1 Introduction

Empirical models have been used to study a wide range of issues in matching markets. These include measuring socio-economic heterogeneity in preferences for schools (Hastings et al., 2009), comparing school assignment mechanisms (Abdulkadiroglu et al., 2017; Agarwal and Somaini, 2018; Calsamiglia et al., 2020), understanding preferences in the marriage market (Chiappori et al., 2012), measuring the effects of market power in the medical residence match (Agarwal, 2015), and documenting discrimination against Medicaid patients in nursing facilities (Gandhi, 2020). The empirical approach taken by these papers is based on first estimating the preferences of the agents in these markets and then using those estimates to make economic conclusions. This chapter provides a unified framework for analyzing agents’ preferences in empirical matching models with non-transferable utility. Our objective to provide a roadmap of the existing literature and highlight avenues for future research.

Unlike the textbook model of consumer choice, agents in a matching market cannot choose with whom to match at posted prices. As examples, marriages are formed by mutual consent; schools do not admit all students willing to pay their tuition; public housing is allocated via waiting lists that use priorities; and employers and job applicants must compete with others in a labor market. Moreover, agents on both sides of the market have preferences over who they transact with, whereas firms rarely care about the identity of a consumer willing to meet their ask. Importantly, prices do not serve as the sole rationing device that equilibrates demand and supply.

Models of two-sided matching with non-transferable utility present a framework for analyzing these markets. They have three main features. First, the market has two sides – workers and firms, or schools and students – with agents on at least one side having preferences over agents on the other. Second, agents form partnerships by mutual consent. And third, the terms of matching are fixed in advance. That is, either an agent on one side of the market cannot...
compensate an agent on the other side to enter into a partnership or such compensation, if any, is determined exogenously.

Estimating preferences in these markets depends crucially on the data that is available and institutional details about how the market is organized. We will consider two common data environments with the goal of identifying and estimating a random utility model further described in Section 2. In the first environment, the researcher only has access to data on realized matches. The second occurs when the researcher has data from a centralized assignment mechanism that elicits preferences in the form of rank-ordered lists of options. The appropriate approach to learning about the preferences of agents differs in these two environments, and the discussion in this chapter is split along these lines.

Learning about preferences when only data on final matches is available requires assuming a notion of equilibrium that results in these matches. The predominant concept used for this purpose is known as pairwise stability (see Roth and Sotomayor, 1992). If a match is stable, then one cannot find a pair of agents, one from each side, that would strictly prefer to match with each other relative to their current partners. This concept is motivated by the idea that agents would have an incentive to break a match that is not stable.

The challenge with using stability is that preferences on either side of the market could determine the final matches. Nonetheless, it has been shown in numerous contexts that this equilibrium concept is useful in learning about preferences. The sorting patterns between the observed characteristics of matched partners contains substantial information about agents’ underlying preferences on both sides of the market. In addition, stability conditions also imply certain useful revealed preference relations. These ideas form the basis for identifying agents’ preferences. Our goal in Section 3 is to outline the various approaches that have been proposed in the literature and provide references for further reading.

In some cases, it is also possible to obtain more direct information on preferences when a centralized assignment mechanism is used to determine allocations. These mechanisms have had a big impact in numerous real-world contexts (Roth, 2018), including in school districts and residency programs around the world. These mechanisms require agents to indicate their preferences over being matched with agents on the other side of the market, which are then used as inputs an algorithm that determines a final assignment. A researcher who knows the algorithm and has access to data on reported preferences has strictly more information than one with access only to data on realized allocations.

A recent literature has developed methods for utilizing these data reported preferences for a wide range of mechanisms. This exercise is straightforward in strategy-proof mechanisms that make it incentive compatible for participants to report their true ordinal preferences, assuming that they follow this recommendation. In other cases, the right approach depends on the properties of the mechanism of interest and what behavioral assumptions are appropriate for the empirical application. Section 4 provides a brief overview of these approaches.
A theme in this chapter is that the flexibility of the preference model that can be identified depends crucially on the available data. As we will discuss below, data only on final matches will typically require stronger independence assumptions than will data on reported preferences. Similarly, our ability to estimate flexible models of preferences in a one-to-one matching market will be more limited than in a many-to-one market, and estimating preferences of agents on both sides of the market will be more demanding than only for one side. Our hope is to provide a guide to these trade-offs on the current research frontier in a unified framework.

This chapter complements the chapter on estimating models with transferable utility in this handbook [editor note: please add chapter number]. In such models, agents can compensate partners for forming matches. In such settings, the set of transfers that supports the match allocation is also part of the equilibrium description, which imposes a different set of implications on the data compared to the models with non-transferable utility discussed in this chapter. We also do not discuss one-sided matching problems, including the roommate problem and matching pairs of patients waiting to receive organ donations and their living but incompatible donors.

Nonetheless, the methods discussed in this chapter bear resemblance to other extensions that we will briefly touch upon. These extensions include dynamic matching markets, matching markets with externalities, and matching models with peer effects. We briefly discuss these extensions in Section 5. Section 5 also provides a taste of the types of analyses that have been conducted using the methods discussed in this chapter.

2 Model

We consider a two-sided matching market in which agents on one side, indexed by $i \in I = \{1, \ldots, I\}$, are assigned to agents on the other side, indexed by $j \in J = \{1, \ldots, J\}$. Agents $i \in I$ can be matched with at most one agent on the other side, but agents $j \in J$ may be matched with $q_j \geq 1$ agents on the other side. Agents on both sides may also remain unmatched.

2.1 Preferences

Agents on each side of the market has preferences for being matched with members of the other, which we will model using a random utility model. Specifically, $i$’s utility if matched with $j$ is denoted $u_{ij}$, and the utility of $j$ from being matched with $i$ will be denoted by $v_{ji}$. These utilities cannot be changed by the agents in the market, since we are interested
in non-transferable utility models. Moreover, the formulation implicitly assumes that the utility an agent receives from a given match does not depend on the other matches in the economy. In particular, in the many-to-one context, the utility \( j \) receives from being matched with \( i \) does not depend on the other agents \( j \) is matched with.

Our goal will be to identify and estimate the joint distributions of the vectors of random utilities \( \mathbf{u}_i = (u_{i1}, \ldots, u_{ij}) \) and \( \mathbf{v}_i = (v_{1i}, \ldots, v_{ji}) \) conditional on observable characteristics. Let \( u_{i0} \) and \( v_{j0} \) denote the utility of remaining unmatched.

We start by considering one side of the market. The most general form for the utility of \( i \) being matched with \( j \) that we will employ is given by

\[
u_{ij} = u(\mathbf{x}_j, \mathbf{z}_i, \xi_j, \epsilon_i) - d_{ij}, \tag{1}\]

where \( \mathbf{z}_i \) and \( \mathbf{x}_j \) are vectors of observed characteristics for \( i \) and \( j \), respectively, and \( d_{ij} \) is a scalar observable that potentially varies with both \( i \) and \( j \). The term \( \epsilon_i \) captures unobserved determinants of agent \( i \)'s preferences. It may be multi-dimensional and include \( j \)-specific taste shocks. The term \( \xi_j \) includes unobserved characteristics of \( j \). The term \( d_{ij} \) is an observable, match-specific characteristic (e.g. a measure of distance between \( i \) and \( j \)) that we will use as a numeraire, and represents a metric for utility.

A random utility model requires scale and location normalizations because choices (under uncertainty) are invariant to a positive, affine tranformation of utilities. Accordingly, we will normalize the value of the outside option to zero, i.e. \( u_{i0} = 0 \). Observe that the unit coefficient on \( d_{ij} \) represents a scale normalization.

Identification and estimation of this model are usually studied in an environment in which the random variables for each \( i \) and each \( j \) are drawn iid from some population distribution. Moreover, we will typically assume the following conditional independence condition of the form:

\[
\epsilon_i \perp d_{ij} \mid \mathbf{z}_i, \mathbf{x}_j, (\xi_j)_{j=1}^J; \tag{2}\]

1Theoretical models of non-transferable utility are also closely related to settings that involve matching with contracts (see Hatfield and Milgrom, 2005; Hatfield et al., 2013). We are not aware of empirical work that directly works with such models. The most closely related work is on models with imperfectly transferable utility, by Galichon et al. (2019).

2This assumption is sometimes referred to as "responsive preferences" (see Roth and Sotomayor, 1992, Chapter 5).

3We refer the reader to Matzkin (2007) for the formal definition of identification that we employ in this chapter.

4Recent results in Allen and Rehbeck (2017) suggest that it may be possible to generalize this specification to allow utility to be non-linear but still separable in \( d_{ij} \).

5The specification above also assumes that all agents dislike increases in \( d_{ij} \). This restriction is not essential in many cases discussed below, and the sign of this coefficient can be estimated.
where \( \mathbf{d}_i = (d_{i1}, \ldots, d_{iJ}) \) and \( \mathbf{x} = (x_1, \ldots, x_J) \). The independence condition (2) assumes that agent \( i \)'s unobserved taste shocks are conditionally independent of the vector of numeraire match-specific characteristics \( \mathbf{d}_i \) given the other observed characteristics of \( i \), \( \mathbf{z}_i \), and the observed vector of observed characteristics for agents on the other side of the market characteristics of market configuration \( \mathbf{x} \). The assumption must be evaluated within each empirical application, and it is typically reasonable if \( \mathbf{x} \) is a sufficiently rich control.\(^6\)

**Example.** (School Choice) In this canonical example, let \( i \) denote a student and \( j \) denote a school. The term \( u_{ij} \) is the utility of student \( i \) from being matched to school \( j \). If we let \( d_{ij} \) be the distance from \( i \)'s resident to \( j \)'s location (as in Abdulkadiroglu et al., 2017, for example), then students’ preferences can be summarized in terms of their "willingness to travel." In this example, the conditional independence assumption requires that the distance to school is independent of other unobserved determinants of preferences for schools. This assumption may be a good approximation if \( \mathbf{z}_i \) includes sufficiently rich data about a student’s achievement, demographics and socio-economic characteristics. Relaxing this assumption would be likely to require a model of residential choice and sorting based on unobserved factors that influence preferences for schools.

