Econometrica, Vol. 62, No. 6 (November, 1994), 1327–1347

TRADE WITH HETEROGENEOUS PRIOR BELIEFS AND ASYMMETRIC INFORMATION

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"No trade" theorems have shown that new information will not lead to trade when agents share the same prior beliefs. This paper explores the structure of no trade theorems with heterogeneous prior beliefs. It is shown how different notions of efficiency under asymmetric information—ex ante, interim, ex post—are related to agents' prior beliefs, as well as incentive compatible and public versions of those efficiency concepts. These efficiency results are used to characterize necessary and sufficient conditions on agents' beliefs for no trade theorems in different trading environments.

KEYWORDS: No trade, common knowledge, efficiency, heterogeneous prior beliefs.

1. INTRODUCTION

"NO TRADE" THEOREMS IN VARIOUS GUISES have established that when agents share the same prior beliefs, they will not trade for purely informational reasons, even in the presence of asymmetric information.[2] This paper explores the question of what differences in prior beliefs lead to trade. The answer to this question will depend on the particular trading environment. "No trade" theorems have been shown to hold in the context of common knowledge trade, incentive compatible trade, and rational expectations equilibria. Here we identify the different varieties of heterogeneous prior beliefs that lead to trade in these environments.

The following example will illustrate the crucial idea that the nature of the differences in prior beliefs required to generate trade depends on the trading environment. Art and Beth are risk-neutral and are considering making a bet on whether the discount rate will go up or down tomorrow. In the absence of private information, they agree on the probability, π, of a rise. But suppose Art has observed a signal correlated with the discount rate movement: if he observes signal u, he believes that the discount rate will rise with probability $a_u > \pi$; if he observes signal d, he believes that the discount rate will rise with probability $a_d < \pi$. Beth has not observed the signal, but if she had, her probabilities of an interest rate rise would be b_u and b_d, respectively.

[1] This paper is a revised and extended version of Chapter II of my dissertation at Yale University (Morris (1991)). I would like to thank my advisor, John Geanakoplos, and thesis committee members, Truman Bewley and David Pearce, for their invaluable advice and support. This version has benefited from comments from fellow students at Yale and seminar participants at Northwestern University, Boston University, Harvard Business School, the Kennedy School, the California Institute of Technology, and the Universities of Pennsylvania, Michigan, California at San Diego, California at Los Angeles, and Iowa. Hal Varian provided me with extremely valuable references. The presentation of the paper has been drastically improved by the detailed and extensive suggestions of a referee and co-editor of this journal. I gratefully acknowledge the financial support of an Anderson Prize Fellowship from the Cowles Foundation.

[2] Rubinstein (1975), Kreps (1977), Milgrom and Stokey (1982), Tirole (1982), and Sebenius and Geanakoplos (1983).

There are gains from trade if either $a_u \neq b_u$ or $a_d \neq b_d$: both agents could be made better off by making a bet which exploits the difference in their posterior beliefs. For example, suppose that $a_u = 2/3$, $a_d = 1/3$, $b_u = 3/4$, and $b_d = 1/4$. Let Art pay Beth \$2 whenever the signal is correct (the discount rises when the signal is u, or the discount rate falls when the signal is d); and let Beth pay Art \$5 if the signal is incorrect. Both agents would wish to accept this bet, whichever signal Art had observed.

But such a bet which exploits differences in beliefs need not be incentive compatible if it depends on private information. In the above numerical example, Art always has an incentive to misreport his signal: his expected gain from telling the truth is \$1/3, but his expected gain from lying is \$8/3. Indeed, there is no bet the agents could make which is incentive compatible.

Thus because of incentive compatibility considerations, heterogeneity of prior beliefs need not induce trade. More generally, the occurrence of trade will depend on the particular trading rules (for example, whether incentive compatibility is required) and the informational environment. We will require different restrictions on beliefs for different kinds of no trade results. We approach the problem of identifying such conditions indirectly, by interpreting no trade results as results about the efficiency of the underlying allocation. There are many different notions of efficiency (in environments with asymmetric information), giving rise to many different no trade results.

Recall that there are two sources of ambiguity in choosing a notion of efficiency. First, there is the timing of the welfare evaluation: ex ante, interim, or ex post? Moving from an ex ante efficiency notion to an interim efficiency notion will lead to a weakening of the necessary and sufficient condition for efficiency from the common prior assumption to the *consistency* requirement of Harsanyi (1967/68). In other words, the weaker notion of efficiency requires a weaker restriction on beliefs (an analogous result is derived for ex post efficiency). The second component of the efficiency notions is the class of reallocations which are considered feasible. It is shown that requiring only incentive efficiency, and efficiency with respect to publicly observed trades, each leads to further, qualitatively different, weakenings of the common prior assumption. Thus it is possible to characterize necessary and sufficient conditions for each notion of efficiency.

Necessary and sufficient conditions for no trade now follow. Incentive compatible trade is related to interim incentive efficiency. The existence of common knowledge trades is related to efficiency with respect to public trades. A slightly more subtle question is whether all rational expectations equilibria entail no trade: this depends on the endogenous revelation of information. However, (unconstrained) interim efficiency is sufficient for no trade in rational expectations equilibria while ex post efficiency with respect to public trades is necessary. Thus results relating efficiency, beliefs, and information are translated into necessary and sufficient conditions for different kinds of no trade theorems.

A number of authors have considered weaker assumptions than the common prior assumption in no trade results. Milgrom and Stokey (1982) noted that "concordant" beliefs—a property which arises in this paper—is a sufficient condition for ex ante efficiency, and thus no trade theorems. Hakansson, Kunkel, and Ohlson (1982) identified necessary and sufficient conditions on beliefs to preclude trade in a competitive setting where all information was public.[3]

The results of this paper parallel those of Geanakoplos (1989). Whereas this paper explored the implications for no trade results of allowing heterogeneous prior beliefs, Geanakoplos considered agents with a common prior whose information was not represented by partitions (and thus could be thought of as "boundedly rational") and identified different necessary and sufficient conditions on those nonpartitional information structures for different kinds of no trade results. Brandenburger, Dekel, and Geanakoplos (1992) shows that there is a formal equivalence between weakening the common prior assumption and weakening the standard information processing assumption. This equivalence can be used to show that this paper gives heterogeneous prior analogues of Geanakoplos' results.[4]

2. EX ANTE EFFICIENCY

In this section, we set up the framework for an exchange economy with asymmetric information and derive results relating conditions for ex ante efficiency to agents' beliefs.

There is a finite set of payoff-relevant states S. There are H agents, labelled 1 to H, and we write $\boldsymbol{H}$ for the set of agents. There are L commodities, and each agent has a concave, strictly increasing, twice differentiable utility function,[5] u_h: $\mathbb{R}^L \times S \to \mathbb{R}$. Each agent h observes a signal in some finite set of possible signals, $t_h \in T_h$. Write T for product of signal sets, $T = T_1 \times \cdots \times T_H$, with typical element $t = (t_1, \ldots, t_H)$.

