Counterfactual Equivalence in Macroeconomics
This Paper

- One insight about dynamic stochastic models with linear representations:
  
  **The Principle of Counterfactual Equivalence**

- A **method** for identifying a set of counterfactually equivalent models

- Tackle a number of **problems** in macroeconomics
Applications

1. How to construct policy counterfactuals that are robust across models?
   - Example: Quantify how fiscal unions contribute to regional stabilization

2. Which micro-foundations matter for analyzing policy changes?
   - Example: Unemployment benefits in search models of the labor market

3. How can we falsify models?
   - Example: Volcker’s policy change falsifies a set of New Keynesian models
Plan for the Talk

1. **Illustrate** the principle of counterfactual equivalence in two examples
   - 3-equation NK model with interest rate policy rule
   - Small Open Economy with transfers policy rule

2. Discuss **methodology** in these examples

3. Show **applications**

4. Relate to **methods** concerned with robustness and misspecification
A Canonical NK model

- 3 equation model of inflation \( \pi_t \), output \( y_t \) and nominal interest rate \( i_t \) in log-deviations from a zero inflation steady state

\[
0 = -E_t(y_{t+1}) - E_t(\pi_{t+1}) + y_t + i_t + E_t[\gamma_{t+1} - \gamma_t]
\]

\[
0 = \beta E_t(\pi_{t+1}) + \kappa y_t - \pi_t + Z_t
\]

\[
0 = -i_t + \theta_y y_t + \theta_{\pi} \pi_t + \epsilon_t
\]

- \( z_t, \gamma_t, \epsilon_t \) independent AR(1)’s

- \( \theta_p, \theta_y \) are policy parameters
Equilibria with Alternative Interest Rate Policy Rules

- **Goal:** analyze alternative interest rate policy rules

- **Standard** approach (e.g., Clarida, Gali, and Gertler (2000)):
  - Estimate/calibrate parameters
  - Solve for equilibria under alternative $\theta_p, \theta_y$
  - Compare equilibria

- But **which** model should we use? When does it matter?
The Principle of Counterfactual Equivalence

Consider the unrestricted set of models

\[
0 = F_{t+1} \begin{bmatrix} y_{t+1} \\ \pi_{t+1} \\ i_{t+1} \end{bmatrix} + (G + \Theta) \begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} + H \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + M \begin{bmatrix} \gamma_t \\ z_t \\ \epsilon_t \end{bmatrix}
\]

\[
0 = - \begin{bmatrix} \gamma_t \\ z_t \\ \epsilon_t \end{bmatrix} + N \begin{bmatrix} \gamma_{t-1} \\ z_{t-1} \\ \epsilon_{t-1} \end{bmatrix} + \Sigma \begin{bmatrix} u_t^\gamma \\ u_t^z \\ u_t^\epsilon \end{bmatrix}
\]

Call \( \Theta \) a policy and \( \xi \equiv \{ F, G, H, M, N, \Sigma \} \) a structure

Let \( \{ P(\xi, \Theta), Q(\xi, \Theta), N \} \) be the unique and stable recursive equilibrium,

\[
\begin{bmatrix} y_t \\ \pi_t \\ i_t \end{bmatrix} = P(\xi, \Theta) \begin{bmatrix} y_{t-1} \\ \pi_{t-1} \\ i_{t-1} \end{bmatrix} + Q(\xi, \Theta) \begin{bmatrix} \gamma_t \\ z_t \\ \epsilon_t \end{bmatrix}
\]
The Principle of Counterfactual Equivalence (cont.)

- **(Observational Equivalence)** Given a benchmark policy $\Theta^0$ and a recursive equilibrium $\{P^0, Q^0, N^0\}$, two models are observationally equivalent if and only if they can be parameterized to generate $P^0, Q^0$ under policy $\Theta^0$

- **(Counterfactual Equivalence)** Given $\{\Theta^0, P^0, Q^0, N^0\}$ and an alternative policy $\Theta^1$, two models are counterfactually equivalent if and only if

1. they are observationally equivalent under the benchmark policy $\Theta^0$
2. they generate identical $P^1, Q^1$ under the alternative policy $\Theta^1$
Counterfactual Equivalence in NK Models

- "Model 1": Non-employed agents in large household
  \[ 0 = \beta E_t(\pi_{t+1}) + \kappa y_t - \pi_t + z_t + \kappa \frac{\gamma}{1 + \gamma} \gamma_t \]

- "Model 2": Behavioral firms as in Gabaix (2017)
  \[ 0 = \beta M^f E_t(\pi_{t+1}) + \kappa m^f y_t - \pi_t + z_t \]

- "Model 3": Working-capital-in-advance
  \[ 0 = \beta E_t(\pi_{t+1}) + \kappa y_t + (\kappa \chi - 1) \pi_t + z_t \]

- **Claim**: All models are observationally equivalent, but only "2" and "3" are counterfactually equivalent wrt to changes in interest rate policy rule.
Identifying a Set of Counterfactually Equivalent Models