While the analysis of identification can often allow for general functional forms, the empirical methods below will typically use additional parametric assumptions to ease the computational burden and to get statistically precise estimates with finite sample sizes. The most convenient functional forms depend on available data and the mechanism or setting being analyzed.

A commonly used parametric form encompassed by the model above assumes that

\[
\begin{align*}
u_{ij} &= \mathbf{x}_j'\beta + \mathbf{x}_j'\bar{\gamma}\mathbf{z}_i + \xi_j + \mathbf{x}_j'\gamma_i + \varepsilon_{ij} - d_{ij},
\end{align*}
\]

where \( \gamma_i \) and \( \varepsilon_{ij} \) are mean-zero, normally distributed random variables with variances to be estimated, and \( \bar{\gamma} \) is a matrix conformable with \( \mathbf{x}_j' \) and \( \mathbf{z}_i \). We denote by \( \theta \) the vector of unknown parameters of the model, namely \( (\beta, \bar{\gamma}, \xi_1, \ldots, \xi_J) \) and the parameters governing the distribution of \( \varepsilon_{ij} \) and \( \gamma_i \). These functional form assumptions may be varied depending on the application.

This formulation is both tractable and flexible. It can capture many determinants of preferences and can be used to measure various aspects of preferences. In particular, the term \( \mathbf{x}_j'\beta \) captures a vertical index of quality that is valued equally by every agent \( i \). The second term, \( \mathbf{x}_j'\bar{\gamma}\mathbf{z}_i \), allows for heterogeneous preferences based on observables. The remaining terms include a vertical unobserved component \( \xi_j \) that is equally valued by every agent \( i \in \mathcal{I} \); a term \( \mathbf{x}_j'\gamma_i \) to allow for heterogeneous values for each component of \( \mathbf{x}_j \); and a term \( \varepsilon_{ij} \) that introduces horizontal differentiation among all options in \( \mathcal{J} \).

\(^6\)Relaxing this assumption is a fruitful avenue for future research. It likely requires augmenting the model to incorporate other sources of exogenous variation and specifying how it affects the data generating process.
In some applications, the preferences on the other side of the market $v_{ji}$ may be known from administrative data or institutional knowledge. For example, many schools and colleges use exam scores to rank students (e.g. Fack et al., 2019; Akyol and Krishna, 2017), while other school districts use different but still well-defined priorities. In these cases, $v_{ji}$ does not need to be estimated.

When $v_{ji}$ is unknown, one can specify an analogous model for the preferences of agents on the other side of the market. Specifically, the utility of agent $j \in J$ for matching with agent $i \in I$ is given by

$$v_{ji} = v(x_j, z_i, \eta_i) - w_{ji},$$

where $\eta_i$ is unobserved and $w_{ji}$ has an interpretation analogous to $d_{ij}$. In this case, we would also normalize $v_{i0}$ to zero and strengthen the independence assumption 2 to

$$(\eta_i, \epsilon_i) \perp (d_i, w_i)|z_i, x, (\xi_j)_{j=1}^J,$$

where $w_i = (w_{i1}, \ldots, w_{ji})$.

A few assumptions in the preference model are worth pointing out. First, there are no externalities. An agent’s utility depends on only her own matches. This rules out preferences for attending school with specific peers or working with specific colleagues, for example. The current approach for capturing such preferences is to include characteristics of the student body from prior years, but capturing such peer effects is an important avenue for future research; see Section 5 for a more detailed discussion.\footnote{Epple et al. (2018) and Allende (2019) are two notable exceptions.} Second, the model abstracts away from costs of acquiring information about the other side of the market by assuming that preferences are well-formed. An exception is Narita (2018), which considers the possibility that preferences evolve after agents receive an initial assignment.

Third, the model assumes that unobservable characteristics are independent of observables. This assumption may be violated for several reasons, including if certain observed characteristics are chosen endogenously by agents in the market. For example, schools may invest in quality by hiring teachers in order to obtain a better set of students. Such endogenously selected characteristics pose two empirical challenges. First, we cannot directly interpret the measured relationship between the observed characteristics and preferences as causal. Second, counterfactual situations that change the allocation mechanism or the market’s competition level may alter the incentives to invest in the characteristic. Methods that account for these sources of endogeneity when analyzing counterfactuals deserve further research.
2.2 Stability and Assignment Mechanisms

In this chapter, we will analyze preferences in two different data environments. In the first environment, we will assume access to information only on final matches, and we will observe the identities of the agents in the economy and their observable characteristics. In the second environment, we will assume access to information on the preferences of each agent as reported to an assignment mechanism. These reports may not be truthful. We now introduce some notation and definitions relevant to each of these cases.

Stability

The most commonly used notion of equilibrium that is used to analyze data from final matches is pairwise stability. Formally, a match is a function \( \mu : I \rightarrow J \cup \{0\} \), where \( \mu(i) \) denotes the agent in \( J \) to which \( i \in I \) is matched, if any. If \( i \) is unmatched, then \( \mu(i) = 0 \).

A match is feasible if \( |\mu^{-1}(j)| \leq q_j \), where \( q_j \) is the capacity of agent \( j \).

**Definition 1.** A feasible match is pairwise stable if

1. The match \( \mu \) is not blocked by an individual: (i) for every \( i \in I \), \( u_{i\mu(i)} \geq u_{i0} = 0 \), and (ii) for every \( j \in J \) and \( i \in \mu^{-1}(j) \), \( v_{ji} \geq v_{j0} = 0 \), and
2. The match \( \mu \) is not blocked by a pair: for all pairs of agents \( i \in I \) and \( j \in J \), if \( \mu(i) \neq j \), either (i) \( u_{ij} \leq u_{i\mu(i)} \), or (ii) \( v_{ji} \leq v_j \) where \( v_j = \min \{ v_{ji'} : i' \in \mu^{-1}(j) \} \) if \( |\mu^{-1}(j)| = q_j \) and \( v_j = 0 \) otherwise.

If a match does not satisfy the first requirement, then some agent has the incentive to unilaterally break a match and stay unmatched. This restriction on observed matches can be justified if agents have the ability to break matches unilaterally. This requirement is akin to individual rationality in mechanism design.

No blocking by a pair requires that there is no pair of agents that would agree to form a new match, which may involve breaking a match with a current partner if the capacity constraint binds. Imposing this requirement is often justified either as an approximation or based on institutional details. For example, the number of blocking pairs may be small if agents have a good sense of who the likely stable match partners are on the other side of the market and are able to explore potential matches with most of them. However, the clearest case for the assumption is if the matching market being studied employs a centralized stable matching mechanism, but for which data on rank-order lists is unavailable (Agarwal, 2015). Alternatively, stable matching may be appropriate if the market clears through a series of cutoffs, as in some college admissions settings (Akyol and Krishna, 2017; Bucarey, 2018).8

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8Indeed, Azevedo and Leshno (2016) and Che and Tercieux (2019) show that a stable match satisfies a cutoff representation. We will present this cutoff representation in Section 3.3.
As is clear from the definition of a stable match, the observed allocation implies certain restrictions on agents’ underlying preferences. The next section will show what we can infer about the distribution of preferences from these inequalities.

Assignment Mechanisms

In settings where a centralized assignment mechanism is used to match agents on the two sides of the market, it may be possible to obtain administrative data on rank-order lists. In such cases, the matches are determined by an algorithm that uses the reported preferences or priorities to determine a match. Let the possible set of reports that agents in \( I \) can make be denoted \( R_I \) and let the set of reports that agents in \( J \) can make be denoted \( R_J \). In cases where one side of the market, say \( J \), uses priorities instead of reports, we let \( R_J \) be the set of possible priorities. A mechanism is a function \( \Phi : R_I \times R_J \times \Omega \to M \), where \( M \) is the set of all feasible matches, and \( \omega \in \Omega \) is a draw from a tie-breaking lottery with support \( \Omega \) if any. The component of the domain \( R_I \) is the space of all report profiles \( R = (R_i)_{i=1}^I \) from side \( I \), and analogously \( R_J \) is the space of all report profiles from side \( J \). As before, a match \( \mu \in M \) is a function from \( I \) to \( J \cup \{0\} \).

An iconic mechanism is based on the Gale and Shapley (1962) Deferred Acceptance algorithm:

**Example. \( I \)-proposing Deferred Acceptance Algorithm**

Step 1: Each agent \( i \) proposes to their highest ranked option. At each option \( j \), proposals from the highest ranked agents according to \( R_j \) up to the capacity \( q_j \), are *tentatively held*. Ties between agents, if any, are resolved according to the value of a tie-breaker. The remaining agents are rejected.

Step \( k > 1 \): In a general step \( k \), agents whose proposals in step \( k-1 \) were rejected propose to their highest-ranked option that has not yet rejected them. At each option \( j \), the previously held proposals are considered along with agents who apply in round \( k \). Proposals from the highest ranked agents according to \( R_j \), up to capacity, are *tentatively held*, with ties resolved according to the value of the tie-breaker. The remaining proposals are rejected.

The algorithm terminates in step \( k \) if no agents are rejected or if all rejected agents have applied to each of their ranked schools. Assignments \( \mu (i) \) are finalized at the options where their applications are currently held if either terminal condition is satisfied. Otherwise, the algorithm proceeds to step \( k + 1 \).

