Automation raises productivity but displaces workers and lowers their earnings.
Motivation

- Automation raises productivity but displacement workers and lowers their earnings.
- Increasing adoption has fueled an active policy debate (Atkison, 2019; Acemoglu et al, 2020).
Automation raises productivity but **displaces workers** and **lowers their earnings**

Increasing adoption has fueled an active **policy debate** (Atkison, 2019; Acemoglu et al, 2020)

**Literature**: Absent redistributive taxes, a government that values **equity** should

1. **Regulate automation** (Guerreiro et al, 2017; Thuemmel, 2018; Costinot-Werning, 2018)

2. **Facilitate labor reallocation** (Jaimovich et al, 2020)
Should the government *intervene* in automation, even if it does not value *equity*?
Should the government intervene in automation, even if it does not value equity?

1. Recognize that displaced workers face two important frictions:
   
   (i) **Slow reallocation**: workers face mobility barriers and may go through unemployment
       Davis-Haltiwanger, 1999; Lee-Wolpin, 2006; Alvarez-Shimer, 2011

   (ii) **Imperfect credit markets**: workers have limited ability to borrow against future incomes
       Jappelli et al, 2010; Chetty, 2008
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2. Incorporate frictions in a model with endog. automation and heterog. agents
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3. Study second-best problem where government controls automation and reallocation
   - Pecuniary externalities that affect workers’ → generic (constrained) inefficiency
   - Gov’t should slow down automation even when it has no preference for redistribution
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2. Incorporate frictions in a *model* with endog. automation and heterog. agents

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   - Pecuniary externalities that affect workers’ → generic *(constrained)* inefficiency
   
   - Gov’t should **slow down automation** even when it has no preference for redistribution

4. **Quantitative**: gross flows *(Moscarini-Vella)* + idiosync. risk *(Aiyagari)* → optimal policy
Environment

Efficient Allocation

Decentralized Equilibrium

Failure of First Welfare Theorem

Optimal Policy

Quantitative Analysis
Continuous time $t \geq 0$
<table>
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<th>Environment</th>
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**Continuous time** \( t \geq 0 \)

<table>
<thead>
<tr>
<th>Occupations</th>
<th>Workers</th>
</tr>
</thead>
</table>

- **Occupations**
  - \( h = A(\phi, \alpha) \geq 0 \)
  - \( h = N(F) = F \star (\mu; \alpha) \)
    - if \( h = A \)
      - \( F(\mu) \equiv F \star (\mu; 0) \)
    - if \( h = N \)
      - \( F \star (1; \alpha) \)
- **Workers**
  - \( F \star (1; \alpha) \) concave in \( \alpha \) (automation cost)
  - Final Good Producer
    - \( G \star (\mu_A, \mu_N; \alpha) \equiv G_n F_h(\mu_h) \)
  - **Workers**
    - \( z \equiv Z \star (s, h, \xi) \) (age, occupation, prod.)
    - \( \mu_A t, \mu_N t \) (in \( t = 0 \))
      - Reallocation afterwards
      - \( U_0 = E_0 Z \exp(-\rho t) c^{1-\sigma} t^{1-\sigma} dt \)
- **Resource constraint**
  - \( Z c(t)(x) d \Lambda = G \star (\mu_A, \mu_N; \alpha) \phi h \mu_h t = Z \{ h(x) \} \xi d \pi t \)
Continuous time $t \geq 0$

**Occupations**

$h = A$ (share $\phi$, degree $\alpha \geq 0$) or $h = N$

**Workers**
Continuous time $t \geq 0$

**Occupations**

$h = A$ (share $\phi$, degree $\alpha \geq 0$) or $h = N$

$$F^h(\mu) = \begin{cases} F^*(\mu; \alpha) & \text{if } h = A \\ F(\mu) \equiv F^*(\mu; 0) & \text{if } h = N \end{cases}$$

**Workers**

$$U_0 = E_0 Z \exp\left( -\rho t c_1 - \sigma t \right) dt$$

**Resources**

$$Z c_t(x) d \Lambda = G^* \mu_A, \mu_N; \alpha$$
Continuous time $t \geq 0$

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$F^* (1; \alpha) \downarrow$ in $\alpha$ (less labor-intensive)

