Motivation

- Automation raises productivity but *displaces workers* and *lowers their earnings*
- Increasing adoption has fueled an active *policy debate* (Atkison, 2019; Acemoglu et al, 2020)
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Two literatures that can justify taxing automation:

(i) Reallocation frictionless/absent
   - Tax automation
     - Guerreiro et al, 2017; Costinot-Werning, 2018

(ii) Govt. has preference for redistribution
   - Automation/reallocation are efficient
     - Diamond 1965; Heathcote et al, 2009; Aguiar et al 2021

(i) Prevent dynamic ineff. or improve insurance
   - Worker displacement/reallocation absent

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**Tax capital**
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Take worker displacement seriously. How should a government respond to automation?

1. Recognize that displaced workers face two important frictions:
   (i) Slow reallocation: workers face mobility barriers and may go through unemployment/retraining
   (Davis-Haltiwanger, 1999; Jacobson et al, 2005; Lee-Wolpin, 2006; Alvarez-Shimer, 2011)
   (ii) Imperfect credit markets: workers have limited ability to borrow against future incomes
   (Jappelli et al, 2010; Chetty, 2008)

2. Incorporate frictions in a model with endogenous automation and heterogeneous agents.

3. Theoretical results:
   (i) Interaction between frictions gives rise to inefficient automation
   (ii) Optimal to tax automation when government lacks instruments to fully alleviate frictions
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4. **Quantitative**: gross flows + idiosync. risk → **welfare gains** from slowing down autom.
Contributions to the Literature


★ Policy interventions on efficiency grounds


★ Normative analysis with incomplete markets


★ Focus on taxation of automation with labor reallocation frictions
Environment

Efficient Allocation

Decentralized Equilibrium

Failure of First Welfare Theorem

Optimal Policy

Quantitative Analysis
Continuous time $t \geq 0$
### Continuous time $t \geq 0$

<table>
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<tr>
<th>Occupations</th>
<th>Workers</th>
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</thead>
</table>

**Environment**

- **Occasions**
  - $h = A(\phi, \text{degree} 0)$
  - $h = N$

- **Final Good Producer**
  - $G^\star(A, N, h)$

- **Workers**
  - $x = f_s; h; g$ (age, occupation, prod.)

- **Resource constraint**
  - $\int c_t(x) \, dt = G^\star(A, N, h)$

- **Reallocation afterwards**
  - $U_0 = \mathbb{E}_0 \left[ \int \exp(-ct) \, dt \right]$
Continuous time $t \geq 0$

Occupations

$h = A$ (share $\phi$, degree $\alpha \geq 0$) or $h = N$

Workers
Environment

Continuous time $t \geq 0$

Occupations

$h = A$ (share $\phi$, degree $\alpha \geq 0$) or $h = N$

$$F^h(\mu) = \begin{cases} F^*(\mu; \alpha) & \text{if } h = A \\ F(\mu) \equiv F^*(\mu; 0) & \text{if } h = N \end{cases}$$

Workers
### Environment

**Continuous time** $t \geq 0$

#### Occupations

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$F^*(1; \alpha) \downarrow$ in $\alpha$ (*less labor-intensive*)

$F^*(1; \alpha)$ concave in $\alpha$ (*automation cost*)
Environment

Continuous time $t \geq 0$

**Occupations**

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**Final Good Producer**

$$G^*(\mu^A, \mu^N; \alpha) \equiv G \left( \left\{ F^h(\mu^h) \right\} \right)$$

(gross complements)

**Workers**

$$x = f_s; h; g(\text{age, occupation, prod.})$$

$U_0 = E_0 \left[ \int \exp \left( ct \right) dt \right]$

Resource constraint

$$\int c(t)(x) \, dt = G^*(\mu^A; \mu^N; \alpha)$$

$$f_h(x) = h g$$
Environment

Continuous time \( t \geq 0 \)

**Occupations**

\( h = A \) (share \( \phi \), degree \( \alpha \geq 0 \)) or \( h = N \)

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\( x = \{ s, h, \xi \} \) (age, occupation, prod.)
**Environment**

Continuous time $t \geq 0$

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### Resource constraint

$$\int c_t(x) \, d\Lambda = G^* (\mu^A, \mu^N; \alpha)$$

$$\phi^h \mu^h_t = \int 1_{\{h(x)=h\}} \xi d\pi_t$$
Reallocation of existing workers is **costly** (Kambourov-Manovskii, Violante, Costinot-Werning)

1. **Permanent cost**: productivity loss $\theta$ due to skill-specificity

$$\xi_t = \begin{cases} 
\lim_{\tau \uparrow t} \xi_\tau & \text{if } h'_t(x) = h \\
(1 - \theta) \times \lim_{\tau \uparrow t} \xi_\tau & \text{otherwise}
\end{cases}$$
Reallocation frictions

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Reallocation of existing workers is **slow** (Davis-Haltiwanger, Alvarez-Shimer). Two reasons:

