Dynamic Oligopoly and Price Stickiness

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Imperfect Competition
Monopolistic competition: continuum of firms (Dixit-Stiglitz)

- simple and tractable
- reigns supreme: trade, macro, growth, …
Imperfect Competition

- **Monopolistic competition**: continuum of firms (Dixit-Stiglitz)
  - simple and tractable
  - reigns supreme: trade, macro, growth, ...

- **Oligopoly**: finite number of firms
  - more realistic and complicated
  - extensive IO literature
  - “rise in market power”: markups, concentration, superstar firms, ...

**Q**: Oligopoly important for macro?
This Paper

- Standard macro models...
  - representative agent, infinite horizon
  - consumption, labor and money
  - nominal rigidities a la Calvo
This Paper

- Standard macro models…
  - representative agent, infinite horizon
  - consumption, labor and money
  - nominal rigidities a la Calvo

- This paper
  - oligopoly with any \( n \) firms
  - general demand structure
    (e.g. Kimball, not just CES)
Challenges and Methods

- Monopolistic Competition
  - best response depends on aggregates…
  - …taken as given (infinitesimal)
- Oligopoly Dynamic Game
  - off-equilibrium deviations…
  - … influence not infinitesimal
- Our paper…
  - innovation: local analysis for small shocks
Literature

- Mongey (2016)

- Rotemberg-Saloner (1986), Rotemberg-Woodford (1992)


- Rational Inattention: Afrouzi (2020)
Setup

- **Households:** consumption, labor, money
- **Firms:** continuum of sectors $s$...
  - $n_s$ firms within sector $s$
  - Calvo: frequency $\lambda_s$ of price change

- Markov equilibrium
- One time, unanticipated "MIT shock" to money
\[ \int_0^\infty e^{-\rho t} U \left( C(t), L(t), \frac{M(t)}{P(t)} \right) \, dt \]

\[ C(t) = G(\{C_s(t)\}_s) \]
\[ C_s(t) = H(c_{s,1}(t), c_{s,2}(t), \ldots, c_{s,n}(t)) \]
\[
\int_0^\infty e^{-\rho t} U \left( C(t), L(t), \frac{M(t)}{P(t)} \right) dt
\]

\[
\int_0^\infty e^{-\rho t} \left( \log(C(t)) - L(t) + \log\left(\frac{M(t)}{P(t)}\right) \right) dt
\]

\[C(t) = G(\{C_s(t)\}_s)\]

\[C_s(t) = H(c_{s,1}(t), c_{s,2}(t), \ldots, c_{s,n}(t))\]

\[C(t) = \exp \int_0^1 \log C_s(t) ds\]

\[\frac{1}{n_s} \sum_{j=1}^{n_s} \Psi \left( \frac{c_{i,s}}{C_s} \right) = 1\]  
(Kimball)
\[ \int_0^\infty e^{-\rho t} U \left( C(t), L(t), \frac{M(t)}{P(t)} \right) dt \]

\[ \int_0^\infty e^{-\rho t} \left( \log(C(t)) - L(t) + \log\left( \frac{M(t)}{P(t)} \right) \right) dt \]

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\[ \frac{1}{n_s} \sum_{j=1}^{n_s} \Psi \left( \frac{c_{i,s}}{C_s} \right) = 1 \quad \text{(Kimball)} \]

\[ P(t)C(t) + \dot{B}(t) + \dot{M}(t) = W(t)L(t) + \tilde{\Pi}(t) + T(t) + r(t)B(t) \]
\[
\int_0^\infty e^{-\rho t} U \left( C(t), L(t), \frac{M(t)}{P(t)} \right) dt = \int_0^\infty e^{-\rho t} \left( \log(C(t)) - L(t) + \log\left( \frac{M(t)}{P(t)} \right) \right) dt
\]

\[
C(t) = G(\{C_s(t)\}_s)
\]
\[
C_s(t) = H(c_{s,1}(t), c_{s,2}(t), \ldots, c_{s,n}(t)) \quad \Rightarrow \quad C(t) = \exp \int_0^1 \log C_s(t) ds
\]
\[
P(t)C(t) + \dot{B}(t) + \dot{M}(t) = W(t)L(t) + \widetilde{\Pi}(t) + T(t) + r(t)B(t)
\]

\[
c_{i,s}(t) = l_{i,s}(t)
\]
\[
E_0 \int_0^\infty e^{-\int_0^t r(s) ds} \widetilde{\Pi}_{i,s}(t) dt
\]
\[
\widetilde{\Pi}_{i,s}(t) = c_{i,s}(t) (p_{i,s}(t) - W(t))
\]
\[
c_{i,s}(t) = C(t)P(t) D_{i,s}(p_s(t))
\]

