{i,t}^h) di\right)$ and $H\left(\{c_{i,j,t}^h\}_{j\in[0,1]}\right) = \left(\int_{j\in[0,1]} (c_{i,j,t}^h)^{\frac{\epsilon-1}{\epsilon}} dj\right)^{\frac{\epsilon}{\epsilon-1}}$.

Firms. Consider firm (i, j) on island i. We assume that production is linear in labor, so that the firm's output in period t is given by

$$y_{i,j,t} = l_{i,j,t}, \tag{4}$$

where $l_{i,j,t}$ is the labor input. The firm's profit is given by $\pi_{i,j,t} = q_{i,j,t}y_{i,j,t} - w_{i,t}l_{i,j,t}$, where $q_{i,j,t}$ is the price of good (i, j) and $w_{i,t}$ is the wage of the local labor market in island i. Note that each firm is a price-setter in the market for the differentiated commodity she produces. But since there are many firms in each island competing for the local labor, each firm is a price taker in the labor market of the island it operates in (and so is the island's representative household). Finally, we assume that the firms of any given island are owned by the local household and distribute their profits to it as dividends. It follows that the dividend income received by household h (also the total dividends in island $i = h$) in period t is given by $e_{h,t} = \int \pi_{h,j,t}dj$.

Budget Constraints. Consider household h. Its income in period t is $w_{h,t}n_t^h + e_{h,t}$, where $w_{h,t}$ is the real wage on the island $i = h$ the household lives in and $e_{h,t} = \int \pi_{h,j,t}dj$ are the corresponding firm profits. Its budget constraint is therefore given by

$$\int_{i\in\mathcal{C}_1^h}\int_{j\in[0,1]} q_{i,j,1}c_{i,j,1}^h dj di = w_{h,1}n_1^h + e_{h,1} + b^h \tag{5}$$

in the first period, and by

$$\int_{i\in\mathcal{C}_2^h}\int_{j\in[0,1]} q_{i,j,2}c_{i,j,2}^h dj di + b^h = w_{h,2}n_2^h + e_{h,2} \tag{6}$$

in the second, where b^h is the net amount of IOUs issued by the household, $\mathcal{C}_1^h$ is the random subset of islands visited in the first period (to be specified shortly), and $\mathcal{C}_2^h = [0, 1]$.

The Question of Interest. Let the aggregate levels of employment, consumption and output be measured by, respectively,

$$\ell_t \equiv \int_{h\in[0,1]} n_t^h dh, \qquad c_t \equiv \int_{h\in[0,1]} c_t^h dh, \qquad \text{and} \qquad y_t \equiv \int y_{i,t}di,$$

with $y_{i,t} \equiv \int y_{i,j,t}dj$ being the measure of output at the island level.[13] The question of interest is how these outcomes vary with $\bar{\beta}$, the aggregate demand shock. In the rest of this section, we first answer this question in a benchmark that represents the standard RBC framework. We then explain the departure made from this benchmark, namely the introduction of an information friction.

[13] Another possible measure of aggregate output is $y_t' \equiv F\left(\{y_{i,t}', \xi_t\}_{i\in[0,1]}\right)$, where $y_{i,t}' \equiv H\left(\{y_{i,j,t}\}_{j\in[0,1]}\right)$. Adopting this alternative measure makes no difference for our purposes, because, in the log-linearized equilibria we study, $\log y_t'$ and $\log y_t$ are equal up to constant. A similar point applies to the measure of consumption.

3.2 Frictionless Benchmark

The model is closed by specifying the matching of consumers and islands in period 1 and the information structure. Let us momentarily assume that consumers visit *all* islands and that $\bar{\beta}$ is commonly known. Without further loss, let us also shut down the idiosyncratic discount-rate and taste shocks. The economy then admits a representative household, as in the standard RBC model.

It is straightforward to verify that, in this benchmark, the equilibrium level of labor in both periods is pinned down by the following condition:

$$v'(\ell_t) = \frac{1}{1+\mu}, \tag{7}$$

where $\mu = \frac{1}{\epsilon-1} \geq 0$ is the monopoly markup. The interpretation is familiar: the marginal disutility of labor is equated to the marginal product of labor times the monopoly distortion. And since $y_1 = \ell_1$, it is immediate that the equilibrium level of output in the first period is given by

$$y_1 = y^* \equiv (v')^{-1}\left(\tfrac{1}{1+\mu}\right), \tag{8}$$

which is invariant to $\bar{\beta}$. The same applies to the second period: $y_2 = y^*$. This verifies that demand shocks fail to generate fluctuations in aggregate employment and output.

Because condition (8) encapsulates equilibrium in the labor market but not optimality in intertemporal consumption choices, it can be interpreted as the aggregate supply of goods in period 1. The aggregate demand, on the other hand, is obtained from the Euler condition of the representative household and can be expressed as follows:

$$Q_1 = \frac{1}{\bar{\beta}}\left(c_1 - v(c_1)\right)^{-\frac{1}{\psi}}, \tag{9}$$

where Q_1 stands for the the real interest rate.[14] Finally, market clearing imposes $c_1 = y_1$. And since y_1 is fixed by (8), we conclude that the equilibrium value of Q_1 moves in the opposite direction than $\bar{\beta}$, by an amount just enough to offset the effect that the exogenous shock would have had on c_1 if Q_1 had been held constant. It is in this sense that the GE adjustment of the real interest rate exactly offsets the PE effect of the aggregate demand shock.

This result does not depend on the GHH specification. It follows more generally from the assumptions of a representative consumer, a constant labor wedge, and time-separable preferences (Barro and King, 1984). It also extends to the presence of idiosyncratic shocks insofar as we focus on a log-linear approximation of the equilibrium, which we do in the rest of the paper.

[14] To obtain (9), take the Euler condition, $U_c(c_1, \ell_1) = Q_1\bar{\beta}U_c(c_2, \ell_2)$, impose $\ell_2 = c_2 = y^*$ and $\ell_1 = c_1$, and normalize $U_c(y^*, y^*) = 1$. Also, note that Q_t is defined shortly as the price index of a basket containing one unit of all the goods produced in period t; by our choice of numeraire, though, $Q_2 = 1$ and Q_1 is the relative price, or the real interest rate.

3.3 The Informational Friction

We depart from the benchmark studied above by introducing an informational friction, which plays a dual role. On the supply side, it lets workers be confused about the inter- and intra-temporal movements in the returns to labor. On the demand side, it lets consumers confuse inter-temporal shifts in aggregate spending for persistent, idiosyncratic income shifts. The latter element requires that either that we relax the assumption that consumers are rational or that we let them visit, and extract information from, only a non-representative sample of the islands in the first period. In the main analysis, we follow the latter route. The rest of this section therefore fills in the details of the random matching of consumers to islands and of the information of all agents (consumers, workers, firms).

Matching. To guarantee that households do not infer the aggregate shock from the prices they transact on in the "short run," we let each household be randomly matched to, and visit, an idiosyncratic and non-representative sample of the islands in the first period. On the other hand, since it is fine for our purposes that the shock becomes commonly known in the "long run," we let the households visit all the islands in the second period. This also justifies our assumption that households can trade IOUs in terms of a basket of all the second-period goods. We next explain what this structure means in terms of the price index faced by a household and the demand faced by each firm.

Let $q_{i,t} = \left(\int_{j\in[0,1]} q_{i,j,t}^{1-\epsilon} dj\right)^{1/1-\epsilon} dj$ be the ideal price index of the goods produced on island i in period t, $q_t^h = \left(\int_{i\in\mathcal{C}_t^h} \xi_i di\right) \exp\left(\int_{i\in\mathcal{C}_t^h} \xi_i \ln(p_i/\xi_i) di / \int_{i\in\mathcal{C}_t^h} \xi_i di\right)$ be the ideal price index of the basket of goods consumed by household h in period t, $d_{i,t} \equiv \int_{j\in[0,1]} \int_{\{h: i\in\mathcal{C}_t^h\}} q_{i,j,t} c_{i,j,t}^h dh dj$ the total spending for the goods produced on island i, and $Q_t = \int_{h\in[0,1]} q_t^h dh$ and $D_t \equiv \int_{i\in[0,1]} d_{i,t} di$ be the corresponding aggregates. By our choice of numeraire, $Q_2 = 1$ and Q_1 can be interpreted as the average real interest rate between the two periods. By the assumption that all households visit all islands and consume all goods in the second period, $q_2^h = Q_2 = 1$ for all h and $d_{i,2} = \xi_i D_2$ for all i. Finally, by the assumption the set of the consumers visiting any given island i is a non-representative sample of all the consumers in the economy and, symmetrically, the set of the islands visited by any given household h is a non-representative sample of the islands, q_1^h and $d_{i,1}$ may deviate from, respectively, Q_1 and $\xi_i D_1$.

In particular, we assume that the matching process is such that, in equilibrium,

$$q_1^h = Q_1 v^h \qquad \text{and} \qquad d_{i,1} = D_1 \xi_i \zeta_i,$$

where $v^h \sim \log\mathcal{N}\left(-\frac{1}{2}\sigma_v^2, \sigma_v^2\right)$ is i.i.d. across h, $\zeta_i \sim \log\mathcal{N}\left(-\frac{1}{2}\sigma_\zeta^2, \sigma_\zeta^2\right)$ is i.i.d. across i, v^h is independent of ζ_i, and (σ_v, σ_ζ) are both positive. This means that the random matching introduces both a household-specific price shock and a island-specific demand shock. The former guarantees that a consumer's knowledge of its own price index is only an imperfect, private signal of the underlying aggregate shock. The latter plays a similar role as the island-specific taste shock, ξ_i.

In Appendix B, we spell out the details of an explicit matching process which gives rise to the random variables v^h and ζ_i assumed above. This makes σ_v and σ_ζ endogenous to the equilibrium, which complicates the analysis without changing the essence. In the main text, we simplify the exposition by treating the random variables v^h and ζ_i as exogenous, similarly to Lorenzoni (2009).

Needless to say, one can imagine many other sources of idiosyncratic variation in consumer prices, firm demand, and household income that can play a similar modeling role as the matching shocks introduced here. What is key for our purposes is that such shocks limit the information that perfectly rational and perfectly attentive agents can extract from their observation of local economic outcomes. Rational inattention (Sims, 2003; Mackowiak and Wiederholt, 2009), sparsity (Gabaix, 2014), and related forms of bounded rationality could be complementary sources of the kind of confusion and mis-coordination we are after in this paper.

Information. We can now spell out the information of the households in the first period in terms of the following two assumptions.

1. The consumer of household h conditions her choice of the consumption bundle $(c_{l,k,1})_{l\in\mathcal{C}_1^h,k\in[0,1]}$ in the first period on the following information set:

$$\mathcal{I}_1^h = \left\{ \beta_1^h; w_{h,1}, e_{h,1}; (q_{l,k,1})_{l\in\mathcal{C}_1^h,k\in[0,1]} \right\}.$$

2. The worker of household h conditions her labor supply in the first period on the following information set:

$$\tilde{\mathcal{I}}_1^h = \left\{ \beta_1^h; w_{h,1}, e_{h,1}; x_1^h \right\},$$

where $x_1^h = q_1^h \varepsilon^h$ is a noisy signal of the price index of the basket consumed by the household, and $\varepsilon^h \sim \log\mathcal{N}\left(0, \sigma_\varepsilon^2\right)$ is independently distributed across households.

The first assumption means that the consumer of each household knows her own preferences and observes perfectly her two sources of income and the prices of the goods she purchases, but does not directly observe the underlying shocks that drive the variation in either her income or the prices of the goods she consumes. This assumption is crucial for the feedback mechanism at the core of our paper and is further discussed in due course.

The second assumption maintains that the worker of each household knows her preferences and her wages, but adds the extra restriction that labor supply only depends on an imperfect signal of the prices of the goods that the household consumes. This assumption can be given at least two interpretations. The first one is in terms of inattention or salience: the household may be less attentive to commodity prices when choosing its labor supply than when making her consumption choices, in part because such prices are less salient in the former context than in the latter. This interpretation requires a departure from full rationality, but is consistent with the evidence from psychology that suggests that the decision maker may not evoke all relevant information for each decision (Anderson, 2009; Bordalo, Gennaioli, and Shleifer, 2017; Lian, 2018), or with the principle "What You See Is All There Is" (Kahneman, 2011). The second interpretation, which we formalize in Section 6.2, avoids any departure from rationality and, instead, enriches the environment so that labor supply depends, not only on concurrent prices that can be perfectly observed, but also on future prices that have to be forecast. Either way, this assumption is crucial for the supply-side, Lucas-like part of our theory, but not for the demand-side feedback mechanism. The latter is instead driven by the inability of the consumers to tell apart the sources of variation in their income.

So far we have spell out the information of the consumers and the workers, but not of the firms. Our baseline model effectively abstracts from any informational friction among the firms by letting them observe, in both periods, the local wage and the demand for their products. Because of the CES specification and the symmetry of the firms within each island i, the demand faced by each firm (i, j) in period t can always be written as

$$\int_{\{h:i\in\mathcal{C}_t^h\}} c_{i,j,t}^h dh = d_{i,t}\left(q_{i,j,t}/q_{i,t}\right)^{-\epsilon} = \tilde{d}_{i,t}\left(q_{i,j,t}\right)^{-\epsilon} \quad \forall i,j,t. \tag{10}$$

where $\tilde{d}_{i,t} \equiv d_{i,t}q_{i,t}^{\epsilon}$ captures the intercept of the demand curve faced by the firm. Knowledge of the entire demand curve is therefore equivalent to knowledge of $\tilde{d}_{i,t}$. As it will become clear, knowledge of this variable together with knowledge of the local wage, $w_{i,t}$, suffices for the firm to make the same employment, production, and pricing choices as those she would have made under full information about the state of Nature. This property is maintained throughout the main analysis, but is relaxed in Section 6.4 so as to accommodate a confidence multiplier among firms in addition to that among consumers.

Equilibrium. The solution concept is Noisy Rational Expectations equilibrium. That is,

1. Each household h chooses its consumption and labor supply so as to maximize expected utility subject to the budget constraint in (5) and (6) and the informational constraints specified above.

2. Each firm (i, j) chooses its demand for labor, its level of production, and the price of its product so as to maximize its expected valuation, subject the demand curve for its product.

3. The goods and labor markets clear: for every island i, every period t, and every realization of uncertainty,

$$\int_{\{h:i\in\mathcal{C}_t^h\}} c_{i,j,t}^h dh = y_{i,j,t}\ \forall j \qquad \text{and} \qquad n_t^i = \int l_{i,j,t}dj. \tag{11}$$

4 Aggregate Supply, Aggregate Demand, and Beliefs

In this section we show how the equilibrium of the economy in the first period can be reduced to two sets of equations, one representing aggregate supply (AS) and another representing aggregate demand (AD). We further show how beliefs enter each block. We thus set the stage for the next section, which completes the equilibrium characterization by studying the fixed point between aggregate supply, aggregate demand, and beliefs. To make the analysis tractable, we log-linearize the equilibrium conditions and, with abuse of notation, henceforth re-interpret all the variables as log-deviations from the deterministic steady state.

