INEFFICIENT AUTOMATION

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Motivation

- Automation raises productivity but *displaces workers and lowers their earnings*

  - Increasing adoption has fueled an active policy debate (Atkison, 2019; Acemoglu et al, 2020)
  - No optimal policy results that take into account frictions faced by displaced workers
  - Two literatures can justify taxing automation
    - (i) Reallocation is frictionless or absent
      - Tax automation (Guerreiro et al, 2017; Costinot-Werning, 2018)
    - (ii) Govt. has preference for redistribution
    - (ii) Automation/reallocation are efficient
      - Tax capital (in the long-run) (Aiyagari 1995; Conesa et al, 2002)
      - Improve efficiency in economies with IM
      - Worker displacement/reallocation absent
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Guerreiro et al, 2017; Costinot-Werning, 2018

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- Two **literatures** can justify taxing automation → **Reallocation is frictionless or absent**

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Take worker displacement seriously. **How should a government respond to automation?**

1. Recognize that displaced workers face two important frictions:
   - (i) Slow reallocation: workers face mobility barriers and may go through unemployment/retraining (Davis-Haltiwanger, 1999; Jacobson et al, 2005; Lee-Wolpin, 2006; Alvarez-Shimer, 2011)
   - (ii) Imperfect credit markets: workers have limited ability to borrow against future incomes (Jappelli et al, 2010; Chetty, 2008)

2. Incorporate frictions in a model with endogenous automation and heterogeneous agents.

3. Theoretical results:
   - (i) Interaction between frictions gives rise to inefficient automation
   - (ii) Optimal to slow down automation while workers reallocate but not tax it in the long-run (even with no preference for redistribution)

4. Quantitative: gross flows + idiosyncratic risk → welfare gains from slowing down automation.
Take worker displacement seriously. **How should a government respond to automation?**

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4. **Quantitative**: gross flows + idiosync. risk → **welfare gains** from slowing down autom.
Outline

Environment

Efficient Allocation

Decentralized Equilibrium

Failure of First Welfare Theorem

Optimal Policy

Quantitative Analysis
Continuous time $t \geq 0$
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- Occupations
- Workers

- Workers $x = \{s, h, \xi\}$ (age, occupation, prod.)
Continuous time $t \geq 0$

**Occupations**

$h = A$ (share $\phi$, degree $\alpha \geq 0$) or $h = N$

**Workers**

$U_0 = E_0 Z \exp(-\rho t) c_1^{-\sigma} t dt$
Environment

Continuous time $t \geq 0$

**Occupations**

$h = A$ (share $\phi$, degree $\alpha \geq 0$) or $h = N$

\[
F^h(\mu) = \begin{cases} 
F^* (\mu; \alpha) & \text{if } h = A \\
F(\mu) \equiv F^* (\mu; 0) & \text{if } h = N
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$F^* (1; \alpha) \downarrow$ in $\alpha$ (less labor-intensive)

$F^* (1; \alpha)$ concave in $\alpha$ (automation cost)

Workers

Final Good Producer $G^* \mu_A, \mu_N; \alpha \equiv G_n F_h(\mu_h)$

Workers $\{s, h, \xi\}$ (age, occupation, prod.)

$\mu_A t, \mu_N t$ ($= 1$ in $t = 0$)

Reallocation afterwards

$$U_0 = E_0 Z \exp(-\rho t) c^{1-\sigma} dt$$

Resource constraint

$$Z c_t(x) d\Lambda = G^* \mu_A, \mu_N; \alpha \phi_h \mu_h t = Z \{h(x) = h\} \xi d\pi_t$$
Environment

Continuous time $t \geq 0$

Occupations

$h = A$ (share $\phi$, degree $\alpha \geq 0$) or $h = N$

$$F^h(\mu) = \begin{cases} F^*(\mu; \alpha) & \text{if } h = A \\ F(\mu) \equiv F^*(\mu; 0) & \text{if } h = N \end{cases}$$

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Final Good Producer

$$G^*(\mu^A, \mu^N; \alpha) \equiv G\left(\left\{F^h(\mu^h)\right\}\right)$$

(gross complements)
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$$G^* (\mu^A, \mu^N; \alpha) \equiv G \left( \left\{ F^h (\mu^h) \right\} \right)$$

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Continuous time $t \geq 0$

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$$F^h(\mu) = \begin{cases} F^*(\mu; \alpha) & \text{if } h = A \\ F(\mu) \equiv F^*(\mu; 0) & \text{if } h = N \end{cases}$$

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$$x = \{s, h, \xi\}$$ (age, occupation, prod.)

$$\left( \mu_t^A, \mu_t^N \right) \begin{cases} = 1 & \text{in } t = 0 \\ \text{Reallocation afterwards} \end{cases}$$
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$U_0 = \mathbb{E}_0 \left[ \int \exp(-\rho t) \frac{c^1_{t \sigma}}{1 - \sigma} dt \right]$
**ENVIRONMENT**

