EXPECTATIONS, NETWORKS, AND CONVENTIONS
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Consider a situation where agents care about matching two targets—uncertainty about both:

- others’ actions;
- a “fundamentally” best action.
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Conventions (in organizations, choice of language, speculative trading...): actions selected in equilibrium when coordination is important.
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Q: How do conventions depend on differences in

(i) information
   (signals)
   beliefs and higher-order beliefs

(ii) interpretation
   (priors)

(ii) coordination concerns
   (interaction)
   networks
Consider a situation where agents care about matching two targets—uncertainty about both:

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- a “fundamentally” best action.

**Conventions** (in organizations, choice of language, speculative trading...): actions selected in equilibrium when coordination is important.

**Q:** How do conventions depend on differences in

(i) information (signals)  (ii) interpretation (priors)  (ii) coordination concerns (interaction)

- beliefs and higher-order beliefs
- networks

**Contribution:** Analyze effects of (i), (ii), (iii) together via reduction of all three to a network. Yields **unification** and **new purely informational results**.
MODEL

Agents

External state

\( i \in \mathbb{N} \)

\( \theta \in \mathcal{Q} \)

\( y^i : \mathcal{Q} \rightarrow [-M,M] \)
MODEL

Agents
External state
i's fundamental
i's types

\( i \in \mathbb{N} \)
\( \theta \in \Theta \)
\( y^i : \Theta \rightarrow [-M,M] \)
\( t^i \in T^i \)
**Model**

Agents

External state

$i$'s fundamental

$i$'s types

\[ i \in N \]

\[ \theta \in \Theta \]

\[ y^i : \Theta \rightarrow [-M,M] \]

\[ t^i \in T^i \]

**Ex post payoff**

\[ u^i = -\beta \sum_{j} \gamma^{ij} (a^i - a^j)^2 \]

\[ -(1-\rho) (a^i - y^i(\theta))^2 \]
**Model**

Agents

External state

i's fundamental

i's types

strategy

belief f'n.

\[ i \in N \]

\[ \Theta \in \Theta \]

\[ y^i : \Theta \rightarrow [-M, M] \]

\[ t^i \in T^i \]

\[ a^i : T^i \rightarrow [-M, M] \]

\[ \pi^i : T^i \rightarrow \Delta(\Theta \times T^{-i}) \]

**Ex post payoff**

\[
U^i = -\beta \sum_{j} \gamma^{ij} (a^i - a^j)^2 \\
-(1-\beta) (a^i - y^i(\theta))^2
\]
MODEL

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External state

i's fundamental

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belief f'n.

\[ i \in N \]

[\Theta \in \Theta] \rightarrow [-M, M]

\[ y^i : \Theta \rightarrow [-M, M] \]

\[ t^i \in T^i \]

[\Theta \times T^{-i}] \rightarrow \Delta (i) \rightarrow E^i \]

\[ a^i : T^i \rightarrow [-M, M] \]

\[ \Pi^i : T^i \rightarrow \Delta (\Theta \times T^{-i}) \]

\[ E^i \rightarrow i's \ expectation \]

ex post payoff

\[ u^i = -\beta \sum_j \gamma^{ij} (a^i - a^j)^2 \]

\[ -(1-\beta) (a^i - y^i(\theta))^2 \]
Model

Agents
External state
i's fundamental
i's types
strategy
belief f'n.

\( i \in N \)
\( \theta \in \Theta \)
\( y^i : \Theta \rightarrow [-M, M] \)
\( t^i \in T^i \)
\( a^i : T^i \rightarrow [-M, M] \)
\( \Pi^i : T^i \rightarrow \Delta(\Theta \times T^{-i}) \)
\( \rightarrow E^i \) i's expectation

\[ U^i = -\beta \sum_j \gamma^{ij} (a^i - a^j)^2 \]
\[ - (1-\beta) (a^i - y^i(\theta))^2 \]

\[ BR^i = \beta \sum_{j \neq i} \gamma^{ij} E^i a^j + (1-\beta) E^i y^i \]

matching others' actions

matching fundamental
Ex. Net Game, Complete Info.

\[ u^i = -\beta \sum_j \gamma^{ij} (a^i - a^i) - (1 - \beta) (a^i - y^i)^2 \]
Ex. Net Game, Complete Info.

\[ u^i = -\beta \sum_j x^{ij} (a^i - a_j) - (1-\beta) (a^i - y^i)^2 \]

Ex. 2 agents, incomplete info

\[ u^i = -\beta (a^i - a_j)^2 - (1-\beta) (a^i - y(\Theta))^2 \]

\( \Theta \in \{ G, B \} \)

\( \rho^i \in \Delta(\Theta) \) i's prior

\( t^i \in \{ g^i, b^i \} \) matches \( \Theta \) w.p. \( q^i \)

\( \Pi^i \) computed via Bayes' rule
**Model**

 Agents

 External state

 i's fundamental

 i's types

 strategy

 belief f'n.

 $\rightarrow E^i$ i's expectation

 $BR^i = \beta \sum_{j \neq i} y^i_j E^i a^j + (1-\beta) E^i y^i$

 matching others' actions

 matching fundamental

 $Ex.$ Net Game, Complete Info.