All uncertainty is now reflected in the finite product space, $\Omega = T \times S$, with typical element (t, s). Agents' beliefs about Ω are a vector of H probability distributions over $T \times S$, i.e. $\pi = (\pi_1, \ldots, \pi_H)$, each $\pi_h \in \Delta(\Omega)$. We want to assume that, while agents may have different prior beliefs, they at least agree on which combinations of signals are possible.[6] Thus let $T^* \subset T$ be the set of possible signals. The following regularity conditions on arbitrary sets of beliefs, $\psi = (\psi_1, \ldots, \psi_H)$, will be important in what follows.

[3] Milgrom and Stokey (1982) identified the same necessary and sufficient condition in the case of public trade. See also Verrechia (1980, 1981). Results of Varian (1989) with private information in a competitive setting reduce to those of Hakansson et al. because all information is revealed in rational expectations equilibrium.

[4] Morris (1991, Chapter IV) explores the relation in detail.

[5] It is convenient to allow utility functions to be defined on negative commodity bundles, so the case of risk-neutral agents betting with each other is a special case. This betting case is treated fully in the Appendix and is used as a step in the proofs of the paper's results.

[6] This assumption is relaxed in Morris (1991, Chapter IV), and qualitatively similar results continue to hold. The main reason for making the assumption is the ambiguity in defining common knowledge without it.

DEFINITION 2.1: Beliefs ψ have *T^*-support* if, for all $h \in \boldsymbol{H}$,

$$\sum_{s \in S} \psi_h(t, s) > 0 \Leftrightarrow t \in T^*.$$

DEFINITION 2.2: Beliefs ψ have *full marginal support* (FMS) if, for all $h' \in \boldsymbol{H}$,

$$\sum_{t_{-h} \in T_{-h}} \sum_{s \in S} \psi_{h'}((t_h, t_{-h}), s) > 0, \quad \text{for all } t_h \in T_h,\ h \in \boldsymbol{H},$$

and

$$\sum_{t \in T} \psi_{h'}(t, s) > 0, \quad \text{for all } s \in S.$$

We assume agents' beliefs π have T^*-support and satisfy (FMS). An allocation e consists of a vector, $e = (e_1, \ldots, e_H)$, where each e_h is a mapping from states to commodity bundles, i.e. e_h: $\Omega \to \mathbb{R}^L$. We say an allocation is payoff-relevant if it depends only on payoff-relevant states S, and not on agents' private information T.

DEFINITION 2.3: An allocation e is *payoff-relevant* if

$$e_h(t, s) = e_h(t', s), \quad \text{for all } t, t' \in T,\ s \in S,\ h \in \boldsymbol{H}.$$

We will be interested in payoff-relevant allocations which are efficient, within the class of payoff-relevant allocations. This involves the more general notion of ex ante domination and feasible trades.

DEFINITION 2.4: Allocation y *ex ante dominates* e if

$$\sum_{t \in T} \sum_{s \in S} \pi_h(t, s) u_h[y_h(t, s), s] \geqslant \sum_{t \in T} \sum_{s \in S} \pi_h(t, s) u_h[e_h(t, s), s],$$

for all $h \in \boldsymbol{H}$, with strict inequality for some $h \in \boldsymbol{H}$.

DEFINITION 2.5: Allocation x is a *feasible trade* if

$$\sum_{h \in \boldsymbol{H}} x_h(t, s) \leqslant 0, \quad \text{for all } t \in T,\ s \in S.$$

DEFINITION 2.6: Allocation e is *initially efficient* if e is payoff-relevant and there does not exist a payoff-relevant feasible trade x such that $e + x$ ex ante dominates x.

Thus an allocation is initially efficient if it is constrained ex ante efficient with respect to payoff-relevant feasible trades. By the second welfare theorem, an initially efficient allocation is also a competitive equilibrium of the exchange economy with S only as the uncertainty space. An initially efficient allocation will also be an incomplete markets competitive equilibrium of the economy with the full uncertainty space before the arrival of any information, where only assets contingent on payoff-relevant states can be traded.

If agents' priors differ, initial efficiency will not necessarily imply ex ante efficiency, since there may be Pareto-improving trades which are not payoff-relevant.

DEFINITION 2.7: Allocation e is *ex ante efficient* if there does not exist a feasible trade x such that $e+x$ ex ante dominates e.

In an initially efficient allocation, there are no gains from making trades contingent on payoff-relevant states. So, in particular, differences in agents' prior beliefs about payoff-relevant states will not lead to gains from trade. Intuitively, beliefs are concordant if they agree about everything except the prior probability of payoff-relevant states.

DEFINITION 2.8: Beliefs ψ are *concordant* if they satisfy (FMS) and

$$\psi_h(t|s) = \psi_{h'}(t|s), \quad \text{for all } t \in T,\ s \in S,\ h, h' \in \boldsymbol{H}.$$

THEOREM 2.1: *An initially efficient allocation is ex ante efficient if and only if beliefs are concordant.*

Theorem 2.1, and all the theorems which follow, are proved by reducing a convex programming problem to a linear algebraic problem, where risk neutral individuals are betting with each other. It is natural to present the theorems in an order different to that in which they are proved, so Theorem 2.1, and all other theorems in the paper, are proved in an Appendix.

DEFINITION 2.9: Allocation e is *incentive compatible* if

$$\sum_{t_{-h} \in T_{-h}} \sum_{s \in S} \pi_h(t,s) u_h[e_h(t,s), s]$$
$$\geqslant \sum_{t_{-h} \in T_{-h}} \sum_{s \in S} \pi_h(t,s) u_h[e_h((t'_h, t_{-h}), s), s],$$
$$\text{for all } t_h, t'_h \in T_h,\ h \in \boldsymbol{H}.$$

DEFINITION 2.10: Allocation e is *ex ante incentive efficient* if e is incentive compatible and there does not exist a feasible trade x such that $e+x$ is incentive compatible and ex ante dominates e.

Notice that if e is initially efficient, it is certainly incentive compatible. So an initially efficient allocation will be ex ante incentive efficient, even when beliefs are not concordant, if any feasible trade x such that $e+x$ ex ante dominates e has the property that it gives at least one type of some agent an incentive to misreport his signal. Intuitively, as the example in the introduction showed, this will occur only if agents take their own signals less seriously than other agents do. This intuition is captured by the following formal definition.

DEFINITION 2.11: Beliefs ψ are a *noisy* version of beliefs θ if, for each h,

$$\psi_h(t,s) = \alpha_h(t_h)\theta_h(t,s) + \sum_{t'_h \in T_h} \beta_h(t'_h, t_h)\psi_h((t'_h, t_{-h}), s)$$

for some $\alpha_h\colon T_h \to (0,1]$, $\beta_h\colon T_h^2 \to \mathbb{R}_+$, for all $t \in T$, $s \in S$, $h \in \boldsymbol{H}$.