4.1 The Long Run

In period 2 ("long run"), the economy features different individual-level outcomes than the frictionless benchmark, not only because of the presence of idiosyncratic shocks to fundamentals, but also because of the mistakes workers and consumers make in period 1. However, because the informational friction disappears and no mistakes are made in period 2, the equilibrium level of aggregate output in that period remains the same as in the frictionless benchmark.

Lemma 1 *In period* 2, *aggregate output is given by* $y_2 = 0$, *as in the frictionless benchmark.*[15]

Given this property, which we verify in the Appendix, the analysis in the sequel focuses exclusively on the determination of the aggregate equilibrium outcomes in period 1 ("short run"). For notation simplicity, we henceforth drop the subscript 1 from period 1 objects.

4.2 Aggregate Supply

Consider the optimal labor supply of household h. Optimality requires that the disutility of labor be equated to the perceived real wage. This gives

$$n^h = \frac{1}{\kappa} \left\{ w_h - \mathbb{E}\left[q^h \middle| \tilde{\mathcal{I}}^h \right] \right\}, \tag{12}$$

where, recall, both w_h and q^h are denominated in terms of the period-2 numeraire. Because $\tilde{\mathcal{I}}^h$ contains only an imperfect signal of q^h, the above condition embeds the idea that workers may have incomplete knowledge of, or may not pay enough attention to, all the relevant prices when making her labor-supply decision. As anticipated, this allows the AS curve to turn elastic from vertical; but whereas Lucas (1972) had aggregate supply increase with the *nominal* price level, we only let it increase with the *real* interest rate.

[15]Note that, because of the notation adopted in this section (i.e., under the log-linearization of the model), $y_2 = 0$ means a zero gap from steady state, which maps to $y_2 = y^*$ under the notation of Section 3.

To see this point and derive the AS curve, we next characterize the optimal behavior of the firms and combine it with labor supply. By taking the optimality conditions of the typical firm and by noting that firms are homogeneous within islands, we infer that all firms on island i make the same employment, production, and pricing choices. With this in mind, the next lemma characterizes the island-level outcomes.

Lemma 2 *Employment, production and prices in island i satisfy the following conditions:*

$$l_i \equiv \int l_{i,j} dj = y_i = d_i - w_i \quad \textit{and} \quad q_i = w_i, \tag{13}$$

where d_i is the total spending on goods produced on island i, *given by*

$$d_i = Q + c + \xi_i + \zeta_i. \tag{14}$$

Condition (13) is straightforward to interpret. The first part says that firms produce more if local demand (d_i) goes up, and less if local costs (w_i) go up. The second part equates the prices set by the firms (q_i) with their marginal costs (w_i) times a constant markup (which disappears in the condition because of the log-linearization).

We are now ready to construct our AS curve. Consider conditions (12), (14) and (13). Substitute the first and the second into the third, aggregate, and solve for y to get:

$$y = \tfrac{1}{1+\kappa} \left\{ Q + c - \mathcal{Q}^e \right\},$$

where $\mathcal{Q}^e \equiv \int \mathbb{E}\left[q^h \middle| \tilde{\mathcal{I}}^h \right] dh$. This condition gives aggregate production as an increasing function of aggregate expenditure, captured here by $Q + c$, and a decreasing function of labor costs, captured here by $\mathcal{Q}^e$. Next, replace c in this condition with y and re-solve for y to reach the following result.

Proposition 1 (Aggregate Supply) *The optimal behavior of the firms, the optimal labor supply of the households, and market clearing imply the following equilibrium restriction:*

$$y = \tfrac{1}{\kappa} \left(Q - \mathcal{Q}^e \right). \tag{15}$$

Because the above result uses optimality on the production side of the economy along with market clearing, but does not use optimality of consumer spending, condition (15) can be interpreted as a description of the aggregate supply in the economy. When workers have perfect information, $\mathcal{Q}^e$ coincides with Q, implying that the AS curve is vertical at $y = 0$.[16] When instead workers have

[16] Note that, because of the notation adopted in this section (i.e., under the log-linearization of the model), $y = 0$ means a zero gap from steady state, which maps to $y = y^*$ under the notation of Section 3.

imperfect information, $\mathcal{Q}^e$ moves less than one-to-one with Q (this will be shown shortly), implying that the effective AS curve is upward sloping. As anticipated, this result represents a non-monetary variant of Lucas (1972), with the real interest rate showing up in place of the nominal price level.

4.3 Consumption and Aggregate Demand

We now turn to optimal consumption and aggregate demand. Consider household h. Her total income in each period coincides with the total revenue collected by the local firms. Letting $i = h$ be the island in which that household lives, we thus have that[17]

$$w_h n^h + e_h = d_i. \tag{16}$$

Using this fact along with the household's budget constraint and the relevant first-order conditions, we reach the following representation of the optimal consumption.

Lemma 3 *Household h's optimal consumption in period 1 is given by*

$$c^h = -\frac{\beta}{1+\beta}\sigma\left(\beta^h + q^h\right) + \frac{1}{1+\beta}\left(d_h - q^h\right) + \frac{\beta}{1+\beta}\mathbb{E}^h\left[d_{h,2}\right] + \frac{\beta}{1+\beta}\varphi\left(n^h - \mathbb{E}^h\left[n_2^h\right]\right), \tag{17}$$

where $\sigma \equiv \psi\left(c^* - v\left(n^*\right)\right)/c^* = \psi\left(1 - \frac{1}{(1+\kappa)(1+\mu)}\right)$ *is the elasticity of intertemporal substitution,* $\varphi = \frac{v(n^*)}{c^*}\left(\kappa+1\right) = \frac{1}{1+\mu} \in [0,1)$ *parametrizes the strength of consumption-labor complementarity embedded in the GHH preference, and* $\mathbb{E}^h\left[\cdot\right] = \mathbb{E}\left[\cdot|\mathcal{I}^h\right]$ *denotes household h's consumer's belief.*

The first term captures the combination of two intertemporal-substitution effects: that for the household's subjective discount-factor shock, β^h; and that of the household-specific real interest rate, q^h. The second and third terms capture the permanent income hypothesis: the second term corresponds to current income, the third term corresponds to expectations of future income. The last term reflects the consumption-labor complementarity implied by the GHH preference specification.

Aggregating the above condition across households gives

$$c = -\frac{\beta}{1+\beta}\sigma\left(\bar{\beta} + Q\right) + \frac{1}{1+\beta}c + \frac{\beta}{1+\beta}\int \mathbb{E}^h\left[d_{h,2}\right]dh + \frac{\beta}{1+\beta}\varphi\int\left(n^h - \mathbb{E}^h\left[n_2^h\right]\right)dh. \tag{18}$$

From labor market clearing in the first period and firm optimality in the second period, we have $n^h = y_h$ and $\mathbb{E}^h\left[n_2^h\right] = \mathbb{E}^h\left[y_{h,2}\right] = \frac{1}{1+\kappa}\mathbb{E}^h\left[d_{h,2}\right]$. Using these facts in the above condition, and solving for c, we arrive at the following result.

[17] Different from the rest of the Section, in (16), each variable represents its original meaning in Section 3, instead of the log-deviations from the deterministic steady state.

Proposition 2 (Aggregate Demand) *The optimal consumption behavior of the households, together with market clearing in both periods and firm optimality only in the second period, implies the following equilibrium restriction:*

$$c = \tilde{y} - \sigma Q + \mathcal{B} + \mathcal{H}. \tag{19}$$

where $\tilde{y} \equiv -\sigma\bar{\beta}$ *captures the exogenous shock, and where*

$$\mathcal{B} \equiv \int \mathbb{E}^h \left[d_{h,2}\right] dh \qquad \text{and} \qquad \mathcal{H} \equiv \varphi \left(y - \tfrac{1}{1+\kappa}\mathcal{B} \right),$$

capture, respectively, the consumers' perceptions of future income ("consumer confidence") and the complementarity between consumption and labor embedded in the GHH specification.

As it will become clear shortly, the dependence of c on $\mathcal{B}$ together with the latter's endogeneity to current activity opens the door to the feedback mechanism that is at the core of our contribution: when the households experience an increase in their current income due to an aggregate discount-factor shock, they may believe, rationally but mistakenly, that their future income will also be higher, and therefore find it optimal to increase their current spending relative to the frictionless benchmark.

The $\mathcal{B}$ term is therefore essential for our purposes. It also encapsulates the most important difference between our contribution and that of Lucas (1972). There, rational misperceptions were affecting aggregate supply but not aggregate demand. This was because the demand side of that model was fixed by the exogenous supply of money.

The $\mathcal{H}$ term, on the other hand, is of no interest to us. This term would disappear if preferences were separable between consumption and leisure. However, such separability would have introduced a wealth effect on labor supply, which instead we have opted to abstract from by adopting GHH preferences, at the expense of introducing a complementarity between consumption and labor. In any event, this term plays no essential role for our results. To simplify the exposition, the main text dispenses with this term by letting the monopoly power, or the labor wedge, be large enough.

Proposition 3 *Consider the limit as* $\mu \to \infty$ *(*$\epsilon \to 1$*). In this limit,* $\sigma \to \psi$*,* $\varphi \to 0$*,* $\mathcal{H} \to 0$*, and the AD curve reduces to the following:*

$$c = \tilde{y} - \sigma Q + \mathcal{B},$$

with $\tilde{y}$ *and* $\mathcal{B}$ *defined as in Proposition 2.*

This limit case, which we henceforth focus on, gives the aggregate demand for today's goods as the sum of three terms. The first term, $\tilde{y}$, measures the exogenous shift caused by the underlying discount-factor shock. The second term, $-\sigma Q$, captures the negative dependence of aggregate demand on the real interest rate. The third term, $\mathcal{B}$, captures its positive dependence on "consumer confidence." The more general case, which adds $\mathcal{H}$ as a fourth term, is studied in Appendix C. This changes certain details, such as the precise value of the threshold $\underline{\lambda}$ appearing in Proposition 7, but does not affect the essence of any of the lessons we deliver next.

4.4 Inference

The preceding two subsections characterized aggregate supply and aggregate demand under rationality but with otherwise arbitrary beliefs: the specific assumptions we made earlier about about the information structure were not invoked in Propositions 1 – 2. We now use these assumptions to put more structure on the two belief-related terms that show up in these propositions, namely $\mathcal{Q}^e$ and $\mathcal{B}$.

Proposition 4 *In any equilibrium, there exist a pair of scalars* $(\lambda_s, \lambda_d) \in (0,1]^2$ *such that: (i) the workers' perceptions of consumer prices satisfy*

$$\mathcal{Q}^e = \lambda_s Q; \tag{20}$$

and (ii) the consumers' perceptions of future income satisfy

$$\mathcal{B} = (1-\lambda_d)\varrho(Q+y), \tag{21}$$

where $\varrho \equiv \frac{\sigma_\xi^2}{\sigma_\zeta^2+\sigma_\xi^2} \in (0,1)$.

To understand this result, note first that, since there is a single aggregate shock, all aggregate variables, including average beliefs, are perfectly co-linear. It follows that the projection of any aggregate variable to any other aggregate variables has no residual, regardless of the level of the informational friction. The latter, however, determines how equilibrium beliefs covary with equilibrium outcomes in response to the underlying variation in $\bar{\beta}$.

Consider first how $\mathcal{Q}^e$ covaries with Q. When the workers are perfectly informed about consumer prices, $\mathcal{Q}^e = Q$. When instead they are imperfectly informed, $\mathcal{Q}^e$ moves less than one-to-one with Q, because they tend to underestimate the movements in these prices. It follows that $\mathcal{Q}^e = \lambda_s Q$ where $\lambda_s \in (0,1)$ represents an inverse measure of the confusion of the workers about the cost of their own consumption basket. That is, a lower λ_s means a larger informational friction on the supply side.

Consider next how $\mathcal{B}$ covaries with $Q+Y$, keeping in mind that the latter measures real aggregate income in period 1 (relative to the period 2 numeraire). In the frictionless benchmark, the consumers understand that variation in $\bar{\beta}$ causes $Q+Y$ to move without any change in period-2 outcomes. It follows, in this benchmark, the projection coefficient of $\mathcal{B}$ on $Q+Y$ is zero. When instead the consumers are imperfectly informed, they confuse the aforementioned movement in $Q+Y$ as idiosyncratic movements in their wages and income; and because the latter are persistent, the consumers end up updating their beliefs about their future income in the same direction. This means that the projection of $\mathcal{B}$ on $Q + Y$ has a positive coefficient, whose magnitude depends inversely on the level of the consumers' confusion. In particular, $\mathcal{B} = (1 - \lambda_d)\varrho(Q + Y)$ where $\varrho \in (0, 1)$ measures, in effect, the perceived persistence of the idiosyncratic income fluctuations and where $\lambda_d \in (0, 1]$ is inversely related to the extent to which consumers are confused and extrapolate from the present to the future.

This highlights the specific structure that our approach puts on "consumer confidence." The literature on multiple equilibria treats, in effect, $\mathcal{B}$ as an exogenous random process. The same is true for the recent works of Angeletos and La'O (2013), Angeletos, Collard, and Dellas (2018), Benhabib, Wang, and Wen (2015) and Huo and Takayama (2015). Our approach, instead, imposes that $\mathcal{B}$ increases with the current equilibrium values of Q and Y, at a rate that depends on how much agents extrapolate from current outcomes to future outcomes. This in turn opens the door to the feedback mechanism at the core of our contribution.

The following remark is worth making. Our model's implication that Q and Y have symmetric effects on $\mathcal{B}$ is an artifact of simplifying assumptions. For instance, if we had allowed for multiple shocks, the consumers could extract different kinds of signals from prices and quantities, and as a result, $\mathcal{B}$ could move differentially with Q and Y.[18] With this in mind, the prediction we wish to retain is that a transitory, demand-driven boom today may trigger excessive optimism about the future (and similarly a recession may trigger excessive pessimism).

4.5 AS and AD

Combining Proposition 4 with Propositions 1 and 3, we arrive at the following characterization of aggregate supply and aggregate demand.

Proposition 5 (AS and AD) *(i) Aggregate Supply (AS) is given by*

$$y = \tfrac{1}{\kappa}(1 - \lambda_s)Q, \tag{22}$$

where a lower value for $\lambda_s \in (0, 1]$ corresponds to larger worker confusion.

(ii) Aggregate Demand (AD) is given by

$$c = \tilde{y} - \sigma Q + (1 - \lambda_d)\varrho\,(Q + y)\,, \tag{23}$$

where $\tilde{y} \equiv -\sigma\bar{\beta}$ measures the demand shock and a lower value for $\lambda_d \in (0, 1]$ corresponds to larger consumer confusion.

[18] The same logic applies to a monetary variant that effectively replaces Q with the nominal price level (and $Q + Y$ with nominal GDP): in such a variant, we expect the dependence of confidence on the nominal price level to be weaker when monetary policy is more volatile.

Market clearing imposes $c = y$. For any given pair (λ_s, λ_d), Proposition 5 therefore allows us to understand the equilibrium determination of aggregate output and the real interest rate as the intersection of the AS and AD curves derived above. But the pair (λ_s, λ_d) is itself endogenous to the equilibrium, because these scalars depend on the precision of the information that can be extracted from the observable outcomes and because this precision is in turn endogenous to the agents' behavior. The complete characterization of the equilibrium therefore involves, not only the intersection of AS and AD, but also the solution of an extra layer of fixed-point problem, one that ties the pair (λ_s, λ_d) to how aggregate output and the real interest rate respond to $\bar{\beta}$.

