Reward and Punishment in a Regime Change Game

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Question

- citizens choose among a repertoire of **contentious performances** (Tilly 08):
  - public meetings, boycotts, strikes, marches, demonstrations, sit-ins, freedom rides, street blockades, suicide bombings, assassination, hijacking, and guerrilla war

- but what contentious performances can be elicited from supporters and how?
  - **pleasure in agency**: "the positive effect associated with self-determination, autonomy, self-esteem, efficacy, and pride that come from the successful assertion of intention" (Wood 03)
  - "depends on the likelihood of success, which in turn depends on the number participating... yet the pleasure in agency is undiminished by the fact that one’s own contribution to the likelihood of victory is vanishingly small"
Question

- **transformational leadership**: ability to create inspirational motivations through a variety of psychological mechanisms (Burns 78)
  - sociology: leaders raise participation by "identification, idealization, elevation of one or more values..." (Snow et al 89)
  - politics: "people-oriented leaders are those who inspire people, give them a sense of identity..." (Goldstone 01)

- our question:
  - what happens when transformative leaders manipulate pleasure in agency in order to influence activists’ choice among a repertoire of contentious performances?

- our answer (in a stylized model)
  - "the organization of the revolutionaries must consist first and foremost of people who make revolutionary activity their profession"....while others (workers) engage in various levels of contentious activities (Lenin 1902)
Key Tradeoffs

- if activists ("citizens") are heterogeneous, then there would be trade-offs in manipulating pleasure in agency ("benefits"); higher benefits from intermediate levels of contentious performances ("effort") will...
  - encourage those who would otherwise have chosen low effort to choose intermediate effort
  - discourage those who might otherwise have chosen high effort

- even if citizens are not heterogenous, at the margin where the revolution is occurring, the citizenry will have heterogeneous beliefs about the likelihood of success
  - if everyone thought revolution was going to succeed, everyone would choose maximum effort and the revolution would succeed
  - if everyone thought revolution was going to fail, everyone would choose minimum effort and the revolution would fail
we will consider a situation where there is an upper bound on benefits...

the upper bound on benefits will imply an upper bound on effort (what would be chosen if you assigned probability 1 to success)

we will identify an optimal reward scheme (mapping from effort to benefits)...

there will be a critical level of effort at which citizens will get the maximal benefit...

benefits will smoothly decline for lower efforts
Model of Coordination...

- continuous action regime change game:
- citizens make a continuous effort decision
- revolution succeeds if aggregate effort exceeds a threshold
- small uncertainty about threshold $\Rightarrow$ unique equilibrium ("global game")
- at critical threshold, there will be uniform distribution among citizens of the probability of success
- citizens trade off costs and success contingent reward scheme
....and Screening

- leader can pick optimal reward scheme
- screening problem because of heterogeneous beliefs about the probability of success
Methodological Contribution

- continuous action regime change game (Guimaraes-Morris 07 is unique applied precursor?)
- global game with screening
- non-standard screening problem: role of maximum choice variable
Extensions and Open Questions

- designing costs instead of benefits (same principle but type dependent participation constraints)
- a game between players choosing costs and benefits respectively
- more standard screening problems
- philanthropy and other applications
Model with Exogenous Benefits and Complete Information

- continuum of citizens with each choosing $e_i \in [0, 1]$
- revolution succeeds if $\int e_i \, di \geq \theta$, where $\theta$ is "regime strength"
- uncontingent cost of effort $C(e)$
  - $C(0) = C'(0) = 0$, $C'(e), C''(e) > 0$ for $e > 0$
- contingent benefit of effort $B(e)$
  - $B(0) = 0$, $B'(e) > 0$
- joint restrictions:
  - $\frac{C(e)}{B(e)}$ strictly increasing, $B''(e) < C''(e)$ for all $e$ and $B(e) < C(e)$ for some $e$
Equilibrium with Exogenous Benefits and Complete Information

- effort $\bar{e}$ if you are sure revolution will occur solves

$$B' (\bar{e}) = C' (\bar{e})$$

**RESULT 1:** There are smallest and largest Nash equilibria where citizens exert effort 0 and $\bar{e}$ respectively
Model with Exogenous Benefits and Incomplete Information

- state $\theta \sim g(\cdot)$, citizen $i$ observes $x_i = \theta + \sigma \varepsilon_i$, where each $\varepsilon_i$ is independent distribution according to $f(\cdot)$
- we will now care about

$$e^*(p) = \arg \max_e pB(e) - C(e)$$

- and we will study what happens as $\sigma \to 0$
Equilibrium with Exogenous Benefits and Incomplete Information

- this is a global game with \textit{continuum actions}
- there is a unique equilibrium in the incomplete information game: Frankel, Morris and Pauzner (2003)
- if (but only if) we have a "regime change game", there is a very nice characterization of that unique equilibrium
RESULT 2: The unique equilibrium will be a "threshold equilibrium" with a critical level of regime strength $\theta^*$ such that in limit of the incomplete information game, revolution succeeds if and only if $\theta > \theta^*$ where

$$\theta^* = \int_{p=0}^{1} e^* (p) \, dp \quad (*)$$

and so

$$\theta^* = \int_{e=0}^{\bar{e}} \left(1 - \frac{C'(e)}{B'(e)}\right) \, de$$

Follows from argument of Guimaeres and Morris (2006)
Statistical Fact

- fix any $\hat{\theta}$
- at $\hat{\theta}$, a citizen with signal $x$ will assign some probability $p$ to $\theta \geq \hat{\theta}$
- STATISTICAL FACT: the distribution of $p$'s in the population is uniform
Statistical Fact Proof

