FISCAL POLICY WITH NON-CONTINGENT DEBT AND THE OPTIMAL MATURITY STRUCTURE*

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Abstract

How should the tax rate and the level of public debt adjust to an adverse fiscal shock? What is the optimal maturity structure of public debt? If the maturity structure is carefully chosen, the ex post variation in the market value of public debt can cover the government against the need to raise taxes or debt should fiscal conditions turn bad. In general, almost every Arrow-Debreu allocation can be implemented with non-contingent debt of different maturities. In a stylized example, the optimal policy is implemented by selling a perpetuity and investing in a short-term asset.

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I. Introduction

What are the properties of optimal fiscal policy when fiscal expenditure and aggregate income are random, taxation is distortionary, and the government can issue only non-contingent bonds? How should the tax rate and the level of public debt adjust to an innovation in fiscal expenditure or national income? How should the government design the maturity structure of public debt in anticipation of future uncertainty in fiscal conditions?

When taxation is distortionary, any random variation in the tax rate reduces social welfare. A benevolent government thus seeks a constant tax rate even in the presence of aggregate uncertainty.[1] But complicating this goal is the requirement that the government satisfy her budget constraint. The present value of taxes must always cover the present value of government expenditure plus the debt burden. Keeping the tax rate constant is only possible when the variation in the realized debt burden perfectly offsets any random variation in fiscal expenditure and the tax base.

The government could easily obtain a constant tax rate if she traded *state-contingent* debt, that is, promises with the private sector that were conditional on the realization of future uncertainty. In reality, however, governments do not customarily issue state-contingent debt, perhaps because the involved contingencies are hard to describe and verify. Instead, governments almost exclusively issue *non-contingent* debt, or bonds

[1] Technically, the marginal social cost of taxation, not the tax rate, must be equal across dates and states of the world. For simplicity, I use the two notions interchangeably.

whose pay-off is independent of the realization of uncertainty. Is it then possible to keep the tax rate constant?

Unfortunately, this is not possible if the government issues only *short-term* debt. The debt burden is then completely independent of the realization of contemporaneous uncertainty, and the government must offset any random variation in the budget with an appropriate adjustment in the tax rate. In fact, a positive innovation in fiscal expenditure or a negative innovation in aggregate income forces the government to raise *both* the tax rate and the level of public debt, since it is optimal to smooth the extra tax burden intertemporally.

The situation is much better if the government issues *long-term* debt. The actual debt burden then depends on contemporaneous interest rates, which in turn depend *endogenously* on the state of the economy. If the maturity structure of public debt is carefully chosen *ex ante*, the *ex post* variation in the market value of outstanding long-term debt may offset the contemporaneous variation in the level of fiscal expenditure or the tax base. In good times, interest rates are relatively low and the market value of long-term debt is relatively high. In bad times, the market value of long-term debt falls and the government enjoys a capital gain, which compensates for the increase in fiscal expenditure or the drop in tax revenues.

The theoretical contribution of this paper is to formalize the above arguments within a standard general-equilibrium economy and establish that the maturity structure of non-contingent debt can replicate state-contingent debt. For general aggregate uncertainty, I prove that the government can implement almost every Arrow-Debreu allocation with non-contingent debt of different maturities.

This paper also offers a practical lesson for public policy: Long-term debt can hedge the budget against fiscal shocks. The optimal maturity structure permits the government to keep a constant tax rate despite any variation in fiscal conditions.

In a stylized example, where shocks occur only to fiscal expenditure, the government implements the optimal policy by selling a perpetuity (long-term debt) and investing in a reserve fund (a short-term asset). In good times, the government rolls over a constant balance in the reserve fund. In bad times, she withdraws from the reserve fund to finance the increase in fiscal expenditure. Higher fiscal expenditure raises real returns, which offset the drop in the balance of the short-term asset. This process insures the government against innovations in fiscal expenditure and insulates the budget from risk of refinancing the debt at high interest rates. Finally, because real returns vary inversely with innovations in output, the same scheme insures against output fluctuations as well.

II. Discussion

The neoclassical literature on optimal fiscal policy is immense and I will not attempt a comprehensive review.[2] I will, however, stress the key elements that distinguish my analysis from the pertinent literature: The restriction that the government can trade only in *non-contingent* debt, and the general-equilibrium approach to the maturity structure

[2]See Barro [1989], Chari and Kehoe [1999], and references therein.

of public debt.

