Commuting, congestion, and employment dispersal in cities with *mixed* land use✩

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Received 24 June 2003; revised 29 December 2003

Abstract

For centuries, cities have been modeled as having centered employment—scarce access to which generates land rent. This paper first presents some new empirical evidence that in US cities actual employment turns out to be almost as dispersed as residences. This range of urban forms is generated with analytic ease in a model that assumes land can have "mixed" rather than exclusive use at any location. The model has firms trading off a central-agglomeration force against the lower wages that accompany shorter commuting distances in peripheral locations. At one extreme, with very high agglomerative forces, employment is approximately centered, with long commute distances and high congestion levels. At the other extreme, lower agglomerative forces lead to employment that is completely dispersed, commute distances of zero and the absence of congestion. Maximizing output-minus-congestion never leads to a pattern of land mixing where employment is fully dispersed. A market model, however, often does if the mix at each location is based on the relative magnitude of the land rent offered from residential as opposed to commercial uses.

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1. Introduction

Since the 19th century, the dominant view of urban land use has been based on the Ricardian rent, monocentric city model. In this model, transportation frictions for commuting or commerce generate a rent “gradient” between a city center and urban periphery. As cities expand in population and density, this entire gradient both shifts

✩ First presented at the 2001 meeting of the Asian Real Estate Society, August 1–4, 2001, Keio University, Tokyo, Japan.
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upward and becomes steeper as travel distances to the center expand and speeds deteriorate. Increased location “scarcity” seems to accompany urban growth.

The first objective of this paper is to review some new empirical evidence that suggests this vision of urban form is largely incorrect. In most US metropolitan areas, recently released data on the spatial distribution of jobs (by place of work) suggest that firms and residences are remarkable well interspersed and that jobs are at best only slightly more centralized than houses. The facts simply do not fit a “monocentric” model. The second objective is to develop a very simply model of urban spatial structure that does fit these facts. The model makes many simplifying assumptions, and has few embellishments, but it can generate city forms that are more consistent with the data. This model has five key features.

- Mixing or interspersed land use is allowed with firms and households able to jointly occupy some “fraction” of land at any location.
- Travel patterns are determined with mixed land use under the assumption that only one-way radial movement is possible. Solow-type congestion then turns out to be highest with fully centralized employment and becomes negligible as firms are fully dispersed and evenly mixed with residences.
- If some form of spatial agglomeration is assumed, it is never “optimal” to fully disperse employment—trading this agglomeration for reduced congestion.
- In a competitive market, rent and wage gradients also will depend on trading agglomeration for congestion.
- In market equilibrium with mixed use, it is assumed that land is occupied in proportion to land rent—rather than deterministically by the highest rent. This often generates fully dispersed employment patterns.

The model developed here brings together a number of previous ideas in the spatial economics literature. Following Mills [17], White [26,27], and Ogawa and Fujita [18], the model motivates employers to seek proximity to residences in order to lower the commuting and hence wages that they pay workers. From Helseley–Sullivan [11] and Anas–Kim [2], the model introduces a spatial agglomeration factor for firms, which provides some rationale for centralization. Finally, Solow-type [22] transportation congestion is introduced for the first time in such models. The innovation of the current paper is to combine all of these together—a synthesis made analytically tractable through the introduction of “mixed” land use.

The paper is organized as follows. The next section reviews the new empirical evidence about the spatial distribution of jobs and people within cities. The following section provides an example of how dispersed employment and mixed land use can generate congested traffic flows between firms and residences. In the fourth section, an exogenous spatial agglomeration factor is introduced, and it is shown that the aggregate net value of urban output is generally maximized by partially dispersing jobs. In Section 5, a market model is developed in which agglomeration and congestion generate equilibrium rent and wage gradients for both firms and households. To allow mixed land use an “idiosyncratic” auction function is assumed that allocates land use based on the relative magnitude of rents from the two uses. The sixth through eighth sections of the paper show how the degree of employment dispersal in the market model varies with the level of urban agglomeration, the
level of transportation infrastructure, and population growth. The final section concludes with a list of suggested future research questions.

2. Empirical evidence on monocentricity

In recent years, there has been a growing empirical literature that suggests employment is far more dispersed than previously thought. Mills [17] estimated employment as well as population density gradients, and claimed that jobs were indeed dispersed—in contradiction to pure a monocentric model. More recent research shows that employment in metropolitan areas around the world, is both continuing to disperse and in some cases clustering into subcenters (Guiliano and Small [9], McMillen–McDonald [14]; Waddell, Berry, and Hoch [24]).

A more detailed analysis of job dispersal has recently become possible with the release of Commerce Department information on employment at place of work—by zip code. Glaeser–Kahn [7] use this data to reestimate the employment density gradients originally observed by Mills [17]. With zip code level data, however, it is possible to construct the exact cumulative spatial distribution of employment from the “center” of each MSA outward. This can then be compared with the similar distribution for residence population. Examples of these two distributions are shown in Figs. 1 and 2, for the two areas in the US that are held up as representing “traditional” as opposed to “newer” cities: the New York and Los Angeles CMSAs. In New York, while employment is slightly more concentrated than population, the closeness of the two distributions is unexpected. In Los Angeles, the distributions of population and employment are virtually identical.

