From Hyperinflation to Stable Prices: Argentina’s evidence on menu cost models

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Overview

▶ Assess several *key robust* predictions of class of menu cost models.

▶ Model makes sharp predictions about differential response to changes in the average rate of inflation at $\pi = 0$ and at $\pi = \infty$.

▶ Evaluate predictions using variation of inflation in Argentina that includes periods of extremely low and high inflation.
  
  ▶ Predictions of the menu cost model refer to:
    
    (1) Frequency of price changes
    
    (2) Dispersion of frequencies of price changes across products
    
    (3) Intensive and extensive price increases/decreases.
    
    (4) Relative price dispersion.
Outline for the talk

1. Briefly review the theoretical model
2. State the key predictions of the model
3. Describe the data
4. Describe the empirical results
Set up of Menu Cost Model

- $F(p - \omega, z)$ per period real profit of monopolistic competitive firm,
  - $p$ log nominal price,
  - $\omega$ log nominal wages: $d\omega = \pi dt$ so that $\pi$ trend of nominal costs—i.e. inflation rate,
  - $z$ shock to profits: $E[dz] = a(z)dt$, and $E[dz^2] = \sigma^2 b(z)^2 dt$.
- $r$ real discount rate,
- $C = \zeta(z)$ fixed cost of changing prices.
Menu Cost Model, Firm’s problem

- Optimal price setting behavior of firms subject to idiosyncratic shocks that affect its profits in an economy with deterministic inflation.

- Choose stopping times and size of price changes $\{\tau_i, \Delta p(\tau_i)\}$

$$V(p - \omega, z) = \max_{\{\tau_i, \Delta p_i\}} \mathbb{E} \left[ \int_0^\infty e^{-rt} F( p(t) - \omega - \pi t, z(t)) \, dt \right.$$ 

$$- \sum_{i=0}^\infty e^{-r\tau_i} \zeta(z(t)) \mid z(0) = z \right]$$

s.t. $p(t) = p + \sum_{i=0}^{\tau_i < t} \Delta p(\tau_i)$ for all $t \geq 0$ and

$$dz(t) = a(z(t)) \, dt + \sigma b(z(t)) \, dW(t) \quad \text{where} \quad z(0) = z.$$
Menu cost model: optimal price policy

- Firm (static) problem with flexible prices
  \[ p_j^*(t) = \arg \max_p F(p - \omega - \pi t, z) \]

- Price gaps \( g_j(t) \equiv p_j(t) - p_j^*(t) \) or markup deviations

- Optimal sS rule described by 3 numbers \( g < g^* < \bar{g} \):
  - If \( g \leq g \) pay fixed cost & increase price so that \( g \to g^* \), thus \( \Delta_p^+ = g^* - g \).
  - If \( g \geq g \) pay fixed cost & decrease price so that \( g \to g^* \), thus \( \Delta_p^- = g - g^* \).

- Aggregating decision across idiosyncratic shocks \( \implies \) invariant distribution:
  - Standard deviation of relative prices \( \bar{\sigma}(\pi, \sigma^2) \)
  - Expected number of adjustment per unit of time \( \lambda_a(\pi, \sigma^2) \)
Low inflation case, no first order effect of inflation

- If the profit function and shocks $z$ are symmetric:
  - the frequency of price adjustments is symmetric around $\pi = 0$.
  - Intuition: frequency of price changes for 1% inflation vs. 1% deflation.

- If $\lambda_a$ and $\bar{\sigma}$ are differentiable w.r.t. $\pi$, at $\pi = 0$, they react to $\pi = 0$ as
  \[
  \frac{\partial \lambda_a}{\partial \pi} = 0 \quad \text{and} \quad \frac{\partial \bar{\sigma}}{\partial \pi} = 0
  \]
  \[
  \frac{\partial \lambda_a^+}{\partial \pi} = -\frac{\partial \lambda_a^-}{\partial \pi} \quad \text{and} \quad \frac{\partial \Delta_a^+}{\partial \pi} = -\frac{\partial \Delta_a^-}{\partial \pi}.
  \]

- Symmetry also implies that the levels at $\pi = 0$:
  \[
  \lambda_a^+ = \lambda_a^- \quad \text{and} \quad \Delta_a^+ = \Delta_a^-
  \]
Low Inflation Case, comments

– Intuition: when $\pi = 0$ and $\sigma > 0$ price changes stem from idiosyncratic shocks so a change in inflation has no first order effects.