This condition requires that, if agent h's beliefs are ψ_h, his posterior beliefs, conditional on observing signal t_h, are a weighted average of what they would have been under θ_h and what they could have been under ψ_h if he had observed a different signal. Thus under ψ_h, it is as if h has observed a noisy version of his own signal under θ_h. A property closely related to noisiness is investigated in more detail in Morris and Shin (1993). We say that beliefs π are noisy concordant if they are a noisy version of concordant beliefs.

THEOREM 2.2: *An initially efficient allocation is ex ante incentive efficient if and only if beliefs are noisy concordant.*

The linear algebraic structure of the noisy beliefs condition is similar to conditions arising in work on linear trading problems (see Myerson (1991, Chapter 10)). This is because the convex problem reduces, under the assumption of initial efficiency, to a linear problem. In our case, the resulting linear conditions on payoffs can be interpreted in terms of agents' beliefs.

We will need a third notion of ex ante efficiency. Is it the case that a re-allocation of resources depending only on public events could make everybody better off in ex ante terms? Public events will be defined in terms of agents' knowledge. Since the relation between knowledge, common knowledge, and public events has been mostly studied in a *partition* representation of private information, we will briefly review the relation in this cross-product representation.

DEFINITION 2.12: Agent h *knows* event $E \subset T$ at signal profile t if all signal profiles he considers possible at t are contained in E. Thus the set of states where h knows event E is defined by:

$$K_h(E) = \{t \in T \mid (t_h, t'_{-h}) \in T^* \Rightarrow (t_h, t'_{-h}) \in E, \quad \text{for all } t'_{-h} \in T_{-h},\ h \in \boldsymbol{H}\}.$$

An event is said to be common knowledge if any statement of the form "h_1 knows that h_2 knows... that h_n knows E" is always true.

DEFINITION 2.13: An event $E \subset T$ is *common knowledge* at signal profile t if, for all integers $n \geqslant 1$, and $f: \{1, \ldots, n\} \to \boldsymbol{H}$,

$$t \in K_{f(1)}\big(K_{f(2)} \ldots \big(K_{f(n)}(E)\big) \ldots\big).$$

DEFINITION 2.14: An event E is *public*[7] if it is known by all agents whenever it is true, i.e. $E \subset K_h E$, for all h.

Which events are public is implicit in the information structure we have written down, since public events are defined as events which, whenever they occur, are known to have occurred. Thus if all such profiles are possible ($T^* = T$), then T is the only public event. It is a consequence of the following well-known lemma (which is implicit in Aumann's (1976) paper) that an event is public if and only if it is common knowledge whenever it occurs.

LEMMA 2.1: *An event $E \subset T$ is common knowledge*[8] *at t if and only if there exists a public event F such that $t \in F \subset E$.*

An allocation is public if it depends only on payoff-relevant states and public events. The easiest way to define this formally is to consider, for each signal profile t, the smallest public event containing t, i.e. write $P(t)$ for the union of the collection, $\{E | t \in E$ and E is public$\}$. Now we have the following definition.

DEFINITION 2.15: An allocation e is *public* if it is measurable with respect to public events, so that

$$x_h(t, s) = x_h(t', s) \quad \text{for all } s \in S,\ t' \in P(t).$$

Notice that, as we would expect, if an allocation is public, it is certainly incentive compatible. The following definition of efficiency with respect to public re-allocations is thus strictly weaker than incentive efficiency. It is essentially equivalent to Wilson's (1978) definition of "coarse efficiency" in a partition setting.

DEFINITION 2.16: Allocation e is *ex ante public efficient* if e is public and there does not exist a feasible trade x such that $e + x$ is public and ex ante dominates e.

Thus e is public efficient if it is efficient with respect to trades depending only on public events. In this case, only differences in agents' beliefs about public events will lead to improving trades.

[7] Events with this property have been variously described as *public* (Milgrom (1981)), *self-evident* (Samet (1990)), or *common truisms* (Binmore and Brandenburger (1990)).

[8] Notice that because some signal profiles t are assigned zero probability, the definition of knowledge used here fails to satisfy the usual property that everything known is true, i.e. $K_h(E) \subset E$, for all agents h and events $E \subset T$. A proof of the lemma for this case appears in Brandenburger and Dekel (1987). Morris (1991, Chapter IV) includes a proof specialized to this finite state space case.

DEFINITION 2.17: Beliefs ψ are a *public* version of beliefs θ if, for each h,

$$\sum_{t \in E} \psi_h(t,s) = \sum_{t \in E} \theta_h(t,s) \quad \text{for all } s \in S \text{ and public events } E.$$

Beliefs are said to be public concordant if they are a public version of concordant beliefs.

THEOREM 2.3: *An initially efficient allocation e is ex ante public efficient if and only if beliefs are public concordant.*

Suppose now that agents were in an initially efficient allocation, and were then informed that they would be observing some signals in the future, but were given the opportunity to trade before observing the signal. Then the ex ante efficiency notions in this section would be pertinent in determining whether they would agree to trade. But the most natural setting to consider the question of when information leads to trade is after agents have observed their information. In this case, interim efficiency notions will be relevant. These are studied in the next section.

3. INTERIM EFFICIENCY AND NO TRADE THEOREMS

In this section, we provide interim analogues to the ex ante efficiency results of the previous section, and relate them to no trade theorems. First we define interim efficiency concepts.

DEFINITION 3.1: *y interim dominates e* if

$$\sum_{t_{-h} \in T_{-h}} \sum_{s \in S} \pi_h(t,s) u_h[y_h(t,s),s] \geqslant \sum_{t_{-h} \in T_{-h}} \sum_{s \in S} \pi_h(t,s) u_h[e_h(t,s),s],$$

for all $t_h \in T_h$, $h \in \boldsymbol{H}$, with strict inequality for some $t_h \in T_h$, $h \in \boldsymbol{H}$.

DEFINITION 3.2: Allocation e is *interim* (*incentive/public*) *efficient* if (e is incentive compatible/public and) there does not exist a feasible trade x such that $e+x$ interim dominates e (and $e+x$ is incentive compatible/public).

Replacing ex ante by interim notions of efficiency will lead to a further kind of weakening of the common prior assumption. Harsanyi (1967/68) said that agents' beliefs were *consistent* if their posterior beliefs could have been derived from the same prior beliefs. For any interim decision problem, only posterior beliefs matter, so clearly any result which is true under the common prior assumption must also be true with consistent prior beliefs. The standard notion of consistency assumes that all information signals are possible (i.e., the full marginal support condition holds), so that posterior beliefs are always well-defined. In what follows, we will also make use of a weaker notion called *weak consistency*, which requires only that prior beliefs assign positive probability to all nonempty public events and that posterior beliefs could have been derived from a common prior whenever they are well-defined (i.e. signals are assigned positive probability).