This extra layer, of course, drops out when information is perfect. The frictionless benchmark studied in Section 3 is indeed nested in the above result by setting $\lambda_s = \lambda_d = 1$. In this benchmark, the AS curve is vertical and, as a result, any variation in $\bar{\beta}$ causes Q to move without any movement in y. Away from that benchmark, the elasticity of the AS curve is pinned down by λ_s alone, in a manner that resembles Lucas (1972). But the equilibrium variation in y and Q also depends on λ_d, because the latter determines how much the variation in real aggregate expenditure $Q + y$ feeds into the consumers' beliefs about future income ($\mathcal{B}$), which in turn feeds into current aggregate demand.

5 Equilibrium Characterization

In this section we complete the characterization of the equilibrium. We proceed in three steps. The first step isolates the supply-side friction. The second step adds the demand-side friction and studies the resulting feedback mechanism between economic activity and consumer beliefs. Both of these steps treat the scalars λ_s and λ_d as given. The third step solves for their equilibrium values.

5.1 Isolating the Supply-Side Effect: A Non-Monetary Version of Lucas (1972)

To better understand the implications of the feedback mechanism we have alluded to, we first analyze a variant of our model that shuts this mechanism down. This variant lets $\lambda_s < 1$ but restricts $\lambda_d = 1$.That is, it preserves the informational friction in the supply side of the economy but removes it from the demand side, thus also isolating the part of our theory that is most closely related to Lucas (1972).

Figure 1 illustrates how this variant economy responds to a sudden decrease in the consumers' willingness to spend, that is, a positive shock in $\bar{\beta}$. Note that the AS and AD curves are now given by, respectively,

$$y = \frac{1}{\kappa}(1 - \lambda_s)Q \qquad \text{and} \qquad y = \tilde{y} - \sigma Q.$$

Unlike the frictionless benchmark, the AS curve is now elastic. The AD curve, on the other hand, remains the same as in the frictionless benchmark. It follows the aforementioned shock causes the same leftward shift in the AD curve as before. But since the AS curve is now elastic, the reduction

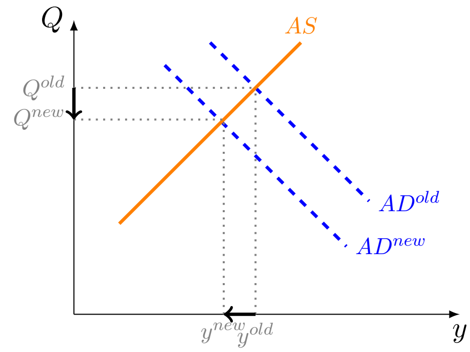

Figure 1: Supply-Side Effect Only

in aggregate demand is no more fully offset by the equilibrium adjustment in the price, Q. Instead, it results to a reduction in the quantity, y. That is, the shock generates a recession, whose magnitude is indeed increasing in the severity of the underlying information friction.

The following proposition summarizes these basic points.

Proposition 6 (Supply Side Only) *Let the friction is present in the supply side but not in the demand side (i.e., $\lambda_s < 1 = \lambda_d$). The equilibrium response of real aggregate output and of the real interest rate are given by, respectively,*

$$y = \gamma\left(\lambda_s\right)\tilde{y} \qquad \text{and} \qquad Q = \omega\left(\lambda_s\right)\tilde{y},$$

where $\tilde{y} \equiv -\sigma\bar{\beta}$ measures the exogenous demand shock,

$$\gamma(\lambda_s) \equiv \frac{\frac{1}{\kappa}(1-\lambda_s)}{\sigma + \frac{1}{\kappa}(1-\lambda_s)} \qquad \text{and} \qquad \omega(\lambda_s) \equiv \frac{1}{\sigma + \frac{1}{\kappa}(1-\lambda_s)}.$$

Furthermore, $\gamma(\lambda_s)$ is strictly decreasing in λ_s, with $\gamma(0) = \frac{(1/\kappa)}{\sigma+(1/\kappa)} > 0$ and $\gamma(1) = 0$, whereas $\omega(\lambda_s)$ is strictly increasing in λ_s, with $\omega(0) = \frac{1}{\sigma+(1/\kappa)} \in (0,1)$ and $\omega(1) = \frac{1}{\sigma}$. That is, a larger informational friction (lower λ_s) among the workers leads to a bigger response in real aggregate output response and a smaller response in the real interest rate.

As anticipated, this result revisits Lucas (1972). Similarly to that paper, the AS curve is non-vertical and the equilibrium quantity responds to the aggregate demand shock because the agents perceive, rationally but incorrectly, an improvement in the returns to labor. But unlike that paper, monetary policy has been kept out of the picture, and the AS curve has been expressed as an increasing function of only the relative price of today's goods. This permits us to think of the analysis conducted here more broadly as a proxy for how demand shocks matter in real, non-monetary models that allow the aggregate supply of today's goods to increase with their relative price. With this in mind, we henceforth concentrate on the role played by consumer beliefs in the demand side of the economy.

5.2 Adding the Demand-Side Effect: A "Confidence Multiplier"

When consumers are confused, aggregate demand becomes

$$y = \tilde{y} - \sigma Q + \mathcal{B},$$

where $\mathcal{B} = (1 - \lambda_d)\varrho(Q + y)$ captures "consumer confidence." Importantly, $\mathcal{B}$ responds endogenously to the aggregate demand shock. Figure 2 illustrates how this changes the behavior of the economy relative to the supply-only variant studied above.

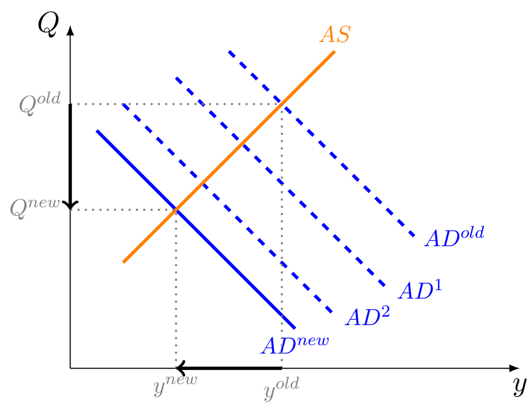

Figure 2: Feedback Mechanism

When the shock hits, aggregate demand falls *even if* we hold $\mathcal{B}$ constant. This is represented in Figure 2 by the shift of the AD curve from AD^{old} to AD^1 and mirrors the shift seen in Figure 1. As already explained, this causes a drop in both Q and y, and therefore also a drop in $Q + y$. That is, aggregate expenditure, denominated in terms of tomorrow's goods, falls. Because the typical household is unable to perfectly disentangle this drop from idiosyncratic income shocks, and because the latter are persistent, the typical household turns pessimist about its future income, which means that $\mathcal{B}$ is negative. As this happens, the AD curve shifts down further, from AD^1 to AD^2. But this causes a further drop in the aggregate expenditure on today's goods, which in turn feeds into additional consumer pessimism, and so on.

Depending on parameters, this feedback mechanism can be explosive: an arbitrarily small negative shock can trigger a cycle between the actual drops in current aggregate expenditure and the consumers' beliefs about future income, whose cumulative effect diverge to infinity. Figure 2 depicts a more palatable scenario in which the cumulative effect remains finite. The next result provides a necessary and sufficient condition for this to be the case and characterizes the cumulative effect.

Proposition 7 (Equilibrium and the Confidence Multiplier) *Let the friction be present in both the supply side and the demand side (i.e., $\lambda_s < 1$ and $\lambda_d < 1$).*

(i) The equilibrium is "stable" in the sense that the aforementioned feedback chain is non-explosive if and only if

$$\lambda_d > \underline{\lambda} \equiv 1 - \frac{1}{\varrho}\left(\frac{\sigma + \frac{1-\lambda_s}{\kappa}}{1 + \frac{1-\lambda_s}{\kappa}}\right). \tag{24}$$

A sufficient condition for this to hold for all $(\lambda_s, \lambda_d) \in (0,1]^2$ is $\sigma \geq 1$.

(ii) In any stable equilibrium, aggregate output and the real interest rate are given by, respectively,

$$y = m(\lambda_d, \lambda_s)\gamma(\lambda_s)\tilde{y} \qquad \text{and} \qquad Q = m(\lambda_d, \lambda_s)\omega(\lambda_s)\tilde{y},$$

where $\gamma(\lambda_s)$ and $\omega(\lambda_s)$ are defined as in Proposition 6 and

$$m(\lambda_d, \lambda_s) \equiv \frac{\sigma + \frac{1-\lambda_s}{\kappa}}{\sigma + \frac{1-\lambda_s}{\kappa} - (1-\lambda_d)\varrho\left(1 + \frac{1-\lambda_s}{\kappa}\right)} > 1 \tag{25}$$

is "the confidence multiplier," namely a multiplier that encapsulates the cumulative effect of the aforementioned feedback chain between aggregate outcomes and consumer beliefs.

Part (i) is explained in the Appendix. The possibility of an explosive feedback loop is not central to the message of our paper. It also stretches the validity of the log-linearized approximation of the equilibrium. For the remainder of the analysis, we thus opt to focus on the more palatable scenario in which the cumulative effect of the feedback mechanism is finite. Nevertheless, the possibility of an explosive feedback loops illustrates the potency of our mechanism.

Part (ii) characterizes the equilibrium responses of aggregate output and the real interest rate to the aggregate demand shock under this scenario. These responses are represented as multiples of the Lucas-like counterparts given in Proposition 6, that is, of the responses that obtain when the information friction is present only in the supply side of the economy. Accordingly, the variable $m(\lambda_d, \lambda_s)$ measures the multiplier introduced by the feedback loop between actual outcomes and consumer confidence.

Proposition 8 (Comperative Statics) *Along any stable equilibrium, the multiplier displays the following comparative statics.*

(i) It is decreasing in λ_d (i.e., increasing in the demand-side friction).

(ii) It is decreasing in λ_s (i.e., increasing in the supply-side friction) if $\sigma > \hat{\sigma}$, invariant to λ_s if $\sigma = \hat{\sigma}$, and increasing in λ_s if $\sigma < \hat{\sigma}$, where the threshold $\hat{\sigma}$ is given by $\hat{\sigma} = 1$.

(iii) It is increasing in ϱ (i.e., increasing in the persistence of income).

(iv) It is decreasing in κ (i.e., increasing in the elasticity of labor supply) if $\sigma > \hat{\sigma}$, invariant to κ if $\sigma = \hat{\sigma}$, and increasing in κ if $\sigma < \hat{\sigma}$, where the threshold $\hat{\sigma}$ is given by $\hat{\sigma} = 1$.

Parts (i) and (iii) are intuitive: the more consumers extrapolate from current outcomes to beliefs of future income, either because they are more confused or because they perceive a higher persistence in their income, the stronger the feedback mechanism.

On the other hand, as stated in part (ii), the multiplier can either increase or decrease with the severity of the supply-side friction. To understand why, note first that the key is how λ_s impacts the equilibrium slope of $Q + y$ with respect to $\tilde{y}$, for this slope controls how big is the the drop in consumer confidence triggered by a negative demand shock. Consider next what this impact is when the feedback mechanism is shut down ($\lambda_d = 1$). The slope of $Q + y$ with respect to $\tilde{y}$ is then given by the following sum:

$$\gamma(\lambda_s) + \omega(\lambda_s) \equiv \frac{1 + \frac{1}{\kappa}(1 - \lambda_s)}{\sigma + \frac{1}{\kappa}(1 - \lambda_s)}.$$

This increases with a larger supply-side friction (a smaller λ_s) if and only if $\sigma > 1$. It follows that that the first round of the feedback mechanism itself increases with a large supply-side friction if and only if $\sigma > 1$. This logic extends to all rounds of the feedback mechanism, explaining part (ii) of the proposition. The same logic also explains part (iv).

5.3 The Fixed Point

In Proposition 7, we take the values of λ_s and λ_d as given. However, the severity of the informational friction depends on how much information workers and consumers can extract from the available market signals. This implies that the values of λ_s and λ_d in our economy are the solution to a fixed point problem that relates the informativeness of these signals to the responsiveness of aggregate output and the real interest rate to the aggregate demand shock: the greater this response, the more precise the information revealed by the aforementioned signals, and the smaller the friction. This fixed point is characterized below.

Proposition 9 *(i) In any stable equilibrium, the pair (λ_s, λ_d) solves the following fixed point problem:*

$$\lambda_d = \frac{\sigma_\delta^{-2} + \sigma_v^{-2}\left(\sigma m\left(\lambda_d, \lambda_s\right)\omega\left(\lambda_s\right)\right)^2 + (\sigma_\xi^2 + \sigma_\zeta^2)^{-1}\left(\sigma m\left(\lambda_d, \lambda_s\right)\left(\omega\left(\lambda_s\right) + \gamma\left(\lambda_s\right)\right)\right)^2}{\sigma_\beta^{-2} + \sigma_\delta^{-2} + \sigma_v^{-2}\left(\sigma m\left(\lambda_d, \lambda_s\right)\omega\left(\lambda_s\right)\right)^2 + (\sigma_\xi^2 + \sigma_\zeta^2)^{-1}\left(\sigma m\left(\lambda_d, \lambda_s\right)\left(\omega\left(\lambda_s\right) + \gamma\left(\lambda_s\right)\right)\right)^2}, \tag{26}$$

$$\lambda_s = \frac{\sigma_\varepsilon^2}{\sigma_v^2 + \sigma_\varepsilon^2}\frac{\sigma_\delta^{-2} + \left(\sigma_v^2 + \sigma_\varepsilon^2\right)^{-1}\left(\sigma m\left(\lambda_d, \lambda_s\right)\omega\left(\lambda_s\right)\right)^2 + (\sigma_\xi^2 + \sigma_\zeta^2)^{-1}\left(\sigma m\left(\lambda_d, \lambda_s\right)\left(\omega\left(\lambda_s\right) + \gamma\left(\lambda_s\right)\right)\right)^2}{\sigma_\beta^{-2} + \sigma_\delta^{-2} + \left(\sigma_v^2 + \sigma_\varepsilon^2\right)^{-1}\left(\sigma m\left(\lambda_d, \lambda_s\right)\omega\left(\lambda_s\right)\right)^2 + (\sigma_\xi^2 + \sigma_\zeta^2)^{-1}\left(\sigma m\left(\lambda_d, \lambda_s\right)\left(\omega\left(\lambda_s\right) + \gamma\left(\lambda_s\right)\right)\right)^2} + \frac{\sigma_v^2}{\sigma_v^2 + \sigma_\varepsilon^2}. \tag{27}$$

(ii) There always exists a solution to (26)-(27) and a stable equilibrium associated with it.

(iii) All stable equilibria satisfy $\lambda_d < 1$ and $\lambda_s < 1$.