Comparing density gradients across different MSAs is complicated because a gradient has at least several different parameters. Instead, this paper proposes a single metric of how “concentrated” the spatial distributions are in Figs. 1 and 2. One takes the area under the cumulative distribution—up to (say) the 98% point for population, and then divides by the distance at that point. Cities that are “fully” concentrated at single central point have a value of unity. A city with an employment density gradient that is inverse to distance has a value of 0.5. A city with uniform employment density has a value less than 0.5. The measure is thus:

\[
\text{Concentration} = \int_0^b \frac{e(t)}{b} \, dt,
\]

where \( e(t) \) is a cumulative fraction of jobs (population) at distance \( t \), \( b \) a distance at which 98% of population live.

In Fig. 3, the value of this measure for both population and employment is shown for 120 MSAs (or CMSAs). Across these areas, either measure of concentration varies from about 0.5 to 0.75 depending on how dense and spread out the population. In a true “monocentric” city, in which all jobs were within say the first 5 miles (out of 50), the employment concentration metric would be close to 0.95 and the population value less than 0.5. There are no cities like this.

Two conclusions are apparent. First, there is a remarkably close correlation between the two measures \( R^2 = 0.73 \). Cities in which population is more spread out—jobs are
as well—and vice versa. Secondly, employment is only slightly more centralized than population. While virtually all MSAs lie above the 45 degree line in Fig. 3, the average concentration measure for employment (0.68) is only slightly greater than for population (0.63).

The widespread evidence on the dispersal of employment receives some corroborating support from an equally voluminous research on the intra-metropolitan spatial variation in wage rates. If jobs are highly centralized, then wage rates obviously will not exhibit
much spatial variation. However, as will be discussed below, with job dispersal, wages rates must vary by location in a manner that exactly compensates workers for commuting to alternative job sites. There is a growing documentation of such variation: Eberts [5], Madden [16], Ihlanfeldt [12], McMillen–Singell [13], Timothy–Wheaton [23].

3. “Mixed” land use and commuting

In order to model dispersed employment, the city constructed here will continue to be circular with location defined as being distance $t$ from some geographic “center.” In the traditional theory of competitive spatial markets, land use at each such location is exclusively of one type—deterministically based on which use offers the highest rent (Alonso [1]). By definition this precludes land use mixing, except possibly in the case where the rent from two uses is identical. Even in the case where rents are “tied,” the exact fraction of land that is assigned to each use is undetermined. As a result, traditional spatial theory tends to create land use patterns in which there are exclusive zones or rings for each use. With rings or multiple employment zones determining the pattern of commuting and rents becomes extremely complex.\footnote{In Braid [4], it is assumed that when rents are “tied” the fraction of land used by employment is constant and equal to the aggregate land share.} Furthermore, the evidence in Figs. 1 and 2 suggests that employment is sufficiently diffused so that exclusivity of use may in fact rarely occur.

As an alternative, this paper adopts the convention of assuming that different uses can cohabit the same location. The fraction of land at each location that is commercial, $F(t)$,
can vary continuously over the 0–1 interval while the remaining fraction, \( 1 - F(t) \) is devoted to residential use.

A simplification in the current model will be the assumption that the consumption of land (per worker), both at both at their place of employment (\( q_f \)) as well as at the location of their residence (\( q_h \)) be fixed and independent of location. In most modern cities, workplace land consumption is far smaller than residential (\( q_f \leq q_h \)), but this is only an observation and is not necessary for the model at hand. Allowing density to be endogenous complicates the model, but should not change the qualitative conclusions derived here. In any case, it will be one of several suggested future extensions.

With fixed density at both residence and workplace, \( e(t) \) and \( h(t) \) are defined as the cumulative number of workers or households who live up to the distance \( t \). Using the \( F(\cdot) \) function from above, these are:

\[
e(t) = \int_0^t 2\pi x \frac{F(x)}{q_f} \, dx, \tag{1}
\]

\[
h(t) = \int_0^t 2\pi x \frac{1 - F(x)}{q_h} \, dx. \tag{2}
\]

The spatial distribution of employment and households will have an outer “edge” or limit. No jobs exist beyond the distance \( b_f \) and no households beyond the distance \( b_h \). The two edge distances \( b_f \) and \( b_h \) are determined so that the cumulative distributions at those points equal the exogenous number of jobs and households \( [E \text{ and } H] \). For convenience, the equality between total jobs and households is assumed:

\[
e(b_f) = E = H = h(b_h), \quad e(0) = 0 = h(0). \tag{3}
\]

An important assumption, that the transportation technology allows only inward radial commuting, results in a very simple and easy characterization of traffic flows. The number of commuters passing each distance in the city is equal to the difference between the cumulative number of workers employed up to that point and the cumulative number of residents living up to that same distance. Since reverse commuting is not allowed, it also follows that workplaces must not be “more dispersed” than residences. Thus the demand (flow) of inward-only commuters \( D(t) \), under this assumption is:

\[
D(t) = e(t) - h(t) \geq 0, \tag{4}
\]

\[
D(b_h) = D(0) = 0,
\]

\[
b_f \leq b_h \quad \text{[so } b_h \text{ is the urban boundary].} \tag{5}
\]