(Golosov-Lucas)
\[
\int_0^\infty e^{-\rho t} U \left( C(t), L(t), \frac{M(t)}{P(t)} \right) dt
\]

\[
\int_0^\infty e^{-\rho t} \left( \log(C(t)) - L(t) + \log\left(\frac{M(t)}{P(t)}\right) \right) dt
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\]

\[
c_{i,s}(t) = l_{i,s}(t)
\]

\[
E_0 \int_0^\infty e^{-\int_0^t r(s) ds} \tilde{\Pi}^{i,s}(t) dt
\]

\[
\tilde{\Pi}^{i,s}(t) = c_{i,s}(t) \left( p_{i,s}(t) - W(t) \right)
\]

\[
c_{i,s}(t) = C(t)P(t) D^{i,s}(p_s(t))
\]

\[\text{Golosov-Lucas}\]

\[\text{Kimball}\]
\[ \int_0^\infty e^{-\rho t} U \left( C(t), L(t), \frac{M(t)}{P(t)} \right) dt \quad \int_0^\infty e^{-\rho t} \left( \log(C(t)) - L(t) + \log\left(\frac{M(t)}{P(t)}\right) \right) dt \]

\[
C(t) = G(\{C_s(t)\}_s) \quad \Rightarrow \quad C(t) = \exp \int_0^1 \log C_s(t) ds
\]

\[
C_s(t) = H(c_{s,1}(t), c_{s,2}(t), \ldots, c_{s,n}(t)) \quad \Rightarrow \quad \frac{1}{n_s} \sum_{j=1}^{n_s} \Psi \left( \frac{c_{i,s}}{C_s} \right) = 1 \quad \text{(Kimball)}
\]

\[
P(t)C(t) + \dot{B}(t) + \dot{M}(t) = W(t)L(t) + \tilde{\Pi}(t) + T(t) + r(t)B(t)
\]

\[
c_{i,s}(t) = l_{i,s}(t)
\]

\[
\mathbb{E}_0 \int_0^\infty e^{-\int_0^t r(s) ds} \tilde{\Pi}^{i,s}(t) dt
\]

\[
\tilde{\Pi}^{i,s}(t) = c_{i,s}(t) (p_{i,s}(t) - W(t))
\]

\[
c_{i,s}(t) = C(t)P(t) D^{i,s}(p_s(t))
\]

Calvo pricing

Poisson arrival

\[ \lambda_s \]
\[
\int_0^\infty e^{-\rho t} U \left( C(t), L(t), \frac{M(t)}{P(t)} \right) dt = \int_0^\infty e^{-\rho t} \left( \log(C(t)) - L(t) + \log\left( \frac{M(t)}{P(t)} \right) \right) dt
\]

\[
C(t) = G(\{C_s(t)\}_s)
\]
\[
C_s(t) = H(c_s,1(t), c_s,2(t), \ldots, c_s,n(t))
\]

\[
P(t)C(t) + \dot{B}(t) + \dot{M}(t) = W(t)L(t) + \widehat{\Pi}(t) + T(t) + r(t)B(t)
\]

\[
c_{i,s}(t) = l_{i,s}(t)
\]

\[
E_0 \int_0^\infty e^{-\int_0^t r(s)ds} \Pi^{i,s}(t) dt
\]

\[
\Pi^{i,s}(t) = c_{i,s}(t) \left( p_{i,s}(t) - W(t) \right)
\]

\[
c_{i,s}(t) = C(t)P(t)D^{i,s}(p_s(t))
\]

\[
P_{i,t}^* = g^{i,s}(p_{-i,s}; t)
\]

