Costly search, information, and competition

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March 2018
Homogenous good markets and price setting

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Homogenous good markets and price setting

- Law of One Price: with more than one seller, firms will compete the price down to cost [Bertrand (1883)]
- 40 years ago, Varian wrote: "Economists have belatedly come to recognize that the "law of one price" is no law at all. Most retail markets are instead characterized by a rather large degree of price dispersion. The challenge to economic theory is to describe how such price dispersion can persist in markets where at least some consumers behave in a rational manner."
40 years on......

- Most retail markets *continue* to be characterized by “a rather large degree of price dispersion”, notwithstanding the invention of the internet.....
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- This smorgasbord of models makes lots of stylized assumptions I don’t know how to observe / what to make of: optimal versus exogenous search, simultaneous versus sequential, consumer vs. search costs, heterogeneity, psychic costs, etc...
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- No metric of how much price dispersion we can explain

- No results on level of prices (e.g., expected price) and welfare
Our Paper

- Single consumer with value 1 for a homogenous good observes a number of price quotes from firms with identical marginal cost (normalized to 0) and buys from the firm with the lowest price quote.
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  1. Exogenous distribution of the number of price quotes (allowing different information among firms about number of quotes).
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- Single consumer with value 1 for a homogenous good observes a number of price quotes from firms with identical marginal cost (normalized to 0) and buys from the firm with the lowest price quote
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- Two cases:
  1. Exogenous distribution of the number of price quotes (allowing different information among firms about number of quotes)
  2. Endogenous distribution of price quotes (e.g., simultaneous and sequential search, costly price posting, information intermediaries)
1. we show the existence of and characterize the highest (under FOSD) distribution of equilibrium prices (across information structures and equilibria).....
Result 1: Fixed Exogenous Distribution of Number of Price Quotes

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2. if the ex ante probability of a single price quote is $\mu$, the expected price / revenue is at most $\sqrt{\mu (2 - \mu)} > \mu$
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3. we show how to attain all bounds
Maximum Revenue

![Graph showing maximum revenue as a function of $\mu(1)$ with two curves: red for 'No information' and blue for 'Rich information'.]
Result 2: Endogenous Distribution of Number of Endogenous Price Quotes

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The bounds continue to hold under reasonable models endogenizing the distribution on price quotes:

1. straightforwardly: under simultaneous price setting with pre-determined information structure (e.g., with ex ante search decisions by consumers, costly price posting and informational intermediary markets)
2. more subtly: under sequential search by consumers
Talk

1. lengthy example to illustrate our results and prior literature on price dispersion in homogenous goods markets
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   3.3 relation to our prior work on auctions and informational robustness
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Example

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Example

- single consumer has value 1 for a single unit of a homogenous good
- two firms have 0 cost of production
- consumer collects a single (monopoly) quote with probability $\frac{1}{2}$, two (competitive) quotes with probability $\frac{1}{2}$
- more precisely, consumer gets quote from firm 1 only with probability $\frac{1}{4}$, firm 2 only with probability $\frac{1}{4}$, and both firms with probability $\frac{1}{4}$. 
Full Information

- if one quote, firm charges monopoly price of 1
if one quote, firm charges monopoly price of 1
if two quotes, both firms charge competitive price of 0
Full Information

- if one quote, firm charges monopoly price of 1
- if two quotes, both firms charge competitive price of 0
- figure 1 plots the equilibrium price distribution $= \text{probability that price is } p \text{ or higher}$. ....
Full Information Price Distribution

The diagram shows the distribution of a variable $S(p)$ over the range of $p$ from 0 to 1. The line represents the full information scenario, indicating a constant value of 0.5 across the range of $p$. The $x$-axis represents $p$, and the $y$-axis represents $S(p)$. The graph is a flat line at 0.5, suggesting no variation with $p$.
No Information

- a firm assigns probability $\frac{1}{3}$ to being the monopolist
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- a firm assigns probability $\frac{1}{3}$ to being the monopolist
- there is a unique symmetric mixed strategy equilibrium where firms randomize over prices on the interval $\left[\frac{1}{3}, 1\right]$ and the probability of choosing price $p$ or above is $F(p) = \frac{1-p}{2p}$
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  \left(\frac{1}{3} + \frac{2}{3}F(p)\right) p = \frac{1}{3}
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  \frac{1}{2} \left( \frac{1-p}{2p} \right) + \frac{1}{2} \left( \frac{1-p}{2p} \right)^2 = \frac{1-p^2}{8p}
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- note that because firms always indifferent to charging monopoly price, expected price = industry revenue = $\frac{1}{2}$, as in no information case
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- note that because firms always indifferent to charging monopoly price, expected price = industry revenue = $\frac{1}{2}$, as in no information case
- this will be a general result about the no information case
No Information Price Distribution
Partially Informed Firms

- consider one special information structure: if market is competitive, then

\[ \alpha \]

there is an equilibrium where uninformed firms charge price 1 and informed firms follow mixed strategy

\[ F(p) = \alpha 1 - p \]

with support \([\alpha 1 - \alpha, 1]\).