These assumptions generate a resulting pattern of travel demand that has several interesting features. First off, it is clear that there exists a land use pattern (a particular

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\[2\] If each household has only one job and each job is filled by only one household, and there is no reverse commuting, then \( D(t) < e(t) - h(t) \) would imply that there is excess labor demand somewhere up to distance \( t \), and excess labor supply beyond \( t \). The reverse implies the opposite inconsistency. Finally, if \( e(t) - h(t) < 0 \) then there are more jobs beyond \( t \) than residences and reverse commuting must be occur.
function \( F(t) \) for which there is no commuting. If at all distances \( t \), \( e(t) = h(t) \), then travel demand is everywhere zero. Using (1) and (2), this is the case if the fraction of land used by firms is always equal to their “share” of overall land demand:

\[
F(t) = \frac{q_f}{q_f + q_h}, \quad \text{implies} \quad e(t) = h(t) \quad \text{for all} \quad t, \quad \text{and} \quad b_f = b_h. \tag{6}
\]

A second observation is that when the land use pattern \( F(t) \) does generate travel demand, then travel demand is at a (local) maximum or minimum at that distance where the marginal number of household equals the marginal number of jobs:

\[
D(t) \quad \text{reaches a local maximum where} \quad \frac{1 - F(t)}{q_h} = \frac{F(t)}{q_f}. \tag{7}
\]

Equation (7) implies that if \( q_f \) is much smaller than \( q_h \), then maximum travel demand tends to occur near to where \( F(t) \) goes to zero—at the edge of employment \( b_f \), regardless of how dispersed that employment is. At this point in the city, the number of residents traveling inward is very high, since no one has yet to be “dropped off” at work.\(^3\)

Finally, travel demand must be converted into travel costs. It is assumed that the marginal cost of travel has a fixed component \( (\beta) \), as well as endogenous congestion. Here an amended version of the Solow [22] congestion function is used wherein the time cost per mile is proportional to demand and inverse to capacity, \( S(t) \):\(^4\)

\[
\frac{\partial C(t)}{\partial t} = C'(t) = \left[ \frac{D(t)}{S(t)} \right]^\alpha + \beta. \tag{8}
\]

Again, for the sake of simplicity, this model assumes that transportation capacity can be provided without using up land and so \( S(t) \) implicitly reflects only road “capital.” This keeps the number of urban land uses to just two (employment, residence).\(^5\)

\(^4\) None of the results here are based on the congestion function having a coefficient of \( \alpha = 1 \), although that will sometimes be assumed in the simulation results. Actual studies (Small [21]) suggest that the ratio assumption is reasonable but with \( \alpha > 1 \) (Small suggests \( \alpha = 2 \)).

\(^5\) This model differs from that of Ogawa and Fujita [18] in two significant respects: they use the traditional \( F = 0, 1 \) assumption which does not permit mixing, and they do not incorporate congestion.
city and hence modeled intra-urban shipments as the aggregation of distances between businesses. In a circular city, such a measure yields the least aggregate shipping costs at the center and the most at the edge. More generally, numerous authors have proposed some more unspecified “urban agglomeration” factor that also works best at the dense urban center (Glaeser–Kahn [7], Rouwendal [20]).

Whatever the source of the agglomeration, if firms are all located at one center, productive efficiency will be highest or production costs lowest. This pattern will also lead to the longest commute trips and hence congestion will be greatest as well. Alternatively, dispersed employment reduces congestion, but also productivity. The objective of the current model is only to examine the impact of such agglomeration and not its generation. Thus this model will assume some function for output/worker $Q(t)$, which declines with distance from the urban center. Greater agglomeration implies a steeper gradient. Making this gradient exogenous again provides great analytic simplicity.

With land consumption fixed, aggregate welfare in a city is simply the difference between aggregate output and the aggregate cost of commuting determined from the city center to edge $(T)$. To aid in the presentation, we make a few additional simplifications. The first is that travel supply is constant and normalized to $S(t) = S = 1$. The second is that $\beta = 0$. Finally, we also assume that the land consumption of each use is the same and again equal to unity, that is $q_f = q_h = 1$. These simplify the presentation, but do not materially change the results. Later they will be relaxed when simulation results are presented.

Incorporating these simplifications with (8), the objective function thus becomes:

$$\max_{F(t)} \int_0^T 2\pi t F(t) Q(t) \, dt - \int_0^T D(t)^{\alpha+1} \, dt.$$  \hfill (9)

Differentiating (4) with respect to $t$ and applying (1) and (2) provides a differential equation that links $F(t)$ to changes in travel demand $D(t)$:

$$D'(t) = 2\pi t \left[ 2F(t) - 1 \right].$$  \hfill (10)

Finally, $F(t)$ and $D(t)$ must be quite tightly constrained:

$$0 \leq F(t) \leq 1, \quad 0 \leq D(t) \leq H, \quad D(0) = D(T) = 0.$$  \hfill (11)

This greatly limits the set of feasible solutions. Since $D(0) = 0$, $D'(0)$ must be $\geq 0$ or else $D(t)$ becomes negative. This in turn requires that $F(0) \geq 1/2$. Similarly, to avoid negativity $D'(T) \leq 0$, which requires that $F(T) \leq 1/2$. In effect, the constrained solution to $F(t)$ must not slope upward. A traditional monocentric city will have a CBD zone exclusively of firms, with $F(t) = 1.0$ over some initial distance and then zero thereafter. At the other extreme, a fully dispersed city has $F(t)$ everywhere equal to 1/2. In-between, employment is more, but not completely, centralized relative to residences.