Calvo pricing
Poisson arrival
Reset strategy

\[
(Golosov-Lucas)
\]

\[
(Kimball)
\]
\[
\int_0^\infty e^{-\rho t} \mathcal{U}\left(C(t), L(t), \frac{M(t)}{P(t)}\right) dt \\
\int_0^\infty e^{-\rho t} \left(\log(C(t)) - L(t) + \log\left(\frac{M(t)}{P(t)}\right)\right) dt \\
C(t) = G\left\{C_s(t)\right\}_s \\
C_s(t) = H(c_{s,1}(t), c_{s,2}(t), \ldots, c_{s,n}(t)) \\
\Rightarrow C(t) = \exp \int_0^1 \log C_s(t) ds \\
\frac{1}{n_s} \sum_{j=1}^{n_s} \Psi \left(\frac{c_{i,s}}{C_s}\right) = 1 \\
(P(t)C(t) + \dot{B}(t) + \dot{M}(t) = W(t)L(t) + \tilde{\Pi}(t) + T(t) + r(t)B(t)

\]

\[
\begin{align*}
\mathbb{E}_0 \int_0^\infty e^{-\int_0^t r(s) ds} \tilde{\Pi}^{i,s}(t) dt \\
\tilde{\Pi}^{i,s}(t) &= c_{i,s}(t) (p_{i,s}(t) - W(t)) \\
c_{i,s}(t) &= C(t)P(t) D^{i,s}(p_s(t)) \\
\end{align*}
\]

Calvo pricing
Poisson arrival
Reset strategy

\[
p^*_i(t) = g^{i,s}(p_{i,s}; t) \\
\{p_{j,s}\}_{j \neq i}
\]
Equilibrium

\{C(t), L(t), M(t), P(t), W(t), r(t)\}

\{c_{i,s}(t), p_{i,s}(t)\}

\{g^{i,s}(p_{-i,s}; t)\}

- **agents:** \(\{P(t), W(t), r(t)\} \rightarrow \max\rightarrow \{C(t), L(t), M(t)\}\)

- **firms:** \(\{C(t), P(t), W(t), r(t)\}\)

\(\bigg\{g^{-i,s}(\cdot; t)\bigg\}\)

\(\rightarrow \max\rightarrow g^{i,s}(p_{-i,s}; t)\)

- **market clearing:**

\[L(t) = \int \sum_{n_s} c_{i,s}(t)\,ds\]
Steady State

- Constant $C, L, M, P, W, r$
Steady State

- Constant $C, L, M, P, W, r$
- Household and market clearing
  
  \[
  C = L \\
  \frac{U_C}{P} \quad \frac{U_L}{W} \quad \frac{U_m}{rP} \\
  r = \rho
  \]
Steady State

- Constant $C, L, M, P, W, r$
- household and market clearing
  \[ C = L \]
  \[ \frac{U_C}{P} = \frac{U_L}{W} = \frac{U_m}{rP} \]
  \[ r = \rho \]
- firms
  \[ \rho V(p) = D^i(p)(p_i - W) + \lambda \sum_j [V(g(p_{-j}), p_{-j}) - V(p)] \]
Steady State

- Constant $C, L, M, P, W, r$
  - household and market clearing
    $$C = L$$
    $$\frac{U_C}{P} = \frac{U_L}{W} = \frac{U_m}{rP}$$
    $$r = \rho$$
  - firms
    $$\rho V(p) = D^i(p)(p_i - W) + \lambda \sum_{j} [V(g(p_{-j}), p_{-j}) - V(p)]$$
    $$g(p_{-i}) \in \arg\max_{p_i} V(p_i, p_{-i})$$
Steady State

- Constant $C, L, M, P, W, r$
  - household and market clearing
    \[ C = L \]
    \[ \frac{U_C}{P} = \frac{U_L}{W} = \frac{U_m}{rP} \]
    \[ r = \rho \]

- firms
  \[ \rho V(p) = D^i(p)(p_i - W) + \lambda \sum_j [V(g(p_{-j}, p_{-j}) - V(p)] \]
  \[ g(p_{-i}) \in \arg \max_{p_i} V(p_i, p_{-i}) \]

- steady state price vector $P = g(P, P, \ldots, P)$
\[ p_i = g(p_{-i}) \]
\( p_i = g(p_{-i}) \)
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\[ p_i = g(p_{-i}) \]
1. Sufficient Statistics
Money Shock

- Starting at steady state…
Money Shock

- Starting at steady state...
- ...unanticipated permanent shock to money...
Money Shock

- Starting at steady state...
- ...unanticipated permanent shock to money...

\[ P(t)C(t) = M(t) \]
Money Shock

- Starting at steady state...
- ...unanticipated permanent shock to money...
- Nominal interest rate unchanged...
Money Shock

- Starting at steady state...
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\[ P(t)C(t) = M(t) \]

- Nominal interest rate unchanged...
Money Shock

- Starting at steady state...
- ...unanticipated permanent shock to money...

\[ P(t)C(t) = M(t) \]

- Nominal interest rate unchanged...
  \[ r(t) = \rho \]

- Wage jumps to new level: \( W = (1 + \delta)W_0 \)
Money Shock

- Starting at steady state...
- ...unanticipated permanent shock to money...