**Continuous time** \( t \geq 0 \)

**Occupations**

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**Resource constraint**

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\int c_t(x) \, d\Lambda = G^* (\mu^A, \mu^N; \alpha)
\]

\[
\phi^h \mu^h = \int 1_{\{h(x) = h\}} \xi \pi_t
\]
Reallocation frictions

- Reallocation of existing workers is **costly** (Kambourov-Manovskii, Violante, Costinot-Werning)

1. **Permanent cost**: productivity loss $\theta$ due to skill-specificity

\[
\xi_t = \begin{cases} 
\lim_{\tau \uparrow t} \xi_\tau & \text{if } h_t'(x) = h \\
(1 - \theta) \times \lim_{\tau \uparrow t} \xi_\tau & \text{otherwise}
\end{cases}
\]
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- Reallocation of existing workers is **slow** (Davis-Haltiwanger, Alvarez-Shimer). Two reasons:

  2. **Random opportunities**: Workers can move across occupations with intensity $\lambda$

  3. **Unemployment/retraining spells**: Enter when moving, and exit at rate $\kappa$
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- Arrival of new workers is **slow** (Rebelo et al., Adão et al.). Rate $\chi$. Choose any occupation.
Outline

Environment

Efficient Allocation

Decentralized Equilibrium

Failure of First Welfare Theorem

Optimal Policy

Quantitative Analysis
First Best Problem

Ex post problem

Ex ante problem

\[ Z + \int_0^\infty e^{-\rho t} u' c N t \Delta t \, dt = 0 \]

\[ \sim (1 - \theta) Y N t - Y A t \]

\[ \Delta t \]

Short unemployment or retraining

Long unemployment or retraining

\[ \Delta \star t \]
First Best Problem

Ex post problem

► Reallocate labor and distribute output

► Close MPLs gap. Stop reallocation at $T_{FB}^{0}$
(No OLG case)

Ex ante problem
Ex post problem

- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T^{FB}_0$ (No OLG case)

\[
\int_{T^{FB}_0}^{+\infty} e^{-\rho t} u' \left( c^N_t \right) \Delta_t dt = 0
\]

where

\[
\Delta_t \equiv (1 - \theta) \left( 1 - e^{-\kappa (t - T^{FB}_0)} \right) Y^N_t - Y^A_t
\]

Cost = Skill loss + unemp

is the IRF of $Y$ to reallocation

Ex ante problem

Choose degree of automation $\alpha$
- Reduce $C$ today, expand $Y$ tomorrow (No OLG case)
First Best Problem

Ex post problem

- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T_0^{FB}$ (No OLG case)

$$\int_{T_0^{FB}}^{+\infty} e^{-\rho t} u' \left( c_t^N \right) \Delta_t dt = 0$$

Ex ante problem

- Choose degree of automation $\alpha$
- Reduce $C$ today, expand $Y$ tomorrow (No OLG case)
**First Best Problem**

**Ex post problem**
- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T_{FB}^{FB}$
  
  \[ \int_{T_{FB}^{FB}}^{+\infty} e^{-\rho t} u' \left( c_t^N \right) \Delta_t dt = 0 \]

(No OLG case)

**Ex ante problem**
- Choose degree of automation $\alpha^{FB}$
- Reduce $C$ today, expand $Y$ tomorrow

**Graph**: 
- Short unemployment or retraining: $\sim (1 - \theta) Y_t^N - Y_t^A$
- Long unemployment or retraining
**FIRST BEST PROBLEM**

**Ex post problem**
- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T_{FB}^0$
  (No OLG case)

$$\int_{T_{FB}^0}^{+\infty} e^{-\rho t} u' \left( c_t^N \right) \Delta_t dt = 0$$

**Ex ante problem**
- Choose degree of automation $\alpha^{FB}$
- Reduce $C$ today, expand $Y$ tomorrow

$$\int_{0}^{+\infty} e^{-\rho t} u' \left( c_t^A \right) \Delta_t^* dt = 0$$

where

$$\Delta_t^* \equiv \frac{\partial}{\partial \alpha} G^* \left( \mu_t^A, \mu_t^N; \alpha^{FB} \right)$$

is the IRF of $Y$ to automation (net of cost)
**First Best Problem**

**Ex post problem**

- Reallocate labor and distribute output
- Close MPLs gap. Stop reallocation at $T^*_0$ (No OLG case)

\[
\int_{T^*_0}^{+\infty} e^{-\rho t} u' \left( c^N_t \right) \Delta_t dt = 0
\]

\[
\sim (1 - \theta) Y^T_N - Y^A_t
\]