The problem in (9)–(11) represents a well-defined optimal control problem, albeit with significant constraints. The Hamiltonian is

$$H = 2\pi t F(t) Q(t) - D(t)^{\alpha+1} + \lambda(t) \left[ 2\pi t \left[ 2F(t) - 1 \right] \right].$$
A solution to the problem involves two general conditions. First, \( F \) must optimize \( H \) at all \( t \), given the required constraints:

\[
\begin{align*}
2\pi t Q(t) + \lambda(t)4\pi t & \leq 0, & \text{if } F(t) = 1, \\
& \geq 0, & \text{if } F(t) = 0, \\
& = 0, & \text{if } F(t) \text{ is within the 0–1 range}.
\end{align*}
\] (12)

The second condition that must hold everywhere with an equality is:

\[
\frac{\partial H}{\partial D(t)} \equiv -(\alpha + 1)D(t)^\alpha = -\lambda'(t), \quad \text{over all } t.
\] (13)

Thus \( \lambda(t) \) must be negative, but also non-decreasing (generally increasing) in \( t \).

Equation (12) can hold as an equality if the optimal \( F(t) \) is in the interior of the constraint range, as well as if it "runs along" either constraining value: 0, 1. A strict inequality can occur only at the constraint boundaries. When (12) holds with an equality then it can be differentiated: this equates \( Q'(t)/2 \) with \( -\lambda'(t) \). Combining this result with (13) and then differentiated again yields the following.

\[
\frac{Q''(t)}{2} = \left[-\alpha(\alpha + 1)D(t)^{\alpha-1}\right]D'(t) \\
= \left[-\alpha(\alpha + 1)D(t)^{\alpha-1}\right][2\pi t][2F(t) - 1].
\] (14)

Equation (14) can hold over any optimal \( F(t) \) that is feasible. However, if the optimal solution has \( Q'(t)/2 \leq -\lambda'(t) \), then (14) will not hold—and this can be true only if the optimal \( F(t) \) is at the constraint boundaries. This observation provides the following results.

**Proposition 1.** Fully dispersed employment is never optimal with a negative productivity gradient.

**Proof.** Full dispersal of employment is possible only if \( F(t) = 1/2 \) everywhere, which then generates a \( D(t) \) that is everywhere equal to zero. Hence from (13) \( \lambda'(t) = 0 \) and \( \lambda \) must be a negative constant. But \( Q(t) \) is not constant, and hence (12) implies that \( F(t) \) will be constrained at 1 and 0 as \( Q(t) \) is greater or less than \( 2\lambda \). Thus it will be impossible for \( F(t) \) to have an interior solution (which must be the case if it everywhere equals \( 1/2 \)).

**Proposition 2.** \( F(0) = 1 \).

**Proof.** Equation (11) requires \( D(0) = 0 \). Thus if \( Q''(0) \) is either positive or negative then Eq. (14) cannot hold at \( t = 0 \). As discussed above, the only values of \( F(t) \) where (14) will not hold is at one of its two constrained values. \( F(0) = 0 \) is ruled out from (10) and (11) and hence \( F(0) \) must equal one.

**Proposition 3.** If \( Q'(t) < 0 \) and \( Q''(t) > 0 \) everywhere (e.g. a negative power or exponential function), then there must exist a CBD: an interval \([0, t^*]\) where \( F(t) = 1.0 \).

**Proof.** Suppose over \( 0 > t > t^* \), \( F(t) \) declines gradually from 1 into the range \([1/2 < F(t) < 1]\), then (12) implies that Eq. (14) holds in this range. For the RHS of (14) to be
positive $F(t)$ must be less than 1/2. Moving away from $t = 0$, $F(t)$ cannot be less than 1/2 since this generates negative $D(t)$ values as $t$ increases from 0. At some $t^*$, $F(t^*)$ must “jump” from 1.0 to a value smaller than 1/2, from which it can decline smoothly. □

**Proposition 4.** Mixed land use is possible everywhere except at $t = 0$ only if $Q''(t) < 0$ for some range of $t < t^*$ and $Q'' > 0$ for $t > t^*$.

