Determinacy without the Taylor Principle

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The Equilibrium Selection Issue in the NK Model

- Can monetary policy regulate AD by adjusting interest rates?

- Important caveat (e.g., Sargent & Wallace):
  - Same nominal interest rate path consistent with multiple bounded eq.
  - Need for equilibrium selection

- Standard approach: Taylor principle (raise rates aggressively with inflation)
  - An off-eq. threat to trigger an explosion in $\pi$ and $y$ (Cochrane)
  - Or a reversion to $M$ regime for large enough deviations (Atkeson, Chari, & Kehoe)

- Alternative: Fiscal Theory of the Price Level (Leeper, Sims, Woodford)
  - An off-eq. threat to blow out the government budget (Kocherlakota & Phelan)
  - Or other interpretations of non-Ricardian fiscal policy (Cochrane, Bassetto)

- Eq. selection debate is a war of “religious beliefs” (Kocherlakota & Phelan)
  - Cannot be guided by empirical evidence and are inherently untestable
This Paper: Determinacy without the Taylor Principle

- Sunspot eq. artifacts of **perfect intertemporal coordination ("infinite chain")**
  - Current agents respond to “irrelevant” sunspots only if future agents respond in a specific way
  - Future agents respond only if they expect agents further in the future respond; and so on.

- Small perturbations in memory/coordination $\Rightarrow$ breaks the infinite chain $\Rightarrow$ determinacy

- **Always selects the standard eq.** (minimum-state-variable eq.)

- Taylor principle perhaps less consequential than previously thought

- No room for FTPL as currently formalized (as an eq. selection device)
  - but **fiscal considerations can matter through the eq. conduct by MP**

- Eases the potential conflict between stabilization and eq. selection
Outline

1 Introduction

2 A Simplified New Keynesian Model

3 The Standard Paradigm

4 Uniqueness with Fading Memory

5 The Generalized Model

6 Observing Past Outcomes

7 Discussion

8 Conclusion
A Simplified Model

- Dynamic IS ($\bar{E}_t[\cdot] = \int E_{i,t}[\cdot] di$ is the average expectation)
  
  $c_t = -\sigma (i_t - \bar{E}_t[\pi_{t+1}]) + \bar{E}_t[c_{t+1}] + \rho_t$

- Phillips curve (static for now, forward looking later)
  
  $\pi_t = \kappa c_t + \xi_t$

- Monetary policy
  
  $i_t = z_t + \phi \pi_t$
An Equivalent Representation

- Substituting monetary policy and Phillips curve in IS curve ⇒
  \[ c_t = \theta_t + \delta \tilde{E}_t [c_{t+1}] \]
  where \( \{\theta_t\} \) is a function of \( \{\rho_t, \xi_t, z_t\} \) and
  \[ \delta = \delta(\phi) \equiv \frac{1 + \kappa \sigma}{1 + \phi \kappa \sigma} \]

- **Taylor principle** holds when
  \[ \phi > 1 \iff \delta < 1 \]

- Equivalent formulation
  \[ \pi_t = \tilde{\theta}_t + \delta \tilde{E}_t [\pi_{t+1}] \]
  ▶ this nests the **flexible price case** \( (i_t = \tilde{E}_t [\pi_{t+1}]) \) with \( \kappa \to \infty \) \( (\delta \to \frac{1}{\phi}) \)
Fundamentals, Sunspots, and the Equilibrium Concept

- Fundamentals:
  \[ \theta_t = \rho \theta_{t-1} + \varepsilon_t, \quad \varepsilon_t \sim_{\text{i.i.d}} \mathcal{N}(0,1) \]
  - In paper: generalization allowing generic state space representations

- Sunspots:
  \[ \eta_t \sim_{\text{i.i.d}} \mathcal{N}(0,1) \]

- State of nature, or (infinite) history, at \( t \):
  \[ h^t = \{ \theta_{t-k}, \eta_{t-k} \}_{k=0}^{\infty} \]

- Equilibrium concept: **REE (based on potentially limited information about \( h^t \))**
  \[ c_t = \sum_{k=0}^{\infty} a_k \eta_{t-k} + \sum_{k=0}^{\infty} \gamma_k \theta_{t-k} \]