- Method of undetermined coefficients: Given \( \{\xi, \Theta^0\} \) \( \Rightarrow \) solve for \( P^0, Q^0 \)

\[
\begin{align*}
F(P^0)^2 + (G + \Theta^0)P^0 + H &= 0 \\
F(Q^0 N + P^0 Q^0) + (G + \Theta^0)Q^0 + M &= 0
\end{align*}
\]

- The mapping from parameters in \( \{\xi, \Theta^0\} \) onto \( P^0, Q^0 \) is highly non-linear

- Hard to determine how policy counterfactuals differ across models
Identifying a Set of Counterfactually Equivalent Models

- However, the reverse mapping from \( \{P^0, Q^0, N, \Theta^0\} \) onto \( \xi \) is **linear**
  
  \[
  F(P^0)^2 + GP^0 + \Theta^0 P^0 + H = 0 \\
  F(Q^0 N + P^0 Q^0) + GQ^0 + \Theta^0 Q^0 + M = 0
  \]

- Imposing enough restrictions on \( \xi \) makes the system exactly determined
- Identifies a set of counterfactually equivalent models wrt to policy \( \Theta \)
- Also, a methodology for constructing **Robust Policy Counterfactuals**

- No need to fully specify a model. Just need to:
  1. Estimate \( \{P^0, Q^0, N, \Theta^0\} \) and impose restrictions, thus identifying \( \xi \)
  2. Given \( \xi \) and alternative policy \( \Theta^1 \), solve the (non-linear) system for \( P^1, Q^1 \)
The Restricted Structure of NK Models

NKPC in Model "1" (non-employed agents)

\[
\begin{bmatrix}
0 & \beta & 0 \\
\kappa & -1 & 0 \\
1 & \frac{\kappa \chi}{1 + \gamma} & 0
\end{bmatrix}
\begin{bmatrix}
y_{t+1} \\
\pi_{t+1} \\
i_{t+1}
\end{bmatrix}
= 0
\]

NKPC in Model "2" (behavioral firms)

\[
\begin{bmatrix}
0 & \beta & 0 \\
\frac{m^f}{M^f} & -\frac{1}{\kappa M^f} & 0 \\
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
y_{t+1} \\
\pi_{t+1} \\
i_{t+1}
\end{bmatrix}
= 0
\]

NKPC in Model "3" (working-capital)

\[
\begin{bmatrix}
0 & \beta & 0 \\
\kappa & \kappa \chi - 1 & 0 \\
1 & 0 & 0
\end{bmatrix}
\begin{bmatrix}
y_{t+1} \\
\pi_{t+1} \\
i_{t+1}
\end{bmatrix}
= 0
\]

- Five restrictions per equation ⇒ identify a counterfactually equivalent set.
- Identical restrictions in Models "2" and "3", but not in Model "1".
A Small Open Economy Model

- Representative firm and infinitely lived household.

- Household period utility: \( e^{-\rho t - \delta t} \left( C_t - \frac{\phi}{1 + \phi} N_t^{\frac{1 + \phi}{1 - \sigma}} \right)^{1 - \sigma} \)

- Save in non-state contingent asset \( B_t \). Fixed rate \( r \).

- Firm maximizes static profits. Technology: \( e^{z_t} (N_t^Y)^\alpha (X_t)^\beta \)

- Intermediate goods \( X_t \) are tradable. Price is 1.

- \( C_t \) and \( N_t \) are not. Price \( P_t \) and wage \( W_t \).

- All markets are competitive.
A Small Open Economy Model (cont.)

- Households receive lump-sum transfers $S_t$ from abroad.

- Interpretation: federal transfers in a fiscal union

- *Transfer Policy Rule* as function of local per-capita variables:

  \[ S_t = (\bar{N}_t)^\vartheta \]

- Sequential budget constraint is:

  \[ P_tC_t + B_t = B_{t-1}(1 + r) + W_tN_t + S_t + \Pi_t + e^{\eta_t} \]

- $\eta_t$ exogenous intermediate good endowment.
Log-linearized equilibrium around Steady State

- Log-linearized equilibrium is characterized by:

\[
0 = \mathbb{E}_t \left[ \frac{\sigma \beta}{\alpha \phi} - 1 \right] (w_{t+1} + n_{t+1} - w_t - n_t) + (\alpha + \beta - 1)(n_{t+1} - n_t)
+ (\beta - 1)(w_{t+1} - w_t) + (1 + \frac{\sigma}{\alpha \phi} - 1) (z_{t+1} - z_t) + \Phi_0(\cdot) - \gamma_{t+1} \right]
\]

(Euler)

\[
\beta w_t = \left( \frac{1 + \phi}{\phi} - (\alpha + \beta) \right) n_t - z_t
\]

(Labor Market)

\[
\frac{B}{S} b_t = \frac{B}{S} (1 + r) b_{t-1} - \frac{X}{S} (w_t + n_t) + \vartheta n_t + \frac{1}{S} \eta_t
\]

(SB)