A mechanism based on the Deferred Acceptance algorithm has two theoretically appealing properties. First, it is strategy-proof for the proposing side if there is no binding limit on the
number of options that can be ranked (Dubins and Freedman, 1981); that is, submitting a rank-order list that coincides with one’s ordinal preferences is a weakly dominant strategy. Second, the resulting assignment is stable if the agents on both sides submit truthful reports. These two properties are useful when trying to deduce preferences.

However, many school choice systems used mechanisms other than one based on Deferred Acceptance (see Chapter [*editor note: add chapter number] of this handbook for examples). Unfortunately, the most commonly used alternatives leave room for the agents to game the system by submitting a rank-order list that does not correspond to their true preferences. Section 4 in this chapter will briefly discuss approaches for empirically analyzing these cases. We refer the reader to Agarwal and Somaini (2020) for a more complete survey of the methods and ideas in this literature.

3 Analysis using Final Matches and Stability

This section reviews different approaches when we only observe data on the realized matches from a single large matching market. Throughout, we will assume that these matches are stable. We will distinguish between three types of markets below.

The first type is one-to-one matching with a large number of agents on each side. A canonical example is the market for marriages between women and men.\(^9\) The second is few-to-one matching. Each agent in \(J\) can match with only a few agents in \(I\). Again, we will assume that both \(I\) and \(J\) are large. The market for medical residents is the canonical example for this type of matching market. Other entry-level labor markets with no salary bargaining can also fit this category if stability is a reasonable assumption. The third type is many-to-one matching. In this type, each agent in \(J\) can match with many agents in \(I\). We assume that \(I\) is large but \(J\) is small. The leading example of this type of setting is the assignment of students to schools.

3.1 One-to-one Matching

Consider a market in which each agent in \(J\) can match with at most one agent in \(I\). Assume that both \(I\) and \(J\) are large. The analysis of this model has been split into two types. The first is based on the canonical single index model (e.g. Becker, 1973) in which each side of the market is differentiated only be a vertical quality index. The second is the case when preferences are heterogeneous so that two agents may have differing preferences over agents on the other side of the market. We discuss each of the two cases below.

\(^9\)In other types of marriages, the matches cannot be described using a bipartite graph and is therefore not two-sided.
3.1.1 Double-Vertical Model

In this model, all agents on one side of the market share the same preferences over all agents on the other side. In our notation, the utility of agent $i$ from matching with $j$ is

$$u_{ij} = u_j = u(x_j) + \xi_j,$$

where we replace the assumption in equation (2) to $\xi_j \perp x_j$. This model omits both observed and unobserved sources of quality heterogeneity, resulting in a desirability index for each agent $j$ denoted by $u_j$. The term $u(x_j)$ is the component explained by observables $x_j$, and $\xi_j$ the unobserved component. The preferences on the other side of the market are analogous:

$$v_{ji} = v_i = v(z_i) + \eta_i,$$

where $\eta_i \perp z_i$. The location for utilities is normalized by either setting the value of the outside option to 0 or picking an arbitrary value $\bar{x}_j$ and setting $u(\cdot)$ to zero at that value. Because the model does not have a quasilinear term $d_{ij}$ for normalizing the scale, we also set the slope of $u(\bar{x}_j)$ with respect to one of its components to one. The normalization on the other side of the market is analogous.

Chiappori et al. (2012) analyze a one-to-one matching model with these preferences. They assume that the researcher has access to data on the observable characteristics of the agents in a matching market. Therefore, they observe the joint distribution $F_{X,Z}$ of the observable characteristics of matched agents. Since they study the marriage market between men and women, we follow their convention in referring to side $I$ as men and to side $J$ as women.

In this model, a match is stable if and only if it exhibits perfect assortative matching on $u_j$ and $v_i$. In such a set of matches, the $t$–th most desirable man matches with the $t$–th most desirable woman. Therefore, if $F_U$ and $F_V$ are the cumulative distribution functions of $u_j$ and $v_i$ respectively, then an agent with characteristics $(x_j, \xi_j)$ is matched with an agent with characteristics $(z_i, \eta_i)$ only if:

$$u(x_j) = F_U^{-1}(F_V(v(z_i) + \eta_i)) - \xi_j. \quad (6)$$

Now, consider two men $i$ and $i'$ with identical values of the observed index, $v(z_i) = v(z_{i'})$. These two men could have different values of $\eta_i$ and therefore their mates may differ. However, if we consider two populations of men, one with observed characteristics $z_i$ and the other with observed characteristics $z_{i'}$, then the distribution of their desirability to women including the $\eta$ terms will be identical. Thus, the two populations of men will have the same marriage prospects and the women they match with will have the same distribution of observed characteristics.

In the terminology employed by Chiappori et al. (2012), this reasoning allows us to identify
“iso-attractiveness profiles” for men by looking at which vertical types end up matching with women with the same distributions of observable characteristics. The same reasoning allow us to identify iso-attractiveness profiles for women. Chiappori et al. (2012) allow $v_i$ to depend on two observable characteristics: body-mass index (BMI) and wages. Figure 1 illustrates these curves for men along these two dimensions.

Formally, they show that for any function $\phi_x$ of observables $x_j$ there exist a function $\phi_v$ of the index $v$ so that $E(\phi_x(x_j)|z_i) = \phi_v(v(z_i))$, where $E[\cdot]$ is the expectation operator. The left hand-side is observable, and the right hand side is a composition of two unknown functions. Differentiating both sides with respect to two components of $z_i$, we can measure the following marginal rate of substitution

$$\frac{\partial v(z_i)/\partial z_{i,k}}{\partial v(z_i)/\partial z_{i,l}} = \frac{\partial E(\phi_x(x_j)|z_i)/\partial z_{i,k}}{\partial E(\phi_x(x_j)|z_i)/\partial z_{i,l}}$$

because the right hand side is observed.

A natural question to ask is whether it is also possible to sort the level curves according to their desirability level. Since the argument above only provides the ratio of derivatives, there is no a priori way to know if desirability is increasing in any specific component. It is therefore necessary to assume that there is a characteristic that is known to be valued monotonically and is desirable.
Chiappori et al. (2012) obtain the marginal rates of substitution between BMI and wages by running seemingly unrelated regressions. Each of these regressions takes a characteristic of the women, BMI or education, and regresses them on the two characteristics of the men with whom they are matched. The coefficients are interpreted as the relative value of each of the men’s characteristics. An analogous set of regressions is run by switching the roles of men and women.\textsuperscript{10,11} They find that men compensate for 1.3 additional BMI points with a 1 percent increase in wages and that women compensate for two BMI points with 1 year of education.

A limitation of the empirical approach described above is that we are only able to assess the relative importance of two different components of the observables. In other words, the marginal rate of substitution between $x_{j,k}$ and $x_{j,k'}$ can be determined for any $k$ and $k'$, but we cannot determine the marginal rate of substitution between $x_{j,k}$ and $\xi_j$. More broadly, it is not possible to determine the contribution of the observables on either side to the overall variation in preferences.\textsuperscript{12} We will discuss how to identify the relative contribution of observables and unobservables when we discuss few-to-one matches below.

A strong restriction in the double-vertical model is that it assumes away heterogeneity in preferences. In fact, the data can reject this assumption in principle. Under the mild condition that $v(z_i)$ is continuous in its arguments, we should be able to find a curve of mens’ characteristics that match with women that have an identical distribution of observed characteristics. The observed data may not possess this feature. The set forms one of the “iso-attractiveness” curves illustrated by figure 1. For example, suppose that men and women prefer partners that are most similar to them. In this case, there may not be a pair of types $z_i$ and $z_i'$ that marry women with the same distribution of observed characteristics. A researcher, in this case, would need to select an alternative model that allows for heterogeneous preferences for empirical analysis.

### 3.1.2 Heterogeneous Preferences

A strong restriction in the model above is that all agents have the same preferences over the agents on the other side of the market. To make progress on relaxing this assumption,\textsuperscript{10}This procedure is justified by assuming that (i) $v(\cdot)$ is linear, and (ii) for the functions $\phi^1_x(x) = x_1$ and $\phi^2_x(x) = x_2$ that isolate each of the characteristics of the men, the corresponding function $\phi^k_v(\cdot)$ for $k \in \{1, 2\}$ are linear in the argument $v$. This second assumption, which jointly restricts $\phi_x(\cdot)$ and $\phi_v(\cdot)$, does not follow from primitives. General estimation methods for this model remains an open question for future research.\textsuperscript{11}An implication of this model is that the coefficients in each of these two regressions must be proportional. Chiappori et al. (2012) do not reject this restriction in their data.\textsuperscript{12}Observe however, that the polar cases when $\eta_i$ and $\xi_j$ are both identically equal to zero for all $i$ and $j$ can be ruled out. This is because equation (6) above reduces to $u(x_j) = F^{-1}_U(F_V(v(z_i)))$. In this case, men with a given set of characteristics $z_i$ match with women whose observables lie exactly on the iso-attractiveness curves described in figure 1.
Menzel (2015) considers a model in which the utilities are parametrized as follows:

\[ u_{ij} = u(x_j, z_i) + \varepsilon_{ij} \]
\[ v_{ji} = v(x_j, z_i) + \eta_{ji} \]

\[ u_{i0} = 0 + \max_{k=1,...,J} \{ \varepsilon_{i0,k} \} \]
\[ v_{j0} = 0 + \max_{k=1,...,J} \{ \eta_{j0,k} \} \]

The error terms \( \varepsilon_{ij}, \varepsilon_{i0,k}, \eta_{ji} \) and \( \eta_{j0,k} \) are independent and identically distributed with an upper tail that is of type I. The paper considers the limit of a sequence of economies indexed by \( J \) with an equal number of agents on each side and \( J \) growing large. Notice that the outside option also becomes more attractive as \( J \) increases. This choice is made in order to make sure that remaining unmatched remains attractive even in a large market with many draws of \( \varepsilon_{ij} \) and \( \eta_{ji} \).