- Focus on **bounded** eq. \( \text{Var}(c_t) \) is finite). Can be justified by escape clauses by ACK.
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The Standard Paradigm

- **FIRE (full information rational expectations)/perfect recall benchmark:**
  \[ c_t = \theta_t + \delta E_t [c_{t+1}] \]
  - \( E_t[\cdot] \) is rational expectation conditional on entire history \( h_t \)

- **The MSV (minimum state variable) solution:**
  \[ c_t = c_t^F \equiv \frac{1}{1 - \delta \rho} \theta_t \]
  - guess and verify \( c_t = \gamma \theta_t \)

- **Is MSV the only solution?**
  - Taylor principle holds when \( \phi > 1 \iff \delta < 1 \)
  - If it does not hold \( \delta > 1 \), solve backward \( \implies \) sunspot and backward looking eq.
The Standard Paradigm

Proposition 1. Perfect Recall Benchmark

- When the Taylor principle is satisfied \(|\delta| < 1\), the MSV equilibrium is the unique one.
- When this principle is violated \(|\delta| > 1\), there exist a continuum of equilibria

\[ c_t = (1 - b)c_t^F + bc_t^B + ac_t^\eta, \]

where

- **Sunspot equilibria** (non-zero solution to \(c_t = \delta E_t [c_{t+1}]\))

\[ c_t^\eta \equiv \sum_{k=0}^{\infty} \delta^{-k} \eta_{t-k} \]

- **Backward fundamental equilibria**

\[ c_t^B \equiv - \sum_{k=1}^{\infty} \delta^{-k} \theta_{t-k} \]
Understanding the Multiplicity

Using the sunspot eq. as an example:

\[ c_t^\eta = \delta E_t [c_{t+1}^\eta] \]

Infinite chain of perfect intertemporal coordination:

- Current agents respond against their intrinsic interest because they expect to be rewarded by future agents

- Future agents themselves respond based on a similar expectation

- …
What’s Next: Breaking the Infinite Chain

What’s next: two perturbations breaking the infinite chain of perfect coordination

Two equivalent representations of the sunspot equilibrium

Sequential: \( c_t^n = \sum_{k=0}^{\infty} \delta^{-k} \eta_{t-k} \)

Recursive: \( c_t^n = \delta^{-1} c_{t-1}^n + \eta_t \)

- \( c_t^n \) needs to respond to distant-past sunspots (directly or indirectly)

First perturbation motivated by the sequential representation
- Fading social memory about \( \eta_{t-k} \) \( \rightarrow \) determinacy

Second perturbation motivated by the recursive representation
- Bounded social memory what drives (a tiny part of) \( c_{t-1}^n \) \( \rightarrow \) determinacy
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The First Perturbation

Memory:
- In each period, a randomly $\lambda \in [0,1]$ of agents are replaced by newborn agents.
- Agents **know fundamentals & sunspots during their lives** but **not before**.
- The period-$t$ information set of an agent born $s$ periods ago is given by

$$I_t^s \equiv \{(\theta_t, \eta_t), \ldots, (\theta_{t-s}, \eta_{t-s})\}$$
The First Perturbation

\[ l_t^s \equiv \{(\theta_t, \eta_t), \ldots, (\theta_{t-s}, \eta_{t-s})\} \]

Interpretation:
- **OLG with “fading” social memory**
  - Consistent with perfect individual recall & standard rational expectations solution concept
  - Equivalent behavioral interpretation: agents are infinitely-lived but have bounded recall

**Standard paradigm:**
- Perfect social memory, nested by \( \lambda = 0 \)

**Properties:**
- For any \( \lambda > 0 \), zero mass of agents has *infinite* memory
  - But as \( \lambda \to 0 \), *almost all agents have arbitrarily long memory*
- Prevent direct knowledge about history of endogenous \( \{c_{t-k}\} \)
  - But as \( \lambda \to 0 \), *arbitrarily well informed long histories of \( \{c_{t-k}\} \)*
Determinacy without the Taylor Principle

**Proposition 2. Determinacy without the Taylor Principle**

With fading social memory, the unique equilibrium is the MSV solution, $c_t = c_t^F$
- Regardless of the value of $\delta$, or equivalently monetary policy $\phi$.
- No matter how slow the memory decay is (how small $\lambda$ is).

