Debt Structure, Monetary Policy, and Investment

Mary Gong
March 8 2021
Growing literature on monetary policy transmission, and differences across firms

This paper: How does debt structure affect firm response?
Growing literature on monetary policy transmission, and differences across firms

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  - Secured vs. unsecured debt: over 85% of firms in sample hold both simultaneously.
Motivation

- Growing literature on monetary policy transmission, and differences across firms
- This paper: How does debt structure affect firm response?
  - Secured vs. unsecured debt: over 85% of firms in sample hold both simultaneously.
  - Secured debt is relatively cheap, but limited by stock of pledgeable assets and existing secured debt
Growing literature on monetary policy transmission, and differences across firms

This paper: How does debt structure affect firm response?

- Secured vs. unsecured debt: over 85% of firms in sample hold both simultaneously.
- Secured debt is relatively cheap, but limited by stock of pledgeable assets and existing secured debt
- Firms with higher pledgeable capacity should be able to better smooth investment when hit with shocks
Firm debt composition

- Firms finance with a mixture of secured and unsecured debt
Firms use unsecured debt before secured capacity is exhausted
Density: fraction pledged vs. fraction secured

![Graph showing the density of fraction pledged vs. fraction secured]
Median fraction pledged across firms

- Fraction of assets pledged co-moves with interest rate

The chart shows the median fraction of assets pledged and the lagged 3-month T-bill rate over various years from 1990 to 2005. The fraction of assets pledged and the lagged 3-month T-bill rate fluctuate over time, indicating a correlation between the two variables.
Change in fraction pledged

- Conditional on debt issuance; median change in fraction pledged co-moves with interest rate.
Main Findings I

- Regress investment rate on the interaction of MP shock and fraction of assets pledged
- Asymmetric response to MP shocks
Main Findings I

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- Asymmetric response to MP shocks
  - For contractionary shocks, firms with higher fraction of assets pledged respond more strongly
Main Findings I

- Regress investment rate on the interaction of MP shock and fraction of assets pledged
- Asymmetric response to MP shocks
  - For contractionary shocks, firms with higher fraction of assets pledged respond more strongly
  - For expansionary shocks, there is no difference in the response of firms based on fraction of assets pledged
Main Findings II

Simple model to understand why firms differ in fraction of assets pledged

Firms preserve secured capacity when there is likely to be a beneficial project in the future.

Response to MP shock depends on whether the firm must issue additional unsecured debt. Stronger response when remaining secured capacity not sufficient and need to use unsecured debt.

Conditional on issuing unsecured debt, response to MP shock is decreasing in level of pre-existing secured debt.

For firms with the same level of pre-existing debt, the less of that debt is secured, the worse is debt overhang.
Main Findings II

Simple model to understand why firms differ in fraction of assets pledged

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  2. Conditional on issuing unsecured debt, response to MP shock is decreasing in level of pre-existing secured debt

    - For firms with same level of pre-existing debt, the less of that debt is secured, the worse is debt overhang
Related Literature


Data

- Panel of non-financial firms from annual Compustat, year 1985-2007; restricted to firms that have some unsecured debt in previous 2 years.
Panel of non-financial firms from annual Compustat, year 1985-2007; restricted to firms that have some unsecured debt in previous 2 years.

- 6051 firms
- 41,354 firm × year observations

Gertler-Karadi monetary policy shocks

- summed to annual frequency
Baseline Regression

\[
\frac{\text{CAPX}_{i,t}}{\text{PPE}_{i,t}} = (\text{PA}_{i,t-1} \times m_p_t) + (\Gamma_{i,t-1} \times m_p_t) + \Gamma_{i,t-1} + \alpha_i + \beta_{st} + \epsilon_{it}
\]

- Pledged Assets

\[
\text{(PA)} = \frac{\text{secured debt}}{\text{PPE} + \text{inventories} + \text{accounts receivable}}
\]
Baseline Regression

\[
\frac{\text{CAPX}_{i,t}}{\text{PPE}_{i,t-1}} = (\text{PA}_{i,t-1} \times \text{mp}_t) + (\Gamma_{i,t-1} \times \text{mp}_t) + \Gamma_{i,t-1} + \alpha_i + \beta_{st} + \epsilon_{it}
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- **Pledged Assets**

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(\text{PA}) = \frac{\text{secured debt}}{\text{PPE} + \text{inventories} + \text{accounts receivable}}
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- **\(\Gamma_{i,t-1} \):** sales growth, market to book ratio, liquidity, leverage, pledged assets, size

- including firm fixed effects and time fixed effects.
Baseline Regression

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\frac{\text{CAPX}_{i,t}}{\text{PPE}_{i,t-1}} = (\text{PA}_{i,t-1} \times m_p_t) + (\Gamma_{i,t-1} \times m_p_t) + \Gamma_{i,t-1} + \alpha_i + \beta_{st} + \epsilon_{it}
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- **Pledged Assets**
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  \]

- **\(\Gamma_{i,t-1}\):** sales growth, market to book ratio, liquidity, leverage, pledged assets, size

- including firm fixed effects and time fixed effects.