**Ex ante problem**

- Choose degree of automation $\alpha^{FB}$
- Reduce $C$ today, expand $Y$ tomorrow

\[
\int_{0}^{+\infty} e^{-\rho t} u' \left( c^A_t \right) \Delta^*_t dt = 0
\]

Output gains $(\alpha, -\mu)$ are complements

Crowding out
Outline

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Quantitative Analysis
### Decentralized Choices

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<th>Firms</th>
<th>Workers</th>
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**Firms**

- Choose automation
- \( \alpha \)
- \( \mu_t \)
- Maximize
- \( Z + \infty \)
- \( Q_t \Pi_t(\mu_t; \alpha) \)
- \( Q_t \): equity priced by unconst. workers

**Workers**

- Choose consumption \( c_t \) and labor supply \( \mu_t \)
- Assets: bonds, incomplete markets
- Workers not insured against automation risk

**Equation**

\[
d a_t(x) = \left[ Y^\star_t(x) + (r_t + \chi) a_t(x) - c_t(x) \right] dt
\]

- Borrowing friction
- \( a_t(x) \geq a \) for some \( a \leq 0 \)
**Decentralized Choices**

**Firms**

Choose automation $\alpha$ + labor demand $\mu_t$

$$\max_{\{\alpha, \mu_t\}} \int_0^{+\infty} Q_t \Pi_t (\mu_t; \alpha) \, dt$$

**Workers**

Choose consumption $c_t$ and labor supply $\mu_t$

Assets: bonds, incomplete markets

Workers not insured against automation risk

$x = \{a, s, h, \xi\}$ (bonds, age, occ., prod.)

$$\text{da}_t (x) = \left[ Y^\star_t (x) + (r_t + \chi) \text{a}_t (x) - c_t (x) \right] dt$$

Borrowing friction $a_t (x) \geq a$ for some $a \leq 0$
Decentralized Choices

**Firms**

Choose automation $\alpha + \text{labor demand } \mu_t$

$$\max_{\{\alpha, \mu_t\}} \int_0^{+\infty} Q_t \Pi_t (\mu_t; \alpha) \, dt$$

**Workers**

Choose cons. $c_t$ and labor supply $\mu_t$
Decentralized Choices

Firms

Choose automation $\alpha +$ labor demand $\mu_t$

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Workers not insured against automation risk

$x = \{a, s, h, \xi\}$ (bonds, age, occ., prod.)

\[
da_t (x) = [\gamma^*_t (x) + (r_t + \chi) a_t (x) - c_t (x)] \, dt
\]
Decentralized Choices

Firms

Choose automation $\alpha$ + labor demand $\mu_t$

$$\max_{\{\alpha, \mu_t\}} \int_0^{+\infty} Q_t \Pi_t (\mu_t; \alpha) \, dt$$

Workers

Choose cons. $c_t$ and labor supply $\mu_t$

**Assets:** bonds, incomplete markets

Workers not insured against automation risk

$x = \{a, s, h, \xi\}$ (bonds, age, occ., prod.)

$$da_t (x) = [\mathcal{Y}_t^* (x) + (r_t + \chi)a_t (x) - c_t (x)] \, dt$$

**Borrowing friction**

$$a_t (x) \geq a \text{ for some } a \leq 0$$
**Decentralized Choices**

**Firms**

Choose automation $\alpha$ + labor demand $\mu_t$

$$\max_{\{\alpha, \mu_t\}} \int_0^{+\infty} Q_t \Pi_t (\mu_t; \alpha) \, dt$$

$Q_t$: equity priced by unconst. workers

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$$a_t (x) \geq a \text{ for some } a \leq 0$$
Outline

Environment
Efficient Allocation
Decentralized Equilibrium
Failure of First Welfare Theorem
Optimal Policy
Quantitative Analysis
Proposition. (Failure of FWT)

1. The laissez-faire equilibrium is inefficient if and only if reallocation frictions \((\lambda, \kappa)\) and borrowing frictions \((a)\) are such that \(a^* (\lambda, \kappa) < a \leq 0\) for threshold \(a^* (\cdot)\).
Proposition. (Failure of FWT)

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2. The threshold \(a^* (\lambda, \kappa) < 0\) if and only if reallocation is slow \((1/\lambda \text{ or } 1/\kappa > 0)\).
Failure of the First Welfare Theorem

**Proposition.** (Failure of FWT)

1. The laissez-faire equilibrium is inefficient if and only if reallocation frictions \((\lambda, \kappa)\) and borrowing frictions \((a)\) are such that \(a^*(\lambda, \kappa) < a \leq 0\) for threshold \(a^*(\cdot)\).

2. The threshold \(a^*(\lambda, \kappa) < 0\) if and only if reallocation is slow \((1/\lambda \text{ or } 1/\kappa > 0)\).