 $-u^i = \beta \sum_j r^i_j (a^i - a^j)^2 + (1-\beta) (a^i - y^i)^2$

 $\Pi^i : T^i \rightarrow \Delta(\Theta \times T^{-i})$

 $\Theta \epsilon \{G, B\}$

 $\rho^i \epsilon \Delta(\Theta)$ i's prior

 $t^i \epsilon \{g^i, b^i\}$ matches o w.p. $q^i$

 $\Pi^i$ computed via Bayes' rule

 $Ex.$ 2 agents, incomplete info

 $-u^i = \beta (a^i - a^j)^2 + (1-\beta) (a^i - y(\theta))^2$
**Model**

Agents

External state

\( i \)'s fundamental

\( i \)'s types

strategy

belief f'n.

\[ i \in \mathcal{N} \]

\( \Theta \in \mathcal{G} \)

\( y_i : \Theta 
\rightarrow [-M, M] \)

\( t^i \in T^i \)

\( a^i : T^i \rightarrow [-M, M] \)

\( \pi^i : T^i \rightarrow \Delta(\Theta \times T^{-i}) \)

\[ \Rightarrow E^i \rightarrow i \)'s expectation \]

\[ BR^i = \beta \sum_{j \neq i} y_{ij} E^i a^j + (1-\beta) E^i y^i \]

Matching others' actions

Matching fundamental

**Fact 1** The game has a unique rationalizable strategy profile.

**Question:** How does play depend on (i) information; (ii) priors; (iii) network?

**Focus:** Conventions: play as \( \beta \rightarrow 1 \).

**Example** 2 agents, incomplete info

\[ u^i = \beta (a^i - a^j)^2 + (1-\beta) (a^i - y(\Theta))^2 \]

\( \Theta \in \{ G, B \} \)

\( \rho^i \in \Delta(\Theta) \rightarrow i \)'s prior

\( t^i \in \{ g^i, b^i \} \) matches \( \Theta \) w.p. \( q^i \)

\( \pi^i \) computed via Bayes' rule
**FACT 1**  The game has a unique rationalizable strategy profile.

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**Example**

2 agents, incomplete info

$$-u^i = \beta (a^i - a^j)^2 + (1-\beta)(a^i - y(\theta))^2$$

$\theta \in \{G, B\}$

$\rho^i \in \Delta(\Theta)$ i’s prior

$\mathcal{t}^i \in \{g^i, b^i\}$ matches $\theta$ w.p. $q^i$

$\Pi^i$ computed via Bayes’ rule
**Key Idea:** Incomplete-info. aspect can be reduced to network aspect

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\[ \theta \in \{ G, B \} \]

\[ \rho^i \in \Delta(\Theta) \text{ i's prior} \]

\[ t^i \in \{ g^i, b^i \} \text{ matches } \theta \text{ w.p. } q^i \]

\[ \Pi^i \text{ computed via Bayes' rule} \]
**Key Idea:** Incomplete-info. aspect can be reduced to network aspect

**Key Device:** “interaction structure”

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$t^i \in \{g^i, b^i\}$ matches $\Theta$ w.p. $q^i$

$\Pi^i$ computed via Bayes' rule
**Key Idea:** Incomplete-info. aspect can be reduced to network aspect

**Key Device:** "interaction structure"

nodes: $S = \bigcup_{i} T^i$

critical function: $B(t^i, t^j) = \gamma^i \Pi^i(t^j | t^i)$

**Fact 1** The game has a unique rationalizable strategy profile.

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**Example** 2 agents, incomplete info

$-u^i = \beta (a^i - a^j)^2 + (1 - \beta)(a^i - y(\theta))^2$

$\Theta \in \{ G, B \}$

$\rho^i \in \Delta(\Theta)$ i’s prior

$t^i \in \{ g^i, b^i \}$ matches $\Theta$ w.p. $q^i$

$\Pi^i$ computed via Bayes’ rule
**Key Idea**: Incomplete-info. aspect can be reduced to network aspect

**Key Device**: "interaction structure"

nodes: $S = \bigcup_i T^i$

edges: $B(t^i, t^j) = \gamma^{ij} \Pi^i(t^j | t^i)$

**Fact 1**: The game has a unique rationalizable strategy profile.

**Question**: How does play depend on (i) information; (ii) priors; (iii) network?

**Focus**: Conventions: play as $\beta \uparrow 1$.

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**Example**: 2 agents, incomplete info

- $u^i = \beta (a^i - a^j)^2 + (1-\beta)(a^i - y(\theta))^2$
- $\theta \in \{G, B\}$
- $\rho^i \in \Delta(\Theta)$ i's prior
- $t^i \in \{g^i, b^i\}$ matches $\theta$ w.p. $q^i$
- $\Pi^i$ computed via Bayes' rule
**Key Idea:** Incomplete-info. aspect can be reduced to network aspect