- **MP shock broken down into contractionary shock (+) and contractionary shocks (-)**
Baseline Regression

- Firms with a higher fraction of assets pledged invest relatively less after a contractionary shock.
- No heterogeneity in response following an expansionary shock.

<table>
<thead>
<tr>
<th></th>
<th>(1) capx rate</th>
<th>(2) capx rate</th>
</tr>
</thead>
<tbody>
<tr>
<td>fraction pledged * shock (c)</td>
<td>-0.0245**</td>
<td>-0.0449**</td>
</tr>
<tr>
<td></td>
<td>(0.006)</td>
<td>(0.007)</td>
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<tr>
<td>fraction pledged * shock (e)</td>
<td>0.00908</td>
<td>0.0126</td>
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<tr>
<td></td>
<td>(0.069)</td>
<td>(0.169)</td>
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<td>leverage * shock (c)</td>
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<td></td>
<td>(0.833)</td>
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<td>liquidity * shock (c)</td>
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<td>leverage * shock (e)</td>
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<tr>
<td></td>
<td>(0.497)</td>
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<tr>
<td>liquidity * shock (e)</td>
<td>0.0276*</td>
<td></td>
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<tr>
<td></td>
<td>(0.015)</td>
<td></td>
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<tr>
<td>Observations</td>
<td>41354</td>
<td>41354</td>
</tr>
<tr>
<td>$R^2$</td>
<td>0.404</td>
<td>0.406</td>
</tr>
</tbody>
</table>

*p-values in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Breakdown by fraction of debt that is secured

1. less than three percent of debt is secured;
2. 3-50 percent of debt is secured;
3. more than 50 percent of debt is secured

⇒ Response is being driven by firms that rely on secured debt for most of their financing

<table>
<thead>
<tr>
<th></th>
<th>(1) &lt; 3%</th>
<th>(2) 3% – 50%</th>
<th>(3) &gt; 50%</th>
</tr>
</thead>
<tbody>
<tr>
<td>fraction pledged * shock (c)</td>
<td>-0.227</td>
<td>-0.0461</td>
<td>-0.0620**</td>
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<tr>
<td></td>
<td>(0.671)</td>
<td>(0.196)</td>
<td>(0.009)</td>
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<tr>
<td>fraction pledged * shock (e)</td>
<td>-0.0534</td>
<td>0.0198</td>
<td>0.0167</td>
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<tr>
<td></td>
<td>(0.821)</td>
<td>(0.128)</td>
<td>(0.155)</td>
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<td>Observations</td>
<td>8727</td>
<td>9995</td>
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<tr>
<td>$R^2$</td>
<td>0.551</td>
<td>0.549</td>
<td>0.552</td>
</tr>
</tbody>
</table>

$p$-values in parentheses
* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$
Long vs short-term debt

Heterogeneity should only be present if secured debt is long term.
Long vs short-term debt

Heterogeneity should only be present if secured debt is long term.

1. more than 50 percent of debt is due in less than one year;

2. less than 50 percent of debt is due in one year

<table>
<thead>
<tr>
<th></th>
<th>(1) mostly short term</th>
<th>(2) mostly long term</th>
</tr>
</thead>
<tbody>
<tr>
<td>fraction pledged * shock (c)</td>
<td>-0.0432 (0.248)</td>
<td>-0.0253** (0.007)</td>
</tr>
<tr>
<td>fraction pledged * shock (e)</td>
<td>-0.00849 (0.806)</td>
<td>0.00851 (0.148)</td>
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<tr>
<td>Observations</td>
<td>6132</td>
<td>31260</td>
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<tr>
<td>$R^2$</td>
<td>0.539</td>
<td>0.469</td>
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</tbody>
</table>

$p$-values in parentheses

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Conditional on issuing unsecured

No qualitative change in results when conditioning on issuances of unsecured debt.