$F^* (1; \alpha)$ concave in $\alpha$ (automation cost)

**Workers**

Final Good Producer $G^\star (\mu_A, \mu_N; \alpha)$

Workers $x = \{s, h, \xi\}$ (age, occupation, prod.)
Continuous time $t \geq 0$

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$$G^* (\mu^A, \mu^N; \alpha) \equiv G \left( \{ F^h (\mu^h) \} \right)$$

(gross complements)
ENVIRONMENT

Continuous time $t \geq 0$

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$x = \{s, h, \xi\}$ (age, occupation, prod.)

$$(\mu^A_t, \mu^N_t) \begin{cases} = 1 & \text{in } t = 0 \\ \text{Reallocation} & \text{afterwards} \end{cases}$$

---

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**Resource constraint**

$$\int c_t(x) d\Lambda = G^* (\mu^A, \mu^N; \alpha)$$

$$\phi^h \mu^h_t = \int 1_{\{h(x)=h\}} \xi d\pi_t$$
Reallocation of existing workers is **costly** (Kambourov-Manovskii, Violante, Costinot-Werning)

1. **Permanent cost**: productivity loss $\theta$ due to skill-specificity

$$\xi_t = \begin{cases} 
\lim_{\tau \uparrow t} \xi_\tau & \text{if } h_t'(x) = h \\
(1 - \theta) \times \lim_{\tau \uparrow t} \xi_\tau & \text{otherwise}
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Reallocation of existing workers is slow (Davis-Haltiwanger, Alvarez-Shimer). Two reasons:

2. **Random opportunities**: Workers can move across occupations with intensity $\lambda$

3. **Unemployment/retraining spells**: Enter when moving, and exit at rate $\kappa$
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Arrival of new workers is **slow** (Rebelo et al., Adão et al.). Rate $\chi$. Choose any occupation.
Environment

Efficient Allocation

Decentralized Equilibrium

Failure of First Welfare Theorem

Optimal Policy

Quantitative Analysis
FIRST BEST PROBLEM

Ex post problem

Ex ante problem

\[ \sum_{t=0}^{\infty} e^{-\rho t} u'(c) N(t) \Delta t = 0 \]

Short unemployment or retraining \[ \sim (1 - \theta) Y_N(t) - Y_A(t) \]

Long unemployment or retraining \[ \sim \]

\[ \sum_{t=0}^{\infty} e^{-\rho t} u'(A(t) \Delta \star t) dt = 0 \]
Ex post problem

- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T_0^{FB}$

Ex ante problem
FIRST BEST PROBLEM

Ex post problem

- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T_{FB}^0$

$$
\int_{T_{FB}^0}^{+\infty} e^{-\rho t} u' \left( c_t^N \right) \Delta_t dt = 0
$$

where

$$
\Delta_t \equiv (1 - \theta) \left( 1 - e^{-\kappa \left( t - T_{FB}^0 \right)} \right) \left( \mathcal{Y}_t^N - \mathcal{Y}_t^A \right)
$$

is the IRF of $Y$ to reallocation

Ex ante problem

Choose degree of automation $\alpha$

Reduce $C$ today, expand $Y$ tomorrow
First Best Problem

Ex post problem

- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T_{FB}^{FB}$

\[
\int_{T_{FB}^{FB}}^{+\infty} e^{-\rho t} u' \left( \frac{c_N}{t} \right) \Delta_t dt = 0
\]

Ex ante problem

Choose degree of automation $\alpha$

Reduce $C$ today, expand $Y$ tomorrow

\[
\int_{T_{FB}^{FB}}^{0} + \infty e^{-\rho t} u' \left( \frac{c_A}{t} \right) \Delta_t^\star dt = 0
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**First Best Problem**

**Ex post problem**

- Reallocate labor and distribute output
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**Ex ante problem**

- Choose degree of automation $\alpha^{FB}$
- Reduce $C$ today, expand $Y$ tomorrow

\[
\sim (1 - \theta) Y_t^N - Y_t^A
\]
**Ex post problem**

- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T_0^{FB}$

$$\int_{T_0^{FB}}^{+\infty} e^{-\rho t} u' \left( c_t^N \right) \Delta_t dt = 0$$