2. **Random opportunities**: Workers can move across occupations with intensity $\lambda$

3. **Unemployment/retraining spells**: Enter when moving, and exit at rate $\kappa$
REALLOCATION FRICTIONS

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- Arrival of new workers is **slow** (Rebelo et al., Adão et al.). Rate $\chi$. Choose any occupation.
Environment

**Efficient Allocation**

Decentralized Equilibrium

Failure of First Welfare Theorem

Optimal Policy

Quantitative Analysis
### First Best Problem

<table>
<thead>
<tr>
<th>Ex post problem</th>
<th>Ex ante problem</th>
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<tr>
<td>Reallocate labor and distribute output</td>
<td>Choose degree of automation</td>
</tr>
<tr>
<td>Close MPLs gap. Stop reallocation at $T_{FB}$</td>
<td>Reduce $C$ today, expand $Y$ tomorrow</td>
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$$\int_{0}^{+1} e^{tu'}(c_{Nt}) \Delta t dt = 0$$

Short unemployment or retraining $\sim (1 - \theta) Y_{Nt} - Y_{At}$

Long unemployment or retraining $\sim t \Delta t$
First Best Problem

Ex post problem

- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T_0^{FB}$

Ex ante problem
First Best Problem

Ex post problem

- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T_{FB}^0$

\[ \int_{T_{FB}^0}^{+\infty} e^{-\rho t} u'(c_t) \Delta_t dt = 0 \]

where

\[ \Delta_t \equiv (1 - \theta) \left(1 - e^{-\kappa(t - T_{FB}^0)}\right) (Y_t^N - Y_t^A) \]

Cost = Skill loss + unemp

is the IRF of $Y$ to reallocation

Ex ante problem

Choose degree of automation $T_{FB}^0$
Reduce $C$ today, expand $Y$ tomorrow

\[ \int_{T_{FB}^0}^{+\infty} e^{-\rho t} u'(c_t) \Delta_t^\star dt = 0 \]
**First Best Problem**

Ex post problem

- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T_{FB}^0$

$$\int_{T_{FB}^0}^{+\infty} e^{-\rho t} u' \left( c_t^N \right) \Delta t dt = 0$$

Ex ante problem

- Choose degree of automation $T_{FB}^0$
- Reduce $C$ today, expand $Y$ tomorrow

$$\int_{T_{FB}^0}^{+\infty} e^{-\rho t} u' \left( c_t^A \right) \Delta t dt = 0$$
**First Best Problem**

**Ex post problem**
- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T_{FB}^*$

\[
\int_{T_{FB}^*}^{+\infty} e^{-\rho t} u' \left( c_t^N \right) \Delta_t dt = 0
\]

Short unemployment or retraining $\sim (1 - \theta) Y^N - Y^A$

Long unemployment or retraining

**Ex ante problem**
- Choose degree of automation $\alpha^{FB}$
- Reduce $C$ today, expand $Y$ tomorrow

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First Best Problem

Ex post problem

- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T_{0}^{FB}$

$$\int_{T_{0}^{FB}}^{+\infty} e^{-\rho t} u'\left(c^N_t\right) \Delta_t dt = 0$$

Ex ante problem

- Choose degree of automation $\alpha^{FB}$
- Reduce $C$ today, expand $Y$ tomorrow

$$\int_{0}^{+\infty} e^{-\rho t} u'\left(c^A_t\right) \Delta^*_t dt = 0$$

where

$$\Delta^*_t \equiv \frac{\partial}{\partial \alpha} G^*\left(\mu^A_t, \mu^N_t; \alpha^{FB}\right)$$

is the IRF of $Y$ to automation (net of cost)
**First Best Problem**

**Ex post problem**

- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T_{FB}^0$

\[
\int_{T_{FB}^0}^{+\infty} e^{-\rho t} u' \left( c^N_t \right) \Delta_t dt = 0
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**Ex ante problem**

- Choose degree of automation $\alpha^{FB}$
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**Graphs**

- Short unemployment or retraining
- Long unemployment or retraining
- Output gains $(\alpha, -\mu)$ are complements
- Crowding out
Environment

Efficient Allocation

Decentralized Equilibrium

Failure of First Welfare Theorem

Optimal Policy

Quantitative Analysis
Decentralized Choices

Firms

Workers

\[ \text{equity priced by unconst. workers} \]

Workers choose consumption \( c \) and labor supply.