- Exogenous processes \( \eta_t, z_t \) and \( \gamma_t \equiv \delta_t - \delta_{t-1} \)

\[
\gamma_t = \rho \gamma \gamma_{t-1} + \sigma \gamma u_{t}^{\gamma} \quad \eta_t = \rho \eta \eta_{t-1} + \sigma \eta \eta_{t}^{\eta} \quad z_t = \rho_z z_{t-1} + \sigma_z z_{t}^{z}
\]
Alternative Models

▶ "Model 1": the one I just described

\[ 0 = \left( \frac{1 + \phi}{\phi} - (\alpha + \beta) \right) n_t - \beta w_t - z_t \]  
(Labor market (1))

▶ "Model 2": sticky wages

\[ \frac{1 - \lambda}{\lambda} (w_t - w_{t-1}) = \left( \frac{1 + \phi}{\phi} - (\alpha + \beta) \right) n_t - \beta w_t - z_t \]  
(Labor Market (2))

▶ "Model 3": durable leisure

\[ \nu \frac{1 + \phi}{\phi} n_{t-1} = \left( \frac{1 + \phi}{\phi} - (\alpha + \beta) \right) n_t - z_t - \beta w_t \]  
(Labor Market (3))
Different Mechanisms, Same Counterfactual...

▶ Intuition in an example with **durability** \((\nu)\) and **wage rigidity** \((\lambda)\)

▶ Simulate \((n_t^0, w_t^0, b_t^0)\) from model with \(\{\nu_0, \lambda_0\}\)

▶ **Claim:**

1. A model with lower durability and higher rigidity can match \((n_t^0, w_t^0, b_t^0)\)
2. Both produce identical counterfactual \((n_t^1, w_t^1, b_t^1)\) if transfer rule is changed.

▶ **Intuition:**

1. - High durability increases employment’s elasticity to shocks (relative to wage’s)
   - High rigidity decreases wage elasticity to shocks (relative to employment’s)
2. - Durability and rigidity only affect marginal decisions, not resources available
   - Transfers only affect available resources, not marginal decisions
Application I: Robust Policy Counterfactuals

▶ **Exercise of interest:** Given data under benchmark policy, construct counterfactual equilibrium under alternative policy

### Paradigms

<table>
<thead>
<tr>
<th></th>
<th>Fully Specified Models</th>
<th>SVARs or FRB/US</th>
</tr>
</thead>
<tbody>
<tr>
<td>Concerns</td>
<td>Robustness</td>
<td>Lucas Critique</td>
</tr>
</tbody>
</table>

▶ **This paper:** A methodology for constructing counterfactuals that are:

1. immune to Lucas Critique
2. robust across models that satisfy identical restrictions
Regional Stabilization in Fiscal Unions

- A fiscal union is a federal tax-and-transfer policy that serves two roles:
  1. Long-run redistribution (e.g., New York has negative transfers net of taxes)
  2. Short-run stabilization (e.g., unemployment insurance)

- I will focus on stabilization role alone

- Characterize set of fiscal union models that satisfy the principle of counterfactual equivalence

- Using US state-level data, construct a counterfactual US economy without fiscal integration
Figure: Net Transfers growth v. Wage Income growth (2006-2010)

Slope=-1.25 (0.18)
Properties of fiscal union models

1. **Transfer policy rule**: Tax-and-transfer system can be summarized as federal lump-sum transfers that are a function of state-level economic variables.

2. **Linear aggregation**: State-level economies in log-deviations from the aggregate union behave to a first-order approximation as if they were small open economies—indeed independent of other states.

3. **3 by 3**: Employment $n_t$, nominal wages $w_t$, and assets $b_t$; and exogenous processes $\{\gamma_t, \eta_t, z_t\}$ are sufficient variables for characterizing the state-level equilibrium in log-deviation from aggregates.

4. **SVAR**: The log-linearized equilibrium has a unique, finite, and stable structural vector autoregression representation.
Unrestricted Set of Fiscal Union Models

Equilibrium in a state in log-deviations from aggregate is characterized by

\[ 0 = (F + \Theta_f)E_t \begin{bmatrix} n_{t+1} \\ w_{t+1} \\ b_{t+1} \end{bmatrix} + (G + \Theta_c) \begin{bmatrix} n_t \\ w_t \\ b_t \end{bmatrix} + (H + \Theta_p) \begin{bmatrix} n_{t-1} \\ w_{t-1} \\ b_{t-1} \end{bmatrix} \]

\[ + L E_t \begin{bmatrix} \gamma_{t+1} \\ z_{t+1} \\ \eta_{t+1} \end{bmatrix} + M \begin{bmatrix} \gamma_t \\ z_t \\ \eta_t \end{bmatrix} \]

\[ + M \begin{bmatrix} \gamma_t \\ z_t \\ \eta_t \end{bmatrix} + N \begin{bmatrix} \gamma_{t-1} \\ z_{t-1} \\ \eta_{t-1} \end{bmatrix} + \Sigma \begin{bmatrix} u_t^\gamma \\ u_t^z \\ u_t^\eta \end{bmatrix} \]

(Euler)
(Labor Market)
(Budget Constraint)

