OA.1 Formal Details Omitted from Section 2

OA.1.1 Strategy Mappings
The map $\sigma_{\delta,\gamma} : \Delta(H_1)^{\Theta} \times \Delta(H_2) \rightarrow \Pi_1 \times \Pi_2$ taking the state in period $t$ to the aggregate strategy profile has component mappings $\sigma_1^{\delta,\gamma} : \Delta(H_1)^{\Theta} \rightarrow \Pi_1$ and $\sigma_2 : \Delta(H_2) \rightarrow \Pi_2$ given by

$$\sigma_1^{\delta,\gamma}(\mu_1)[s,m|\theta] = \sum_{h_1 : x_1^{\delta,\gamma}(h_1) = (s,m)} \mu_\theta[h_1] \text{ for all } s \in S, m \in M, \theta \in \Theta,$$

$$\sigma_2(\mu_2)[a|s,m] = \sum_{h_2 : y_2(h_2) = a} \mu_2[h_2] \text{ for all } a \in A, s \in S, m \in M.$$

OA.1.2 Update Rule
We define the various component mappings of the rule, $f_{\delta,\gamma}^{:\delta,\gamma} : \Delta(H_1)^{\Theta} \times \Delta(H_2) \rightarrow \Delta(H_1)^{\Theta} \times \Delta(H_2)$, taking the state in period $t$ to the state in period $t + 1$. The mapping $f_{\theta}^{\delta,\gamma} : \Delta(H_1)^{\Theta} \times \Delta(H_2) \rightarrow \Delta(H_1)$ taking the state in period $t$ to the distribution over type $\theta$ sender histories at period $t + 1$ is given by

$$f_{\theta}^{\delta,\gamma}(\mu)[\emptyset] = 1 - \gamma_1,$$

$$f_{\theta}^{\delta,\gamma}(\mu)[(h_1, (s, m, a))] = \gamma_1 \mu_\theta[h_1] i_\theta^{\delta,\gamma}(h_1, s, m) \sigma_2(\mu)[a|s,m] \forall h_1 \in H_1, s \in S, m \in M, a \in A.$$
where \((h_1, (s, m, a)) \in H_1\) is the concatenation of the history \(h_1 \in H_1\) with a period where the sender plays \((s, m)\) and the receiver responds with \(a\), and \(i(h_1, s, m) \mathbb{I}_{(\alpha s_1^{-1} - (s, m))}(h_1)\) equals 1 if a type \(\theta\) sender with history \(h_1\) plays \((s, m)\) under policy \(x_\theta\) and equals 0 otherwise. Likewise, the mapping \(f_2^{\alpha s_1^{-1} - (s, m)} : \Delta(H_1)^{\Theta} \times \Delta(H_2) \to \Delta(H_1)\) taking the state in period \(t\) to the distribution over receiver histories at period \(t + 1\) is given by

\[
f_2^{\alpha s_1^{-1} - (s, m)}(\mu)[\emptyset] = 1 - \gamma_2, \\
f_2^{\alpha s_1^{-1} - (s, m)}(\mu)[(h_2, (\theta, s, m))] = \gamma_2 \mu_2[h_2] \lambda(\theta) \sigma_1^{\alpha s_1^{-1} - (s, m)}(s, m|\theta) \forall h_2 \in H_2, \theta \in \Theta, s \in S, m \in M,
\]

where \((h_2, (\theta, s, m)) \in H_2\) is the concatenation of the history \(h_2 \in H_2\) with a period where the receiver is matched with a type \(\theta\) sender who plays \((s, m)\).

### OA.1.3 Aggregate Response Mappings

Here we define the aggregate response mappings, \(R_1^{\alpha s_1^{-1} - (s, m)} : \Pi \to \Pi_1\) and \(R_2^{\alpha s_1^{-1} - (s, m)} : \Pi_1 \to \Pi_2\). To do so, we first define two mappings, \(L_1^{\alpha s_1^{-1} - (s, m)} : \Pi \to \Delta(H_1)^{\Theta}\) and \(L_2^{\alpha s_1^{-1} - (s, m)} : \Pi_1 \to \Delta(H_2)\). \(L_1^{\alpha s_1^{-1} - (s, m)}\) takes a fixed receiver aggregate behavior strategy and outputs the shares of the sender types with the various possible histories. For each \(\theta \in \Theta\), let

\[
L_\theta^{\alpha s_1^{-1} - (s, m)}(\pi_2)[\emptyset] = 1 - \gamma_1, \\
L_\theta^{\alpha s_1^{-1} - (s, m)}(\pi_2)[h_1] = (1 - \gamma_1) \gamma_1^{h_1} \times_{t \leq |h_1|} i_\theta^{\alpha s_1^{-1} - (s, m)}(h(t), h(t)[X]) \pi_2[h(t)[A]|h(t)[X]] \text{ if } h_1 \neq \emptyset,
\]

where \(|h_1|\) is the length of history \(h_1 \in H_1\), \(h(t)\) denotes the \(t\)-th observation in history \(h_1\), \(h(t) = (h(0), ..., h(t - 1))\), \(h(t)[A]\) denotes the receiver action played in the \(t\)-th observation of history \(h_1\), and \(h(t)[X]\) denotes the sender signal-message pair played in the \(t\)-th observation of history \(h_1\). \(L_2^{\alpha s_1^{-1} - (s, m)}\) takes a fixed sender aggregate behavior strategy and gives the shares of receivers with the various possible histories according to

\[
L_2^{\alpha s_1^{-1} - (s, m)}(\pi_1)[\emptyset] = 1 - \gamma_2, \\
L_2^{\alpha s_1^{-1} - (s, m)}(\pi_1)[h_2] = (1 - \gamma_2) \gamma_2^{h_2} \times_{t \leq |h_2|} \lambda(h(t)[\Theta]) \pi_1[h(t)[X]|h(t)[\Theta]] \text{ if } h_2 \neq \emptyset,
\]
where \(|h_2|\) is the length of history \(h_2 \in \mathcal{H}_2\), \(h_2(t)\) denotes the \(t\)-th observation in history \(h_2\), \(h_2(t)[\Theta]\) denotes the sender type in the \(t\)-th observation of history \(h_2\), and \(h_2(t)[X]\) denotes the sender signal-message pair played in the \(t\)-th observation of history \(h_2\).

\(\mathcal{R}^{\delta,\gamma}_1\) is then given by \(\mathcal{R}^{\delta,\gamma}_1 = \sigma^{\delta,\gamma}_1(\mathcal{L}^{\delta,\gamma}_1(\pi_2))\), and \(\mathcal{R}^{\gamma}_2\) is given by \(\mathcal{R}^{\gamma}_2 = \sigma_2(\mathcal{L}^{\gamma}_2(\pi_2))\).

### OA.2 Equivalent Definition of JCE

We show that it would be equivalent to define JCE by setting \(\Theta^\dagger(s, \pi) = \{\theta \in \Theta : \tilde{D}^0_\theta(s, \pi) \nleq \cup_{\theta' \neq \theta} \tilde{D}_{\theta'}(s, \pi)\}\), rather than \(\Theta^\dagger(s, \pi) = \{\theta \in \Theta : \tilde{D}_\theta(s, \pi) \cup \tilde{D}^0_\theta(s, \pi) \nleq \cup_{\theta' \neq \theta} \tilde{D}_{\theta'}(s, \pi)\}\).

For every \(s \in S\) and \(\pi \in \Pi_1 \times \Pi_2\), let

\(\Theta^\dagger(s, \pi) = \{\theta \in \Theta : \tilde{D}^0_\theta(s, \pi) \nleq \cup_{\theta' \neq \theta} \tilde{D}_{\theta'}(s, \pi)\}\)

be the set of types \(\theta\) where there is some mixed receiver action \(\alpha \in \Delta(BR(\Theta, s))\) that makes \(\theta\) indifferent between \((s, \alpha)\) and their outcome under \(\pi\) and makes no other type \(\theta'\) strictly prefer \((s, \alpha)\) to their outcome under \(\pi\). Additionally, let

\[
\Theta'(s, \pi) = \begin{cases} 
\Theta^\dagger(s, \pi) & \text{if } \Theta^\dagger(s, \pi) \neq \emptyset \\
\emptyset & \text{if } \Theta^\dagger(s, \pi) = \emptyset 
\end{cases}
\]

**Proposition OA 1.** If \(\pi\) is a PBE-H, then \(\overline{\Theta}(s, \pi) = \overline{\Theta}'(s, \pi)\) for all \(s \in S\).

**Proof.** Fix PBE-H \(\pi\). We will argue that \(\Theta^\dagger(s, \pi) = \Theta^\dagger(s, \pi)\), which gives \(\overline{\Theta}'(s, \pi) = \overline{\Theta}(s, \pi)\).

First, suppose that \(\theta \in \Theta^\dagger(s, \pi)\). Then by definition, \(\tilde{D}^0_\theta(s, \pi) \nleq \cup_{\theta' \neq \theta} \tilde{D}_{\theta'}(s, \pi)\). Hence, \(\tilde{D}_\theta(s, \pi) \cup \tilde{D}^0_\theta(s, \pi) \nleq \cup_{\theta' \neq \theta} \tilde{D}_{\theta'}(s, \pi)\), so \(\theta \in \Theta^\dagger(s, \pi)\).

Now, suppose that \(\theta \in \Theta^\dagger(s, \pi)\). Then by definition, \(\tilde{D}_\theta(s, \pi) \cup \tilde{D}^0_\theta(s, \pi) \nleq \cup_{\theta' \neq \theta} \tilde{D}_{\theta'}(s, \pi)\). Thus, there is some \(\alpha \in \Delta(BR(\Theta, s))\) such that \(u_1(\theta, s, \alpha) \geq u_1(\theta, \pi)\) and \(u_1(\theta, s, \alpha) \leq u_1(\theta', \pi)\) for all \(\theta' \neq \theta\). Since \(\pi\) is a PBE-H, there is also some \(\alpha' \in \Delta(BR(\Theta, s))\) such
that $u_1(\theta', s, \alpha') \leq u_1(\theta, \pi)$ for all $\theta' \in \Theta$. By continuity, there is some $\nu \in [0, 1]$ and $\alpha'' = \nu \alpha + (1 - \nu) \alpha'$ such that $u_1(\theta, s, \alpha'') = u_1(\theta, \pi)$, while $u_1(\theta', s, \alpha'') \leq u_1(\theta', \pi)$ for all $\theta' \neq \theta$. As $\alpha'' \in \Delta(BR(\Theta, s))$, it follows that $\tilde{D}^0_\theta(s, \pi) \not\subseteq \cup_{\theta' \neq \theta} \tilde{D}_{\theta'}(s, \pi)$, so $\theta \in \Theta^\dagger(s, \pi)$.

\section{OA.3 Proof of Proposition 9}

**Proposition 9.** If $\pi$ is a uniformly justified JCE in a strictly monotonic signaling game, it induces the same distribution over $\Theta \times S \times A$ as a stable profile for all non-doctrinaire priors $g_1, g_2$, including those that do not satisfy initial trust.

**Proof.** Because $\pi$ is a uniformly justified JCE in a strictly monotonic signaling game, $\pi_2(\cdot|s, m) = \pi_2(\cdot|s, m')$ for all $s \in S$ and $m, m' \in M$ such that $(s, m), (s, m') \in X^\text{on}$. Thus, for every $s \in S^\text{on}$, there is some $a_s \in A$ such that $\pi_2(a_s|s, m) = 1$ for all $(s, m) \in X^\text{on}$. For all $s \in S^\text{off}$, fix some $a_s \in BR(\Theta(s, \pi), s)$.

Our construction modifies the aggregate receiver response so that the response to any $s$ is $a_s$ with high probability unless the aggregate sender play is such that each type $\theta \in \Theta$ uses $s_{\theta}$ with sufficiently high probability. We show that the fixed points of this modified aggregate response mapping correspond to fixed points of the true aggregate response mapping in the iterated limit where $\gamma_1 \to 1$ then $\gamma_2 \to 1$. Moreover, we show that the limit of these steady state profiles induce the same distribution over $\Theta \times S \times A$ as $\pi$.