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nodes: $S = \bigcup T^i$

edges: $B(t^i, t^j) = \gamma^{ij} \Pi^i(t^j | t^i)$

**Fact 1:** The game has a unique rationalizable strategy profile.

**Question:** How does play depend on (i) information; (ii) priors; (iii) network?

**Focus:** Conventions: play as $\beta \uparrow 1$.

**Fact 2:** Let $a = (a^1, ..., a^{1m})^T$

$a_{eqm} = (1-\beta) (I - \beta B)^{-1} f$

where $f(t^i) = (E_i y^i)(t^i)$

**Example:** 2 agents, incomplete info

$-u^i = \beta (a^i - a^j)^2 + (1-\beta)(a^i - y(\Theta))^2$

$\Theta \in \{G, B\}$

$\rho^i \in \Delta(\Theta)$ $i$’s prior

$t^i \in \{g^i, b^i\}$ matches $\Theta$ w.p. $q^i$

$\Pi^i$ computed via Bayes’s rule
**Key Idea:** Incomplete-info. aspect can be reduced to network aspect

**Key Device:** “interaction structure”

\[ S = \bigcup_i T^i \quad B(t^i, t^j) = \gamma^{ij} \pi^i(t^j | t^i) \]

**Fact 1:** The game has a unique rationalizable strategy profile.

**Question:** How does play depend on (i) information; (ii) priors; (iii) network?

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**Fact 2:** Let \( a = (a^1, ..., a^m)^T \)

\[ a_{eqm} = (1-\beta)(I - \beta B)^{-1} f \]

where \( f(t^i) = (E_i y^i)(t^i) \)

**Proof**

\[ -u^i = \beta (a^i - a^j)^2 + (1-\beta)(a^i - y(\theta))^2 \]

\( \Theta \in \{ G, B \} \)

\( \rho^i \in \Delta(\Theta) \) i’s prior

\( t^i \in \{ g^i, b^i \} \) matches \( \Theta \) w.p. \( q^i \)

\( \pi^i \) computed via Bayes’ rule
**KEY IDEA:** Incomplete-info. aspect can be reduced to network aspect

**KEY DEVICE:** "interaction structure"

nodes: \( S = \bigcup_{i} T^{i} \)
edges: \( B(t^{i}, t^{j}) = \gamma^{ij} \Pi^{i}(t^{j} | t^{i}) \)

**FACT 1** The game has a unique rationalizable strategy profile.

**QUESTION:** How does play depend on (i) information; (ii) priors; (iii) network?

**Focus:** Conventions: play as \( \beta \rightarrow 1 \).

**FACT 2:** Let \( a = (a^{1}, ..., a^{lm})^{T} \)
\( a_{eqm} = (1 - \beta)(I - \beta B)^{-1} f \)
where \( f(t^{i}) = (E^{i} y^{i})(t^{i}) \)

**Proof**

\[
BR^{i} = \beta \sum_{j \neq i} \gamma^{ij} E^{i} a^{j} + (1 - \beta) E^{i} y^{i}
\]

**Example 2 agents, incomplete info**

\[
-u^{i} = \beta (a^{i} - a^{j})^{2} + (1 - \beta) (a^{i} - y(\Theta))^{2}
\]
\( \Theta \in \{ G, B \} \)
\( \rho^{i} \in \Delta(\Theta) \) i's prior
\( t^{i} \in \{ g^{i}, b^{i} \} \) matches \( \Theta \) w.p. \( q^{i} \)
\( \Pi^{i} \) computed via Bayes' rule.
**Key Idea:** Incomplete-info. aspect can be reduced to network aspect

**Key Device:** "interaction structure"

\[ S = \bigcup_{i} T^i \quad B(t^i, t^j) = \gamma^{ij} \pi^i(t^j | t^i) \]

**Fact 1:** The game has a unique rationalizable strategy profile.

**Question:** How does play depend on (i) information; (ii) priors; (iii) network?

**Focus:** Conventions: play as \( \beta \uparrow 1 \).

**Fact 2:** Let \( a = (a^1, ..., a^m)^T \)

\[ a_{eq} = (1-\beta)(I - \beta B)^{-1}f \]

where \( f(t^i) = (E^i y^i)(t^i) \)

**Proof**

\[ BR^i = \beta \sum_{j \neq i} \gamma^{ij} E^i a^j + (1-\beta) E^i y^i \]

\[ a = BR(a) \iff a = \beta Ba + (1-\beta)f \]

**Example:** 2 agents, incomplete info

\[ u^i = \beta (a^i - a^j)^2 + (1-\beta)(a^i - y^i)^2 \]

\( \theta \in \{ G, B \} \)

\( \rho^i \in \Delta(\Theta) \) i's prior

\( t^i \in \{ g^i, b^i \} \) matches \( \theta \) w.p. \( q^i \)

\( \pi^i \) computed via Bayes' rule
**Key Idea:** Incomplete-info. aspect can be reduced to network aspect

**Key Device:** “interaction structure”

- **Nodes:** \( S = \bigcup T^i \)
- **Edges:** \( B(t^i, t^j) = \gamma_{ij} \Pi_i(t^j | t^i) \)