Part (i) characterizes the equilibrium values of λ_d and λ_s. Condition (26) encapsulates the signal extraction problem of the consumer and reflects the fact that the consumer has three sources of information: her own discount rate, $\beta^h = \bar{\beta} + \delta^h$; the local demand, $d_h = Q + y + \xi_h + \zeta_h = -\sigma m(\lambda_d, \lambda_s)(\omega(\lambda_s) + \gamma(\lambda_s))\bar{\beta} + \xi_h + \zeta_h$, which maps to her own income; and her consumer price index, $q^h = -\sigma m(\lambda_d, \lambda_s)\omega(\lambda_s)\bar{\beta} + v_h$. Condition (27) encapsulates from the signal extraction problem of the worker and reflects the fact that the typical worker also has three sources of information: her own discount rate; the local wage, which is determined by the local demand; and the signal about consumer prices, $x^h = q^h + \varepsilon^h$.

Part (ii) establishes that the fixed point defined by condition (26) and (27) always admits a solution that also satisfies condition (24). That is, there always exists a stable equilibrium of the type we have studied in the preceding analysis.

The possibility of multiple equilibria, associated with multiple solutions to the signal-extraction problem, has not been ruled out.[19] This, however, is not of interest here. For our purposes, the key observation the one made in part (iii) of Proposition 9: even if there are multiple equilibria, all of them feature a non-zero friction, which in turn guarantees that the qualitative lessons delivered earlier apply regardless of which equilibrium is selected.

5.4 PE and GE

We close this section by offering a complementary interpretation of our results in terms of the PE and the GE effects of aggregate demand shocks.

The failure of intertemporal-preference or other aggregate-demand shocks to generate business cycles in the baseline RBC model can be understood as the product of offsetting PE and GE effects. To understand what we mean, consider an exogenous discount-factor shock that hits only a measure-zero subset of the households and suppose that the latter determine the demand faced by a particular, measure-zero subset of the firms. In equilibrium, such a shock has negative effects on the spending of the affected households, the real returns of the affected firms, and the real income of the workers employed in these firms.

When the shock is aggregate, the aforementioned effects remain present and can be interpreted as the partial-equilibrium (PE) effects of the shock. But now there is also an additional set of effects, which captures the GE adjustment of the entire economy. In the frictionless RBC model, this adjustment manifests in the real interest rate: as the aggregate demand for today's goods fall, their relative price also falls. This endogenous price response in turn offsets the exogenous drop in demand.

[19] Such a possibility has been documented by Benhabib, Wang, and Wen (2015), Gaballo (2017), and Chahrour and Gaballo (2018) in different environments. Here, numerical simulations suggest a unique solution to the fixed point described above, which treats the variances of the sampling shocks as exogenous. However, in the extension studied in Appendix B, these variances are endogenous, the relevant fixed point is more convoluted, and the possibility of multiple equilibria cannot be ruled out.

The first element of theory, namely our non-monetary version of Lucas (1972), arrests this countervailing, "neoclassical," GE mechanism by letting aggregate supply increase with the real interest. The second element, on the other hand, introduces a novel GE mechanism in the form of a positive feedback loop between economic activity and consumer beliefs.

This mechanism works in the opposite direction than the aforementioned, neoclassical GE mechanism and effectively transforms the economy from a game of strategic substitutability to a game of strategic complementarity. In the RBC model, the GE response in the real interest rate represents a form of strategic substitutability, which offsets the underlying PE effects. By contrast, the "confidence multiplier" put forward in our paper represents a form of strategic complementarity, which amplifies the underlying PE effects.

6 Extensions and Discussion

In this section we offer a few extensions and discuss the robustness and the empirical plausibility of our results. Subsection 6.1 explores the implications of our theory for the effects of fiscal stimuli. Subsection 6.2 elaborates on the possible origins and the empirical plausibility of the belief distortion underlying our confidence multiplier. Subsection 6.3 turns attention to the reasons that give us a positively-slopped aggregate supply. Subsection 6.4 finally shifts the focus from *consumer* confidence to *producer* confidence.

6.1 Government Spending

We now explore the implications of our analysis for the macroeconomic effects of government spending. In particular, we show that a front-loaded fiscal stimuli boosts consumer confidence and triggers an expansion, whereas the opposite is true for a back-loaded fiscal stimuli.

The model is the same, except that we shut down the shocks to the household's discount factor and, instead, introduce shocks to government spending. In particular, we assume that, in each period, the government purchases a basket of all the goods in the economy and uses that to produce a public good, whose quantity we denote by G_t. We treat G_t as exogenous and random. Specifically, we assume that G_t is drawn from $\mathcal{N}(0, \sigma_G^2)$, i.i.d. across t, and independent of any other shock. We also let the composition of government spending mirror that of private spending, that is, we impose

$$\frac{q_{i,j,t} g_{i,j,t}}{Q_t G_t} = \frac{\int_{\{h: i \in \mathcal{C}_t^h\}} q_{i,j,t} c_{i,j,t}^h dh}{Q_t C_t},$$

where $g_{i,j,t}$ is the government's purchase of variety j from island i in period t.[20]

[20] This restriction involves equilibrium outcomes in the right hand side, but can be derived from first principles by interpret-

We let government spending be financed by lump-sum taxation and, without further loss, impose budget balance in each period. At the same time, we prevent one's own tax burden be a perfect signal of the aggregate level of government spending by introducing idiosyncratic tax shocks. Specifically, we let the lump-sum tax levied to household h in period t be given by $T_t^h = T_t + \Delta_t^h$, where T_t is the average tax and $\Delta_t^h \sim N\left(0, \sigma_\Delta^2\right)$ is i.i.d. across h and t, and independent from any other shock. The average tax is such that the government budget is balanced:

$$T_t = \int_{i\in[0,1]} \int_{j\in[0,1]} q_{i,j,t} g_{i,j,t} dj di. \tag{28}$$

We assume that both G_1 and G_2 are realized in period 1, which means that one can think of G_2 as a "news shock," but preclude either one from being commonly known. We finally close the model by letting the information structure be the same as in the baseline model, except for the following modification: in period 1, the knowledge of β^h is replaced with the knowledge of the pair (T_1^h, T_2^h).[21] This pair captures the exogenous information that the household has about the underlying fiscal shocks, just as β^h captured the exogenous information that the household had about the underlying aggregate consumer-spending shock in our baseline model.

A detailed characterization of the equilibrium can be found in the proof of Proposition 10 in the Appendix. As in the baseline model, we log-linearize the equilibrium conditions, re-interpret all the variables as log-deviations from their steady-state counterparts,[22] shut down the GHH complementarity,[23] and drop the subscript 1 from y_1 and Q_1. Following similar steps as in Section 5, we finally reach the following result.

Proposition 10 *(i) Aggregate supply and aggregate demand are given by, respectively,*

$$y = \tfrac{1}{\kappa}\left(Q - \mathcal{Q}^e\right) \qquad \textit{and} \qquad y = G_1 - G_2 - \sigma Q + \mathcal{B}, \tag{29}$$

where $\mathcal{Q}^e$ and $\mathcal{B}$ are defined as before. Moreover, Proposition 4 still holds.

ing G_t as the quantity of a public good produced with a CES composite of all the varieties, letting this CES composite mirror that describing private preferences, and finally having the government optimally choose its purchases of all the variates so as to minimize the cost of producing the exogenously given quantity, G_t, of the public good.

[21] That is, the worker's and the consumer's information sets in period 1 are given by, respectively, $\tilde{\mathcal{I}}_1^h = \left\{T_1^h, T_2^h; w_{h,1}, e_{h,1}; x_1^h\right\}$ and $\mathcal{I}_1^h = \left\{T_1^h, T_2^h; w_{h,1}, e_{h,1}; (q_{l,k,1})_{l\in\mathcal{C}_1^h, k\in[0,1]}\right\}$; and everything becomes commonly known in period 2. Also, nothing essential changes if we replace the perfect observation of T_2^h with the observation of a noisy private signal about it.

[22] The following exception applies: G_t, T_t, $g_{i,t}$ and T_t^h henceforth represent, respectively, G_t/y^*, T_t/y^*, $g_{i,t}/y^*$, and T_t^h/y^*, where y^* is the steady-state (also, complete-information) value of aggregate output. This is a standard trick in the literature on fiscal multipliers (e.g., Woodford, 2011) and it simply takes care of the issue that the log-deviation of the government spending is not well defined when its steady-state value is 0.

[23] That is, we set $\phi = 0$, or $\mu = \infty$.

(ii) In any stable equilibrium,

$$y = m(\lambda_d, \lambda_s)\gamma(\lambda_s)(G_1 - G_2),$$

where m and γ are defined as in Proposition 7 and where, as before, λ_s and λ_d measure the supply-side and demand-side friction, respectively.

Part (i) states that the effective AS and AD curves are the same as in our baseline model, except for one difference: the effective aggregate demand shock is now given by $\tilde{y} = G_1 - G_2$ rather than $\tilde{y} = -\sigma\bar{\beta}$. Part (ii) is then immediate and verifies the claim made in the beginning of this section that a front-loaded fiscal stimulus (an increase in G_1) is expansionary while a back-loaded one (an increase in G_2) is contractionary.

To understand the full content of this result, consider first the case in which the informational friction is absent. This case is nested in part (i) by letting $\mathcal{Q}^e = Q$ and $\mathcal{B} = 0$. The former means that the AS curve is vertical, the latter means that the AD curve reduces to $y = G_1 - G_2 - \sigma Q$. This makes clear that, even with perfect information, the aggregate demand for today's goods increases with the *current* level of government spending but falls with the *future* level of government spending. This is because G_1 and G_2 have the same crowding-out effect on private spending—an increase in either of them reduces net-of-taxes permanent income, thus also reducing c for given prices—but G_1 directly adds to the aggregate demand for today's goods while G_2 does not.

With a vertical AS curve, G_1 and G_2 only have opposite effects on the real interest rate but no effect on real output. But as soon as the AS curve is positively sloped, an increase in G_1 raises both y and Q, whereas an increase in G_2 have the opposite effect. Furthermore, as long as the consumers are confused, our "confidence multiplier" kicks in, amplifying both the expansionary effects of current government spending and the contractionary effects of future government spending. Furthermore, as shown next, a large enough friction suffices for a front-loaded fiscal stimulus to crowd *in* private consumption, or equivalently the fiscal multiplier to exceed one.

Proposition 11 *Suppose that the equilibrium is stable and that $(1 - \lambda_d)\left(1 + \frac{1-\lambda_s}{\kappa}\right) > \frac{\sigma}{\varrho}$. Then,*

$$\frac{dy}{dG_1} = m(\lambda_d, \lambda_s)\gamma(\lambda_s) > 1.$$

Our theory can thus make sense of the following narrative, without the need for nominal rigidity and accommodating monetary policy (as in the New Keynesian model). When G_1 increases, each firm experiences an increase in the demand for its product, each worker experiences an increase in the demand for his labor, and each household experiences an increase in its income. Because the workers are confused, this leads to an increase in aggregate supply, which alone suffices for aggregate output to increase with the increase in G_1. But because the consumers are also confused, this leads to a boost in consumer confidence and an increase in consumer spending, which in turn triggers a series of further increases in aggregate employment and income. With a back-loaded stimulus (an increase in G_2), on the other hand, all these effects are reversed.

Because the multiplier is highest when consumers are most confused, our theory also suggests that front-loaded fiscal stimuli may be particularly effective in periods of heightened uncertainty, or following unusual events. To capture this, think of an "uncertainty shock" as an exogenous reduction in λ_d and λ_s. In our model, such a shock maps to a higher response of y_1 to G_1. This in turn suggests an intriguing connection between our work and the recent literature on uncertainty shocks, and a testable implication, whose investigation we leave open for future work.

6.2 Belief Overreaction and Confidence

In this subsection, we illustrate how a simple, possibly irrational, form of belief extrapolation can offer a complementary micro-foundation of the belief distortion that opens the door to our "confidence multiplier." We also discuss how this connects to existing evidence.

Consider a fully rational and fully informed household. Her belief of $d_{h,2}$ moves in the same direction as $d_{h,1}$ in response to an idiosyncratic income shock, whereas it does not move at all in response to an aggregate discount-rate shock.[24] The preceding analysis has departed from this benchmark, and has allowed beliefs to overreact in response to the aggregate discount-rate shock, by letting consumers be rationally confused about which kind of shock drives the movements in $d_{h,1}$. An alternative that delivers the same results lets the consumers extrapolate from the present to the future according to the following heuristic:

$$\mathbb{E}_1^h \left[d_{h,2} \right] = \Lambda_d d_{h,1},$$

for some $\Lambda_d > 0$. A similar heuristic is used in Barberis, Greenwood, Jin, and Shleifer (2015), albeit in a different context, as a proxy for extrapolative beliefs. Here, this gives us

$$\mathcal{B} \equiv \int \mathbb{E}_1^h \left[d_{h,2} \right] dh = \Lambda_d \left(Q + y \right). \tag{30}$$

It is then immediate that the analysis goes through provided we replace $(1 - \lambda_d)\varrho$ with Λ_d. The only difference is that Λ_d is no more pinned down by the signal-extraction problem characterized in Proposition 9. What is thus lost is the idea that the informational friction, and hence also the confidence multiplier, may be higher in periods of high uncertainty. But the big picture remains unchanged.

As noted in the Introduction, Greenwood and Shleifer (2014), Gennaioli, Ma, and Shleifer (2016)

[24]Recall that $d_{h,t}$ captures both the demand faced by all firms in island $i = h$ and the income made by the household who lives in that island at period t.

and Rozsypal and Schlafmann (2018) offer evidence in support of extrapolation and overreaction in the relevant kind of beliefs. Specifically, Rozsypal and Schlafmann (2018) use micro data on household income expectations, and find that consumers over-extrapolate from their current income to expectations of future income, as they overestimate the persistence of their income process. Households with currently high income turn out to be too optimistic about their future income, while households with currently low income turn out to be too pessimistic about their future income. Gennaioli, Ma, and Shleifer (2016) turn the focus to the firm side. They find that firm CFOs over-extrapolate from current earnings to expectations of future earnings. Greenwood and Shleifer (2014) find that investors over-extrapolate from current stock returns to expectations of future stock returns.

It is an open question whether this kind of evidence speaks more to irrationality, to informational friction, or to both, as for example argued by Broer and Kohlhas (2018). But as long as the relevant beliefs—those of consumers about their income and those of firms about their earnings—overreact, our "confidence multiplier" is likely to kick in. Another open question is whether such overreaction is a universal phenomenon, or one that applies only to specific shocks or specific times.

6.3 Revisiting Aggregate Supply

Our baseline model features an asymmetry between the information sets upon which consumption and labor-supply decisions were made: whereas consumption decisions during period 1 were based on perfect information of the prices of the purchased goods, labor supply was based on imperfect information of these prices. Taking literally, this means that the household is a team of two selfs, a worker-self and a consumer-self, and that the consumer-self does not "talk to" the worker-self, so that the decisions of the later are based on a subset of the information of the former.

Such an interpretation is consistent with a central theme of behavioral economics: a decision maker may not fully incorporate all the relevant information when making each economic decision (Anderson, 2009), either because certain pieces of information are more salient than others (Bordalo, Gennaioli, and Shleifer, 2013) and the principle "What You See Is All There Is" (Kahneman, 2011), or because of other related departures from full rationality. Such departures include bounded recall (Kahana, 2012; Bordalo, Gennaioli, and Shleifer, 2017), "narrow bracketing" (Tversky and Kahneman, 1981; Read, Loewenstein, and Rabin, 1999), and "narrow thinking" (Lian, 2018). In our context, such departures can help justify why the household decides on her consumption and labor choices separately and why she may be less attentive to commodity prices when choosing labor supply than when making her consumption choices.

