

Determinacy without the Taylor Principle

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Outline

- 1 Introduction
- 2 A Simplified New Keynesian Model
- 3 The Standard Paradigm
- 4 Uniqueness with Fading Memory
- 5 The Full NK Model and FTPL
- 6 Observation of Past Outcomes
- 7 Discussion
- 8 Conclusion

A Perennial Issue

- Important questions (trickier than they look):
 - ▶ What determines the price level?
 - ▶ Can monetary policy regulate AD and inflation?
 - ▶ Does the ZLB trigger a deflationary spiral?
- Basic problem (back to Sargent & Wallace):
 - ▶ same path for $R \Rightarrow$ multiple equilibrium paths for π and y
 - ▶ need equilibrium selection
 - ▶ different selections \Rightarrow different answers to above questions

The (Confusing?) State of the Art

- Two alternatives in the theory:
 - ▶ **Taylor principle** (TP)
 - ★ π and y pinned down by interest-rate policy
 - ★ good news for the Fed's role
 - ★ but: TP = an off-eq. threat to blow up the economy? (Cochrane)
 - ▶ **Fiscal Theory of the Price Level** (FTPL)
 - ★ π and y pinned down by gov debt and deficits
 - ★ upsets lessons of the NK model (Cochrane)
 - ★ but: FTPL = an off-eq. threat to blow out the gov budget?
(Kocherlakota & Phelan, Buiter, etc)
- Debate hard to settle
 - ▶ both approaches are equally logical (within the current paradigm)
 - ▶ both boil down to off-eq assumptions, which can not be tested
 - ▶ choice between the two = a choice of religion! (Kocherlakota & Phelan)

This Paper

- Recast the NK economy as a game among current and future consumers
 - ▶ intertemporal Keynesian cross, plus feedbacks via Phillips curve and MP response
- All equilibria but one hinge on strong intertemporal coordination:
 - ▶ current consumers respond to sunspots, or payoff-irrelevant histories, only because they expect future consumers to do the same and in a very specific way
 - ▶ future consumers do it because expect agents further in the future to do the same; so on.
 - ▶ sunspots must have “infinite memory” in the economy

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- All equilibria but one hinge on strong intertemporal coordination:
 - ▶ current consumers respond to sunspots, or payoff-irrelevant histories, only because they expect future consumers to do the same and in a very specific way
 - ▶ future consumers do it because expect agents further in the future to do the same; so on.
 - ▶ sunspots must have “infinite memory” in the economy
- **Tiny friction in memory/coordination \Rightarrow breaks this infinite chain \Rightarrow \Rightarrow Unique equilibrium**
 - ▶ coincides with what is known as the model’s fundamental/MSV solution
 - ▶ but does not require Taylor principle
- **No room for FTPL** (at least as currently formulated)
 - ▶ non-Ricardian assumption = equil non-existence (logical impossibility, not religious choice)

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A simplified NK economy

- Overlapping generations of consumers, each living two periods:

$$u(C_{i,t}^1) + \beta u(C_{i,t+1}^2) e^{-\rho t}$$

- Budgets:

$$C_{i,t}^1 + B_{i,t} = Y_t \quad \text{and} \quad C_{i,t+1}^2 = Y_{t+1} + \frac{I_t}{\Pi_{t+1}} B_{i,t}$$

- ▶ Young and old earn the same income Y_t (for simplicity)
- ▶ Young are “strategic”: they optimally choose consumption and saving, given expectations of Y and Π , which will map to expectations of the choices of other consumers
- ▶ Old are “robots”: their consumption mechanically adjusts to meet budget

Intertemporal Keynesian Cross, or DIS

- Log-linearized optimal consumption for the young:

$$c_{i,t}^1 = E_{i,t} \left[\frac{1}{1+\beta} y_t + \frac{\beta}{1+\beta} y_{t+1} - \frac{\beta}{1+\beta} \sigma(i_t - \pi_{t+1} - \rho_t) \right]$$