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<td>fraction pledged * shock (c)</td>
<td>-0.0448**</td>
<td>-0.0804*</td>
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<tr>
<td></td>
<td>(0.008)</td>
<td>(0.013)</td>
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<tr>
<td>fraction pledged * shock (e)</td>
<td>0.0122</td>
<td>0.0245</td>
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<tr>
<td></td>
<td>(0.242)</td>
<td>(0.101)</td>
</tr>
<tr>
<td>leverage * shock (c)</td>
<td>0.0103</td>
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<tr>
<td></td>
<td>(0.806)</td>
<td></td>
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<td>liquidity * shock (c)</td>
<td>0.0878</td>
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<td></td>
<td>(0.329)</td>
<td></td>
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<tr>
<td>leverage * shock (e)</td>
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<td>-0.0241</td>
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<tr>
<td></td>
<td></td>
<td>(0.144)</td>
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<tr>
<td>liquidity * shock (e)</td>
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<td>-0.0239</td>
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<td>(0.457)</td>
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<td>Observations</td>
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<td>11019</td>
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<tr>
<td>$R^2$</td>
<td>0.538</td>
<td>0.544</td>
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</tbody>
</table>

$p$-values in parentheses
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Firms with a higher fraction of assets pledged appear to be more constrained when it comes to retiring debt.

<table>
<thead>
<tr>
<th></th>
<th>(1) change in debt</th>
<th>(2) change secured</th>
<th>(3) debt issuance</th>
<th>(4) debt reduction</th>
</tr>
</thead>
<tbody>
<tr>
<td>fraction pledged * shock (c)</td>
<td>-0.175</td>
<td>-0.0105</td>
<td>-0.144</td>
<td>-0.138**</td>
</tr>
<tr>
<td></td>
<td>(0.158)</td>
<td>(0.864)</td>
<td>(0.171)</td>
<td>(0.007)</td>
</tr>
<tr>
<td>fraction pledged * shock (e)</td>
<td>0.113</td>
<td>0.0235</td>
<td>0.0500</td>
<td>0.0366</td>
</tr>
<tr>
<td></td>
<td>(0.099)</td>
<td>(0.411)</td>
<td>(0.363)</td>
<td>(0.145)</td>
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<tr>
<td>Observations</td>
<td>41353</td>
<td>40214</td>
<td>39249</td>
<td>40270</td>
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<tr>
<td>$R^2$</td>
<td>0.350</td>
<td>0.329</td>
<td>0.576</td>
<td>0.607</td>
</tr>
</tbody>
</table>

$p$-values in parentheses

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Want secured debt to be determined at the time of MP shock

⇒ three periods \( t = 0, 1, 2 \)
Model Setup: the Ingredients

- Want secured debt to be determined at the time of MP shock
  → three periods $t = 0, 1, 2$

- Firm has pledgeable capacity
  → project at $t = 0$, can use as collateral for secured debt.
Model Setup: the Ingredients

- Want secured debt to be determined at the time of MP shock
  \[ \Rightarrow \text{three periods } t = 0, 1, 2 \]

- Firm has pledgeable capacity
  \[ \Rightarrow \text{project at } t = 0, \text{ can use as collateral for secured debt.} \]

- Want to analyze firm investment response to shock
  \[ \Rightarrow \text{project at time } t = 1, \text{ look at change in required success probability} \]
Model Setup: the Ingredients

- Want secured debt to be determined at the time of MP shock
  \[ \implies \text{three periods } t = 0, 1, 2 \]

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- Want to analyze firm investment response to shock
  \[ \implies \text{project at time } t = 1, \text{ look at change in required success probability} \]

- Want tradeoff between using secured or unsecured debt at time \( t = 0 \)
  \[ \implies \text{success probability of time } 1 \text{ project is unknown at time } 0 \]
Model Setup: the Ingredients

- Want secured debt to be determined at the time of MP shock
  \[\Rightarrow\] three periods \( t = 0, 1, 2 \)

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- Want to analyze firm investment response to shock
  \[\Rightarrow\] project at time \( t = 1 \), look at change in required success probability

- Want tradeoff between using secured or unsecured debt at time \( t = 0 \)
  \[\Rightarrow\] success probability of time 1 project is unknown at time 0

- Want long term debt
  \[\Rightarrow\] Projects all pay off at time \( t = 2 \).
Model Setup: The firm’s projects

- Time 0: safe interest rate \( r_0 = 0 \) (normalization)

\[
\begin{align*}
t & = 0 & t & = 2 \\
I_0 & \quad p_0 \quad \sigma_{0,H} I_0 > I_0 \\
 & \quad 1 - p_0 \quad \sigma_{0,L} I_0 < I_0
\end{align*}
\]
Model Setup: The firm’s projects

- Time 0: safe interest rate $r_0 = 0$ (normalization)

  $t = 0$  
  $t = 2$

  ![Diagram for Time 0]