Since Lucas and Stokey [1983], the standard paradigm on optimal fiscal policy has maintained the assumption that markets are complete and the government can trade *state-contingent* debt. State-contingent debt works as an *insurance* contract that permits the government to sustain a constant tax rate in the face of aggregate uncertainty.

There are two disturbing facts, however, about the standard paradigm. First, the maturity structure is irrelevant for tax smoothing when state-contingent debt is available. The standard paradigm thus remains silent about the role of the maturity structure.[3] Second, the assumption of state-contingent debt is counterfactual. The standard paradigm thus appears vacuous for a world where governments do not customarily issue state-contingent debt.

Motivated from the last fact, Marcet, Sargent, and Seppala [2000] take the economy of Lucas and Stokey [1983] and restrict the government to issue only risk-free debt, that is, short-term non-contingent debt. Their findings affirm the "random-walk hypothesis" of Barro [1979]. The rate of taxation and the level of debt *permanently* increase after an adverse fiscal shock.

The critical difference in this paper is that the government has the option to issue *long-term* non-contingent debt. The market value of long-term debt varies with equilibrium interest rates. The endogenous variation in the debt burden insures the government

[3] This paper does not consider the alternative role that the maturity structure can play in ensuring time consistency, as in Lucas and Stokey [1983].

against the need to raise either the tax rate or the level of debt when fiscal conditions turn bad.

It is worth noting that the maturity structure can replicate state-contingent debt not only when state-contingent debt is unavailable *by assumption*, but also when market incompleteness is *endogenous*. Suppose that default is not possible and the state of the economy is not verifiable. If the government writes a contingent debt contract with the private sector, she is tempted ex post to misrepresent the state (e.g., the exact level of fiscal spending), in order to pay the lowest possible amount to the private sector. Private agents, however, are rational and anticipate that the government will be tempted to cheat. Thus, the optimal contingent contract may not be feasible. The maturity structure does not face this problem, simply because it is a contract that prescibes *unconditional* payments on specific dates.

Finally, by translating the optimal contingent contract to non-contingent debt, I provide a concrete theory for the optimal maturity structure of public debt, on the basis of Ramsey's [1927] principle for optimal taxation. I identify the optimal maturity structure as the one that insures against fiscal shocks and permits the government to implement an invariant tax rate.[4]

[4]Because nominal debt makes the real debt burden contingent on inflation, one could argue that nominal debt can replicate state-contingent debt. The problem with this argument, however, is that variation in the inflation rate is distortionary. In fact, just as it is optimal to keep that tax rate invariant, it is optimal to keep the inflation rate invariant as well. In that case, nominal and real debt coincide. And then, the optimal maturity structure permits the government to keep both the tax rate and the

My discussion on optimal debt management is related to Roley [1979], Bohn [1990], and Barro [1995]. However, they model interest rates as exogenous and take a simple portfolio approach. I instead take a genuine general-equilibrium approach. General equilibrium allows me to prove that the asset span of non-contingent debt is generically complete and provides sound theoretical foundations for the optimal maturity structure. Finally, the insight that long-term debt can provide insurance appears also in Gale [1990]. However, he considers an orthogonal dimension – how long-term debt facilitates inter-generational risk-sharing in an OLG economy – and does not provide a general result about the relation between state-contingent and non-contingent debt.

The rest of the paper is organized as follows: Section III introduces the model and characterizes what policies are implementable with and without state-contingent debt. Section IV proves that the maturity structure can replicate state-contingent debt and defines the optimal maturity structure under general aggregate uncertainty. Section V focuses on uncertainty in fiscal expenditure and discusses some further insights. Section VI concludes. All proofs are deferred to Appendix A.

III. The Economy

The model is a neoclassical, stochastic production economy similar to Lucas and Stokey [1983]. The economy has a representative private agent and a government. The govern-

inflation rate constant.

ment imposes a linear income tax which distorts the labor-leisure choice of the private agent. Government spending, labor productivity, and tastes are exogenous and stochastic.