Because $\pi$ is a uniformly justified JCE in a strictly monotonic signaling game, there is an $\varepsilon > 0$ such that the following two properties hold. First, when $\pi_2(a_s|s, m) \geq 1 - \varepsilon$ for all $s$, playing $s_{\theta}$ paired with message $m$ is strictly better for type $\theta$ than playing any other $s' \neq s_{\theta}$ paired with any $m'$. Second, if $\pi_1(s_{\theta}, m|\theta) \geq 1 - \varepsilon$ for every $\theta \in \Theta$, it is strictly optimal for the receiver to respond to $(s, m)$ with $a_s$ for every $s \in S^\text{on}$. Fix such an $\varepsilon$.

Let $\kappa : \mathbb{R} \to [0, 1]$ be a continuous function such that $\kappa(z) = 0$ for all $z \leq 0$ and
\( \kappa(z) = 1 \) for all \( z \geq 1 \). \( \kappa(z) = \max\{\min\{z, 1\}, 0\} \) is an example of such a \( \kappa \). Also, let \( \phi : \Pi_1 \times \Pi_2 \rightarrow \Pi_2 \) be the mapping

\[
\phi(\pi_1, \pi_2)(s, m) = \left(1 - \kappa \left( 2 \left( \min_{\theta \in \Theta} \pi_1(s_\theta | \theta) - 1 + \varepsilon \right) \right) \right) \mathbb{1}_{a_s}(\cdot) + \kappa \left( 2 \left( \min_{\theta \in \Theta} \pi_1(s_\theta | \theta) - 1 + \varepsilon \right) \right) \pi_2(s, m)
\]

for all \( s \in S \) and \( m \in M \). Note that \( \phi \) is continuous. Additionally, \( \phi(\pi_1, \pi_2)(a_s | s, m) = 1 \) when \( \pi_1(s_\theta | \theta) \leq 1 - \varepsilon \) for some \( \theta \in \Theta \), and \( \phi(\pi_1, \pi_2) = \pi_2 \) when \( \pi_1(s_\theta | \theta) \geq 1 - \varepsilon / 2 \) for all \( \theta \in \Theta \).

Consider the correspondence \( \tilde{\Theta}^{\delta, \gamma_1, \gamma_2} : \Pi_1 \times \Pi_2 \rightarrow \Pi_1 \times \Pi_2 \) given by \( \tilde{\Theta}^{\delta, \gamma_1, \gamma_2}(\pi) = (\Theta^{\delta, \gamma_1}(\pi_2), \phi(\pi_1, \Theta^{\delta, \gamma_2} \pi)) \). Since \( \tilde{\Theta}^{\delta, \gamma_1, \gamma_2} \) is continuous, Brouwer’s fixed point theorem guarantees the existence of a fixed point \( (\pi_1^{\delta, \gamma_1, \gamma_2}, \pi_2^{\delta, \gamma_1, \gamma_2}) \). We will establish that, in the iterated limit where \( \gamma_1 \rightarrow 1 \) then \( \delta \rightarrow 1 \) then \( \gamma_2 \rightarrow 1 \), \( \pi^{\delta, \gamma_1, \gamma_2} = (\pi_1^{\delta, \gamma_1, \gamma_2}, \pi_2^{\delta, \gamma_1, \gamma_2}) \) induces the same distribution over \( \Theta \times S \times A \) as \( \pi \). Towards this end, consider a sequence \( \{\gamma_{2,j}\}_{j \in \mathbb{N}} \), sequences \( \{\delta_{j,k}\}_{j,k \in \mathbb{N}} \), and sequences \( \{\gamma_{1,j,k,l}\}_{j,k,l \in \mathbb{N}} \) such that (1) \( \lim_{j \rightarrow \infty} \gamma_{2,j} = 1 \), (2) \( \lim_{k \rightarrow \infty} \delta_{j,k} = 1 \) for all \( j \), (3) \( \lim_{l \rightarrow \infty} \gamma_{1,j,k,l} = 1 \) for all \( j, k \), and (4) \( \lim_{l \rightarrow \infty} \lim_{k \rightarrow \infty} \lim_{j \rightarrow \infty} \pi^{\delta_{j,k}, \gamma_{1,j,k,l}, \gamma_{2,j}} = \pi' \) for some \( \pi' = (\pi_1', \pi_2') \in \Pi_1 \times \Pi_2 \).

We first establish that \( \pi_1'(s_\theta | \theta) \geq 1 - \varepsilon \) for all \( \theta \in \Theta \). If instead there were some \( \theta \in \Theta \) such that \( \pi'(s_\theta | \theta) < 1 - \varepsilon \), then by construction, \( \pi_2'(a_s | s, m) \geq 1 - \varepsilon \) for all \( s \in S \) and \( m \in M \). Lemma 8 thus requires that \( \pi_1'(s_\theta | \theta) = 1 \) for all \( \theta \in \Theta \), which is a contradiction.

Next we show that \( \pi_2'(a_s | s, m) = 1 \) for all \( s \in S^{on} \) and \( m \in M \) such that \( \pi_1'(s, m | \theta) > 0 \) for some \( \theta \in \Theta \). Fix \( s \in S^{on} \). Consider \( m, m' \in M \) such that \( \pi_1'(s, m | \theta) > 0 \) and \( \pi_1'(s, m' | \theta') > 0 \) for some \( \theta, \theta' \in \Theta \). The construction of \( \tilde{\Theta}^{\delta, \gamma_1, \gamma_2} \), along with an almost identical argument to the one which proves Lemma 2, implies that there exists some \( \xi \in [0, 1] \) and \( \alpha, \alpha' \in MBR(\Theta, s) \) such that \( \pi_2'(s, m) = (1 - \xi) \mathbb{1}_{a_s}(\cdot) + \xi \alpha \) and \( \pi_2'(s, m') = (1 - \xi) \mathbb{1}_{a_s}(\cdot) + \xi \alpha' \). In fact, \( \alpha \) and \( \alpha' \) must be optimal responses to \( s \) under the posterior distributions obtained by updating \( \lambda \) under Bayes’ rule using \( \{\pi_1'(s, m | \theta)\}_{\theta \in \Theta} \) and \( \{\pi_1'(s, m' | \theta)\}_{\theta \in \Theta} \), respectively. Because the game is strictly monotonic, Lemma 8 implies that \( \alpha = \alpha' \). Thus, for a given \( s \), \( \pi_2'(s, m) \) is the same for all \( m \in M \) for
which there is a \( \theta' \in \Theta \) such that \( \pi'_1(s, m|\theta') > 0 \). Combining this with the fact that 
\[ \pi'_1(s_0|\theta) \geq 1 - \varepsilon \] 
for all \( \theta \), it follows that \( \pi'_2(a_s|s, m) = 1 \) for all \( m \in M \) such that \( \pi'_1(s, m|\theta) > 0 \) for some \( \theta \in \Theta \).

Since \( \pi'_2(a_s|s, m) = 1 \) for all \( s \in S^m \) and \( m \in M \) such that \( \pi'_1(s, m|\theta) > 0 \) for some \( \theta \in \Theta \), it follows from Lemma 8 that \( \pi'_1(s|\theta) = 0 \) whenever \( s \in S^m \) and \( s \neq s_0 \).

We now show that for all \( \theta \in \Theta \), \( \pi'_1(s|\theta) = 0 \) for all \( s \in S^{off} \). Note that, because \( \pi_1(s, \theta) > 0 \) for all \( \theta \in \Theta \) and \( \pi_2(a_s|s, m) = 1 \) for all \( \theta \in \Theta \) and \( m \in M \) where \( \pi_1(s, m|\theta) > 0 \), Lemma 8 implies that \( u_1(\theta, \pi') = u_1(\theta, s_0, a_{s_0}) = u_1(\theta, s, \pi) \) for all \( \theta \in \Theta \).

Additionally, Lemma 8 requires that \( u_1(\theta, s, \pi'_2(\cdot|s, m)) \leq u_1(\theta, \pi') = u_1(\theta, s, \pi) \) for all \( \theta \in \Theta \), \( s \in S \), and \( m \in M \). Now, suppose that there is some \( s \in S^{off} \) and \( m \in M \) such that \( \pi'_1(s, m|\theta) > 0 \) for some \( \theta \in \Theta \). There are two possible cases: (1) There is some \( \theta \notin \Theta(s, \pi) \) such that \( \pi'_1(s, m|\theta) > 0 \), and (2) All \( \theta \) with \( \pi'_1(s, m|\theta) > 0 \) belong to \( \Theta(s, \pi) \). In Case (1), because \( \pi'_2(\cdot|s, m) \in \Delta(BR(\Theta, s)) \), there must be some \( \theta' \in \Theta(s, \pi) \) such that \( u_1(\theta', s, \pi'_2(\cdot|s, m)) > u_1(\theta', s, \pi) \), which is a contradiction. In Case (2), the construction of \( \mathcal{R}^{\delta, \gamma_1, \gamma_2} \), combined with an almost identical argument to the one behind Lemma 2, implies that \( \pi'_2(\cdot|s, m) \in \Delta(BR(\Theta(s, \pi), s)) \). Since \( \pi \) is a uniformly justified JCE, it follows that \( u_1(\theta, s, \pi'_2(\cdot|s, m)) < u_1(\theta, \pi) \) for all \( \theta \in \Theta \), but this, along with Lemma 8, implies that \( \pi'_1(s, m|\theta) = 0 \) for all \( \theta \in \Theta \), a contradiction.

It follows that \( \pi'_1(s_0|\theta) = 1 \) for all \( \theta \) and \( \pi'_2(a_s|s, m) = 1 \) for all \( s \in S^m \) and \( m \in M \) such that \( \pi'_1(s, m|\theta) > 0 \) for some \( \theta \in \Theta \). Thus, \( \pi^{\delta, \gamma_1, \gamma_2} \) induces the same distribution over \( \Theta \times S \times A \) as \( \pi \) in the iterated limit where first \( \gamma_1 \to 1 \) then \( \delta \to 1 \) then \( \gamma_2 \to 1 \). Moreover, since \( \pi'_1(s_0|\theta) = 1 \) for all \( \theta \in \Theta \), \( \pi^{\delta, \gamma_1, \gamma_2}_2 = \phi(\pi^{\delta, \gamma_1, \gamma_2}_1, \mathcal{R}^{\delta, \gamma_1, \gamma_2}_2(\pi^{\delta, \gamma_1, \gamma_2}_1)) = \mathcal{R}^{\gamma_2}_2(\pi^{\delta, \gamma_1, \gamma_2}_1) \) in the iterated limit. Thus, \( \pi^{\delta, \gamma_1, \gamma_2} \) is a fixed point of \( \mathcal{R}^{\delta, \gamma_1, \gamma_2}_2 \) in the iterated limit, which means that \( \pi' \) is a stable profile. 

\[ \blacksquare \]

**OA.4 Proof of Proposition 5**

**Proposition 5.** The game in Example 1 has stable profiles where both types play Out with probability 1, even though this is not the outcome of a PBE.
Proof. We specify that the receiver prior $g_2$ is a Dirichlet distribution with initial weight 1 on $(\theta_1, In, m_{In, \theta_1})$ and $1/2$ on $(\theta_2, In, m_{In, \theta_1})$, and, for all other messages $m \neq m_{In, \theta_1}$, initial weight $1/2$ on $(\theta_1, In, m)$ and 1 on $(\theta_2, In, m)$. This means that initial trust is satisfied: When a receiver first encounters a sender who plays $(In, m_{In, \theta})$, the probability they place on the receiver having type $\theta$ is $2/3$ so $BR(\theta, In)$ is optimal.

We claim first that if a receiver has encountered past plays of $(In, m)$ and all such plays have been by senders with the same type $\theta$, then the receiver will respond to the next instance of $(In, m)$ with $BR(\theta, In)$. We demonstrate this for the case $m = m_{\theta_1}$; analogous arguments handle the other case. If this message has only ever been sent by $\theta_1$, the receiver’s belief about the sender’s type after $(In, m_{\theta_1})$ must put probability at least $(1 + 1)/(1 + 1 + .5) = 4/5$ on $\theta_1$, which makes $a_1$ the unique receiver best response. When $\theta = \theta_2$, the receiver’s conditional distribution over the sender’s type after $(In, m_{\theta_1})$ must put probability at least $(1 + .5)/(1 + 1 + .5) = 3/5$ on $\theta_2$, which makes $a_2$ the unique receiver best response.