SVAR representation

\[ \begin{bmatrix} n_t \\ w_t \\ b_t \end{bmatrix} = \rho_1 \begin{bmatrix} n_{t-1} \\ w_{t-1} \\ b_{t-1} \end{bmatrix} + \rho_2 \begin{bmatrix} n_{t-2} \\ w_{t-2} \\ b_{t-2} \end{bmatrix} + Q \Sigma \begin{bmatrix} u_t^\gamma \\ u_t^z \\ u_t^\eta \end{bmatrix} \]
Policy and structural restrictions

- The policy $\Theta$ is: $\Theta_c = \begin{bmatrix} 0 & 0 & 0 \\ \vartheta_n & \vartheta_w & 0 \end{bmatrix}$, $\Theta_p = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & \vartheta_b \end{bmatrix}$, $\Theta_f = 0_{3,3}$

- The structural restrictions defining a set of models are:

$$F = \begin{bmatrix} f_{11} & f_{12} & 0 \\ f_{21} & f_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix}; \ G = \begin{bmatrix} g_{11} & g_{12} & 0 \\ g_{21} & g_{22} & 0 \\ g_{31} & g_{32} & g_{33} \end{bmatrix}; \ H = \begin{bmatrix} h_{11} & 0 & 0 \\ h_{21} & h_{22} & 0 \\ 0 & 0 & h_{33} \end{bmatrix};$$

$$L = \begin{bmatrix} 1 & l_{12} & l_{13} \\ 0 & 1 & l_{23} \\ 0 & 0 & 0 \end{bmatrix}; \ M = \begin{bmatrix} 0 & m_{12} & m_{13} \\ 0 & m_{22} & m_{23} \\ 0 & 0 & m_{33} \end{bmatrix}; \ N = \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ 0 & n_{32} & n_{33} \end{bmatrix}$$

- Essentially, "no wealth effects" in (Euler) and (Labor Market)

- (Budget Constraint) is more restricted (e.g., no expectational terms). I will use it to identify Q following Beraja, Hurst, and Ospina (2016).
Inputs for Robust Transfer Policy Counterfactual

- **Goal**: construct counterfactual US without fiscal integration

- Corresponds to an economy with $\Theta = 0$

- We need:
  1. Data on employment $n_t$, wages $w_t$, and assets $b_t$ in the US
  2. Data on federal taxes and transfers in the US
  3. Estimate transfer policy rule $\{\vartheta_n, \vartheta_w, \vartheta_b\}$
  4. Estimate SVAR to get recursive equilibrium
Data

- Yearly US state level data from 2006 to 2012.
- Wages from ACS and Census.
- Employment rate from BLS.
- Federal taxes and transfers from BEA.
- Assets is own construction.
### Table: Transfers Policy Rule Estimates

<table>
<thead>
<tr>
<th>Model</th>
<th>$\vartheta_n$</th>
<th>$\vartheta_w$</th>
<th>$\vartheta_b$</th>
<th>$\vartheta_{w+n}$</th>
<th>$R^2$</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>OLS</strong></td>
<td>$-1.6^{**}$</td>
<td>$-0.9^{*}$</td>
<td>$-0.03$</td>
<td>.</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>(0.5)</td>
<td>(0.7)</td>
<td>(0.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>IV w/ shocks (1)</strong></td>
<td>$-1.3^{*}$</td>
<td>$-1.4^{*}$</td>
<td>$-0.02$</td>
<td>.</td>
<td>0.41</td>
</tr>
<tr>
<td></td>
<td>(0.7)</td>
<td>(0.8)</td>
<td>(0.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>IV w/ house prices (1)</strong></td>
<td>.</td>
<td>.</td>
<td>$-0.03$</td>
<td>$-1.1^{**}$</td>
<td>0.42</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>(0.02)</td>
<td>(0.4)</td>
<td></td>
</tr>
<tr>
<td><strong>IV w/ house prices and shocks (1)</strong></td>
<td>$-1.4^{*}$</td>
<td>$-1.2^{*}$</td>
<td>$-0.02$</td>
<td>.</td>
<td>0.43</td>
</tr>
<tr>
<td></td>
<td>(0.6)</td>
<td>(0.6)</td>
<td>(0.02)</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>(2)</strong></td>
<td>$-1.3^{*}$</td>
<td>$-1.4^{*}$</td>
<td>$0.01$</td>
<td>.</td>
<td>0.38</td>
</tr>
<tr>
<td></td>
<td>(0.7)</td>
<td>(0.7)</td>
<td>(0.08)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Note:** Numbers in parenthesis are OLS (or second stage) standard errors. Variables with ’**’ are significant at a 5% level. Variables with ’***’ are significant at 1%. All variables are state log-growth rates between 2006 and 2010. $b_{t-1}$ is exogenous in (1) and endogenous in (2).
Regional SVAR