– Assumptions

  ▶ Approximate symmetry is consequence of low adjustment cost $C$.

  ▶ If cost $C$ small, then $p$ is close to $p^*(z)$.

  ▶ 2nd order approx. $F(\cdot, z)$ around $p^*(z)$.

– Example: quadratic profit function and $dz = -a z \, dt + \sigma \, b \, dW$
Elasticities for high inflation

- We want to characterize elasticities for high $\pi$ keeping $\sigma^2$ constant.

- First to consider $\sigma^2 = 0$ and $\pi > 0$: Sheshinski-Weiss.
  - $sS$: when relative price hits $s < p^*$ adjust to $S > p^*$.
  - Time between adjustments $(S - s)/\pi = 1/\lambda_a$.
  - Log of relative prices, uniform on $S - s$, so $\bar{\sigma} = (S - s)/\sqrt{12}$
  - Either as approximation for small fixec cost

  \[
  \lim_{C \downarrow 0} \frac{\pi}{\lambda_a} \frac{\partial \lambda_a}{\partial \pi} = \frac{2}{3} \text{ and } \lim_{C \downarrow 0} \frac{\pi}{\bar{\sigma}} \frac{\partial \bar{\sigma}}{\partial \pi} = \frac{1}{3} .
  \]

- or exactly for all cost $C$ if $F(\cdot)$ is quadratic around $p^*$ and $r = 0$: 
High Inflation Case, comments

Does $\sigma = 0, \pi > 0$ have the same elasticity as $\sigma > 0, \pi = \infty$?

Yes, only ratio $\pi / \sigma^2$ matters!

- Multiply $r$, drift $a(\cdot)$, $\sigma^2, \pi$ by $k > 0$ change units of time.
- Thus $\lambda_a$ is multiplied by $k$, i.e. measured time in months vs years.
- Set $r = 0$ : maximizes expected average profits, and
- Set $a(\cdot) = 0$, so shocks are permanent,
- Thus $\lambda_a(\pi, \sigma^2)$ is homogenous of degree one in $(\pi, \sigma^2)$

$$\lim_{\sigma^2 > 0, \pi \to \infty} \frac{\pi}{\lambda_a(\pi, \sigma^2)} \frac{\partial \lambda_a(\pi, \sigma^2)}{\partial \pi} = \lim_{\pi > 0, \sigma^2 \to 0} \frac{\pi}{\lambda_a(\pi, \sigma^2)} \frac{\partial \lambda_a(\pi, \sigma^2)}{\partial \pi}$$

- Intuition: As $\pi \to \infty$ effect of $\sigma$ negligible
Dispersion of the Frequency of Price Changes ($\lambda$) and Inflation ($\pi$)

- As inflation becomes large the dispersion of $\lambda$s across goods falls.

- Interpretation: the common aggregate shock swamps heterogeneity across goods.
  - As $\pi/\sigma^2$ becomes large the model with idiosyncratic converges to Sheshinski-Weiss

- Two models that differ on volatility $\sigma^2_1 > \sigma^2_1$.
  - At $\pi = 0$ then $\lambda_a (\pi, \sigma^2_1) > \lambda_a (\pi, \sigma^2_2)$
  - On the other extreme for very large $\pi \to \infty$:

$$\lim_{\pi \to \infty} \frac{\lambda_a (\pi, \sigma^2_1)}{\lambda_a (\pi, \sigma^2_2)} \to 1$$
Decomposition of Changes on Inflation

- General decomposition:

\[ \pi = \Delta^+ \lambda_a^+ - \Delta^+ \lambda_a^- \]

- Low Inflation, i.e. as \( \pi \rightarrow 0 \), differentiating w.r.t. \( \pi \):

\[ 1 = \left( \frac{\partial \Delta^+_p}{\partial \pi} \lambda_a^+ - \frac{\partial \Delta^-_p}{\partial \pi} \lambda_a^- \right) \left( \frac{1}{10} \right) + \left( \frac{\partial \lambda_a^+}{\partial \pi} \Delta^+_p - \frac{\partial \lambda_a^-}{\partial \pi} \Delta^-_p \right) \left( \frac{9}{10} \right) \]