We focus on steady state profiles in which, for every $m \in M$, the aggregate probability that a receiver responds to $(In, m)$ with $a_3$ is less than $1/4$. Under such responses, it can never be weakly optimal for both types to play $In$ with the same message. To see this, note that

$$u_1(\theta_1, In, \alpha) + u_2(\theta_2, In, \alpha) = -\alpha[a_1] - \alpha[a_2] + 2\alpha[a_3] = -1 + 3\alpha[a_3],$$

which is strictly negative whenever $\alpha[a_3] \leq 1/4$. We argue that such steady state profiles exist in the iterated limit where $\gamma_1 \to 1$ then $\delta \to 1$ then $\gamma_2 \to 1$ and that the corresponding aggregate probability that either sender type plays $In$ converges to 0.

Let $\chi : \Delta(A) \Rightarrow \Delta(A)$ be the correspondence given by

$$\chi(\alpha) = \begin{cases} \{\alpha\} & \text{if } \alpha[a_3] \leq \frac{1}{4} \\ \{\alpha' \in \Delta(A) : \alpha'[a_3] = \frac{1}{4}\} & \text{if } \alpha[a_3] > \frac{1}{4} \end{cases},$$
and let \( \rho : \Pi_2 \Rightarrow \Pi_2 \) be the correspondence given by

\[
\rho(\pi_2) = \{ \pi'_2 \in \Pi_2 : \pi'_2(\cdot|In, m) \in \chi(\pi_2(\cdot|In, m)) \ \forall m \in M \}.
\]

Note that \( \rho \) is upper hemicontinuous and coincides with the identity correspondence whenever \( \pi_2(\cdot|In, m) \leq 1/4 \) for all \( m \).

Consider the correspondence \( \tilde{\mathcal{R}}^\delta_{\gamma_1,\gamma_2} : \Pi_1 \times \Pi_2 \Rightarrow \Pi_1 \times \Pi_2 \) given by \( \tilde{\mathcal{R}}^\delta_{\gamma_1,\gamma_2}(\pi_1, \pi_2) = \{ (\pi'_1, \pi'_2) \in \Pi_1 \times \Pi_2 : \pi'_1 = \mathcal{R}_1^\delta_{\gamma_1}(\pi_2) \text{ and } \pi'_2 \in \rho(\mathcal{R}_2^{\gamma_2}(\pi_1)) \} \). Since \( \mathcal{R} \) is upper hemicontinuous, Kakutani’s fixed point theorem guarantees the existence of a fixed point \( (\tilde{\pi}'_1, \tilde{\pi}'_2) \approx (\tilde{\pi}'_1, \tilde{\pi}'_2) \). As \( \pi^\delta_{\gamma_1,\gamma_2}(a_3|s, m) \leq 1/4 \) for all \( (s, m) \) by construction, Lemma 8 implies that, for all \( \gamma_2 \in [0, 1) \) and \( (s, m) \), either \( \lim_{\gamma_1 \to 1} \pi^\delta_{\gamma_1,\gamma_2}[In, m|\theta_1] = 0 \) or \( \lim_{\gamma_1 \to 1} \pi^\delta_{\gamma_1,\gamma_2}[In, m|\theta_2] = 0 \). This means that, as \( \gamma_1 \to 1 \) then \( \delta \to 1 \), the probability that a receiver encounters senders with both types that pair \( In \) with the same message \( m \) approaches 0. Since a receiver would only ever play \( a_3 \) in response to \( (In, m) \) if they have previously encountered senders of both types play \( (In, m) \), this means that \( \lim_{\gamma_1 \to 1} \lim_{\delta \to 1} \mathcal{R}_2^{\gamma_2}(\pi^\delta_{\gamma_1,\gamma_2}(a_3|In, m) = 0 \) for all \( m \in M \). Since \( \rho(\pi_2) = \{ \pi_2 \} \) if \( \pi_2(a_3|In, m) \leq 1/4 \) for all \( m \), \( \pi^\delta_{\gamma_1,\gamma_2} = \rho(\mathcal{R}_2^{\gamma_2}(\pi^\delta_{\gamma_1,\gamma_2})) = \mathcal{R}_2^{\gamma_2}(\pi^\delta_{\gamma_1,\gamma_2}) \) for fixed \( \gamma_2 \in [0, 1) \) when \( \delta \) is sufficiently close to 1 and, given \( \delta \), \( \gamma_1 \) is sufficiently close to 1. Thus, for fixed \( \gamma_2 \in [0, 1) \), \( (\pi^\delta_{\gamma_1,\gamma_2}, \pi^\delta_{\gamma_1,\gamma_2}) \) is a fixed point of \( \tilde{\mathcal{R}}^\delta_{\gamma_1,\gamma_2} \) when \( \delta \) is sufficiently close to 1 and, given \( \delta \), \( \gamma_1 \) sufficiently close to 1.

To show that \( \lim_{\gamma_2 \to 1} \lim_{\delta \to 1} \lim_{\gamma_1 \to 1} \pi^\delta_{\gamma_1,\gamma_2}[In] = 0 \), suppose towards a contradiction that there is a sequence of receiver continuation probabilities \( \{ \gamma_{2,j} \}_{j \in \mathbb{N}} \), a collection of sequences of sender discount factors \( \{ \delta_{j,k} \}_{j,k \in \mathbb{N}} \), and a collection of sequences of sender continuation probabilities \( \{ \gamma_{1,j,k,l} \}_{j,k,l \in \mathbb{N}} \) such that (a) \( \lim_{j \to \infty} \gamma_{2,j} = 1 \), (b) \( \lim_{k \to \infty} \delta_{j,k} = 1 \) for all \( j \), (c) \( \lim_{l \to \infty} \gamma_{1,j,k,l} = 1 \) for all \( j, k, l \), and (d) \( \lim_{j \to \infty} \lim_{k \to \infty} \lim_{l \to \infty} \pi^\delta_{j,k,j,k,l,j,l,\gamma_{1,j,k,l}}[In, m|\theta] > 0 \) for some \( \theta \in \Theta \) and \( m \in M \). Without loss of generality, take \( \theta = \theta_1 \). By what we have shown, it must be that \( \lim_{k \to \infty} \lim_{l \to \infty} \pi^\delta_{j,k,j,k,l,j,l,\gamma_{1,j,k,l}}[In, m|\theta_2] = 0 \) for all sufficiently large \( j \). Combining this with \( \lim_{j \to \infty} \lim_{k \to \infty} \lim_{l \to \infty} \pi^\delta_{j,k,j,k,l,j,l,\gamma_{1,j,k,l}}[In, m|\theta_1] > 0 \) and \( \lim_{j \to \infty} \gamma_{2,j} = 1 \) gives
\[
\lim_{j \to \infty} \lim_{k \to \infty} \lim_{l \to \infty} \pi_2^{\delta_j, k, \gamma_1, j, k, l, \gamma_2, j} (a_1 | s, m) = 1, \text{ because with probability 1 every receiver encounters a type } \theta_1 \text{ sender playing } (In, m) \text{ but never encounters a type } \theta_2 \text{ sender playing } (In, m). \text{ However, since } u_1(\theta_1, In, a_1) < 0, \lim_{j \to \infty} \lim_{k \to \infty} \lim_{l \to \infty} \pi_2^{\delta_j, k, \gamma_1, j, k, l, \gamma_2, j} (a_1 | s, m) = 1 \text{ combined with Lemma 8 requires } \lim_{j \to \infty} \lim_{k \to \infty} \lim_{l \to \infty} \pi_2^{\delta_j, k, \gamma_1, j, k, l, \gamma_2, j} [In, m | \theta_1] = 0, \text{ a contradiction.} \]

### OA.5 Proof of Lemma 7

**Lemma 7.** If \( \pi \) is a JCE, then for every \( s \in S \), either

1. \( \Theta^\dagger(s, \pi) \neq \emptyset \), or
2. \( u_1(\theta, s, a) < u_1(\theta, \pi) \) for all \( \theta \in \Theta \) and \( a \in BR(\Theta, s) \).

**Proof.** Let \( \pi \) be a JCE. Fix \( s \in S \) and suppose that \( \Theta^\dagger(s, \pi) = \emptyset \). Let \( A_- = \{ \alpha \in \Delta(BR(\Theta, s)) : u_1(\theta, s, \alpha) < u_1(\theta, \pi) \ \forall \theta \in \Theta \} \) be the set of mixtures over receiver best responses that make playing \( s \) strictly worse for every type than their outcome under \( \pi \). Similarly, let \( A_+ = \{ \alpha \in \Delta(BR(\Theta, s)) : \exists \theta \in \Theta \text{ s.t. } u_1(\theta, s, \alpha) > u_1(\theta, \pi) \} \) be the set of mixtures over receiver best responses that make some type strictly better off by playing \( s \) than receiving their outcome under \( \pi \). \( A_- \) and \( A_+ \) are disjoint open subsets of \( \Delta(BR(\Theta, s)) \), and \( A_- \cup A_+ = \Delta(BR(\Theta, s)) \text{ since } \Theta^\dagger(s, \pi) = \emptyset \). As \( \Delta(BR(\Theta, s)) \) is connected, either \( A_- = \Delta(BR(\Theta, s)) \) or \( \Delta(BR(\Theta, s)) = A_+ \). \( \Delta(BR(\Theta, s)) = A_+ \) is not possible when \( \pi \) is a JCE since then, for every \( \alpha \in \Delta(BR(\overline{\Theta}(s, \pi), s)) \), there is some \( \theta \) such that \( u_1(\theta, s, \alpha) > u_1(\theta, \pi) \). Therefore, \( \Delta(BR(\Theta, s)) = A_- \), which gives \( u_1(\theta, s, a) < u_1(\theta, \pi) \) for all \( a \in BR(\Theta, s) \). \[\Box\]

### OA.6 Analysis of Example 2

**Proposition OA 2.** If \( \pi \) is a JCE in the game in Example 2:

1. \( \pi_1(s = 0 | \theta = 1) > 0 \), and \( \pi_2(a = 20 | s = 0, m) = 1 \) for all \( m \) such that \( \pi_1(s = 0, m) > 0 \),
2. $\pi_1(s = 20|\theta = 2) > 0$, and $\pi_2(a = 40|s = 20, m) = 1$ for all $m \in M$ such that $\pi_1(s = 20, m) > 0$, and
   
   3. $\pi_1(s = 60|\theta = 3) = 1$, and $\pi_2(a = 60|s = 60, m) = 1$ for all $m \in M$ such that $\pi_1(s = 60, m) > 0$.

Condition 1 says that the $\theta = 1$ sender plays $s = 0$ with positive probability, and that the receiver responds with $a = 20$ to every on-path signal-message pair with $s = 0$. Condition 2 says that the $\theta = 2$ sender plays $s = 20$ with positive probability, and that the receiver responds with $a = 40$ to every on-path signal-message pair with $s = 20$. Condition 3 says that the $\theta = 3$ sender plays $s = 60$ with positive probability, and that the receiver responds with $a = 60$ to every on-path signal-message pair with $s = 60$.

Proof. We first establish that in any JCE $\pi$, the receiver’s response to each signal-message pair played by $\theta = 3$ has expected value of at least $60 - 10/3$. Suppose otherwise that there is some signal-message pair $(s, m)$ that $\theta = 3$ plays which induces a receiver response with expected value $\tilde{a} < 60 - 10/3$. It must be that $s < 100$ as 100 is a strictly dominated signal for types other than $\theta = 3$ and the receiver’s response to any signal-message pair played only by $\theta = 3$ must be 60 since $BR(3, s) = \{60\}$ for all $s$. Thus, $s + 10 \in S$. Note that $u_1(3, \pi) = 3\tilde{a} - s$, while $u_1(\theta, \pi) \leq \theta\tilde{a} - s$ for $\theta = 1$ and $\theta = 2$. Since $u_1(3, s + 10, a) = 3a - s - 10$, we have that $u_1(3, s + 10, a) \geq u_1(3, \pi)$ if and only if $a \geq \tilde{a} + 10/3$, with the inequality strict for all $a > \tilde{a} + 10/3$. Moreover, $u_1(\theta, s + 10, a) \geq u_1(\theta, \pi)$ for $\theta = 1$ or $\theta = 2$ only if $u_1(\theta, s + 10, a) = \theta a - s - 10 \geq \theta\tilde{a} - s$, which requires $a \geq \tilde{a} + 5$. Thus, $\Theta(s + 10, \pi) = \{3\}$, so the only justified response to $s + 10$ is 60. As this is strictly greater than $\tilde{a} + 10/3$ when $\tilde{a} < 60 - 10/3$, it follows that in every JCE, any signal-message pair played $\theta = 3$ must induce a receiver response with expected value at least $60 - 10/3$.