DEFINITION 3.3: Beliefs ψ have *public support*, if, for all $h \in \boldsymbol{H}$,

$$\sum_{t \in E} \sum_{s \in S} \psi_h(t, s) > 0, \quad \text{for all nonempty public events } E.$$

DEFINITION 3.4: Beliefs ψ are a *weakly consistent* version of θ if ψ and θ have public support and, for each h,

$$\sum_{t_{-h} \in T_{-h}} \sum_{s \in S} \psi_h((t_h, t_{-h}), s) > 0 \quad \text{and} \quad \sum_{t_{-h} \in T_{-h}} \sum_{s \in S} \theta_h((t_h, t_{-h}), s) > 0$$
$$\Rightarrow \psi_h[t_{-h}, s | t_h] = \theta_h[t_{-h}, s | t_h], \quad \text{for all } t_{-h} \in T_{-h},\ s \in S.$$

DEFINITION 3.5: Beliefs ψ are a *consistent* version of θ if ψ and θ satisfy (FMS) and ψ are a weakly consistent version of θ.

Thus beliefs ψ are a consistent version of θ if ψ_h and θ_h imply the same posterior probabilities for agent h after observing his signal. Say that beliefs θ are *common* if $\theta_h = \theta_{h'}$, for all $h, h' \in \boldsymbol{H}$. Beliefs ψ are a consistent version of some common beliefs θ if and only if they satisfy Harsanyi's notion of consistency. Beliefs are said to be noisy consistent concordant if they are a noisy version of beliefs which are a consistent version of beliefs which are concordant.

THEOREM 3.1: *An initially efficient allocation e is interim (incentive/public) efficient if beliefs are (noisy/public) consistent concordant. If initially efficient allocation e is interim (incentive/public) efficient, then beliefs are (noisy/public) weakly consistent concordant.*

Thus there is a small wedge between the necessary and sufficient conditions of the theorem. The proof in the Appendix makes clear that if utility functions are linear (and thus agents are risk-neutral), the sufficient condition for efficiency is also necessary in each case. On the other hand, if utility functions are strictly concave, the necessary condition is also sufficient.

We want to interpret the efficiency results in Theorem 3.1 in terms of no trade results. When would agents in an initially efficient allocation be prepared to trade with each other after the arrival of information? Say that there is incentive compatible trade if there exists a feasible trade, contingent on payoff-relevant states and messages agents report, such that all agents are prepared to accept trade contingent on some signal profile, and some agent is made strictly better off? By the revelation principle, we can restrict attention to truth telling mechanisms. Holmström and Myerson (1983) emphasized that in general interim incentive efficiency does not imply that there do not exist incentive compatible trades. Nor is the converse (in general) true. But because initially efficient allocations are by definition incentive compatible, there is incentive compatible trade from such allocations if and only if they are interim incentive efficient.

COROLLARY 3.1: *A necessary condition for incentive compatible trade from an initially efficient allocation is that beliefs are not noisy consistent concordant. A sufficient condition is that beliefs are not noisy weakly consistent concordant.*

A formal statement of this result was given in a previous version of this paper (Morris (1992)). Incentive compatible trades depend on agents' signals as well as payoff-relevant states. Suppose agents were restricted to make payoff-relevant trades and trade took place publicly, so that trade could not take place unless, at some signal profile, it was common knowledge that the trade was interim Pareto improving and some agent strictly benefited. Clearly, if an allocation is not interim public efficient, there exists such common knowledge trade. On the other hand, if there exists such common knowledge trade, then there exists a public event where that trade is interim Pareto-improving. Then a new trade which is equal to the original trade on that public event, and equal to zero elsewhere, interim dominates no trade, so the initial allocation is not interim public efficient. Thus we have the following corollary.

COROLLARY 3.2: *A necessary condition for common knowledge trade from an initially efficient allocation is that beliefs are not public consistent concordant. A sufficient condition is that beliefs are not public weakly consistent concordant.*

It is important to note that the information structure is taken as given in the statement of Corollary 3.2. Any realistic trading mechanism will have some dynamic component and thus some potential for information to be revealed during the course of the trading process. It is possible to restate both Corollary 3.2 and other no trade theorems in this paper in terms of the final information structure at the time that trade takes place.

4. EX POST EFFICIENCY AND RATIONAL EXPECTATIONS EQUILIBRIA

Generically, all private information is revealed in rational expectations equilibria (Radner (1979), Grossman (1981)). Thus ex post efficiency concepts will be relevant in evaluating whether initially efficient allocations are supportable as rational expectations equilibria after the arrival of information. This section develops the ex post analogues of earlier unconstrained and public efficiency results and uses them to study the effect of new information on competitive equilibria. (Ex post incentive efficiency is not meaningful and is omitted.)

DEFINITION 4.1: *y ex post dominates e* if

$$\sum_{s \in S} \pi_h(t,s) u_h[y_h(t,s),s] \geqslant \sum_{s \in S} \pi_h(t,s) u_h[e_h(t,s),s],$$

for all $t \in T$, $h \in \boldsymbol{H}$, with strict inequality for some $t \in T$, $h \in \boldsymbol{H}$.

DEFINITION 4.2: Allocation e is *ex post (public) efficient* if (e is public) and there does not exist a feasible trade x such that ($e+x$ is public) and ex post dominates e.

If all private information was revealed, agents' prior beliefs about signal profiles would be irrelevant. Only beliefs conditional on fully revealed private information would matter and could lead to trade. This is captured by the following, final, weakening of the common prior assumption.

DEFINITION 4.3: Beliefs ψ are a *revelation consistent* version of θ if ψ and θ satisfy (FMS) and, for each h,

$$\psi_h(s|t) = \theta_h(s|t), \quad \text{for all } s \in S,\ t \in T.$$

THEOREM 4.1: *An initially efficient allocation is ex post (public) efficient if and only if beliefs are (public) revelation consistent concordant.*

It was noted in Section 2 that, for any initially efficient allocation, there exist prices that constitute a competitive equilibrium of the economy with state space S (and without signals T). Suppose the economy was initially in such a competitive equilibrium, and new information arrived. A standard competitive solution concept after the arrival of private information would be rational expectations equilibrium, where agents are price takers and may learn about other agents' information via prices in equilibrium.[9]

DEFINITION 4.4: An allocation e is a *rational expectations equilibrium* [REE] if there exists a price vector q: $T \times S \to \mathbb{R}_+^L$ such that:

(1) $q(t',s) = q(t,s)$ for all $s \in S$, and $t'_h = t_h \Rightarrow e_h(t',s) = e_h(t,s)$ for all $s \in S$.

(2) There does not exist allocation x such that

(i) $q(t',s) = q(t,s)$ for all $s \in S$, and $t'_h = t_h \Rightarrow x_h(t',s) = x_h(t,s)$ for all $s \in S$.

(ii) $\sum_{s \in S} q(t,s) x_h(t,s) \leqslant 0$, for all $t \in T$, $h \in \boldsymbol{H}$.

(iii) $e+x$ ex ante dominates e.

[9] The notion of rational expectations equilibrium is problematic in the context. In a finite agent model, the price taking assumption is always problematic. As a co-editor of this journal pointed out, it is especially problematic in the context of asymmetric information environments where strategic information revelation becomes an issue and large number arguments do not necessarily help (see Gul and Postlewaite (1992)). But the purpose of this paper is to consider standard no trade results under heterogeneous prior beliefs, not to improve on existing no trade results.