For these reasons, we welcome a behavioral re-interpretation of the supply side of our model—just as we welcome a behavioral re-interpretation of our "confidence multiplier" along the lines discussed in the previous subsection. That said, neither part of our theory strictly needs a departure from rationality. For the demand side, this is already evident from our baseline model. To illustrate why this is also true for the supply side, we now consider a variant of our baseline model that drops the aforementioned asymmetry between consumption and labor-supply decisions and lets the consumer-self and the worker-self share the same information.

The key idea is to split the first period into two sub-periods, henceforth called "morning" and "afternoon;" to let a portion of the first-period consumption and labor-supply decisions be made in each sub-period; and finally to let information be the same between consumer-self and the worker-self but different across the two sub-periods. This is meant to capture the idea that information arrives slowly over time and that both consumption and labor supply ("in the morning") depend on the expectation of prices that have not yet been observed ("afternoon prices"), and helps rationalize the informational friction superimposed in the supply side of our baseline model.

Let us spell out the details. The preferences of household $h \in [0,1]$ are now represented by

$$\mathcal{U}^h = U\left(c^h_{m,1}, c^h_{a,1}, n^h_{m,1}, n^h_{a,1}\right) + \beta^h U\left(c^h_{m,2}, c^h_{a,2}, n^h_{m,2}, n^h_{a,2}\right), \tag{31}$$

where $c_{s,t}$ and $n_{s,t}$ denote consumption and labor supply in sub-period $s \in \{m, a\}$ of period $t \in \{1,2\}$, $s = m$ stands for "morning," $s = a$ stands for "afternoon," and the per-period utility is given by

$$U\left(c_m, c_a, n_m, n_a\right) = \frac{\left(c_m^{1/2} c_a^{1/2} - v\left(n_a\right) - v\left(n_m\right)\right)^{1-\frac{1}{\psi}}}{1-\frac{1}{\psi}} \qquad \text{and} \qquad v(n) = \frac{n^{1+\kappa}}{1+\kappa}. \tag{32}$$

In each sub-period, the effective consumption is given by the same nested CES aggregator as in (3). For simplicity, production is separable across the two sub-periods, so that the output of a firm j on island i in sub-period s of period t is given by $y_{i,j,s,t} = l_{i,j,s,t}$, and labor markets clear if and only if $\int l_{i,j,s,t} = n^i_{s,t}$ for all i,s, t.[25]

Turning to the information structure, we let the aggregate shock $\bar{\beta}$ and all the endogenous outcomes become commonly known as early as in the afternoon of period 1. We next let both the consumption and the labor-supply decisions in the morning of period 1 depend on the *same* information, and specify that information as follows:

$$\mathcal{I}^h_{m,1} = \left\{\beta^h_1; w_{h,m,1}, e_{h,m,1}; \left(q_{l,k,m,1}\right)_{l\in\mathcal{C}^h_{m,1}, k\in[0,1]}\right\}.$$

Apart from these modifications, every other aspect of the environment remains exactly the same as in the baseline model.[26] For simplicity, we shut down the GHH complementarity and assume a

[25] Although it seems more elegant to split both periods into two sub-periods, the results presented below only need this to be true for the first period.

[26] We use the $\mathcal{C}^h_{s,t}$ to denote the set of islands in which the household h "visits" in sub-period s of period t. Similar to the baseline model, we have $\mathcal{C}^h_{m,1}, \mathcal{C}^h_{a,1} \subset [0,1]$ and $\mathcal{C}^h_{m,2} = \mathcal{C}^h_{a,2} = [0,1]$. $\mathcal{C}^h_{m,1}$ and $\mathcal{C}^h_{a,1}$ are identically drawn across households

unitary elasticity of intertemporal substitution (i.e., we let $\mu \to \infty$ and $\sigma \to \psi = 1$). As in the baseline model, we log-linearize the equilibrium conditions, re-interpret all the variables as log-deviations from their steady-state counterparts, and drop the subscript 1 from period 1 objects.

The detailed characterization of the equilibrium can be found in the Appendix. Here, we only state the main result regarding the determination of aggregate output in period 1.

Proposition 12 *In any stable equilibrium, there exists a scalar $\lambda \in (0,1)$, which is inversely related to the severity of the informational friction, such that aggregate output is given by*

$$y = -\hat{m}(\lambda)\hat{\gamma}(\lambda)\bar{\beta}$$

where

$$\hat{\gamma}(\lambda) \equiv \frac{\left(1+\frac{1}{2(1+\kappa)-1}\right)\left(\frac{1-\lambda}{2\kappa}\right)}{2+\frac{1}{\kappa}\left(\frac{2\kappa+1-\lambda}{2\kappa+1}\right)} > 0$$

is the equilibrium response obtained when consumer confusion is assumed away and

$$\hat{m}(\lambda) \equiv 1 + \frac{2\beta\varrho}{2\beta(1-\varrho)+\lambda(1+2\beta\varrho)} > 1 \tag{33}$$

is the multiplier that encapsulates the feedback chain between economic activity and consumer beliefs.

To read this result, note the following basic point. In the baseline model, the decisions underlying aggregate demand and the aggregate supply were based on different information sets. As a result, a different scalar was used to measure the severity of the informational friction on each side for the economy, namely λ_d for the demand side and λ_s for the supply side. In the present variant, instead, a single scalar λ measures the severity of the informational friction on both sides, precisely because we have let the decisions in both sides be conditioned on the same information set. The next result then verifies that the bottom line is the same as that in our baseline model: a larger friction generates a larger output response via both a more elastic aggregate supply and a larger confidence multiplier.

Proposition 13 *Both $\hat{\gamma}(\lambda)$ and $\hat{m}(\lambda)$ decrease in λ.*

Finally, we wish to iterate the following point. Although we have found it conceptually appealing to abstract from monetary policy and let information friction be the source of both a non-vertical aggregate supply and the confidence multiplier, these elements can be separated. For instance, we can remove the informational friction in the supply side, preserve monetary non-neutrality, and nevertheless obtain an aggregate supply for today's goods that increases with their relative price by embedding

and sub-periods. Also similar to the baseline model, ξ_i and ζ_i parametrize the island specific demand shocks. Using $d_{i,s,t}$ to denote the total spending for the goods produced on island i in sub-period s of period t and $D_{s,t}$ the corresponding aggregates, we have $d_{i,s,1} = D_{s,1}\xi_i\zeta_i$ and $d_{i,s,2} = \xi_i D_{s,2}$.

other real frictions, such as those considered in Beaudry and Portier (2013), Guerrieri and Lorenzoni (2017), and Bai, Rıos-Rull, and Storesletten (2017). Alternatively, we can allow monetary policy to be non-neutral, by adding either sticky prices or nominal confusion, and recast aggregate supply as an increasing function of the nominal price level. In all these cases, our confidence multiplier is likely to survive, provided of course that consumers are confused.

6.4 Producer Confidence

We now consider a different variant of our baseline model, one that shifts the focus from *consumer* confidence to *producer* confidence. This is achieved by allowing production to be forward looking and therefore to respond, not only to current demand, but also to beliefs about future demand.

The most natural way to make production forward looking is to let firms choose certain inputs, such as capital, prior to observing realized demand. To keep the analysis tractable, we capture this possibility here by assuming that the second-period labor input has to be chosen in the first period, along with the first-period labor input. One can think of this either as an extreme form of adjustment cost in labor, or as a proxy for a variety of forward-looking decisions made by the firms, including investment decisions. We next let the information of firm i in the first period be

$$\mathcal{I}^f_{i,1} = \left\{ \tilde{d}_{i,1}, w_{i,1} \right\},$$

where $\tilde{d}_{i,1}$ is the intercept of the local demand, defined as in condition (10), and $w_{i,1}$ is the local wage. This means that the firm observes the local demand for its product and the local cost, but not the aggregate state of the economy. The information of the workers and the consumers remains as in the baseline model, and so does every other aspect of the environment.[27]

Proposition 14 *(i) In the first period, aggregate supply and aggregate demand are given by, respectively,*

$$y = \tfrac{1}{\kappa}\left(Q - \mathcal{Q}^e\right) \qquad \text{and} \qquad y = -\sigma\left(\bar{\beta} + Q\right) + \mathcal{B},$$

where $\mathcal{Q}^e$ and $\mathcal{B}$ are defined as in the baselined model.

(ii) In the second period, the equilibrium aggregate output is given by

$$y_2 = \tfrac{1}{1+\kappa}\mathcal{B}^f \equiv \tfrac{1}{1+\kappa} \int \mathbb{E}\left[d_{i,2} | \mathcal{I}^f_{i,1}\right] di.$$

Part (i) says that the structure of aggregate demand and aggregate supply remains essentially the same as in the baseline model. Part (ii) states that the equilibrium level of output in the second

[27] As in the baseline model, we log-linearize the equilibrium conditions, re-interpret all the variables as log-deviations from their steady-state counterparts, shut down the GHH complementarity (i.e., we let $\mu \to \infty$ and $\sigma \to \psi$), and drop the subscript 1 from y_1 and Q_1.

period increases with "producer confidence" as measured by $\mathcal{B}^f$, the average first-period beliefs of the producers about their future demand. This reflects the fact the second-period labor input is a form of investment, chosen one period in advance on the basis of the firms' beliefs about future demand.

This opens the door to an additional feedback mechanism. Consider a positive demand shock (a drop in $\bar{\beta}$). As consumers get optimistic, they drive current demand up. This makes firms, not only produce more today, but also update upward their beliefs about future demand. Consumer optimism therefore feeds into producer optimism. This in turn makes firms to "invest" more today, in the form of committing on a high level of employment and output for tomorrow. But as this happens, the consumers come to expect an even higher level of income tomorrow. Producer optimism therefore feeds back to even more consumer optimism. And so on.

This logic is verified in the next result.

Proposition 15 *In any stable equilibrium, the following properties hold:*

(i) Current and future output co-move, that is,

$$Cov(y_1, y_2) > 0$$

(ii) Relative to the baseline model, the current output response is amplified, that is,

$$\left| \frac{dy_1}{d\tilde{y}} \right| > m(\lambda_d, \lambda_s)\gamma(\lambda_s),$$

where γ and m are the same objects as those obtained in the baseline model.

Part (i) hints that our model has the ability to generate positive co-movement between consumption, employment, and "investment." This is, of course, subject to the caveat that "investment" herein means choosing future employment and output in advance. In a more realistic variant in which "investment" means forgoing current consumption, there will be a familiar countervailing force: as the consumers try to consume more, the real interest rate goes up, discouraging investment. However, to the extent the boost in producer optimism is sufficiently large, equilibrium investment may still increase, as in the stylized example studied here.

Part (ii) shows how the present model produces an even larger "confidence multiplier" than in our baseline model. This is because of the additional feedback between consumer and producer confidence discussed above. Needless to say, if we shut down the confusion among the consumers but preserve it among the firms, the confidence multiplier is reduced but does not vanish. In short, although our paper has focused on consumer confidence, producer confidence can play a similar, and indeed reinforcing, role.

7 Conclusion

This paper revisited the elementary question of why fluctuations in consumer spending appear to drive business cycles. At the core of our theory was an informational or behavioral friction that played a dual role. First, it allowed the aggregate supply of today's goods to increase with their relative price. And second, it introduced a positive feedback mechanism between actual economic activity and consumer beliefs about the future—what we called the "confidence multiplier."

The first element was, in effect, a non-monetary version of Lucas (1972) and allowed us to accommodate demand-driven fluctuations while keeping monetary policy out of the picture. The second element, on the other hand, allowed us to shed light on the role that consumer confidence may be playing over the business cycle as a propagation mechanism. This element seems empirically relevant and also robust to whether the underlying confusion is the product of an informational friction, as assumed here, or of the type of extrapolative beliefs advanced in Greenwood and Shleifer (2014) and Gennaioli, Ma, and Shleifer (2016).

An obvious direction for future work is the inclusion of our "confidence multiplier" in the New Keynesian model. Such an extension would give up on the goal of disentangling demand-driven business cycles from nominal rigidity, but would permit the study of the interaction of our mechanism with monetary policy and, possibly, a quantitative evaluation.

Finally, whereas there is evidence of belief overreaction in line with our theory, it is unclear whether such overreaction varies across different shocks or business-cycle episodes. Additional empirical work is therefore needed before a quantitative implementation.

Appendix A: Proofs

For clarity, in the Appendix, we will not drop the subscript 1 from period 1 objects.

Proof of Lemma 1. Optimality in the labor supply decision of each household gives

$$n_2^h = \frac{1}{\kappa}\left\{w_{h,2} - Q_2\right\} \quad \forall h. \tag{34}$$

Optimality in the production decision of each firm gives

$$q_{i,2} = w_{i,2} \quad \forall i.$$

Aggregating the above two conditions, together with the technology in (4) and labor market cleaning, we have

$$y_2 = \int n_2^h dh = 0.$$

Proof of Lemma 2. Because the demand curve each firm faces takes the form of (10), the prices set by the firms are equal to their marginal costs times a constant markup, which disappears in the condition below because of the log-linearization:

$$q_{i,1} = w_{i,1}.$$

Substituting this into the definition of $d_{i,1}$ in (14) and using the technology in (4), we have

$$l_{i,1} = y_{i,1} = d_{i,1} - w_{i,1}.$$

Proof of Proposition 1. For each island i, using labor market clearing in (11), the optimal labor supply in (12), and the firm optimality in (13), we have

$$y_{i,1} = \tfrac{1}{1+\kappa}\left\{d_{i,1} - \mathbb{E}\left[q_t^i \middle| \tilde{\mathcal{I}}_1^i\right]\right\}.$$

Aggregating this condition, we have

$$y_1 = \tfrac{1}{1+\kappa}\left\{Q_1 + c_1 - \mathcal{Q}_1^e\right\},$$

where $\mathcal{Q}_1^e \equiv \int \mathbb{E}\left[q_1^h \middle| \tilde{\mathcal{I}}_1^h\right] dh$. Replacing c_1 with y_1 and re-solving for y_1 proves Proposition 1.

Proof of Lemma 3. The household h's intertemporal optimality implies:

$$\frac{c^* - v(n^*)}{c^*} c_1^h - \frac{v(n^*)}{c^*} (\kappa+1) n_1^h = E_1^h \left[\frac{c^* - v(n^*)}{c^*} c_2^h - \frac{v(n^*)}{c^*} (\kappa+1) n_2^h \right] - \psi \left[\beta_1^h + q_1^h \right], \quad (35)$$

where we use the fact that $E_1^h \left[q_2^h \right] = E_1^h \left[Q_2 \right] = 0$. The household's budget constraint in (5) and (6), together with her income in (16), implies:

$$q_1^h + c_1^h + \beta \left(q_2^h + c_2^h \right) = d_{h,1} + \beta d_{h,2}. \quad (36)$$

Using (36) to replace c_2^h in (35), we arrive at (17), with $\sigma \equiv \psi \left(c^* - v(n^*) \right) / c^*$ and $\varphi = \frac{v(n^*)}{c^*} (\kappa + 1)$. Finally, in steady state, from (7), we have $n^* = y^* = (1+\mu)^{-\frac{1}{\kappa}}$. As a result, $\sigma = \psi \left(1 - \frac{1}{(1+\kappa)(1+\mu)} \right)$ and $\varphi = \frac{1}{1+\mu}$.