**Fact 2:** Let \( a = (a^i, \ldots, a^{1\text{st}})^T \)

\[ a_{\text{eqm}} = (1-\beta)(I - \beta B)^{-1} f \]

where \( f(t^i) = (E^i y^i)(t^i) \)

**Fact 1** The game has a unique rationalizable strategy profile.

**Question:** How does play depend on (i) information; (ii) priors; (iii) network?

**Focus:** Conventions: play as \( \beta \to 1 \).

**Example:** 2 agents, incomplete info

\[-u^i = \beta (a^i - a^j)^2 + (1-\beta)(a^i - y(\theta))^2 \]

\( \theta \in \{ G, B \} \)

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\( t^i \in \{ g^i, b^i \} \) matches \( \theta \) w.p. \( q^i \)

\( \Pi^i \) computed via Bayes’ rule
**Key Idea:** Incomplete-info. aspect can be reduced to network aspect

**Key Device:** “interaction structure”

\[ S = \bigcup_i T^i \quad B(t^i, t^j) = \gamma_{ij} \Pi_i(t^j | t^i) \]

**Fact 1:** The game has a unique rationalizable strategy profile.

**Question:** How does play depend on (i) information; (ii) priors; (iii) network?

**Focus:** Conventions: play as \( \beta \uparrow 1 \).

**Ex:** 2 agents, incomplete info

\[ -u^i = \beta (a^i - a^j)^2 + (1-\beta) (a^i - y(\theta))^2 \]

\( \theta \in \{G, B\} \)

\( \rho^i \in \Delta(\Theta) i’s \ prior \)

\( t^i \in \{g^i, b^i\} \) matches \( \theta \) w.p. \( q^i \)

**Prop 0:** If \( B \) str. connected, then as \( \beta \uparrow 1 \),

\( \forall i \ a^i(t^i) \rightarrow c(y^i, \bar{\Pi}, \Gamma) : “the \ convention” \)
**Key Device:** "interaction structure"

**Nodes:**

\[ S = \bigcup_i T^i \]

**Edges:**

\[ B(t^i, t^j) = \gamma^{ij} \Pi^i (t^j | t^i) \]

**Prop 0:** If \( B \) is strongly connected, then as \( \beta \uparrow 1 \),

\[ \forall i \, \alpha_i(t^i) \rightarrow c(\vec{y}; \vec{\pi}, \Gamma) : "the \ convention" \]
**Key Device:** "interaction structure"

\[ S = \bigcup_i T^i \quad B(t^i, t^j) = \gamma^{ij} \pi^i(t^j | t^i) \]

**Prop 1**

\[ c(\vec{y}; \vec{\pi}, \vec{\Gamma}) = \sum_{t^i \in S} p(t^i) f(t^i) \]

where \( p \) is unique \( p \in \Delta(S) \) s.t. \( pB = p \)

I.e., \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.

**Prop 0:** If \( B \) str. connected, then as \( \beta \uparrow 1 \),

\[ \forall i \quad a_i(t^i) \to c(\vec{y}; \vec{\pi}, \vec{\Gamma}) : "the\ convention" \]
**Prop 1** \[ c(\bar{y}; \bar{\pi}, \bar{\mu}) = \sum_{t^i \in S} p(t^i)f(t^i) \]

where \( p \) is unique \( p \in \Delta(S) \) s.t. \( pB = p \)

i.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.
**KEY DEVICE:** “interaction structure”

nodes: \( S = \bigcup_i T^i \)

edges: \( B(t^i, t^j) = \gamma^j \pi^i(t^j | t^i) \)

**CONTAGION OF OPTIMISM**

Suppose each \( i \) is certain each counterparty has \( E^j y \geq E^i y + \delta \), unless \( E^i y \geq \overline{f} \) – then \( E^j y \geq E^i y \).

Then \( c(y; \pi, \Gamma) \geq \overline{f} \)

Reason: for \( t^i \) s.t. \( f(t^i) < \overline{f} \), the \( B \) process can only move upward.

**Prop 1**

\[
c(y; \pi, \Gamma) = \sum_{t^i \in S} p(t^i) f(t^i)
\]

where \( p \) is unique \( p \in \Delta(S) \) s.t. \( pB = p \)

i.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.
KEY DEVICE: "interaction structure"

nodes: \[ S = \bigcup_i T^i \]
edges: \[ B(t^i, t^j) = \gamma^j \pi^i (t^j | t^i) \]

CONTAGION OF OPTIMISM

Suppose each \( i \) is certain each counterparty has \( E^i y > E^{i'} y + \delta \), unless \( E^i y > \overline{f} \) then \( E^i y \geq E^{i'} y \).

Then \[ c(y; \pi, \Gamma) \geq \overline{f} \]

Reason: for \( t^i \) s.t. \( f(t^i) < \overline{f} \), the \( B \) process can only move upward.

\[ \text{Prop 1: } c(y; \pi, \Gamma) = \sum_{t^i \in S} p(t^i) f(t^i) \]

where \( p \) is unique \( p \in \Delta(S) \) s.t. \( p B = p \)

i.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.
**Key Device:** "interaction structure"

\[ S = \bigcup_i T_i \quad B(t^i, t^j) = \gamma^{ij} \Pi^i(t^j | t^i) \]

**Contagion of Optimism**

Suppose each \( i \) is second-order optimistic (on avg)

\[ \sum_j \pi^i_j E_j y > E^i y + \delta, \text{ unless } E^i y \geq \bar{f} \]

Then "" \( \geq E^i y \).