**Ex ante problem**

- Choose degree of automation $\alpha^{FB}$
- Reduce $C$ today, expand $Y$ tomorrow

$$\int_0^{+\infty} e^{-\rho t} u' \left( c_t^A \right) \Delta_t^{*} dt = 0$$

where

$$\Delta_t^{*} \equiv \frac{\partial}{\partial \alpha} G^* \left( \mu_t^A, \mu_t^N; \alpha^{FB} \right)$$

is the IRF of $Y$ to automation (net of cost)
**First Best Problem**

### Ex post problem

- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T_{FB}^0$

$$\int_{T_{FB}^0}^{+\infty} e^{-\rho t} u'(c_t^N) \Delta_t dt = 0$$

### Ex ante problem

- Choose degree of automation $\alpha_{FB}^*$
- Reduce $C$ today, expand $Y$ tomorrow

$$\int_{0}^{+\infty} e^{-\rho t} u'(c_t^A) \Delta_t^* dt = 0$$
Firms

Workers

Equilibrium
Firms

Choose automation $\alpha$ + labor demand $\mu_t$

$$\max_{\{\alpha, \mu_t\}} \int_0^{+\infty} Q_t \Pi_t(\mu_t; \alpha) \, dt$$

Workers

Assets: bonds, incomplete markets

But, workers insured against mobility risk ($\lambda$)

$$x = \{a, s, h, \xi\}$$ (bonds, age, occ., prod.)

$$da_t(x) = h_\xi w_h t + \Pi_t + r_t a_t(x) - c_t(x)$$

$$a_t(x) \geq a$$ for some $a \leq 0$
**Firms**

Choose automation $\alpha$ + labor demand $\mu_t$

$$\max_{\{\alpha, \mu_t\}} \int_0^{+\infty} Q_t \Pi_t (\mu_t; \alpha) \, dt$$

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Environment

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Choose automation $\alpha + \text{ labor demand } \mu_t$

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**Assets:** bonds, incomplete markets

But, workers insured against mobility risk ($\lambda$)

$$x = \{a, s, h, \xi\} \text{ (bonds, age, occ., prod.)}$$

$$da_t (x) = \left[ \xi w^h_t + \Pi_t + r_t a_t (x) - c_t (x) \right] \, dt$$
**Firms**

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$$a_t (x) \geq a \ \text{for some } a \leq 0$$
ENVIRONMENT

Firms
Choose automation $\alpha$ + labor demand $\mu_t$

$$\max_{\{\alpha, \mu_t\}} \int_0^{+\infty} Q_t \Pi_t (\mu_t; \alpha) \, dt$$

$$Q_t = e^{-\rho t} u^\prime (c_{0,t}^N)$$

(equity priced by unconstrained workers)

Workers

**Assets:** bonds, incomplete markets

But, workers insured against mobility risk ($\lambda$)

$$x = \{ a, s, h, \xi \} \text{ (bonds, age, occ., prod.)}$$

$$da_t (x) = \left[ \xi w_t^h + \Pi + r_t a_t (x) - c_t (x) \right] \, dt$$

$$a_t (x) \geq a \text{ for some } a \leq 0$$
Environment

Efficient Allocation

Decentralized Equilibrium

Failure of First Welfare Theorem

Optimal Policy

Quantitative Analysis
Proposition. (Failure of FWT)

1. The laissez-faire equilibrium is inefficient if and only if reallocation frictions \((\lambda, \kappa)\) and borrowing frictions \((a)\) are such that \(a^*(\lambda, \kappa) < a \leq 0\) for threshold \(a^*(\cdot)\).
Proposition. (Failure of FWT)

1. The laissez-faire equilibrium is inefficient if and only if reallocation frictions \((\lambda, \kappa)\) and borrowing frictions \((a)\) are such that \(a^*(\lambda, \kappa) < a \leq 0\) for threshold \(a^*(\cdot)\).