Proof of Proposition 2. It follows from (18), $n_1^h = y_{h,1}$, and $E_1^h[n_2^h] = E_1^h[y_{h,2}] = \frac{1}{1+\kappa} E_1^h[d_{h,2}]$.

Proof of Proposition 3. From Lemma 3, we know $\sigma = \psi \left(1 - \frac{1}{(1+\kappa)(1+\mu)} \right)$ and $\varphi = \frac{1}{1+\mu}$. When $\mu \to \infty$, we have $\varphi \to 0$ and $\sigma \to \psi$. Proposition 3 then follows from Proposition 2.

Proof of Proposition 4. Projecting $\mathbb{E} \left[q_1^h \middle| \tilde{\mathcal{I}}_1^h \right]$ onto Q_1, we have

$$\mathbb{E} \left[q_1^h \middle| \tilde{\mathcal{I}}_1^h \right] = \lambda_s Q_1 + \varpi^h,$$

with ϖ^h orthogonal to Q_1 and $\lambda_s = \frac{Cov\left(P_1, \mathbb{E}\left[q_1^h \middle| \tilde{\mathcal{I}}_1^h \right]\right)}{Var(P_1)}$. Aggregating leads to (20). The fact that $\lambda_s \in (0,1]$ follows from Proposition 9 below.

Projecting $\mathbb{E}_1^h \left[Q_1 \right]$ onto Q_1, we have

$$\mathbb{E}_1^h \left[Q_1 \right] = \lambda_d Q_1 + \omega^h, \quad (37)$$

with ω^h orthogonal to Q_1 and $\lambda_d = \frac{Cov\left(P_1, \mathbb{E}_1^h[P_1]\right)}{Var(P_1)} \in (0,1]$. Aggregating the above condition, together with the fact that Q_1 and Y_1 are linear functions of the underlying shock $\bar{\beta}$, we have

$$\int \mathbb{E}_1^h \left[Q_1 \right] dh = \lambda_d Q_1 \quad \text{and} \quad \int \mathbb{E}_1^h \left[Y_1 \right] dh = \lambda_d Y_1. \quad (38)$$

Then, note that

$$\mathbb{E}_1^h \left[d_{h,2} \right] = \mathbb{E}_1^h \left[\xi_i \right] = \varrho \mathbb{E}_1^h \left[\xi_h + \zeta_h \right] = \varrho \left(d_{h,1} - \mathbb{E}_1^h \left[Q_1 + Y_1 \right] \right),$$

where $\varrho \equiv \frac{\sigma_\xi^2}{\sigma_\zeta^2 + \sigma_\xi^2} \in (0,1)$. Aggregating the above condition and combining with (38) gives (21).

Proof of Proposition 5. It follows from combining Proposition 4 with Propositions 1 and 3.

Proof of Proposition 6. Finding the intersection of the AS and AD curves, given by, respectively,

$$y_1 = \frac{1-\lambda_s}{\kappa} Q_1 \qquad \text{and} \qquad y_1 = -\sigma\left(\bar{\beta} + Q_1\right),$$

results in the formula for $\gamma(\lambda_s)$ and $\omega(\lambda_s)$ in Proposition 6. The comparative statics in Proposition 6 follows from the formula.

Proof of Proposition 7. Note that from the discussion at the end of 5.2, a demand shock $\tilde{y}$ triggers a feedback chain between aggregate income, consumer beliefs, and aggregate spending. The first round of this chain has AD change by $\chi \cdot \tilde{y}$, where

$$\chi = (1-\lambda_d)\varrho \frac{1+\frac{1}{\kappa}(1-\lambda_s)}{\sigma + \frac{1}{\kappa}(1-\lambda_s)}.$$

By induction, for any $k \geq 1$, the k-th round change is $\chi^k \cdot \tilde{y}$. For the feedback loop to be non-explosive, we need $\chi < 1$, which is equivalent to condition (24). Provided that this condition is satisfied, using the AS curve in (22) together with the AD curve in (23), we arrive at the expression of the equilibrium output and price response given in Proposition 7.

Proof of Proposition 8. The non-explosive condition in (24) means that the denominator in the expression of $m(\lambda_d, \lambda_s)$ in (25) is positive. As a result, $m(\lambda_d, \lambda_s)$ is strictly decreasing in λ_d. Now, let us rewrite the multiplier as

$$m(\lambda_d, \lambda_s) = \frac{1}{1-(1-\lambda_d)\varrho \frac{1+\frac{1-\lambda_s}{\kappa}}{\sigma+\frac{1-\lambda_s}{\kappa}}}.$$

We can then see that $m(\lambda_d, \lambda_s)$ is strictly decreasing in λ_s if $\sigma > 1$, invariant to it if $\sigma = 1$, and increasing in it if $\sigma < 1$. The comparative statics with respect to ϱ and κ are then also immediate.

Proof of Proposition 9. The consumer has three sources of information about $\bar{\beta}$: her own discount rate, $\beta^h = \bar{\beta} + \delta^h$; the local demand,[28]

$$d_{h,1} = -\sigma m(\lambda_d, \lambda_s)\left(\omega(\lambda_s) + \gamma(\lambda_s)\right)\bar{\beta} + \xi_h + \zeta_h,$$

[28]From (13), (16) and the fact that $n_1^h = l_{h,1}$, we know that the consumer know knowing $d_{h,1}$ is equivalent to her knowing $w_{h,1}$ and $e_{h,1}$.

which maps to her own income; and her consumer price index,

$$q_1^h = -\sigma m(\lambda_d, \lambda_s)\,\omega(\lambda_s)\,\bar{\beta} + v_h.$$

Because δ^h, $\xi_h + \zeta_h$, and v_h are independent, we have

$$\begin{aligned}\mathbb{E}_1^h[\bar{\beta}] = &\frac{\sigma_\delta^{-2}\beta_1^h + \sigma_v^{-2}(\sigma m(\lambda_d,\lambda_s)\,\omega(\lambda_s))^2\left(-\frac{q_1^h}{\sigma m(\lambda_d,\lambda_s)\omega(\lambda_s)}\right)}{\sigma_\beta^{-2} + \sigma_\delta^{-2} + \sigma_v^{-2}(\sigma m(\lambda_d,\lambda_s)\,\omega(\lambda_s))^2 + \left(\sigma_\zeta^2 + \sigma_\xi^2\right)^{-1}(\sigma m(\lambda_d,\lambda_s)(\omega(\lambda_s)+\gamma(\lambda_s)))^2} \\ &+ \frac{\left(\sigma_\zeta^2 + \sigma_\xi^2\right)^{-1}(\sigma m(\lambda_d,\lambda_s)(\omega(\lambda_s)+\gamma(\lambda_s)))^2\left(-\frac{d_{h,1}}{\sigma m(\lambda_d,\lambda_s)(\omega(\lambda_s)+\gamma(\lambda_s))}\right)}{\sigma_\beta^{-2} + \sigma_\delta^{-2} + \sigma_v^{-2}(\sigma m(\lambda_d,\lambda_s)\,\omega(\lambda_s))^2 + \left(\sigma_\zeta^2 + \sigma_\xi^2\right)^{-1}(\sigma m(\lambda_d,\lambda_s)(\omega(\lambda_s)+\gamma(\lambda_s)))^2},\end{aligned}$$

Aggregating over h, we have

$$\mathbb{E}_1^h[\bar{\beta}] = \frac{\sigma_\delta^{-2} + \sigma_v^{-2}(\sigma m(\lambda_d,\lambda_s)\,\omega(\lambda_s))^2 + \left(\sigma_\xi^2 + \sigma_\zeta^2\right)^{-1}(\sigma m(\lambda_d,\lambda_s)(\omega(\lambda_s)+\gamma(\lambda_s)))^2}{\sigma_\beta^{-2} + \sigma_\delta^{-2} + \sigma_v^{-2}(\sigma m(\lambda_d,\lambda_s)\,\omega(\lambda_s))^2 + \left(\sigma_\zeta^2 + \sigma_\xi^2\right)^{-1}(\sigma m(\lambda_d,\lambda_s)(\omega(\lambda_s)+\gamma(\lambda_s)))^2}\bar{\beta}.$$

As a result,

$$\lambda_d = \frac{\sigma_\delta^{-2} + \sigma_v^{-2}(\sigma m(\lambda_d,\lambda_s)\,\omega(\lambda_s))^2 + \left(\sigma_\xi^2 + \sigma_\zeta^2\right)^{-1}(\sigma m(\lambda_d,\lambda_s)(\omega(\lambda_s)+\gamma(\lambda_s)))^2}{\sigma_\beta^{-2} + \sigma_\delta^{-2} + \sigma_v^{-2}(\sigma m(\lambda_d,\lambda_s)\,\omega(\lambda_s))^2 + \left(\sigma_\zeta^2 + \sigma_\xi^2\right)^{-1}(\sigma m(\lambda_d,\lambda_s)(\omega(\lambda_s)+\gamma(\lambda_s)))^2} \in (0,1).$$

The typical worker also has three sources of information: her own discount rate, $\beta^h = \bar{\beta} + \delta^h$; the local demand,[29]

$$d_{h,1} = -\sigma m(\lambda_d,\lambda_s)(\omega(\lambda_s)+\gamma(\lambda_s))\,\bar{\beta} + \xi_h + \zeta_h;$$

and the signal about consumer prices,

$$x_1^h = -\sigma m(\lambda_d,\lambda_s)\,\omega(\lambda_s)\,\bar{\beta} + v_h + \varepsilon_1^h.$$

Similarly to above, we therefore have

$$\int \mathbb{E}\left[\bar{\beta}\middle|\tilde{\mathcal{I}}_1^h\right]dh = \frac{\sigma_\delta^{-2} + \left(\sigma_v^2+\sigma_\varepsilon^2\right)^{-1}(\sigma m(\lambda_d,\lambda_s)\,\omega(\lambda_s))^2 + \left(\sigma_\zeta^2 + \sigma_\xi^2\right)^{-1}(\sigma m(\lambda_d,\lambda_s)(\omega(\lambda_s)+\gamma(\lambda_s)))^2}{\sigma_\beta^{-2} + \sigma_\delta^{-2} + \left(\sigma_v^2+\sigma_\varepsilon^2\right)^{-1}(\sigma m(\lambda_d,\lambda_s)\,\omega(\lambda_s))^2 + \left(\sigma_\zeta^2 + \sigma_\xi^2\right)^{-1}(\sigma m(\lambda_d,\lambda_s)(\omega(\lambda_s)+\gamma(\lambda_s)))^2}\bar{\beta}.$$

As a result,

$$\int \mathbb{E}\left[Q_1\middle|\tilde{\mathcal{I}}_1^h\right]dh = \frac{\sigma_\delta^{-2} + \left(\sigma_v^2+\sigma_\varepsilon^2\right)^{-1}(\sigma m(\lambda_d,\lambda_s)\,\omega(\lambda_s))^2 + \left(\sigma_\zeta^2 + \sigma_\xi^2\right)^{-1}(\sigma m(\lambda_d,\lambda_s)(\omega(\lambda_s)+\gamma(\lambda_s)))^2}{\sigma_\beta^{-2} + \sigma_\delta^{-2} + \left(\sigma_v^2+\sigma_\varepsilon^2\right)^{-1}(\sigma m(\lambda_d,\lambda_s)\,\omega(\lambda_s))^2 + \left(\sigma_\zeta^2 + \sigma_\xi^2\right)^{-1}(\sigma m(\lambda_d,\lambda_s)(\omega(\lambda_s)+\gamma(\lambda_s)))^2}Q_1.$$

[29] From (13) and the fact that $n_1^h = l_{h,1}$, we know that the firm know knowing $d_{h,1}$ is equivalent to her knowing $w_{h,1}$.

Then, notice that

$$\mathbb{E}\left[q_1^h\middle|\tilde{\mathcal{I}}_1^h\right]=\mathbb{E}\left[Q_1|\tilde{\mathcal{I}}_1^h\right]+\frac{\sigma_v^2}{\sigma_v^2+\sigma_\varepsilon^2}\left(x_1^h-\mathbb{E}\left[Q_1|\tilde{\mathcal{I}}_1^h\right]\right).$$

Aggregating over h, we have

$$\int\mathbb{E}\left[q_1^h\middle|\tilde{\mathcal{I}}_1^h\right]dh=\frac{1}{\sigma_v^2+\sigma_\varepsilon^2}\left(\frac{\sigma_\varepsilon^2\left(\sigma_\delta^{-2}+\left(\sigma_v^2+\sigma_\varepsilon^2\right)^{-1}\left(\sigma m\left(\lambda_d,\lambda_s\right)\omega\left(\lambda_s\right)\right)^2+\left(\sigma_\zeta^2+\sigma_\xi^2\right)^{-1}\left(\sigma m\left(\lambda_d,\lambda_s\right)\left(\omega\left(\lambda_s\right)+\gamma\left(\lambda_s\right)\right)\right)^2\right)}{\sigma_\beta^{-2}+\sigma_\delta^{-2}+\left(\sigma_v^2+\sigma_\varepsilon^2\right)^{-1}\left(\sigma m\left(\lambda_d,\lambda_s\right)\omega\left(\lambda_s\right)\right)^2+\left(\sigma_\zeta^2+\sigma_\xi^2\right)^{-1}\left(\sigma m\left(\lambda_d,\lambda_s\right)\left(\omega\left(\lambda_s\right)+\gamma\left(\lambda_s\right)\right)\right)^2}+\sigma_v^2\right)Q_1.$$

As a result,

$$\lambda_s=\frac{\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2}\frac{\sigma_\delta^{-2}+\left(\sigma_v^2+\sigma_\varepsilon^2\right)^{-1}\left(\sigma m\left(\lambda_d,\lambda_s\right)\omega\left(\lambda_s\right)\right)^2+\left(\sigma_\zeta^2+\sigma_\xi^2\right)^{-1}\left(\sigma m\left(\lambda_d,\lambda_s\right)\left(\omega\left(\lambda_s\right)+\gamma\left(\lambda_s\right)\right)\right)^2}{\sigma_\beta^{-2}+\sigma_\delta^{-2}+\left(\sigma_v^2+\sigma_\varepsilon^2\right)^{-1}\left(\sigma m\left(\lambda_d,\lambda_s\right)\omega\left(\lambda_s\right)\right)^2+\left(\sigma_\zeta^2+\sigma_\xi^2\right)^{-1}\left(\sigma m\left(\lambda_d,\lambda_s\right)\left(\omega\left(\lambda_s\right)+\gamma\left(\lambda_s\right)\right)\right)^2}+\frac{\sigma_v^2}{\sigma_v^2+\sigma_\varepsilon^2}\in(0,1).$$

We have thus proved parts (i) and (iii) of the Proposition. To prove part (ii), first note that from Proposition 8, the RHS of (26) is decreasing in λ_d in the stable region, while the LHS of (26) is increasing in the stability region. Then note that, for each $\lambda_s\in(0,1)$, the RHS of (26) is smaller than the LHS when $\lambda_d=1$ and the RHS of (26) is larger than the LHS when $\lambda_d=\max\left(\underline{\lambda},0\right)$. Together, for each $\lambda_s\in(0,1)$, in the stable region, we can find a unique $\lambda_d\equiv g\left(\lambda_s\right)\in(0,1)$ such that (26) holds.