- verify:

informed firms payoff to charging price \(p\) is

\[ (\alpha 1 - \alpha + 1 - 2\alpha) F(p) = \alpha \]

condition (1) ensures that uninformed attaches a higher probability to facing monopoly price than uninformed firm
Partially Informed Firms

- consider one special information structure: if market is competitive, then
  - with probability $\alpha$, firm 1 only is told
  - with probability $\alpha$, firm 2 only is told
  - with probability $1 - 2\alpha$, both firms are told

There is an equilibrium where uninformed firms charge price $1$ and informed firms follow mixed strategy $F(p) = \alpha - 2\alpha p$ with support $[\alpha - \alpha, 1]$. Verify:

Informed firms payoff to charging price $p$ is

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- highest prices (across all information structures and equilibria) arise in this equilibrium where (1) holds with equality and

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- result 1: analogous result for general distributions over number of quotes
Price Distributions
Endogenizing the Number of Quotes 1: Simultaneous Search

- All consumers observe at least one price quote
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- At a cost/benefit $c$, they may decide (ex ante) to observe an additional price quote
Endogenizing the Number of Quotes 1: Simultaneous Search

- All consumers observe at least one price quote
- At a cost/benefit $c$, they may decide (ex ante) to observe an additional price quote
- There may be a distribution of $c$ in the population
Three Cases

1. like Diamond (1971): same strictly positive cost for everyone in the population
Three Cases

1. like Diamond (1971): same strictly positive cost for everyone in the population
   ▶ there is a unique equilibrium where no one gets a second quote ($\mu = 1$) and the monopoly price is charged

2. like Varian (1980): $c = \infty$ for proportion $\frac{1}{2}$ and $c \leq 0$ for proportion $\frac{1}{2}$ ("shoppers")
   ▶ our (no information) equilibrium is played

3. more generally: full support density on $c$ with prob.
   $c \geq \frac{1}{2} (\ln 3 - 1)$ equal to $\frac{1}{2}$
   ▶ equilibrium where consumers by second quote only if $c \geq \frac{1}{2} (\ln 3 - 1) \approx 0.05$.
   ▶ verify: expected price from two quotes is $\frac{1}{2}$; expected price from one quote is $\frac{1}{2} \ln 3 \approx 0.55$.

▶ In each case, endogeneity of $\mu$ does not change pricing
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- In each case, endogeneity of $\mu$ does not change pricing.
Endogenizing the Number of Quotes 2: Posting Prices

- firm pays cost \( \frac{1}{2} \left( 1 - \frac{1}{\sqrt{2}} \right) \) to have price advertised on public site
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- consumer buys at lowest posted price; if no price posted, he is randomly assigned to a firm
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- there will be symmetric equilibrium where
- non-advertising firms charge the monopoly price.
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  - advertising firms follow a symmetric mixed strategy $F(p) = (\sqrt{2} - 1) \frac{1-p}{p}$ with support $\left[\frac{\sqrt{2}-1}{\sqrt{2}}, 1\right]$
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  - non-advertising firms charge the monopoly price.
- see plot....
like Baye and Morgan (2001), information intermediary charges firms pays a advertising fee $c > 0$ to have prices advertised on his site; consumer pays an access fee $\kappa \geq 0$ to observe
Endogenizing the Number of Quotes 3: Information Intermediaries

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  - non-advertising firms charge the monopoly price.
Endogenizing the Number of Quotes 4: Sequential Search

- like Stahl (1989), proportion $\frac{1}{2}$ of consumers observe one quote and can then choose (after observing the price) to pay $c > 0$ to get a second price; proportion $\frac{1}{2}$ of consumers (“shoppers”) will always get two price quotes
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- In equilibrium:

  \[ F(p) = r - p \]
  
  with support $[\frac{r}{3}, r]$. Where $r = c_1 - \frac{1}{2} \ln 3$.

  Verify: expected price in second search is $\frac{1}{2} r \ln 3$, so gain from search is $r \left( 1 - \frac{1}{2} \ln 3 \right)$.

