Crises: Equilibrium Shifts and Large Shocks

Stephen Morris (Princeton) and Muhamet Yildiz (MIT)

Cowles Lunch Talk
February 2018
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Widely credited with having shifted the Eurozone economy from a "bad equilibrium" (high sovereign debt spreads and growing fiscal deficits mutually reinforcing each other); to a "good equilibrium" (with low spreads and sustainable fiscal policy).
explaining equilibrium shifts

in many economic (and other) settings...

- have convincing explanations/models of strategic complementarities giving rise to self-fulfilling outcomes
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E.g., sovereign debt markets, financial crises, revolutions
Consider a setting where...

- Fundamentals hit a critical boundary (we will see how this boundary is determined)
- There is a large enough shock to fundamentals - even if the shock does not take us to the critical boundary (we will see how big this jump must be)

We explain when shifts must occur but allow for multiplicity and hysteresis in many scenarios.
Consider a setting where...

- a coordination game is played every period whose payoffs depend on a "fundamental state"
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- the fundamental state evolves according to an exogenous random process

We ask: which informational events (must) trigger equilibrium switches?

We identify two distinct scenarios that must trigger equilibrium shifts:

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• Key strategic implication:
  • with no or small shocks, can keep doing same thing as before because you may rationally be confident that others will do so
  • with large shocks,
    • not rational for marginal player to be confident of others’ behavior; uniform rank beliefs select "risk dominant" equilibrium
Both levels and change predict shifts.
Distinctive Predictions

1. Both levels and change predict shifts.
2. Don’t always play risk dominant equilibrium, but switches only to risk dominant equilibrium.
Part 1 (Analysis): Individual Rationalizable Behavior in a Static Coordination Game with Incomplete Information

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- Our main large shock result relies on fat tails (c.f., large normal prior, normal noise global game literature)
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Part 2 (Interpretation): Aggregate Behavior in Dynamic Coordination Game

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- Static coordination game played repeatedly under evolving fundamentals and fat-tailed prior on common innovations
- Assume hysteresis: follow majority play from previous period if rationalizable, otherwise
- Majority behavior switches in response to either extreme enough level of fundamentals or a large shock
a continuum of players
Complete Information Game

- a continuum of players
- each player decides to "invest" or "not invest"
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Complete Information Game

- a continuum of players
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- "return to investing" $\times$
- invest if the return exceeds the expected proportion of others not investing
- formally, payoff to not investing is 0 and payoff to investing is $x + \alpha - 1$, where $\alpha$ is the proportion of other players investing
Complete Information Game Equilibria

- Equilibria...
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- Equilibria...
  - if $x > 1$, players have a dominant strategy to invest
  - if $x < 0$, players have a dominant strategy to not invest
  - if $0 < x < 1$, “all invest” and “all not invest” are both equilibria
  - Terminology: the risk dominant action is the one that would be chosen by a player with a uniform belief over the proportion of others who will invest.
  - if $x > 1/2$, “all invest” is the risk dominant equilibrium
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• common prior mean return is $y$
• agent $i$ has return to investment is $x_i = y + \sigma z_i$ where
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Incomplete Information / Heterogeneous Returns

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  - parameter $\sigma > 0$ measures "shock sensitivity"
  - agent $i$’s shock $z_i$ has two components, $z_i = \eta + \varepsilon_i$
    - a common shock $\eta$
    - an idiosyncratic shock $\varepsilon_i$
1 thick tailed common shocks: $\eta$ is distributed according to density $g$ with thick (regularly varying) tails, i.e.,

$$\lim_{\lambda \to \infty} \frac{g(\lambda \eta)}{g(\lambda \eta')} \in (0, \infty) \text{ for all } \eta, \eta'$$
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2. **thinner tailed idiosyncratic shocks**: $\varepsilon$ is distributed according to log-concave density $f$ (i.e., log $f$ is concave)
Rank belief: what probability does an agent assign to a representative agent having a lower return than his own?

\[ R(z) \equiv \Pr(z_j \leq z | z_i = z) = \frac{\int F(\varepsilon) f(\varepsilon) g(z - \varepsilon) d\varepsilon}{\int f(\varepsilon) g(z - \varepsilon) d\varepsilon} \]

Equivalently, what is an agent’s expectation of the proportion of other agents with lower returns?
• \( f \) is standard normal distribution \( N(0, 1) \)
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  • variance of $\eta$ is unknown and distributed with inverse $\chi^2$
Rank Beliefs in the Leading Example

Figure: Rank belief function $R$. 