2. The threshold \(a^*(\lambda, \kappa) < 0\) if and only if reallocation is slow \((1/\lambda \text{ or } 1/\kappa > 0)\).
## Failure of the First Welfare Theorem

**Proposition.** (Failure of FWT)

1. The laissez-faire equilibrium is inefficient if and only if **reallocation frictions** $(\lambda, \kappa)$ and **borrowing frictions** $(a)$ are such that $a^* (\lambda, \kappa) < a \leq 0$ for threshold $a^* (\cdot)$.

2. The threshold $a^* (\lambda, \kappa) < 0$ if and only if reallocation is slow $(1/\lambda \text{ or } 1/\kappa > 0)$.

- **Interaction** between reallocation and borrowing frictions is key
- **Efficient cases:** instant realloc. (Costinot-Werning) or no borrowing frictions (Guerreiro et al)
Nature of the Inefficiency

Distortions at the laissez-faire

- Tight constraint
- Slow reallocation
- Reallocation (PE)
- Consumption (PE)
- \( a^* (\lambda) \)
- \( \hat{a} (\lambda) \)

Workers expect income to improve as they reallocate → Motive for borrowing

\[ \frac{1}{\lambda} \]
Distortions at the laissez-faire

Average income

Workers expect income to improve as they reallocate → Motive for borrowing
Nature of the Inefficiency

Distortions at the laissez-faire

Average income

Workers expect income to improve as they reallocate → Motive for borrowing
**Why Is Automation Inefficient?**

- **Automation.** Compare the optimality conditions

  (first best)

  \[
  \int_{0}^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \Delta_t^* dt = 0
  \]

  (laissez-faire)

  \[
  \int_{0}^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \Delta_t^* dt = 0
  \]

  Partly overlook that benefits of automation take time to realize.

  Rationale for redistribution.

- **Constraints**
Why Is Automation Inefficient?

- **Automation.** Compare the optimality conditions

  \[
  \int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \Delta_t^* dt = 0
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- **Automation** \( \rightarrow u'(c_{0,t}^A) > u'(c_{0,t}^N) \) \( \rightarrow \) Rationale for redistribution
**Why Is Automation Inefficient?**

- **Automation.** Compare the optimality conditions

  (first best) \[ \int_{0}^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \Delta^*_t dt = 0 \]

  (laissez-faire) \[ \int_{0}^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \Delta^*_t dt = 0 \]

- Automation \[ \rightarrow \quad u'(c_{0,t}^A) > u'(c_{0,t}^N) \rightarrow \text{Rationale for redistribution} \]

- No borrowing constraints \[ \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} = \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \rightarrow \text{Laissez-faire = First best} \]
Why Is Automation Inefficient?

- Automation. Compare the optimality conditions

  (first best) \[ \int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \Delta^*_t dt = 0 \]

  (laissez-faire) \[ \int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \Delta^*_t dt = 0 \]

- Automation \[ \rightarrow u'(c_{0,t}^A) > u'(c_{0,t}^N) \rightarrow \text{Rationale for redistribution} \]

- Borrowing constraints \[ \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} < \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \rightarrow \text{Laissez-faire} \neq \text{First best} \]
**Why Is Automation Inefficient?**

- **Automation.** Compare the optimality conditions

  \[
  \int_0^{\infty} e^{-\rho t} \frac{u'(c_{0,t})}{u'(c_{0,0})} \Delta_t^* dt = 0
  \]

  \[
  \int_0^{\infty} e^{-\rho t} \frac{u'(c_{N,t})}{u'(c_{0,0})} \Delta_t^* dt = 0
  \]

  \[
  \int_0^{\infty} e^{-\rho t} \frac{u'(c_{0,t})}{u'(c_{0,0})} \Delta_t^* dt = 0
  \]

- **Automation** → \( u'(c_{0,t}) > u'(c_{0,0}) \) → Rationale for redistribution

- **Borrowing constraints** → \( \frac{u'(c_{0,t})}{u'(c_{0,0})} < \frac{u'(c_{N,t})}{u'(c_{0,0})} \) → Laissez-faire ≠ First best

Firms are **too patient.** Partly overlook that benefits of automation take time to realize.
Environment

Efficient Allocation

Decentralized Equilibrium

Failure of First Welfare Theorem

Optimal Policy

Quantitative Analysis
Novel rationale for curbing automation even without preference for redistribution
Novel rationale for curbing automation even without preference for redistribution

