Corrigendum for Proposition 3 of Pathak and Sönmez (2013)

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August 2020

We are grateful to Somouaogo Bonkoungou and Alexander Nesterov (Bonkoungou and Nesterov, 2020) for pointing out an error in the proof of Proposition 3 of Pathak and Sönmez (2013). Proposition 3 states:

**Proposition 3:** Suppose there are at least \( k \) schools, where \( k > 1 \). Then \( FPF^k \) is more manipulable than \( GS^k \).

The statement is only true when \( k = 2 \). As shown by Bonkoungou and Nesterov (2020), for \( k > 2 \), our proof did not consider the case where some student can block \( FPF^k(P,\succ) \) with an equal preference school.

Proposition 3 is the only result affected by our error. In the special case where all schools have a First-Preferences-First admissions rule, the FPF mechanism reduces to the Boston mechanism. In Pathak and Sönmez (2013), we use this fact to derive Proposition 1 and Corollary 2 as a consequence of Proposition 3. We restate these results:

**Proposition 1:** Suppose there are at least \( k \) schools and let \( k > 1 \). The old Chicago mechanism \((Chi^k)\) is more manipulable than truncated serial-dictatorship \((Sd^k)\) CPS adopted in 2009.

**Corollary 2:** Suppose there are at least \( k \) schools where \( k > 1 \). Then \( \beta^k \) is more manipulable than \( GS^k \).

Even though these results were derived as a consequence of Proposition 3, they are unaffected by our error. In fact, Proposition 1 can also be derived from Theorem 1 of Pathak and Sönmez (2013). Theorem 1 states:

**Theorem 1:** Suppose each student has a complete rank ordering and \( k > 1 \). The old CPS mechanism \((Chi^k)\) is at least as manipulable as any weakly stable mechanism.
Since the truncated serial dictatorship is weakly stable, this result can be used to establish Proposition 1.

Earlier versions of Pathak and Sönmez (2013) provided a direct proof of Corollary 2. The 2008 version of our paper which also had a direct and relatively simple proof of Corollary 2 and pre-dates our knowledge of the English reforms. That paper is linked here: http://web.stanford.edu/group/SITE/archive/SITE_2008/segment_2/papers/pathak_compare.pdf. This argument also directly implies Proposition 1 of Pathak and Sönmez (2013), since the Chicago mechanism is a special case of the Boston mechanism where each school uses the same test-score tie-breaker. Hence, the only result affected by our error is Proposition 3.

Our error in Proposition 3 affects some of the text describing Table 1, which includes 55 instances in England where an LEA moved from a truncated version of the FPF mechanism to a truncated version of Gale Shapley as a result of the 2007 English ban of First Preference First Admissions.

In constructing that table, we defaulted to describing an LEA’s assignment mechanism as an FPF mechanism when we were not able to independently verify that all schools used FPF admissions criteria. When all schools use FPF admissions criteria, the assignment mechanism simplifies to the Boston mechanism. Table 1 lists 5 LEAs where the system moved from Boston to Gale-Shapley, which is unaffected by our error.

We have since corresponded with John Coldron, who conducted a survey of English municipalities at that time period, and he generously shared the micro data from Coldron (2006). This data has more information than we previously had access to and includes school-level admissions rules, which allow us to more comprehensively tabulate which LEAs had used the Boston mechanism. This tabulation shows that Corollary 2 applies in many more cases than we knew about when constructing Table 1. According to this data, a total of 17 LEAs moved from truncated version of Boston to truncated versions of Gale-Shapley as a result of the 2007 Ban.

REFERENCES

Coldron, John. 2006. “Survey of Local Education Authorities with First Preferences or Equal Preferences.” Sheffield UK: Sheffield Hallam University, Centre for Education Research.