Which efficiency notion is relevant for rational expectations equilibria? Clearly interim efficiency will be sufficient for an initially efficient allocation to remain a REE after the revelation of information. But interim efficiency is not necessary for two reasons. Revelation of information means that interim gains from trade need not be exploited. For example, conditional on full revelation of information, ex post efficiency is sufficient for an initially efficient allocation to be a REE. On the other hand, trades are restricted to depend on an agent's equilibrium private information, which may *not* be fully revealed. This means that even incentive compatible trades may not be informationally feasible. However, public trades are always informationally feasible. Thus any initially efficient allocation which is a REE must be ex post public efficient.[10]

COROLLARY 4.1: *An initially efficient allocation is a rational expectations equilibrium if beliefs are consistent concordant. If an initially efficient allocation is a rational expectations equilibrium, beliefs are public revelation consistent concordant.*

A couple of examples will illustrate why consistent concordant beliefs (and thus interim efficiency) are not a necessary condition. Consider first the example in the introduction. There were two risk-neutral agents, both of whom assigned probability π to a discount rate rise tomorrow. One agent Art has observed a signal and believes that the likelihood of a discount rate rise is $a_u > \pi$, if the signal was u, and $a_d < \pi$, if the signal was d. The posterior beliefs of the uninformed agent, Beth, would have been b_u and b_d, respectively. Suppose now that $a_u = a_d$, so that Art in effect ignores his information. There is a rational expectations equilibrium in this situation where the relative price of a dollar if the discount rate rises, relative to the price of a dollar does not rise, is $\pi/(1-\pi)$. Both Art and Beth do not want to trade at such prices. But since we placed no restrictions on b_u and b_d, this situation need not be interim incentive efficient.

In that example, the initial environment is interim public efficient, but this need not be true in general. Consider a situation with risk neutral agents where, conditional on all private information, all agents agree on the probability of each payoff-relevant state (so that beliefs are revelation consistent). Clearly there are no ex post gains from trade. But suppose that these common conditional beliefs differ across all signal profiles. Then clearly there exists a rational expectations equilibrium, with full revelation of private information. But agents could have sufficiently different beliefs about the ex ante likelihood of different signal profiles to ensure that the initial situation is not interim public efficient.

[10] Laffont (1985) gives a general analysis of the efficiency properties of rational expectations equilibria (i.e., including those that are not initially efficient).

5. CONCLUSION

Despite the subtle and powerful implications of no trade theorems, both academic and lay analysts continue to believe that much observed trading volume in financial and other markets is driven by the arrival of new information. Thus Ross (1989) has written that

> "It is difficult to imagine that the volume of trade in security markets has very much to do with the modest amount of trading required to accomplish the continual and gradual portfolio balancing inherent in our current intertemporal models. It seems clear that the only way to explain the volume of trade is with a model that is at one and the same time appealingly rational and yet permits divergent and changing opinions in a fashion that is other than ad hoc."

Assorted ad hoc ploys to get around no trade theorems—such as assuming unmodelled "noise traders"[11]—have been used in the literature. This paper is motivated by the conviction that to understand the role of information in trade, it is necessary to explicitly address the most important source of trade in response to new information, which is differences in prior beliefs leading to different interpretations of the new information.

The paper illustrates that it is not the case (as is often claimed) that "anything can happen" if the common prior assumption is relaxed.[12] Let us conclude by summarizing intuitively some of the results of the paper about what kind of differences in beliefs do *not* lead to trade. Differences in prior beliefs of observing one's own signal will not lead to trade. If it is possible to make trades contingent on some event prior to the arrival of information, then differences in prior beliefs about that event will not lead to trade. If trade is to be incentive compatible, differences of beliefs where agents undervalue their own signal will not lead to trade. If trade is to be common knowledge, differences in beliefs about events which are not publicly revealed will not lead to trade.

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Manuscript received May, 1992; final revision received February, 1994.

APPENDIX

LEMMA A1 (Farkas' Lemma): *Suppose I and J are finite sets and S is a subset of I (i.e. $S \subset I$). Then there exist $\{x_j\}_{j \in J}$ solving*

$$\sum_{j \in J} a_{ij} x_j \geqslant 0, \quad \text{for all } i \in I, \text{ strict for all } i \in S,$$

if and only if there does not exist $\lambda = \{\lambda_i\}_{i \in I} \geqslant 0$ such that (i) $\sum_{i \in I} \lambda_i a_{ij} = 0$, for all $j \in J$, and (ii) $\sum_{i \in S} \lambda_i > 0$.

[11] See Black (1986).

[12] See Bernheim (1986) and Aumann (1987) for discussions of the common prior assumption in economic theory. Chapter V of Morris (1991) presents a general view of why and how the implications of heterogeneous prior beliefs can be usefully studied.

PROOF: A standard argument (see, for example, Gale (1960, p. 44)).

COROLLARY A1: *Suppose I and J are finite sets and C is a collection of subsets of I (i.e. C is a subset of the power set of I, $C \subset 2^I$). Then there exist $\{x_j\}_{j \in I}$ solving*

$$\sum_{j \in J} a_{ij} x_j \geqslant 0, \quad \text{for all } i \in I, \text{ strict for all } i \in S, \text{ for some } S \in C$$

if and only if there does not exist $\lambda = \{\lambda_i\}_{i \in I} \geqslant 0$ such that (i) $\sum_{i \in I} \lambda_i a_{ij} = 0$, for all $j \in J$, and (ii) $\sum_{i \in S} \lambda_i > 0$, for all $S \in C$.

PROOF: Suppose there exist such x. Then nonexistence of such λ follows immediately from Lemma A1. On the other hand, suppose there do not exist such x. Then, by Lemma A1, for each $S \in C$, there exists $\lambda(S)$ satisfying (i) and $\lambda_i(S) > 0$ for some $i \in S$. Now $\lambda = \sum_{S \in C} \lambda(S)$ satisfies (i) and (ii) above. *Q.E.D.*

The Betting Case

The proofs will make extensive use of the *betting case*, when there is only one good $[L = 1]$ and each agent is risk neutral, i.e.

$$u_h(x, s) = x, \quad \text{for all } x \in \mathbb{R}, s \in S.$$

In the betting case, endowments do not matter and dominance, incentive compatibility and feasibility constraints simplify to the following linear constraints:

[EAD] $e + x$ ex ante dominates e if

$$\sum_{t \in T} \sum_{s \in S} \pi_h(t, s) x_h(t, s) \geqslant 0, \quad \text{for all } h \in \boldsymbol{H}, \text{with strict inequality for some } h \in \boldsymbol{H}.$$