- **Interaction** between reallocation and borrowing frictions is key
- **Efficient cases:** instant realloc. (Costinot-Werning) or no borrowing frictions (Guerreiro et al)
Nature of the Inefficiency

Distortions at the laissez-faire

Workers expect income to improve as they reallocate.

Motive for borrowing

\[ a^* (\lambda) \]

Slow reallocation

Tight constraint
Workers expect income to improve as they reallocate → Motive for borrowing
Why Is Automation Inefficient?

- Automation. Compare the optimality conditions
  (first best)
  \[ \int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \Delta_t^* dt = 0 \]

  (laissez-faire)
  \[ \int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \Delta_t^* dt = 0 \]

  where \( \Delta_t^* \) is the IRF of \( Y \) to automation.
Why Is Automation Inefficient?

▶ Automation. Compare the optimality conditions

(first best) \[ \int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \Delta_t^* dt = 0 \]

(laissez-faire) \[ \int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \Delta_t^* dt = 0 \]

▶ Automation \[ u'(c_{0,t}^A) > u'(c_{0,t}^N) \rightarrow \] Rationale for redistribution
Why Is Automation Inefficient?

- Automation. Compare the optimality conditions
  (first best) (laissez-faire)

\[
\int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \Delta^*_t dt = 0 \quad \int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \Delta^*_t dt = 0
\]

- Automation
  \[u'(c_{0,t}^A) > u'(c_{0,t}^N) \rightarrow \text{Rationale for redistribution}\]

- No borrowing constraints
  \[\frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} = \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \rightarrow \text{Laissez-faire = First best}\]
Why Is Automation Inefficient?

- Automation. Compare the optimality conditions (first best) vs. (laissez-faire)

\[ \int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t})}{u'(c_{0,0})} \Delta^*_t dt = 0 \]

\[ \int_0^{+\infty} e^{-\rho t} \frac{u'(c_{N,t})}{u'(c_{N,0})} \Delta^*_t dt = 0 \]

- Automation

\[ u'(c_{0,t}) > u'(c_{N,t}) \rightarrow \text{Rationale for redistribution} \]

- Borrowing constraints

\[ \frac{u'(c_{0,t})}{u'(c_{0,0})} < \frac{u'(c_{N,t})}{u'(c_{N,0})} \rightarrow \text{Laissez-faire} \neq \text{First best} \]
Why Is Automation Inefficient?

- Automation. Compare the optimality conditions
  (first best) \[ \int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t})}{u'(c_{0,0})} \Delta_t^* dt = 0 \]
  (laissez-faire) \[ \int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \Delta_t^* dt = 0 \]

- Automation \[ u'(c_{0,t}^A) > u'(c_{0,t}^N) \rightarrow \text{Rationale for redistribution} \]

- Borrowing constraints \[ \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} < \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \rightarrow \text{Laissez-faire} \neq \text{First best} \]

Firms fail to internalize that displaced workers have a limited ability to smooth consumption while they reallocate.
Outline

Environment

Efficient Allocation

Decentralized Equilibrium

Failure of First Welfare Theorem

Optimal Policy

Quantitative Analysis
How should a government respond to automation? Depends on the *tools* available.
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- Suppose: **tax on automation** $\tau^\alpha +$ arbitrary transfers/taxes to redistribute
How should a government respond to automation? Depends on the tools available

- Suppose: tax on automation $\tau^\alpha +$ arbitrary transfers/taxes to redistribute
- Wedge between first best and laissez-faire optimality condition

$$\tau^\alpha = \int_0^{+\infty} e^{-\rho t} \left( \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} - \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \right) \Delta^*_t dt$$
How should a government respond to automation? Depends on the tools available

- Suppose: tax on automation $\tau^\alpha +$ arbitrary transfers/taxes to redistribute

- Wedge between first best and laissez-faire optimality condition

$$\tau^\alpha = \int_0^{+\infty} e^{-\rho t} \left( \frac{u'(c_{N,0,t})}{u'(c_{0,0}^N)} - \frac{u'(c_{A,0,t})}{u'(c_{0,0}^A)} \right) \Delta_t^* dt$$

When is $\tau^\alpha = 0$? Redistributive tools → alleviate borrowing cons. and close MRS gap
How should a government respond to automation? Depends on the tools available

- Suppose: tax on automation $\tau^\alpha$ + arbitrary transfers/taxes to redistribute
- Wedge between first best and laissez-faire optimality condition

$$
\tau^\alpha = \int_0^{+\infty} e^{-\rho t} \left( \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} - \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \right) \Delta^*_t dt
$$

1. Worker/time-specific lump sum transfers $\rightarrow$ implement any first best (SWT holds)
   Info req’s? Take-up? Unint. conseq’s? (Piketty-Saez, 2013; Schochet, 2002; Crépon and van der Berg, 2016)
How should a government respond to automation? Depends on the tools available