Rank belief function for $t$ distribution
$R$ is differentiable and satisfies:

- **symmetry**: $R(-z) = 1 - R(z)$; in particular, $R(0) = 1/2$. 
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- **symmetry**: $R(-z) = 1 - R(z)$; in particular, $R(0) = 1/2$.
- **single crossing at $1/2$**: $R(z) > 1/2 > R(-z)$ whenever $z > 0$. 
\( R \) is differentiable and satisfies:

- **symmetry**: \( R(-z) = 1 - R(z) \); in particular, \( R(0) = 1/2 \).
- **single crossing at 1/2**: \( R(z) > 1/2 > R(-z) \) whenever \( z > 0 \).
- **limit uniform rank beliefs**: \( R(z) \to \frac{1}{2} \) as \( z \to \infty \).
Fat-Tails Assumption—Motivation

- model uncertainty:
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  - e.g., variance uncertainty + normal $\Rightarrow$ t distribution
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  - e.g. income, prices, financial asset returns, exchange rates, GDP, ...
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  - e.g. income, prices, financial asset returns, exchange rates, GDP, ...
- present in many commonly used statistical models (e.g. GARCH, stochastic volatility)
- limit uniform rank beliefs as a primitive assumption?
Figure: Rank belief function under normal idiosyncratic shocks and normal or exponential common shocks
• Suppose agents follow a "cutoff" strategy, with each agent investing if his shock $z_i$ is above some critical threshold $\hat{Z}$.
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• an agent with shock $\hat{Z}$ agent is indifferent between investing and not investing when

$$y + \sigma \hat{Z} = R(\hat{Z})$$

(1) is a necessary condition for a $\hat{Z}$-cutoff equilibrium also sufficient because log-concavity of $f$ implies that when an agent has a high return, she has a higher (w.r.t. FOSD) belief about other player's return.
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Following graph plots $y + \sigma \hat{z}$ (in blue) and $R(\hat{z})$ (in red).
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Let $z^{**}$ be largest solution to (1)
Unique Rationalizable Play

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- Corresponds to equilibrium with the least investment (invest only if $z \geq z^{**}$)
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• Invest is uniquely rationalizable if and only if $z > z^{**}$
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• Corresponds to equilibrium with the least investment (invest only if $z \geq z^{**}$)
• Invest is uniquely rationalizable if and only if $z > z^{**}$
• PROOF: Let $\bar{z}$ be the largest shock at which not invest is rationalizable and suppose $\bar{z} > z^{**}$. The payoff to investing is at least

\[
\underbrace{y + \sigma \bar{z}} \quad \text{own return} \quad \underbrace{R(\bar{z})} \quad \text{proportion of others not investing} > 0,
\]

a contradiction.
Let $\bar{R}$ be the maximum possible rank belief:

$$\bar{R} = \max_{z \geq 0} R(z)$$

**Proposition**

*Invest is uniquely rationalizable whenever $x > \bar{R}$*

- equivalently, invest is uniquely rationalizable if $z > \frac{\bar{R} - y}{\sigma}$
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- equivalently, invest is uniquely rationalizable if $z > \frac{\overline{R}-y}{\sigma}$
- for sufficiently high returns, it doesn’t matter how you got there
- observe that $\frac{1}{2} < \overline{R} < 1$; thus this criterion is intermediate between risk dominance and dominant strategies
For each $x \in \left(\frac{1}{2}, R\right]$, define critical shock size $\bar{z}(x)$ to be the largest shock at which the rank belief is $x$:

$$\bar{z}(x) = \max R^{-1}(x)$$

Proposition

*Invest is uniquely rationalizable if $x \in \left(\frac{1}{2}, R\right]$ and $z > \bar{z}(x)$*
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**Proposition**

*Invest is uniquely rationalizable if* $x \in \left(\frac{1}{2}, R\right]$ *and* $z > \bar{z}(x)$

• for intermediate returns, whether invest is uniquely rationalizable depends on whether there was a positive shock
• Invest will be uniquely rationalizable at fundamentals $x_i$ if reached via a large shock (left panel) but not if reached by a small shock (right panel)
Proposition

*Invest is uniquely rationalizable whenever $x > \frac{1}{2}$ and $y > \bar{y}$*
Ex Ante Level Sufficient Condition

- Let $\bar{y}$ be the critical level of fundamentals at which returns will exceed the rank belief whenever shocks are positive.
- Formally, define $\bar{y}$ to be the largest $y$ such that

$$R(z) \geq y + \sigma z$$

for some $z$.