Change in inflation due to size \quad \text{Change in inflation due to frequency}

- High Inflation, i.e. as \( \pi \rightarrow \infty \) (using \( \lambda_a^- = 0 \)) elasticities w.r.t. \( \log \pi \):

\[ 1 = \left( \frac{\partial \log \Delta^+_p}{\partial \log \pi} \right) \left( \frac{1}{3} \right) + \left( \frac{\partial \log \lambda_a^+}{\partial \log \pi} \right) \left( \frac{2}{3} \right) \]

Elasticity inflation due to size \quad \text{Elasticity inflation due to frequency}
Underlying raw data for the Argentine CPI

- December 1988-September 1997
- 8,618,345 price quotes for *items*.
- *item* : good/service of a determined brand sold in a specific outlet in a specific period of time.
- Goods/services are divided into two groups:
  - *Homogeneous*: barley bread, chicken, lettuce, etc.
  - *Differentiated*: moccasin shoes, utilities, tourism, and professional services.
- 302 of prices collected every month (56% exp.)
- 233 of prices collected every two weeks (44% exp.)
- On average across the 9 years there are 166 outlets per good
Inflation and the Frequency of Price Adjustment

- Estimator of the frequency of price changes
  \[ \lambda_t = -\ln (1 - \text{Fraction outlets change price between } t \text{ and } t - 1) \]

- Results
  - The derivative of the frequency of price changes with respect to inflation are very for low inflation rates (but derivative of difference of increases minus decreases is large)
  - The elasticity of the frequency of price changes with respect to inflation are between \( \left[ \frac{1}{2}, \frac{2}{3} \right] \) for high inflation rates.

- Relation to existing literature.
  - The level of the estimated frequency of price changes and its relation to inflation are consistent with other studies
  - Due to the range of inflation in our sample we span and extend existing international evidence.
Figure: Estimated Frequency of Price Changes $\lambda$ and Expected Inflation

Simple estimator $\hat{\lambda}_t = -\log(1 - f_t)$, where $f_t$ fraction of outlets that changed price in period $t$. 
Figure: Frequency of price changes $\lambda$ and the inflation rate (pooled simple estimator)

% change in $\lambda$ of increasing $\pi$ from 0 to 1% = 0.04
Elasticity for high inflation = 0.53

Fitted line:

$$\log \lambda = a + \epsilon \min \{\pi - \pi^c, 0\} + \nu(\min \{\pi - \pi^c, 0\})^2 + \eta \max \{\log \pi - \log \pi^c, 0\}$$
we plot $-\log(1 - f)$, $f =$ reported frequency of price changes in each study.
Inflation vs Extensive/Intensive Margins of Price Increases/Decreases

- **Extensive Margin**
  - Fraction of Price Increases and Decreases is similar for low inflation rates
  - Fraction of Price Increases converges to $\lambda$ for high inflation rates
  - Fraction of Price Decreases converges to 0 for high inflation rates

- **Intensive Margin**
  - Magnitude of Price Increases and Decreases is similar for low inflation rates
  - Magnitude of Price Increases is increasing in $\pi$ for high inflation rates
  - Magnitude of Price Decreases is (weakly) increasing in $\pi$ for high infl. rates
Figure: Frequency vs Inflation and Difference in frequencies vs inflation
Figure: Extensive Margin of Price Changes, frequency of price increases and decreases

Homogeneous goods

Absolute Value of Annual Inflation Rate (log points)

Monthly frequency of price changes

- 2 years
- 1 year
- 6 months
- 1 month
- 1 week

- total
- increases
- decreases

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Figure: Intensive Margin of Price Changes, avg. price increases and decreases
Dispersion across Frequency of Price Changes ($\lambda$’s) decreases with Inflation ($\pi$), asymptotic result on ratio

Figure: Estimates of $\lambda$ by product. Homogeneous goods sampled twice a month

Frequency of Price Changes and Expected Inflation at a 5-digit disaggregation level

$1200 \times |\log P(t) - \log P(t-1)|$ abs. value of c.c. % annual inflation rate

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Relative Price Dispersion and Inflation

Comparative static results of model reviewed above imply:

- Zero elasticity to inflation on dispersion at low inflation.
- 1/3 elasticity of dispersion to inflation at high inflation (lower elasticity if shocks are persistent)

Interpretation: model applies for pricing across outlets.