Now, rewrite (27) as

$$\lambda_s=\frac{\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2}\frac{\sigma_\delta^{-2}+\left(\sigma_v^2+\sigma_\varepsilon^2\right)^{-1}\left(\sigma m\left(g\left(\lambda_s\right),\lambda_s\right)\omega\left(\lambda_s\right)\right)^2+\left(\sigma_\zeta^2+\sigma_\xi^2\right)^{-1}\left(\sigma m\left(g\left(\lambda_s\right),\lambda_s\right)\left(\omega\left(\lambda_s\right)+\gamma\left(\lambda_s\right)\right)\right)^2}{\sigma_\beta^{-2}+\sigma_\delta^{-2}+\left(\sigma_v^2+\sigma_\varepsilon^2\right)^{-1}\left(\sigma m\left(g\left(\lambda_s\right),\lambda_s\right)\omega\left(\lambda_s\right)\right)^2+\left(\sigma_\zeta^2+\sigma_\xi^2\right)^{-1}\left(\sigma m\left(g\left(\lambda_s\right),\lambda_s\right)\left(\omega\left(\lambda_s\right)+\gamma\left(\lambda_s\right)\right)\right)^2}+\frac{\sigma_v^2}{\sigma_v^2+\sigma_\varepsilon^2}. \tag{39}$$

The RHS of (39) is smaller than the LHS when $\lambda_s=1$ and the RHS of (39) is larger than the LHS when $\lambda_s=0$. As a result, we can find a λ_s^* such that (39) holds. The pair $\left(\lambda_s^*,g\left(\lambda_s^*\right)\right)$ then solves (26) and (27). and satisfy the stable condition in (24). As a result, a stable equilibrium exists.

Proof of Proposition 10. The supply side and the AS curve is the same as the benchmark model. At the demand side, household h's optimal consumption in period 1 is given by

$$c_1^h=-\tfrac{\beta}{1+\beta}\sigma\left(\beta^h+q_1^h\right)+\tfrac{1}{1+\beta}\left(d_{h,1}-T_1^h-q_1^h\right)+\tfrac{\beta}{1+\beta}\left(\mathbb{E}_1^h\left[d_{h,2}\right]-T_2^h\right),$$

where $d_{i,1}=Q_1+y_1+\xi_i+\zeta_i$ and $d_{i,2}=Q_2+y_2+\xi_i$ denote local demand that includes the government spending. Aggregating and using the government budget constraint in (28), we have

$$c_1=y_1-G_1=-\sigma\left(\bar{\beta}+Q_1\right)+\mathcal{B}-G_2,$$

where $\mathcal{B}\equiv\int\mathbb{E}_1^h\left[d_{h,2}\right]dh$. Part (i) of the Proposition then follows directly.

Comparing the AS and AD curves in Proposition 10 and those in the benchmark model, we find that they are same, except G_1-G_2 replace the role of $-\sigma\bar{\beta}$ as the aggregate demand shock. The

Proposition 4 also remain to hold.[30] As a result, in any stable equilibrium,

$$y_1 = m(\lambda_d, \lambda_s)\gamma(\lambda_s)(G_1 - G_2),$$

where m and γ are defined as in Proposition 7.

Proof of Proposition 11. It follows from the formulas of $m(\lambda_d, \lambda_s)$ and $\gamma(\lambda_s)$ in Proposition 7.

Proof of Proposition 12. Similar to the baseline model, the second period outcomes (both morning and afternoon) replicate the frictionless benchmark. We thus analyze only the first period.

The optimal labor supply for household h in the morning of period 1 is given by

$$n_{m,1}^h = \frac{1}{\kappa}\left\{w_{h,m,1} - \mathbb{E}\left[q_1^h \middle| \mathcal{I}_{m,1}^h\right]\right\},$$

where $q_1^h = \frac{1}{2}q_{m,1}^h + \frac{1}{2}q_{a,1}^h$ captures the the ideal price index of the basket of goods consumed by household h in the entire period 1. The optimal labor supply for household h in the afternoon of period 1 is given by

$$n_{a,1}^h = \frac{1}{\kappa}\left(w_{h,a,1} - q_{a,1}^h + c_1^h - c_{a,1}^h\right).$$

Firm optimality and labor market clearing imply that, for $s \in \{m, a\}$,

$$q_{i,s,1} = w_{i,s,1} \quad \text{and} \quad n_{s,1}^h = y_{i,s,1} = d_{i,s,1} - w_{i,s,1}.$$

Together with optimal labor supply conditions above, we have

$$y_{i,m,1} = \tfrac{1}{1+\kappa}\left\{d_{i,m,1} - \mathbb{E}\left[q_1^h \middle| \mathcal{I}_{m,1}^h\right]\right\} \quad \text{and} \quad y_{i,a,1} = \tfrac{1}{1+\kappa}\left\{d_{i,a,1} - \left(q_{a,1}^h + c_1^h - c_{a,1}^h\right)\right\}.$$

Aggregating, we have the following AS relations in the morning and in the afternoon:

$$y_{m,1} = \frac{1}{2\kappa}\left(Q_{m,1} - \int \mathbb{E}_m^h\left[Q_{a,1}\right] dh\right) \quad \text{and} \quad y_{a,1} = \frac{1}{2(1+\kappa)-1} y_{m,1}. \tag{40}$$

We now turn to the demand side. Similar to Lemma 3, together with the fact that $\psi = 1$ and that the consumption over morning and afternoon takes a Cobb-Douglas form, we have

$$c_{m,1}^h = -q_{m,1}^h - \tfrac{\beta}{1+\beta}\beta^h + \tfrac{1}{1+\beta}E_m^h\left[\frac{1}{2}d_{h,m,1} + \frac{1}{2}d_{h,a,1}\right] + \tfrac{\beta}{1+\beta}E_m^h\left[\frac{1}{2}d_{h,m,2} + \frac{1}{2}d_{h,a,2}\right].$$

[30] Note that in both the benchmark model and the model considered here, $y_2 = 0$ due to the GHH preference.

Aggregating, we have[31]

$$c_{m,1} = -Q_{m,1} - \frac{\frac{\beta}{1+\beta}}{1-\frac{1}{2(1+\beta)}}\bar{\beta} + \frac{1}{2(1+\beta)}\int E_m^h\left[Q_{a,1}+y_{a,1}\right]dh + \frac{\frac{1}{2(1+\beta)}}{1-\frac{1}{2(1+\beta)}}\int E_m^h\left[\zeta_h\right]dh + \int E_m^h\left[\xi_h\right]dh. \tag{41}$$

As information is complete in the afternoon, together with the facts that $\psi = 1$ and that the consumption over morning and afternoon takes a Cobb-Douglas form, we have

$$c_{a,1} = -Q_{a,1} - \bar{\beta}. \tag{42}$$

Similarly to Proposition 4, we can find a $\lambda \in (0,1)$ that captures the degree of information friction about the aggregate shock in the morning:

$$\int \mathbb{E}_m^h\left[Q_{a,1}\right]dh = \lambda Q_{a,1} \quad \text{and} \quad \int E_m^h\left[Q_{a,1}+y_{a,1}\right] = \lambda\left(Q_{a,1}+y_{a,1}\right). \tag{43}$$

If we shut down information friction at the demand side, $\int E_m^h\left[Q_{a,1}+y_{a,1}\right]dh = -\bar{\beta}$ and $\int E_m^h\left[\xi_h\right]dh = \int E_m^h\left[\zeta_h\right]dh = 0$, the AD curve in the morning (41) becomes $c_{m,1} = -Q_{m,1} - \bar{\beta}$. Together with the AS curves in (40) and goods market clearing, we have

$$y_{m,1} = -\frac{\frac{1}{2\kappa}\left(1-\lambda\right)}{1+\frac{1}{2\kappa}\left(\frac{2\kappa+1-\lambda}{2\kappa+1}\right)}\bar{\beta} \quad \text{and} \quad y_1 = \frac{1}{2}\left(1+\frac{1}{2\left(1+\kappa\right)-1}\right)y_{m,1}.$$

As a result, $y_1 = \hat{\gamma}\left(\lambda\right)\tilde{y}$, with $\tilde{y} \equiv -\bar{\beta}$ and

$$\hat{\gamma}\left(\lambda\right) \equiv \frac{\left(1+\frac{1}{2(1+\kappa)-1}\right)\left(\frac{1-\lambda}{2\kappa}\right)}{2+\frac{1}{\kappa}\left(\frac{2\kappa+1-\lambda}{2\kappa+1}\right)}.$$

When the information friction is present in the demand side, conditions (40)-(43), together with the fact that

$$\int E_m^h\left[\xi_h\right]dh = \varrho\left(1-\lambda\right)\left(P_m+Y_m\right) \quad \text{and} \quad \int E_m^h\left[\zeta_h\right]dh = \left(1-\varrho\right)\left(1-\lambda\right)\left(P_m+Y_m\right),$$

give

$$y_{m,1} = -\left(1+\frac{2\beta\varrho}{2\beta\left(1-\varrho\right)+\lambda(1+2\beta\varrho)}\right)\frac{\frac{1}{2\kappa}\left(1-\lambda\right)}{1+\frac{1}{2\kappa}\left(\frac{2\kappa+1-\lambda}{2\kappa+1}\right)}\bar{\beta} \quad \text{and} \quad y_1 = \frac{1}{2}\left(1+\frac{1}{2\left(1+\kappa\right)-1}\right)y_{m,1}.$$

[31] Note that, from Footnote 26, we know $d_{h,s,1} = D_{s,1} + \xi_h + \zeta_h$ and $d_{h,s,2} = D_{s,2} + \xi_h$ for $s \in \{m, a\}$.

As result, $y_1 = \hat{m}(\lambda)\hat{\gamma}(\lambda)\tilde{y}$, with

$$\hat{m}(\lambda) \equiv 1 + \frac{2\beta\varrho}{2\beta(1-\varrho) + \lambda(1+2\beta\varrho)}.$$

Proof of Proposition 13. It follows from the formulas of $\hat{\gamma}(\lambda)$ and $\hat{m}(\lambda)$ in Proposition 12.

Proof of Proposition 14. Part (i) follows from the benchmark model. For part (ii), note that firm optimality means that

$$\mathbb{E}\left[p_{i,2}|\mathcal{I}_{i,1}^f\right] = \mathbb{E}\left[w_{i,2}|\mathcal{I}_{i,1}^f\right].$$

Together with the fact that $d_{i,2} = y_{i,2} + p_{i,2}$ and $w_{i,2} = \kappa y_{i,2}$ (optimal labor supply), we have

$$y_{i,2} = \tfrac{1}{1+\kappa}\mathbb{E}\left[d_{i,2}|\mathcal{I}_{i,1}^f\right].$$

Aggregating gives

$$y_2 = \tfrac{1}{1+\kappa}\mathcal{B}^f \equiv \tfrac{1}{1+\kappa}\int \mathbb{E}\left[d_{i,2}|\mathcal{I}_{i,1}^f\right] di.$$

Proof of Proposition 15. Similar to Proposition 4, we can find a $\lambda_f \in (0,1)$ that parametrizes the informational friction on the firm side:

$$\int \mathbb{E}\left[Q_1 + y_1|\mathcal{I}_{i,1}^f\right] di = \lambda_f(Q_1 + y_1).$$

Condition (20) in Proposition 4 remains true. But as y_2 can now be non-zero, we have

$$\mathcal{B} \equiv \int \mathbb{E}_1^h[d_{h,2}]\,dh = \int \mathbb{E}_1^h[\xi_h]\,dh + \int \mathbb{E}_1^h[y_2]\,dh = (1-\lambda_d)\varrho(Q_1+y_1) + \lambda_d y_2,$$

and

$$\mathcal{B}^f = \int \mathbb{E}\left[d_{i,2}|\mathcal{I}_{i,1}^f\right] di = \int \mathbb{E}\left[\xi_i|\mathcal{I}_{i,1}^f\right] di + \int \mathbb{E}_1^i[y_2]\,di = (1-\lambda_f)\varrho(Q_1+y_1) + \lambda_f y_2.$$

Together with Proposition 15, we have

$$y_1 = \tfrac{1}{\kappa}(1-\lambda_s)Q_1 \quad \text{and} \quad y_1 = -\sigma\left(\bar{\beta} + Q_1\right) + (1-\lambda_d)\varrho(Q_1+y_1) + \lambda_d y_2, \tag{44}$$

and

$$y_2 = \tfrac{1}{1+\kappa}\left((1-\lambda_f)\varrho(Q_1+y_1) + \lambda_f y_2\right).$$

Solving y_2, we have

$$y_2 = \frac{(1-\lambda_f)\varrho}{1+\kappa-\lambda_f}(Q_1 + y_1) = \frac{(1-\lambda_f)\varrho}{1+\kappa-\lambda_f}(1+\frac{\kappa}{1-\lambda_s})y_1.$$

This proves part (i) of the proposition.

Using the above condition to replace y_2 in (44), we then have

$$y_1 = -\sigma\left(\bar{\beta} + Q_1\right) + \left[(1-\lambda_d)\varrho + \lambda_d \frac{(1-\lambda_f)\varrho}{1+\kappa-\lambda_f}\right](Q_1 + y_1). \tag{45}$$

The stability condition is then given by

$$\left[(1-\lambda_d)\varrho + \lambda_d \frac{(1-\lambda_f)\varrho}{1+\kappa-\lambda_f}\right] \frac{1+\frac{1}{\kappa}(1-\lambda_s)}{\sigma + \frac{1}{\kappa}(1-\lambda_s)} < 1.$$

Condition (45) together with the AS curve then gives

$$y_1 = m^{new}(\lambda_d, \lambda_s)\gamma(\lambda_s)\tilde{y} \qquad \text{and} \qquad Q_1 = m^{new}(\lambda_d, \lambda_s)\omega(\lambda_s)\tilde{y},$$

where

$$m^{new}(\lambda_d, \lambda_s) = \frac{\sigma + \frac{1-\lambda_s}{\kappa}}{\sigma + \frac{1-\lambda_s}{\kappa} - \left[(1-\lambda_d)\varrho + \lambda_d \frac{(1-\lambda_f)\varrho}{1+\kappa-\lambda_f}\right]\left(1 + \frac{1-\lambda_s}{\kappa}\right)} > m(\lambda_d, \lambda_s).$$

Appendix B: Explicit Matching Process

In this Appendix, we consider an explicit matching process which gives rise to the random variables v^h and ζ_i assumed in Section 3 and makes σ_v and σ_ζ endogenous to the equilibrium. This complicates the fixed-point problem whose solution pins down the values of λ_d and λ_s, but does not affect the essence of our results.