Result #1.
Equilibrium transition after shock $\delta$ satisfies steady-state policies...

$$\hat{p}_{i,s} = g(\hat{p}_{i-s})$$

with $\hat{p}_{i,s} = p_{i,s}/(1 + \delta)$
\[ p_i = g(p_{-i}) \]
$p_i = g(p_{-i})$
$p_i = g(p_{-i})$
\[ p_i = g(p_{-i}) \]
Money Shock

- Starting at steady state...
- ...unanticipated permanent shock to money...

Result #2.

\[
\log P(t) - \log \bar{P} = -\delta \ e^{-\lambda(1-B)t}
\]

\[
B = (n - 1) \frac{\partial g^i}{\partial p_j}(\bar{p})
\]
The diagram illustrates the relationship between $p_i$ and $p_{-i}$, with the equation $p_i = g(p_{-i})$. It shows that a steeper policy leads to slower convergence.
Money Shock

- Starting at steady state...
- ...unanticipated permanent shock to money...

Result #2.

\[ \log P(t) - \log \bar{P} = -\delta \int_s \zeta_s e^{-\lambda_s (1 - B_s) t} \, ds \]

\[ B_s = (n - 1) \frac{\partial g^i}{\partial p_j}(\bar{p}_s) \]

Adding Heterogeneity!
Metrics for Stickiness

- Cumulative Output

$$\int_{0}^{\infty} e^{-rt} \log \left( \frac{C(t)}{\bar{C}} \right) dt = \delta \int_{s} \frac{\zeta_{s} ds}{r + \lambda_{s} (1 - B_{s})}$$

- Half Life: $\log(2) \cdot h$

$$h = \frac{1}{\lambda(1 - B)}$$

- Phillips Curve?

$$\dot{\pi}(t) = \rho \pi(t) - \kappa m c(t)$$

$$\pi(t) = \kappa \int e^{-\rho s} mc(t + s) ds$$
Metrics for Stickiness

Result #3
After M shock

\[ \dot{\pi}(t) = \rho \pi(t) - \kappa mc(t) \]

- Phillips Curve?

\[ \dot{\pi}(t) = \rho \pi(t) - \kappa mc(t) \]
\[ \pi(t) = \kappa \int e^{-\rho s} mc(t + s) ds \]
Result #3
After M shock

\[ \dot{\pi}(t) = \rho \pi(t) - \kappa mc(t) \]

\[ \kappa = \hat{\lambda}(\rho + \hat{\lambda}) \]
\[ \hat{\lambda} = \lambda(1 - B) \]

- Phillips Curve?

\[ \dot{\pi}(t) = \rho \pi(t) - \kappa mc(t) \]

\[ \pi(t) = \kappa \int e^{-\rho s} mc(t + s) ds \]
Result #3
After M shock

\[ \dot{\pi}(t) = \rho \pi(t) - \kappa mc(t) \]

\[ \kappa = \hat{\lambda}(\rho + \hat{\lambda}) \]

\[ \hat{\lambda} = \lambda(1 - B) \]

\[ \kappa \approx \frac{1}{h^2} \]

- Phillips Curve?

\[ \dot{\pi}(t) = \rho \pi(t) - \kappa mc(t) \]

\[ \pi(t) = \kappa \int e^{-\rho s} mc(t + s) ds \]
Sufficient Statistic

Result #4

\[ B = \frac{1 + \frac{\rho}{\lambda}}{1 + \frac{1}{(n-1)[(\epsilon-1)(\mu-1)-1]}} \]

\[ \mu = \frac{P}{W} \]

\[ \epsilon = -\frac{\partial \log D^i}{\partial \log p_i} \]
**Result #4**

\[ B = \frac{1 + \frac{\rho}{\lambda}}{1 + \frac{1}{(n-1)[(e-1)(\mu-1)-1]}} \]

\[ \mu = \frac{P}{W} \]

\[ \epsilon = \frac{-\partial \log D^i}{\partial \log p_i} \]