Under these assumptions, Menzel (2015) shows that the limiting probability density function of the types of agents matched with each other, denoted \( f(x, z) \), has a very tractable functional form. Specifically, we get that

\[ \log \frac{f(x, z)}{f(\star, z)f(x, \star)} = \exp \left( u(x, z) + v(x, z) \right) \]

where \( f(x, \star) \) is the density of agents on side \( \mathcal{I} \) remaining unmatched, and analogously, \( f(\star, z) \) is the density of agents on side \( \mathcal{J} \) remaining unmatched. This convenient functional form is derived from the core insight that if some set \( J_i \) is willing to match with agent \( i \), then the probability \( i \) gets matched with \( j \) is the probability that \( j \) is \( i \)'s most preferred option in the set \( J_i \). And, similarly, \( i \) must be \( j \)'s most preferred option. Each of these probabilities are given by a logit-like formula in a large market, and are therefore proportional to \( \exp (u(x, z)) \) and \( \exp (v(x, z)) \) for sides \( \mathcal{I} \) and \( \mathcal{J} \) respectively. Hence, \( f(x, z) \) is proportional to the product \( \exp (u(x, z) + v(x, z)) \). The probability of remaining unmatched provides the right normalizing constant.

Another approach, due to Sorensen (2007), is to assume that matches depend only on the joint surplus \( S(x_j, z_i, \varepsilon_{ij}, \eta_{ji}) = u(x_j, z_i) + v(x_j, z_i) + \varepsilon_{ij} + \eta_{ji} \), but that the partners split this surplus via Nash bargaining after the match is formed. That is, side \( \mathcal{I} \) receives a fraction \( \lambda S(x_j, z_i, \varepsilon_{ij}, \eta_{ji}) \) from a realized match and side \( \mathcal{J} \) receives \( (1 - \lambda) S(x_j, z_i, \varepsilon_{ij}, \eta_{ji}) \) for some \( \lambda \in [0,1] \). Using the terminology of Niederle and Yariv (2009), this model exhibits aligned preferences, resulting in a unique pairwise stable match. Sorensen (2007) uses a Bayesian approach to estimate the joint surplus in the market for venture capital, targeting the joint surplus function \( S(x_j, z_i, \varepsilon_{ij}, \eta_{ji}) \) directly.

These results suggest a different limitation of data from one-to-one matches, this time in a model with heterogeneous preferences. Namely, only the sum of the surplus on the two sides of the market is identified. The difficulty lies in trying to determine whether the preferences on side \( \mathcal{I} \) or on side \( \mathcal{J} \) are driving the observed matches.

In early work, Logan et al. (2008) used a Bayesian approach to estimate heterogeneous preferences of
3.2 Few-to-one Matching

The discussion above demonstrates that only some features of preferences are identified given only data on final matches in a one-to-one matching market. In the double-vertical case, only the functions $u(\cdot)$ and $v(\cdot)$ can be identified, but the distribution of the unobserved determinants of match utility, $\eta$ and $\xi$, cannot. In the case of heterogeneous preferences, only the sum of payoffs, $u_{ij} + v_{ji}$, has been shown to be identified. However, answers to certain questions may require that the distribution of preferences on both sides be separately identified. For example, it is not sufficient to only know $u(\cdot)$ if we are interested in assessing the probability that a type $x$ is preferred to $x'$. This probability depends on the full distribution of preferences.

One conjecture is that it is not possible to identify preferences on both sides of the market in a one-to-one matching market. Diamond and Agarwal (2017) prove this result for the case of double-vertical preferences. As argued in Section 3.1.1, it is possible to learn the functions $u(\cdot)$ and $v(\cdot)$ under mild restrictions. However, if there are unobserved determinants of preferences on either side of the market, then the matching will not be perfectly assortative in these indices. This is because the match is assortative on $u_j = u(x_j) + \xi_j$ and $v_i = v(z_i) + \eta_i$, not only on the components $u(x_j)$ and $v(z_i)$ that can be predicted by observables. However, the data can be rationalized by either setting $\xi_j \equiv 0$ for all $j$ or $\eta_i \equiv 0$ for all $i$. This result follows because the double-vertical model only places a single restriction on behavior expressed in equation (6), but there are two unobservables in the model, $\xi_j$ and $\eta_i$. In other words, the matches are determined by unobserved determinants of preferences on both sides of the market, making them hard to disentangle.

Diamond and Agarwal (2017) go on to show that this problem can be solved in many-to-one matching markets, since a setting in which each agent $j$ can match with multiple agents $i$ on the other side has significantly more information than a market with one-to-one matching. An iconic example is the National Residence Program, where the allocation is determined according to a variant of the deferred acceptance algorithm (Roth and Peranson, 1999). While each resident is assigned to a single program, each program matches with several residents. The number of residents to which each program $j$ is matched can be as low as two. For this reason, we term such markets few-to-one matching markets.

As before, if preferences on both sides are vertical, matches are stable if and only if they exhibit perfect sorting. In other words, in any stable match, the most preferred residents are allocated to the most preferred hospital until its vacancies are filled. The second most preferred hospital takes the most preferred remaining residents and so on.

More formally, in such a market, consider a pair of residents $i$ and $i'$ matched to the same both men and women over their partners. However, we are not aware of results that show identification of this model.
hospital $j$. Equation (6) generalizes to:

$$u(z_i) = F_U^{-1}(F_V(v(x_j) + \xi_j)) - \eta_i$$  
$$u(z_i') = F_U^{-1}(F_V(v(x_j) + \xi_j)) - \eta_i'$$  

(7)

As in the marriage market problem, the lack of perfect sorting based on observables indicates the presence of the errors $\eta_i$ and $\xi_j$. Now however, the composition of the incoming class in each program provides additional information about the contribution of each of the error terms. The expressions 7 suggest that dispersion in the $\eta$ terms, the unobserved shocks affecting residents’ desirabilities, will cause a program to admit residents with heterogeneous observable determinants of human capital. Thus, the unobservables $\eta_i$ contribute to the variance in the observable characteristics of residents within each program.

This model can be estimated using a simulated minimum distance estimator (Agarwal, 2015; Diamond and Agarwal, 2017). The method consists of the following steps. First, define a set of moments of the data $m$ that we will try to match with our model. Second, fix a vector of parameters $\theta = \{\beta, \gamma, \sigma_\eta, \sigma_\xi\}$ for the model and use them to simulate stable matches and obtain a simulated set of moments $m(\theta)$ as a function of the parameters. Third, compute the distance between the simulated moments and the moments observed in the data, e.g. $\|m - m(\theta)\|_W = \sqrt{(m - m(\theta))' W (m - m(\theta))}$. Fourth, search over $\theta$ to minimize the distance.$^{14}$

Agarwal (2015) uses three sets of moments for estimation. The first set of moments summarizes the general sorting patterns of residents across programs. Recall that $x_j$ and $z_i$ are column vectors; thus, $x_j z_i'$ is a matrix. Averaging this matrix over all matches yields:

$$\frac{1}{I} \sum_{i \in I} \sum_{j \in J} 1 \{\mu(i) = j\} x_j z_i'.$$

The second set of moments compute the withn-program variances of resident observables for each component $z_{i,\ell}$ of $z_i$:

$$\frac{1}{I} \sum_{i \in I} (z_{i,\ell} - \bar{z}_{i,\ell})^2,$$

where $\bar{z}_i$ is the vector of average characteristic values of $i$’s peers, that is, of residents allocated to the same program. The third set of moments computes the correlation between residents’ characteristics and the average characteristics of the residents’ peers for each set of components $z_{i,\ell}$ and $z_{i,k}$ for $k \neq \ell$:

$$\frac{1}{I} \sum_{i \in I} z_{i,\ell} \bar{z}_{i,k}.$$

$^{14}$Train (2009) provides an overview of best practices.
where $\hat{z}_i$ is the average characteristics of $i$’s peers excluding $i$.

The first set of moments summarizes the same type of information about the allocation as the regression coefficients in Chiappori et al. (2012).\(^{15}\) Both summarize the aggregate sorting patterns based on observable characteristics. The second and third set of moments include additional information that is required to identify the contribution of each of the error terms.

An open question in the few-to-one matching literature is whether it is possible to relax the assumptions of the double vertical model. Agarwal (2015) assumes that programs have vertical preferences over doctors, but doctors’ preferences over programs include some horizontal components. For example, the paper observes that doctors are more likely to be allocated to programs in their birth state or medical school state. This feature cannot be rationalized by a double-vertical model. Instead, it indicates a resident’s geographical preference for training close to home.

Estimates of this model using data on matches from the market for family medicine residents show that several non-salary observables and unobservables make significant contributions to the programs’ desirability indices, implying that residents are willing to forego higher salaries for training at more desirable programs. This preference gives desirable programs market power, allowing them to levy an implicit tuition of over $23,000 through a markdown in salaries.

### 3.3 Many-to-one Matching

We now consider settings in which agents on side $J$ can match with a large number of agents on the side $I$, while the number of agents on $J$ is small. The canonical examples for such settings are school and college admissions. We will therefore refer to agents on side $I$ as students and the agents on side $J$ as schools.

There are two relevant types of data in these settings. The first is when the preferences or priorities used by schools to admit students are known. That is, the researcher can directly ascertain how two students will be ranked, possibly up to a random tie-breaker. For example, many school districts grant priority to students in their walk-zone and to students who have siblings already enrolled, and many college systems prioritize students only using high school grades or entrance exam scores. In this case, the researcher only needs to estimate the preferences of students for the schools.