Assets: bonds, incomplete markets

But, workers insured against mobility risk \( (x) = f(a); s; h; g(bonds, age, occ., prod.) \)

\[
\int_{0}^{t} Q_t \, dt \]

\[
A_t(x) = [Y_t(x) + (r_t + a_t(x)) c_t(x)] \, dt
\]

for some \( a_0 \)

Definition of Equilibrium
Decentralized Choices

Firms

Choose automation $\alpha$ + labor demand $\mu_t$

$$\max_{\{\alpha, \mu_t\}} \int_0^{+\infty} Q_t \Pi_t (\mu_t; \alpha) \, dt$$

Workers
Decentralized Choices

Firms
Choose automation $\alpha$ + labor demand $\mu_t$

$$\max_{\{\alpha, \mu_t\}} \int_0^{+\infty} Q_t \Pi_t (\mu_t; \alpha) \, dt$$

Workers
Choose consumption $c$ and labor supply $\mu_t$

Assets: bonds, incomplete markets
But, workers insured against mobility risk ($x = f_a(s; h; g(bonds, age, occ., prod.))$

$$d a_t (x) = \left[ Y^* t(x) + (r_t + a_t(x)) c_t(x) \right] dt$$
Decentralized Choices

Firms

Choose automation $\alpha$ + labor demand $\mu_t$

$$\max_{\{\alpha, \mu_t\}} \int_0^{+\infty} Q_t \Pi_t (\mu_t; \alpha) \, dt$$

Workers

Choose consumption $c$ and labor supply $\mu_t$

**Assets:** bonds, incomplete markets

But, workers insured against mobility risk ($\lambda$)
Decentralized Choices

Firms

Choose automation $\alpha$ + labor demand $\mu_t$

$$\max_{\{\alpha, \mu_t\}} \int_0^{+\infty} Q_t \Pi_t (\mu_t; \alpha) \, dt$$

Workers

Choose consumption $c$ and labor supply $\mu_t$

Assets: bonds, incomplete markets
But, workers insured against mobility risk ($\lambda$)

$x = \{a, s, h, \xi\}$ (bonds, age, occ., prod.)
**Decentralized Choices**

**Firms**

Choose automation $\alpha$ + labor demand $\mu_t$

$$\max_{\{\alpha, \mu_t\}} \int_0^{+\infty} Q_t \Pi_t (\mu_t; \alpha) \, dt$$

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Choose consumption $c$ and labor supply $\mu_t$

**Assets:** bonds, incomplete markets

But, workers insured against mobility risk ($\lambda$)

$$x = \{a, s, h, \xi\} \text{ (bonds, age, occ., prod.)}$$

$$da_t (x) = [\mathcal{Y}_t^* (x) + (r_t + \chi)a_t (x) - c_t (x)] \, dt$$
Decentralized Choices

Firms

Choose automation $\alpha +$ labor demand $\mu_t$

$$\max_{\{\alpha, \mu_t\}} \int_0^{+\infty} Q_t \Pi_t (\mu_t; \alpha) \, dt$$

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$$a_t (x) \geq a \text{ for some } a \leq 0$$
**Decentralized Choices**

**Firms**

Choose automation $\alpha + \text{labor demand } \mu_t$

$$\max_{\{\alpha, \mu_t\}} \int_0^{+\infty} Q_{t} \Pi_{t} (\mu_t; \alpha) \, dt$$

$Q_t$: equity priced by unconst. workers

**Workers**

Choose consumption $c$ and labor supply $\mu_t$

**Assets:** bonds, incomplete markets

But, workers insured against mobility risk ($\lambda$)

$$x = \{a, s, h, \xi\} \text{ (bonds, age, occ., prod.)}$$

$$da_t (x) = [\gamma^*_t (x) + (r_t + \chi)a_t (x) - c_t (x)] \, dt$$

$$a_t (x) \geq a \text{ for some } a \leq 0$$
Outline

Environment
Efficient Allocation
Decentralized Equilibrium
Failure of First Welfare Theorem
Optimal Policy
Quantitative Analysis
Proposition. (Failure of FWT)

1. The laissez-faire equilibrium is inefficient if and only if reallocation frictions \((\lambda, \kappa)\) and borrowing frictions \((a)\) are such that \(a^* (\lambda, \kappa) < a \leq 0\) for threshold \(a^*(\cdot)\).
**Proposition. (Failure of FWT)**

1. The laissez-faire equilibrium is inefficient if and only if **reallocation frictions** $(\lambda, \kappa)$ and **borrowing frictions** $(a)$ are such that $a^*(\lambda, \kappa) < a \leq 0$ for threshold $a^*(\cdot)$.

2. The threshold $a^*(\lambda, \kappa) < 0$ if and only if reallocation is slow ($1/\lambda$ or $1/\kappa > 0$).
Proposition. (Failure of FWT)

1. The laissez-faire equilibrium is inefficient if and only if reallocation frictions \((\lambda, \kappa)\) and borrowing frictions \((a)\) are such that \(a^* (\lambda, \kappa) < a \leq 0\) for threshold \(a^* (\cdot)\).

2. The threshold \(a^* (\lambda, \kappa) < 0\) if and only if reallocation is slow \((1/\lambda \text{ or } 1/\kappa > 0)\).

- Interaction between reallocation and borrowing frictions is key
- Efficient cases: instant realloc. (Costinot-Werning) or no borrowing frictions (Guerreiro et al)
Distortions at the laissez-faire

Workers expect income to improve as they reallocate!