- Intuition… (reverse causality)
  - Nash markup \( \Longleftrightarrow B = 0 \)
  - higher markup \( \Longleftrightarrow \) rivals mimic my price (high \( B \))

\[
\frac{\mu - 1}{\mu^{\text{Nash}} - 1} = 1 + \frac{1}{n-1} \cdot \frac{B}{1-B}
\]
\[ p_i = g(p_{-i}) \]
\[ p_i = g(p_{-i}) \]
Monopolistic competition + Static oligopoly: markup only depends on local elasticity

Dynamic oligopoly: conditional on elasticity, markup depends on $n, \theta, \lambda$...
2. Counterfactuals
Solving MPE
Solving MPE

- Previous: stickiness from observed steady-state markup
Solving MPE

- **Previous:** stickiness from observed steady-state markup
- **Now:** Comparative statics…
  - counterfactuals: do not know steady state
  - must solve MPE
  - challenging: large state for large $n$
Solving MPE

- **Previous**: stickiness from observed steady-state markup
- **Now**: Comparative statics…
  - counterfactuals: do not know steady state
  - must solve MPE
  - challenging: large state for large $n$

- **Our Method**…
  - solve exact approximate model i.e. demand system
  - benefit: tractable and flexible
  - check approximation with other methods
Solving MPE

- **Previous:** stickiness from observed steady-state markup
- **Now:** Comparative statics...
  - counterfactuals: do not know steady state
  - must solve MPE
  - challenging: large state for large $n$

- **Our Method…**
  - solve exact approximate model i.e. demand system
  - benefit: tractable and flexible
  - check approximation with other methods
- IO literature: other approximations ("oblivious" equilibria)
Kimball Demand

\[ d^i, s(p_i, s(t)) \quad \leftrightarrow \quad \frac{1}{n} \sum \Psi \left( \frac{c_i}{C} \right) = 1 \]

\[ \epsilon = - \frac{\partial \log d^i}{\partial \log p_i} \quad \Sigma = \frac{\partial \log \epsilon}{\partial \log p_i} \]

\[ \eta = - \frac{\Psi'(x)}{x \Psi''(x)} \quad \theta = - \frac{\partial \eta}{\partial x} \]
Kimball Demand

\[
d^{i,s}(p_i, s(t)) \quad \leftrightarrow \quad \frac{1}{n} \sum \psi\left(\frac{c_i}{C}\right) = 1
\]

\[
e = -\frac{\partial \log d^i}{\partial \log p_i} \quad \sum = \frac{\partial \log \epsilon}{\partial \log p_i}
\]

\[
\eta = -\frac{\psi'(x)}{x\psi''(x)} \quad \theta = -\frac{\partial \eta}{\partial x}
\]

\[
\epsilon = \left(1 - \frac{1}{n}\right) \eta + \frac{1}{n} \omega
\]

\[
\sum = \frac{n - 1}{n} \cdot \frac{(n - 2)\theta \eta + \eta^2 - (1 + \omega) \eta + \omega}{(n - 1) \eta + \omega}
\]
Kimball Demand

\[ d^{i,s}(p_i,s(t)) \iff \frac{1}{n} \sum \Psi\left(\frac{c_i}{C}\right) = 1 \]

\[
\epsilon = -\frac{\partial \log d^i}{\partial \log p_i} \quad \Sigma = \frac{\partial \log \epsilon}{\partial \log p_i} \\
\eta = -\frac{\Psi'(x)}{x\Psi''(x)} \quad \theta = -\frac{\partial \eta}{\partial x}
\]

\[
\epsilon = \left(1 - \frac{1}{n}\right) \eta + \frac{1}{n} \omega \\
\Sigma = \frac{n - 1}{n} \cdot \frac{(n - 2)\theta \eta + \eta^2 - (1 + \omega) \eta + \omega}{(n - 1) \eta + \omega}
\]

\[
\epsilon = \eta \quad \Sigma = \theta \quad n \to \infty
\]
Method

- 2 equations in 2 unknowns...
  - Sufficient statistic formula...
    \[ B = B(\mu, \epsilon, n, \lambda/\rho) \]
  - One extra equation...
    \[ \mu = \mu(B, \epsilon, \Sigma, n, \lambda/\rho) \]

- Verified: good approximation!
- More general: k-order derivatives of demand (see paper)
Changing n
Changing n
What to hold fixed?