The second case is more challenging as we need to estimate preferences on both sides of the market. This case is relevant to college admissions systems and entry-level job settings in which the rules used by agents on side $J$ are unknown.

\(^{15}\)Sorting patterns and simulation-based estimation methods have also been used in Boyd et al. (2013) to estimate the preferences of teachers for working at various schools. Although Boyd et al. (2013) have access to data from many-to-one matches, they do not use this information to construct the latter two sets of moments. As a result, their approach may be susceptible to the non-identification issues discussed above.
In both cases, we consider the problem when only data on final matches are available, assuming that pairwise stability is satisfied. As before, this assumption requires justifications based on theory and institutional background on the process used in the market to assign students to schools. The main implication of the assumption is that the stable matches can be characterized by a cutoff rule. Azevedo and Leshno (2016) show that in a stable match, each student $i$ is assigned to her most preferred school in the set

$$S(v_i; p) = \{j : v_{ji} \geq p_j\},$$

where $v_i = (v_{1i}, \ldots, v_{Ji})$. The vector of cutoffs $p = (p_1, \ldots, p_J)$ is set so that the total number of students $i$ with school $j$ as their preferred option within the set $S(v_i; p)$ does not exceed the capacity $q_j$ at the school if $p_j > 0$. Moreover, in a market with an infinite number of students and a fixed number of schools, the stable match and the corresponding cutoffs $p_j$ are unique.

### 3.3.1 Known Priorities: School Choice

Suppose the researcher knows each student’s eligibility score for each school, denoted $v_{ji}$, up to a tie-breaker and the final assignment is stable. That is, $v_{ji} = v_j(z_i, \eta_i)$ where the function $v_j(\cdot)$ and the distribution of the tie-breaker $\eta_i$ are known. The cutoff scores $p_j$ can be computed as the lowest eligibility score $v_{ji}$ of a student that was matched to school $j$ if the school does not have spare capacity. Otherwise, the cutoff $p_j$ is equal to 0. The goal is then to estimate and identify the specification of preferences defined in equation (1).

This model is used by Fack et al. (2019) to study high school admissions in Paris, which are determined by a deferred acceptance mechanism, and by Akyol and Krishna (2017) to study Turkish high schools that use an entrance exam to make admissions decisions. This assumption can also be used to study higher education settings that use an entrance exam. For example, Bucarey (2018) uses stability to estimate preferences for colleges in Chile.

To see what can be learned with this information and the final assignments, consider the case with only two schools, 1 and 2, and an outside option 0. Figure 2 shows five regions of utilities denoted by Roman numerals. Each region implies different ordinal preferences except for region V, which pools the cases when $u_{i0} > u_{i1} > u_{i2}$ and $u_{i0} > u_{i2} > u_{i1}$. A student who is eligible for both schools will be assigned to school 1 if her utilities belong to either region I or II. Therefore, the share of students assigned to school 1 amongst those eligible for both schools is an estimate of the total probability mass of the distribution of utilities in regions I and II. Similarly, the share assigned to school 2 is an estimate of the total probability mass in regions III and IV.

A student eligible only for school 1 can either be assigned to that school or remain unassigned. In the former case, we can infer that $u_{i0} < u_{i1}$ which is the darkly-shaded region in figure 3.
In the later case, we infer $u_{i1} < u_{i0}$ which is shaded lightly. The share of students assigned to school 1 amongst these students is an estimate of the total probability in regions I, II and III of figure 2.

These arguments are similar to those for standard consumer choice models but differ crucially in that not all students are assigned to their first choice school. In this context, a student’s choice set is constrained by her eligibility. Thus, observed assignments provide no information about preferences for schools that are not in a student’s choice set. Learning about the full distribution of ordinal preferences for students with a vector of eligibility score $v_i$ will require extrapolation using data from students with larger choice sets. Fack et al. (2019) perform this extrapolation by assuming that the unobserved determinants of preferences in equation (1) are conditionally independent of eligibility given the observables included in the model. Formally, they require that

$$\epsilon_i \perp v_i | z_i, d_i, \{x_j, \xi_j\}_{j=1}^J.$$  \hfill (8)

This assumption may be a reasonable approximation if $z_i$ contains a rich set of student characteristics but can be violated, for example if eligibility scores are correlated with both unobserved student ability and unobserved preference parameters.

Under this assumption, the probability of each observed assignment can be used to construct a likelihood function given a parametrization of utilities. Specifically, let $F_{U^*}$ denote the joint cdf of the random vector $u_i^*$ with the $j$-th element equal to $u(x_j, z_i, \xi_j, \epsilon_i)$. We will drop the conditioning on $z_i, \{x_j, \xi_j\}_{j=1}^J$ for notational simplicity. The independence assumptions
in equations (2) and (8) obviate the need to condition on \( d_i \) and \( v_i \). Under this assumption, the probability that \( i \) is assigned to \( j \) given the parameter \( F_{U^*} \) can be written as

\[
P(\mu(i) = j | v_i = v, p, d_i = d; F_{U^*}) = \int 1 \left\{ u^*_j - d_j \geq u^*_{j'} - d_{j'} \quad \text{for all} \quad j' \in S(v; p) \right\} dF_{U^*}.
\]

This expression enables estimation via maximum likelihood or other likelihood-based methods. Specific functional forms that are convenient for estimation are further discussed in the next section and in Agarwal and Somaini (2020).

This expression shows that the preference shifter \( d \) plays a crucial role in identification. Under our assumptions, \( d \) changes the desirability of each school exogenously and, as a consequence, changes the schools to which students are assigned. This source of variation provides a wealth of information about agents’ preferences. Consider the probability that \( \mu(i) = 0 \), which is equal to the probability that \( u^*_i - d_i \) belongs to region V in figure 2. This probability is identified in the two-school case if both schools are in the choice set or if the assumption (8) above holds. It is equal to:

\[
P(\mu(i) = 0 | d_i = d) = P(u^*_i - d_i \leq 0) = F_{U^*}(d).
\]

Thus, we identify \( F_{U^*}(d) \) by the share of students in region V for \( d_i = d \). Variation in \( d \) allows us to identify \( F_{U^*} \) evaluated at different values. Finally, equations (1) and (2) imply that the joint cdf of \( u_i = (u_{i1}, \ldots, u_{iJ}) \) conditional on \( d \) is given by \( F_{U|d}(u) = F_{U^*}(u + d) \).
implying that the former is nonparametrically identified.\footnote{It is also possible to develop the same identification argument using any other region in figure 2. We choose region V because it is the negative orthant, which results in simpler expressions. Therefore, this model is over-identified.} 

### 3.3.2 Unknown Priorities: College Admissions

We now consider the implications of stability in many-to-one matching environments where preferences on both sides of the market have to be estimated. We will use college admissions as the leading example. The empirical challenge is not limited to estimating preferences for colleges. Because college preferences are unknown, it is not possible to make the same revealed preference arguments for students that we derived in the school choice context.

Nonetheless, there is a considerable amount of information available in the matches. Consider the simple case with $J = 2$. If student $i$ is observed attending college $j = 1$, then we can make the following claims:

- Student $i$ prefers college 1 to remaining unassigned: $u_{i1} \geq 0$.
- Student $i$ clears the threshold for college 1: $v_{i1} \geq p_1$.
- Either student $i$ prefers college 1 to college 2, or student $i$ does not clear the threshold for college 2: $u_{i1} \geq u_{i2}$ or $v_{i2} < p_2$.

These restrictions define a set in a four-dimensional space that rationalizes the allocation of $i$ to college 1.

Agarwal and Somaini (in progress) show how to learn about preferences on both sides of the market simultaneously for the model described by equations (1) and (4). In the model discussed in section 3.3.1, variation in $d_i$ is used to identify the joint distribution of the $J$-dimensional vector of students’ preferences $(u_{i1}, ..., u_{iJ})$. Similarly, exogenous variation in $d_i$ and $w_i$ can be used to nonparametrically identify the joint distribution of the $2J$ dimensional vector $(u_{i1}, ..., u_{iJ}, v_{i1}, ..., v_{iJ})$ conditional on all observables, up to appropriate scale and location normalizations. A closely related prior argument in Sun (2019) shows a similar result under more stringent restrictions on equation (1).\footnote{Specifically, Sun (2019) assumes that $u_{ij} = u(x_j, z_i, \xi_j) - d_{ij} + \epsilon_{ij}$ and $v_{ji} = v(x_j, z_i, \xi_j) - w_{ji} + \eta_{ij}$, whereas Agarwal and Somaini (in progress) can work with the general case in which $u_{ij} = g(d_{ij})$ and $v_{ji} = v(x_j, z_i, \xi_j, \eta_i, w_{ji})$ for a general function $g(\cdot)$.

This joint distribution allows for a host of economic phenomena based on unobservable factors. For example, correlation between $u_{ij}$ and $u_{ij'}$ implies that colleges $j$ and $j'$ are closer substitutes, i.e. students who like one tend to also like the other one; correlation between $v_{ji}$ and $v_{j'i}$ suggests that colleges $j$ and $j'$ tend to prefer the same set of students; and correlation between $u_{ij}$ and $v_{ji}$ suggests that students tend to like colleges that also like them.
A detail about the location normalization in this model is worth noting. As before, it is possible to normalize $u_{i0}$ to zero for all $i$ and $v_{j0}$ to zero for all $j$. Moreover, if the researcher has information on the capacity of each college, then it is possible in principle to learn the distribution of $v_{ji}$ for a college that does not fill its seats. This inference is based on students who have characteristics $d_i$ that indicate strong preferences for college $j$, but did not attend the college and therefore must have been unacceptable to the college. Unfortunately, it is not possible to use a similar reasoning for colleges that do not have spare capacity. For students who strongly prefer college $j$ but were not admitted, we can only deduce that $v_{ji} < p_j$, and we cannot determine the location of $v_{ji}$ because $p_j$ is not observed. One alternative is to set $p_j = 0$ and to treat these colleges in the same way as those with spare capacity to obtain the distribution of $v_{ji}$. However, the data are also consistent with any $p_j > 0$ and a distribution of $v_{ji}$ that is shifted by $p_j$. This ambiguity prevents us from identifying the location parameter of $v_{ji}$ when capacity is not known or when we know that capacity limits are binding. Nonetheless, the distribution of the difference $v_{ji} - p_j$ is identified in either case.