  - $l_0$  
    - $p_0$ \( \rightarrow \sigma_{0, H} l_0 > l_0 \)
    - $1 - p_0$ \( \rightarrow \sigma_{0, L} l_0 < l_0 \)

- Time 1: safe interest rate $r_1$

  $t = 1$  
  $t = 2$

  ![Diagram for Time 1]

  - $l_1$  
    - $p_1$ \( \rightarrow \sigma_{1, H} l_1 > l_1 \)
    - $1 - p_1$ \( \rightarrow 0 \)
Model Setup: The firm’s projects

- Time 0: safe interest rate $r_0 = 0$ (normalization)
  
  $t = 0$ \hspace{1cm} $t = 2$

  $l_0 \rightarrow p_0 \rightarrow \sigma_{o,H}l_0 > l_0$
  $\downarrow$

  $1 - p_0 \rightarrow \sigma_{o,L}l_0 < l_0$

- Time 1: safe interest rate $r_1$

  $t = 1$ \hspace{1cm} $t = 2$

  $l_1 \rightarrow p_1 \rightarrow \sigma_{1,H}l_1 > l_1$
  $\downarrow$

  $1 - p_1 \rightarrow 0$

- Constraint on secured debt:

  $$D_0^s + D_1^s(1 + r_1) \leq \sigma_{o,L}l_0$$
Additional Assumptions

▶ The two projects are independent.

▶ time $t = 0$ decisions cannot be changed at time $t = 1$:
  ▶ At time 1, the firm takes $D_0^s$, $D_0^u$, and $r_0^u$ as given.

▶ The firm only needs one project to succeed in order to not go bankrupt

\[
\sigma_o, H l_0 \gg l_0 + l_1, \quad \sigma_1, H l_1 \gg l_0 + l_1
\]
Solving backwards: Firm’s problem at time $t = 1$

- The firm is faced with a project with upfront cost $l_1$, pays off at $t = 2$
  - with probability $p_1$: project succeeds and returns $\sigma_{1,H}l_1$
  - with probability $1 - p_1$: project fails and returns $0$. 

Solving backwards: Firm’s problem at time $t = 1$

- The firm is faced with a project with upfront cost $l_1$, pays off at $t = 2$
  - with probability $p_1$: project succeeds and returns $\sigma_{1,H} l_1$
  - with probability $1 - p_1$: project fails and returns 0.

- If the firm has any remaining secured capacity, $\sigma_{0,L} l_0 - D_0^s > 0$, can issue more secured debt.

- The firm can also issue unsecured debt to a risk-neutral lender, who has outside option with return rate $r_1$. 
  
  Details
Firm’s problem at time $t = 1$

- **Result:** The firm uses the *least* amount of unsecured debt possible to finance the project.
Firm’s problem at time $t = 1$

- **Result:** The firm uses the *least* amount of unsecured debt possible to finance the project
  - Intuition: Secured debt at $t = 1$ decreases payment to $t = 0$ unsecured creditors in case of bankruptcy, but $r^u_0$ fixed.
Firm’s problem at time $t = 1$

- **Result:** The firm uses the *least* amount of unsecured debt possible to finance the project
  - Intuition: Secured debt at $t = 1$ decreases payment to $t = 0$ unsecured creditors in case of bankruptcy, but $r^d_0$ fixed.

- The firm takes on the additional project if it increases the expected return at $t = 2$.

- This defines a threshold success probability $p_1(D^s_0; r_1)$.
  - $\implies$ if the realized $p_1$ is higher than $p_1(D^s_0; r_1)$, the firm takes on the second project.
Firm’s decision at $t = 0$.

- Investment project requiring $l_0$ of funding, pays off at $t = 2$.
  - with probability $p_0$: project pays $\sigma_{o,H}l_0 > l_0$
  - with probability $1 - p_0$: project pays $\sigma_{o,L}l_0 > l_0 \implies$ this portion can be pledged for secured debt.
Firm’s decision at $t = 0$.

- Investment project requiring $l_0$ of funding, pays off at $t = 2$.
  - with probability $p_0$: project pays $\sigma_{0,H}l_0 > l_0$
  - with probability $1 - p_0$: project pays $\sigma_{0,L}l_0 > l_0 \implies$ this portion can be pledged for secured debt.

- The firm knows in the next period, will have another project requiring $l_1$ of funding, and will return $\sigma_{1,H}l_1$ if it succeeds and 0 otherwise
  - $p_1$ is unknown, but firm believes it is distributed $p_1 \sim \varphi(p_1)$.