**Proof sketch:** focusing on responses to $\eta_0 (a_t)$.
- “Twin” economy with perfect memory but modified best response:
  
  $$c_t = \theta_t + \delta \bar{E}_t [c_{t+1}] \implies c_t = \delta \mu_t E_t [c_{t+1}],$$

  where $\mu_t = (1 - \lambda)^t \to 0$ is the proportion of agents remembering $\eta_0$ at $t$.
- But $\delta \mu_t < 1$ eventually, so always determinacy.
I can see the current sunspot very clearly

It would make sense to react if all future agents will keep responding to it \textit{in perpetuity}

But I worry that agents \textit{far in the future will fail to do so}
  ▶ either because they will have forgotten it
  ▶ or because they may worry that agents further into the future will not react to it

It therefore makes sense to ignore the sunspot
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A Micro-funded NK Model

- A micro-founded IS curve robust to incomplete information

\[ c_t = -\beta \omega \sigma \left\{ \sum_{k=0}^{+\infty} (\beta \omega)^k \bar{E}_t [i_{t+k} - \pi_{t+k+1}] \right\} + (1 - \beta \omega) \left\{ \sum_{k=0}^{+\infty} (\beta \omega)^k \bar{E}_t [c_{t+k}] \right\} + \rho_t \]

- \( \omega = 1 - \lambda \) is the survival probability (as the OLG structure above)
- embeds individual optimality + market clearing + budgets
- reduces to the RA Euler equation (plus transversality) when \( \bar{E}_t [\cdot] = E_t [\cdot] \)

- Standard dynamic NKPC

\[ \pi_t = \kappa c_t + \beta E_t [\pi_{t+1}] + \xi_t \]

- Monetary policy

\[ i_t = z_t + \phi_c c_t + \phi_\pi \pi_t \]
The Generalized Model and Nesting

- The generalized model

\[ c_t = \theta_t + \bar{E}_t \left[ \sum_{k=0}^{+\infty} \delta_k c_{t+k} \right] \]

- only requires that the sum \( \sum_{k=0}^{\infty} |\delta_k| \) is finite

- Nests the previous micro-founded NK with

\[ \delta_k = (1 - \beta \omega - \beta \omega \sigma \phi_c) (\beta \omega)^k + \omega \sigma \kappa \left( -\phi_{\pi \beta} + (1 - \omega \phi_{\pi \beta}) \frac{1 - \omega^k}{1 - \omega} \right) \beta^k. \]
Proposition 3. Fading Memory Rules out Sunspot Volatility

With fading social memory ($\lambda > 0$), the equilibrium is unique and is given by the MSV solution.

**Proof sketch:** focusing on response to $\eta_0 (a_t)$.

- “Twin” economy with perfect memory but modified best response:

$$
c_t = \theta_t + \bar{E}_t \left[ \sum_{k=0}^{+\infty} \delta_k c_{t+k} \right] \quad \Longrightarrow \quad c_t = \mu_t E_t \left[ \sum_{k=0}^{+\infty} \delta_k c_{t+k} \right],
$$

where $\mu_t \rightarrow 0$ is the proportion of agents remembering $\eta_0$ at $t$.

- But $\mu_t (\sum_{k=0}^{\infty} |\delta_k|) < 1$ eventually, so always determinacy

- Effective complementary $< 1$, uniquely pinned down by iterating of best responses
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Observing Past Outcomes

- Baseline: preclude *direct* observation of past outcomes, such as $c_{t-1}$

- But note: agents have *almost perfect* knowledge of past outcomes
  - for any $T$, almost all agents learn $\{c_{t-1}, ..., c_{t-T}\}$ nearly perfectly as $\lambda \to 0$

- Still, what if perfectly observing past outcomes?
  - Could long memory of sunspots and past fundamentals be efficiently “stored” in short memory of past outcomes?