Methods for estimating this model are still a subject of ongoing research. Sun (2019) proposes a method based on first estimating the probability that a student with characteristics $(z_i, w_i, d_i)$ is matched with college $j$, and then using the derivatives of these probabilities to estimate the utility functions. Agarwal and Somaini (in progress) develop a Gibbs sampling method based on a probit model. In principle, a simulated minimum distance estimator similar to the one used in Agarwal (2015) offers another approach. The interested reader should consult these references for further details.

4 Analysis using Reported Preferences

Many centralized matching algorithms ask participants to submit a report that is used as an input to determine a match. Correspondingly, a well-developed literature has taken advantage of the additional information contained in these reports and derived methods to estimate agents’ preferences based on these reports. Methods for analyzing preferences in this richer data environment are more flexible and comparatively better developed than the cases described in the previous section.

As in the many-to-one matching case, we call agents on side $I$ students and agents on side $J$ schools, since most of the applications studying rank-ordered data have been in the context of school choice. This focus corresponds to the widespread use of centralized mechanisms to assign students to schools. However, the methods discussed below are generally applicable to other settings where a researcher can obtain data on preferences.\footnote{For example, Hitsch et al. (2010) estimate preferences in an online dating context by analyzing the decision to contact a potential date. They interpret the decision to contact a potential date as indicative of}
Our goal will be to estimate the preferences of students for schools. An analogous exercise allows analysis of the preferences of schools for students when rank-ordered data is available.\textsuperscript{19}

The discussion below is brief and focuses on the main differences from the previous section. Agarwal and Somaini (2020) provides a more thorough review of methods for estimating preferences in school choice settings.

### 4.1 Truthful Reports

An important goal when designing assignment mechanisms is strategy-proofness (Roth, 1982; Abdulkadiroglu and Sonmez, 2003; Azevedo and Budish, 2018). In such a mechanism, no student can benefit by submitting a list that does not rank schools in order of their true preferences. Strategy-proofness of a school choice mechanism can also enable an empirical strategy if agents understand it and follow this recommendation.\textsuperscript{20} Specifically, if agent \(i\) ranks \(j\) above \(j'\), then we can infer that

\[ u_{ij} > u_{ij'} \]

It is less clear how to treat schools that are not ranked on the list. One approach is to assume that students rank all schools that are acceptable, i.e. preferable to the outside option. In this case, if \(j\) is the lowest-ranked school, then \(u_{ij} > u_{i0} > u_{ij'}\) if \(j'\) is not ranked. In this model, the various rank-order lists partition the space of utilities, as shown in figure 4 for when \(J = 2\). The five regions in the figure correspond to the various ways in which two schools can be ranked, including the possibility that only one school or an empty list is submitted.

Observe that the rank-order lists provide richer information about preferences than in standard discrete choice models in which a consumer picks only their favorite product. Specifically, if a consumer picks option 1 in a standard discrete choice setting, then we can only deduce that the consumer’s utilities are in either the region labelled “Rank 1” or “Rank 1>2” in figure 4, but we cannot distinguish between these two regions. The richer information in ordered lists can help identify heterogeneity in preferences (Beggs et al., 1981; Berry et al., high utility. Their approach allows them to estimate flexible preferences for men and women.

\textsuperscript{19}Such an analysis may require combining multiple approaches illustrated below. For example, the Gale-Shapley mechanism is strategy-proof for the proposing side but not for the accepting side. In fact, Roth (1982) shows that there are no stable matching mechanisms that are strategy-proof for agents on both sides of the market. Thus, one may need to use techniques for truthful reports to estimate preferences for the proposing side and account for strategic behavior when estimating preferences for the accepting side.

\textsuperscript{20}Evidence from both experiments and the field suggests that students are more likely to report their preferences truthfully when interacting with a strategy-proof mechanism (Chen and Sonmez, 2006; de Haan et al., 2018). Nonetheless, comprehending that a mechanism is strategy-proof may be complicated (Li, 2017) and some students are liable to mistakenly submit rankings that are not truthful (Shorrer and Sovago, 2019; Rees-Jones, 2018; Hassidim et al., n.d.).
2004). In the school choice context, students often rank many more schools, allowing for very rich specifications for the distribution of utilities (see Abdulkadiroglu et al., 2017, for example). Allowing for such heterogeneity is important for accurately estimating the value of improving assignments.

As in Section 3.3.1, our goal is to identify the cdf $F_{U^*}$, the joint cdf of the random vector $u_i^*$ with the $j$-th element equal to $u(x_j, z_i, \xi_j, \epsilon_i)$. We drop the explicit conditioning on $z_i, \{x_j, \xi_j\}_{j=1}^{J}$ for notational simplicity and assume the condition in equation (2) holds. Under this assumption, the probability that $i$ submits the rank-order list $R = (j_1, j_2, \ldots, j_J)$ can be written as

$$\mathbb{P}(R|d = d; F_{U^*}) = \int 1 \left\{ u_{jk}^* - d_{jk} \geq u_{jk+1}^* - d_{jk+1} \text{ for all } k \in \{1, \ldots, J-1\} \right\} dF_{U^*}.$$ 

Convenient functional forms for estimating this model via maximum likelihood are further discussed in Section 4.3 below.

An important point to note is that we can do away with the independence assumption in equation (8) required in Section 3.3.1. This advantage arises because our ability to deduce whether or not $u_{ij} > u_{ij'}$ does not depend on the endogenous choice set of the agent as it did in Section 3.3.1. This is the case because the model assumes that agents report preferences truthfully irrespective of the preferences of agents on the other side of the market.
4.2 Manipulable Mechanisms

There are many school districts that use non-strategy-proof mechanisms. The widely criticized but still commonly used Immediate Acceptance mechanism, for example, prioritizes students who rank a school higher, generating strategic incentives. Lab studies (Chen and Sonmez, 2006), survey data (de Haan et al., 2018) and signs of strategic reporting in administrative data (Agarwal and Somaini, 2018) suggest that students do respond to these incentives.

To understand what can be learned from reports in manipulable mechanisms, it is useful to think about reports as actions in a game. Each action is associated with an expected payoff. If agents maximize expected utility, we can infer that the observed report yields the highest expected payoff. This approach assumes a considerable degree of sophistication as it requires agents to perform two cognitively demanding tasks. First, they have to be able to calculate the expected payoff for each possible report. Second, they have to maximize over all possible reports. We focus on the case where agents have rational expectations and can optimize before discussing extensions.

Let $L_R \in \Delta^J$ be a probability vector representing an agent’s beliefs about the probabilities with which she will be assigned to each of the $J$ schools if she submits the report $R \in \mathcal{R}_T$. The expected utility of this report is $u_i \cdot L_R$. If we observe the report $R_i$ from student $i$, then optimality implies that $u_i \cdot L_{R_i} \geq u_i \cdot L_R$ for all $R \in \mathcal{R}_T$. Let $C_{R_i}$ be the set of utilities $u_i$ such that the report $R_i$ maximizes expected utility. This set is a convex cone in the space of utilities that contains the origin. Moreover, the collection of sets $C_R$ for $R \in \mathcal{R}_T$ partitions the space. Figure 5 illustrates these sets for our simplified case with two schools. In this example, $u_{R,R'}$ represents utilities for which the student is indifferent between submitting $R$ and $R'$. Similarly, a student with utilities given by $u_{R,R''}$ is indifferent between $R$ and $R''$. The students with utility vectors in the set $C_R$ (weakly) prefer $R$ to the other reports.

The discussion above implicitly assumes that the vectors $L_R$ for $R \in \mathcal{R}_T$ are known to the analyst. In practice, they have to be estimated. Under rational expectations, these beliefs are objective assignment probabilities. Agarwal and Somaini (2018) noticed that almost all of the mechanisms used in practice can be described using a cutoff structure analogous to the one that applies to stable allocations. The distribution of these cutoffs in equilibrium determines the objective assignment probabilities. Thus, instead of estimating $L_R$, one can estimate the cutoff distribution instead which is a lower dimensional object. The cutoff structure is also useful to estimate beliefs under alternative assumptions on the belief formation process.

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21 More precisely, every $u \in \mathbb{R}^J$ belongs to the interior of at most one of the sets in the collection and it belongs to at least one set $C_R$. There is one exception. If two reports $R_i$ and $R'_i$ result in the same vector of probabilities, then the sets $C_{R_i}$ and $C_{R'_i}$ will be identical to each other.
In this model, the probability that $i$ submits the rank-order list $R_i$ can be written as

$$\mathbb{P}(R_i|d_i = d; F_{U^*}) = \int \{ (u^* - d) \cdot L_{R_i} \geq (u^* - d) \cdot L_R \text{ for all } R \in R_T \} \text{d}F_{U^*}.$$  

This expression follows because $(u^* - d) \cdot L_{R_i}$ is the expected utility from reporting $R_i$, which must be larger than the expected utility from any alternative report $R$. This expression forms the basis of estimation via maximum likelihood. We provide further details in Section 4.3.