- Choosing $D^s_0$ defines $p_1(D^s_0; r_1)$, the firm chooses $D^s_0$ to maximize expected $t = 2$ profits.
Firm’s decision at \( t = 0 \): Optimal \( D_0^s \)

Two cases

- \( D_0^s \) less than threshold \( \overline{D_0^s} = \sigma_o L_0 - (1 + r_1)l_1 \)
  
  ▶ No further unsecured debt at \( t = 1 \).

  ▶ **Result:** Composition of secured and unsecured debt does not matter below this threshold

  \( V(D_0^s) \) does not change with \( D_0^s \).

- \( D_0^s \) above threshold

  ▶ The firm will take on more unsecured debt at \( t = 1 \).

  ▶ **Result:** Composition of debt does matter above this threshold

  \( V(D_0^s) \) does change with \( D_0^s \).
Firm’s decision at $t = 0$: Optimal $D^S_0$

Two cases

- $D^S_0$ less than threshold $\overline{D}^S_0 = \sigma_0, L_0 - (1 + r_1)l_1$
  - No further unsecured debt at $t = 1$.
  - **Result:** Composition of secured and unsecured debt does not matter below this threshold $V(D^S_0)$ does not change with $D^S_0$.

- $D^S_0$ above threshold $\overline{D}^S_0$
  - The firm will take on more unsecured debt at $t = 1$.
  - **Result:** Composition of debt does matter above this threshold $V(D^S_0)$ does change with $D^S_0$. 

Case 1: $D_s^O < \overline{D_s^O}$

Result:
Composition of secured and unsecured debt does not matter

Intuition:

- Varying $D_s^O$ below threshold does not change composition of financing for the next project.
Result:
Composition of secured and unsecured debt does not matter

Intuition:

- Varying $D_o^s$ below threshold does not change composition of financing for the next project.

- For the lender: when the firm is bankrupt, the unsecured lender gets $\sigma_o L_i - D^s$
  - increasing $D_o^s$ by 1 decreases payment by 1 when firm bankrupt.
Case 1: $D^s_0 < \overline{D^s_0}$

Result:
Composition of secured and unsecured debt does not matter

Intuition:

- Varying $D^s_0$ below threshold does not change composition of financing for the next project.

- For the lender: when the firm is bankrupt, the unsecured lender gets $\sigma_{0,L} I_0 - D^s$
  - increasing $D^s_0$ by 1 decreases payment by 1 when firm bankrupt.

- interest rate determines tradeoff between secured and unsecured. If increase $D^s_0$ by one unit
  - firm saves $r^u_0$ from unsecured debt interest payments
  - costs firm $D^u_0 \frac{\partial r^u_0}{\partial D^s_0}$.
Case 1: $D^s_O < \overline{D}^s_O$

Result:
Composition of secured and unsecured debt does not matter

Intuition:

- Varying $D^s_O$ below threshold does not change composition of financing for the next project.

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- interest rate determines tradeoff between secured and unsecured. If increase $D^s_O$ by one unit
  - firm saves $r^u_O$ from unsecured debt interest payments
  - costs firm $D^u_O \frac{\partial r^u_O}{\partial D^s_O}$.

- $r^u_O = D^u_O \frac{\partial r^u_O}{\partial D^s_O}$ for all $D^s_O$ in this region.
Case 2: $D_s^O \geq \overline{D}_s^O$

- Firm knows that there is some probability it will take on project at time 1 $\implies$ will need to issue unsecured debt

$$D_1^u = I_1 - \frac{\sigma_{0,L} l_0 - D_s^O}{1 + r_1} = D_1^s$$

- Now, when increasing $D_s^O$
  - firm saves $r_o^u$ from unsecured debt interest payments
  - costs firm $D_o^u \frac{\partial r_o^u}{\partial D_s^O}$.
  - costs firm because will need to increase $D_1^u$ and the cost of financing tomorrow’s project.

**Result:** At optimum $D_s^O$ in this region,

$$r_o^u - D_o^u \frac{\partial r_o^u}{\partial D_s^O} > 0$$
Case 2: $D^s_0 \geq \overline{D}^s_0$, the lender’s perspective

Why does changing $D^s_0$ can change the value of the firm?

- $D^s_0$ defines a threshold probability $p_1(D^s_0; r_1)$, for which the firm is indifferent between taking on the second project or not.
Case 2: $D^s_0 \geq \overline{D^s_0}$, the lender’s perspective

Why does changing $D^s_0$ can change the value of the firm?