- Market clearing + old and young earn same income \Rightarrow

$$\int c_{i,t}^1 di = \int c_{i,t}^2 di = c_t = y_t$$

- Combining \Rightarrow

$$c_t = \bar{E}_t \left[\frac{1}{1+\beta} c_t + \frac{\beta}{1+\beta} c_{t+1} - \frac{\beta}{1+\beta} \sigma(i_t - \pi_{t+1} - \rho_t) \right]$$

- With FIRE, this reduces to RA's Euler:

$$c_t = \mathbb{E}_t[c_{t+1}] - \sigma(i_t - \mathbb{E}_t[\pi_{t+1}] - \rho_t)$$

- More generally: an example of an “intertemporal Keynesian cross”

Our Three-Equation Model

- 1 DIS, from above:

$$c_t = \bar{E}_t \left[\frac{1}{1+\beta} c_t + \frac{\beta}{1+\beta} c_{t+1} - \frac{\beta}{1+\beta} \sigma(i_t - \pi_{t+1} - \rho_t) \right] \quad (\text{DIS})$$

- ▶ richer version, with infinitely-lived or OLG, later

- 2 Phillips curve:

$$\pi_t = \kappa c_t + \xi_t \quad (\text{PC})$$

- ▶ ad hoc and static for now, micro-founded and forward-looking later

- 3 Taylor rule:

$$i_t = z_t + \phi \pi_t \quad (\text{MP})$$

- ▶ nests pegs with $\phi = 0$, allows both $\phi < 1$ and $\phi > 1$

From 3 equations to 1, and a game-theoretic prism

- Solving (PC) and (MP) for π_t and i_t , and substituting in (DIS) \Rightarrow

$$c_t = \bar{E}_t [(1 - \delta_0)\theta_t + \delta_0 c_t + \delta_1 c_{t+1}]$$

- ▶ θ_t is a transformation of (ρ_t, ξ_t, z_t) , $\delta_0 \equiv \frac{1 - \beta\sigma\phi\kappa}{1 + \beta} < 1$, and $\delta_1 \equiv \frac{\beta + \beta\sigma\kappa}{1 + \beta} > 0$
- NK economy = a game among consumers
 - ▶ δ_0 and δ_1 measure strategic complementarity within and across time
 - ▶ summarize all GE feedbacks: income \leftrightarrow spending, output \leftrightarrow inflation, MP response

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- NK economy = a game among consumers
 - ▶ δ_0 and δ_1 measure strategic complementarity within and across time
 - ▶ summarize all GE feedbacks: income \leftrightarrow spending, output \leftrightarrow inflation, MP response
- With FIRE, this reduces to:

$$c_t = \theta_t + \delta \mathbb{E}_t [c_{t+1}]$$

$$\delta \equiv \frac{\delta_1}{1 - \delta_0} = \frac{1 + \kappa\sigma}{1 + \phi\kappa\sigma} > 0$$

- Under this prism, Taylor principle translates as follows:

$$\phi > 1 \Leftrightarrow \delta < 1 \text{ ("forward-stable")} \Leftrightarrow \delta_0 + \delta_1 < 1 \text{ ("weak complementarity")}$$

Fundamentals, Sunspots, and Equilibrium Concept

- Fundamentals:

$$\theta_t = \rho\theta_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim_{\text{i.i.d.}} \mathcal{N}(0, 1)$$

- Sunspots:

$$\eta_t \sim_{\text{i.i.d.}} \mathcal{N}(0, 1)$$

- In paper: general stochasticity

- State of nature, or history, at t :

$$h^t = \{\theta_{t-k}, \eta_{t-k}\}_{k=0}^{\infty}$$

- Solution concept: **Rational Expectations Equilibrium (REE)**
 - ▶ but, based on potentially limited information about h^t (this will prove crucial)
 - ▶ plus, linearity and stationarity (for tractability):

$$c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k} + \sum_{k=0}^{\infty} \gamma_k \theta_{t-k}$$