Then \( c(y; \pi, \Gamma) \geq f / (1 + \epsilon / \delta) \)

Reason: for \( t^i \) s.t. \( f(t^i) < \bar{f} \), B process moves upward on average.

**Prop 1**

\[ c(y; \pi, \Gamma) = \sum_{t^i \in S} p(t^i) f(t^i) \]

where \( p \) is unique \( p \in \Delta(S) \) s.t. \( pB = p \)

i.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.
**KEY DEVICE:** “interaction structure”

**nodes:**

\[ S = \bigcup_i T^i \]

**edges:**

\[ B(t^i, t^j) = \gamma^j \Pi^i(t^j | t^i) \]

**Contagion of Optimism**

Suppose each \( i \) is second-order optimistic

\[ \sum y^j \epsilon ^i E^j y \geq \epsilon ^i y + \delta, \]

unless \( E ^i y \geq \bar{f} - \)

then “ \( * \) \( \geq \) \( E ^i y \).

Then \( c(y; \bar{\pi}, \bar{\Gamma}) \geq \bar{f} / (1 + \epsilon / \delta) \)

Reason: for \( t^i \) s.t. \( f(t^i) < \bar{f} \), \( B \) process moves upward on average

**Prop 1**

\[ c(y; \bar{\pi}, \bar{\Gamma}) = \sum_{t^i \in S} p(t^i) f(t^i) \]

where \( p \) is unique \( p \in \Delta(S) \) s.t. \( pB = p \)

i.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.

**Proof:** Take MC \( W_1, W_2, \ldots \), with ergodic dist \( p \). Suppose \( \exists \delta, \epsilon \) s.t.

\[ f(s) < \bar{f} \Rightarrow E_{W_0 = s}[f(W_1)] \geq f(s) + \delta \]

\[ f(s) \geq \bar{f} \Rightarrow E_{W_0 = s}[f(W_1)] \geq f(s) - \epsilon \]

Then \( p\{s; f(s) \geq \bar{f}\} \geq \frac{1}{1 + \epsilon / \delta} \)

Follows from

\[ E_{W_0 = p}[W_1 - W_0] = 0 \]
Contagion of Optimism

Suppose each $i$ is second-order optimistic

$$\sum_{i} E^i E^j y \geq E^i y + \delta,$$

unless $E^i y \geq \bar{f} - \varepsilon$

then $\Rightarrow E^i y.$

Then $c(y; \bar{\pi}, \Gamma) \geq \bar{f} / (1 + \varepsilon/\delta)$

Reason: for $t^i$ s.t. $f(t^i) < \bar{f}, B$ process moves upward on average.

Proof: Take MC $W_1, W_2, \ldots$, with ergodic dist $p$. Suppose $\exists \delta, \varepsilon$ s.t.

$$f(s) < \bar{f} \Rightarrow E_{W_0 = s}[f(W_1)] \geq f(s) + \delta$$

$$f(s) \geq \bar{f} \Rightarrow E_{W_0 = s}[f(W_1)] \geq f(s) - \varepsilon$$

Then $p(s; f(s) < \bar{f}) \geq \frac{1}{1 + \varepsilon/\delta}$

Follows from

$$E_{W_0 \sim p}[W_1 - W_0] = 0$$
**Key Idea:** Incomplete-info. aspect can be reduced to network aspect → analyze how info. struct. matters.

**Key Device:** "interaction structure"

- **Nodes:** $S = \bigcup T^i$
- **Edges:** $B(t^i, t^j) = \gamma^{ij} \pi^i(t^j | t^i)$

**Applications**

1. Contagion of Optimism
2. (Pseudo) Common Prior
   - Influence $\propto$ net centrality
3. Tyranny of least-informed
**Common Priors & Influence**

**Def.** Common priors over signals (CPs)

\[ \Pi^i \text{ all compatible w/ } \hat{\mu} \in \Delta(T) \]

\[ \exists \text{ priors } (\mu^i)_{i \in \mathbb{N}} \text{ s.t. } \]

\[ \mu^i(t') \Pi^i(t^j | t^i) = \mu^i(t^j) \Pi^j(t^i | t^j) \]

**Prop 1.**

\[ c(y^j; \bar{\pi}, \bar{\pi}) = \sum_{t^i \in S} p(t^i) f(t^i) \]

where \( p \) is unique \( p \in \Delta(S) \) s.t. \( p \cdot B = p \)

i.e., \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.
**Common Priors & Influence**

**Def:** Common priors over signals (CPs)

$\pi^i$ all compatible w/ a $\hat{\mu} \in \Delta(T)$

$\Rightarrow$

$\exists$ priors $(\mu^i)_{i \in \mathbb{N}}$ s.t.