[ID] $e + x$ interim dominates e if

$$\sum_{t_{-h} \in T_{-h}} \sum_{s \in S} \pi_h(t, s) x_h(t, s) \geqslant 0, \quad \text{for all } t_h \in T_k, h \in \boldsymbol{H},$$

$$\text{with strict inequality for some } t_k \in T_h, h \in \boldsymbol{H}.$$

[EPD] $e + x$ ex post dominates e if

$$\sum_{s \in S} \pi_h(t, s) x_h(t, s) \geqslant 0, \quad \text{for all } t \in T, h \in \boldsymbol{H},$$

$$\text{with strict inequality for some } t \in T, h \in \boldsymbol{H}.$$

[IC] $e + x$ is incentive compatible if

$$\sum_{t_{-h} \in T_{-h}} \sum_{s \in S} \pi_h(t, s) x_h(t, s) \geqslant \sum_{t_{-h} \in T_{-h}} \sum_{s \in S} \pi_h(t, s) x_h((t'_h, t_{-h}), s),$$

$$\text{for all } t_h, t'_h \in T_h, h \in \boldsymbol{H}.$$

[P] If e is public, $e + x$ is public if

$$x_h(t, s) = x_h(t', s) \quad \text{for all } s \in S, t' \in P(t).$$

[F] x is a trade if

$$\sum_{h \in \boldsymbol{H}} x_h(t, s) \leqslant 0, \quad \text{for all } t \in T, s \in S.$$

DEFINITION A1: Beliefs ψ are *common* if

$$\psi_h(t, s) = \psi_{h'}(t, s), \quad \text{for all } t \in T, s \in S, h, h' \in \boldsymbol{H}.$$

APPENDIX

LEMMA A2 (Betting Case Efficiency): *The following is true of any allocation e in the betting case:*

(i) *e is ex ante efficient if and only if beliefs are common.*
(ii) *e is ex ante incentive efficient if and only if beliefs are a noisy version of some common beliefs.*
(iii) *e is ex ante public efficient if and only if beliefs are a public version of some common beliefs.*
(iv) *e is interim efficient if and only if beliefs are consistent.*
(v) *e is interim incentive efficient if and only if beliefs are a noisy version of some consistent beliefs.*
(vi) *e is interim public efficient if and only if beliefs are a public version of some consistent beliefs.*
(vii) *e is ex post efficient if and only if beliefs are revelation consistent.*
(viii) *e is ex post incentive efficient if and only if beliefs are a noisy version of some revelation consistent beliefs.*
(ix) *e is ex post public efficient if and only if beliefs are a public version of some revelation consistent beliefs.*

Notice that, since endowments do not matter in the betting case, the above results are independent of whether e is initially efficient or not.

PROOF: (i) Does there exist trade x satisfying [EAD] and [F]? By Corollary A1, if and only if there do not exist, for each h, $c_h \in \mathbb{R}_{++}$ and $k\colon T \times S \to \mathbb{R}_+$ such that

$$c_h \pi_h(t,s) = k(t,s), \quad \text{for all } t \in T,\ s \in S,\ h \in \boldsymbol{H}.$$

Now let

$$k^* = \sum_{t \in T} \sum_{s \in S} k(t,s).$$

Thus we must have $c_h = k^*$, for each h. Now define

$$\psi(t,s) = \frac{k(t,s)}{k^*}, \quad \text{for all } t \in T,\ s \in S.$$

Thus there exists prior ψ such that

$$\pi_h(t,s) = \psi(t,s), \quad \text{for all } t \in T,\ s \in S,\ h \in \boldsymbol{H}.$$

(ii) Does there exist trade x satisfying [EAD], [IC], and [F]? By Corollary A1, if and only if there do not exist, for each h, $c_h \in \mathbb{R}_{++}$ and $d_h\colon T_h^2 \to \mathbb{R}_+$, and $k\colon T \times S \to \mathbb{R}_+$ such that

$$c_h \pi_h(t,s) + \sum_{t'_h \in T_h} \{ d_h(t_h, t'_h) \pi_h(t,s) - d_h(t'_h, t_h) \pi_h((t'_h, t_{-h}), s) \} = k(t,s), \tag{A.1}$$
$$\text{for all } t \in T,\ s \in S,\ h \in \boldsymbol{H}.$$

We want to show that this is equivalent to the requirement beliefs are a noisy version of some common beliefs, i.e.

$$\pi_h(t,s) = \alpha_h(t_h) \psi(t,s) + \sum_{t'_h \in T_h} \beta_h(t'_h, t_h) \pi_h((t'_h, t_{-h}), s), \quad \text{for all } t \in T,\ s \in S,\ h \in \boldsymbol{H} \tag{A.2}$$
$$\text{for some } \alpha_h\colon T_h \to (0,1],\ \beta_h\colon T_h^2 \to \mathbb{R}_+,\ \psi \in \Delta(\Omega), \text{ for all } t \in T,\ s \in S,\ h \in \boldsymbol{H}.$$

Making the following substitutions in equation (A.1) and re-arranging gives (A.2):

$$k^* = \sum_{t \in T} \sum_{s \in S} k(t,s), \quad \psi(t,s) = \frac{k(t,s)}{k^*}, \quad \text{for all } t \in T,\ s \in S.$$

$$\alpha_h(t_h) = \frac{k^*}{k^* + \sum_{t'_h \in T_h} d_h(t_h, t'_h)}, \quad \text{for all } t_h \in T_h,$$

and

$$\beta_k(t'_h, t_h) = \frac{d_h(t'_h, t_h)}{k^* + \sum_{t'_h \in T_h} d_h(t_h, t'_h)}, \quad \text{for all } t_h, t'_h \in T_h.$$

Making the following substitution in equation (A.2) and re-arranging gives (A.1):

$$d_h(t'_h, t_h) = \frac{\beta_h(t'_h, t_h)}{\alpha_h(t_h)}.$$

(iii) Does there exist trade x satisfying [EAD], [P], and [F]? By Corollary A1, if and only if there do not exist, for each h, $c_h \in \mathbb{R}_{++}$ and k: $T \times S \to \mathbb{R}_+$ such that

$$\sum_{t \in E} c_h \pi_h(t,s) = \sum_{t \in E} k(t,s), \quad \text{for all public events } E,\ s \in S,\ h \in \boldsymbol{H}.$$

Normalizing k as before, there must exist a prior ψ such that

$$\sum_{t \in E} \pi_h(t,s) = \sum_{t \in T} \psi(t,s), \quad \text{for all public events } E,\ s \in S,\ h \in \boldsymbol{H}.$$

(iv) Does there exist trade x satisfying [ID] and [F]? By Corollary A1, if and only if there do not exist, for each h, c_h: $T_h \to \mathbb{R}_{++}$ and k: $T \times S \to \mathbb{R}_+$ such that

$$c_h(t_h)\pi_h(t,s) = k(t,s), \quad \text{for all } t \in T,\ s \in S,\ h \in \boldsymbol{H}.$$

Now let

$$k^* = \sum_{t \in T} \sum_{s \in S} k(t,s) = \sum_{t \in T} \sum_{s \in S} c_h(t_h)\pi_h(t,s), \quad \text{for all } h \in \boldsymbol{H}.$$