  See plot for $c = \frac{1}{2}$ and so $r \approx 0.74$. 
Endogenizing the Number of Quotes 4: Sequential Search

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- In equilibrium:
  - non-shoppers only get one quote;
  - firms follow mixed strategy $F(p) = \frac{r-p}{2p}$ with support $\left[\frac{r}{3}, r\right]$ where $r = \frac{c}{1-\frac{1}{2}\ln 3}$.
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Price Distribution
Fundamentals

- One representative consumer
Fundamentals

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- $N$ firms, indexed by $k \in \mathcal{N} = \{1, \ldots, N\}$
Fundamentals

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Fundamentals

- One representative consumer
- $N$ firms, indexed by $k \in \mathcal{N} = \{1, \ldots, N\}$
- Firms produce a perfectly homogenous good
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- Common production cost normalized to 0
Timing

- Consumer observes $n$ prices
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- Distribution of number of price quotes $\mu \in \Delta (\{1, \ldots, N\})$
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- Randomly drawn set of firms is $\tilde{N}$
Timing

- Consumer observes $n$ prices
- Distribution of number of price quotes $\mu \in \Delta (\{1, \ldots, N\})$
- Randomly drawn set of firms is $\tilde{N}$
- Firms $k \in \tilde{N}$ quote prices $p_k$
Consumer observes \( n \) prices

Distribution of number of price quotes \( \mu \in \Delta \left( \{1, \ldots, N\} \right) \)

Randomly drawn set of firms is \( \tilde{N} \)

Firms \( k \in \tilde{N} \) quote prices \( p_k \)

Consumer buys from a firm with lowest price

\[
p^* = \min_{k \in \tilde{N}} p_k
\]

(Break ties uniformly)
Information & firms’ strategies

- We assume firm $k$ gets a signal $s_k \in S$
Information & firms’ strategies

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- Distribution of $s_{\tilde{N}} = (s_k)_{k \in \tilde{N}}$ is given by $\pi(s_{\tilde{N}} | n)$
Information & firms’ strategies

- We assume firm $k$ gets a signal $s_k \in S$
- Distribution of $s_{\tilde{N}} = (s_k)_{k \in \tilde{N}}$ is given by $\pi(s_{\tilde{N}}|n)$
- For now, assume symmetry: distribution depends on the number of the firms, but not their identities
Conditional on observing signal $s_k$, firm $k$ sets prices according to

$$F(p|s_k) = \text{probability that } p_k \geq p$$
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Strategies and Equilibrium

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Strategies and Equilibrium

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$$F(p|s_k) = \text{probability that } p_k \geq p$$

- NB an upper cumulative distribution
- NB assuming firms use symmetric strategies
- Firms want to maximize price times probability of sale
- $F$ is an equilibrium if for all $k$ and $F'_k$,

$$R_k(\sigma, F) \geq R_k(\sigma, F'_k, F_{-k})$$
Sale price distribution

- For a strategy $F$, let $S(p|n)$ denote the probability the sale price is at least $p$, conditional on $n$ firms quoted:

$$S(p|n) = \int_{s \in S^n} \times_k F(p|s_k) \pi(ds|n)$$
Sale price distribution

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$$S(p|n) = \int_{s \in S^n} \times_k F(p|s_k) \pi(ds|n)$$

- Also let

$$S(p) = \sum_{n=1}^{N} \mu(n) S(p|n)$$

denote the ex ante distribution of the sale price
A constraint on sales

Theorem

In any equilibrium,

\[ p \sum_{n=1}^{N} \mu(n) n S(p \mid n) \leq \int_{x=p}^{\infty} xS(dx). \]
A constraint on sales

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- This inequality will drive the rest of the analysis
A constraint on sales

**Theorem**

*In any equilibrium,*

\[ p \sum_{n=1}^{N} \mu(n)n S(p|n) \leq \int_{x=p}^{\infty} xS(dx). \]

- This inequality will drive the rest of the analysis
- Will give a proof sketch
Equilibrium surplus

- The $S(p|n)$ are rich enough objects to compute the equilibrium revenue of a representative firm.
Equilibrium surplus

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- Since the model is symmetric, when $n$ firms are active, there is an $n/N$ chance that firm $k$ is active.
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Equilibrium surplus

- The $S(p|n)$ are rich enough objects to compute the equilibrium revenue of a representative firm.
- Since the model is symmetric, when $n$ firms are active, there is an $n/N$ chance that firm $k$ is active.
- Conditional on being active, there is a $1/n$ chance that firm $k$ has the lowest price.
- Hence, equilibrium surplus must be

$$
\frac{1}{N} \sum_{n=1}^{N} \mu(n) \int_{x=0}^{1} xS(dx|n)
$$
A class of deviations