$\mu^i(t^j) \pi^i(t^j | t^i) = \mu^i(t^j) \Pi^j(t^i | t^j)$

**Def:** $e(\Gamma)$ is defined as unique $e \in \Delta(\Gamma)$ s.t. $e \Gamma = e$.

Prop 1

$c(y; \bar{\pi}, \bar{\pi}) = \sum_{t^i \in \mathbb{S}} p(t^i) f(t^i)$

where $p$ is unique $p \in \Delta(S)$ s.t. $p B = p$

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**Common Priors & Influence**

**Def** common priors over signals (CPs)

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**Def** \( e(\Gamma) \) is defined as unique \( e \in \Delta(\Gamma) \) s.t. \( e \Gamma = e \).

**Prop 1**

\[ c(\mathbf{y}^i; \bar{\pi}, \bar{\mu}) = \sum_{t^i \in \mathcal{S}} p(t^i)f(t^i) \]

where \( p \) is unique \( p \in \Delta(\mathcal{S}) \) s.t. \( pB = p \)

i.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.

**Lemma** \( \forall i \)

\[ \sum_{t^i} p(t^i) = e^i \]
**Common Priors & Influence**

**Def** Common priors over signals (CPS)

\( \Pi^i \) all compatible w/ a \( \hat{\mu} \in \Delta(T) \)

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**Def** \( e(\Gamma) \) is defined as unique \( e \in \Delta(\Gamma) \) s.t. \( e \Gamma = e \).

**Prop 2** CPS \( \Rightarrow \) \( c = \sum_i e^i E^i \bar{y}^i \)

where

\[ E^i[\bar{y}^i] = \sum_i \mu(t^i) E^i[\bar{y}^i | t^i] \]

**Prop 1** \( c(\bar{y}; \Pi, \bar{p}) = \sum_{t^i \in S} p(t^i) f(t^i) \)

where \( p \) is unique \( p \in \Delta(S) \) s.t. \( p \Gamma = p \)

i.e. \( p \) is the stationary distribution of \( \Gamma \), viewed as a Markov chain.

**Lemma** \( \forall i \)

\[ \sum_{t^i} p(t^i) = e^i \]
**Common Priors & Influence**

Def common priors over signals (CPs): \( \Pi^i \) all compatible \( \omega \) a \( \hat{\omega} \in \Delta(T) \) \[ \Rightarrow \exists \text{ priors } (\mu^i)_{i \in N} \text{ s.t.} \]

\[ \mu^i(t^j) \Pi^i(t^j \mid t^i) = \mu^i(t^i) \Pi^j(t^i \mid t^j) \]

Def \( e(\Gamma) \) is defined as unique \( e \in \Delta(\Gamma) \text{ s.t. } e \Gamma = e \).

Prop 1: \( c(\bar{y}; \bar{\Pi}, \bar{\Pi}) = \sum_{t^i \in S} \Pi(t^i) f(t^i) \)

where \( \Pi \) is unique \( \Pi \in \Delta(S) \) s.t. \( \Pi B = \Pi \)

I.e. \( \Pi \) is the stationary distribution of \( B \), viewed as a Markov chain.

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Calvó-Armengol, de Marti, and Prat (TE 2015) “Communication and Influence”

Bergemann, Heumann, Morris (2017) "Information and Interaction"

Myatt and Wallace (2017), “Information Acquisition and Use by Networked Players”
Higher-Order Average Expectations

\[ \chi_{t^i}^{i} (1) = E^i [y^i | t^i] \]

1st - order expectation of \( y^i \) given \( i \)'s info

\[ \chi_{t^i}^{i} (2) = \sum_j r^{ij} E^i [x^j (1) | t^i] \]

2nd - order avg. expectation an average of 1st - order exp. given \( i \)'s info
Higher-Order Average Expectations

\[ \chi_{t^i}^{i}(1) = E^i[y^i|t^i] \]

1\textsuperscript{st}-order expectation of \( y^i \) given i's info

\[ \chi_{t^i}^{i}(n+1) = \sum_j \gamma^{ij} E^i[x^{j(n)}|t^i] \]

\((n+1)\textsuperscript{th}\)-order avg. expectation an average of \( n\textsuperscript{th}\)-order exp. given i's info
Higher-Order Average Expectations

\[ \chi^i_{t^i}(1) = E^i[y^i|t^i] \]

1st-order expectation of \( y^i \) given \( i \)'s info

\[ \chi^i_{t^i}(n+1) = \sum_j \gamma^{ij} E^i[x^{i(n)}|t^i] \]

\( (n+1) \)th-order avg. expectation an average of \( n \)th-order exp. given \( i \)'s info

Relation to Game

\[ a_{eqm} = (1-\beta)(I-\beta B)^{-1}f = (1-\beta) \sum_{n=0}^{\infty} \beta^n B^n f \]
Higher-Order Average Expectations

\[ x_{ti}^i(1) = E^i[y_i^i|t_i^i] \]

1st-order expectation of \( y^i \) given i's info

\[ x_{t_i}^i(n+1) = \sum_j y_{ij} E^i[x_{t_i}^i(n)|t_i^i] \]