- $D^s_0$ defines a threshold probability $p_1(D^s_0; r_1)$, for which the firm is indifferent between taking on the second project or not

- But, lender is not necessarily indifferent at $p_1(D^s_0; r_1)$. Taking on the next project
  - increases the probability the lender is paid in full
  - decreases payment to the lender when both projects fail
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- **Result:**
  - $V(D^s_0)$ is increasing in $D^s_0$ if the lender strictly prefers the firm to not take on the project at $p_1(D^s_0; r_1)$
  - $V(D^s_0)$ is decreasing in $D^s_0$ if the lender strictly prefers the firm to take on the project at $p_1(D^s_0; r_1)$
Case 2: $D^s_o \geq \overline{D^s_o}$, the lender’s perspective

- Suppose that at some $D^s_o$, the lender strictly prefers the firm to not take on the next project when $p_1 = p_{-1}(D^s_o; r_1)$,
  - Lender will try to increase $p_{-1}(D^s_o; r_1)$.
  - Since $p_{-1}(D^s_o; r_1)$ is increasing in $D^s_o$, the lender will set the interest rate schedule so that the value of the firm is increasing in $D^s_o$. 

  At the optimal $D^s_o$, and at $p_1 = p_{-1}(D^s_o; r_1)$ the lender is indifferent between the firm taking on the next project or not. The firm’s incentive at time $t_1$ is aligned with lender.
Case 2: $D_{O}^{s} \geq \bar{D}_{O}^{s}$, the lender’s perspective

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- Similarly, if the lender strictly prefers the firm to take on the next project, will try to decrease $p_{-1}(D_{O}^{s}; r_{1})$ by making the firm decrease $D_{O}^{s}$. 


Case 2: $D^s_0 \geq \overline{D^s_0}$, the lender’s perspective

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At the optimal $D^s_0$, and at $p_1(D^s_0; r_1)$ the lender is indifferent between the firm taking on the next project or not. The firm’s incentive at time 1 is aligned with lender.
Example:
Shock to $r_1$: interest rate higher than expected.

**Result:** Near threshold, response to MP shock is larger when project requires unsecured financing

\[ \left. \frac{\partial p_1(D_0^s; r_1)}{\partial r_1} \right|_{(D_0^u > 0)} > \left. \frac{\partial p_1(D_0^s; r_1)}{\partial r_1} \right|_{(D_0^u = 0)} > 0 \]

Intuition: Change in cost of financing higher with unsecured debt
Shock to $r_1$: interest rate higher than expected.

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Intuition: Change in cost of financing higher with unsecured debt

- *(D_s^0 small): Without unsecured debt, cost of project is*
  \[D_1 = I_1(1 + r_1)\]
  \[\frac{\partial D_1}{\partial r_1} = I_1\]
Shock to $r_1$: interest rate higher than expected.

**Result:** Near threshold, response to MP shock is larger when project requires unsecured financing

\[
\frac{\partial p_1(D^s_o; r_1)}{\partial r_1} \bigg|_{(D^u_o > 0)} > \frac{\partial p_1(D^s_o; r_1)}{\partial r_1} \bigg|_{(D^u_o = 0)} > 0
\]

Intuition: Change in cost of financing higher with unsecured debt

- **(D^s_o small):** Without unsecured debt, cost of project is $D_1 = l_1(1 + r_1)$

  \[
  \frac{\partial D_1}{\partial r_1} = l_1
  \]

- **(D^s_o large):** With unsecured debt, cost is $D_1 = l_1(1 + r_1) + D^u_1(r^u_1 - r_1)$ and

  \[
  \frac{\partial D_1}{\partial r_1} = l_1 + D^u_1 \frac{\partial (r^u_1 - r_1)}{\partial r_1}
  \]
Across the threshold, the change in required success probability given a shock to $r_1$ is increasing in $D_0^s$. 
Shock to $r_1$: At the threshold

For any firm located exactly at the threshold (before MP shock), there is an asymmetric response to MP shocks.

$$D^s_0 = \sigma_0 l_0 - (1 + r_1)l_1$$

- **Contractionary shock**: as if the firm is pushed to the right of threshold $\Rightarrow$ reacts more strongly to MP shock

- **Expansionary shock**: as if firm is pushed to the left of threshold $\Rightarrow$ reacts less strongly to the MP shock
Shock to $r_1$: Cross derivative in the region $D^s_0 < \overline{D}^s_0$

Result.
In the region where $D^s_0 < \overline{D}^s_0$, if $D^s_0$ was optimally chosen, no heterogeneity in response to $r_1$ shock across firms with different $D^s_0$.

\[
\frac{\partial^2 p_1(D^s_0; r_1)}{\partial r_1 \partial D^s_0} = 0
\]
Shock to $r_1$: Cross derivative in the region $D^s_0 < \overline{D}^s_0$

Result.
In the region where $D^s_0 < \overline{D}^s_0$, if $D^s_0$ was optimally chosen, no heterogeneity in response to $r_1$ shock across firms with different $D^s_0$.