**Proof.** Mixed use requires (12) be an equality and hence (14) hold. To have $F''(t) < 0$ over all $t$ (so jobs are more centralized than residences) requires $F(t) > 1/2$ for $t < t^*$. From (14) these necessitate that $Q''(0)$ must be negative. At some point $t^*$, $F(t^*) = 1/2$. From there outward $F(t) > 1/2$ and (14) requires $Q'' > 0$. For there to be partial mixing everywhere, $-Q'(t)$ must rise and then fall—matching the spatial pattern of traffic demand $D(t)$ which always does likewise. Similarly $Q'(t)$ must match up with $-D'(t)$, being at first negative and then positive. □

These results are quite powerful for they seem to imply that fully interspersed land use is never optimal—agglomeration always gives rise to some degree of centrality of jobs. Furthermore, “exclusive” central business zones are normally optimal unless the particular pattern of agglomeration exactly matches that of traffic demand.

5. Wage and rent gradients in a market model of mixed land use

The model above shows that with a range of “agglomeration” functions it can be optimal to have employment fully or at least “partially” centralized. The next step is to examine how a competitive private land market might produce a similar range of land use patterns. This necessitates determining market land rents.

With land consumption fixed, household utility depends only on net income after receiving wages, paying for travel costs, and consuming land (paying rent). With identical households, equilibrium then requires that net income be constant across locations. Of course in this model, households make two location decisions: where to live and where to work. There thus must exist two “price” gradients to make households indifferent about each decision.

For households at a fixed place of residence, all alternative workplaces must yield them identical net income. Since rent is fixed by residence place, choice of workplace impacts net income through commuting. For indifference, a wage gradient $W(t)$ must exist and vary directly with the marginal cost of travel:

$$\frac{\partial W(t)}{\partial t} = W'(t) = -C'(t), \quad W(0) = W_0. \quad (15)$$

For households at a fixed place of employment, all alternative residential choices must be equally attractive. This requires a land rent gradient $R_h(t)$ that varies with the marginal cost of travel and everywhere is greater than the reservation rent for land ($A$), except at the edge of residential development:

$$\frac{\partial R_h(t)}{\partial t} = R'_h(t) = -C'(t)/q_h, \quad R_h(b_h) = A. \quad (16)$$
Turning to firms, equilibrium will require that the location of jobs be equally profitable at all locations. Thus it must be true that worker productivity, wages and firm land rents all exactly offset each other. This yields a firm land rent gradient $R_f(t)$ that obeys the following conditions:

$$\frac{\partial R_f(t)}{\partial t} \equiv R'_f(t) = \begin{cases} Q'(t) - W'(t), & q_f \\ Q'(t) + C'(t), & q_f \end{cases}, \text{ from (9)}, \quad R_f(b_f) = A. \tag{17}$$

Thus if the decline in productivity is larger than marginal travel costs, firm rent gradients decrease with distance. From (7)–(8), the pattern of congestion suggests this is most likely to occur at the very center ($t = 0$) and at the residential edge ($t = b_h$). On the other hand, if marginal travel costs are larger than the decline in productivity, then firm rent gradients might increase with distance. Again from (7)–(8), this is most likely at or near the edge of employment $b_f$.

Thus if Eq. (15) through (17) hold, both firms and households can be in location equilibrium. The rent and wage gradients that insure this, however, depend crucially on congestion, which in turn, depends on the travel flows that result from the pattern of land use mixing. Thus a rule is needed in which rents determine land use in a manner that allows mixing.

It is clear that the traditional competitive assumption that land goes to its “highest use” will create perfect segregation of uses, or in the case of “ties” perfect mixing of an undetermined degree. As an alternative, this paper adopts the convention of assuming that there are random or idiosyncratic effects that allow different uses to cohabit the same location. The fraction of land at each location that is commercial, $F(t)$ then will depend on the relative magnitude of the rent levels for each use.\(^6\)

There are a number of functions that one could use to model some kind of “idiosyncratic land use competition,” including for example logistic choice. Here a very simple form is illustrated, again for analytic ease. The only requirement is that the function $F(t)$ map a pair of positive values for commercial and residential land rents $[R_f(t), R_h(t)]$ into the zero–one interval. The function in (18) assumes that with equal Ricardian rent, the two uses get equal (50%) land use assignment:\(^7\)

Equal rents imply equal shares: $F(t) = \begin{cases} R_f(t)/\left[R_f(t) + R_h(t)\right], & \text{if } R_f(t) \leq A. \end{cases}$

\(^6\) There are two reasonable arguments that can be made for land use mixing: that there exists some unmeasured “other” location dimension, or that there are true random variations in utility or production. An example of the first would be the tendency of commerce to occupy the first floor of urban buildings while residences exist above. The unmeasured impact of foot traffic in the vertical dimension generates this kind of “mixing.” Commercial preferences for corners, frontage, etc. operate similarly. True random effects would generate stochastic rents. In this case, each use would have a probabilistic rent distribution for any location and application of the highest use principle would yield probabilistically mixed land use. See Anas [3] for a similar idea.

\(^7\) A simple alternative would be $F(t) = q_f R_f(t)/(q_f R_f(t) + q_h R_h(t))$. Here, with equal rents land would be allocated in proportion to its aggregate usage. If the allocation is stochastic-based, however, it seems more reasonable to assume that with equal rents, each use stands an equal chance of acquiring land.
Thus if there are two non-negative rent gradients over the dimension \((t)\), then the above function can be used to determine the aggregate allocations of land to each of the two uses.