Measure the dispersion of relative prices through the residual variance in a regression of prices at each time, store and good on a rich set of fixed effects.


## Regressions

$$\Rightarrow$$

Residual variance of price levels per period

<table>
<thead>
<tr>
<th>Models $i$: indicate dummies</th>
<th># of dummies</th>
<th>Adj-$R^2$</th>
<th>Elast at $\pi = 100%$</th>
<th>Elast at $\pi = 500%$</th>
<th>Elast at $\pi = 700%$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1: time</td>
<td>212</td>
<td>0.751</td>
<td>0.03</td>
<td>0.21</td>
<td>0.31</td>
</tr>
<tr>
<td>2: time + good + store</td>
<td>4,978</td>
<td>0.982</td>
<td>0.06</td>
<td>0.26</td>
<td>0.34</td>
</tr>
<tr>
<td>3: time + good × store</td>
<td>74,755</td>
<td>0.987</td>
<td>0.14</td>
<td>0.35</td>
<td>0.37</td>
</tr>
<tr>
<td>4: time + good × store × non-subsspell</td>
<td>153,896</td>
<td>0.989</td>
<td>0.16</td>
<td>0.37</td>
<td>0.38</td>
</tr>
<tr>
<td>5: time × store + time × good + good × store × non-subsspell</td>
<td>464,505</td>
<td>0.996</td>
<td>0.13</td>
<td>0.30</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Price observations in each regression 5,497,452 for 233 outlets w/prices collected twice a month, 212 periods.

Cost of inflation due to price dispersion $\approx$

Increase in variance $\times$ elasticity of substitution / 2
Figure: Average Dispersion of Relative Prices and Inflation
Sensitivity Analysis (Skip)

- Missing data, substitutions and sales.
- Different aggregation methods.
- Contemporaneous versus expected inflation.
- Estimator (missing price changes at high inflation).
- Dynamics of Disinflation during convertibility
- Time aggregation at high inflation
Conclusions

- Empirical analysis of the effect of Inflation on price dynamics guided by the menu cost model of price setting.

- Unique data set that spans periods of sustained inflation rates ranging from 0 to over 5000% per year.

- Several key prediction of the model are consistent with the Argentine data.
Figure: Symmetry assumption on profit Function $F(p - \omega, z)$
No 1st order effect of inflation w/symmetry at $\pi = 0$

Let $Z = [-\bar{z}, \bar{z}]$, define $p^*(z) = \arg \max_x F(x, z)$ & normalize $p^*(0) = 0$.

Assume that $F(\cdot)$ and $a(\cdot), b(\cdot)$ are symmetric:

- $a(z) = -a(-z) \leq 0$ and $b(z) = b(-z) > 0$ for all $z \in [0, \bar{z}]$
- $p^*(z) = -p^*(-z) \geq 0$ for all $z \in [0, \bar{z}]$
- $F(\hat{p} + p^*(z), z) = F(-\hat{p} + p^*(-z), -z) + f(z)$ for all $z \in [0, \bar{z}]$ and all $\hat{p}$.

Then if $\lambda_a$ and $\bar{\sigma}$ are differentiable w.r.t. $\pi$:

$$\frac{\partial \lambda_a}{\partial \pi} = 0, \quad \frac{\partial \bar{\sigma}}{\partial \pi} = 0$$
and

$$\frac{\partial \mathbb{E}[V]}{\partial \pi} = 0$$

at $\pi = 0$.

Example: positive coefficients $a_0, b_0, d_0, c_0, f_0$:

$a(z) = -a_0 z$, $b(z) = b_0$, $F(p, z) = d_0 - c_0 (p - z)^2 - f_0 z$ so $p^*(z) = z$