An immediate implication of this is that there must be some signal-message pair that $\theta = 2$ sends with positive probability that $\theta = 3$ does not send. The reason is that the receiver’s best responses to distributions where the relative weight on $\theta = 2$ versus $\theta = 3$ exceeds that under the prior are all no more than 50.
We now show that the receiver’s response to each signal-message pair played by \( \theta = 2 \) but not by \( \theta = 3 \) must have an expected value between 35 and 40. Since every receiver action strictly higher than 40 is strictly dominated whenever the probability of \( \theta = 3 \) is 0, we need only show that the expected value of the receiver response to any signal-message pair played by \( \theta = 2 \) must exceed 35. Suppose otherwise that there is some signal-message pair \((s, m)\) that \( \theta = 2 \) plays but \( \theta = 3 \) does not play which induces a receiver response with expected value \( \tilde{a} < 35 \). Again, it must be that \( s < 100 \), so \( s + 10 \in S \). Note that \( u_1(2, \pi) = 2\tilde{a} - s \), while \( u_1(1, \pi) \leq \tilde{a} - s \). Since \( u_1(2, s + 10, a) = 2a - s - 10 \), we have that \( u_1(2, s + 10, a) \geq u_1(2, \pi) \) if and only if \( a \geq \tilde{a} + 5 \), with the inequality strict for all \( a > \tilde{a} + 5 \). Moreover, \( u_1(1, s + 10, a) \geq u_1(1, \pi) \) only if \( u_1(1, s + 10, a) = a - s - 10 \geq \tilde{a} - s \), which requires \( a \geq \tilde{a} + 10 \). Thus, \( 1 \notin \Theta(s + 10, \pi) \), so justified responses to \( s + 10 \) must weakly exceed 40. As this is strictly greater than \( \tilde{a} + 5 \) when \( \tilde{a} < 35 \), it follows that in every JCE, any signal-message pair played by \( \theta = 2 \) must induce a receiver response with expected value at least 35.

There must be some signal-message pair that only \( \theta = 1 \) plays. To see this, first observe that there can be no signal-message pair played by both \( \theta = 1 \) and \( \theta = 3 \). If there were some signal-message pair \((s, m)\) played by both \( \theta = 1 \) and \( \theta = 3 \), the expected value of the receiver response \( \tilde{a} \) must be less than 50, because increasing differences in \( \theta \) and \( a \) in the sender utility function implies that every signal-message pair played by \( \theta = 2 \) must induce a receiver response with this same expected value \( \tilde{a} \). This implies that \( \tilde{a} \leq 50 \) since the receiver’s best responses to distributions where the weight on \( \theta = 2 \) exceeds that under the prior are all no more than 50. However, this contradicts the fact that every signal-message played by \( \theta = 3 \) must induce a receiver response with expected value weakly greater than \( 60 - 10/3 \). Additionally, \( \theta = 1 \) cannot only play signal-message pairs that are also played by \( \theta = 2 \). Otherwise, there would be some signal-message pair played by \( \theta = 2 \) that induces a receiver response with expected value weakly less than 30, which has already been shown to not be possible. This follows from the fact that the receiver’s best responses to distributions where the weight on \( \theta = 3 \) is 0 and the relative weight of \( \theta = 1 \) to \( \theta = 2 \) exceeds that under the
prior are all no more than 30.

Moreover, for every signal-message pair that only \( \theta = 1 \) plays, \( s = 0 \) and the receiver responds with \( a = 20 \). That the receiver responds with \( a = 20 \) to every signal-message pair that only \( \theta = 1 \) plays follows from the fact that \( BR(1, s) = \{20\} \) for all \( s \). So the payoff \( \theta = 1 \) obtains from a signal-message pair \((s, m)\) that only \( \theta = 1 \) plays is \( 20 - s \), which is strictly less than 20 for all \( s > 0 \). However, \( \theta = 1 \) can secure a payoff of 20 by simply playing \( s = 0 \), since every \( a < 20 \) is strictly dominated for the receiver. This, \( s = 0 \) for every signal-message pair that only \( \theta = 1 \) plays.

We now argue that, for every signal-message pair played by \( \theta = 2 \) but not by \( \theta = 3 \), \( s = 20 \) and the receiver responds with \( a = 40 \). We have previously established that the expected value of the receiver’s response, \( \tilde{a} \), to such a signal-message pair, \((s, m)\), must be between 35 and 40. For \( \tilde{a} < 40 \) to hold, it must be that \( \theta = 1 \) also plays this signal-message pair. This requires \( u_1(1, s, \tilde{a}) = \tilde{a} - s = u_1(1, \pi) \). As previously established, \( u_1(1, \pi) = 20 \), so it must be that \( s = \tilde{a} - 20 \). However, there is no \( \tilde{a} \in [35, 40) \) such that \( \tilde{a} - 20 \in \mathcal{S} \). Therefore, \( \tilde{a} = 40 \) so the receiver’s response is necessarily 40 since the receiver never responds to any on-path signal-message pair with a mixture over non-adjacent actions. From \( u_1(1, s, 40) = 40 - s \leq 20 = u_1(1, \pi) \), we obtain \( s \geq 20 \). All that remains is to rule out \( s \geq 30 \). If \( s \geq 30 \), \( u_1(1, s - 10, a) = a - s + 10 \geq 20 = u_1(1, \pi) \) if and only if \( a \geq 40 \). On the other hand, \( u_1(2, s - 10, a) = 2a - s + 10 \geq 80 - s = u_1(2, \pi) \) if and only if \( a \geq 35 \), with the inequality strict for all \( a > 35 \). Thus, \( 1 \notin \tilde{\Theta}(s - 10, \pi) \), so justified responses to \( s + 10 \) must weakly exceed 40. It follows that \( s = 20 \).

Finally, we show that, for every signal-message pair played by \( \theta = 3 \), \( s = 60 \) and the receiver responds with \( a = 60 \). We have previously established that the expected value of the receiver’s response, \( \tilde{a} \), to such a signal-message pair, \((s, m)\), must be between 60 – 10/3 and 60. For \( \tilde{a} < 60 \) to hold, it must be that \( \theta = 2 \) also plays this signal-message pair. This requires \( u_1(2, s, \tilde{a}) = 2\tilde{a} - s = u_1(2, \pi) \). As previously established, \( u_1(2, \pi) = 60 \), so it must be that \( s = 2\tilde{a} - 60 \). However, there is no \( \tilde{a} \in [60 - 10/3, 60) \) such that \( 2\tilde{a} - 60 \in \mathcal{S} \). Therefore, \( \tilde{a} = 60 \) so the receiver’s response is necessarily 60. From \( u_1(2, s, 60) = 120 - s \leq 60 = u_1(2, \pi) \), we obtain \( s \geq 60 \). All that remains is to
rule out $s > 60$. If $s \geq 70$, $u_1(\theta, s - 10, a) = \theta a - s + 10 \geq u_1(\theta, \pi)$ for either $\theta = 1$ or $\theta = 2$ requires that $a \geq 60$. On the other hand, $u_1(3, s - 10, a) = 3a - s + 10 \geq 180 - s = u_1(3, \pi)$ if and only if $a \geq 170/3$, with the inequality strict for all $a > 170/3$. Thus, $\Theta(s - 10, \pi) = \{3\}$, so the only justified response to $s - 10$ is 60. It follows that $s = 60$. ■

**OA.7 Omitted Examples**

**OA.7.1 Example Where Stability Does Not Imply the Intuitive Criterion without Initially Suggestible Receivers**

*Example OA 1.* The sender’s type space is $\Theta = \{\theta_1, \theta_2\}$, and $\lambda(\theta_1) = \lambda(\theta_2) = 1/2$. The sender’s signal space is $S = \{In, Out\}$, and the receiver’s action space is $A = \{a_1, a_2\}$. The payoffs to the sender and receiver are given in Table 1 below.

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<table>
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<th>$a_1$</th>
<th>$a_2$</th>
<th></th>
<th>$a_1$</th>
<th>$a_2$</th>
</tr>
</thead>
<tbody>
<tr>
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<td>$Out$</td>
<td>$-1, -1$</td>
<td>$0, 0$</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 1: The payoffs for Example OA 1.

*Out* strictly dominates *In* for type $\theta_2$, so $\theta_2$ plays *Out* in every equilibrium of this game. However, there are equilibria in which $\theta_1$ plays *In* and equilibria in which $\theta_1$ plays *Out*. The equilibria where $\theta_1$ plays *Out* do not survive the Intuitive Criterion since $a_1$ is the receiver’s unique best response to *In* when the sender’s type is $\theta_1$, and $\theta_1$ obtains a strictly higher payoff from $(In_1, a_1)$ than from playing *Out*.

We show that, when $g_2$ is such that a receiver plays $a_2$ when they first encounter a sender playing $(In, m)$ for every message $m \in M$, there are stable profiles in which $\theta_1$ plays *Out*.

We focus on steady state profiles in which the aggregate probability that a receiver responds to $(In, m)$ with $a_1$ is less than $1/3$ for every message $m \in M$, which makes it
strictly optimal for type $\theta_1$ senders to play $Out$. We show that, for fixed $\gamma_2 \in [0, 1)$, such steady state profiles exist, and, moreover, that the corresponding aggregate probability that a type $\theta_1$ sender plays $In$ approaches 0 as $\gamma_1 \to 1$ and then $\delta \to 1$.

Let $\psi : \Pi_2 \to \Pi_2$ be the mapping given by

$$\psi(\pi_2)(a_1|In, m) = \min\left\{\pi_2(a_1|In, m), \frac{1}{3}\right\} \forall m \in M.$$ 

Note that $\psi$ is continuous and coincides with the identity mapping whenever $\pi_2(a_1|In, m) \leq 1/3$ for all $m$.

Consider the mapping $\tilde{R}^{\delta,\gamma_1,\gamma_2} : \Pi_1 \times \Pi_2 \to \Pi_1 \times \Pi_2$ given by $\tilde{R}^{\delta,\gamma_1,\gamma_2}(\pi_1, \pi_2) = (R^{\delta,\gamma_1}(\pi_2), \psi(R^{\gamma_2}(\pi_1)))$. Since $\tilde{R}^{\delta,\gamma_1,\gamma_2}$ is continuous, Brouwer’s fixed point theorem guarantees the existence of a fixed point $(\pi_1^{\delta,\gamma_1,\gamma_2}, \pi_2^{\delta,\gamma_1,\gamma_2})$. As $\pi_2^{\delta,\gamma_1,\gamma_2}(a_1|In, m) \leq 1/3$ for all $m$ by construction, Lemma 8 implies that $\lim_{\delta \to 1} \lim_{\gamma_1 \to 1} \pi_1^{\delta,\gamma_1,\gamma_2}[In] = 0$ for all $\gamma_2 \in [0, 1)$. Furthermore, because $g_2$ is such that every receiver would play $a_2$ at a first encounter with a sender playing $(In, m)$, $\lim_{\delta \to 1} \lim_{\gamma_1 \to 1} R^{\gamma_2}(\pi_1^{\delta,\gamma_1,\gamma_2})(a_1|In, m) = 0$ for all $m, \gamma_2 \in [0, 1)$, so the $\pi_2(a_1|In, m) \leq 1/3$ constraint does not bind when $\delta$ is sufficiently close to 1 and, given $\delta$, $\gamma_1$ is sufficiently close to 1. Formally, since $\pi_2^{\delta,\gamma_1,\gamma_2} \neq R^{\gamma_2}(\pi_1^{\delta,\gamma_1,\gamma_2})$ only if $R^{\gamma_2}(\pi_1^{\delta,\gamma_1,\gamma_2})(a_1|In, m) > 1/3$ for some $m$, we have that, for fixed $\gamma_2 \in [0, 1)$, $\pi_2^{\delta,\gamma_1,\gamma_2} = R^{\gamma_2}(\pi_1^{\delta,\gamma_1,\gamma_2})$ for $\delta$ sufficiently close to 1 and, given $\delta$, $\gamma_1$ sufficiently close to 1. Combining this with the fact that $\pi_1^{\delta,\gamma_1,\gamma_2} = R^{\delta,\gamma_1}(\pi_2^{\delta,\gamma_1,\gamma_2})$ for all $\gamma_1, \gamma_2 \in [0, 1)$, it follows that, for fixed $\gamma_2 \in [0, 1)$, $\pi_1^{\delta,\gamma_1,\gamma_2} = R^{\delta,\gamma_1}(\pi_2^{\delta,\gamma_1,\gamma_2})$ is a fixed point of $\tilde{R}^{\delta,\gamma_1,\gamma_2}$ for $\delta$ sufficiently close to 1 and, given $\delta$, $\gamma_1$ sufficiently close to 1. Since $\lim_{\delta \to 1} \lim_{\gamma_2 \to 1} \lim_{\gamma_1 \to 1} \pi_1^{\delta,\gamma_1,\gamma_2}[In] = 0$, we conclude that there are stable profiles in which both types plays $Out$. \qed