- Suppose: tax on automation $\tau^\alpha +$ arbitrary transfers/taxes to redistribute
- Wedge between first best and laissez-faire optimality condition

$$\tau^\alpha = \int_0^{+\infty} e^{-\rho t} \left( \frac{u'(c_{0,t}^N)}{u'(c_{0,0})} - \frac{u'(c_{0,t}^A)}{u'(c_{0,0})} \right) \Delta_t^* dt$$

2. **Symmetric lump sum transf.** (UBI) → govt. borrows for workers → restore efficiency
   
How should a government respond to automation? Depends on the tools available

- Suppose: **tax on automation** $\tau^\alpha +$ arbitrary transfers/taxes to redistribute

- Wedge between first best and laissez-faire optimality condition

$$\tau^\alpha = \int_0^{+\infty} e^{-\rho t} \left( \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} - \frac{u'(c_{0,t}^A)}{u'(c_{0,0}^A)} \right) \Delta^*_t dt$$

3. **Non-linear income taxes or unemp. insurance** → help but **do not restore efficiency**

Heterogeneity **within** occupations swamps heterogeneity **between** occupations (as in quantitative model)
Second best tools: tax automation (ex ante) + labor market interventions (ex post)

No social insurance for now, reintroduced in quantitative model
Constrained Ramsey problem

- **Second best tools**: tax automation (ex ante) + labor market interventions (ex post)
  
  No social insurance for now, reintroduced in quantitative model

- **Tractability**: hand-to-mouth workers \((a \to 0)\), no OLG \((\chi = 0)\)
Constrained Ramsey problem

- **Second best tools**: tax automation (*ex ante*) + labor market interventions (*ex post*)
  
  No social insurance for now, reintroduced in quantitative model

- **Tractability**: hand-to-mouth workers ($a \to 0$), no OLG ($\chi = 0$)

- **Primal problem**: control automation $\alpha$ and reallocation $T_0$

\[
\max_{\{\alpha, T_0, \mu_t, c_t\}} \sum_h \phi^n h^n \int_0^{+\infty} \exp (-\rho t) u \left( c_t^h \right) dt
\]

subject to workers’ budget constraints, the law of motion of labor, firms choosing labor optimally, and market clearing.
Government’s optimality conditions to automate ($\alpha$) and reallocate ($T_0$)

\[
\int_{0}^{+\infty} \exp(-\rho t) u'(c_{0,t}^N) \Delta_{t}^* dt = -\Phi^* (\alpha^{SB}, T_0^{SB}, \eta)
\]

\[
\int_{T_0^{SB}}^{+\infty} \exp(-\rho t) u'(c_{0,t}^A) \Delta_{t} dt = -\Phi (\alpha^{SB}, T_0^{SB}; \eta)
\]

\text{laissé-faire} \quad \text{pecuniary externalities}
Government’s optimality conditions to **automate** ($\alpha$) and **reallocate** ($T_0$)

\[
\int_0^{+\infty} \exp(-\rho t) u'(c_{0,t}^N) \Delta_t^* dt = -\Phi^* (\alpha^{SB}, T_0^{SB}; \eta)
\]

\[
\int_{T_0^{SB}}^{+\infty} \exp(-\rho t) u'(c_{0,t}^A) \Delta_t dt = -\Phi (\alpha^{SB}, T_0^{SB}; \eta)
\]

**Proposition.** (Constrained inefficiency)

Fix weights $\eta$. Then, there is always a small perturbation of the technology $G^*(\cdot)$ such that either $\Phi^*(\cdot) \neq 0$ or $\Phi(\cdot) \neq 0$ — i.e., the equilibrium is *generically* constrained inefficient.
No pref. for redistribution: weights $\eta^\text{effic}$ so that distributional terms cancel out

 Guarantee that the government would not distort an efficient allocation for redistributive reasons
No pref. for redistribution: weights $\eta_{\text{effic}}$ so that distributional terms cancel out

Guarantee that the government would not distort an efficient allocation for redistributive reasons

$\int_{0}^{+\infty} e^{-\rho t} \sum_{h} \phi^{h} \eta^{h,\text{effic}} \frac{u'(c^{h}_{0,t})}{u'(c^{h}_{0,0})} \Delta^{*} dt = 0$

$\int_{0}^{+\infty} e^{-\rho t} \frac{u'(c^{N}_{0,t})}{u'(c^{N}_{0,0})} \Delta^{*} dt = 0$
**TAXING AUTOMATION ON EFFICIENCY GROUNDS**

- **No pref. for redistribution**: weights $\eta^{\text{effic}}$ so that distributional terms cancel out