Now suppose firm $k$ uniform deviation down to $p$: Whenever you would set a price $p_k \geq p$, set a price of $p$ instead.
A class of deviations

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- Claim: deviator’s surplus is:

$$
\frac{1}{N} \sum_{n=1}^{N} \mu(n) \left[ \int_{x=0}^{p} xS(dx|n) + npS(p|n) \right]
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- Deviator wins at a price \( p \) whenever he is active and the equilibrium sale price would have been above \( p \).
  Happens with probability \( \mu(n)(n/N)S(p|n) \).
A class of deviations

- Now suppose firm $k$ *uniform deviation down to $p$*: Whenever you would set a price $p_k \geq p$, set a price of $p$ instead

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- Deviator wins at a price $p$ whenever he is active and the equilibrium sale price would have been above $p$
  Happens with probability $\mu(n)(n/N)S(p|n)$

- On the other hand, if the equilibrium sale price is $x < p$, then the outcome is the same as it would have been in equilibrium (since firm $k$’s price is unchanged as well)
A necessary condition for equilibrium is that firms wouldn't want to uniformly deviate down, i.e.,

\[
\frac{1}{N} \sum_{n=1}^{N} \mu(n) \int_{x=0}^{\nu} xS(dx|n) \geq \frac{1}{N} \sum_{n=1}^{N} \mu(n) \left[ \int_{x=0}^{p} xS(dx|n) + npS(p|n) \right]
\]
Conclusion of proof

▶ A necessary condition for equilibrium is that firms wouldn’t want to uniformly deviate down, i.e.,

\[
\frac{1}{N} \sum_{n=1}^{N} \mu(n) \int_{x=0}^{V} xS(dx|n) \geq \frac{1}{N} \sum_{n=1}^{N} \mu(n) \left[ \int_{x=0}^{P} xS(dx|n) + npS(p|n) \right]
\]

▶ Rearranging yields our result
No information warm up

- Suppose that $|S| = 1$, so firms get no information about consumers
Suppose that $|S| = 1$, so firms get no information about consumers.

Then $F(p|s) = F(p)$, and $S(p|n) = (F(p))^n$.
No information warm up

- Suppose that $|S| = 1$, so firms get no information about consumers.
- Then $F(p|s) = F(p)$, and $S(p|n) = (F(p))^n$.
- Our inequality reduces to

$$p \sum_{n=1}^{N} \mu(n)(n-1)(F(p))^n \leq \sum_{n=1}^{N} \mu(n) \int_{x=p}^{1} (F(x))^n dx.$$
An upper bound on prices under no information

Proposition

There exists a highest price distribution $\overline{F}$ that satisfies the uniform downward incentive constraints under no information. This distribution satisfies all of the constraints as equalities wherever $\overline{F}(p) < 1$. 

Proof:

If there is a largest element, it must satisfy all of the constraints as equalities when $\overline{F}(p) < 1$, for otherwise we could define $\overline{F}'(p)$ as the minimum of 1 and the solution to $\sum_{n=1}^{N} \mu(n)(n-1) \overline{F}'(p) = \sum_{n=1}^{N} \mu(n) \int_{v(x)=p} \overline{F}(x) \, dx$.

Must have $\overline{F}'(p) \geq \overline{F}(p)$, and strictly so when the constraint is slack.

Moreover, the right-hand side has increased with $\overline{F}'$, so that $\overline{F}'$ must be feasible.
An upper bound on prices under no information

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Proof: If there is a largest element, it must satisfy all of the constraints as equalities when $F(p) < 1$, for otherwise we could define $F'(p)$ as the minimum of 1 and the solution to

$$p \sum_{n=1}^{N} \mu(n)(n-1)(F'(p))^n = \sum_{n=1}^{N} \mu(n) \int_{x=p}^{v} (F(x))^n \, dx$$
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An upper bound on prices under no information

**Proposition**

*There exists a highest price distribution $\overline{F}$ that satisfies the uniform downward incentive constraints under no information. This distribution satisfies all of the constraints as equalities wherever $\overline{F}(p) < 1$.***

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\[
p \sum_{n=1}^{N} \mu(n)(n-1)(F'(p))^n = \sum_{n=1}^{N} \mu(n) \int_{x=p}^{v} (F(x))^n dx
\]

- Must have $F'(p) \geq F(p)$, and strictly so when the constraint is slack

- Moreover, the right-hand side has increased with $F'$, so that $F'$ must be feasible
Proof, conclusion