How should government intervene? Depends on the tools available

1. **Lump-sum** transfers → first best (the SWT holds)
2. **Limited tools** ⇒ automation (*ex ante*) and reallocation (*ex-post*) → second best
   Implementation: taxes that depend on time, no need to know which occup. are automated
Novel rationale for curbing automation even without preference for redistribution

- How should government intervene? Depends on the tools available
  1. **Lump-sum** transfers $\rightarrow$ first best (the SWT holds)
  2. **Limited tools** $\rightsquigarrow$ **automation** (*ex ante*) and **reallocation** (*ex-post*) $\rightarrow$ second best
     Implementation: taxes that depend on time, no need to know which occup. are automated

- Tractability: hand-to-mouth workers ($a \rightarrow 0$), no OLG ($\chi = 0$)
Government’s optimality conditions to **automate** ($\alpha$) and **reallocate** ($T_0$)

\[
\int_{0}^{+\infty} \exp\left(- (\rho + \chi) t \right) u'(c_{0,t}^N) \Delta^*_t \, dt = - \Phi^* (\alpha^{SB}, T_0^{SB}; \eta)
\]

\[
\int_{T_0^{SB}}^{+\infty} \exp\left(- (\rho + \chi) t \right) u'(c_{0,t}^A) \Delta_t \, dt = - \Phi (\alpha^{SB}, T_0^{SB}; \eta)
\]

\{ **laissez-faire** \}

\{ **pecuniary externalities** \}
Government’s optimality conditions to automate ($\alpha$) and reallocate ($T_0$)

\[
\int_0^{+\infty} \exp\left(-\left(\rho + \chi\right)t\right) u'\left(c_{0,t}^N\right) \Delta_t^* dt = -\Phi^*\left(\alpha^{SB}, T_0^{SB}; \eta\right)
\]

\[
\int_{T_0^{SB}}^{+\infty} \exp\left(-\left(\rho + \chi\right)t\right) u'\left(c_{0,t}^A\right) \Delta_t dt = -\Phi\left(\alpha^{SB}, T_0^{SB}; \eta\right)
\]

\text{laissé-faire}

\text{pecuniary externalities}

**Proposition.** (Constrained inefficiency)

Fix weights $\eta$. Then, there is always a small perturbation of the technology $G^*(\cdot)$ such that either $\Phi^*(\cdot) \neq 0$ or $\Phi(\cdot) \neq 0$ — i.e., the equilibrium is *generically* constrained inefficient.
No pref. for redistribution: weights $\eta^{\text{effic}}$ so that distributional terms cancel out

Guarantees that the government would not distort an efficient allocation for redistributive reasons
OPTIMAL INTERVENTION

- **No pref. for redistribution**: weights $\eta_{\text{effic}}$ so that distributional terms cancel out
  Guarantees that the government would not distort an efficient allocation for redistributive reasons

- **Trade-off**: reduce consumption ($\Delta_t^* < 0$) today, expand output ($\Delta_t^* > 0$) tomorrow
OPTIMAL INTERVENTION

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(Second best)\hspace{2cm} (laissez-faire)

\[
\int_0^{+\infty} e^{-\rho t} \sum_h \phi^h \eta^{h,\text{effic}} u' \left( \mathcal{Y}_{0,t}^h + \Pi_t \right) \Delta_t^* dt = 0 \hspace{2cm} \int_0^{+\infty} e^{-\rho t} u' \left( \mathcal{Y}_{0,t}^N + \Pi_t \right) \Delta_t^* dt = 0
\]
**Optimal Intervention**

- **No pref. for redistribution**: weights $\eta^{\text{effic}}$ so that distributional terms cancel out
  
  Guarantees that the government would not distort an efficient allocation for redistributive reasons

- **Trade-off**: reduce consumption ($\Delta^*_t < 0$) today, expand output ($\Delta^*_t > 0$) tomorrow