- ▶ plus, bounded (can be justified by escape clauses)

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The Standard Paradigm

- FIRE / representative-agent benchmark:

$$c_t = \theta_t + \delta \mathbb{E}_t [c_{t+1}]$$

- ▶ $\mathbb{E}_t[\cdot]$ is rational expectation conditional on entire history h^t
- The fundamental or minimum state variable (MSV) solution:
 - ▶ equation is only forward looking, plus θ_t is sufficient statistic for the future
 - ▶ so guess and verify $c_t = \gamma \theta_t$, to find

$$c_t = c_t^F \equiv \frac{1}{1 - \delta\rho} \theta_t$$

- Is MSV the only equilibrium?

The Standard Paradigm

Proposition. FIRE/RA Benchmark

- When $\phi > 1$ (Taylor principle), the MSV solution is the unique equilibrium
- When $\phi < 1$, there exist a **continuum of equilibria**, given by

$$c_t = (1 - b)c_t^F + bc_t^B + ac_t^\eta,$$

where $a, b \in \mathbb{R}$ are arbitrary scalars and where

$$c_t^\eta \equiv \sum_{k=0}^{\infty} \delta^{-k} \eta_{t-k} \quad \text{and} \quad c_t^B \equiv - \sum_{k=1}^{\infty} \delta^{-k} \theta_{t-k}$$

- c_t^η is a **sunspot equilibrium**
 - ▶ a non-zero solution to $c_t = \delta \mathbb{E}_t [c_{t+1}]$
- c_t^B is a **backward-looking, pseudo-fundamental equilibrium**
 - ▶ the product of solving $c_t = \theta_t + \delta c_{t+1}$ backwards

Understanding the Multiplicity

- Equilibrium condition (back-shifted)

$$c_{t-1} = \theta_{t-1} + \delta \mathbb{E}_{t-1} [c_t]$$

- Solving backwards (when $\delta > 1$):

$$\begin{aligned}\mathbb{E}_{t-1}[c_t] &= \delta^{-1}(c_{t-1} - \theta_{t-1}) \\ c_t &= \delta^{-1}(c_{t-1} - \theta_{t-1}) + \eta_t \\ c_t &= \underbrace{-\sum_{k=1}^{\infty} \delta^{-k} \theta_{t-k}}_{\substack{\text{backward-looking} \\ \text{pseudo-fundamental}}} + \underbrace{\sum_{k=0}^{\infty} \delta^{-k} \eta_{t-k}}_{\text{sunspot}}\end{aligned}$$

- **Infinite chain based on “thin air”:**

- ▶ Current agents ignore current fundamental and respond to **payoff-irrelevant histories** because they **expect to be “rewarded” appropriately by future agents**
- ▶ Future agents themselves respond on the basis of a similar expectation

What's Next: Two Perturbations Breaking the Infinite Chain

- Two equivalent representations of non-fundamental equilibria:

(Sequential Form)

$$c_t = - \sum_{k=1}^{\infty} \delta^{-k} \theta_{t-k} + \sum_{k=0}^{\infty} \delta^{-k} \eta_{t-k}$$

(Recursive Form)

$$c_t = \delta^{-1}(c_{t-1} - \theta_{t-1}) + \eta_t$$

- Two takes on infinite chain : c_t needs to respond to payoff-irrelevant distant past
 - ▶ directly in Sequential Form
 - ▶ indirectly in Recursive Form
- Two perturbations that make infinite chain unravel:
 - ▶ Fading social memory about $\eta_{t-k} \implies$ determinacy
 - ▶ Bounded social memory of what drives $c_{t-1} \implies$ determinacy

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The First Perturbation

Assumption. Fading Social Memory

In every t , information set are given by

$$I_{i,t} = \{(\theta_t, \eta_t), \dots, (\theta_{t-s}, \eta_{t-s})\},$$

where $s \in \{0, 1, \dots\}$ is drawn from a **geometric distribution** with $\lambda \in (0, 1)$.