Let $\mathcal{F}$ be the set of $F$’s satisfying the uniform downward constraint.
Proof, conclusion

- Let $\mathcal{F}$ be the set of $F$’s satisfying the uniform downward constraint.
- Then the pointwise supremum of the $F$’s, denoted $\overline{F}$, is finite and also satisfies the constraints, since

\[
p \sum_{n=1}^{N} \mu(n)(n-1)(\overline{F}(p))^n = \sup_{F \in \mathcal{F}} p \sum_{n=1}^{N} \mu(n)(n-1)(F(p))^n
\]

\[
\leq \sup_{F \in \mathcal{F}} \sum_{n=1}^{N} \mu(n) \int_{x=p}^{v} (F(x))^n \, dx
\]

\[
= \sup_{F \in \mathcal{F}} \sum_{n=1}^{N} \mu(n) \int_{x=p}^{v} (\overline{F}(x))^n \, dx
\]

and hence is the largest element of $\mathcal{F}$ \qed
Equilibrium

Theorem
Equilibrium

1. The unique equilibrium is $\overline{F}$. 

Theorem
Equilibrium

Theorem

1. The unique equilibrium is $F$.
2. The expected price in any equilibrium is $\mu(1)$. 
Proof

- Can directly verify that $\bar{F}$ is an equilibrium. Firm is indifferent between all prices.
Proof

- Can directly verify that $F$ is an equilibrium. Firm is indifferent between all prices.
- Standard FPA auction arguments imply uniqueness
Proof

- Can directly verify that $F$ is an equilibrium. Firm is indifferent between all prices.
- Standard FPA auction arguments imply uniqueness
- Firms are also indifferent between charging monopoly price, proving (2) expected price
Uniform example

- Suppose $\mu(n) = 1/N$ for $n \in \{1, \ldots, N\}$
Uniform example

- Suppose $\mu(n) = 1/N$ for $n \in \{1, \ldots, N\}$
- Set $v = 1$
Uniform example

- Suppose $\mu(n) = 1/N$ for $n \in \{1, \ldots, N\}$
- Set $\nu = 1$
- No closed form solutions but easy to compute numerically
Uniform example

\[ \tilde{F}(p) \]

\begin{figure}
\centering
\includegraphics[width=\textwidth]{uniform_example}
\end{figure}
Uniform example

\[ S(p) \]

\[ \begin{align*}
S(p) & \quad \text{at different } N \\
N=2 & \quad \text{at } S(p) = 0.9 \quad \text{Price} = 0.1 \quad \text{to} \quad 0.9 \\
N=3 & \quad \text{at } S(p) = 0.8 \quad \text{Price} = 0.2 \quad \text{to} \quad 0.8 \\
N=4 & \quad \text{at } S(p) = 0.7 \quad \text{Price} = 0.3 \quad \text{to} \quad 0.7 \\
N=5 & \quad \text{at } S(p) = 0.6 \quad \text{Price} = 0.4 \quad \text{to} \quad 0.6 \\
N=10 & \quad \text{at } S(p) = 0.5 \quad \text{Price} = 0.5 \quad \text{to} \quad 0.5 \\
\end{align*} \]
General information

- If firms get partial information about \(n\), then the \(S(p|n)\) can vary more flexibly
General information

- If firms get partial information about $n$, then the $S(p|n)$ can vary more flexibly.
- We again derive an upper bound, and now show that it is attained in an equilibrium for some information structure.
Proposition

There exists a highest price distribution \( \bar{S}(p) \) that can be induced by \( S(p|n) \) satisfying the uniform downward incentive constraints. The inducing distributions \( \bar{S}(p|n) \) satisfy the constraints as equalities whenever \( \bar{S}(p) < 1 \).
Generalized bounds

Proposition

There exists a highest price distribution $\bar{S}(p)$ that can be induced by $S(p|n)$ satisfying the uniform downward incentive constraints. The inducing distributions $\bar{S}(p|n)$ satisfy the constraints as equalities whenever $\bar{S}(p) < 1$.