- When $Q(\xi, \Theta)$ is non-singular, there is a SVAR representation of the recursive equilibrium

$$x_t = \rho_1(\xi, \Theta)x_{t-1} + \rho_2(\xi, \Theta)x_{t-2} + Q(\xi, \Theta)u_t$$

where

$$\rho_1(\xi, \Theta) \equiv P(\xi, \Theta) + Q(\xi, \Theta)NQ(\xi, \Theta)^{-1}$$
$$\rho_2(\xi, \Theta) \equiv (P(\xi, \Theta) - \rho_1(\xi, \Theta))P(\xi, \Theta)$$
$$V(\xi, \Theta) \equiv \text{Var}(Q(\xi, \Theta)u_t) = Q(\xi, \Theta)\Sigma\Sigma'Q(\xi, \Theta)'$$

- Estimate reduced-form VAR via OLS equation by equation
- All states pooled together. Common VAR for all.
- $n_t, w_t, b_t$ measured in log-growth rates from 2006
Obtaining $P, Q, N$

- Given estimated VAR, solve $\rho_2 = (X - \rho_1)X$

- Problem: there are two solutions with all eigenvalues inside unit circle, $P$ and $QNQ^{-1} = \rho_1 - P$

- Also, we need to identify $Q$. How? Use structural restrictions.
Obtaining $P, Q, N$

- Given a solution to $\rho_2 = (P - \rho_1)P$ and $G_{33} \equiv \frac{B}{S} = 2.25$, to match the median net worth to revenues ratio across states in the US in 2006, get $G_{31}, G_{32}, H_{33}$

\[
\begin{bmatrix}
G_{31} + \vartheta_n & G_{32} + \vartheta_w & G_{33}
\end{bmatrix} P + \begin{bmatrix}
0 & 0 & H_{33} + \vartheta_b
\end{bmatrix} = 0_{1.3}
\]

- Given coefficients in budget constraint and reduced form VAR, identify $Q$

\[
\begin{bmatrix}
G_{31} + \vartheta_n & G_{32} + \vartheta_w & G_{33}
\end{bmatrix} Q \begin{bmatrix}
1 & 0 \\
0 & 1 \\
0 & 0
\end{bmatrix} = \begin{bmatrix}
0 & 0
\end{bmatrix}
\]

\[
\left(\begin{bmatrix}
G_{31} + \vartheta_n & G_{32} + \vartheta_w & G_{33}
\end{bmatrix} \rho_1 + \begin{bmatrix}
0 & 0 & H_{33} + \vartheta_b
\end{bmatrix}\right) Q \begin{bmatrix}
1 & 0 \\
0 & 0
\end{bmatrix} = 0
\]

\[QQ' = V\]

- Given $Q$, obtain $N = Q^{-1}(\rho_1 - P)Q$. Check that $N$ satisfies restrictions. If not, pick different solution and repeat all.
Figure: Impulse response to a Discount rate shock $\gamma_t$
Main Quantitative Findings

- **Thought experiment:** in 2006 it is announced that US will not have transfer rule anymore. But state-level shock realizations are the same.

- Employment growth (’06-’10) cross-state standard deviation was 2.6%

- It would have been 3.5% without a federal transfers policy rule

- Similar findings for ’08 and ’09, and in the stationary distribution
Figure: Employment Change from Fiscal Integration by State in 2010

Ordered states by employment in 2010

Employment gain (%)
Sensitivity: Distortionary taxes

- Tax rate per unit of labor income $w_t + n_t$ is $\tau_t = -(1 + \vartheta_n)n_t - (1 + \vartheta_w)w_t$

- Assume only current variables are affected in (Labor Market) equation

$$0 = f_{21} \mathbb{E}_t[n_{t+1}] + f_{22} \mathbb{E}_t[w_{t+1}] + \left( g_{21} + \frac{\bar{\tau} \vartheta_n}{1 - \bar{\tau} \vartheta_n} (1 + \vartheta_n) \right) n_t$$

$$+ \left( g_{22} + \frac{\bar{\tau} \vartheta_n}{1 - \bar{\tau} \vartheta_n} (1 + \vartheta_w) \right) w_t + h_{21} n_{t-1} + h_{22} w_{t-1}$$

$$+ \mathbb{E}_t[z_{t+1}] + l_{23} \mathbb{E}_t[\eta_{t+1}] + m_{22} z_t + m_{23} \eta_t$$

- Set $\tau = 0.17$ to match average tax rate in the US.

- Construct counterfactual where the second line in $\Theta$ incorporates this

- Doesn’t change results much. Intuition: $\vartheta_n \approx \vartheta_w \approx -1$
Application II: Which Micro-Foundations Matter?

- When building models to analyze policy rules, we have to make choices
  - For example, which extensions of a canonical model are promising?

- Also, why is it that papers using seemingly different models obtain similar quantitative answers? What features of such models make it so?