- For example, the recursive formulation of the sunspot equilibrium (turn off $\theta_t$ briefly)

  $$c_t = \eta_t + \delta^{-1}c_{t-1}$$

- Perfect memory of $c_{t-1}$ suffice as the memory of the history of sunspots
  - sunspot equilibria strike back?
Storing Memory in Endogenous Outcomes

- Still takes a strong, **fragile**, form of **intertemporal coordination**
  - Current agents respond because they expect future **respond in a perfect way**
  - Infinite chain of coordination ···

- Add i.i.d. fundamental shocks $\zeta_t \in [-\varepsilon, \varepsilon]$ (arbitrarily small) known only to $t$
  \[ c_t = \zeta_t + \delta \tilde{E}_t [c_{t+1}] \]

- For a sunspot eq, requires **perfect knowledge of $\zeta_t$ at $t+1$**
  \[ c_{t+1} = \eta_{t+1} + \delta^{-1} (c_t - \zeta_t) \]

- But if $\zeta_t$ unknown to agents at $t+1$, the sunspot equilibrium collapses
The Second Perturbation

- Bring back fundamentals $\theta_t$ with arbitrarily small. i.i.d. perturbations $\zeta_t \in [-\varepsilon, \varepsilon]$

$$c_t = \theta_t + \zeta_t + \delta \mathbb{E}[c_{t+1}|I_t]$$

- A representative agent in each period, with info set

$$I_t = \{\zeta_t\} \cup \{\theta_{t-K}, \ldots, \theta_{t-1}\} \cup \{\eta_{t-K}, \ldots, \eta_{t-1}\} \cup \{c_{t-1}, \ldots, c_{t-K}\}$$

  - Long memory of past sunspots, fundamentals, & outcomes for arbitrarily large but finite $K$
  - But knowledge of only current $\zeta_t$ & no memory of past $\zeta$s

Proposition 5. Storing Memory in Endogenous Outcomes

With above info. structure, regardless of $\delta$, there is a unique equilibrium and is given by $c_t = c_t^F + \zeta_t$, where $c_t^F$ is the same MSV solution as before.

- Break the infinite chain $\Rightarrow$ MSV as the unique eq
Fiscal Theory of Price Level (FTPL)

- Essence of the FTPL: **non-Ricardian fiscal policy**
  - primary surplus do respond enough to public debt level
  - An off-equilibrium threat to blow out the government budget (Kocherlakota & Phelan)
  - Or other interpretations (Cochrane, Bassetto)

- Standard paradigm: FTPL perfectly logical with “passive MP” ($\phi < 1$)
  - concur with **passive-monetary and active-fiscal regime in Leeper (1991)**

- Our contribution: no need/space for eq selection from FTPL
  - **underscores the fragility of existing formalization of FTPL**
  - but allow fiscal considerations to matter on eq. through conduct of MP
Consider the **Ramsey optimum**. How can monetary policy uniquely implement it?

If the monetary authority **observes the underlying shocks**, uniquely implemented with:

\[
i_t = i_t^o + \phi (\pi_t - \pi_t^o),
\]

where \(i_t^o\) and \(\pi_t^o\) are rates and inflation in the optimum and \(\phi > 1\).

What if the monetary authority **does not observe the underlying shocks**?

- implemented through feedback rules?

\[
i_t = \phi \pi_t
\]

Two conflicting roles

- **Stabilization** (\(\phi < 1\) possible in the Ramsey optimum)
- **Eq. selection** (\(\phi > 1\) necessary in the standard paradigm)

Here: Liberates the **stabilization role** of monetary policy from its **eq. selection role**
Alternative Boundedly-Rational Solution Concepts

- Group 1: relax REE but maintain a “fix point” between expectations & actual eq.
  - e.g., Cognitive discounting in Gabaix (20); Diagnostic expectations in Bordalo et. al (20)
  - may shrink the determinacy region but the indeterminacy problem remains

- Group 2: completely shuts down the “fix point”
  - e.g. level-k thinking (Garcia-Schmidt & Woodford, 19; Farhi & Werning, 19)
  - produces a unique solution but opens a new issue
  - whenever $\phi < 1$, Level-k solution becomes infinitely sensitive to Level-0 behavior
Conclusion

- **Main lesson:** NK indeterminacy/FTPL hinge on **strong info assumptions**

- **A small friction in memory & intertemporal coordination** can result in **determinacy**

- Taylor principle perhaps less consequential than previously thought
  - more crucial: boundedness (commitment to rule out large deviations)

- No room for FTPL as currently formalized (as an eq. selection device)
  - but **fiscal considerations can matter if internalized by MP**
  - Model MP-FP interaction as a game of between monetary & fiscal authority?