- The logic is somewhat different from the earlier proof, because we now have more flexibility in choosing the distributions
Maximizing $S(p)$

- Recall the incentive constraints:

$$p \sum_{n=1}^{N} \mu(n)(n-1) S(p|n) \leq \sum_{n=1}^{N} \mu(n) \int_{x=p}^{1} S(x|n) dx$$
Maximizing $S(p)$

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- Imagine constructing a solution downward from $p = 1$
Maximizing $S(p)$

- Recall the incentive constraints:

$$p \sum_{n=1}^{N} \mu(n) (n - 1) S(p|n) \leq \sum_{n=1}^{N} \mu(n) \int_{x=p}^{1} S(x|n) dx$$

- Imagine constructing a solution downward from $p = 1$
- There’s slack on the RHS that can be “allocated” to $S(p|n)$
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- Imagine constructing a solution downward from $p = 1$
- There’s slack on the RHS that can be “allocated” to $S(p|n)$
- Because $S(p|n)$ has weight $n-1$ on the LHS, distributions with smaller $n$ are “cheaper” to use
  
  Suggests we should first use $S(p|n)$ with lower $n$
Maximizing $S(p)$

- Recall the incentive constraints:

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- Imagine constructing a solution downward from $p = 1$
- There's slack on the RHS that can be "allocated" to $S(p|n)$
- Because $S(p|n)$ has weight $n - 1$ on the LHS, distributions with smaller $n$ are "cheaper" to use
  Suggests we should first use $S(p|n)$ with lower $n$
- Indeed, $S(p|1)$ only appears on the right, so can set $S(p|1) \equiv 1$
More generally, we can show that any feasible $S(p|n)$ is dominated by one that is \textit{monotonic}:

$$S(p|n) > 0 \implies S(p|n') = 1 \text{ for all } n' < n$$
More generally, we can show that any feasible $S(p|n)$ is dominated by one that is \emph{monotonic}:

$$S(p|n) > 0 \implies S(p|n') = 1 \text{ for all } n' < n$$

The argument is omitted for brevity, but basically if this constraint is violated, we can move a little mass from the larger $n$ to the smaller $n'$ and it increases $S(p)$
Monotonic solutions

- More generally, we can show that any feasible $S(p|n)$ is dominated by one that is monotonic:

$$S(p|n) > 0 \implies S(p|n') = 1 \text{ for all } n' < n$$

- The argument is omitted for brevity, but basically if this constraint is violated, we can move a little mass from the larger $n$ to the smaller $n'$ and it increases $S(p)$

- Once we restrict attention to monotonic solutions, we can use a similar trick as before to show that the set of $S(p)$ that can generated by $S(p|n)$ satisfying the uniform downward constraints is a complete semi-lattice
Explicit formulae

- In fact, we have a closed form solution for the highest distribution:

\[
S(p|n) = \begin{cases} 
\frac{p_n - p_{n-1}}{T_n - 1} & \text{for } n > 2, \\
p_1 = p_0 = 1 & \text{and for } n > 2, 
\end{cases}
\]

where

\[
T_n = \sum_{k=1}^{n} k \mu(k)
\]
Explicit formulae

- In fact, we have a closed form solution for the highest distribution:
- The support of $S(p|n)$ is an interval $[p_n, p_{n-1}]$, with $p_1 = p_0 = 1$ and for $n > 2$,

$$p_n = p_{n-1} \left( \frac{T_{n-1}}{T_n} \right)^{\frac{n-1}{n}}$$

where

$$T_n = \sum_{k=1}^{n} k \mu(k)$$
Explicit formulae

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  where

  $$T_n = \sum_{k=1}^{n} k \mu(k)$$

- The distributions are

  $$\tilde{S}(p|n) = \frac{\left( \frac{p_n}{p} \right)^{\frac{n}{n-1}} - \left( \frac{p_n}{p_{n-1}} \right)^{\frac{n}{n-1}}}{1 - \left( \frac{p_n}{p_{n-1}} \right)^{\frac{n}{n-1}}}$$
Intuition for derivation

- Given that we have ordered supports, can rearrange the incentive constraint to

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\mu(n) \left( \int_{x=p}^{v} S(x|n)dx - p(n-1)S(p|n) \right) \\
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- \( S(p|n) \) satisfies the boundary condition \( S(p_{n-1}|n) = 0 \), and \( p_n \) is defined through \( S(p_n|n) = 1 \)
Uniform example revisited

\[ \tilde{S}(p|n) \] for \( N = 5 \)
Uniform example revisited

$\bar{S}(p)$ for various $N$
Uniform example revisited

\[ \tilde{S}(p) \] for \( N = 5 \) comparing no info and general info
Bounds attained

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3. The expected price is under the maximum distribution of prices is in the interval \( \left[ \mu(1), \sqrt{\mu(1)(2 - \mu(1))} \right] \)
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- This is upper bound on expected price if we shift distribution to \( n \geq 3 \), holding \( \mu(1) \) fixed
More Formally: Firms’ information