  (second best) \hspace{2cm} (laissez-faire)

  $$\int_0^{+\infty} e^{-\rho t} \sum_h \phi^h \eta^{h,\text{effic}} u' \left( \mathcal{Y}^h_{0,t} + \Pi_t \right) \Delta^*_t dt = 0$$

  $$\int_0^{+\infty} e^{-\rho t} u' \left( \mathcal{Y}^N_{0,t} + \Pi_t \right) \Delta^*_t dt = 0$$

- **Government** is *more impatient* than the firm — priced by unconstrained workers only
No pref. for redistribution: weights $\eta^{\text{effic}}$ so that distributional terms cancel out
Guarantees that the government would not distort an efficient allocation for redistributive reasons

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(second best)                              (laissez-faire)

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Government is more impatient than the firm — priced by unconstrained workers only

Optimal policy: Tax automation $\tau^{\alpha} > 0$
Optimal Intervention

▶ No pref. for redistribution: weights $\eta^{\text{effic}}$ so that distributional terms cancel out

Guarantees that the government would not distort an efficient allocation for redistributive reasons

▶ Trade-off: reduce consumption ($\Delta^*_t < 0$) today, expand output ($\Delta^*_t > 0$) tomorrow

(Second best) \hspace{1cm} (laissez-faire)

$$\int_0^{+\infty} e^{-\rho t} \sum_h \phi^h \eta^{h,\text{effic}} u' \left( \mathcal{Y}_{0,t}^h + \Pi_t \right) \Delta^*_t dt = 0$$

$$\int_0^{+\infty} e^{-\rho t} u' \left( \mathcal{Y}_{0,t}^N + \Pi_t \right) \Delta^*_t dt = 0$$

Prevents excessive investment in automation and raises consumption early on in the transition, precisely when displaced workers are borrowing-constrained
Active labor market interventions might not be available (Heckman et al., Card et al.)
Active labor market interventions might not be available (Heckman et al., Card et al.)

The government uses automation ($\alpha$) as a proxy for reallocation ($T_0$)

$$\int_{T_0}^{+\infty} e^{-(\rho+\chi)t} \sum_h \phi^h \eta^h u' \left( \gamma_{0,t}^h + \Pi_t \right) \left( \Delta_t^* + T_0' (\alpha^{SB}) \Delta_t \right) dt = 0$$

so that

- Short unempl/retraining spells ($1/\kappa$ low) $\rightarrow$ tax $\alpha$ more
- Long unempl/retraining spells ($1/\kappa$ high) $\rightarrow$ tax $\alpha$ less
Environment

Efficient Allocation

Decentralized Equilibrium

Failure of First Welfare Theorem

Optimal Policy

Quantitative Analysis
Firm

task-based framework – Acemoglu-Autor

\[ y^h_t = F\left(\mu^h_t; \alpha^h_t\right) = A^h \left(\alpha^h_t + \mu^h_t\right)^{1-n} \]

quadratic adjustment costs – \( \omega (x_t/\alpha_t)^2 \alpha_t \)

\[ d\alpha^A_t = (x_t - \delta\alpha^A_t) \, dt \quad \alpha^N_t = 0 \]
**Firm**

Task-based framework – Acemoglu-Autor

\[ y_t^h = F \left( \mu_t^h; \alpha_t^h \right) = A_t^h \left( \alpha_t^h + \mu_t^h \right)^{1-\eta} \]

Quadratic adjustment costs – \( \omega \left( x_t / \alpha_t \right)^2 \alpha_t \)

\[ d\alpha_t^A = (x_t - \delta \alpha_t^A) \, dt \quad \alpha_t^N = 0 \]

**Workers**

Gross flows – Kambourov-Manovskii

\[ S_t \left( x \right) = \frac{(1 - \phi) \exp \left( \frac{V_N^t \left( x' \left( N; x \right) \right)}{\gamma} \right)}{\sum_{h'} \phi_{h'} \exp \left( \frac{V_{h'}^t \left( x' \left( h'; x \right) \right)}{\gamma} \right)} \]