Motive for borrowing
Workers expect income to improve as they reallocate → Motive for borrowing
Distortions at the laissez-faire

Workers expect income to improve as they reallocate → Motive for borrowing
Why Is Automation Inefficient?

- **Automation.** Compare the optimality conditions

\[
\int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \Delta_t^* dt = 0
\]

\[
\int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \Delta_t^* dt = 0
\]

where \(\Delta_t^*\) is the IRF of \(Y\) to automation.
Why Is Automation Inefficient?

- Automation. Compare the optimality conditions

\[
\int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \Delta_t^* dt = 0
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\[
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\]

- Automation \[ u'(c_{0,t}^A) > u'(c_{0,t}^N) \] → Rationale for redistribution
Why Is Automation Inefficient?

- Automation. Compare the optimality conditions

\[
\begin{align*}
\int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \Delta_t^* dt &= 0 \\
\int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \Delta_t^* dt &= 0
\end{align*}
\]

- Automation \[ u'(c_{0,t}^A) > u'(c_{0,t}^N) \] \rightarrow Rationale for redistribution

- No borrowing constraints \[ \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} = \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \] \rightarrow Laissez-faire = First best
Why Is Automation Inefficient?

- **Automation.** Compare the optimality conditions

  (first best) \( \int_{0}^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \Delta^*_t dt = 0 \)

  (laissez-faire) \( \int_{0}^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \Delta^*_t dt = 0 \)

- Automation \( \rightarrow u'(c_{0,t}^A) > u'(c_{0,t}^N) \rightarrow \) Rationale for redistribution

- Borrowing constraints \( \rightarrow \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} < \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \rightarrow \) Laissez-faire \( \neq \) First best
Why Is Automation Inefficient?

▶ Automation. Compare the optimality conditions

\[(\text{first best})\]

\[
\int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \Delta^*_t dt = 0
\]

\[(\text{laissez-faire})\]

\[
\int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \Delta^*_t dt = 0
\]

▶ Automation  \quad \rightarrow \quad u'(c_{0,t}^A) > u'(c_{0,t}^N) \rightarrow \text{Rationale for redistribution}

▶ Borrowing constraints  \quad \rightarrow \quad \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} < \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \rightarrow \text{Laissez-faire} \neq \text{First best}

**Firms are too patient.** Partly overlook that benefits of automation take time to realize.
Environment

Efficient Allocation

Decentralized Equilibrium

Failure of First Welfare Theorem

Optimal Policy

Quantitative Analysis
How should a government respond to automation? Depends on the tools available.
How should a government respond to automation? Depends on the tools available

- Suppose: tax on automation $\tau^\alpha +$ arbitrary transfers/taxes to redistribute
- When can we implement a first best with $\tau^\alpha = 0$?
Optimal policy

How should a government respond to automation? Depends on the tools available

- Suppose: tax on automation $\tau^\alpha$ + arbitrary transfers/taxes to redistribute

- When can we implement a first best with $\tau^\alpha = 0$?

- Wedge between first best and laissez-faire optimality condition

$$\tau^\alpha = \int_0^{+\infty} e^{-\rho t} \left( \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} - \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \right) \Delta_t^* dt$$
How should a government respond to automation? Depends on the tools available.

- Suppose: tax on automation $\tau^\alpha +$ arbitrary transfers/taxes to redistribute
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Tools to redistribute income $\rightarrow$ alleviate borrowing cons. and close MRS gap.
Optimal policy

How should a government respond to automation? Depends on the tools available

- Suppose: tax on automation $\tau^\alpha +$ arbitrary transfers/taxes to redistribute
- When can we implement a first best with $\tau^\alpha = 0$?
- Wedge between first best and laissez-faire optimality condition

$$
\tau^\alpha = \int_0^{+\infty} e^{-\rho t} \left( \frac{u'(c^N_{0,t})}{u'(c^N_{0,0})} - \frac{u'(c^A_{0,t})}{u'(c^A_{0,0})} \right) \Delta^*_t dt
$$

1. Worker/time-specific lump sum transfers $\to$ implement any first best (SWT holds)

Informational requirements? (Piketty-Saez, 2013; Guerreiro et al., 2017; Costinot-Werning, 2018)
How should a government respond to automation? Depends on the tools available

- Suppose: tax on automation $\tau^\alpha +$ arbitrary transfers/taxes to redistribute
- When can we implement a first best with $\tau^\alpha = 0$?
- Wedge between first best and laissez-faire optimality condition

$$\tau^\alpha = \int_0^{+\infty} e^{-\rho t} \left( \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} - \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \right) \Delta^*_t dt$$

2. Symmetric lump sum transf. (UBI) $\rightarrow$ govt. borrows for workers $\rightarrow$ restore efficiency

Fiscal cost? Govt. debt limits? (Araujo-Woodford, 2015; Daruich-Fernandez, 2020; Guner et al., 2021)
How should a government respond to automation? Depends on the tools available