With the specification of the market assignment function, a positive model can be theoretically “closed.” An equilibrium solution can be imagined with the following mappings. Equation (18) takes the two rent gradients \([R_f, R_h]\) and maps them with the \(F(t)\) function into land use shares at each location. Equations (1) through (8) take land use shares and determine the pattern of travel \([e(t), h(t), D(t), and C(t)]\). Finally, Eqs. (15) through (17) take the pattern of travel congestion and map it back to the rent gradients.

While it is quite straightforward to find reasonable equilibrium market solutions, it may not always be the case that one exists, at least without further restrictions than those above. A particular troublesome problem can arise for seemingly reasonable representations of \(C(t)\) and \(Q(t)\). At points of maximum congestion it is possible that \(C' > -Q'\) and firm bid rents will locally slope up with greater distance. Since according to (17) this is likely to happen near to the edge of employment, it can become difficult to find a solution to \(b_f\).

While it is difficult to prove that an equilibrium always exists with mixed land use, it is quite easy to find particular market solutions, and to demonstrate that these require certain conditions. As a first step, it is useful to examine the range of simulated land use patterns that are possible with the equations from this section. Two patterns are possible: partially mixed use (over some range of locations), and fully mixed land use over all locations (dispersed employment). In the first, firms are more centralized than households, but there is a range of locations where both uses exist. In the latter, firms and households are perfectly and evenly intermingled. Each of these outcomes can be generated with particular combinations of urban productivity, transport capacity, and population size.

6. The dispersal of employment and agglomeration

When an idiosyncratic land use function is used, such as (18), then some degree of land use mixing occurs by definition. The degree of employment dispersal, and hence land use mixing, however, will depend heavily on the level of agglomeration or on how rapidly productivity declines with distance.

**Proposition 5.** Lower agglomeration generates greater employment dispersal in mixed use cities.

**Proof by contradiction.** In a city that has a partially mixed land use pattern, \(b_f < b_h\), and \(F(t) < 1\), for \(t < b_f\). With less agglomeration, the absolute value of \(Q'(t)\) is less at all locations. If greater employment centralization were to result, then \(b_f\) would have to be smaller, requiring \(F(t)\) be higher over at least part of the interval \([0, b_f]\). From (1)–(8), however, this would cause \(C'\) to be greater over this same interval. From (16) and (17), lower \(Q'\) and higher \(C'\) when combined with a smaller \(b_f\) will reduce commercial rent levels \(R_f(t)\), and increase residential rents \(R_h(t)\) over this interval. With (18) this will reduce rather than increase \(F(t)\) over \(t = 0, b_f\). Through the reverse argument, the converse holds as well—greater agglomeration cannot lead to employment dispersal. With
less agglomeration, firm rent gradients are flatter and the firm employment boundary farther. □

At the extreme case, fully dispersed employment does not require the absence of agglomeration, but rather only that agglomeration must be such as to exactly offset the exogenous travel costs that occur when congestion is nil.

**Proposition 6.** A fully dispersed employment equilibrium is possible with the land use mix function (18).

**Proof.** Full employment dispersal requires that $F(t) = q_f/(q_f + q_h)$ and $e(t) = h(t)$ over all $t$. Hence $b_f = b_h$, $D(t) = 0$, and $C'(t) = \beta$ (no congestion, only exogenous travel costs).

If the ratio $C'/Q' = q_f^2/[q_f^2 + q_h^2]$, while $C' = \beta$, and hence $Q'$ is constant across locations, as well, then all the gradients also have constant slopes: $W'(t) = -\beta$, $R'_h = -\beta/q_h$, and $R'_f = -\beta q_f/(q_f + q_h)$. Furthermore, it is necessary for $A = 0$, so that rent levels become proportional to their slopes. Then using (18), if $F(t) = R_f(t)/[R_f(t) + R_h(t)]$, it is also true that $F(t) = q_f/(q_f + q_h)$. □

The important assumption in this example is that for any given city, the ratio $C'/Q'$ is equal to the residential share of squared land consumptions across the two uses. While this may seem restrictive, it is actually quite realistic and other assumptions can also generate a fully dispersed city. A most interesting point is that even with no commuting and congestion, some rent and wage gradient based upon free-flow travel costs is “necessary” to insure that employment and residential locations remain dispersed.