- The Principle of Counterfactual equivalence offers guidance
Unemployment Benefits in Labor Search Models

- Canonical model is Mortensen-Pissarides
- Discrete-time version with Nash Bargaining and free-entry of firms
- Equations characterizing wages $w_t$ and vacancy-unemployment ratio $\vartheta_t$

\[
0 = -\frac{1}{1-\phi} w_t + \frac{\phi}{1-\phi} y_t + \frac{\phi c}{1-\phi} \vartheta_t + z + b_t \quad \text{(Wage Setting (1))}
\]

\[
0 = -\frac{c}{q(\vartheta_t)} + \beta E_t \left[ y_{t+1} - w_{t+1} + \frac{(1-s)c}{q(\vartheta_{t+1})} \right] \quad \text{(Job Creation (1))}
\]

- $y_t$: productivity of the match, $q(\theta_t)$: vacancy-filling probability
- $b_t$: unemployment benefits; $c$: vacancy posting cost
Imagine we want to evaluate policy rule \( b_t = \tilde{b}_t + (\frac{\varnothing}{\varnothing})^\Theta \)

\( \tilde{b}_t \) is an exogenous policy shock

\( \Theta > 0 \) governs how generous benefits are when labor market slackness is above or below its long run level

Suppose we log-linearize, estimate and solve the model for several values of \( \Theta \)

We find that it has negligible effects on the equilibrium.

Is this robust across many models? What extensions are promising for generating non-negligible effects?
The Canonical Model Semi-structure

- The equilibrium equations are consistent with the following semi-structure

\[
F = \begin{bmatrix}
    0 & 0 \\
    f_{21} & f_{22}
\end{bmatrix};
\quad G = \begin{bmatrix}
    g_{11} & g_{12} \\
    0 & g_{22}
\end{bmatrix};
\quad H = \begin{bmatrix}
    0 & 0 \\
    0 & 0
\end{bmatrix};
\quad L = \begin{bmatrix}
    0 & 0 \\
    0 & l_{22}
\end{bmatrix};
\quad M = \begin{bmatrix}
    m_{12} & 0 \\
    0 & 0
\end{bmatrix}
\]

- "Stare" at the exclusion restrictions

- Wage Setting (1) equation is rather restricted compared to Job Creation (1) equation

- Models that change how wages are set seem promising

- Models that don’t significantly change the incentives to create jobs will be counterfactually equivalent to the canonical model
Wage Bargaining Protocols

- Replace Nash Bargaining with an *ad-hoc* wage rule ala Hall (2005)

\[ 0 = -w_t + z_t + \lambda y_t + (1 - \lambda)w_{t-1} \quad \text{(Wage Setting (2))} \]

where \( \lambda \) governs how "sticky" wages are.

- Or, alternating-offer-wage-bargaining protocol in Christiano et.al. (2016)

\[ 0 = -\omega_2 \left( 1 - \beta \left( 1 - s - \vartheta_t q(\vartheta_t) \right) \right) \gamma + (1 + \omega_1)(w_t - z_t) - (\omega_1 + \omega_3)(y_t - z_t) \]
\[ + \beta \left( 1 - s - \vartheta_t q(\vartheta_t) \right) \omega_3 \mathbb{E}_t [y_{t+1} - z_{t+1}] \quad \text{(Wage Setting (3))} \]

where \( \gamma \) is the cost of delay in bargaining

- Semi-structures generated by these models satisfy different exclusion restrictions than the canonical model

- Thus, they belong to different Counterfactually Equivalent Sets with respect to changes in unemployment benefits policy.
Financial Accelerator Model

- Wasmer-Weil (2004): frictional credit market. Firms and creditors search
- Firms have to be matched to a creditor before posting vacancy
- Value of vacancy is $K > 0$

$$0 = -\frac{1}{1-\phi} w_t + \frac{\phi}{1-\phi} y_t + \frac{\phi}{1-\phi} (c + (1 - \beta(1 - q(\varphi_t)))K) \varphi_t + z + b_t$$

(Wage Setting (4))

$$0 = -\frac{1}{q(\varphi_t)} (c + (1 - \beta(1 - q(\varphi_t)))K)$$

$$+ \beta \mathbb{E}_t \left[ y_{t+1} - w_{t+1} + \frac{1-s}{q(\varphi_{t+1})} (c + (1 - \beta(1 - q(\varphi_{t+1})))K) \right]$$

(Job Creation (4))

- Similar to canonical model but with endogenous vacancy-posting cost.
- However, they are counterfactually equivalent because they satisfy identical exclusion restrictions
Endogenous Recruiting Intensity

- Firms choose recruiting intensity $e$. In the spirit of Gavazza et.al. (2016).
- Cost of posting a vacancy is a well-behaved function $c(e, \vartheta)$.
- Probability of filling the vacancy is $q(\vartheta \bar{e})e$, where $\bar{e}$ is the average recruiting intensity in the economy.
- In equilibrium, all firms choose identical intensity as a function of $\vartheta$ alone.

$$0 = -\frac{1}{1-\phi} w_t + \frac{\phi}{1-\phi} y_t + \frac{\phi}{1-\phi} c_e(e(\vartheta_t), \vartheta_t) \vartheta_t + z_t \quad \text{(Wage Setting (5))}$$

$$0 = -\frac{c_e(e(\vartheta_t), \vartheta_t)}{q(\vartheta_t e(\vartheta_t))} + \beta \mathbb{E}_t \left[ y_{t+1} - w_{t+1} + \frac{(1-s)c_e(e(\vartheta_{t+1}), \vartheta_{t+1})}{q(\vartheta_{t+1} e(\vartheta_{t+1}))} \right] \quad \text{(Job Creation (5))}$$

- Again, this model is counterfactually equivalent to the canonical model.
Application III: Falsifying a Set of Models

- So far...study counterfactuals for unobserved policy changes

- But suppose we observe equilibrium before and after a policy change
  - E.g., Catalunya secession?