Uninsured risk – Huggett-Aiyagari

\[ dz_t^T = -\rho_z z_t^T \, dt + \sigma_z dW_t \]

\[ z_t^P = (1 - \theta) z_{t,-}^P \text{ when moving} \]
QUANTITATIVE MODEL

Firm
task-based framework – Acemoglu-Autor

\[ y_t^h = F\left(\mu_t^h; \alpha_t^h\right) = A^h \left(\alpha_t^h + \mu_t^h\right)^{1-\eta} \]

quadratic adjustment costs \( - \omega \left(x_t/\alpha_t\right)^2 \alpha_t \)

\[ d\alpha_t^A = (x_t - \delta\alpha_t^A) \, dt \quad \alpha_t^N = 0 \]

Calibration
internal (7) and external (13)

Workers
gross flows – Kambourov-Manovskii

\[ S_t(x) = \frac{(1 - \phi) \exp\left(\frac{V^N_t(x'(N;x))}{\gamma}\right)}{\sum_{h'} \phi^{h'} \exp\left(\frac{V^{h'}_t(x'(h';x))}{\gamma}\right)} \]

uninsured risk – Huggett-Aiyagari

\[ dz_t^T = -\rho_z z_t^T dt + \sigma_z dW_t \]
\[ z_t^P = (1 - \theta) z_{t,-}^P \text{ when moving} \]
Objective: The government maximizes

\[ W(\eta) \equiv \int_{-\infty}^{+\infty} \int \eta_t(x) \, V_{t}^{\text{birth}}(x) \, d\pi_t(x) \, dt \]

Second best: Vary the speed of automation \( \varphi \)

\[ \alpha_t = \alpha_0 + (1 - e^{-\varphi t}) \left( \alpha^{\text{CE}} - \alpha_0 \right) \]

Implementation: \( \{\tau^x_t\} \) on investment, rebated lump sum to workers
Table 1: Welfare Gains

<table>
<thead>
<tr>
<th>Half-life (years)</th>
<th>40</th>
<th>70</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>$W(\eta_{\text{effic}}; r_t)$</td>
<td>0%</td>
<td>2.3%</td>
<td>3.2%</td>
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<td>$W(\eta_{\text{util}}; r_t)$</td>
<td>0%</td>
<td>2.8%</td>
<td>3.9%</td>
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- *Within* generations: utilitarian or efficient $1/V_t(\cdot)$ weights
- *Across* generations: discount with $r_t$
### Table 1: Welfare Gains

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- **Within** generations: utilitarian or efficient $1/V'_t(\cdot)$ weights
- **Across** generations: discount with $r_t$
- **Next**: ...
Workers displaced by automation would like to borrow as they reallocate slowly.
Takeaways

- Workers displaced by automation would like to borrow as they reallocate slowly.
- Borrowing constraints create wedge between interest rate and MRS of workers.
Workers displaced by automation would like to borrow as they reallocate slowly.

Borrowing constraints create wedge between interest rate and MRS of workers.

Firms are effectively too patient when investing in automation.
Takeaways

- Workers displaced by automation would like to borrow as they reallocate slowly.
- Borrowing constraints create a wedge between interest rate and MRS of workers.
- Firms are effectively too patient when investing in automation.

The government should slow down automation on efficiency grounds, even when it has no preference for redistribution.
Competitive equilibrium. Set of

1. Allocation for consumption \( \{c_t\} \) and labor supplies \( \{\mu_t, \Theta_t\} \), and automation \( \alpha \)
2. Interest rate \( \{r_t\} \), wages \( \{w_t^A, w_t^N\} \) and profits \( \{\Pi_t\} \)

such that

1. Workers consume, save and move optimally given prices
2. Representative firm chooses automation and hires labor optimally given prices
3. Labor markets clear and resource constraint holds
BORROWING

Distortions at the laissez-faire

Distributional effects

- Tight constraint
- Slow reallocation
- Consumption (PE)
- Reallocation (PE)
- Slow reallocation
- Production inefficiency
- Stopping time with $a \to -\infty$
- Constraint binds for some $t > T_0$
- Constraint slack for all $t > T_0$
- Constraint not binding

Back
**COMPETITIVE EQUILIBRIUM**

▶ **Incomes:**

\[ \mathcal{Y}_t^* (x) = \Pi_t + (1 - \tau_t) \times \begin{cases} 
\xi \exp (z) w_t^h & \text{if } e = E \\
 b \xi \exp (z) w_t^{-h} & \text{if } e = U 
\end{cases} \]

where \( b \) is replacement rate during unemployment.