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for some $z$.
- For small $\sigma$, $\bar{y} \approx \bar{R}$

**Proposition**

*Invest is uniquely rationalizable whenever $x > \frac{1}{2}$ and $y > \bar{y}$*
• For small $\sigma$, sufficient conditions are also necessary...

**Proposition**

*If $R$ is single peaked and $y \leq \bar{R} - \sigma \bar{z}(\bar{R}) \leq \bar{y}$, invest is uniquely rationalizable only if (i) $x > \bar{R}$ or (ii) $x > \frac{1}{2}$ and $z > \bar{z}(x)$*
• For small $\sigma$, sufficient conditions are also necessary....
• We get a partial converse under two additional restrictions:

Proposition

If $R$ is single peaked and $y \leq \bar{R} - \sigma \bar{z}(\bar{R}) \leq \bar{y}$, invest is uniquely rationalizable only if (i) $x > \bar{R}$ or (ii) $x > \frac{1}{2}$ and $z > \bar{z}(x)$
Call $\theta = y + \sigma \eta$ the *fundamental state*; fundamental state is the population mean return and also the median agent’s return.

**Proposition**

Invest is uniquely rationalizable for the majority if it is risk dominant ($\theta > \frac{1}{2}$) and, in addition, (i) the realized fundamentals are sufficiently high ($\theta > \bar{R}$), or (ii) the expected fundamentals were sufficiently high ($y > \bar{y}$), or (iii) the shock is sufficiently high $\eta > \bar{z}(\theta)$. 
• Infinite horizon game played in every period $t = 0, 1, \ldots$
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• Enter each period with mean $y_t$
Dynamic Game

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- Draw $x_{it} = \theta + \sigma \varepsilon_i = y + \sigma \eta + \sigma \varepsilon_i$
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• Draw $x_{it} = \theta + \sigma \varepsilon_i = y + \sigma \eta + \sigma \varepsilon_i$
• Play static game
• Period $t$ play and $\theta_t$ become common knowledge
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- Period $t$ play and $\theta_t$ become common knowledge
- Let $y_{t+1} = Y(\theta_t)$ for $t = 0, 1, ...$
Dynamic Game

- Infinite horizon game played in every period $t = 0, 1, \ldots$
- Enter each period with mean $y_t$
- Draw $\theta_t = y_t + \sigma \eta_t$
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- Period $t$ play and $\theta_t$ become common knowledge
- Let $y_{t+1} = Y(\theta_t)$ for $t = 0, 1, \ldots$
  - for example, random walk ($y_{t+1} = \theta_t$) or reversion to the mean ($y_{t+1} = \frac{1}{2} + \kappa (\theta_t - \frac{1}{2})$)
Equilibria of the Dynamic Game

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- A special *hysteresis equilibrium*:
  - was there majority investment in the previous period?
  - if yes, invest whenever rationalizable
  - if not, do not invest whenever rationalizable
**Proposition**

*Shifts to majority investment will occur whenever invest is risk dominant* \((\theta_t > \frac{1}{2})\) *and, in addition, (i) the realized fundamentals are sufficiently high* \((\theta_t > \bar{R})\), *(ii) the expected fundamentals were sufficiently high* \((y_t > \bar{y})\) *or the shock was sufficiently high* \(\eta_t > \bar{z}(\theta_t)\).*
• Methodological:

- Large shocks imply uniform rank beliefs and selection, even without unique predictions, leaving role for hysteresis, culture, or whatever...
- Significant events may shift equilibria because there is NOT common knowledge of how to interpret them.

Substantive:
- Slow news release is good if you want to stay in current equilibrium (and vice versa).
- Simple mechanism that can be plugged into richer models.
Takeaways

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  - rank beliefs matter
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• Invest is uniquely rationalizable if \( x > \bar{R} \) or if \( x \in (R_\infty, \bar{R}] \) and \( z > \bar{z}(x) \) ....

• No role for shocks with monotone rank beliefs and \( R_\infty = 1 \) (e.g., normality)
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• SMALL SHOCKS PROPOSITION: Under multiplicity condition, there exists \( \Delta > 0 \) such that whenever
  \(|x - y| \leq \Delta\), invest is uniquely rationalizable if and only if \( y > \bar{y}\).
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• If a bad equilibrium is being played, and fundamentals are heading up, it is better to have fundamentals jump up (or good news released in chunks)
• Equilibrium shifts occur when triggered by common knowledge events
Competing Hypothesis? Coordination and Common Knowledge

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  - folk argument
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