Specifically, we use an exogenously drawn random variable $m_i \sim \log\mathcal{N}\left(-\frac{1}{2}\sigma_m^2, \sigma_m^2\right)$ to capture the measure of households who consume goods produced on island i. This means, for each pair of islands (i,j), $\frac{\left|h:i\in\mathcal{C}_1^h\right|}{\left|h:j\in\mathcal{C}_1^h\right|} = \frac{m_i}{m_j}$. Then, we let the household h be assigned with an unobservable sampling shock $\omega^h \sim \log\mathcal{N}\left(-\frac{1}{2}\sigma_\omega^2, \sigma_\omega^2\right)$. The nature selects a random subset of islands $\mathcal{C}_1^h \in [0,1]$ of fixed measure which the household h consumes the products. The matching satisfies the following property: the distribution of $\log m_i$ for the goods in $\mathcal{C}_1^h$ is normal with mean $\log\omega^h$ and variance $\sigma_{m|\omega}^2$. Using the law of total variance, we have $\sigma_{m|\omega}^2 + \sigma_\omega^2 = \sigma_m^2$. I then use $\chi = \sigma_\omega^2/\sigma_m^2 \in [0,1]$ to reflect the degree of heterogeneity in consumption baskets. For households who consume on island i, their average ω^h is then given by have $E_{h:i\in\mathcal{C}_1^h}[\log\omega^h] = \chi\log m_i$. For each island i, m_i and ω^i is independent from each other and all other random variables in the economy.

From now on, as in the baseline model, we re-interpret all the variables as log-deviations from the deterministic steady state and shut down the GHH complementarity (i.e., we let $\mu \to \infty$ and $\sigma \to \psi$). We now provide a link between the exogenous shocks m_i and ω^h defined here and the endogenous objects ζ_i and v^h defined in Section 3.

We guess that

$$q_{i,1} = Q_1 + \phi^q_\xi \xi_i + \phi^q_m m_i + \phi^q_\omega \omega^i + \phi^q_\varepsilon \varepsilon^i + \phi^q_\delta \delta^i.$$

The ideal price index of the goods consumed by household h is then given by

$$q^h_1 = Q_1 + \phi^q_m \omega^h.$$

It follows that the household-specific sampling shock v_i used in main text is given by

$$v_i = \phi^q_m \omega^h,$$

and its variance is given by

$$\sigma^2_v = \left(\phi^q_m\right)^2 \sigma^2_\omega. \tag{46}$$

Among households who consume on island i, we then have

$$E_{h:i\in\mathcal{C}^h_1}[q^h_1] = Q_1 + \chi \phi^q_m m_i.$$

Together with Lemma 3, we infer that these households' average spending in period 1 is given by

$$\begin{aligned}
E_{h:i\in\mathcal{C}^h_1}[q^h_1 + c^h_1] &= \tfrac{\beta}{1+\beta}\left(1-\sigma\right) E_{h:i\in\mathcal{C}^h_1}[q^h_1] - \tfrac{\beta}{1+\beta}\sigma\bar{\beta} + \tfrac{1}{1+\beta}\left(Q_1 + c_1\right) + \tfrac{\beta}{1+\beta} E_{h:i\in\mathcal{C}^h_1}\left[E^h_1\left[d_{h,2}\right]\right] \\
&= \tfrac{\beta}{1+\beta}\left(1-\sigma\right) E_{h:i\in\mathcal{C}^h_1}[q^h_1] - \tfrac{\beta}{1+\beta}\sigma\bar{\beta} + \tfrac{1}{1+\beta}\left(Q_1 + c_1\right) + \tfrac{\beta}{1+\beta} E_{h:i\in\mathcal{C}^h_1}\left[\varrho\left(d_{h,1} - \mathbb{E}^h_1\left[Q_1 + Y_1\right]\right)\right] \\
&= Q_1 + c_1 + \tfrac{\beta}{1+\beta}\left(1-\sigma\right)\chi\phi^q_m m_i - \tfrac{\beta}{1+\beta}\varrho\lambda^q\chi\phi^q_m m_i,
\end{aligned}$$

where, following the proof of Proposition 4,

$$\lambda^q = \frac{\sigma_v^{-2}\sigma^2 m^2\left(\lambda_d, \lambda_s\right)\omega\left(\lambda_s\right)\left(\omega\left(\lambda_s\right) + \gamma\left(\lambda_s\right)\right)}{\sigma_\beta^{-2} + \sigma_\delta^{-2} + \sigma_v^{-2}\left(\sigma m\left(\lambda_d, \lambda_s\right)\omega\left(\lambda_s\right)\right)^2 + (\sigma^2_\zeta + \sigma^2_\xi)^{-1}\sigma^2\left(m\left(\lambda_d, \lambda_s\right)\left(\omega\left(\lambda_s\right) + \gamma\left(\lambda_s\right)\right)\right)^2} \tag{47}$$

captures how household h's belief $\mathbb{E}^h_1\left[Q_1 + Y_1\right]$ moves with respect changes in q^h_1.

As a result, the total spending for the goods produced on island i is given by

$$d_{i,1} = E_{h:i\in\mathcal{C}^h_1}[q^h_1 + c^h_1] + \xi_i + m_i = Q_1 + y_1 + \xi_i + \left(1 + \tfrac{\beta}{1+\beta}\left(1-\sigma\right)\chi\phi^q_m - \tfrac{\beta}{1+\beta}\varrho\lambda^q\chi\phi^q_m\right) m_i.$$

The island-specific sampling shock ζ_i used in main text is then given by

$$\zeta_i = \left(1 + \tfrac{\beta}{1+\beta}\left(1-\sigma\right)\chi\phi_m^q - \tfrac{\beta}{1+\beta}\varrho\lambda^q\chi\phi_m^q\right)m_i.$$

and its variance is given by

$$\sigma_\zeta^2 = \left(1 + \tfrac{\beta}{1+\beta}\left(1-\sigma\right)\chi\phi_m^q - \tfrac{\beta}{1+\beta}\varrho\lambda^q\chi\phi_m^q\right)^2\sigma_\omega^2 \tag{48}$$

with

$$\varrho \equiv \frac{\sigma_\xi^2}{\sigma_\zeta^2+\sigma_\xi^2} = \frac{\sigma_\xi^2}{\left(1 + \tfrac{\beta}{1+\beta}\left(1-\sigma\right)\chi\phi_m^q - \tfrac{\beta}{1+\beta}\varrho\lambda^q\chi\phi_m^q\right)^2\sigma_\omega^2+\sigma_\xi^2}. \tag{49}$$

Using $d_{i,1} = q_{i,1} + y_{i,1}$, we also have

$$y_{i,1} = y_1 + \left(1-\phi_\xi^q\right)\xi_i + \left(1 + \tfrac{\beta}{1+\beta}\left(1-\sigma\right)\chi\phi_m^q - \tfrac{\beta}{1+\beta}\varrho\lambda^q\chi\phi_m^q - \phi_m^q\right)m_i - \phi_\omega^q\omega^i - \phi_\varepsilon^q\varepsilon^i - \phi_\delta^q\delta^i.$$

From the supply side,

$$y_{i,1} = \frac{1}{\kappa}\left\{q_{i,1} - \mathbb{E}\left[q_1^i\middle|\tilde{\mathcal{I}}_1^i\right]\right\}.$$

Comparing coefficient of m_i on each side, we then have

$$\phi_m^q = \kappa\left(1 + \tfrac{\beta}{1+\beta}\left(1-\sigma\right)\chi\phi_m^q - \tfrac{\beta}{1+\beta}\varrho\lambda^q\chi\phi_m^q - \phi_m^q\right) + \lambda^d\left(1 + \tfrac{\beta}{1+\beta}\left(1-\sigma\right)\chi\phi_m^q - \tfrac{\beta}{1+\beta}\varrho\lambda^q\chi\phi_m^q\right), \tag{50}$$

where, following the proof of Proposition 4,

$$\lambda^d = \frac{\sigma_\varepsilon^2}{\sigma_v^2+\sigma_\varepsilon^2}\frac{(\sigma_\xi^2+\sigma_\zeta^2)^{-1}\sigma^2 m\left(\lambda_d,\lambda_s\right)^2\omega\left(\lambda_s\right)\left(\omega\left(\lambda_s\right)+\gamma\left(\lambda_s\right)\right)}{\sigma_\beta^{-2}+\sigma_\delta^{-2}+\sigma_v^{-2}\left(\sigma m\left(\lambda_d,\lambda_s\right)\omega\left(\lambda_s\right)\right)^2+(\sigma_\xi^2+\sigma_\zeta^2)^{-1}\sigma^2\left(m\left(\lambda_d,\lambda_s\right)\left(\omega\left(\lambda_s\right)+\gamma\left(\lambda_s\right)\right)\right)^2} \tag{51}$$

captures how the worker on island i's belief $\mathbb{E}\left[q_1^i\middle|\tilde{\mathcal{I}}_1^i\right]$ moves with respect changes in the local demand (income) $d_{i,1}$.

The system of equations comprised by condition (26), (27), and (46)-(51) fully characterizes the equilibrium determination of the vector $(\lambda_s, \lambda_d, \lambda^q, \lambda^d, \phi_m^p, \sigma_\zeta^2, \sigma_\xi^2, \varrho)$. This system is highly non-linear, precluding a sharp analysis of the determinacy of its solution or the comparative statics with respect to the exogenous parameters. However, because λ_m and λ_d lie between 0 and 1 in *any* solution to this system, our main results remain unaffected.

Appendix C: GHH complementarity ($\varphi \neq 0$)

In the main text, we shut down the effect stemming from the GHH complementarity between consumption and labor by considering the limit case in which $\varphi = 0$ (or $\mu = \infty$). We now consider the more general case in which this effect is present and, by the same token, the term $\mathcal{H}$ showing up in aggregate demand (19) is non-zero.

The analysis parallels the one in the previous subsections, except that we now have to take into account the equilibrium determination of $\mathcal{H}$. As a result, Proposition 16 is modified as follows.

Proposition 16 *Consider the general case with $\mu < \infty$, or $\varphi > 0$.*

(i) The equilibrium is stable in the sense that feedback chain is non-explosive if and only if

$$\lambda^d > \underline{\lambda} \equiv 1 - \frac{1}{\left(1 - \frac{1}{(1+\kappa)(1+\mu)}\right)\varrho} \frac{\sigma + \frac{1-\lambda_s}{\kappa}\frac{\mu}{\mu+1}}{1 + \frac{1-\lambda_s}{\kappa}}. \tag{52}$$

A sufficient condition for (52) to be true for all $\lambda_s, \lambda_d \in (0, 1]$ is $\sigma \geq 1$.

(ii) In any stable equilibrium, aggregate output and the real interest rate are given by, respectively,

$$y = m(\lambda_d, \lambda_s)\gamma(\lambda_s)\tilde{y} \qquad \text{and} \qquad Q = m(\lambda_d, \lambda_s)\omega(\lambda_s)\tilde{y},$$

where $\tilde{y} \equiv -\sigma\bar{\beta}$ measures the exogenous demand shock,

$$\gamma(\lambda_s) = \frac{\frac{1-\lambda_s}{\kappa}}{\sigma + \frac{1-\lambda_s}{\kappa}\frac{\mu}{\mu+1}} \quad \text{and} \quad \omega(\lambda_s) = \frac{1}{\sigma + \frac{1-\lambda_s}{\kappa}\frac{\mu}{\mu+1}}$$

describe the equilibrium responses when only the supply side friction is present, and

$$m(\lambda_d, \lambda_s) \equiv \frac{\sigma + \frac{1-\lambda_s}{\kappa}\frac{\mu}{\mu+1}}{\sigma + \frac{1-\lambda_s}{\kappa}\frac{\mu}{\mu+1} - (1-\lambda_d)\left(1 - \frac{1}{(1+\kappa)(1+\mu)}\right)\varrho\left(1 + \frac{1-\lambda_s}{\kappa}\right)} > 1 \tag{53}$$

is the confidence multiplier.

Comparing this result to Proposition 16, we see that the essence remains the same. The only differences are in the condition for non-explosiveness and in the formulas for γ, ω and μ. The next result verifies that the comparative statics of the multiplier with respect to the informational friction remain qualitatively the same, too.

Proposition 17 *In any stable equilibrium, the comparative statics stated in Proposition 8 continue to hold, modulo the adjustment of the threshold $\hat{\sigma}$ to $\hat{\sigma} = \frac{\mu}{\mu+1} < 1$.*

The rest of this appendix provides the proofs of the above two results.

Proof of Proposition 16. The AS curve is still given by (22). The AD curve follows from (19) and (21):

$$y_1 = -\frac{\sigma}{1-\varphi}\left(\bar{\beta}+Q_1\right) + \frac{1}{1-\varphi}\left(1-\frac{\varphi}{1+\kappa}\right)(1-\lambda_d)\varrho\left(Q_1+y_1\right).$$

When only the supply side friction is present ($\lambda_d = 1$), together with the AS curve in (22), we have

$$y_1 = \gamma(\lambda_s)\tilde{y} \qquad \text{and} \qquad Q_1 = \omega(\lambda_s)\tilde{y},$$

where $\tilde{y} \equiv -\sigma\bar{\beta}$, $\gamma(\lambda_s) = \frac{\frac{1-\lambda_s}{\kappa}}{\sigma+\frac{1-\lambda_s}{\kappa}\frac{\mu}{\mu+1}}$, $\omega(\lambda_s) = \frac{1}{\sigma+\frac{1-\lambda_s}{\kappa}\frac{\mu}{\mu+1}}$, and I use the formula $\varphi = \frac{1}{1+\mu}$.

Now, we turn to the case in which the demand side friction is present ($\lambda_d < 1$). Similarly to the case in which $\varphi = 0$, it easy to check that, for any $k \geq 1$, the k-th step of the feedback loop of the aggregate demand shock $\bar{\beta}$ has AD change by $-\chi^k \frac{\sigma}{1-\varphi}\bar{\beta}$, where χ is now given by

$$\chi \equiv \left(1-\frac{1}{(1+\kappa)(1+\mu)}\right)(1-\lambda_d)\varrho\left(\frac{\frac{1-\lambda_s}{\kappa}+1}{\sigma+\frac{1-\lambda_s}{\kappa}\frac{\mu}{\mu+1}}\right).$$

For the feedback loop to be non-explosive, it must be that $\chi < 1$, which gives condition (24).

In this region, using the AS curve in (22) together with the AD curve in (23), we arrive at the expression of the equilibrium output and price response in Proposition 16.

Proof of Proposition 17. The non-explosive condition in (24) means that the denominator in the expression of $m(\lambda_d, \lambda_s)$ in (25) is positive. As a result, $m(\lambda_d, \lambda_s)$ is strictly decreasing in λ_d. Now, let us rewrite

$$m(\lambda_d, \lambda_s) = \frac{1}{1-(1-\lambda_d)\left(1-\frac{1}{(1+\kappa)(1+\mu)}\right)\varrho\left(\frac{1+\frac{1-\lambda_s}{\kappa}}{\sigma+\frac{1-\lambda_s}{\kappa}\frac{\mu}{\mu+1}}\right)}.$$

The comparative statics with respect to λ_s, ϱ and κ are then immediate.

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