**OA.7.2 Example Where D1 Does Not Imply JCE**

*Example OA 2.* The sender’s type space is $\Theta = \{\theta_1, \theta_2, \theta_3\}$, and $\lambda(\theta_1) = \lambda(\theta_2) = \lambda(\theta_3) = 1/3$. The sender’s signal space is $S = \{In, Out\}$, and the receiver’s action space is $A = \{a_1, a_2, a_3\}$. The payoffs to the sender and receiver are given in Table 2.
Every type playing $Out$ is a D1 equilibrium outcome. To see this, let $\pi$ denote a strategy profile in which every type plays $Out$ and note that $\{\alpha \in MBR(\Theta, In) : \alpha[a_3] < 1/2\} \subset D_{\theta_3}(In, \pi)$, while $\{\nu a_2 + (1 - \nu)a_3 : \nu \in [0,1]\} \cap D_{\theta_1}(In, \pi) = \emptyset$ and $\{\nu a_1 + (1 - \nu)a_3 : \nu \in [0,1]\} \cap D_{\theta_2}(In, \pi) = \emptyset$. Thus, $D_{\theta_3}(In, \pi) \not\subset D_{\theta_1}(In, \pi)$ and $D_{\theta_3}(In, \pi) \not\subset D_{\theta_2}(In, \pi)$, so the receiver responding to $In$ with $a_3$, which deters all types from playing $In$, is consistent with D1 since $BR(\theta_3, In) = \{a_3\}$.

However, JCE rules out profiles in which every sender type plays $Out$. Again, let $\pi$ denote such a strategy profile. $\tilde{D}_{\theta_3}(In, \pi) \cup \tilde{D}^0_{\theta_3}(In, \pi) = \{\alpha \in \Delta(A) : \alpha[a_3] \leq 1/2\}$. Since

$$u_1(\theta_1, In, \alpha) + u_1(\theta_2, In, \alpha) = 3\alpha[a_1] + 3\alpha[a_2] - 2\alpha[a_3] = 3 - 5\alpha[a_3],$$

it follows that $\{\alpha \in \Delta(A) : \alpha[a_3] \leq 1/2\} \subset \tilde{D}_{\theta_1}(In, \pi) \cup \tilde{D}_{\theta_2}(In, \pi)$. Note that $BR(\{\theta_1, \theta_2\}, In) = \{a_1, a_2\}$, so no such profile can be a JCE since type $\theta_3$ cannot be deterred from playing $In$ by either $a_1$ or $a_2$, as $u_1(\theta_3, In, a_1) = u_1(\theta_3, In, a_2) = 1 > 0$. □
OA.7.3 Example Where Stability Does Not Imply D1

Example OA 3. The sender’s type space is \( \Theta = \{\theta_1, \theta_2\} \), and \( \lambda(\theta_1) = \lambda(\theta_2) = 1/2 \). The sender’s signal space is \( S = \{In, Out\} \), and the receiver’s action space is \( A = \{a_1, a_2, a_3\} \). The payoffs to the sender and receiver are given in Table 3 below.

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<td>0,0</td>
<td>0,0</td>
<td>Out</td>
<td>0,0</td>
<td>0,0</td>
<td>0,0</td>
</tr>
</tbody>
</table>

Table 3: The payoffs for Example OA 3.

Every type playing Out is both a PBE outcome and a JCE outcome. To see it is a PBE, note that both types are deterred from playing In when the receiver responds with \( a_2 \), which is the best response to \( \theta_2 \) playing In. Moreover, \( a_2 \) is a justified response to In since \( u_1(\theta_2, In, (1/2)a_1 + (1/2)a_2) = 0 \) and \( u_1(\theta_1, In, (1/2)a_1 + (1/2)a_2) < 0 \), which imply that \( \theta_2 \) is a justified type.

However, every type playing Out is not a D1 equilibrium outcome. To see this, note that

\[
MBR(\Theta, In) = \{ \alpha \in \Delta(A) : \alpha[a_1] = 0 \text{ or } \alpha[a_2] = 0 \}.
\]

Let \( \pi \) denote a strategy profile in which every type plays Out. As \( D_{\theta_2}(In, \pi) \cup D_{\theta_2}^0(\Theta, In) = \{ \alpha \in MBR(\Theta, In) : \alpha[a_2] = 0 \} \) and \( D_{\theta_1}(In, \pi) = \{ \alpha \in MBR(\Theta, In) : \alpha[a_2] < 1/10 \} \), we have that \( D_{\theta_2}(In, \pi) \cup D_{\theta_2}^0(\Theta, In, \pi) \subset D_{\theta_1}(In, \pi) \). Since \( \{a_1\} = BR(\theta_1, In) \) and neither type is deterred from playing In by \( a_1 \), it follows that no such strategy profile is D1.

We now show that there are stable profiles in which all types play Out. We specify that the receiver prior \( g_2 \) is a Dirichlet distribution with initial weight 1 on \( (\theta_1, In, m_{In,\theta_1}) \) and 1/4 on \( (\theta_2, In, m_{In,\theta_2}) \), and, for all other messages \( m \neq m_{In,\theta_1} \), initial weight 1/4 on \( (\theta_1, In, m) \) and 1 on \( (\theta_2, In, m) \). Note that initial trust is satisfied: When a receiver first encounters a sender who plays \( (In, m_{In,\theta}) \), the probability they
place on the receiver having type $\theta$ is $4/5$, so $BR(\theta, \text{In})$ is optimal.

We observe that $a_2$ is the receiver’s unique best response to $\text{In}$ under any distribution that puts weakly more weight on $\theta_2$ than the prior. Additionally, if a receiver has encountered past play of $(\text{In}, m)$ and all such plays have been by senders with type $\theta_2$, then the receiver will respond to the next instance of $(\text{In}, m)$ with $a_2$. To see that this holds for the case $m = m_{\text{In}, \theta_1}$, note that the receiver’s conditional distribution over the sender’s type after $(\text{In}, m_{\text{In}, \theta_1})$ must put probability at least $5/9$ on $\theta_2$. Analogous arguments handle the other cases.

We focus on steady state profiles in which, for every $m \in M$, the aggregate probability that a receiver responds to $(\text{In}, m)$ with $a_3$ is less than $1/10$. Under such responses, whenever it is weakly optimal for $\theta_1$ to play $\text{In}$, it must be strictly optimal for $\theta_2$ to do so. To see this, note that

$$u_1(\theta_1, \text{In}, \alpha) = 3\alpha[a_1] + 2.1\alpha[a_3] - 2,$$

so $\alpha[a_1] \geq 2/3 - 7/10\alpha[a_3]$ whenever $u_1(\theta_1, \text{In}, \alpha) \geq 0$. Additionally,

$$u_1(\theta_2, \text{In}, \alpha) = 2\alpha[a_1] + \alpha[a_3] - 1,$$

which is strictly positive whenever $\alpha[a_1] \geq 2/3 - 7/10\alpha[a_3]$ and $\alpha[a_3] \leq 1/10$. We argue that such steady state profiles exist in the iterated limit where $\gamma_1 \rightarrow 1$, then $\delta \rightarrow 1$, and then $\gamma_2 \rightarrow 1$, and that the corresponding aggregate probability that either sender type plays $\text{In}$ converges to $0$.

Let $\chi : \Delta(A) \rightrightarrows \Delta(A)$ be the correspondence given by

$$\chi(\alpha) = \begin{cases} 
\{\alpha\} & \text{if } \alpha[a_3] \leq \frac{1}{10} \\
\{\alpha' \in \Delta(A) : \alpha'[a_3] = \frac{1}{10}\} & \text{if } \alpha[a_3] > \frac{1}{10},
\end{cases}$$

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and let \( \rho : \Pi_2 \Rightarrow \Pi_2 \) be the correspondence given by

\[
\rho(\pi_2) = \{ \pi'_2 \in \Pi_2 : \pi'_2(\cdot|In, m) \in \chi(\pi_2(\cdot|In, m)) \forall m \in M \}.
\]

Note that \( \rho \) is upper hemicontinuous and coincides with the identity correspondence whenever \( \pi_2(a_3|In, m) \leq 1/10 \) for all \( m \).

Consider the correspondence \( \mathcal{R}^{\delta, \gamma_1, \gamma_2} : \Pi_1 \times \Pi_2 \Rightarrow \Pi_1 \times \Pi_2 \) given by

\[
\mathcal{R}^{\delta, \gamma_1, \gamma_2}(\pi_1, \pi_2) = \{ (\pi'_1, \pi'_2) \in \Pi_1 \times \Pi_2 : \pi'_1 = \mathcal{R}^{\delta, \gamma_1}(\pi_2) \text{ and } \pi'_2 \in \rho(\mathcal{R}^{\gamma_2}(\pi_1)) \}.
\]

Since \( \mathcal{R} \) is upper hemicontinuous, Kakutani’s fixed point theorem guarantees the existence of a fixed point \( (\pi_1^{\delta, \gamma_1, \gamma_2}, \pi_2^{\delta, \gamma_1, \gamma_2}) \).

We establish that \( \lim_{\gamma_1 \to 1} \lim_{\delta \to 1} \lim_{\gamma_1 \to 1} \pi_1^{\delta, \gamma_1, \gamma_2}[In|\theta_1] = 0 \). Suppose towards a contradiction that there is a sequence of receiver continuation probabilities \( \{\gamma_{2,j}\}_{j \in \mathbb{N}} \), a collection of sequences of sender discount factors \( \{\delta_{j,k}\}_{j,k \in \mathbb{N}} \), and a collection of sequences of sender continuation probabilities \( \{\gamma_{1,j,k,l}\}_{j,k,l \in \mathbb{N}} \) such that (a) \( \lim_{j \to \infty} \gamma_{2,j} = 1 \), (b) \( \lim_{k \to \infty} \delta_{j,k} = 1 \) for all \( j \), (c) \( \lim_{l \to \infty} \gamma_{1,j,k,l} = 1 \) for all \( j, k, l \), (d) \( \lim_{j \to \infty} \lim_{k \to \infty} \lim_{l \to \infty} \pi_1^{\delta_{j,k}, \gamma_{1,j,k,l}, \gamma_{2,j}}[In, m|\theta] \) exists for all \( \theta \in \Theta \) and \( m \in M \), and (e) \( \lim_{j \to \infty} \lim_{k \to \infty} \lim_{l \to \infty} \pi_1^{\delta_{j,k}, \gamma_{1,j,k,l}, \gamma_{2,j}}[In|\theta_1] > 0 \). Then since \( \pi_2^{\delta, \gamma_1, \gamma_2}(a_3|In, m) \leq 1/10 \) for all \( m \in M \), Lemma 8 implies that \( \lim_{j \to \infty} \lim_{k \to \infty} \lim_{l \to \infty} \pi_1^{\delta_{j,k}, \gamma_{1,j,k,l}, \gamma_{2,j}}[In|\theta_2] = 1 \). Therefore, there exists some \( m \in M \) such that \( \lim_{j \to \infty} \lim_{k \to \infty} \lim_{l \to \infty} \pi_1^{\delta_{j,k}, \gamma_{1,j,k,l}, \gamma_{2,j}}[In|\theta_2] > 0 \) and \( \lim_{j \to \infty} \lim_{k \to \infty} \lim_{l \to \infty} \pi_1^{\delta_{j,k}, \gamma_{1,j,k,l}, \gamma_{2,j}}[In|\theta_2] \geq \lim_{j \to \infty} \lim_{k \to \infty} \lim_{l \to \infty} \pi_1^{\delta_{j,k}, \gamma_{1,j,k,l}, \gamma_{2,j}}[In|\theta_1] \).