- Compare Robust Policy Counterfactual with observed equilibrium

- If too far apart $\implies$ reject set of models generated observed equilibrium

- Example: Volcker’s policy change to falsify set of New Keynesian models
Log-likelihood ratio test

- Assume recursive equilibrium has VAR representation

- Let $e_t$ be reduced-form shocks, with theoretical distribution $g(e_t | \xi, \Theta)$

- Suppose we have observations of $x_t$ under policy $\Theta_0$ and $\Theta_1$

- Construct test-statistic comparing predicted and observed $e_t$ under $\Theta_1$
Falsifying a set of models via a log-likelihood ratio test

1. Using the observations under policy $\Theta_0$, obtain maximum-likelihood estimates \{\hat{\rho}_1^0, \hat{\rho}_2^0, \hat{\nu}_0\}.

2. Imposing a set of restrictions $R^*$ on the structure $\xi$, construct a Robust Policy Counterfactual associated with $\Theta_1$. In particular, obtain maximum-likelihood estimates \{\xi^*, \rho_1(\xi^*, \Theta_1), \rho_2(\xi^*, \Theta_1)\} and, thus, a predicted distribution $g(e_t|\xi^*, \Theta_1)$ from the restricted set of models.

3. Using the observations under policy $\Theta_1$, estimate via maximum likelihood the distribution $\hat{g}_1(e_t)$ of the reduced-form shocks corresponding to the unrestricted set of models.

4. Construct the log-likelihood ratio $LR = -2 \sum_{t=1}^{T_1} \log \left( \frac{g(e_t|\xi^*, \Theta_1)}{\hat{g}_1(e_t)} \right)$
Falsifying a set of models via a log-likelihood ratio test

- We wish to test the null hypothesis $H_0 : \xi = \xi^\ast$.

- When $H_0$ is true and $T_1 \to \infty$, $LR$ converges in distribution to $\chi_r^2$ distribution with degrees of freedom $r$ equal to the number of restrictions in $R^\ast$.

- Reject $H_0$ at significance level $\alpha$ whenever $P(\chi_r^2 > LR) \leq \alpha$. 
Falsifying NK Models with Volcker’s Policy Change

- Clarida et.al. (2000) shows how US interest rate policy rule changed following Volcker’s appointment, stabilizing inflation and output

- They estimate before and after interest rate rules. Then, compare equilibria under both rules in a canonical New Keynesian model

- Here, use this policy experiment to falsify a larger set of NK models (including such canonical model)

- **Claim:** These models are inconsistent with the behavior of output, inflation, and interest rates both before and after the policy change
A Canonical New Keynesian Model

- 3 equation model of inflation $\pi_t$, output $y_t$ and nominal interest rate $r_t$ in log-deviations from a zero inflation steady state

$$
0 = \begin{bmatrix}
1 & \frac{1}{\sigma} & 0 \\
0 & -\frac{\delta}{\lambda} & 0 \\
0 & 0 & 0
\end{bmatrix}
\mathbb{E}_t
\begin{bmatrix}
y_{t+1} \\
\pi_{t+1} \\
r_{t+1}
\end{bmatrix}
+ \begin{bmatrix}
-1 & 0 & -\frac{1}{\sigma} \\
-1 & \frac{1}{\lambda} & 0 \\
\gamma & \beta & -\frac{1}{1-\rho}
\end{bmatrix}
\begin{bmatrix}
y_t \\
\pi_t \\
r_t
\end{bmatrix}
+ \begin{bmatrix}
0 & 0 & 0 \\
0 & 0 & \rho \\
0 & 0 & \frac{1-\rho}{1-\rho}
\end{bmatrix}
\begin{bmatrix}
y_{t-1} \\
\pi_{t-1} \\
r_{t-1}
\end{bmatrix}
+ \begin{bmatrix}
0 & 1 & 0 \\
1 & 0 & 0 \\
0 & 0 & 1
\end{bmatrix}
\mathbb{E}_t
\begin{bmatrix}
z_t \\
g_t \\
\epsilon_t
\end{bmatrix}
$$

(Euler) (NKPC) (Policy)