- The firms get discrete signals in $S = \{1, \ldots, N\}$
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- The firms get discrete signals in $S = \{1, \ldots, N\}$
- The distribution of signals is generated by first taking independent draws from $\alpha(\cdot | n)$ which has support on $\{1, \ldots, n\}$, and throwing out signal profiles where no bidder gets a signal of $n$
The formulae are:

\[ \alpha(k|n) = \frac{T_{k-1}p_{k-1}}{T_{n-1}p_{n-1}} \left( \left( \frac{p_{k-1}}{p_k} \right)^{\frac{1}{k-1}} - 1 \right) \]

and

\[ \pi(s_{\tilde{N}}|n) = \frac{1}{1 - (1 - \alpha(n|n))^n} \times_{k \in \tilde{N}} \pi(s_k|n) \]

if \( s_k = n \) for at least one \( k \in \tilde{N} \), and \( \pi(s|n) = 0 \) otherwise.
A firm that gets signal $k$ randomizes according to

$$F(p|n) = \left( \frac{p_n}{p} \right)^{1\over n-1} - \left( \frac{p_n}{p_{n-1}} \right)^{1\over n-1}$$

One can show through some algebra and use of the binomial theorem that these strategies are an equilibrium. They induce the distributions $S(p|n)$.

The proof that these strategies are an equilibrium shows that bidder surplus is quasiconcave in $p$ conditional on $s_k$, and flat in $[p_{s_{k}}, p_{s_{k}-1}]$. 
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- The probability that the sale price is at least $p$ when there are $n$ active firms is

\[
\frac{1}{1 - (1 - \alpha_n)^n} \sum_{k=1}^{n} \binom{n}{k} (\alpha_n F(p|n))^k (1 - \alpha_n)^{n-k}
\]

\[
= \frac{(1 - \alpha_n + \alpha_n F(p|n))^n - 1}{1 - (1 - \alpha_n)^n}
\]

\[
= S(p|n)
\]
Endogenizing Price Quotes: Simultaneous Price Setting with predetermined information

- consider an arbitrary game in which, first the consumer, firms and other parties take actions that influence which firms’ prices the consumer will observe; and then the firms choose prices
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- this class of games embeds simultaneous search, costly price posting, information intermediaries
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- firms simultaneously choose prices (without observing each other’s prices)
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Our Bound?

- Our bound still holds lower prices always lead to less search.
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- Our bound still holds lower prices always lead to less search.
- More formally, if $p(h_t)$ is the lowest price in history $h_t$, we would like to show that if $p(h_t) \leq p(h'_t)$ and continuing to search is a best response at $h_t$ for $\theta$, then continuing to search is a strict best response at $h'_t$ for $\theta$. 
Relation to Auctions

- the exogenous number of price quotes problem corresponds to the first price auction with two possible values, 0 or 1

- our 2017 Econometrica paper derives the analogous distribution when bidders may not know their own values

- today's problem is harder and the result is less general
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Asymmetric Equilibria, Symmetric Distribution on $n$

$\mu(1) = \mu(2) = 1/2$
Asymmetric Distribution and Equilibrium

\[ \mu_1(1) = \frac{3}{8}, \mu_2(1) = \frac{1}{8}, \mu(2) = \frac{1}{2} \]
Heterogeneous Costs and the minimum expected price

- Homogenous cost case....
Heterogeneous Costs and the minimum expected price

- Homogenous cost case....
  - We have a characterization of maximum expected price for arbitrary $\mu$

Minimum expected price was $\mu(1)$. 

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Minimum Expected Price with heterogeneous costs

- Now suppose firms have independent cost drawn according to density $g_k$
Minimum Expected Price with heterogeneous costs

- Now suppose firms have independent cost drawn according to density $g_k$
- If firm $k$ had cost $c_k$ and expected all other firms to charge at cost, his payoff to charging $p$ would be

$$
(p - c_k) \prod_{k' \neq k} (1 - G_{k'}(p))
$$

and his ex ante “competitive rent” would be

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\int_{c_k} \left[ \max_p (p - c_k) \prod_{k' \neq k} (1 - G_{k'}(p)) \right] g_k(c_k) \, dc_k
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the minimum expected price is the sum of the minimum expected cost and the firms' competitive rents
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Conclusion

- Price predictions based on distribution of number of price quotes (abstracting from details of where it came from)
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- Information structure determines prices
Conclusion

- Price predictions based on distribution of number of price quotes (abstracting from details of where it came from)
- Information structure determines prices
- Tight upper bound on price distribution, tight upper and lower bounds on expected prices
Further topics

- First Price Auction
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- Some cheap extensions
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- Some cheap extensions
- Asymptotics
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First Price Auction