- Suppose: tax on automation $\tau^\alpha +$ arbitrary transfers/taxes to redistribute
- When can we implement a first best with $\tau^\alpha = 0$?
- Wedge between first best and laissez-faire optimality condition

$$\tau^\alpha = \int_0^{+\infty} e^{-\rho t} \left( \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} - \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \right) \Delta_t^* dt$$

3. Non-linear income taxes or unemp. insurance → help but do not restore efficiency

Heterogeneity within occupations swamps heterogeneity between occupations (as in quant model)
Constrained Ramsey problem

- **Second best tools**: tax automation *(ex ante)* + labor market interventions *(ex-post)*

  Taxes/subsidies that depend on time, not worker-types. No social insurance for now.
Second best tools: tax automation (ex ante) + labor market interventions (ex-post)

Taxes/subsidies that depend on time, not worker-types. No social insurance for now.

Tractability: hand-to-mouth workers ($a \to 0$), no OLG ($\chi = 0$)
Constrained Ramsey problem

- **Second best tools**: tax automation *(ex ante)* + labor market interventions *(ex-post)*
  
  Taxes/subsidies that depend on time, not worker-types. No social insurance for now.

- **Tractability**: hand-to-mouth workers *(a → 0)*, no OLG *(χ = 0)*

- **Primal problem**: control automation \( \alpha \) and reallocation \( T_0 \)

\[
\max_{\{\alpha, T_0, \mu, c_t\}} \sum_h \phi h^n \int_0^{+\infty} \exp(-\rho t) u\left(c_t^h\right) dt
\]

subject to workers’ budget constraints, the law of motion of labor, firms choosing labor optimally, and market clearing.
Constrained inefficiency

- Government’s optimality conditions to automate ($\alpha$) and reallocate ($T_0$)

\[
\int_0^{+\infty} \exp(-\rho t) u'(c_{0,t}^N) \Delta_t^* dt = -\Phi^* (\alpha^{SB}, T_0^{SB}; \eta)
\]

\[
\int_{T_0^{SB}}^{+\infty} \exp(-\rho t) u'(c_{0,t}^A) \Delta_t dt = -\Phi (\alpha^{SB}, T_0^{SB}; \eta)
\]

\{ \text{laissez-faire} \}

\{ \text{pecuniary externalities} \}
Proposition. (Constrained inefficiency)

Fix weights $\eta$. Then, there is always a small perturbation of the technology $G^*(\cdot)$ such that either $\Phi^*(\cdot) \neq 0$ or $\Phi(\cdot) \neq 0$ — i.e., the equilibrium is generically constrained inefficient.
No pref. for redistribution: weights $\eta^\text{effic}$ so that distributional terms cancel out

Guarantees that the government would not distort an efficient allocation for redistributive reasons.
Taxing automation on efficiency grounds

- No pref. for redistribution: weights $\eta^{\text{effic}}$ so that distributional terms cancel out

Guarantees that the government would not distort an efficient allocation for redistributive reasons

(second best)

$$\int_{0}^{+\infty} e^{-\rho t} \sum_{h} \phi^{h} \eta^{h,\text{effic}} \frac{u'(c_{0,t}^{h})}{u'(c_{0,0}^{h})} \Delta^{*} dt = 0$$

(laissez-faire)

$$\int_{0}^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^{N})}{u'(c_{0,0}^{N})} \Delta^{*} dt = 0$$
Taxing automation on efficiency grounds

▶ No pref. for redistribution: weights $\eta^{\text{effic}}$ so that distributional terms cancel out

Guarantees that the government would not distort an efficient allocation for redistributive reasons

\[
\int_0^{+\infty} e^{-\rho t} \sum_h \phi^h \eta^{h,\text{effic}} \frac{u'(c^h_{0,t})}{u'(c^h_{0,0})} \Delta^*_t dt = 0
\]

(second best)

\[
\int_0^{+\infty} e^{-\rho t} \frac{u'(c^N_{0,t})}{u'(c^N_{0,0})} \Delta^*_t dt = 0
\]

(laissez-faire)

1. The response of output to automation $\Delta^*_t$ is back-loaded

Figure
**Taxing automation on efficiency grounds**

- **No pref. for redistribution**: weights $\eta^{\text{effic}}$ so that distributional terms cancel out.
  
  Guarantees that the government would not distort an efficient allocation for redistributive reasons.

\[
\int_0^{+\infty} e^{-\rho t} \sum_h \phi^h \eta^{h, \text{effic}} \frac{u'(c^h_{0,t})}{u'(c^h_{0,0})} \Delta^*_t dt = 0
\]

\[
\int_0^{+\infty} e^{-\rho t} \frac{u'(c^N_{0,t})}{u'(c^N_{0,0})} \Delta^*_t dt = 0
\]

1. The response of output to automation $\Delta^*_t$ is back-loaded.

2. **Government** is *more impatient* than the firm — priced by unconstrained workers only.
TAXING AUTOMATION ON EFFICIENCY GROUNDS