  Guarantee that the government would not distort an efficient allocation for redistributive reasons

\[
\int_0^{+\infty} e^{-\rho t} \sum_h \phi^h \eta^{h, \text{effic}} \frac{u'(c_{0,t}^h)}{u'(c_{0,0}^h)} \Delta_t^* dt = 0
\]

\[
\int_0^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^N)}{u'(c_{0,0}^N)} \Delta_t^* dt = 0
\]

1. The response of output to automation $\Delta_t^*$ is **back-loaded**

---

**Figure**
No pref. for redistribution: weights $\eta^{\text{effic}}$ so that distributional terms cancel out

Guarantee that the government would not distort an efficient allocation for redistributive reasons

(second best)

$$\int_0^{+\infty} e^{-\rho t} \sum_h \phi^h \eta^{h,\text{effic}} \frac{u'(c^h_{0,t})}{u'(c^h_{0,0})} \Delta_t^* dt = 0$$

(laissez-faire)

$$\int_0^{+\infty} e^{-\rho t} u'(c^N_{0,t}) \frac{u'(c^N_{0,0})}{\Delta_t^*} dt = 0$$

1. The response of output to automation $\Delta_t^*$ is back-loaded

2. Government is more impatient than the firm — priced by unconstrained workers only

Firms (partly) overlook that the gains from automation take time to materialize.
No pref. for redistribution: weights $\eta^{\text{effic}}$ so that distributional terms cancel out.

Guarantee that the government would not distort an efficient allocation for redistributive reasons.

(second best)

\[
\int_{0}^{+\infty} e^{-\rho t} \sum_{h} \phi_{h}^{h,\text{effic}} \frac{u'(c_{0,t}^{h})}{u'(c_{0,0}^{h})} \Delta_{t}^{*} dt = 0
\]

(laissez-faire)

\[
\int_{0}^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^{N})}{u'(c_{0,0}^{N})} \Delta_{t}^{*} dt = 0
\]

1. The response of output to automation $\Delta_{t}^{*}$ is back-loaded

2. Government is more impatient than the firm — priced by unconstrained workers only.

Firms (partly) overlook that the gains from automation take time to materialize.

$\rightarrow$ Optimal to tax automation on efficiency grounds.
No pref. for redistribution: weights $\eta^{\text{effic}}$ so that distributional terms cancel out

Guarantee that the government would not distort an efficient allocation for redistributive reasons

(second best) \[ \int_{0}^{+\infty} e^{-\rho t} \sum_{h} \phi^{h} \eta^{h,\text{effic}} \frac{u'(c_{0,t}^{h})}{u'(c_{0,0}^{h})} \Delta^{*} dt = 0 \]

(laissez-faire) \[ \int_{0}^{+\infty} e^{-\rho t} \frac{u'(c_{0,t}^{N})}{u'(c_{0,0}^{N})} \Delta^{*} dt = 0 \]

The optimal tax on automation improves aggregate efficiency. It raises consumption early on in the transition, precisely when displaced workers value it more.
Active labor market interventions might not be available (Heckman et al., Card et al.)
Active labor market interventions might not be available \textit{(Heckman et al., Card et al.)}

The government uses automation ($\alpha$) as a proxy for reallocation ($T_0$)

\[
\int_0^{+\infty} e^{-\rho t} \sum_h \phi^h \eta^h \frac{u'(c_{0,t})}{u'(c_{0,0})} (\Delta_t^* + T'_0 (\alpha^{SB}) \phi^A \lambda \exp (-\lambda T_0 (\alpha^{SB})) \mathbf{1}_{\{t > T_0(\alpha^{SB})\}} \Delta_t) \, dt = 0
\]

so that

Short unempl/retraining spells ($1/\kappa$ low) $\rightarrow$ tax $\alpha$ more

Long unempl/retraining spells ($1/\kappa$ high) $\rightarrow$ tax $\alpha$ less
EXTENSION II: EQUITY CONCERNS

\[ MRS^A = MRS^N \]

\[ LF = SB^{effic} \]

\[ SB^{utilit} \]

\[ MU^A = MU^N \]

Equity

Efficiency

Automation ↓

Automation ↓
Extension III: Slowing down automation

▶ Tax capital in the long-run → improve insurance or prevent capital overaccumulation

(Aiyagari, 1995; Conesa et al., 2009; Dávila et al., 2012)
Extension III: Slowing down automation

▶ Tax capital in the long-run → improve insurance or prevent capital overaccumulation (Aiyagari, 1995; Conesa et al., 2009; Dávila et al., 2012)

▶ Rationale for taxing automation is distinct

1. Does not rely on uninsured income risk (or overlapping generations)

2. Slow down automation only while labor reallocates and workers are borrowing constrained. Not optimal to tax it in the long-run.
Extension III: Slowing down automation

- Tax capital in the long-run → improve insurance or prevent capital overaccumulation (Aiyagari, 1995; Conesa et al., 2009; Dávila et al., 2012)