Figures 4–5 illustrate a mixed land use city that is extremely close to looking like a traditional centralized monocentric city. There are 2 million households (and workers), firm land consumption is 0.0001 square miles per worker and household land consumption is 0.0005 square miles per worker. Exogenous marginal transport costs are set to $\beta = 100$ and transportation capacity is set so as to be constant across distance. The business zone extends to ring 96 and the city edge is at ring 195 (rings are tenths of miles). Congestion is zero at the very center and residential edge, and reaches a maximum at the edge of the business zone (Fig. 4) where congestion costs are approximately $350 per mile. This edge is determined by where the firm rent gradient equals the reservation rent for land (Fig. 5), and firm rents are based on a spatial decline in worker productivity that is assumed to be about twice the maximum marginal travel cost ($Q' = -1200$). This is sufficient so that firm rent gradients are much steeper than those of households (Fig. 5). As a result that portion of the city with mixed use (to ring 96) has on average almost 80% of its land used by firms. In this simulation, aggregate travel expenditures are 5.4 billion, and central residential rents reach 105 million per square mile.8

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8 In the top figure of each pair, population and employment depict the normalized variables: $h(t)/H, e(t)/E$. Travel costs are $C' = 25D(t)/S(t) + \beta$. Transport capacity is set to be uniform over $t$: $S(t) = 125600$. Transport costs are in yearly $/mile, and rents are in yearly $/square mile. The annual opportunity rent of land at the urban
Figures 6–7 illustrate a similar mixed land use city, but one where the decline in productivity is a quarter as much, $-300$ as opposed to $-1200$ per mile. The employment border moves out (from 96 to 155) and the fraction of land inside this border that is devoted to commercial use decreases to an average of 40%. Because of greater land use mixing, the necessity to commute is lower, and thereby the marginal cost of travel (congestion) is as well. In the aggregate, travel expenditure is down sharply—to 2.5 billion—less than half

edge is $A = 1000000$ per square mile. Finally, in various simulations worker productivity is assumed to decline linearly with distance in the range: $\phi = -100$ to $-1200$. 
of its value in the more centralized city (Figs. 4–5). Part of this expenditure reduction is directly due to less aggregate travel, with the remainder due to the secondary impact of this on the marginal cost of travel. With lower marginal travel costs, central residential rents drop, here to 78 million.

Finally in Figs. 8–9, a city with fully dispersed employment is illustrated. This is simply accomplished by setting the level of agglomeration at that specified in Proposition 6. There is no commuting and hence no congestion, so marginal travel costs everywhere are equal to $\beta = 100$. At each location, firms occupy 17% of land and residences 83%. Even with exogenous marginal travel costs, aggregate travel expenditures are equal to zero since no
one has to travel! In the fully dispersed city, the absence of congestion generates a large difference in land rents—central residential rents now are only 37 million.\(^9\)

These three simulations show how lower agglomeration disperses employment, reduces commuting and hence congestion costs and thereby reduces residential land rent. Commercial land rents also are greatly reduced, from 700 million down to only 7 million.

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\(^9\) The mixed use city simulations in Figs. 4–7 use the same \(F(t)\) function as in the fully dispersed simulation results of Figs. 8–9. Only the values of agglomeration and exogenous travel costs are changed, so that partial mixing occurs rather than fully dispersed employment.
but much of this is due directly to the reduction in agglomeration that generates the increasingly more dispersed employment patterns. It is interesting that both a wage and rent gradients continue to exist in the fully dispersed city—even though there is no travel, and congestion is absent. This is to insure that firms and households remain fully dispersed. As long as travel has some cost—even without congestion—a wage and rent gradient are necessary to keep economic agents completely interspersed.

7. The dispersal of employment and transportation capacity

While changes in agglomeration alter the slope of a firm’s rent gradient, changes in transportation capacity alter the rent gradients for both commercial and residential uses, through impacting congestion. As expanded capacity improves travel flows, both firms and residences move further “apart” to realize other location advantages.

Proposition 7. Expanded transport capacity generates employment centralization.

Proof by contradiction. In a mixed land use pattern, \( b_f < b_h \), and \( F(t) < 1 \), for \( t > b_f \). With a uniform expansion of \( S(t) \), and no change in land use, \( C'(t) \) will decrease everywhere. From (16)–(17), this causes \( R'_f \) to increase at all locations, while \( R'_h \) is less at all locations. With fixed aggregate land consumption, the urban boundary \( b_h \) is fixed and thus \( R_h(t) \) will be lower at all locations. Suppose \( b_f \) were to increase, then \( R_f(t) \) would be higher at all locations. From (18), these rent changes would cause \( F(t) \) to be greater at the same time that \( b_f \) increases—violating (1). To meet conditions (1)–(2), \( b_f \) must contract as \( F(t) \) increases—the definition of employment centralization.

Figures 10–11 illustrate a simulated city with mixed employment that is in all respects identical to that in Figs. 6–7, except that the value of transport capacity \( S(t) \) has been doubled at each location. The employment border moves in (from 155 to 128) and the fraction of land inside this border that is devoted to commercial use increases. Central commercial rents increase while residential rents decrease.

The land use changes in Figs. 10–11 provide an interesting version of “Braess’ paradox”: adding transportation capacity may actually increase average travel expenditures—in the absence of congestion-based user pricing (Small [21]).10 In the base case simulation of Figs. 6–7, marginal travel costs per mile are higher, but job dispersal keeps the aggregate miles of travel low. The combination of the two generates aggregate travel expenditure of 2.5 billion. By adding capacity, the city in Figs. 10–11 has lower marginal travel costs, but the more centralized land use pattern leads to longer trips. The product of longer but speedier travel actually leads to greater travel expenditure—about 10% higher, at 2.7 billion. With the same population, average trip times are higher as well.