Several extensions have been based on this approach, but under alternative behavioral assumptions. Kapor et al. (2020) propose estimating $L_R$ by surveying agents. They surveyed families participating in the school choice mechanism in New Haven and found significant differences between elicited and objective assignment probabilities. He (2017) and Hwang (2016) do not impose all the conditions imposed by optimality. Instead, they derive a few intuitive conditions that reports have to satisfy and use the implied revealed preference relations to estimate preferences. Their approaches result in incomplete models of behavior that do not admit maximum likelihood methods. Agarwal and Somaini (2018) and Calsamiglia et al. (2020) estimate mixture models in which some agents behave optimally while some others behave naively, i.e., they report their true ordinal preferences even if it is in their interest to report something else. A more detailed survey of methods for incomplete and mixture models is provided in Agarwal and Somaini (2020).
4.3 Parametrizations

A common feature of the models described in sections 3.3.2, 4.1 and 4.2 is that they result in linear restrictions on the vector of utilities \( u_i \) for each agent \( i \). This simple structure allows for likelihood-based estimation methods. These methods typically employ specific functional form and distributional assumptions on equation (1) in order to limit the dimension of parameters to be estimated. The two most commonly used functional forms are based on logit and probit errors.

**Logit Models**  Consider the special case of equation (3) in which

\[
 u_{ij} = \delta_j + \bar{x}_j \bar{\gamma} z_i - d_{ij} + \varepsilon_{ij} \tag{9}
\]

and \( u_{i0} = \varepsilon_{i0} \), where \( \varepsilon_{ij} \) follows an extreme-value type I distribution with location parameter 0 and scale parameter \( \sigma \). In addition to the distributional assumption on \( \varepsilon_{ij} \), this specification excludes the terms \( \gamma_i \) and folds \( x_j \beta + \xi_j \) into the fixed effect \( \delta_j \). Fack et al. (2019) used this parametric form to estimate high school preferences in Paris under both stability and truth-telling. For notational convenience, collect the parameters of the model in the vector \( \theta = (\delta, \bar{\gamma}, \sigma) \).

When final assignments are stable (as in Section 3.3.2) and agents’ preferences on only one side need to be estimated, the probability that student \( i \) is assigned to school \( j \), conditional on the observable characteristics, the cutoffs \( p \), and the model’s parameters, is given by

\[
 P(\mu (i) = j | v_i, x_j, z_i, p; \theta) = \frac{\exp \left( \frac{1}{\sigma} (\delta_j + \bar{x}_j \bar{\gamma} z_i - d_{ij}) \right)}{1 + \sum_k 1 \{ k \in S(v_i, p) \} \exp \left( \frac{1}{\sigma} (\delta_k + \bar{x}_k \bar{\gamma} z_i - d_{ik}) \right)}, \tag{10}
\]

where \( v_i \) is the observed eligibility score vector for student \( i \). The parameters \( \theta \) can be estimated by Maximum Likelihood. This formula is similar to the logit choice probabilities from the standard discrete choice model (McFadden, 1973; Train, 2009). The only difference is that the summation in the denominator only includes terms for schools that are achievable by the student.

This functional form is also useful when rank-order lists are assumed to be truthful (as in Section 4.1). The probability that student \( i \) submits the rank-order list \( R_i = (j_1, j_2, \ldots, j_J) \) is given by

\[
 P(R_i | x_j, z_i; \theta) = \prod_{k=1}^{J} \frac{\exp \left( \frac{1}{\sigma} (\delta_{jk} + \bar{x}_{jk} \bar{\gamma} z_i - d_{ijk}) \right)}{1 + \sum_{j' \neq j_k \text{ for } k' < k} \exp \left( \frac{1}{\sigma} (\delta_{j'k} + \bar{x}_{j'k} \bar{\gamma} z_i - d_{ij'}) \right)} \tag{11}
\]

The term corresponding to \( k = 1 \) is the probability that the school ranked first, \( j_1 \), has the highest utility. Thus, the term corresponding to the general \( k \) is the probability that
the school ranked in position $k$ has the highest utility amongst the schools not ranked any higher. The parameters $\theta$ can be estimated by Maximum Likelihood. The comparison between equations (10) and (11) reveals that each rank-order list contains strictly more information than the observed assignment. Using data on reports will typically yield more precise estimates and allow for more flexible parameterizations than using only data on allocations.$^{22}$

The logit model has closed form expressions for the cases of stable matchings and truth-telling reports, but not for the case of manipulable mechanisms. We discuss an alternative parametrization below, based on the probit model, that is useful in this case. In many contexts, we expect that a student who ranks a school with, say, good math outcomes at the highest position will also rank other schools with good math outcomes near the top of their list. Such patterns motivate introducing the random coefficients $\gamma_i$ in equation (9). In these models, students with a high coefficient on a particular school characteristic will tend to rank many schools with high values of that characteristic. To incorporate this heterogeneity, the likelihood functions need to be modified. For example, assume that $\gamma_i$ is assumed to be distributed $\gamma_i \sim \mathcal{N}(0, \Sigma_{\gamma})$ with density $\phi(\cdot; \Sigma_{\gamma})$, as is common practice (see Berry et al., 1995, for example). In the case with truthful preferences, the likelihood is now

$$P(R_i | x_j, z_i; \theta, \Sigma_{\gamma}) = \int P(R_i | x_j, z_i, \gamma; \theta) \phi(\gamma; \Sigma_{\gamma}) d\gamma. \quad (12)$$

An analogous change is required in the case when the analysis is conducted assuming that only final matches are observed, and are stable.

A challenge with specifications that include random coefficients is that closed-form expressions for the probabilities are not typically available. Estimation techniques for these models typically require simulation, even in the simpler discrete choice context. Provided that the number of random coefficients is small, this expression can be approximated by numerical integration or simulation methods. However, approximation error in this integral can result in bias in the final estimates if the objective function is non-linear in the approximation error. We refer the reader to Train (2009) for recommendations and results on simulation-based estimators.

**Probit Models** Another popular approach is based on the probit model with random coefficients. In this model, we specify

$$u_{ij} = \delta_j + x_{j} \gamma_i z_i + x_{j} \gamma_i - d_{ij} + \varepsilon_{ij}, \quad (13)$$

---

$^{22}$Relying on rank-ordered data requires a specific model of student behavior. Artemov et al. (2020) argues that relying on the stability of the allocation yields results that are robust to misspecifications of the model of behavior that could bias approaches that rely on reports.
where
\[ \gamma_i \sim \mathcal{N}(0, \Sigma) \quad \text{and} \quad \varepsilon_{ij} \sim \mathcal{N}(0, \sigma^2) . \]

The parameters of the model can be estimated using a Markov chain Monte Carlo (MCMC) technique called a Gibbs sampler with an appropriate conjugate prior distribution for the parameters \( \theta \). This method generates a Markov chain by iterating between drawing the parameters of the model (including the random coefficients) conditional on simulated utilities \( u_{ij} \) and drawing the utilities \( u_{ij} \) conditional on the parameters \( \theta \) and the observed assignments or reports.\(^{23}\) That is, the sampler iterates through the following three steps:

\[ u_{ij} | u_{i,-j}, \gamma_i, \theta; data \quad \text{for each } j \]
\[ \gamma_i | u_i, \theta; data \]
\[ \theta | \{u_i, \gamma_i\}_{i=1}^N ; data , \]

where \( u_{i,-j} = (u_{i1}, \ldots, u_{ij-1}, u_{ij+1}, \ldots, u_{iJ}) \). The procedure can be started from an arbitrary value of the parameters and \( u_i \). Iterating through this procedure results in a long sequence of simulated draws which is a Markov chain. A portion from the beginning of the chain is discarded and the distribution of the remainder can be used to compute both the point estimates and credible sets (Bayesian variants of confidence intervals) simultaneously. This Bayesian technique yields estimates that are asymptotically equivalent to the maximum likelihood estimator (van der Vaart, 2000, Theorem 10.1 (Bernstein-von Mises)).\(^{24}\) These features have made the probit model popular for discrete choice models in the marketing literature.

The second and third steps are standard and the same across the models discussed in sections 3.3, 4.1 and 4.2. Specifically, assuming a normal prior on \( \theta \), \( \gamma_i \) is also normally distributed given the conditional distributions above. Within \( \theta \), the coefficients \( \hat{\gamma}_i \) and \( \delta_j \) are normally distributed if their prior distributions are normal, and the covariances \( \Sigma \) and \( \Sigma_{\gamma} \) follow inverse-Wishart distributions.

The first step draws the utility of each school conditional on the current draws of the utilities of the other schools and the observed rank-order list or outcome. The assumptions of the model imply that \( u_{ij} | u_{i,-j}, \gamma_i, \theta; data \) has a truncated normal distribution with truncation points determined by the observed outcome or the report. There are three cases corresponding to data on stable allocations, truthful reports, and manipulable mechanisms.

The first case considers the model in section 3.3 where matches are stable. The truncation bounds are determined by the region in the space of utility vectors implied by the observed assignment. For example, take a student that was assigned to school 1 when their choice set contained both schools. In this case, their utility belongs to the sets I and II in figure 2.

\(^{23}\)Drawing the utilities in this second step is known as data augmentation.

\(^{24}\)We refer the reader to Gelman and Rubin (1992) for a textbook treatment of Gibbs sampling and to McCulloch and Rossi (1994) for a discussion more specific to discrete choice models.
Conditional on $u_{i1}$, we draw $u_{i2}$ from a normal truncated above by $u_{i1}$ to ensure that $u_{i1} \geq u_{i2}$. Given the new draw of $u_{i2}$, we now draw $u_{i1}$ from a normal truncated below by $\min(0, u_{i2})$ which ensures that $u_{i1} \geq u_{i2}$ and $u_{i1} \geq 0$. Therefore, the Gibbs’ sampler generates draws of $u_i$ that always lie in the set that rationalizes the observed allocation given the choice set.