- i.e., $s = 0$ with prob λ , $s = 1$ with prob $(1 - \lambda)\lambda$, ... $s = k$ with prob $(1 - \lambda)^k \lambda$

Interpretation in terms of **social transmission of information**:

- at every t , the typical young consumer learns current shocks (θ_t, η_t)
- with prob. λ , she learns nothing more;
- with prob. $1 - \lambda$, she inherits the info of a random old consumer

The First Perturbation

Social memory:

- $\mu_k \equiv$ mass of agents that knows histories of length k or higher $= (1 - \lambda)^k$

RA benchmark, nested by $\lambda = 0$:

- All agents have infinite memory (and this is common knowledge)
- Defines *infinite* social memory, and *perfect* coordination

Key properties for $\lambda > 0$:

- Zero mass has truly *infinite* memory
- But as $\lambda \rightarrow 0$, **almost all** consumers **have arbitrarily long memory**
- No direct knowledge of endogenous outcomes, or of the actions of others
- But as $\lambda \rightarrow 0$, **arbitrarily well informed** about arbitrarily long histories of $\{c_{t-k}, \pi_{t-k}\}$

Main Result: Determinacy without Taylor Principle

- Bottom line: $\lambda > 0$ small enough = nearly infinite social memory = nearly RA
- Still: for any $\lambda > 0$, no matter how small, $\mu_k \rightarrow 0$ as $k \rightarrow \infty$

Proposition. Determinacy with fading social memory

The MSV solution is the unique equilibrium

- Regardless of ϕ (e.g., even with pegs)
- No matter how slow the memory decay is (how small $\lambda > 0$ is).

Proof Sketch

- Abstract from fundamentals, let only shock be η_0 (effectively, focus on IRF of c_t to η_0):

$$c_t = \alpha_t \eta_0 \quad \text{for some coefficient } \alpha_t \equiv \frac{\partial c_t}{\partial \eta_0}$$

- Use this to rewrite key equilibrium equation as

$$c_t = \delta_0 \underbrace{\alpha_t \bar{E}_t[\eta_0]}_{\bar{E}_t[c_t]} + \delta_1 \underbrace{\alpha_{t+1} \bar{E}_t[\eta_0]}_{\bar{E}_t[c_{t+1}]}$$

- Next, use $\bar{E}_t[\eta_0] = \mu_t \eta_0$ to get

$$c_t = \delta_0 \underbrace{\mu_t \alpha_t \eta_0}_{c_t} + \delta_1 \underbrace{\mu_t \alpha_{t+1} \eta_0}_{\mathbb{E}_t[c_{t+1}]}$$

where $\mu_t = (1 - \lambda)^t$ is mass remembering η_0 at t .

- Maps to “twin” economy with infinite memory ($\lambda = 0$) but modified best response:

$$c_t = (\mu_t \delta_0) c_t + (\mu_t \delta_1) \mathbb{E}[c_{t+1}]$$

Proof Sketch

- Maps to “twin” economy with infinite memory ($\lambda = 0$) but modified best response:

$$c_t = (\mu_t \delta_0) c_t + (\mu_t \delta_1) \mathbb{E}[c_{t+1}]$$

- ▶ $\lim_{t \rightarrow \infty} \mu_t = 0 \Rightarrow \mu_t \delta_0 + \mu_t \delta_1 < 1$ for t large enough
- ▶ perceived strategic complementarity eventually fades \implies determinacy

Proof Sketch

- Maps to “twin” economy with infinite memory ($\lambda = 0$) but modified best response:

$$c_t = (\mu_t \delta_0) c_t + (\mu_t \delta_1) \mathbb{E}[c_{t+1}]$$

- ▶ $\lim_{t \rightarrow \infty} \mu_t = 0 \Rightarrow \mu_\tau \delta_0 + \mu_\tau \delta_1 < 1$ for τ large enough
 - ▶ perceived strategic complementarity eventually fades \implies determinacy
- Equivalently, RA economy with modified eigenvalue:

$$c_t = \tilde{\delta}_t \mathbb{E}[c_{t+1}] \quad \text{with} \quad \tilde{\delta}_t \equiv \frac{\mu_t \delta_1}{1 - \mu_t \delta_0}$$