1. Preferences \((\eta, \theta)\)

3. Calibrate: evidence on pass-through

5. Local elasticities of demand \((\epsilon, \Sigma)\)
Changing n

- What to hold fixed?
  1. Preferences \((\eta, \theta)\)
  3. Calibrate: evidence on pass-through
  5. Local elasticities of demand \((\epsilon, \Sigma)\)
Half-Life

$\theta = 0$ (CES)
Kimball Demand

Half-life

$\theta = 15$

$\theta = 10$

$\theta = 5$

$\theta = 0$ (CES)
Kimball Demand

- Low $\theta$: greatest stickiness at $n=2$
- High $\theta$: lowest stickiness at $n=2$!
- Duopoly is knife-edge: half-life stuck at CES level…
  … Kimball can’t help $n=2$!
Changing n
Changing n
Changing n

- What to hold fixed?
  1. Preferences \((\eta, \theta)\)
  3. Calibrate: evidence on pass-through
  5. Local elasticities of demand \((\epsilon, \Sigma)\)
Changing $n$

- What to hold fixed?
  1. Preferences $(\eta, \theta)$
  3. Calibrate: evidence on pass-through
  5. Local elasticities of demand $(\epsilon, \Sigma)$
Pass-Through

- Amiti-Itskhoki-Konings
- Evidence own-cost pass-through...
  - high for small firms
  - low for large firms
- Here: Fix elasticity, set super-elasticity to match...

\[ \text{pass-through} = f(\text{market share}) \]
Pass-Through

$$\text{pass-through} = f(\text{market share})$$
Half-life

- National HHI 0.05 to 0.1 (e.g., Gutierrez-Philippon): MP 15% stronger
\[ \Delta \log p_{it} = \hat{\alpha} \Delta \log mc_{it} + \hat{B} \frac{\sum_{j \neq i} \Delta \log p_{jt}}{n - 1} + u_{it} \]

\[ \hat{\alpha} \approx \frac{1}{1 + s(\eta - 1)} \]
\[ \Delta \log p_{it} = \hat{\alpha} \Delta \log mc_{it} + \hat{B} \frac{\sum_{j \neq i} \Delta \log p_{jt}}{n - 1} + u_{it} \]

\[ \hat{\alpha} \approx \frac{1}{1 + s(\eta - 1)} \]

\[ \tilde{p}_i = \alpha \tilde{mc}_i + B \frac{\sum_{j \neq i} \tilde{p}_j}{n - 1} + \gamma \sum_{j \neq i} \tilde{mc}_j \]
\[ \Delta \log p_{it} = \hat{\alpha} \Delta \log mc_{it} + \hat{B} \frac{\sum_{j \neq i} \Delta \log p_{jt}}{n - 1} + u_{it} \]

\[ \hat{\alpha} \approx \frac{1}{1 + s(\eta - 1)} \]

\[ \tilde{p}_i = \alpha \tilde{mc}_i + B \frac{\sum_{j \neq i} \tilde{p}_j}{n - 1} + \gamma \sum_{j \neq i} \tilde{mc}_j \]

\[ \hat{\alpha} = \frac{n\alpha + B - 1}{\alpha + B + n - 2} \]
3. Inspecting Mechanism
Inspecting Mechanism

\[ \mu W(t) \]

- Firm 1
- Firm 2
- \( P(t) \)
- \( \mu W(t) \)
Inspecting Mechanism

- Two effects with finite $n$...
  - feedback: firm i cares about others’ prices
  - strategic: firm i can affect others’ prices
Inspecting Mechanism

- Two effects with finite $n$...
  - **feedback**: firm $i$ cares about others’ prices
  - **strategic**: firm $i$ can affect others’ prices

- Feedback effect with $n = \infty$
  - inputs from other firms
  - Kimball (1995) demand

\[ \mu W(t) \]
Comparing...  
- Markov with \( n \) firms  
- Naive equilibrium with \( n \) firms

Equivalent to \( n = \infty \) with modified Kimball preferences to match elasticity and superelasticity
Small strategic effects

\[ \frac{h}{h_{\text{Naive}}} \]

- **AIK**
- \( \theta = 10 \)
- \( \theta = 0 \) (CES)
Small strategic effects

Takeaway:
Naive good fit,
i.e. mostly feedback effects,
strategic effects small
Back to Kimball Demand