- **No pref. for redistribution**: weights $\eta^{\text{effic}}$ so that distributional terms cancel out

Guarantees that the government would not distort an efficient allocation for redistributive reasons

\[
\int_{0}^{+\infty} e^{-\rho t} \sum_{h} \phi_{h}^{\text{effic}} \frac{u'(c_{0,t}^{h})}{u'(c_{0,0}^{h})} \Delta^{*} dt = 0
\]

(second best)

\[
\int_{0}^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^{N})}{u'(c_{0,0}^{N})} \Delta^{*} dt = 0
\]

(laissez-faire)

1. The response of output to automation $\Delta^{*}$ is **back-loaded**

2. **Government** is *more impatient* than the firm — priced by unconstrained workers only

→ Optimal to **tax automation** on efficiency grounds

Figure
Taxing automation on efficiency grounds

- **No pref. for redistribution**: weights $\eta^{\text{effic}}$ so that distributional terms cancel out

Guarantees that the government would not distort an efficient allocation for redistributive reasons

\[
\int_{0}^{+\infty} e^{-rt} \sum_{h} \phi^{h} \eta^{h,\text{effic}} \frac{u'(c_{0,h}^{h})}{u'(c_{0,0}^{h})} \Delta^{*}_{t} dt = 0
\]

\[
\int_{0}^{+\infty} e^{-rt} \frac{u'(c_{0,t}^{N})}{u'(c_{0,0}^{N})} \Delta^{*}_{t} dt = 0
\]

Taxing automation prevents excessive investment and raises consumption early on in the transition, precisely when displaced workers are borrowing-constrained
Active labor market interventions might not be available (Heckman et al., Card et al.)
Extension I: No Labor Market Intervention

- Active labor market interventions might not be available (Heckman et al., Card et al.)

- The government uses automation ($\alpha$) as a proxy for reallocation ($T_0$)

$$\int_0^{+\infty} e^{-\rho t} \sum_h \phi^h \eta^h \frac{u'(c_{0,t}^h)}{u'(c_{0,0}^h)} (\Delta_t^* + T_0' (\alpha^{SB}) \Delta_t) dt = 0$$

so that

- Short unempl/retraining spells (1/$\kappa$ low) $\rightarrow$ tax $\alpha$ more

- Long unempl/retraining spells (1/$\kappa$ high) $\rightarrow$ tax $\alpha$ less
EXTENSION II: EQUITY CONCERNS

\[ \text{MRS}^A = \text{MRS}^N \]

\[ \text{LF} = \text{SB}^{\text{effic}} \]

\[ \text{Automation} \downarrow \]

\[ \text{Automation} \downarrow \]
Extension III: Gradual Automation

- Tax capital in the long-run → Improve insurance or prevent dynamic inefficiency
  (Aiyagari, 1995; Chamley, 2001; Conesa et al., 2009; Dávila et al., 2021; Aguiar et al.; 2021)
Extension III: Gradual Automation

- Tax capital in the long-run → Improve insurance or prevent dynamic inefficiency
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- Rationale for taxing automation is distinct
  1. Does not rely on uninsured income risk
  2. Tax only during the transition, but not in the long-run once labor reallocation is complete
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- To clarify 2., extend model so that automation takes place gradually

\[
\frac{d\alpha_t}{dt} = (x_t - \delta\alpha_t)
\]

Law of motion

\[
Y_t = G^* (\mu_t; \alpha_t) - x_t\alpha_t - \Omega (x_t/\alpha_t) \alpha_t
\]

Output net of investment costs
Extension III: Gradual Automation

- Tax capital in the long-run → Improve insurance or prevent dynamic inefficiency
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- To clarify 2., extend model so that automation takes place gradually

\[ d\alpha_t = (x_t - \delta \alpha_t) \, dt; \]
\[ Y_t = G^* (\mu_t; \alpha_t) - x_t \alpha_t - \Omega (x_t / \alpha_t) \alpha_t \]
\[ \text{Law of motion} \quad \text{Output net of investment costs} \]

- Workers are unconstrained in the long-run \( \iff \alpha_t^{LF} / \alpha_t^{FB} \rightarrow 1 \) as \( t \rightarrow +\infty \)
Environment

Efficient Allocation

Decentralized Equilibrium

Failure of First Welfare Theorem

Optimal Policy

Quantitative Analysis
Firm task-based framework – Acemoglu-Autor

\[ y_t^h = F(\mu_t^h; \alpha_t^h) = A^h \left( \alpha_t^h + \mu_t^h \right)^{1-\eta} \]

quadratic adjustment costs \(- \omega \left( x_t / \alpha_t \right)^2 \alpha_t \)

\[ d\alpha_t^A = (x_t - \delta \alpha_t^A) \, dt \quad \alpha_t^N = 0 \]
Firm

- task-based framework – Acemoglu-Autor

\[ y^h_t = F\left(\mu^h_t; \alpha^h_t\right) = A^h \left(\alpha^h_t + \mu^h_t\right)^{1-\eta} \]