- ▶ $\lim_{t \rightarrow \infty} \mu_t = 0 \Rightarrow \tilde{\delta}_\tau < 1$ for τ large enough $\implies c_t = 0$ is unique outcome for $t \geq \tau$
- ▶ by backward induction starting at $\tau \implies c_t = 0$ is unique outcome for $t < \tau$ as well

Logic

- I can see the current sunspot very clearly
- It would make sense to react if all future agents will keep responding to it **in perpetuity**
- But I worry that agents **far in the future will fail to do so**
 - ▶ either because they will have forgotten it
 - ▶ or because they may worry that agents further into the future will not react to it
- It therefore makes sense to ignore the sunspot

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The Full NK Model: AD Side

- Let consumers be long lived but imperfectly informed or inattentive
- Consumer optimality (basically, PIH):

$$c_{i,t} = E_{i,t} \left[(1 - \beta) a_{i,t} - \beta \sigma \sum_{k=0}^{+\infty} \beta^k (i_{t+k} - \pi_{t+k+1}) + (1 - \beta) \sum_{k=0}^{+\infty} \beta^k (y_{t+k} - \tau_{t+k}) \right]$$

- Gov's intertemporal budget:

$$b_t - p_{t-1} = \sum_{k=0}^{+\infty} \beta^k (\tau_{t+k} - g_{t+k})$$

- Market clearing:

$$\int a_{i,t} di = b_t - p_{t-1} \quad \text{and} \quad y_t = c_t \equiv \int c_{i,t} di$$

- Put together \implies intertemporal Keynesian cross:

$$c_t = -\beta \sigma \left\{ \sum_{k=0}^{+\infty} \beta^k \bar{E}_t [i_{t+k} - \pi_{t+k+1}] \right\} + (1 - \beta) \left\{ \sum_{k=0}^{+\infty} \beta^k \bar{E}_t [c_{t+k}] \right\}$$

The Full NK Model: its three equations

- Intertemporal Keynesian cross (proper DIS):

$$c_t = -\beta\sigma \left\{ \sum_{k=0}^{+\infty} \beta^k \bar{E}_t [i_{t+k} - \pi_{t+k+1}] \right\} + (1-\beta) \left\{ \sum_{k=0}^{+\infty} \beta^k \bar{E}_t [c_{t+k}] \right\}$$

- ▶ reduces to RA's Euler (plus transversality) under full information
 - ▶ summarizes consumer optimality and market clearing more generally (think HANK)
 - ▶ in paper: robust to idiosyncratic shocks as long as there are no "misperceptions"
- Standard forward-looking NKPC:

$$\pi_t = \kappa c_t + \beta E_t [\pi_{t+1}] + \xi_t$$

- ▶ unlike consumers, firms are perfectly informed (can be relaxed)
- Monetary policy:

$$i_t = z_t + \phi_c c_t + \phi_\pi \pi_t$$

- ▶ for arbitrary $\phi_c, \phi_\pi \geq 0$ (nests pegs with $\phi_c = \phi_\pi = 0$)

The Generalized Result

- From three equations to one:
 - ▶ solve NKPC and MP for (π_t, i_t) as a function of $\{c_{t+k}\}_{k=0}^{\infty}$
 - ▶ replace in DIS to get an equation of the form

$$c_t = \theta_t + \bar{E}_t \left[\sum_{k=0}^{+\infty} \delta_k c_{t+k} \right]$$

for some coefficients $\{\delta_k\}_{k=0}^{\infty}$ that encapsulate all the GE feedbacks

- Once again: a game among the consumers, albeit with more complicated best response
 - ▶ math is more complicated, but basic logic is same.
 - ▶ precise values of δ_k don't matter, result extends as long as $\delta_0 < 1$ and $\sum_{k=0}^{\infty} |\delta_k| < \infty$

Proposition. Determinacy with fading social memory

Maintain our assumption about information, for any $\lambda > 0$.