Now define

$$\pi'_k(t,s) = \frac{c_h(t_h)\pi_h(t,s)}{k'}, \qquad \psi(t,s) = \frac{k(t,s)}{k^*}, \quad \text{for all } t \in T,\ s \in S.$$

Thus beliefs π are a consistent version of π' and there exists prior ψ such that

$$\pi'_h(t,s) = \psi(t,s), \quad \text{for all } t \in T,\ s \in S,\ h \in \boldsymbol{H}.$$

(vii) Does there exist trade x satisfying [EPD] and [F]? By Corollary A1, if and only if there do not exist, for each h, c_h: $T \to \mathbb{R}_{++}$ and k: $T \times S \to \mathbb{R}_+$ such that

$$c_h(t)\pi_h(t,s) = k(t,s), \quad \text{for all } t \in T,\ s \in S,\ h \in \boldsymbol{H}.$$

Now let

$$k^* = \sum_{t \in T} \sum_{s \in S} k(t,s) = \sum_{t \in T} \sum_{s \in S} c_h(t)\pi_h(t,s), \quad \text{for all } h \in \boldsymbol{H}.$$

Now define

$$\pi'_h(t,s) = \frac{c_h(t)\pi_h(t,s)}{k^*}, \quad \psi(t,s) = \frac{k(t,s)}{k^*}, \quad \text{for all } t \in T,\ s \in S.$$

Thus beliefs π are a revelation consistent version of π' and there exists prior ψ such that

$$\pi'_h(t,s) = \psi(t,s), \quad \text{for all } t \in T,\ s \in S,\ h \in \boldsymbol{H}.$$

Proofs of (v), (vi), (viii), and (ix) are natural extensions. The converse of each argument is immediate. *Q.E.D.*

To relate the general case to the betting case, some stricter conditions will be required.

DEFINITION A2: *y strongly ex ante dominates e* if

$$\sum_{t \in T} \sum_{s \in S} \pi_h(t,s) u_h[y_h(t,s), s] > \sum_{t \in T} \sum_{s \in S} \pi_h(t,s) u_h[e_h(t,s), s], \quad \text{for all } h \in \boldsymbol{H}.$$

All constraints are now required to hold with strict inequality.

DEFINITION A3: *y strongly interim dominates e* if

$$\sum_{t_{-h}\in T_{-h}} \sum_{s\in S} \pi_h(t,s)u_h[y_h(t,s),s] \geqslant \sum_{t_{-h}\in T_{-h}} \sum_{s\in S} \pi_h(t,s)u_h[e_h(t,s),s],$$

$$\text{for all } t_h \in T_h,\ h \in \boldsymbol{H},$$

with strict inequality for all $h \in \boldsymbol{H}$, t_h s.t. $(t_h, t_{-h}) \in E$ for some $t_{-h} \in T_{-h}$, for some nonempty public event E.

All constraints are now required to hold with strict inequality on some public event.

DEFINITION A4: *y strongly ex post dominates e* if

$$\sum_{s\in S} \pi_h(t,s)u_h[y_h(t,s),s] \geqslant \sum_{s\in S} \pi_h(t,s)u_h[e_h(t,s),s], \quad \text{for all } t \in T,\ h \in \boldsymbol{H},$$

with strict inequality for all $h \in \boldsymbol{H}$, for some $t \in T$.

All constraints are now required to hold with strict inequality on some signal profile.

DEFINITION A5: *e is strongly incentive compatible* if

$$\sum_{t_{-h}\in T_{-h}} \sum_{s\in S} \pi_h(t,s)u_h[e_h(t,s),s] \geqslant \sum_{t_{-h}\in T_{-h}} \sum_{s\in S} \pi_h(t,s)u_h[e_h((t'_h,t_{-h}),s),s],$$

$$\text{for all } t_h, t'_h \in T_h,\ h \in \boldsymbol{H},$$

with strict inequality for all $t_h, t'_h \in T_h$ such that $\pi_h(t_h|t_{-h},s) \neq \pi_h(t'_h|t_{-h},s)$, for some $t_{-h} \in T_{-h}$, $s \in S$.

Strong incentive compatibility requires that incentive compatibility constraints hold strictly unless posterior beliefs are the same given two signals, so that the constraint can never hold strictly.

DEFINITION A6: *e* is *strongly interim* (*incentive/public*) *efficient* if *e* is (incentive compatible/public) and there does not exist a trade x such that $e+x$ is (strongly incentive compatible/public) and strongly interim dominates e.

LEMMA A3: *In the betting case, an allocation is strongly interim* (*incentive/public*) *efficient if and only if beliefs are* (*noisy/public*) *weakly consistent*.

PROOF: First, suppose strong incentive compatibility was not required in the definition of strong incentive efficiency. The argument then goes through as for Lemma A2, but now instead of each $c_h(t_h)$ strictly positive, we now require only each $c_h(t_h)$ nonnegative and

$$\sum_{t\in E} c_h(t_h)\pi_h(t,s) > 0, \quad \text{for all public events } E,\ h \in \boldsymbol{H}.$$

This leads to the replacement of consistency by weak consistency.

Now suppose that there does exist an incentive compatible trade which strongly interim dominates 0. Then, by Corollary A1, there exists a public event E such that, if c_h: $T_h \to \mathbb{R}_+$ and d_h: $T_h^2 \to \mathbb{R}_+$ and k: $T \times S \to \mathbb{R}_+$ satisfy

$$c_h(t_h)\pi_h(t,s) + \sum_{t'_h\in T_h} d_h(t_h,t'_h)\pi_h(t,s) - d_h(t'_h,t_h)\pi_h((t'_h,t_{-h}),s) = k(t,s),$$

$$\text{for all } t \in T,\ s \in S,\ h \in \boldsymbol{H}.$$

Then

$$\sum_{t\in E} c_h(t_h)\pi_h(t,s) = \sum_{t\in E} k(t,s) = 0, \quad \text{for all } s \in S.$$

Suppose also that there does *not* exist a *strongly* incentive compatible trade x that strongly interim dominates 0. Now, again by Corollary A1, there must exist, for each h, d_h: $T_h^2 \to \mathbb{R}_+$, with $d_h(t_h, t'_h) > 0$ for some h and t_h, t_h, (with $\pi_h(t_{-h}, s \mid t_h) \neq \pi_h(t_{-h}, s \mid t'_h)$ for some $t_{-h} \in T_{-h}$, $s \in S$), such that

$$\sum_{t'_h \in T_h} d_h(t_h, t'_h)\pi_h(t, s) = \sum_{t'_h \in T_h} d_h(t'_h, t_h)\pi_h((t'_h, t_{-h}), s), \quad \text{for all } t \in E,\ s \in S,\ h \in \boldsymbol{H}.$$

This leads to a contradiction. So if there exist an incentive compatible trade strongly interim dominating 0, then there exists a strongly incentive compatible trade strongly interim dominating 0. *Q.E.D.*

Definition A7: e is *strongly ex ante* (*incentive/public*) *efficient* if (e is incentive compatible/public and) there does not exist a trade x such that $e + x$ is (strongly incentive compatible/public) and strongly ex ante dominates e.