- fix any \( \hat{\theta} \) and \( p \)
- player assigns probability \( p \) to \( \theta \geq \hat{\theta} \) when \( x^* - \hat{\theta} = F^{-1}(p) \)
  - \( x^* - \varepsilon \geq \hat{\theta} \) or \( \varepsilon \leq x^* - \hat{\theta} \)
  - this occurs with probability \( F \left( x^* - \hat{\theta} \right) \)
- the probability an observer assigns to \( x \leq x^* \) is \( F \left( x^* - \hat{\theta} \right) \)
  - \( x \leq x^* \) if \( \hat{\theta} + \varepsilon \leq x^* \); prob \( F \left( x^* - \hat{\theta} \right) \) event
- so probability observer assigns to \( p \) is \( F \left( F^{-1}(p) \right) = p \)
Endogenous Benefits

- Leader chooses $B : \mathbb{R}_+ \rightarrow [0, M]$ to maximize the probability of revolution, i.e., $\theta^*$
- motivation: see introduction
- modelling choices: many alternatives
Recall Two Insights (from introduction)

- If supporters ("citizens") are heterogeneous, then there would be trade-offs in manipulating benefits; higher benefits from intermediate levels of effort will...
  - Encourage those who would otherwise have chosen low effort to choose intermediate effort
  - Discourage those who might otherwise have chosen high effort
- Even if citizens are not heterogeneous, at the margin where the revolution is occurring, the citizenry will have heterogeneous beliefs about the likelihood of success
  - If everyone thought revolution was going to succeed, everyone would choose maximum effort and the revolution would succeed
  - If everyone thought revolution was going to fail, everyone would choose minimum effort and the revolution would fail
Leader chooses $B$ to maximize

$$\theta^* = \int_{p=0}^{1} e^*(p) \, dp \quad (*)$$

subject to $e^*$ being the effort levels induced by $B$
Screening Problem

- Standard revelation principle / optimal screening argument can re-write problem as:

**MAXIMIZATION PROBLEM**: Leader chooses \( B, e \) to maximize

\[
\theta^* = \int_{p=0}^{1} e(p) \, dp \quad (*)
\]

subject to...

\[
pB(p) - C(e(p)) \geq 0
\]
\[
pB(p) - C(e(p)) \geq pB(p') - C(e(p'))
\]
\[
B(p) \in [0, M]
\]
Buyer Seller Screening Problem

- with $C(\cdot)$ is linear, as if a buyer of type $p$ is buying a good of quality $B$ from a seller for price $e$, where only constraint on $B$ is its maximum
- could solve this problem with essentially standard pointwise methods
- non-linear $C(\cdot)$ creates complications
RESULT 3: Suppose that $C$ is strictly convex. Then the optimal $B(\cdot)$ is continuous, weakly increasing and - for some $0 < \hat{e} < 1$ - strictly convex on $[0, \hat{e}]$ and equal to $M$ above $\hat{e}$. The optimal $e$ is continuous, strictly increasing on $[0, 1/2]$ and equal to $\hat{e}$ constant on $[1/2, 1]$.

- Why bunching?
  - Suppose that $0 < B'(e) < C'(e)$ on an interval $[e_1, e_2]$; no one will choose effort in the interval $[e_1, e_2]$
  - Must have $B'(e) \geq C'(e)$ for unless $B'$ is zero or $e = \bar{e}$
Example: Optimal Benefits

- $M = 1$ and $c(e) = e^n$ for some $n \geq 1$
Example: Optimal Effort

- $M = 1$ and $c(e) = e^n$ for some $n \geq 1$
RESULT 3: Suppose that $C$ is strictly convex. Then the optimal $B(\cdot)$ is continuous, weakly increasing and - for some $0 < \hat{e} < 1$ - strictly convex on $[0, \hat{e}]$ and equal to $M$ above $\hat{e}$. The optimal $e$ is continuous, strictly increasing on $[0, 1/2]$ and equal to $\hat{e}$ constant on $[1/2, 1]$.

Why bunching?

- Suppose that $0 < B'(e) < C'(e)$ on an interval $[e_1, e_2]$; no one will choose effort in the interval $[e_1, e_2]$.
- Must have $B'(e) \geq C'(e)$ for unless $B'$ is zero or $e$ except at the top.
- Bunching at the top.
Extensions and Open Questions

- designing costs instead of benefits (same principle but type dependent participation constraints)
- a game between players choosing costs and benefits respectively
- more standard screening problems
- philanthropy and other applications
Conclusion

- Methodology:
  - incorporating continuous actions and screening into models of revolution (many novel comparative statics we could study)

- Substantive:
  - subtle tradeoffs in design of optimal pleasure in agency
  - in support of the vanguard