By Lemma 2, this implies that \( \lim_{j \to \infty} \lim_{k \to \infty} \lim_{l \to \infty} \mathcal{R}^{\gamma_{1,j,k,l}, \gamma_{2,j}}(\pi_1^{\delta_{j,k}, \gamma_{1,j,k,l}, \gamma_{2,j}})(a_3|In, m) = 1 \). Since \( \chi(\pi_2(\cdot|In, m)) = \{ \pi_2(\cdot|In, m) \} \) if \( \pi_2(a_3|In, m) \leq 1/10 \), it follows that \( \lim_{j \to \infty} \lim_{k \to \infty} \lim_{l \to \infty} \pi_1^{\delta_{j,k}, \gamma_{1,j,k,l}, \gamma_{2,j}}(\pi_1)(a_3|In, m) = 1 \). However, by Lemma 8, this requires that \( \lim_{j \to \infty} \lim_{k \to \infty} \lim_{l \to \infty} \pi_1^{\delta_{j,k}, \gamma_{1,j,k,l}, \gamma_{2,j}}[In, m] = 0 \) must hold, a contradiction.

A very similar argument establishes that \( \lim_{\gamma_2 \to 1} \lim_{\delta \to 1} \lim_{\gamma_1 \to 1} \pi_1^{\delta, \gamma_1, \gamma_2}[In|\theta_2] = 0 \). Thus, \( \lim_{\gamma_2 \to 1} \lim_{\delta \to 1} \lim_{\gamma_1 \to 1} \pi_1^{\delta, \gamma_1, \gamma_2}[In] = 0 \). Since a receiver will only play \( a_3 \) in response to some \( (In, m) \) if they have previously encountered a sender playing \( (In, m) \), we have that, for all sufficiently high \( \gamma_2 \), \( \mathcal{R}^{\gamma_{1,j,k,l}, \gamma_{2,j}}(\pi_1^{\gamma_{1,j,k,l}, \gamma_{2,j}})(a_3|In, m) \leq 1/10 \) for all
\( m \in M \) when \( \delta \) is sufficiently high and, given \( \delta, \gamma_1 \) is sufficiently high. Since \( \rho(\pi_2) = \{\pi_2\} \) if \( \pi_2(a_3|In,m) \leq 1/10 \) for all \( m \), \( \pi^*_{2,\gamma_1,\gamma_2} = \rho(\mathcal{R}_{2,\gamma_1,\gamma_2})(\pi^*_{1,\gamma_1,\gamma_2}) = \mathcal{R}_{2,\gamma_2}(\pi^*_{1,\gamma_1,\gamma_2}) \) for fixed, sufficiently high \( \gamma_2 \in [0,1) \) when \( \delta \) is sufficiently close to 1 and, given \( \delta, \gamma_1 \) is sufficiently close to 1. Thus, for fixed, sufficiently high \( \gamma_2 \in [0,1) \), \( (\pi^*_{1,\gamma_1,\gamma_2}, \pi^*_{2,\gamma_1,\gamma_2}) \) is a fixed point of \( \mathcal{R}_{\gamma_1,\gamma_2} \) when \( \delta \) is sufficiently close to 1 and, given \( \delta, \gamma_1 \) is sufficiently close to 1. We conclude that there are stable profiles in which every type plays Out. \( \square \)

### OA.8 Stability Under Alternative Assumptions

#### OA.8.1 Weakening Initial Trust

Here we discuss a refinement satisfied by all stable profiles under an alternative assumption to initial trust. Suppose that receivers know the payoff functions of the senders, as in Fudenberg and He [2020]. Then receivers who are long-lived may feel that they have acquired a good sense of each sender type’s equilibrium payoff. Suppose that such a receiver encounters a sender playing a pair \((s,m,\tilde{\Theta})\) that the receiver has not previously seen types outside of \(\tilde{\Theta}\) play. If the receiver believes that only types in \(\tilde{\Theta}\) could improve their outcome by deviating to \(s\) when the receiver’s response is contained in \(BR(s,\tilde{\Theta})\), we assume the receiver finds such a message credible and respond accordingly.\(^1\)

As before, any stable profile must be a PBE-H. Moreover, stability also imposes additional conditions for profiles \(\pi\) that are on-path strict for the receiver or are such that the sender types’ payoffs would not be changed if the receiver deviated.\(^2\) For such a profile to be stable, it must be that, for every signal \(s\) where \(u_1(\theta,s,a) < u_1(\theta,\pi)\) for all \(a \in BR(\overline{\Theta}(s,\pi),s)\) and \(\theta \not\in \overline{\Theta}(s,\pi)\), there is some \(m \in M\) such that \(\pi_2(\cdot|s,m) \in \Delta(BR(\overline{\Theta}(s,\pi),s))\). Aside from the qualifying condition \(u_1(\theta,s,a) < u_1(\theta,\pi)\) for all

---

\(^1\)The receiver responding to “credible” statements in this way is similar to the motivation underlying “credible robust neologisms” in Clark [2020].

\(^2\)These restrictions on \(\pi\) guarantee that a typical receiver agent will learn the equilibrium payoffs of the sender types with high probability.
\( a \in BR(\Theta(s, \pi), s) \) and \( \theta \not\in \overline{\Theta}(s, \pi) \), this requirement is the same as Condition 2 of Definition 3.

This refinement is weaker than JCE, strictly so in some games, including some co-monotonic games. Thus, it preserves the equilibria we focus on in Examples 1 and 3. This refinement does not make the same predictions as JCE does in Example 2; however, if the game were altered to have finer action spaces that sufficiently approximate a continuum, then this refinement, like JCE, would select only equilibria that are close to the least-cost separating equilibrium. The D1 equilibrium in Example OA.7.2 satisfies this refinement, but there are other games in which this refinement rules out D1 equilibria.

**OA.8.2 Costs of Lying**

Suppose that we allow the sender’s utility function \( u_1 : \Theta \times S \times M \times A \rightarrow \mathbb{R} \) to depend on the sender’s message \( m \) in the following way: For all \( \theta \in \Theta \) and \( \Theta', \Theta'' \subseteq \Theta \) such that \( \theta \in \Theta' \cap \Theta'' \), and \( \Theta''' \subseteq \Theta \) such that \( \theta \not\in \Theta''' \), \( u_1(\theta, s, m, \Theta', a) = u_1(\theta, s, m, \Theta'', a) \) for all \( s \in S \) and \( a \in A \). Here lying is weakly costly for the sender in that, for a given \( s \) and \( a \), the sender gets a lower payoff from a message that represents a set of types to which they do not belong. For simplicity, we assume that all messages that represent a set of types containing a sender’s type give the sender the same payoff.

For each signal \( s \), message \( m \), and profile \( \pi \), we will define a set of types \( \overline{\Theta}(s, m, \pi) \) that is analogous to the set of justified types in our main setting where \( m \) does not impact payoffs. To do this, first set

\[
\tilde{D}_\theta(s, m, \pi) = \{ \alpha \in \Delta(BR(\Theta, s)) : u_1(\theta, s, m, \alpha) > u_1(\theta, \pi) \},
\]

\[
\tilde{D}_\theta^0(s, m, \pi) = \{ \alpha \in \Delta(BR(\Theta, s)) : u_1(\theta, s, m, \alpha) = u_1(\theta, \pi) \},
\]

and

\[
\Theta^1(s, m, \pi) = \{ \theta \in \Theta : \tilde{D}_\theta(s, m, \pi) \cup \tilde{D}_\theta^0(s, m, \pi) \not\subseteq \cup_{\theta' \neq \theta} \tilde{D}_{\theta'}(s, m, \pi) \}
\]
Then let
\[
\Theta(s, m, \pi) = \begin{cases} 
\Theta^\dagger(s, m, \pi) & \text{if } \Theta^\dagger(s, m, \pi) \neq \emptyset \\
\Theta & \text{if } \Theta^\dagger(s, m, \pi) = \emptyset 
\end{cases}
\]

Under initial trust, any stable profile \(\pi\) must satisfy the following requirement:
\[
\pi_2(\cdot | s, m, \Theta(s, m, \pi)) \in \Delta(BR(\Theta(s, m, \pi), s)) \text{ for all } s \in S.
\]
When the sender’s message is payoff irrelevant, \(\Theta(s, m, \pi) = \Theta(s, \pi)\), so this requirement implies Condition 2 of Definition 3. While lying costs make it less appealing for a non-justified type to falsely represent themselves as justified, they can change the set of equilibria, so it is hard to give a precise summary of their effect in general games.

**OA.8.3 Strengthening Initial Trust**

Suppose that we strengthen initial trust to require that for any \(s \in S\) and \(\tilde{\Theta}, \tilde{\Theta}' \subseteq \Theta\), if the receiver has never seen a type outside of \(\tilde{\Theta} \cup \tilde{\Theta}'\) play \((s, m, \tilde{\Theta})\), then their response to a first instance of \((s, m, \tilde{\Theta})\) will belong to \(BR(\tilde{\Theta} \cup \tilde{\Theta}', s)\). This means that a receiver who has only observed types in \(\tilde{\Theta}'\) deceitfully play \((s, m, \tilde{\Theta})\) puts high probability on the sender type being in either \(\tilde{\Theta}\) or \(\tilde{\Theta}'\) after observing this signal-message pair. This seems plausible; however, we focus on initial trust because of JCE is simpler and easier to apply than its iterated version.

The stable profiles then satisfy an iterated version of JCE, which itself is stronger than the *Iterated Intuitive Criterion* (Cho and Kreps [1987]) and *co-divinity* (Sobel, Stole, and Zapater [1990]). Moreover, it is not nested with NWBR, but it is weaker than the refinement obtained by iteratively applying NWBR.

Fix \(s \in S\) and \(\pi \in \Pi_1 \times \Pi_2\). Consider the following iterated version of the JCE procedure for computing the set of justified types. Initialize \(\bar{\Theta}^0(s, \pi) = \Theta(s, \pi)\). For
\( n \in \{1, 2, 3, \ldots \} \), let

\[
\tilde{D}_0^n(s, \pi) = \{ \alpha \in \Delta(\Theta^{-1}_n(s, \pi), s) : u_1(\theta, s, \alpha) > u_1(\theta, \pi) \},
\]

\[
\tilde{D}_0^0(s, \pi) = \{ \alpha \in \Delta(\Theta^{-1}_n(s, \pi), s) : u_1(\theta, s, \alpha) = u_1(\theta, \pi) \},
\]

\[
\Theta^{\dagger, n}_n(s, \pi) = \{ \theta \in \Theta : \tilde{D}_0^n(s, \pi) \cup \tilde{D}_0^0(s, \pi) \not\subseteq \cup_{\theta' \neq \theta} \tilde{D}_\theta(s, \pi) \},
\]

\[
\Theta^n(s, \pi) = \begin{cases} 
\Theta^{\dagger, n}_n(s, \pi) & \text{if } \Theta^{\dagger, n}_n(s, \pi) \neq \emptyset \\
\Theta^{n-1}(s, \pi) & \text{if } \Theta^{\dagger, n}_n(s, \pi) = \emptyset
\end{cases}.
\]

Set \( \Theta^\infty(s, \pi) = \cap_{n \in \mathbb{N}} \Theta^n(s, \pi) \). Note that \( \Theta^{n+1}(s, \pi) \subseteq \Theta^n(s, \pi) \) for all \( n \) and that \( \Theta^\infty(s, \pi) \subseteq \Theta^0(s, \pi) = \Theta(s, \pi) \).

Under this strengthening of initial trust, every stable profile \( \pi \) must satisfy the following requirement: For every signal \( s \), there is some \( m \in M \) such that \( \pi_2(\cdot|s, m) \in \Delta(\Theta^\infty(s, \pi), s)) \).

### OA.9 Stability Under a More General Limit

In this section, we study steady state aggregate play in the more general limit where first \( \gamma_1 \) tends to 1, and then \( \delta \) and \( \gamma_2 \) tend to 1, without any restrictions on the relative speed with which \( \delta \) and \( \gamma_2 \) converge. Formally, we consider \( \lim_{(\delta, \gamma_2) \to (1, 1)} \lim_{\gamma_1 \to 1} \Pi^*(g, \delta, \gamma_1, \gamma_2) \). We will call these the stable* profiles.