▶ **Assets:**

Workers trade riskless bonds, and annuities (Blanchard-Yaari)

▶ **Fiscal policy:**

Constant debt / GDP, adjusts distorsionary tax \( \{\tau_t\} \)

▶ **Resource constraint:**

\[
\int c_t (x) \, d\pi_t + x_t + \omega \left( \frac{x_t}{\alpha_t} \right)^2 \alpha_t = G^* \left( \left\{ \int 1_{\{h(x) = h\}} \xi d\pi_t \right\} \phi^h \right) + b \int 1_{\{e = U\}} \mathcal{Y}_t (x) \, d\pi_t,
\]
Table 2: External Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibration</th>
<th>Target / Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>EIS (inverse)</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Death rate</td>
<td>1/45</td>
<td>Average working life of 45 years</td>
</tr>
<tr>
<td>$\eta$</td>
<td>Initial labor share</td>
<td>0.36</td>
<td>1970 labor share (BLS)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.1</td>
<td>Graetz-Michaels (2018)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Elasticity of substitution across occs.</td>
<td>0.75</td>
<td>Buera-Kaboski (2011)</td>
</tr>
<tr>
<td>$1/\kappa$</td>
<td>Average unemployment duration</td>
<td>1/3.2</td>
<td>Alvarez-Shimer (2011)</td>
</tr>
<tr>
<td>$1 - \kappa^*$</td>
<td>Probability of return move</td>
<td>0.44</td>
<td>Carillo-Tudela-Visschers (2020)</td>
</tr>
<tr>
<td>$1 - \theta$</td>
<td>Productivity loss from relocation</td>
<td>0.18</td>
<td>Kambourov-Manovskii (2009)</td>
</tr>
</tbody>
</table>
- **Parameters**: External calibration (13) and internal calibration (7)

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<tr>
<th>Parameter</th>
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<td>$\rho$</td>
<td>Discount rate</td>
<td>0.13 2% real interest rate</td>
</tr>
<tr>
<td>$A$</td>
<td>Productivities</td>
<td>0.89, 1.26 Initial output</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Adjustment cost</td>
<td>16 Routine empl. share 2015</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Fraction of automated occupations</td>
<td>0.53 Routine empl. share 1970</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Mobility hazard</td>
<td>0.49 Occupational mobility 1970</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Fréchet parameter</td>
<td>Elasticity of labor supply</td>
</tr>
</tbody>
</table>
Parameters: External calibration (13) and internal calibration (7)

Table 3: Internal Calibration

<table>
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</thead>
<tbody>
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<td>$\rho$</td>
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<td>0.13</td>
<td>2% real interest rate</td>
</tr>
<tr>
<td>$A^A, A^N$</td>
<td>Productivities</td>
<td>0.89, 1.26</td>
<td>Initial output (1)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Adjustment cost</td>
<td>16</td>
<td>Routine empl. share 2015</td>
</tr>
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<tr>
<td>$\gamma$</td>
<td>Fréchet parameter</td>
<td>0.06</td>
<td>Elasticity of labor supply</td>
</tr>
</tbody>
</table>
## Table 4: External Calibration

<table>
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<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibration</th>
<th>Target / Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$a$</td>
<td>Borrowing limit</td>
<td>0</td>
<td>Auclert et al. (2018)</td>
</tr>
<tr>
<td>$B/Y$</td>
<td>Government debt / GDP</td>
<td>0.26</td>
<td>Liquid assets / GDP</td>
</tr>
<tr>
<td>$\rho_z$</td>
<td>Mean reversion (inc)</td>
<td>0.9775</td>
<td>Floden-Linde (2001)</td>
</tr>
<tr>
<td>$\sigma_z$</td>
<td>Volatility (inc)</td>
<td>0.1025</td>
<td>Floden-Linde (2001)</td>
</tr>
<tr>
<td>$b$</td>
<td>Replacement rate while unemployed</td>
<td>0.4</td>
<td>Ganong et al. (2020)</td>
</tr>
</tbody>
</table>
AGGREGATE ALLOCATIONS

Mass of workers in $h = A$

Profit share

No automation