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- To clarify 2., extend model so that automation takes place gradually

\[
d\alpha_t = (x_t - \delta \alpha_t) \, dt; \quad Y_t = G^* (\mu_t; \alpha_t) - x_t \alpha_t - \Omega (x_t/\alpha_t) \alpha_t
\]

Law of motion; Output net of investment costs
Extension III: Slowing down automation

- Tax capital in the long-run → improve insurance or prevent capital overaccumulation
  (Aiyagari, 1995; Conesa et al., 2009; Dávila et al., 2012)

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\]

Law of motion
Output net of investment costs

- Workers are unconstrained in the long-run \( \iff \alpha_t^{LF}/\alpha_t^{FB} \to 1 \) as \( t \to +\infty \)
Environment

Efficient Allocation

Decentralized Equilibrium

Failure of First Welfare Theorem

Optimal Policy

Quantitative Analysis
Firm

task-based framework – Acemoglu-Autor

\[ y_t^h = F\left(\mu_t^h; \alpha_t^h\right) = A_t^h \left( \varphi_t^h \alpha_t^h + \mu_t^h \right)^{1-\eta} \]

quadratic adjustment costs – \( \omega \left( x_t / \alpha_t \right)^2 \alpha_t \)

\[ d\alpha_t^A = (x_t - \delta \alpha_t^A) \, dt \quad \alpha_t^N = 0 \]
Quantitative Model

Firm

task-based framework – Acemoglu-Autor

\[ y^h_t = F\left(\mu^h_t, \alpha^h_t\right) = A^h \left(\varphi^h \alpha^h_t + \mu^h_t\right)^{1-\eta} \]

quadratic adjustment costs \[ \omega \left(x_t/\alpha_t\right)^2 \alpha_t \]

\[ d\alpha^A_t = (x_t - \delta\alpha^A_t) \, dt \quad \alpha^N_t = 0 \]

Workers

gross flows – Kambourov-Manovskii

\[ S_t(x) = \frac{(1 - \phi) \exp\left(V^N_t(x'(N;x))\right)}{\sum_{h'} \phi^{h'} \exp\left(V^{h'}_t(x'(h';x))\right)} \]

uninsured risk – Huggett-Aiyagari

\[ dz^T_t = -\rho_z z^T_t dt + \sigma_z dW_t \]

\[ z^P_t = (1 - \theta) z^P_{t,-} \text{ when moving} \]
**Quantitative Model**

**Firm**

- **task-based framework** – Acemoglu-Autor

\[
y^h_t = F\left(\mu^h_t; \alpha^h_t\right) = A^h \left(\phi^h \alpha^h_t + \mu^h_t\right)^{1-\eta}
\]

- **quadratic adjustment costs** – \( \omega \left(x_t/\alpha_t\right)^2 \alpha_t \)

\[
d\alpha^A_t = \left(x_t - \delta\alpha^A_t\right) dt \quad \alpha^N_t = 0
\]

**Workers**

- **gross flows** – Kambourov-Manovskii

\[
S_t(x) = \frac{(1 - \phi) \exp\left(\frac{V^N_t(x(N;x))}{\gamma}\right)}{\sum_{h'} \phi^{h'} \exp\left(\frac{V^{h'}_t(x(h';x))}{\gamma}\right)}
\]

- **uninsured risk** – Huggett-Aiyagari

\[
dz^I_t = -\rho_z z^I_t dt + \sigma_z dW_t
\]

\[
z^P_t = (1 - \theta) z^P_{t-1} \text{ when moving}
\]
Second Best Policies and Welfare

- **Objective:** The government maximizes

\[ W(\eta) \equiv \int_{-\infty}^{+\infty} \int \eta_t(x) V^\text{birth}_t(x) d\pi_t(x) dt \]

- **Second best:** Choose \( \{\tau_t^x\} \) on investment, rebated to firm owners.
## Table 1: Welfare Gains $\Delta W$ from Second Best Interventions

<table>
<thead>
<tr>
<th></th>
<th>Alternative calibrations</th>
<th>Alternative policies</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Benchmark</td>
<td>Long unempl.</td>
</tr>
<tr>
<td>Efficiency</td>
<td>3.8%</td>
<td>3.5%</td>
</tr>
<tr>
<td>Utilitarian</td>
<td>5.9%</td>
<td>5.8%</td>
</tr>
</tbody>
</table>

Note: ‘Long unempl.’ and ‘High liquid.’ are calibrations with $1/\kappa = 2$ and $-B/Y = 1.4$. ‘Transfers’ gives $10k$ to automated workers at time $t = 0$ financed with government debt.
Two **novel results** in economies where automated workers face reallocation and borrowing **frictions**

1. Automation is **inefficient** when frictions are sufficiently severe  
   Firms fail to internalize that automated workers have a limited ability to smooth consumption