**Definition OA 1.** Strategy profile \( \pi \) is **stable* if there is a sequence \( \{\delta_j\}_{j \in \mathbb{N}} \to 1 \), sequence \( \{\gamma_2,j\}_{j \in \mathbb{N}} \to 1 \), and sequences \( \{\gamma_1,j,k\}_{j, k \in \mathbb{N}} \) with \( \lim_{k \to \infty} \gamma_1,j,k = 1 \) for all \( j \), such that \( \pi = \lim_{j \to \infty} \lim_{k \to \infty} \pi_{j,k} \) for some sequence \( \pi_{j,k} \in \Pi^*(g, \delta_{1,j}, \gamma_1,j,k, \gamma_2,j) \).

Since every stable profile is also stable*, it follows that stable* profiles exist.

**Corollary OA 1.** **Stable* strategy profiles exist.

As with stability, there is a strong relationship between the stable* profiles and the set of JCE.
Definition OA 2. Strategy profile \( \pi \) has strong incentives if, for every off-path \( s \) and \( \theta \not\in \Theta(s, \pi) \), there is some on-path \((s', m')\) such that \( u_1(\theta, s', a) > u_1(\theta, s, \pi_2(\cdot|s, m_s, \eta(s, \pi))) \) for all \( a \in BR(p(\cdot|s', m'), s') \), where \( p(\cdot|s', m') \) is the posterior belief given \((s', m')\) obtained from \( \pi_1 \) and Bayes’ rule.

A strategy profile has strong incentives if for every off-path \( s \), every type would obtain a strictly lower payoff from playing \((s, m_s, \Theta(s, \pi))\) than they would from playing some on-path signal-message pair when the receiver responds with any best response to the corresponding posterior.

Theorem OA 1. Suppose that the density of the prior of the sender agents is everywhere positive. If \( \pi \) is stable* and has strong incentives, then it is a JCE.

Theorem OA 1 says that a profile with strong incentives can be stable* only if it is a JCE. The assumption of strong incentives is vacuous if all signals are played with positive probability in \( \pi \). Also, note that \( u_1(\theta, \pi) > u_1(\theta, s, m_s, \eta(s, \pi)) \) for an arbitrary signal \( s \) and profile \( \pi \) whenever \( \theta \not\in \Theta(s, \pi) \). Thus, every profile that is on-path strict for the receiver has strong incentives.\(^3\)

The remainder of this section is devoted to the proof of Theorem OA 1. The argument that every stable* profile is a PBE-H proceeds very similarly to that for the stable profiles. The following lemma affirms the optimality of the aggregate sender play given the aggregate receiver play.

Lemma OA 1. Suppose that \( \pi \) is stable*. Then for each \( \theta \in \Theta \), \( \pi_1(\cdot|\theta) \) puts support only on those sender signal-message pairs that are optimal for type \( \theta \) under the receiver behavior strategy \( \pi_2 \).

The next lemma shows that aggregate receiver play is a best response to (on-path) aggregate play by the senders in a stable* profile.

\(^3\)Another sufficient condition is that no sender type would be hurt if the receiver were to change their response to some on-path signal-message pair, as is the case when all types choose an “exit” option.
**Lemma OA 2.** Suppose that \( \pi \) is stable*. Then for any sender signal-message pair \((s, m) \in S \times M \) that occurs with positive probability under \( \pi \), \( \pi_2(\cdot|s, m) \) puts support only on receiver actions that are best-responses to \( s \) and the posterior belief induced by \( \lambda \) and \( \{\pi_1(s, m|\theta)\}_{\theta \in \Theta} \) under Bayes’ rule.

We omit the proofs of Lemma OA 1 and Lemma OA 2, which are quite similar to the proofs of Lemma 1 and Lemma 2, respectively.

Lemma OA 3 below shows that when \( \pi \) is a stable profile that has strong incentives, the aggregate receiver response to any \((s, m, \Theta(s, \pi))\) must be supported on \( BR(\Theta(s, \pi), s) \).

**Lemma OA 3.** Suppose that \( \pi \) is stable* and has strong incentives. Then \( \pi_2(\cdot|s, m, \Theta(s, \pi)) \in \Delta(BR(\Theta(s, \pi), s)) \) for all \( s \in S \).

We prove Lemma OA 3 in the following subsection, but first we use Lemmas OA 1, OA 2, and OA 3 to prove Theorem OA 1.

*Proof of Theorem OA 1.* Lemma OA 1 implies Condition 1 of the definition of PBE-H, and Lemma OA 2 implies Condition 2. As before, Condition 3 of Definition 1 follows from the fact that the receivers in our model myopically optimize. Finally, the additional condition in Definition 3 follows from Lemma OA 3 and the assumption that \( \pi \) has strong incentives. ■

**OA.9.1 Proof of Lemma OA 3**

The following lemma relates the receiver’s continuation parameter to the probability the aggregate receiver response to any on-path signal-message pair places on the corresponding receiver best responses.

**Lemma OA 4.** Fix a strategy profile \( \pi \). Let \( X_{on} \) be the set of sender signal-message pairs that are on-path under \( \pi_1 \), and let \( p(\cdot|s, m) \) be the posterior belief given \((s, m) \in X_{on} \) that is obtained from \( \pi_1 \) and Bayes’ rule. There are \( \nu, \eta > 0 \) such that, for every
\[\pi'_1 \in \Pi_1 \text{ satisfying } \max_{(\theta,s,m) \in \Theta \times S \times M} |\pi'_1(s,m|\theta) - \pi_1(s,m|\theta)| < \nu \text{ and all } \delta, \gamma_1, \gamma_2 \in [0,1),\]
\[\mathcal{R}^{\gamma_2}_2(\pi'_1)(BR(p(\cdot|s,m),s)|(s,m)) \geq 1 - \eta(1 - \gamma_2)\]
for all \((s,m) \in X^\omega\).

Proof. Let \(q(\theta, s, m) = \lambda(\theta)\pi_1(s,m|\theta)\) be the distribution over sender types, signals, and messages induced by \(\lambda\) and \(\pi_1\). For \(\varepsilon > 0\), let \(Q_{\varepsilon} = \{q' \in \Delta(\Theta \times S \times M) : \max_{(\theta,s,m) \in \Theta \times S \times M} |q'(\theta,s,m) - q(\theta,s,m)| \leq \varepsilon\}\). By upper hemicontinuity, there exists \(\varepsilon > 0\) such that every receiver whose belief \(\tilde{g}_2 \in \Delta(\Delta(\Theta \times S \times M))\) puts probability at least \(1 - \varepsilon\) on \(Q_{\varepsilon}\) will respond to every \((s,m) \in X^\omega\) with some action belonging to \(BR(p(\cdot|s,m),s)\).

Given the non-doctrinaire prior \(g_2\), Theorem 4.2 of Diaconis and Freedman [1990] implies that there is some \(T > 0\) such that a receiver who has lived more than \(T\) periods assigns posterior probability of at least \(1 - \varepsilon\) to probability distributions \(q'\) within \(\varepsilon/3\) distance (in the sup-norm metric) of the empirical distribution they have observed.

We provide a lower bound on the share of receivers who have lived more than \(T\) periods and who have observed an empirical distribution within \(\varepsilon/3\) distance of the true distribution \(q' \in \Delta(\Theta \times S \times M)\). By Hoeffding’s inequality, the probability that the fraction of \((\theta,s,m)\) observations is outside of \([q'(\theta,s,m) - \varepsilon/3, q'(\theta,s,m) + \varepsilon/3]\) for a receiver with \(t\) observations is less than \(2e^{-\frac{2\varepsilon^2}{9}t}\), so the probability that the empirical distribution of a receiver with \(t\) observations is greater than \(\varepsilon/3\) distance from \(q'\) is no more than \(2|S||M|e^{-\frac{2\varepsilon^2}{9}t}\). Thus, the share of receivers who have lived longer than \(T\) periods and who have observed an empirical distribution within \(\varepsilon/3\) distance of \(q'\) is
at least
\[
\sum_{t=T}^{\infty} (1 - \gamma_2) \gamma_2^t \left( 1 - 2|S||M|e^{-\frac{2\varepsilon^2}{9}} \right) = \gamma_2^T - \frac{2|S||M|(1 - \gamma_2) \gamma_2^T e^{-\frac{2\varepsilon^2}{9}}}{1 - \gamma_2 e^{-\frac{2\varepsilon^2}{9}}},
\]
\[
= 1 - \left( \frac{1 - \gamma_2^T}{1 - \gamma_2} + \frac{2|S||M|\gamma_2^T e^{-\frac{2\varepsilon^2}{9}}}{1 - \gamma_2 e^{-\frac{2\varepsilon^2}{9}}} \right) (1 - \gamma_2),
\]
\[
\geq 1 - \left( T + \frac{2|S||M|}{1 - e^{-\frac{2\varepsilon^2}{9}}} \right) (1 - \gamma_2),
\]
where the inequality follows from the facts that \((1 - \gamma_2^T)/(1 - \gamma_2) < T\) and \(\gamma_2^T e^{-\frac{2\varepsilon^2}{9}}/(1 - \gamma_2 e^{-\frac{2\varepsilon^2}{9}}) < 1/(1 - e^{-\frac{2\varepsilon^2}{9}})\) for all \(\gamma_2 \in [0, 1)\).

Let \(\eta = T + 2|S||M|/\left(1 - e^{-\frac{2\varepsilon^2}{9}}\right)\), and let \(\nu > 0\) be such that, for every \(\pi'_1 \in \Pi_1\) satisfying \(\max_{(\theta, s, m) \in \Theta \times S \times M} |\pi'_1(s, m|\theta) - \pi_1(s, m|\theta)| < \nu\), the corresponding distribution over sender types, signals, and messages belongs to \(Q_{\varepsilon/3}\). It follows from the arguments above that, for all \(\pi'_1\) within \(\nu\) distance (in the sup-norm metric) of \(\pi_1\), the steady-state share of receivers who respond to each \((s, m) \in X^m\) with some element of \(BR(p^*(s, m), s)\) is at least \(1 - \eta(1 - \gamma_2)\). ■

The next lemma builds on Lemma OA 4 to show that, in a sequence of steady states converging to a stable* profile with strong incentives, the ratio of the aggregate probability of a non-justified type playing \((s, m, \pi(s, \pi))\) to the expected lifetime of a receiver agent approaches 0.

**Lemma OA 5.** Fix a stable* strategy profile \(\pi\) with strong incentives. Let \(\{\pi_{j, k} \in \Pi^*(g, \delta_j, \gamma_{1, j}, \gamma_{2, j})\}_{j, k} \in \mathbb{N}\) be a sequence of steady state profiles such that \(\lim_{j \to \infty} \lim_{k \to \infty} \pi_{j, k} = \pi\), where \(\lim_{j \to \infty} \delta_j = 1\), \(\lim_{j \to \infty} \gamma_{2, j} = 1\), and \(\lim_{k \to \infty} \delta_{j, k} = 1\) for all \(j\). For every \(\varepsilon > 0\), there exists some \(J \in \mathbb{N}\) and function \(K : \mathbb{N} \to \mathbb{N}\) such that

\[
\pi_{1, j, k}(s, m, \pi(s, \pi)|\theta) \leq \varepsilon(1 - \gamma_{2, j})
\]

for all \(s, \theta \notin \overline{\Theta}(s, \pi), j > J,\) and \(k > K(j)\).
Proof. By Lemma OA 4 and the fact that \( \lim_{j \to \infty} \lim_{k \to \infty} \pi_{j,k} = \pi \), there exists some \( \eta > 0 \), \( J' \in \mathbb{N} \), and function \( K' : \mathbb{N} \to \mathbb{N} \) such that

\[
\pi_{2,j,k}(BR(p(\cdot|s,m), s)|(s,m)) \geq 1 - \eta(1 - \gamma_{2,j}) \quad (1)
\]

for all \((s,m)\) on-path under \( \pi_1 \), \( j > J' \), and \( k > K'(j) \).

Fix a signal \( s \) and type \( \theta \) such that \( \theta \not\in \mathcal{E}(s, \pi) \). Since \( \pi \) has strong incentives, there is some \((s',m')\) that is on-path under \( \pi_1 \) such that \( u_1(\theta, s', a) > u_1(\theta, s, \pi_2(\cdot|s,m,\mathcal{E}(s,\pi))) \) for all \( a \in BR(p(\cdot|s',m'), s') \). For any \( \alpha \in \Delta(A) \) and \( z > 0 \), let \( \mathcal{A}_{(\alpha,z)} = \{ \alpha' \in \Delta(A) : \max_{a \in A} |\alpha'[a] - \alpha[a]| \leq z \} \) be the set of mixtures over \( A \) that are no greater than \( z \) away from \( \alpha \) in the sup-norm metric. Let \( \nu > 0 \) be such that

\[
(1 - \nu)u_1(\theta, s', a) + \nu \min_{a' \in A} u_1(\theta, s', a') > u_1(\theta, s, \alpha) + \nu \quad (2)
\]

for all \( a \in BR(p(\cdot|s',m'), s') \) and \( \alpha \in \mathcal{A}_{(\pi_2(\cdot|s,m,\mathcal{E}(s,\pi)),\nu)} \).