$$\frac{\partial^2 p_1(D^s_0; r_1)}{\partial r_1 \partial D^s_0} = 0$$

Intuition:
- $r^u_0$ is fixed, so will not respond to $r_1$.
- All firms in this region have $D^s_1 = l_1$, so face the same change in the cost of financing the second project.
Shock to $r_1$: Cross derivative in the region $D^s_0 > \overline{D^s_0}$

Result.
If $D^s_0 > \overline{D^s_0}$, and $D^s_0$ optimally chosen, then firms with higher level of secured debt respond less to the shock:

$$\frac{\partial^2 p_1(D^s_0; r_1)}{\partial r_1 \partial D^s_0} < 0$$
Result.
Firms with higher level of secured debt respond less to the shock.
Shock to $r_1$: Cross derivative in the region $D_s^s > D_s^s$

**Result.**
Firms with higher level of secured debt respond less to the shock.

- Higher $D_s^s$ means that the $t = 0$ lender is less worried about their claim being diluted at time 1 with new secured debt issuances.
Result.
Firms with higher level of secured debt respond less to the shock.

- Higher $D^s_0$ means that the $t = 0$ lender is less worried about their claim being diluted at time 1 with new secured debt issuances.
  - Firms with higher $D^s_0$ have more interest savings on $t = 0$ unsecured debt. Recall

$$r^u_0 - D^u_0 \frac{\partial r^u_0}{D^s_0} > 0$$

- The firm cares more about these interest rate savings if it takes on the next project.
Result.
Firms with higher level of secured debt respond less to the shock.

- Higher $D^s_0$ means that the $t = 0$ lender is less worried about their claim being diluted at time 1 with new secured debt issuances.
  - Firms with higher $D^s_0$ have more interest savings on $t = 0$ unsecured debt. Recall
    \[ r^u_0 - D^u_0 \frac{\partial r^u_0}{D^s_0} > 0 \]
    
    - The firm cares more about these interest rate savings if it takes on the next project.

- For firms with higher $D^s_0$, increase in cost of financing due to $r_1$ shock is offset by the savings in interest payments on $t = 0$ debt.
If there are firms on both sides of threshold, then running an unconditional regression would produce empirical result:

\[ \Rightarrow \text{higher secured debt implies stronger response.} \]
Reconciling with Empirics

- If there are firms on both sides of threshold, then running an unconditional regression would produce empirical result: 
  \[ \implies \text{higher secured debt implies stronger response.} \]

- Conditional on taking on unsecured debt, model predictions differ from data following MP shock
  
  - Data: Higher level of secured debt still produces stronger response
  
  - Model: Higher level of secured debt produces weaker response

- If \( t = 1 \) lender is risk averse, with risk aversion increasing in \( D_1^u \), then can get different result.
Conclusion

- Considerable heterogeneity across firms in composition of debt (secured vs. unsecured), and the fraction of assets pledged
  \(\Rightarrow\) Many firms use unsecured debt before exhausting secured capacity.

- Firms with higher fraction of assets pledged as collateral respond more strongly to contractionary shocks.
Conclusion

- Considerable heterogeneity across firms in composition of debt (secured vs. unsecured), and the fraction of assets pledged. Many firms use unsecured debt before exhausting secured capacity.

- Firms with higher fraction of assets pledged as collateral respond more strongly to contractionary shocks.

- Model shows that firms needing unsecured debt to finance project react more to MP shocks because of a higher increase in cost of financing.

- Conditional on using unsecured debt, firms with more securitizable capacity respond more strongly to shock due to debt overhang.
  - This may no longer hold if the lender is risk averse and aversion increasing in level of unsecured debt.
risk-neutral lender, interest rate

\[ p(1 + r_1^u)D_1^u + (1 - p) \left( \sigma_{o,H}l_o - D_o^s - (1 + r_1)D_1^s \right) \mathcal{F} = (1 + r_1^u)D_1^u \]

Bankruptcy: assets split proportionally

where \( \mathcal{F} \) is the fraction of unsecured debt owed to \( t = 1 \) lender

\[ \mathcal{F} \equiv \frac{(1 + r_1^u)D_1^u}{(1 + r_1^u)D_1^u + (1 + r_o^u)D_o^u} \]

\[ p = p_o p_1 + p_1 (1 - p_o) + p_o (1 - p_1) \]