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- Equivalent to solving for Bayes correlated equilibrium

Bergemann, Brooks and Morris (2017) solved for the (easier) case where bidders do not necessarily know their own value

Today's problem with exogenous $\mu$ is isomorphic to this problem: normalize low valuation to 0, low valuation bidders always bid 0, $\mu(n)$ is the probability that there are $n$ high valuation bidders
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- Bidders know their values but the analyst does not know what they know about others’ values.
- What can happen in equilibrium?
- Equivalent to solving for Bayes correlated equilibrium.
- Today’s problem with exogenous $\mu$ is isomorphic to this problem: normalize low valuation to 0, low valuation bidders always bid 0, $\mu(n)$ is the probability that there are $n$ high valuation bidders.
- Bergemann, Brooks and Morris (2017) solved for the (easier) case where bidders do not necessarily know their own value.
Value uncertainty

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- We can treat the associated revenue levels as the price in the baseline model and everything goes through
Noisy search

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▶ Firms’ incentive constraints are independent of how we rationalize consumer behavior
Asymmetric strategies and maximum revenue

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- We know that the analysis of the bounds under general info doesn’t change if we allow firms to use different strategies.
- A simple convexity argument says that it is WLOG to restrict attention to symmetric information and symmetric strategies.
- Even in the symmetric case, symmetry is WLOG, since all equilibria have to be symmetric (Maskin and Riley 1983).
Asymptotics

- Consider a sequence of economies with $N = 1, 2, \ldots$ with quote distributions $\mu^N$
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  - If $\mu^N(1)$ goes to zero, profits converge to zero, and we obtain a competitive limit;
  - If $\mu^N(1)$ is bounded away from zero, then total firm profits will be positive, even if the expected number of searches goes to infinity
Going dynamic

- Extension we’re most interested in: allowing for sequential search
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- Now suppose search occurs over time $t = 1, \ldots, T$

At each period $t$, consumer chooses to search $n_t$ firms
- Draw a random subset $N_t$ of the unsearched firms with $|N_t| = n_t$ (i.e., no replacement)
- Firms in $N_t$ get (possibly correlated) signals about $(\theta, n_1, \ldots, n_t)$, set prices
- After seeing prices, consumer either chooses to buy at current lowest price, or continue searching

$\sigma(\cdot|\theta, n_1, \ldots, n_T, \{p_k|k \in N_T, T \leq t\}) \in \Delta(\{0, 1, \ldots, N - \sum_{\tau \leq T} n_\tau\})$
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New issues

- Basically, we think we should be able to generalize our results using the following logic:
  - Cutting prices creates an incentive for consumers to search less, because search is costly, and the outside option has gotten better.
  - Less search leads to less competition, and an even higher distribution of the lowest price of firms.
  - Thus, in a sequential search model, the gains from deviating down would be weakly greater than the static gains, and hence our uniform downward incentive constraint is a necessary condition.
  - Can still rationalize search as simultaneous, with bang-bang costs.
  - This is the most collusive search cost model, because it creates the weakest incentives for firms to cut prices.
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- But for general information, lower prices may lead the consumer’s behavior to shift in a way that makes the probability of a sale go down
- This would weaken incentives to deviate down, and support even higher prices than the ones we construct
A “sufficient” condition

- For now, we know our result goes through whenever the consumer’s search strategy leads to less search when prices are lower, in a strong sense:

\[ h_t \text{ and } h'_t \text{ are two histories, where } h_t \text{ has a lower minimum price, then for all } n_{t+1} > 0, \sigma(n_{t+1} | h_t) < \sigma(n_{t+1} | h'_t) \]

We are looking for conditions on primitives (information and costs) under which this will be true, or the weaker condition that price cuts lead to less competition

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Other welfare objectives

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- For example, what are the possible weighted sums of firms’ profits, or profit and consumer surplus?
- A bit hard to think about consumer surplus, since we don’t know search costs, but we can think about profits...
Minimizing revenue

Theorem

*The no-information costly search model minimizes total profits, even when firms can have general information.*
Minimizing revenue

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- Firms can always set a price \( p = v \) and only make a sale at that price when the consumer has value \( v \), and guarantee themselves

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- NB Would also be achieved under complete information
The set of possible profit profiles

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- *(Mention connection with first-price auction?)*
The set of firms’ profits when \( p_1 = p_2 = 2/9, \ p_0 = 1/9 \)
Asymmetric firms

$p_1 = 0.1$, $p_2 = 0.4$, $r = 0.5$

Four type equilibria
Three type equilibria
Final words

Thank you!