The second case is for truthful reports in section 4.1, first applied in Abdulkadiroglu et al. (2017). The truncation bounds are determined by the region in the utility space implied by the observed rank-order list. The utility of the school ranked in the first position is drawn from a normal distribution truncated below by the current draw of the utility of the second highest ranked school. The utility of the $k$th ranked school is drawn from a normal distribution truncated below and above by the utilities of the $(k + 1)$st and $(k - 1)$st ranked schools, respectively. Finally, the utility of the lowest ranked school is drawn from a normal truncated below by zero and above by the utility of the second lowest ranked school.

The third and final case is for manipulable mechanisms, developed in Agarwal and Somaini (2018). The sets for this case, illustrated in figure 5, are less disciplined than those that arise in the truthful reporting case. As a result, logit models do not yield tractable closed form solutions for the probability of observing a given report. Random coefficient probit models estimated using MCMC, however, do not require nice closed form solutions. Instead they require tractable methods for sampling utilities within the set that rationalizes the observed report. The fact that the bounds are determined by linear inequalities makes the sampling procedure simple.
Figure 6 illustrates the procedure for an agent that reports $R$. Starting from some initial draw $u_i(t)$, the draws of $u_1$ such that $(u_1(t), u_2(t))$ belongs to $C_R$ are represented by the horizontal dotted line. Thus, we draw $u_1|u_2(t), \gamma_i, \theta$ from a normal truncated below at the value of $u_1$ where the dotted horizontal line intersects the ray $u_{R,R'}$ and above by the value of $u_1$ where the dotted horizontal line intersects the ray $u_{R,R''}$. Let the draw be $u_i(t+1)$. Figure 6 represents this draw with a vertical dotted line. Next, we draw $u_2|u_1(t+1), \gamma_i, \theta$ from a normal truncated below at the value of $u_2$ where the dotted vertical line intersects the ray $u_{R,R''}$ and above by the value of $u_2$ where the dotted horizontal line intersects the ray $u_{R,R'}$. Let this draw be $u_i(t+1)$. The point $u_i(t+1) = (u_{i1(t+1)}, u_{i2(t+1)})$ is the new utility draw. Thus, the Gibbs’ sampler in this case is not more difficult to implement than the case of truthful reporting.

5 Applications, Extensions and Open Questions

The empirical analysis of matching markets and related markets is still at a relatively early stage of development. This section highlights topics and areas of research that are ripe for study or have become active recently.

5.1 Applications

Comparing Mechanisms The theory on matching mechanisms has informed the implementation of coordinated mechanisms in various settings including school choice (Abdulkadiroglu and Sonmez, 2003; Pathak, 2017) and the National Medical Residency Program (Roth and Peranson, 1999). Quantifying the benefits of centralization requires credible estimates of agents’ preferences. The methods proposed in this chapter can be used for this purpose. For example, Abdulkadiroglu et al. (2017) use the implementation of the New York City High School assignment system, which is based on the Deferred Acceptance algorithm, to quantify the welfare effects of coordinated school assignment. They find that, following the reform that centralized the assignment process, students were placed at more desirable schools and were more likely to enroll in their assigned school. Moreover, exits from the public school system fell. Their analysis also compared the new DA-based system to the old system and alternatives using a distance-metric utility function. The empirical model was estimated assuming truthful reports after the reforms. On a scale ranging from a no-choice neighborhood assignment to the utilitarian optimal, the new system realized 80% of the potential gains, whereas the old system achieved one-third at most. Other ordinal mechanisms studied in the theoretical literature were within a few percentage points, suggesting that the primary gains arise from coordinating assignments.

In addition to the comparison between uncoordinated assignment systems and those coordinated via a centralized mechanism, the methods developed here have been used to compare
various coordinated mechanisms. A number of papers have compared the Deferred Acceptance mechanism with alternatives. Most of the work has compared this celebrated mechanism to the Immediate Acceptance mechanism (also known as the Boston Mechanism). The common finding in this literature is that the Immediate Acceptance mechanism yields higher utility to students in the best case (Agarwal and Somaini, 2018; Calsamiglia et al., 2020), but that this conclusion is not robust if students make mistakes (Agarwal and Somaini, 2018; Kapor et al., 2020). We point the reader to Agarwal and Somaini (2020) for a more detailed review of this literature.

**Rationing and Redistribution** Regulated prices and capacity constraints can result in rationing and redistribution. Allocation mechanisms that do not use prices affect both the level and distribution of welfare. Distributional concerns are particularly important in education (Hastings et al., 2009; Calsamiglia et al., 2020), health care (Agarwal et al., 2020), and social assistance (Waldinger, 2020). In these settings, the design of an effective allocation mechanism needs to consider ethical and political constraints alongside traditional issues surrounding efficiency and incentives. For example, school districts often implement quotas to equalize access to high-quality education, and publicly provided health care may need to ensure that all receive adequate treatment. Preference estimates are a key tool for understanding the performance and distributional consequences of various mechanisms. They also help understand the welfare and distributional effects of alternative designs.

The tools described above are also useful when centralized assignment mechanisms are not used. For example, rationing also occurs if providers can choose who to treat based on their preferences. As in other matching markets, preferences on both sides of the market determine the final allocation.²⁵ Gandhi (2020) takes this view when studying the market for nursing homes in California. The paper finds that providers tend to discriminate against Medicaid-eligible patients with lengthy anticipated stays because they are not profitable. These considerations may be important in other health care settings as well.

### 5.2 Extensions

There are several directions in which the methods discussed in this chapter can be extended. We discuss a few below:

**Dynamics** The models described in this chapter are static, in that all agents arrive to the market simultaneously and match once and for all. However, there are environments like

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²⁵The key feature is that, unlike standard consumers, patients cannot choose their most preferred healthcare provider at posted prices. Instead, the provider can exercise discretion on who to treat. Therefore, insurance contracts may also result in rationing.
the markets for child care, public housing, and organ transplants in which agents or units on one side of the market arrive over time, while agents on the other side can wait. These allocation systems often use waitlists to prioritize agents on the waiting side. Agents on the waiting side have to decide whether to match with a unit that has just arrived. This decision is informative about agents’ preferences in the same way that reports are for static allocation systems. Leveraging this insight, Agarwal et al. (2020) studied the system that allocates deceased donor kidneys in the U.S.; Waldinger (2020) analyzed public housing allocation; Reeling and Verdier (2020) analyzed bear-hunting licenses; and Liu et al. (2019) studied ride-sharing markets.

**Externalities – Peer Effects and Competition**  A central assumption in the framework described above is that each agent has preferences only over who they match with. There is relatively little work on externalities, whereby the matches of others also affect an agent’s payoffs. There are at least two important reasons why the matches of others may be important.

The first reason can be classified as peer effects. For example, students may derive utility from their classmates, and workers may have preferences over their co-workers. In this case, i’s preference over j can depend on the set $\mu^{-1}(j)$. There is limited work in education markets that have addressed these issues (e.g. Epple et al., 2018; Allende, 2019). The typical approach here is to assume preferences for aggregate statistics of the composition of the student body in equilibrium.

The second reason is due to competitive effects. These are particularly important in settings concerning industrial organization. For example, Uetake and Watanabe (2019) model an entry game in the banking industry using the tools of two-sided matching games in which a bank can enter a market by merging with an incumbent. In this entry game, payoffs are affected by the competitor banks that match in the market. Similarly, Vissing (2018) models the market for oil drilling leases as a matching game between oil companies and landlords that hold mineral rights. In this model, the terms that an oil company can negotiate depends on their overall market presence. A challenge in these settings is in finding an appropriate notion of stability that allows for externalities to be present.\textsuperscript{26}

**6 Conclusion**

Estimating preferences is a crucial first step towards understanding the effects of policy interventions in a matching market. These estimates enable both positive and normative

\textsuperscript{26}Conditions for existence of stable matchings with externalities is an active area of theoretical research. We refer the reader to Pycia and Yenmez (2019), Fisher and Hafalir (2016), and references within for some recent results.
analyses. Specifically, they can be used to predict how agents will behave after an intervention is implemented and how the allocation will change. Preference estimates are central to evaluating the welfare and distributional effects of such interventions.

However, standard tools for estimating consumer demand are not directly applicable in matching markets since prices do not clear the market. Instead, agents choose among an individualized set of options determined by the agents on the other side of the market that are willing to match with them. This choice set depends on the agent’s desirability to the other side of the market. These features require the development of a new analytical toolset.

The appropriate toolset depends on whether the researcher has access to data only on final matches, or also to data on preferences submitted to an allocation mechanism. This chapter discussed methods for both cases, outlining the assumptions required in order to make progress in each environment.

A message of our survey is that the data and institutional environment dictate how flexible a preference model can be estimated. The most severe restrictions need to be placed on preferences in one-to-one matching markets. The current set of results only justify either the sum of payoffs or a model with homogeneous vertical preferences on both sides to be estimated. More flexible models of preferences can be learned from markets with many-to-one matches. However, only data on reported preferences enable the estimation of the most flexible models of preferences.

While the methods discussed in this chapter have focussed on two-sided matching, similar models or their extensions are useful in analyzing a number of related problems. For example, they are applicable to environments where rationing occurs due to capacity constraints. These features describe many education and healthcare markets, ranging from college admissions to the assignment of patients to nursing homes. Extensions of the model have also been used to describe the market for oil drilling leases and the allocation of organs to patients on a waiting list. We believe that there are many other applications and settings where insights from this rapidly growing literature can be applied to further our understanding.

References