**Result:** When \( D_1^u > 0 \), the unsecured interest is

\[ p(1 + r_1^u) = (1 + r_1)\mu(D_1) \]
Choose $D_0^s$ to maximize

$$V(D_0^s) = \begin{cases} 
  p_0 (\sigma_{0,H} l_0 - D_0) & \mathbb{P}(p_1 \leq p_1(D_0^s; r_1)) \\
  \int_{p_1(D_0^s; r_1)}^{1} (p_0 \sigma_{0,H} l_0 + p_1 \sigma_{1,H} l_1 - p(D_0 + D_1)) \varphi(p_1) dp_1 & \text{payoff with both projects}
\end{cases}$$

where $p_1(D_0^s; r_1)$ is the threshold success probability at which the firm is indifferent between taking on the next project and not taking it on.
Firm’s problem at time $t = 0$: derivatives

When $D^s_0 < \sigma_0 L_0 - (1 + r_1)l_1$,

$$\frac{\partial V}{\partial D^s_0} = \left( r^u_0 - D^u_0 \frac{\partial r^u_0}{\partial D^s_0} \right) \left( p_o + (1 - p_o) \int_{p_1(D^s_0; r_1)(D^s_0, r_1)}^1 p_1 \varphi(p_1) dp_1 \right)$$

probability not bankrupt

When $D^s_0 > \sigma_0 L_0 - (1 + r_1)l_1$,

$$\frac{\partial V}{\partial D^s_0} = \left( r^u_0 - D^u_0 \frac{\partial r^u_0}{\partial D^s_0} \right) \left( p_o + (1 - p_o) \int_{p_1(D^s_0; r_1)(D^s_0, r_1)}^1 p_1 \varphi(p_1) dp_1 \right)$$

savings on interest payments for $D^u_0$

$$- (1 - p_o) \int_{p_1(D^s_0; r_1)}^1 (1 - p_1) \varphi(p_1) dp_1$$

Cost from $t = 1$ unsecured debt

$$C = \left( 1 - (1 - p_o)(1 - p_1) \right) \cdot D^u_1 (r^u_1 - r_1) \quad \Rightarrow \quad \frac{\partial C}{\partial D^s_0} = (1 - p_o)(1 - p_1)$$

probability pay unsecured debt
Lender at time $t = 0$

If firm doesn’t take on second project, gets in expectation

$$RD_0 = p_o (1 + r_o^u) D_o^u + (1 - p_o) (\sigma_o L_o - D_o^s)$$

If lender takes on the second project with $p_1(D_o^s; r_1)$, lender gets in expectation

$$RD_{01} = (1 + r_o^u) D_o^u (1 - (1 - p_o)(1 - p_1(D_o^s; r_1)))$$

It follows then that

$$\text{sgn} \left( \frac{\partial V}{\partial D_o^s} \right) = \text{sgn} (RD_0 - RD_{01})$$
Shock to \( r_1 \): the realized interest rate is higher than expected.

- \( D^s_0 \) below \( \overline{D}^s_0 \): No additional unsecured debt \( D^u_1 = 0 \)

\[
\frac{\partial p_1 (D^s_0; r_1)}{\partial r_1} = \frac{l_1 (1 - (1 - p_0) (1 - p_1 (D^s_0; r_1)))}{\sigma_1, H l_1 - (1 - p_0) (l_1 (1 + r_1) + I_0 + D^u_0)},
\]

- Firm needs unsecured debt to finance project:

\[
\frac{\partial p_1 (D^s_0; r_1)}{\partial r_1} = \frac{l_1}{\sigma_1, H l_1 - (1 - p_0) (l_1 (1 + r_1) + I_0 + D^u_0)},
\]

- **Result:** Near the threshold:

\[
\left. \frac{\partial p_1 (D^s_0; r_1)}{\partial r_1} \right|_{(D^u_0 > 0)} > \left. \frac{\partial p_1 (D^s_0; r_1)}{\partial r_1} \right|_{(D^u_0 = 0)} > 0
\]
Shock to $r_1$, within the two regions

When $D^s_o < \overline{D}^s_o$: no difference in debt overhang

$$\frac{\partial^2 p_1(D^s_o; r_1)}{\partial r_1 \partial D^s_o} = - \frac{(1 - p_o) l_1}{D(D^s_o, D^s_1)^2} \left( r^u_o - \frac{D^u_o}{D^s_o} \frac{\partial r^u_o}{D^s_o} \right) (p_o + 2p_1(D^s_o; r_1)(1 - p_o))$$

When $D^s_o \geq \overline{D}^s_o$: higher $D^s_o$ means less debt overhang

$$\frac{\partial^2 p_1(D^s_o; r_1)}{\partial r_1 \partial D^s_o} = - \frac{(1 - p_o) l_1}{D(D^s_o, D^s_1)^2} \left( 1 + r^u_o - \frac{D^u_o}{D^s_o} \frac{\partial r^u_o}{D^s_o} \right)$$