Half-life

\[ \theta < \frac{(\eta - 1)^2}{\eta + 1} \]
• Small strategic effects…
  • … use naive model for comparative statics
  • half-life decreases with $n$ if $\theta < \frac{(\eta - 1)^2}{\eta + 1}$
Naive and Static Nash

- Naive…
  - ignore own impact
  - anticipation of dynamics of future
- Static Nash…
  - best response to fixed prices
  - simple function of primitives

- **In paper:** provide useful formula…

\[ B^{\text{Naive}} = f \left( B^{\text{Nash}}, \frac{\lambda}{\rho} \right) \]
Changing n
Changing n
Changing n

- **What to hold fixed?**
  1. Preferences \((\eta, \theta)\)
  3. Calibrate: evidence on pass-through
  5. Local elasticities of demand \((\epsilon, \Sigma)\)
Changing n

- What to hold fixed?
  1. Preferences \((\eta, \theta)\)

  3. Calibrate: evidence on pass-through

  5. Local elasticities of demand \((\epsilon, \Sigma)\)

Related to Naive/Kimball Results
Heterogeneity

- Heterogeneity...
  - across sectors
  - within sector (extension)
Sectoral Heterogeneity

Cum. $Y$

$\lambda_3$
Sectoral Heterogeneity

- Cumulative output effect is proportional to

\[
\mathbb{E} \left[ \frac{1}{\lambda_s} \right] \mathbb{E} \left[ \frac{1}{1 - B_s} \right] + \text{Cov} \left( \frac{1}{\lambda_s}, \frac{1}{1 - B_s} \right)
\]
Sectoral Heterogeneity

- Cumulative output effect is proportional to

\[ \mathbb{E} \left[ \frac{1}{\lambda_s} \right] \mathbb{E} \left[ \frac{1}{1 - B_s} \right] + \text{Cov} \left( \frac{1}{\lambda_s}, \frac{1}{1 - B_s} \right) \]

- Example: Two sectors, \( n = 3 \) and \( n = 20 \), keeping average duration fixed at 1
Within Sector Heterogeneity

Half-life

- Symmetric firms
- Heterog. firms
Within Sector Heterogeneity

Half-life

- Symmetric firms
- Heterog. firms

$2^5$
Within Sector Heterogeneity

- Many ways to attain a given HHI instead of $1/n$
Many ways to attain a given HHI instead of $1/n$

Example:
- 25 firms, 2 type of firms, 23 type A, 2 type B
- vary relative productivity A vs B

Half-life

Graph showing the relationship between $n$ and half-life for Symmetric firms and Heterog. firms.
4. Phillips Curve
Phillips Curve

\[ \pi(t) = \int_0^\infty \gamma^{mc}(s) mc(t + s) \, ds \]
\[ + \int_0^\infty \gamma^c(s) c(t + s) \, ds \]
\[ + \int_0^\infty \gamma^R(s) (R(t + s) - \rho) \, ds \]

\[ \gamma^{mc}(s), \gamma^c(s), \gamma^R(s) = \text{linear combinations of } \{e^{-\nu_j s}\}^7_{j=1} \]

- **Oligopolistic NKPC**
  - Higher order ODE (≤ 7): inflation persistence
  - Not just Marginal Cost (mc): demand (c), interest rates (R)
  - Generally, not equivalent to lower \( \lambda \)
Phillips Curve

- Standard NKPC
  \[ \dot{\pi} = 0.05\pi - 1.05mc \]

- Oligopoly with \( n = 3 \) (AIK calibration)
  - MPE
    \[ \dot{\pi} = 0.07\pi - 0.27mc \]
    \[ + 1.33\ddot{\pi} + 0.44mic + 0.03(r - \rho) \]
  - Naive
    \[ \dot{\pi} = 0.05\pi - 0.25mc \]
Phillips Curve

- Standard NKPC
  \[ \dot{\pi} = 0.05\pi - 1.05mc \]

- Oligopoly with \( n = 3 \) (AIK calibration)
  - MPE
    \[ \dot{\pi} = 0.07\pi - 0.27mc \]
    \[ +1.33\ddot{\pi} + 0.44mic + 0.03(r - \rho) \]
  - Naive
    \[ \dot{\pi} = 0.05\pi - 0.25mc \]