- quadratic adjustment costs \(-\omega \left(x_t/\alpha_t\right)^2\alpha_t\)

\[ d\alpha^A_t = (x_t - \delta\alpha^A_t)\, dt \quad \alpha^N_t = 0 \]

Workers

- gross flows – Kambourov-Manovskii

\[ S_t(x) = \frac{(1 - \phi) \exp\left(\frac{V_N^t(x'(N;x))}{\gamma}\right)}{\sum_{h'} \phi^{h'} \exp\left(\frac{V_{t}^{h'}(x'(h';x))}{\gamma}\right)} \]

- uninsured risk – Huggett-Aiyagari

\[ dz^T_t = -\rho_z z^T_t \, dt + \sigma_z dW_t \]

\[ z^P_t = (1 - \theta) z^P_{t-} \text{ when moving} \]
**Quantitative Model**

**Firm**

- task-based framework – Acemoglu-Autor

\[
y^h_t = F\left(\mu^h_t; \alpha^h_t\right) = A^h \left(\alpha^h_t + \mu^h_t\right)^{1-\eta}
\]

- quadratic adjustment costs \(-\omega (x_t/\alpha_t)^2 \alpha_t\)

\[
d\alpha_t^A = (x_t - \delta \alpha_t^A) \, dt \quad \alpha_t^N = 0
\]

**Workers**

- gross flows – Kambourov-Manovskii

\[
S_t(x) = \frac{(1 - \phi) \exp\left(\frac{V^N_t\left(x'(N;x)\right)}{\gamma}\right)}{\sum_{h'} \phi^{h'} \exp\left(\frac{V^{h'}_{t}\left(x'(h';x)\right)}{\gamma}\right)}
\]

- uninsured risk – Huggett-Aiyagari

\[
dz_t^T = -\rho z_t^T dt + \sigma_z dW_t
\]

\[
z_t^P = (1 - \theta) z_{t-} \text{ when moving}
\]
Automation

Insurance
+ Dynamic inefficiency

Inefficient transition

Laissez-faire
Constrained efficiency
Objective: The government maximizes

$$\mathcal{W}(\eta) \equiv \int_{-\infty}^{+\infty} \int \eta_t(x) V^\text{birth}_t(x) \, d\pi_t(x) \, dt$$

Second best: Vary the speed of automation $\varphi$

$$\alpha_t = \alpha_0 + (1 - e^{-\varphi t}) (\alpha^\text{CE}_0 - \alpha_0)$$

Implementation: $\{\tau^x_t\}$ on investment, rebated to firm owners
**Table 1: Welfare Gains**

<table>
<thead>
<tr>
<th>Half-life (years)</th>
<th>40</th>
<th>70</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\mathcal{W}(\eta^{\text{effic}}; r_t)$</td>
<td>0%</td>
<td>2.3%</td>
<td>3.2%</td>
</tr>
<tr>
<td>$\mathcal{W}(\eta^{\text{utilit}}; r_t)$</td>
<td>0%</td>
<td>2.8%</td>
<td>3.9%</td>
</tr>
</tbody>
</table>

- Within generations: utilitarian or efficient $1/V_t(\cdot)$ weights
- Across generations: discount with $r_t$
Workers displaced by automation would like to borrow as they reallocate slowly.
Takeaways

- Workers displaced by automation would like to borrow as they reallocate slowly.

- Borrowing constraints create wedge between interest rate and MRS of workers.
Takeaways

- Workers displaced by automation would like to borrow as they reallocate slowly.
- Borrowing constraints create wedge between interest rate and MRS of workers.
- Firms are effectively too patient when investing in automation.

The government should slow down automation on efficiency grounds, even when it has no preference for redistribution.
Workers displaced by automation would like to borrow as they reallocate slowly.

Borrowing constraints create wedge between interest rate and MRS of workers.

Firms are effectively too patient when investing in automation.

The government should slow down automation on efficiency grounds, even when it has no preference for redistribution.
Competitive equilibrium. Set of

1. Allocation for consumption \( \{c_t\} \) and labor supplies \( \{\mu_t, \Theta_t\} \), and automation \( \alpha \)
2. Interest rate \( \{r_t\} \), wages \( \{w_t^A, w_t^N\} \) and profits \( \{\Pi_t\} \)

such that

1. Workers consume, save and move optimally given prices
2. Representative firm chooses automation and hires labor optimally given prices
3. Labor markets clear and resource constraint holds
Borrowing

Distortions at the laissez-faire

Distributional effects

Constraint not binding
Constraint slack for all $t > T_0$
Constraint binds for some $t > T_0$
Prod. inefficiency
Stopping time with $a \rightarrow -\infty$

Tight constraint
Slow reallocation

$\hat{a}(\lambda)$
Consumption (PE)
Reallocation (PE)