Then, the **MSV solution** is again the **unique equilibrium**

Fiscal Theory of Price Level

- Conventional approach:
 - ▶ assume Taylor principle \implies select MSV solution
 - ▶ debt and deficits do not appear in $DIS+NKPC+MP \implies$
 - ★ Ricardian equivalence
 - ★ fiscal regime does not enter MSV
- Known as “monetary-active regime” (Leeper) or “Ricardian regime” (Woodford)

Fiscal Theory of Price Level

- Alternative approach: FTPL

- ▶ suppose the MSV solution is such that

$$\frac{B_{t-1}}{P_t^{(MSV)}} \neq PVS_t$$

- ▶ then MSV cannot be a “full” equilibrium
- ▶ but if drop Taylor principle, another solution to DIS+NKPC+MP such that

$$\frac{B_{t-1}}{P_t^{(FTPL)}} = PVS_t$$

and this is the “right” equilibrium

- ▶ pins $\{P_t\}$ and hence also $\{c_t, \pi_t, i_t\}$ as a function of debt and deficits
- Known as “fiscal-active regime” (Leeper) or “nonRicardian regime” (Woodford)
- Breaks Ricardian equivalence
 - ▶ not by liquidity constraints, finite lives etc
 - ▶ but rather by force of equilibrium selection (“threat of equil non-existence”)

Our Take on the FTPL

- Our result \Rightarrow **no room for “non-Ricardian” regime**
 - ▶ even if Taylor principle does not hold ($\phi_\pi < 1$)
 - ▶ $\{c_t, \pi_t, i_t\}$ necessarily pinned down by MSV solution to DIS+NKPC+MP
 - ▶ Ricardian equivalence *has* to hold
 - ▶ fiscal policy *has* to adjust to make sure

$$\frac{B_{t-1}}{P_t^{(MSV)}} = PVS_t$$

- **Non-Ricardian assumption = actual equil non-existence**
 - ▶ to paraphrase Kocherlakota & Phelan:
 - ▶ “rejecting the non-Ricardian assumption is a logical necessity, not a religious choice”
- But of course: fiscal policy may still matter if MP internalizes fiscal conditions
 - ▶ perhaps this is what the FTPL *should* be about?

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Observation of Past Outcomes

- So far:
 - ▶ preclude *direct* knowledge of past outcomes, such as c_{t-1} or π_{t-1}
 - ▶ but as $\lambda \rightarrow 0$, *almost perfect but private and indirect* knowledge of past outcomes
- But what about *direct* knowledge of past outcomes?
- Shut down fundamentals (tentatively) and consider recursive representation of pure sunspot equilibrium:

$$c_t = \delta^{-1} c_{t-1} + \eta_t$$

- This can be supported by

$$l_{i,t} = \{\eta_t, c_{t-1}\} \quad \text{instead of} \quad l_{i,t} = \{\eta_t, \eta_{t-1}, \eta_{t-2}, \dots\}$$

- ▶ c_{t-1} alone serves as a sufficient statistic (“storage device”) for infinite memory
- Is our take on “infinite memory” misleading?
- No! It’s the simple recursive logic that is misleading!

Illustrative Example

- **Private signals** of the past aggregate outcome (turn off θ_t tentatively)

$$I_{i,t} = \{\eta_t, s_{i,t}\}, \quad s_{it} = c_{t-1} + \varepsilon_{i,t}$$

where $\varepsilon_{i,t} \sim \mathcal{N}(0, \sigma)$ is i.i.d. and independent of η_t

Proposition. Noisy Recursive Equilibria

- When $\sigma = 0$, there exist multiple sunspot equilibria.
 - When instead $\sigma > 0$, no matter how small, **sunspot equilibria cease to exist**, and the MSV solution is the unique equilibrium.
-
- Illustrates fragility of *simplest* recursive equilibria
 - But what about more complicated cases, with more info about past outcomes?