Good approximation?
3-Eq Oligopoly NK

- Euler equation + Taylor Rule
  \[ \dot{c} = \sigma^{-1} (r - \pi - \rho - \epsilon^r) \]
  \[ r = \rho + \phi \pi + \epsilon^m \]

- AR(1) $\epsilon^r$, $\epsilon^m$ shocks fit basic Phillips curve…
  - exactly with $\kappa \approx \kappa^{Naive}$

- Other shocks fit very well…
  - zero lower bound
  - News shocks
3-Eq Oligopoly NK

- Euler equation + Taylor Rule
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- AR(1) $\epsilon^r, \epsilon^m$ shocks fit basic Phillips curve…
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Takeaway: basic NK Phillips curve excellent approximation!
Figure 8: Effective slope of the Phillips curve $\hat{\kappa}$, strategic vs. naive oligopoly.

Note: Left panel: $\hat{\kappa} = x + r y + s \cdot p(0) c(0)$ as a function of $n$ following an interest rate shock $#m_0$ with decay $x = 10$ under AIK calibration. Solid black line: (strategic) oligopolistic Phillips curve (19). Dashed gray line: $\kappa_{Naive}$. Right panel: Ratio $\hat{\kappa}/\kappa_{Naive}$ as a function of $n$. The left panel of Figure 8 compares $\hat{\kappa}$ to $\kappa_{Naive}$ as a function of $n$ (under the AIK calibration); the right panel shows the ratio $\hat{\kappa}/\kappa_{Naive}$ as a measure of strategic effects. The message is consistent with what we found for permanent money shocks: concentration amplifies monetary non-neutrality by a significant amount. The left panel shows that a large part of the amplification can again be explained by feedback effects, that is, through the lens of the naive model. For low $n$ strategic effects are more substantial than we found earlier: the naive model actually underestimates the effective slope $\hat{\kappa}$ by around 30% when $n = 3$. But strategic effects vanish rapidly as $n$ increases. One can recover the permanent money shock case from Proposition 3 by setting $x = l (1 + B)$ since then $f_p p + #m = 0$ so $R(t)$ is unchanged. The shock is very transitory as the exponential decay $x$ is set at 10; more persistent shocks bring $\hat{\kappa}/\kappa_{Naive}$ even closer to 1.
News and ZLB

Figure A.8: Impulse responses for consumption and inflation following date-0 news about monetary policy shock happening at \( t_{\text{shock}} \) indicated by the vertical line.

Note: \( n = 3 \) firms with AIK calibration. Solid black line: Strategic oligopoly. Dashed gray line: Naive model.

\( c \) and \( p \) denote log-deviations from steady state values in \%. 

\( n = 3 \) firms
News and ZLB

$t_{\text{shock}} = 1$

$t_{\text{shock}} = 2$

$t_{\text{shock}} = 3$

$n = 3$ firms

Figure A.8: Impulse responses for consumption and inflation following date-0 news about monetary policy shock happening at $t_{\text{shock}}$ indicated by the vertical line.

Note: $n = 3$ firms with AIK calibration. Solid black line: Strategic oligopoly. Dashed gray line: Naive model. $c$ and $\pi$ denote log-deviations from steady state values in %.

Figure A.9: Date-0 consumption and inflation in a liquidity trap lasting from $t = 0$ to $t = T$, for different values of $T$.

Note: $n = 3$. Solid black line: $n = 3$. Dotted gray line: $n = \cdots$. $c$ and $\pi$ denote log-deviations from steady state values in %.

Figure A.10: Date-0 consumption and inflation in a liquidity trap lasting from $t = 0$ to $t = T$, for different values of $T$.

Note: $n = 3$ firms with AIK calibration. Solid black line: Strategic oligopoly. Dashed gray line: Naive model. $c$ and $\pi$ denote log-deviations from steady state values in %.
Summary

Results...

1. Oligopoly tractable!
2. Sufficient statistic formula
3. Comparative Statics in n: big amplification when calibrated to pass-through
4. Naive/Kimball connection
5. Standard NK Phillips curve good fit
Teaser: Future Work
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- Non-Markov equilibria? Trigger strategies
Teaser: Future Work

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- Abreu-Pearce-Stachetti methods?
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- Folk-theorem $\rho \to 0$ then collusive outcome
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Finding: CES + Collusion
Teaser: Future Work

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**Finding:** CES + Collusion