(n+1)th-order avg. expectation
an average of nth-order
exp. given i's info

Relation to Game

\[ \alpha_{eqm} = (1 - \beta)(I - \beta B)^{-1} f = (1 - \beta) \sum_{n=0}^{\infty} \beta^n B^n f \]
Higher-Order Average Expectations

\[
\chi^{i}_{t} (1) = E^{i} [y^{i} | t^{i}]
\]

1\textsuperscript{st}-order expectation of \(y^{i}\) given \(i\)'s info

\[
\chi^{i}_{t} (n+1) = \sum_{j} \gamma^{ij} E^{i} [x^{j(n)} | t^{i}]
\]

\((n+1)\textsuperscript{th}\)-order avg. expectation of \(n\textsuperscript{th}\)-order exp. given \(i\)'s info

Relation to Game

\[
\alpha_{eqm} = (1-\beta) (I - \beta B)^{-1} f = (1-\beta) \sum_{n=0}^{\infty} \beta^n B^n f
\]

Samet (JET 98) “Iterated Expectation and Common Priors”

Our companion paper: “Higher-Order Expectations”
Higher-Order Average Expectations

\[ x^i_{t^i}(1) = \mathbb{E}^i[y^i | t^i] \]

\[ x^i_{t^i}(n+1) = \sum_j \gamma^{ij} \mathbb{E}^i[x^{j(i)} | t^i] \]

Relation to Game

\[ a_{eqm} = (1-\beta)(I - \beta B)^{-1} f = (1-\beta) \sum_{n=0}^{\infty} \beta^n B^n f \]

\[ [B^n f](t^i) = x^i_{t^i}(n+1) \]

Samet (JET 98) “Iterated Expectation and Common Priors”

Our companion paper: “Higher-Order Expectations”
Tyranny of Least-Informed

Prop 3 Suppose $q^i \leq 1 - \delta$ at least $\delta$-noisy for all $i \neq 1$ $q^i \geq 1 - \epsilon$ at most $\epsilon$-noisy

Prop 1
\[ c(\vec{y}; \vec{\pi}, \vec{\rho}) = \sum_{t \in S} p(t^i)f(t^i) \]
where $p$ is unique $p \in \Delta(S)$ s.t. $p B = p$
I.e. $p$ is the stationary distribution of $B$, viewed as a Markov chain.

Example 2 agents, incomplete info
\[ -u^i = \beta (a^i - a^j)^2 + (1 - \beta) (a^i - y(\theta))^2 \]
\[ \theta \in \{ \theta_1, \ldots, \theta_k \} \]
\[ \rho^i e \Delta(\Theta) \text{ i's prior} \]
\[ t^i e \{ t^i_1, \ldots, t^i_k \} \text{ matches } \theta \text{ w.p. } q^i \]
Otherwise full support noise.
TYRANNY OF LEAST-INFORMED

**Prop 3** Suppose \( q^i \leq 1 - \delta \) at least \( \delta \)-noisy for all \( i \neq 1 \) \( q^i \geq 1 - \varepsilon \) at most \( \varepsilon \)-noisy.

Then
\[
|c(y; \bar{\pi}) - E_{\bar{\pi}}^1 [y] | \leq k \cdot \frac{\varepsilon}{\delta}
\]

**Prop 1**
\[
c(y; \bar{\pi}, \bar{\nu}) = \sum_{t \in S} p(t') f(t')
\]
where \( p \) is unique \( p \in \Delta(S) \) s.t. \( p B = p \)
i.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.

\[\boxed{\text{Ex}}\]
2 agents, incomplete info

\[-u^i = \beta (a^i - a^j)^2 + (1 - \beta) (a^i - y(\theta))^2\]
\( \theta \in \{\theta_1, \ldots, \theta_k\} \)
\( \rho^i \in \Delta(\Theta) \quad i \text{'s prior} \)
\( t^i \in \{t_1^i, \ldots, t_k^i\} \text{ matches } \theta \text{ w.p. } q^i \)
Otherwise full support noise.
Prop 3. Suppose \( q^i \leq 1 - \delta \) \( \delta \)-noisy for all \( i \neq 1 \), \( q^i \geq 1 - \epsilon \) \( \epsilon \)-noisy. Then

\[
\left| c(y; \bar{\pi}) - E_{\bar{\pi}} c^1[y] \right| \leq K \cdot \frac{\epsilon}{\delta}
\]

Prop 1. \( c(y; \bar{\pi}, \bar{\nu}) = \sum_{t \in S} p(t) f(t) \)

where \( p \) is unique \( p \in \Delta(S) \) s.t. \( pB = p \)

i.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.