Lemma A4: *In the betting case, e is strongly ex ante* (*incentive/public*) *efficient if and only if e is ex ante* (*incentive/public*) *efficient.*

Proof: Similar argument to Lemma A3.

Definition A8: e is *strongly ex post* (*public*) *efficient* if (e is public and) there does not exist a (public) trade x such that $e + x$ strongly ex post dominates e.

Lemma A5: *In the betting case, e is strongly ex post* (*public*) *efficient if and only if e is ex post* (*public*) *efficient.*

Proof: Similar argument to Lemma A3.

Relating the Betting Case to the General Case

Consider again the general case with L goods and concave utility.

Lemma A6 (Conditions for Initial Efficiency): *An initial allocation is initially efficient if and only if there exists $k \in \mathbb{R}^H_{++}$ and prices q: $S \to \mathbb{R}^L_+$ such that*

$$k_h \frac{\partial u_h}{\partial x}[e_h(s), s] \sum_{t \in T} \pi_h(t, s) = q(s), \quad \text{for all } s \in S,\ h \in \boldsymbol{H}. \tag{A.3}$$

Proof: Standard argument from first order conditions.

Notice that an allocation is initially efficient in the betting case if and only if agents have common beliefs about payoff-relevant events, i.e.,

$$\sum_{t \in T} \pi_h(t, s) = \sum_{t \in T} \pi_{h'}(t, s), \quad \text{for all } s \in S,\ h, h' \in \boldsymbol{H}.$$

Proof of Theorem 3.1 (Interim Case): Now consider an L good initially efficient allocation e, with agent beliefs π, satisfying equation (A.3) above. We will compare this with the betting case with beliefs π', where $\pi'_h(t, s) = \pi_h(t \mid s)\phi(s)$, for all $t \in T$, $s \in S$, $h \in \boldsymbol{H}$, for some strictly positive probability distribution on $S, \phi \in \Delta_{++}(S)$. Observe that beliefs π are concordant if and only if beliefs π' are common.

For any L good trade, consider the one good trade y where

$$y_h(t, s) = k_h \left\{ \sum_{t' \in T} \pi_h(t', s) \right\} \{ u_h[e_h(s) + x_h(t, s), s] - u_h[e_h(s), s] \}.$$

If x is a feasible trade, then y is a feasible trade. If $e+x$ interim dominates e in the L good case, then y ex ante (interim/ex post) dominates 0 in the betting case. If x is incentive compatible in the L good case, then y is incentive compatible in the betting case. If x is public, then y is public.

Thus if allocation e is not interim (incentive/public) efficient in the L good case, then no allocation is interim (incentive/public) efficient in the betting case.

Thus if beliefs π are (noisy/public) consistent concordant, then beliefs π' are (a noisy/public version of) consistent beliefs; then (by Lemma A2) some allocation is (incentive/public) interim efficient in the betting case; then e is interim (incentive/public) efficient in the L good case.

Conversely, for any one good trade y, consider the L good trade x where, for some public event E,

$$x_{h1}(t,s)=\varepsilon y_h(t,s), \quad \text{if } t\in E,$$

$$x_{hl}(t,s)=0 \quad \text{if } l\neq 1 \text{ or } t\notin E.$$

If y is a feasible trade, then x is a feasible trade. For $\varepsilon>0$ sufficiently close to zero, if y strongly interim dominates 0 in the betting case, and E is the public event when interim domination constraints are strict, then $e+x$ strongly interim dominates e in the L good case. If y satisfies strong incentive compatibility in the betting case, then x satisfies incentive compatibility in the L good case. If y is public, then x is public.

Thus if any allocation is not strongly interim (incentive/public) efficient in the betting case, allocation e is not (incentive/public) efficient in the L good case.

Thus if allocation e is (incentive/public) efficient in the L good case, then any allocation is strongly (incentive/public) efficient in the betting case, and (by Lemma A3) beliefs π' are (noisy/public) weakly consistent. Thus beliefs π are (noisy/public) weakly consistent concordant. *Q.E.D.*

PROOF OF THEOREMS 2.1, 2.2, 2.3 (Ex Ante Case): Same argument as for interim case, except with trade x defined by

$$x_{h1}(t,s)=\varepsilon y_h(t,s),$$

$$x_{hl}(t,s)=0 \quad \text{if } l\neq 1.$$

Lemma A4 ensures that theorems are tight.

PROOF OF THEOREM 4.1 (Ex Post Case): Same argument as for interim case, except with trade defined by

$$x_{h1}(t,s)=\varepsilon y_h(t,s), \quad \text{if } t=t^*,$$

$$x_{hl}(t,s)=0 \quad \text{if } l\neq 1 \text{ or } t\neq t^*,$$

where t^* is the signal profile where ex post domination constraints hold strictly.

Lemma A5 ensures that the theorem is tight.

PROOF OF COROLLARY 4.1: Suppose allocation e is initially efficient and beliefs are consistent concordant. Then, by Lemma A6, there exist $k\in\mathbb{R}^H_{++}$, prices r: $S\to\mathbb{R}^L_+$, and, for each h, c_h: $T_h\to\mathbb{R}_+$, such that

$$k_h\frac{\partial u_h}{\partial x}[e_h(s),s]\sum_{t\in T}\pi_h(t,s)=r(s), \quad \text{for all } s\in S,$$

$$c_h(t_h)\pi_h(t,s)=\psi(t,s), \quad \text{for all } t\in T,\ s\in S,\ h\in \boldsymbol{H}.$$

Now the following prices constitute a rational expectations equilibrium:

$$q(t,s)=r(s)\psi(t|s), \quad \text{for all } t\in T,\ s\in S.$$

Conversely, suppose initially efficient allocation e is a rational expectations equilibrium with prices q. Write $Q_h(t)$ for the set of signal profiles considered possible by agent h is equilibrium, i.e.

$$Q_h(t) = \{t' \in T^* \mid t'_h = t_h \text{ and } q(t', s) = q(t, s), \text{ for all } s \in S\}.$$

Now equilibrium requires that, for each h, there exists λ_h: $T \to \mathbb{R}_{++}$, with λ_h measurable with respect to Q_h, such that

$$\lambda_h(t) \frac{\partial u_h}{\partial x}[e_h(s), s] \sum_{t' \in Q_h(t)} \pi_h(t', s) = \sum_{t' \in Q_h(t)} q(t', s), \quad \text{for all } t \in T, s \in S, h \in \boldsymbol{H}.$$

But since $Q_h(t) \subset P(t)$, for all $t \in T$, summation gives

$$\frac{\partial u_h}{\partial x}[e_h(s), s] \sum_{t' \in P(t)} \lambda_h(t') \pi_h(t', s) = \sum_{t' \in P(t)} q(t', s), \quad \text{for all } t \in T, s \in S, h \in \boldsymbol{H}.$$

Under assumption that e is initially efficient, this implies that beliefs are public revelation consistent concordant. *Q.E.D.*

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