Suppose that a sender has played \((s',m')\) at least \( N > 0 \) times. Combining Equation 1 with Lemma A.1 of Fudenberg and Levine [2006] implies that the probability that the fraction of times the sender observed a receiver play something outside of \( BR(p(\cdot|s',m')) \) in response to \((s',m')\) exceeds \( \nu/2 \) is no more than \( 2^{11}\eta(1 - \gamma_{2,j})/(3\nu^4N) \). For a fixed \( \varepsilon > 0 \), let \( N_{(s',m')} \) be such that \( 2^{11}\eta/(3\nu^4N_{(s',m')}) < \varepsilon/4 \). For such an \( N_{(s',m')} \), it follows that \( 2^{11}\eta(1 - \gamma_{2,j})/(3\nu^4N_{(s',m')}) < \varepsilon(1 - \gamma_{2,j})/4 \).

By the assumption that the sender’s prior has a density \( g_1(\pi_2) \) that is everywhere positive and continuous in \( \pi_2 \in \Pi_2 \), we can find a lower bound on the probability that certain senders put on the receiver aggregate response to \((s',m')\) playing an element of \( BR(p(\cdot|s',m'), s') \) with probability at least \( 1 - \nu \). In particular, we will show there is a lower bound \( \zeta > 0 \) on the probability that the aggregate receiver response to \((s',m')\) puts probability at least \( 1 - \nu \) on \( BR(p(\cdot|s',m'), s') \) as determined by two classes of sender agents: (1) a sender agent who has played \((s',m')\) fewer than \( N_{(s',m')} \) times, and (2) a sender agent who has played \((s',m')\) more than \( N_{(s',m')} \) times and observed...
a response in $BR(p(\cdot|s', m'), s')$ greater than a fraction $1 - \nu/2$ of the times. From the preceding paragraph, the share of sender agents who fall into either of these two classes exceeds $1 - \varepsilon(1 - \gamma_{2,j})/4$.

Consider a sender who, for each $a \in A$, has $n_a$ observations of a receiver responding to $(s', m')$ with $a$. Then such a sender puts probability at least

$$\min_{\pi_2 \in \Pi_2} g_1(\pi_2) \int_{\{a \in \Delta(A) : \alpha[BR(p(\cdot|s', m'), s')] \geq 1 - \nu\}} \prod_{a \in A} \alpha[a]^{n_a}$$

$$\max_{\pi_2 \in \Pi_2} g_1(\pi_2) \int_{\Delta(A)} \prod_{a \in A} \alpha[a]^{n_a}$$

on the set of aggregate receiver responses to $(s', m')$ that have probability weakly greater than $1 - \nu$ on $BR(p(\cdot|s', m'), s')$. This expression is uniformly bounded away from 0 when there are fewer than $N_{(s', m')}$ observations. Moreover, Theorem 4.2 of Diaconis and Freedman [1990] implies that this expression is uniformly bounded away from 0 when there are more than $N_{(s', m')}^*$ observations and the fraction of these observations where the receiver responding with some element of $BR(p(\cdot|s', m'), s')$ exceeds $1 - \nu/2$.

By similar arguments, there is some $N' \in \mathbb{N}$ such that, for a sender who has played $(s, m, s', \Theta(s, \pi))$ at least $N'_{s,j}$ times, the sender’s expectation of the aggregate receiver response to $(s, m, s', \Theta(s, \pi))$ is within $\nu/3$ (in the sup-norm metric) of the empirical response the sender has observed. Moreover, by the law of large numbers, for any $j \in \mathbb{N}$, we can choose some $N'_{s,j} > N'_{s,j}$ to be such that there is a probability no greater than $\varepsilon(1 - \gamma_{2,j})/4$ that the empirical response to $(s, m, s', \Theta(s, \pi))$ observed by a sender who has played $(s, m, s', \Theta(s, \pi))$ at least $N'_{s,j}$ times is more than $\nu/3$ away from the aggregate receiver response $\pi_{2,j, k}(\cdot|s, m, s', \Theta(s, \pi))$. Let $J'' \in \mathbb{N}$ and $K'' : \mathbb{N} \rightarrow \mathbb{N}$ be such that

$$\max_{a \in A} |\pi_{2,j, k}(a|s, m, s, \Theta(s, \pi)) - \pi_2(a|s, m, s, \Theta(s, \pi))| < \nu/3$$

for all $j > J''$ and $k > K''(j)$. It follows that, for all such $j$ and $k$, the probability that $A(\pi_{2,j, k}(\cdot|s, m, s, \Theta(s, \pi)), \nu)$ contains the expectation of the aggregate receiver response to $(s, m, s', \Theta(s, \pi))$, as evaluated by a sender who has played $(s, m, s', \Theta(s, \pi))$ at least $N'_{s,j}$ times, exceeds $1 - \varepsilon(1 - \gamma_{2,j})/4$. 

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Consider a sender belief \( \tilde{g}_1 \in \Delta(\Pi_2) \) that satisfies

\[
\tilde{g}_1(\pi_2(BR(p(\cdot|s', m'), s'|s', m')) \geq 1 - \nu) \geq \zeta,
\]

\[
\tilde{g}_1(\pi_2(\cdot|s, m_s, \bar{v}(s, \pi)) \in A_\pi(\cdot|s, m_s, \bar{v}(s, \pi), \nu)) \geq 1 - \frac{1}{2}\zeta.
\]

The first inequality says that the belief puts probability at least \( \zeta \) on aggregate receiver responses to \((s', m')\) that play an element of \( BR(p(\cdot|s', m'), s') \) with probability weakly greater than \( 1 - \nu \). The second inequality says that the belief puts probability at least \( 1 - \frac{1}{2}\zeta \) on the aggregate receiver response to \((s, m_s, \Theta(s, \pi))\) belonging to \( A_\pi(\cdot|s, m_s, \bar{v}(s, \pi), \nu) \).

By Equation 2, all beliefs satisfying the conditions in (3) must put probability at least \( \zeta/2 \) on aggregate receiver behavior strategies where playing \((s', m')\) gives a type \( \theta \) sender an expected payoff at least \( \nu \) greater than that from playing \((s, m_s, \bar{v}(s, \pi))\).

For a type \( \theta \) sender with any belief that satisfies (3), the expected total lifetime payoff from the optimal policy exceeds the expected total lifetime payoff from only playing \((s, m_s, \bar{v}(s, \pi))\) by an amount bounded away from 0 when \( \delta \) and \( \gamma_1 \) are sufficiently high. In particular, for \( \delta \) and \( \gamma_1 \) sufficiently close to 1, the difference in the the expected payoff from the optimal policy and that from repeatedly playing \((s, m_s, \bar{v}(s, \pi))\) exceeds \( c = \zeta\nu/4 > 0 \). Let \( J'' \in \mathbb{N} \) and \( K'' : \mathbb{N} \to \mathbb{N} \) be such that, whenever \( j > J'' \) and \( k > K''(j) \), \( \delta_j \) and \( \gamma_{1,j,k} \) are sufficiently close to 1 so that this gap in the expected payoffs holds. Then, the version of Corollary 5.5 of Fudenberg and Levine [1993] presented in Fudenberg and He [2018] implies that, for every \( j > J'' \), there is some \( N''_{s,j} \) such that the share of type \( \theta \) sender agents who have a belief satisfying the conditions in (3), have played \((s, m_s, \bar{v}(s, \pi))\) more than \( N''_{s,j} \) times, and are set to play \((s, m_s, \bar{v}(s, \pi))\) in the current period is less than \( \varepsilon(1 - \gamma_{2,j})/4 \) for all \( k > K''(j) \).

Let \( J = \max\{J', J'', J''\} \), \( K(j) = \max\{K'(j), K''(j), K'''(j)\} \) for all \( j > J \), and \( N_{s,j} = \max\{N'_s, N''_s\} \) for all \( j > J \). Combining the preceding results shows that, when \( j > J \) and \( k > K(j) \), the share of type \( \theta \) sender agents who have played \((s, m_s, \bar{v}(s, \pi))\) more than \( N_{s,j} \) times and are set to play \((s, m_s, \bar{v}(s, \pi))\) in the current period is no more than \( 3\varepsilon(1 - \gamma_{2,j})/4 \). Additionally, using the version of Lemma 5.7 of Fudenberg and
Levine [1993] presented in Fudenberg and He [2018], it follows that, for all \( j > J \), \( K(j) \) can also be chosen so that \( \pi_{1,j,k}(s, m_s, \Theta(s, \pi)) \) exceeds the share of type \( \theta \) sender agents who have played \( (s, m_s, \Theta(s, \pi)) \) more than \( N''_{s,j} \) times and are set to play \( (s, m_s, \Theta(s, \pi)) \) in the current period by no more than \( \varepsilon(1 - \gamma_2) / 4 \) when \( k > K(j) \). Thus, we conclude that \( \pi_{1,j,k}(s, m_s, \Theta(s, \pi)) \leq \varepsilon(1 - \gamma_2) \) for all \( j > J \) and \( k > K(j) \). ■

The proof of Lemma OA 3 uses Lemma OA 5 to show that, in a sequence of steady states converging to a stable* profile with strong incentives, the probability that a receiver encounters a non-justified sender type playing \( (s, m_s, \Theta(s, \pi)) \) over the course of their lifetime converges to 0. Initial trust then ensures that the aggregate receiver response to each \( (s, m_s, \Theta(s, \pi)) \) is justified.

**Proof of Lemma OA 3.** Let \( \{\pi_{j,k} \in \Pi^*(g, \delta_j, \gamma_{1,j,k}, \gamma_{2,j})\}_{j,k \in \mathbb{N}} \) be a sequence of steady state profiles such that \( \lim_{j \to \infty} \lim_{k \to \infty} \pi_{j,k} = \pi \), where \( \lim_{j \to \infty} \delta_j = 1 \), \( \lim_{j \to \infty} \gamma_{2,j} = 1 \), and \( \lim_{k \to \infty} \delta_{j,k} = 1 \) for all \( j \). By Lemma OA 5, for any \( \varepsilon > 0 \), there exists some \( J \in \mathbb{N} \) and some function \( K : \mathbb{N} \to \mathbb{N} \) such that \( \pi_{1,j,k}(s, m_s, \Theta(s, \pi)) \leq \varepsilon(1 - \gamma_2) / \lambda(\theta) \) for all \( \theta \notin \Theta(s, \pi), j > J \), and \( k > K(j) \). Thus, when \( j > J \) and \( k > K(j) \), the probability that a receiver agent in a given period encounters a sender type outside of \( \Theta(s, \pi) \) playing \( (s, m_s, \Theta(s, \pi)) \) is no greater than \( \varepsilon(1 - \gamma_2) \). It follows that, when \( j > J \) and \( k > K(j) \), the probability that a receiver agent never encounters a sender type outside of \( \Theta(s, \pi) \) playing \( (s, m_s, \Theta(s, \pi)) \) over the course of their lifetime is at least

\[
\sum_{t=0}^{\infty} (1 - \gamma_{2,j}) \gamma_{2,j}^t (1 - \varepsilon(1 - \gamma_{2,j}))^t = \frac{1}{1 + \gamma_{2,j} \varepsilon}.
\]

Receivers who have never observed the signal-message pair \( (s, m_s, \Theta(s, \pi)) \) played by a type outside of \( \Theta(s, \pi) \) would respond to this pair with an action belonging to \( BR(\Theta(s, \pi), s) \). Thus,

\[
\pi_2(BR(\Theta(s, \pi), s) | s, m_s, \Theta(s, \pi)) = \lim_{j \to \infty} \lim_{k \to \infty} \pi_{2,j,k}(BR(\Theta(s, \pi), s) | s, m_s, \Theta(s, \pi)) \geq 1/(1 + \varepsilon).
\]
Since this holds for all \( \epsilon > 0 \), we have that \( \pi_2(BR(\Theta(s, \pi), s)|s, m_{s,\Theta(s, \pi)}) = 1 \). □

References