Example 2 agents, incomplete info

\(-u^i = \beta (a^i - a^j)^2 + (1 - \beta) (a^i - y(\theta))^2\)

\( \theta \in \{\theta_1, \ldots, \theta_K\} \)

\( p^i \in \Delta(\Theta) \) \( i \)'s prior

\( t^i \in \{t^i_1, \ldots, t^i_K\} \) matches \( \theta \) w.p. \( q^i \)

Otherwise full support noise.
Prop 3  Suppose $q^i \leq 1 - \delta$ at least $\delta$-noisy for all $i \neq 1$ $q^i \geq 1 - \varepsilon$ at most $\varepsilon$-noisy

Then

$$|c(y; \pi) - E_{\pi^1}^y[y] | \leq k \cdot \frac{\varepsilon}{\delta}$$

Prop 1  $c(y; \pi, \pi^1) = \sum_{t \in S} p(t^i) f(t^i)$

where $p$ is unique $p \in \Delta(\mathcal{S})$ s.t. $pB = p$

i.e. $p$ is the stationary distribution of $B$, viewed as a Markov chain.
Tyranny of Least-Informed

Prop 3. Suppose \( q^1 \leq 1 - \delta \) at least \( \delta \)-noisy
for all \( i \neq 1 \) \( q^i \geq 1 - \varepsilon \) at most \( \varepsilon \)-noisy

Then
\[
| c(y; \hat{\pi}) - \mathbb{E}^{\hat{\pi}}[y] | \leq \kappa \cdot \frac{\varepsilon}{\delta}
\]

Proof Idea

0. Define artificial \( \hat{\pi} \):
   - each \( i \neq 1 \) knows \( \Theta \)
   - 1's info. unchanged

Prop 1. \( c(y; \pi, \bar{\pi}) = \sum_{t' \in S} p(t') f(t') \)
where \( p \) is unique \( p \in \Delta(S) \) s.t. \( pB = p \)
I.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.
Tyranny of Least-Informed

Prop 3 Suppose $q^i \leq 1 - \delta$ at least $\delta$-noisy
for all $i \neq 1$ $q^i \geq 1 - \epsilon$ at most $\epsilon$-noisy

Then

$$|c(y; \tilde{\pi}) - \mathbb{E}^\tilde{\pi}[y]| \leq k \cdot \frac{\epsilon}{\delta}$$

Proof Idea

0. Define artificial $\hat{\pi}$:
  - each $i \neq 1$ knows $\Theta$
  - 1's info. unchanged

1. $c(y; \hat{\pi}) = \mathbb{E}^{\hat{\pi}}[y]$

Reason: $\hat{\pi}$ satisfies CPS with 1's prior.
**Tyranny of Least-Informed**

**Prop 3** Suppose \( q^1 \leq 1 - \delta \) \( \delta \)-noisy at least for all \( i \neq 1 \) \( q^i \geq 1 - \varepsilon \) \( \varepsilon \)-noisy.

Then

\[
|c(y; \hat{\pi}) - |E_{\hat{\pi}}^1[y]| | \leq k \cdot \frac{\varepsilon}{\delta}
\]

**Proof Idea**

0. Define artificial \( \hat{\pi} \):
   - each \( i \neq 1 \) knows \( \Theta \)
   - 1's info. unchanged

1. \( c(y; \hat{\pi}) = |E_{\hat{\pi}}^1[y]| \)
   Reason: \( \hat{\pi} \) satisfies CPS with 1's prior.

2. \( p(B_{\hat{\pi}}) \approx p(B_{\pi}) \)
   Reason: if \( \| B_{\pi} - B_{\hat{\pi}} \| \) small compared to mean first passage times in \( B_{\hat{\pi}} \), then \( \approx \) holds.

**Prop 1**

\[
c(y; \bar{\pi}, \bar{\pi}) = \sum_{t' \in S} p(t') f(t')
\]

where \( p \) is unique \( p \in \Delta(S) \) s.t. \( pB = p \)

i.e. \( p \) is the stationary distribution of \( B \), viewed as a Markov chain.
Prop 3
Suppose $q^1 \leq 1 - \delta$ at least $\delta$-noisy,
for all $i \neq 1$ $q^i \geq 1 - \varepsilon$ at most $\varepsilon$-noisy.

Then
\[ |c(y; \pi) - \mathbb{E}^\pi [y] | \leq k \cdot \frac{\varepsilon}{\delta} \]

Proof Idea

0. Define artificial $\hat{\pi}$:
   - each $i \neq 1$ knows $\Theta$
   - $1$'s info. unchanged

1. $c(y; \hat{\pi}) = \mathbb{E}^\pi [y]$
   Reason: $\hat{\pi}$ satisfies CPS with 1's prior.

2. $p(B_{\hat{\pi}}) \approx p(B_{\pi})$
   Reason: if $\| B_{\hat{\pi}} - B_{\pi} \|$ small compared to mean first passage times in $B_{\hat{\pi}}$ then $\approx$ holds.

Cho and Meyer (00) “Markov chain sensitivity measured by mean first passage times”
Interaction structure captures (interim) beliefs and network simultaneously: a method for studying how behavior depends on

(i) information (signals)  (ii) interpretation (priors)  (ii) coordination concerns (interaction)

General characterization of conventions in terms of eigenvector centrality in interaction structure. Reduction to a complete-information network game.

Illustrate with three applications.

Contagion of optimism – small local bias (in common direction) leads to extreme conventions.

Under common prior over signals, agents’ prior expectations matter in proportion to their centrality in the network $\Gamma$ only.

Under common interpretation of signals and precise private information, get tyranny of the least-informed.