2. Optimal to **slow down automation** while workers reallocate, but not tax it in the long-run
Two novel results in economies where automated workers face reallocation and borrowing frictions

1. Automation is inefficient when frictions are sufficiently severe
   Firms fail to internalize that automated workers have a limited ability to smooth consumption

2. Optimal to slow down automation while workers reallocate, but not tax it in the long-run

Quantitatively, slowing down automation achieves substantial efficiency and welfare gains, even when the government can implement generous transfers
Competitive equilibrium. Set of

1. Allocation for consumption \( \{c_t\} \) and labor supplies \( \{\mu_t, \Theta_t\} \), and automation \( \alpha \)
2. Interest rate \( \{r_t\} \), wages \( \{w_t^A, w_t^N\} \) and profits \( \{\Pi_t\} \)

such that

1. Workers consume, save and move optimally given prices
2. Representative firm chooses automation and hires labor optimally given prices
3. Labor markets clear and resource constraint holds
Borrowing

Distortions at the laissez-faire

- Consumption (PE)
- Reallocation (PE)
- Slow reallocation
- Tight constraint

Distributional effects

- Constraint not binding
- Prod. inefficiency
- Constraint slack for all $t > T_0$
- Constraint binds for some $t > T_0$
Response of output to automation

Output gains \((\alpha, -\mu)\) are complements

Crowding out

\[ \Delta_t^* \]

\[ 0 \]

\[ t \]
COMPETITIVE EQUILIBRIUM

- Incomes:
  \[ Y_t(x) = T_t \left( \xi \exp(z) w^h_t + \exp(z) \Pi_t \right) \]
  where \( T_t(y) = y - \psi_0 \cdot y^{1-\psi_1} \) captures the non-linear tax schedule.

- Assets:
  Workers trade riskless bonds, and annuities (Blanchard-Yaari)

- Fiscal policy:
  Constant government spending and debt to GDP, adjusts taxes

- Resource constraint:
  \[ \int a_t(x) \, d\pi_t = -B_t, \]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibration</th>
<th>Target / Source</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma$</td>
<td>EIS (inverse)</td>
<td>2</td>
<td>-</td>
</tr>
<tr>
<td>$\chi$</td>
<td>Death rate</td>
<td>1/50</td>
<td>Average working life of 50 years</td>
</tr>
<tr>
<td>$1 - \eta$</td>
<td>Initial labor share</td>
<td>0.64</td>
<td>1970 labor share (BLS)</td>
</tr>
<tr>
<td>$\delta$</td>
<td>Depreciation rate</td>
<td>0.1</td>
<td>Graetz-Michaels (2018)</td>
</tr>
<tr>
<td>$\nu$</td>
<td>Elasticity of substitution across occs.</td>
<td>0.75</td>
<td>Buera-Kaboski (2011)</td>
</tr>
<tr>
<td>$1/\kappa$</td>
<td>Average unemployment duration</td>
<td>1/3.2</td>
<td>Alvarez-Shimer (2011)</td>
</tr>
<tr>
<td>$\theta$</td>
<td>Productivity loss from relocation</td>
<td>0.18</td>
<td>Kambourov-Manovskii (2009)</td>
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<tr>
<td>$a$</td>
<td>Borrowing limit</td>
<td>0</td>
<td>Auclert et al (2018)</td>
</tr>
<tr>
<td>$\phi_0, \phi_1, -B/Y$</td>
<td>Government</td>
<td>0.35, 0.18, 0.26</td>
<td>Heathcote et al (2017), Kaplan et al (2018)</td>
</tr>
<tr>
<td>$\rho_z, \sigma_z, b$</td>
<td>Income</td>
<td>0.023, 0.102, 0.4</td>
<td>Floden-Lindé (2001), Shimer (2005)</td>
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</table>
**Table 3:** Internal Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Calibration</th>
<th>Target / Source</th>
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</thead>
<tbody>
<tr>
<td>$\rho$</td>
<td>Discount rate</td>
<td>0.148</td>
<td>2% real interest rate</td>
</tr>
<tr>
<td>$A_A, A_N$</td>
<td>Productivities</td>
<td>0.89, 1.23</td>
<td>Initial output (1)</td>
</tr>
<tr>
<td>$\omega$</td>
<td>Adjustment cost</td>
<td>2</td>
<td>Half-life of automation</td>
</tr>
<tr>
<td>$\phi$</td>
<td>Fraction of automated occupations</td>
<td>0.53</td>
<td>Routine occs. share 1970</td>
</tr>
<tr>
<td>$\lambda$</td>
<td>Mobility hazard</td>
<td>0.42</td>
<td>Occupational mobility 1970</td>
</tr>
<tr>
<td>$\gamma$</td>
<td>Fréchet parameter</td>
<td>0.053</td>
<td>Elasticity of labor supply</td>
</tr>
</tbody>
</table>