$0 \quad 1/\lambda$
$1/\lambda_0$
$t$

$0 \quad S_0 \quad S_1 \quad S'_0 \quad S'_1$

$\hat{a}(\lambda)$
$a(\lambda)$
$a^*(\lambda)$

$\hat{a}(\lambda)$
Consumption (PE)
Reallocation (PE)

$0 \quad 1/\lambda$
$1/\lambda_0$
$t$

$\hat{a}(\lambda)$
Consumption (PE)
Reallocation (PE)

$0 \quad 1/\lambda$
$1/\lambda_0$
$t$

$\hat{a}(\lambda)$
Consumption (PE)
Reallocation (PE)
Output gains $(\alpha, -\mu)$ are complements.
**Competitive Equilibrium**

- **Incomes:**

\[ \mathcal{Y}_t^\star (x) = \Pi_t + (1 - \tau_t) \times \begin{cases} \xi \exp (Z) W^h_t & \text{if } e = E \\ b\xi \exp (Z) W^{-h}_t & \text{if } e = U \end{cases} \]

where \( b \) is replacement rate during unemployment.

- **Assets:**

  Workers trade riskless bonds, and annuities (Blanchard-Yaari)

- **Fiscal policy:**

  Constant debt / GDP, adjusts distorsionary tax \( \{\tau_t\} \)

- **Resource constraint:**

\[
\int c_t (x) \, d\pi_t + x_t + \omega \left( \frac{x_t}{\alpha_t} \right)^2 \alpha_t = G^* \left( \int \mathbf{1}_{\{h(x) = h\}} \xi d\pi_t \phi^h \right) + b \int \mathbf{1}_{\{e = U\}} \tilde{\mathcal{Y}} (x) \, d\pi_t,
\]
## Table 2: External Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibration</th>
<th>Target / Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>EIS (inverse)</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Death rate</td>
<td>1/45</td>
<td>Average working life of 45 years</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Initial labor share</td>
<td>0.36</td>
<td>1970 labor share (BLS)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.1</td>
<td>Graetz-Michaels (2018)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Elasticity of substitution across occs.</td>
<td>0.75</td>
<td>Buera-Kaboski (2011)</td>
</tr>
<tr>
<td>$1/\kappa$</td>
<td>Average unemployment duration</td>
<td>1/3.2</td>
<td>Alvarez-Shimer (2011)</td>
</tr>
<tr>
<td>$1 - \kappa^*$</td>
<td>Probability of return move</td>
<td>0.44</td>
<td>Carillo-Tudela-Visschers (2020)</td>
</tr>
<tr>
<td>$1 - \theta$</td>
<td>Productivity loss from relocation</td>
<td>0.18</td>
<td>Kambourov-Manovskii (2009)</td>
</tr>
</tbody>
</table>
Calibration

- **Parameters**: External calibration (13) and internal calibration (7)

<table>
<thead>
<tr>
<th>Table 3: Internal Calibration</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Parameter</strong></td>
</tr>
<tr>
<td>Discount rate</td>
</tr>
<tr>
<td>Productivities</td>
</tr>
<tr>
<td>Adjustment cost</td>
</tr>
<tr>
<td>Fraction of automated occupations</td>
</tr>
<tr>
<td>Mobility hazard</td>
</tr>
<tr>
<td>Fréchet parameter</td>
</tr>
</tbody>
</table>
**Parameters:** External calibration (13) and internal calibration (7)

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibration</th>
<th>Target / Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Discount rate</td>
<td>0.13</td>
<td>2% real interest rate</td>
</tr>
<tr>
<td>$A^A, A^N$</td>
<td>Productivities</td>
<td>0.89, 1.26</td>
<td>Initial output (1)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Adjustment cost</td>
<td>16</td>
<td>Routine empl. share 2015</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Fraction of automated occupations</td>
<td>0.53</td>
<td>Routine empl. share 1970</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Mobility hazard</td>
<td>0.49</td>
<td>Occupational mobility 1970</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Fréchet parameter</td>
<td>0.06</td>
<td>Elasticity of labor supply</td>
</tr>
<tr>
<td>Parameter</td>
<td>Description</td>
<td>Calibration</td>
<td>Target / Source</td>
</tr>
<tr>
<td>-----------</td>
<td>------------------------------------------</td>
<td>-------------</td>
<td>------------------------------</td>
</tr>
<tr>
<td>$\alpha$</td>
<td>Borrowing limit</td>
<td>0</td>
<td>Auclert et al. (2018)</td>
</tr>
<tr>
<td>$B/Y$</td>
<td>Government debt / GDP</td>
<td>0.26</td>
<td>Liquid assets / GDP</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Mean reversion (inc)</td>
<td>0.9775</td>
<td>Floden-Linde (2001)</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Volatility (inc)</td>
<td>0.1025</td>
<td>Floden-Linde (2001)</td>
</tr>
<tr>
<td>$b$</td>
<td>Replacement rate while unemployed</td>
<td>0.4</td>
<td>Ganong et al. (2020)</td>
</tr>
</tbody>
</table>
Mass of workers in $h = A$

Profit share

No automation