10 If each driver’s actions are based on minimizing total system costs, as they would be with congestion pricing, then the envelope theorem applies and the general equilibrium impact on total system costs of adding capacity equals the partial impact—always beneficial. In the real world, where efficient congestion pricing is absent, there are many documented examples of the paradox.
8. The dispersal of employment and travel demand

Increasing travel demand occurs in the real world for any number of reasons: increased car ownership with greater income, a rise in the proportion of households with two working heads, etc. In this model we increase travel demand by expanding the metropolitan population—while keeping the urban boundary fixed. In effect, $E$ and $H$ increase through comparable decreases in $q_h$ and $q_f$ (increased density), while $b_h$ and $b_f$ remain fixed. This
will proportionately increase the aggregate number of commute trips each day. The supply of transportation capacity remains fixed.

**Proposition 8.** Increased travel demand (greater density) generates greater employment dispersal.

**Proof by contradiction.** Consider starting from a city whose land use pattern is neither fully dispersed nor fully centralized. In this case \( b_f < b_h \), and \( F(t) < 1 \), for \( t < b_f \). If \( E \) and \( H \) expand internally by a factor \( (\gamma > 1) \), because \( q_h \) and \( q_f \) decrease by \( 1/\gamma \), then at each \( t \), \( e(t) \), \( h(t) \) and the difference \( D(t) \) grow by \( \gamma \). Without any expansion of
travel capacity, $C'(t)$ increases everywhere. Together with the scaled decreases in $q_f, q_h$, this causes $R'_f$ to decrease relative to $R'_h$ at all locations (in absolute value). Given that $b_h$ is fixed, and that $b_f < b_h$, this reduces $R_f(t)$ relative to $R_h(t)$. From (18), these rent changes would cause $F(t)$ to be less at all locations up to $b_f$. To meet conditions (2)–(3), $b_f$ must expand as $F(t)$ is lower. An expansion of $b_f$ and decrease in $F(t)$ for $t < b_f$ is the definition of greater employment dispersal.

Figures 12–13 illustrate a simulated city with a mixed land use pattern that has half the population, but also half the density levels, of the city in Figs. 6–7. Since the total land consumption of each use is the same, so will be the outer residential edge of the city. However, the increase in population and density (moving from Figs. 12–13 back to Figs. 6–7) causes the employment boundary to increase from 126 to 155, and thus the employment distribution to flatten. Congestion is everywhere slightly greater, but employment dispersal leads to shorter trips (Gordon–Richardson [8]). In the Figs. 12–13, aggregate travel expenditure is 1.37 billion, which is more than half of the 2.5 billion in Figs. 6–7. Thus the employment dispersal that accompanies the (two-fold) increases in travel demand leads to lower average travel expenditure even though congestion is worse.

9. Conclusions and extensions

It is tempting to conclude from the results above that there is a divergence between market outcomes and optimal land use. In the market simulations it is easy to obtain fully dispersed employment using a simple linearly declining agglomeration function. In this case, however it is never optimal to have jobs completely dispersed. Nor with linearity is it optimal to have “smooth” mixing. The problem, however, is that directly comparing the two models is somewhat misleading, since the market model has a particular “idiosyncratic” mixing function—which the optimal model does not.

The paper has only begun to analyze cities where land use mixing can occur. The list of extensions is quite long. First off, more realistic two-way commuting might be examined, since clearly all travel modes have this feature. This would seem to further encourage job dispersal. Next, density or land consumption could be made endogenous, and this gives an additional dimension of adjustment to job dispersal decisions. Private land consumption decisions by households are not efficient, and the residential case has been well studied (Wheaton [25]), but the density decision of firms should impact congestion as well.

The empirical implications of the model, however are perhaps most interesting. Job dispersal per se should, if the model is correct, lead both to lower marginal commuting costs as well as travel distances. There are two pieces of research on this question. Hamilton [10] first argued that cities do have very dispersed employment—and that the pattern was inconsistent with the observed high commuting times. Richardson and Kumar [19] undertake a more thorough statistical analysis using a rough measure of job dispersal. Their cross section results show that more dispersed cities, ceteris paribus, do have lower average commute times. Both papers, however take observed employment dispersal as exogenous.

In a follow up paper to their original model, Fujita and Ogawa [6] argue that as city population increases and traffic congestion worsens, there is a greater likelihood (in their
model) of a second employment center forming. In the model developed here, greater traffic congestion (through increased demand or reduced supply) leads directly to employment decentralization. Thus in any cross section of US cities, it may be equally likely that longer trip times produce employment dispersal as well as that dispersed employment reduces trip lengths.

To overcome this shortcoming, McMillen and Smith [15] have recently undertaken a similar cross section analysis of city employment dispersal. They find that larger cities and cities where instrumented travel demand is higher do in fact have more subcenters (such as Fujita predicts). This finding corroborates as well the comparative statics of the model here, wherein such differences generate a greater degree of continuous employment dispersal. Clearly further empirical work is needed. It is important to not only determine if instrumented travel demand causes employment dispersal, but also if instrumented employment dispersal reduces travel. In the latter question, it also would be important to determine if job dispersal independently reduces trip distances and/or the average speed of travel.

References