- $z_t, g_t, \epsilon_t$ independent AR(1)s

- $\gamma, \beta, \rho$ are policy parameters
A Set of New Keynesian Models

- Canonical model belongs to a larger set of models described by policy $\Theta$ and semi-structure $\xi^*$

$$
\begin{align*}
\Theta_f &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};
\Theta_c &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \gamma & \beta & -\frac{1}{1-\rho} \end{bmatrix};
\Theta_p &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1-\rho \end{bmatrix}.
\end{align*}
$$

$$
\begin{align*}
F &= \begin{bmatrix} f_{11} & f_{12} & 0 \\ 0 & f_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix};
G &= \begin{bmatrix} g_{11} & 0 & g_{13} \\ g_{21} & g_{22} & 0 \\ 0 & 0 & 0 \end{bmatrix};
H &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};
\end{align*}
$$

$$
\begin{align*}
L &= \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix};
M &= \begin{bmatrix} 0 & 1 & 0 \\ 1 & m_{22} & 0 \\ 0 & 0 & m_{33} \end{bmatrix};
N &= \begin{bmatrix} n_{11} & n_{12} & n_{13} \\ n_{21} & n_{22} & n_{23} \\ 0 & n_{32} & n_{33} \end{bmatrix}.
\end{align*}
$$

- For example, canonical model has $m_{22} = 0$. An extension with $m_{22} \neq 0$:
  - Large family of agents in the households. A fraction of the agents $g_t$ are non-employed. They consume the transfers from the family alone.
  - Then, $g_t$ generates variation in MRS between consumption and labor
Volcker's policy change falsifies NK models

- Quarterly data on real GDP, inflation and nominal interest rates
- Two periods: 1960:1 to 1979:2 (i.e., pre-Volcker) and 1982:4 to 1996:4 (i.e., pos-Volcker)
- Policy parameters before and after Volcker come from Clarida et. al.
- Assume that the structural shocks are normally distributed $\Rightarrow$
  log-LR test compares $V^1$ in the pos-Volcker period to predicted $V(\xi^*, \Theta^1)$
- Log-LR=94 $\Rightarrow$ rejecting $H_0$ at all standard levels of significance.
- Note: $\chi^2$ tests are known for "not rejecting enough" (i.e., low power)
Comparison to related ideas

- Policy shocks in SVARs (e.g., Sims’s "Macroeconomics and Reality")

- "Sufficient statistics" (e.g., ACR (2012); Alvarez, Bihan, and Lippi (2014); Auclert (2014))

- Business cycles accounting (Chari, Kehoe, McGrattan (2007))

- Perturbation to SVARs and DSGEs (Del Negro and Schorfheide (2009))
Conclusions

- A new insight about linear dynamic stochastic models
- Allows us to identify sets of counterfactually equivalent models
  1. Quantitative policy counterfactuals that are robust across models
  2. Guiding researchers when building models to study policy changes
  3. Falsifying a set of models using policy experiments
- Limitations: (a) linearity (?) and (b) welfare (?)
- Other applications:
  - Agents within a model that are uncertain about "true" model of the economy
Appendix
Wage Data

- Use data from the 2000 Census and 01-12 American Community Surveys.
- Restrict sample to males, age 21-55, currently employed, working at least 30 hours per week, worked 48 weeks in prior year
- Measure hourly wage as earnings divided by hours
- Earnings: both wage and salary, and business
- Hours: (Total weeks)\times(Usual hours worked per week reported)
- To adjust wages, I regress log wage rate on age dummies (6 groups), education dummies (5 groups), citizenship dummies (3 groups), a black dummy, and usual hours worked dummies (4 groups).
  - Do this separately for each year.
  - Take residuals from regression add constant back and average by state.
Employment Data

- BLS reports annual employment and population counts per state.

- The employment rate is measured as the employment to population ratio.
Transfers Data

- Federal transfers include: retirement, disability and medical benefits; income maintenance benefits; unemployment insurance compensation; veteran benefits; federal education and training assistance.

- Federal taxes include: withheld personal income; estimated payments; final settlements.

- Federal taxes exclude: Excise, Medicare and Social security.
Assets construction

- Asset accumulation equation:

\[ Assets_t = Assets_{t-1}(1 + R_t) + NGDP_t - Consumption_t + NetTransfers_t \]

- \( R_t \) chosen to match aggregate Household Net Worth yearly growth.

- Assets by state in 2006 from Mian, Rao and Sufi (2013)

- NGDP, Consumption and Transfers by state from BEA.
Sensitivity: Distorsionary taxes

- Tax rate per unit of labor income $w_t + n_t$ is $\tau_t = -(1 + \vartheta_n)n_t - (1 + \vartheta_w)w_t$

- Assume only current variables are affected in (Labor Market) equation

$$0 = f_{21}E_t[n_{t+1}] + f_{22}E_t[w_{t+1}] + \left( g_{21} + \frac{\bar{\tau}\vartheta_n}{1 - \bar{\tau}\vartheta_n}(1 + \vartheta_n) \right) n_t$$

$$+ \left( g_{22} + \frac{\bar{\tau}\vartheta_n}{1 - \bar{\tau}\vartheta_n}(1 + \vartheta_w) \right) w_t + h_{21}n_{t-1} + h_{22}w_{t-1}$$

$$+ E_t[z_{t+1}] + l_{23}E_t[\eta_{t+1}] + m_{22}z_t + m_{23}\eta_t$$

- Set $\tau = 0.17$ to match average tax rate in the US.

- Construct counterfactual where the second line in $\Theta$ incorporates this

- Doesn’t change results much. Intuition: $\vartheta_n \approx \vartheta_w \approx -1$