The Second Perturbation

- Let's even insist on *perfect* and *public* knowledge of $\{c_{t-1}, \dots, c_{t-K}\}$ for K finite but arbitrarily large
- But accommodate uncertainty (and no revelation) about a small driver of past outcomes
- Formally, add small fundamental shock that is known at t but unknown at $\tau \geq t+1$.

▶ best responses:

$$c_t = (1 - \delta_0)(\theta_t + \zeta_t) + \delta_0 \bar{E}_t [c_t] + \delta_1 \bar{E}_t [c_{t+1}]$$

- ▶ payoff perturbation: $\zeta_t \sim_{\text{i.i.d.}} U[-\varepsilon, +\varepsilon]$, for $\varepsilon > 0$ but arbitrarily small
- ▶ information: for all t and all i ,

$$I_{i,t} = I_t = \{\zeta_t\} \cup \{\theta_t, \dots, \theta_{t-K}\} \cup \{\eta_t, \dots, \eta_{t-K}\} \cup \{\dots, c_{t-K}\}$$

- Equivalently: representative but “forgetful” agent in each period ($I_t \not\subseteq I_{t+1}$)

The Second Perturbation

Proposition. Determinacy even with CK of past outcomes

For any $\varepsilon > 0$, not matter how small, there is a **unique equilibrium** and is given by $c_t = c_t^F + \zeta_t$, where c_t^F is the same **MSV solution** as before.

- Intuition:

- ▶ to support infinite chain, the agents at $t+1$ must know how to “reward” the agents at t
- ▶ in particular, they must be able to play

$$c_{t+1} = \delta^{-1}(c_t - \theta_t - \zeta_t) + \eta_{t+1}$$

- ▶ but if they don't know ζ_t , they don't know the “preferences” of the agents at t , and they can't reward them in the needed way
- ▶ and because the agents at t understand this, the chain breaks at t (and before)

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Feedback Rules and Ramsey Implementation

- Consider the **Ramsey optimum**. How can monetary policy uniquely implement it?
- If the monetary authority **observes the underlying shocks**, uniquely implemented with:

$$i_t = i_t^o + \phi(\pi_t - \pi_t^o),$$

where i_t^o and π_t^o are rates and inflation in the optimum and $\phi > 1$.

- What if the monetary authority **does not observe the underlying shocks**?
 - ▶ implemented through feedback rules?

$$i_t = \phi \pi_t$$

- Two conflicting roles
 - ▶ **Stabilization** ($\phi < 1$ possible in the Ramsey optimum)
 - ▶ **Eq. selection** ($\phi > 1$ necessary in the standard paradigm)
- Here: Liberates the **stabilization role** of monetary policy from **its eq. selection role**

Alternative Boundedly-Rational Solution Concepts

① Relax REE but maintain a “fixed point” between expectations & actual

- ▶ e.g., Cognitive discounting in Gabaix (20); Diagnostic expectations in Bordalo et. al (20)
- ▶ may change the parameter region ϕ for which there is indeterminacy
- ▶ but the indeterminacy problem remains, and so does the “war” with the FTPL

- ▶ also: a version of our logic extends to such solutions concepts as well

② Completely shut down the “fixed point”

- ▶ e.g. Level-k thinking (Garcia-Schmidt & Woodford, 19; Farhi & Werning, 19)
- ▶ produces a unique solution but opens a new issue
- ▶ whenever $\phi < 1$, Level-k solution becomes infinitely sensitive to Level-0 behavior
- ▶ so doesn't really resolve the issue, but complements our approach in solidifying the logical foundations of the conventional solution

Outline

- 1 Introduction
- 2 A Simplified New Keynesian Model
- 3 The Standard Paradigm
- 4 Uniqueness with Fading Memory
- 5 The Full NK Model and FTPL
- 6 Observation of Past Outcomes
- 7 Discussion
- 8 Conclusion**

Conclusion (1)

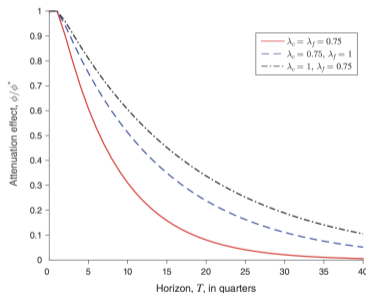
- NK indeterminacy/FTPL hinge on **strong info assumptions**
- **A small friction in memory & intertemporal coordination** can result in **determinacy**
 - ▶ equilibrium same as conventional, but not equi selection conondrum
- Taylor principle perhaps less consequential than previously thought
 - ▶ more crucial: boundedness (commitment to rule out large deviations)
 - ▶ and: stabilization role of high ϕ
- **No room for FTPL** as currently formulated
 - ▶ non-Ricardian assumption = equil non-existence even if MP is passive
 - ▶ but **fiscal considerations can matter if internalized by MP**
 - ▶ model MP-FP interaction as a game of between monetary & fiscal authority?

Bonus: Revisiting the MSV Solution

- So far: θ_t is perfectly and commonly known \Rightarrow MSV solution unaffected by friction
- How does c_t^F change if θ_t is not perfectly and commonly known?
- That's the question studied by existing literature
 - ▶ Woodford, Mankiw-Reis, Mackowiac-Wiederholt
 - ▶ my earlier work with Chen and Zhen
 - ▶ that is, this literature assumes away the determinacy issue by imposing TP and focuses on “noising up” the fundamental solution
- Key difference: noise has to be large to affect MSV
- But common theme:
 - ▶ info friction attenuates strategic complementarity/GE feedback
 - ▶ here: lower them enough to kill self-fulfilling equilibria
 - ▶ earlier work: reduce GE multipliers for forward guidance, monetary shocks etc

Illustration 1: Angeletos & Lian (AER 2018)

- Liquidity trap of length T , forward guidance about i_t after T
 - ▶ akin to a news shock at $t = 0$ for θ_{T+1}
- Add idios noise in observation of this news (higher-order uncertainty)
- Study how effect varies with T



- Why? Because higher T maps to a large GE multiplier in the standard model, which is greatly attenuated by the noise/higher-order uncertainty

Illustration 2: Angeletos & Huo (AER 2021)

- Normal times (away from ZLB), stationary environment
- Main result: adding noise to MSV is *as if* adding two behavioral distortions:

$$c_t = \theta_t + \omega_f \delta E_t[c_{t+1}] + \omega_b c_{t-1}$$

with $\omega_f < 1$ (“myopia”) and $\omega_b > 0$ (“anchoring”)

- Also: both distortions increase with GE params such as MPC/slope of Keynesian cross
 - ▶ HANK meets HOB

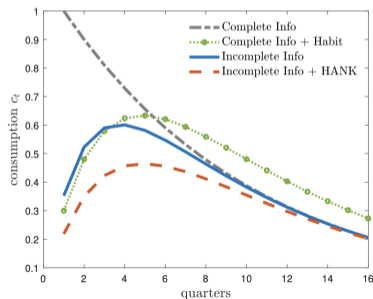


FIGURE 4. RESPONSE OF CONSUMPTION TO LOWER INTEREST RATES

Conclusion (2): Two Birds with One Stone

- This paper:
 - ▶ resolve classic indeterminacy issue
 - ▶ reinforce logical foundations of conventional solution
 - ▶ mute FTPL critique
- Complementary previous work:
 - ▶ improve empirical properties of standard solution
 - ▶ reduce forward-guidance puzzle etc
 - ▶ interaction with HANK etc