A. Preferences, Technology, and Uncertainty

Time is discrete, indexed by $t \in \mathbb{N} \equiv \{0, 1, ...\}$. The exogenous stochastic disturbances in period t are summarized by a random variable $s_t \in \mathbb{S}$. s_t is the 'state' at date t. For convenience, $\mathbb{S} = \{1, 2, ..., S\}$. S is thus the number of possible states. I let $s^t \equiv (s_1, ..., s_t) \in \mathbb{S}^t$, $\forall t \geq 1$. I call s^{t-1} the 'history' and $s^t \equiv (s^{t-1}, s_t)$ the 'event' at date t. $C_t(s^t), X_t(s^t), L_t(s^t), Y_t(s^t), G_t(s_t)$, and $\tau_t(s^t)$ denote, respectively, aggregate consumption, leisure, labor, output, government spending, and the tax rate at date t and event s^t. Finally, $\mu(s^t)$ denotes the unconditional probability of s^t, and $\sigma^n(s^t)$ denotes the set of events at date n that are consistent with a given event s^t at date $t : \sigma^t(s^t) \equiv \{s^t\}$, $\sigma^n(s^t) \equiv \{s^t\} \times \mathbb{S}^{n-t}$ if $n > t$, and $\sigma^n(s^t) \equiv \{s^n \in \mathbb{S}^n | s^t \in \sigma^t(s^n)\}$ if $n < t$.

There is a representative private agent. His preferences are standard, given recursively by $\mathbb{E}_t \mathcal{U}_t = U_t(C_t, X_t, s_t) + \beta \cdot \mathbb{E}_t \mathcal{U}_{t+1}, \forall t \geq 1$. $\mathbb{E}_t$ denotes the expectation operator conditional on s^t. β is the constant discount factor. $U_t\left(C, X, s\right)$ denotes the utility from consumption C and leisure X at date t and state s. $U_t : \mathbb{R}^2_{++} \times \mathbb{S} \rightarrow \mathbb{R}$ is increasing, strictly concave, and smooth, and satisfies the Inada conditions. I frequently use the shortcuts $U(t)$ or $U_t(s^t)$ for $U_t(C_t(s^t), X_t(s^t), s_t)$.

The technology frontier is also separable across time and states. The resource constraint at (t, s^t) is:

$$C_t(s^t) + G_t(s_t) = Y_t(s^t) = F_t(L_t(s^t), s_t) \tag{1}$$

$G_t : \mathbb{S} \to \mathbb{R}_+$ gives the level of fiscal expenditure at date t as a function of the contemporaneous state. $F_t : \mathbb{R}_+ \times \mathbb{S} \to \mathbb{R}_+$ is a neoclassical production function, with $\partial F_t / \partial L > 0 \geq \partial^2 F_t / \partial L^2$. Through its dependence on s_t, the technology specification accommodates exogenous productivity and endowment shocks, as well as productive government services. Finally, total time is normalized to one: $X_t(s^t) = 1 - L_t(s^t)$.

To complete the description of the economy, I need to specify the type of assets the government trades with the private agent. I consider two asset structures. The first corresponds to *complete Arrow-Debreu markets*: The government is permitted to issue state-contingent debt. The second introduces *incomplete markets*: The government is restricted to trade only non-contingent bonds of N different maturities.

B. Arrow-Debreu Markets and State-Contingent Debt

The complete-markets economy is defined by the collection $\mathbf{E} = (\{U_t, F_t, G_t\}_{t=0}^{\infty}, \beta, \mu, \mathbb{S})$. In every period, the government trades a complete contingent debt contract with the private agent. For $s^{t+1} = (s^t, s_{t+1})$, let $d_t(s^{t+1})$ denote the amount of the consumable good that the government promises to deliver to the private agent at $t+1$ if event s^{t+1} (state s_{t+1}) occurs. The contingent contract at (t, s^t) is defined by $D_t(s^t) = (d_t(s^t, s))_{s \in \mathbb{S}} \in \mathbb{R}^S$. Let $q_t(s^{t+1})$ denote the price of an Arrow security issued at $t, s^t = \sigma^t(s^{t-1})$, which promises to pay one unit of the consumption good at $t+1$ if event s^{t+1} occurs. Then, the value of $D_t(s^t)$ at the date of issue is $\sum_{s \in \mathbb{S}} q_t(s^t, s) d_t(s^t, s)$.

Consider the fiscal budget at $t, s^t = (s^{t-1}, s_t)$. The government inherits a debt obligation $d_{t-1}(s^t)$, pays $G_t(s_t)$ in fiscal expenditures, collects $\tau_t(s^t) Y_t(s^t)$ in tax revenues, and receives $\sum_{s \in \mathbb{S}} q_t(s^t, s) d_t(s^t, s)$ from the sale of $D_t(s^t)$. Thus, the budget at (t, s^t) is

$$d_{t-1}(s^t) = \left[\tau_t(s^t) Y_t(s^t) - G_t(s^t)\right] + \sum_{s \in \mathbb{S}} q_t(s^t, s) d_t(s^t, s). \tag{2}$$

The consumer's budget at (t, s^t) is symmetrically given by:

$$C_t(s^t) + \sum_{s \in \mathbb{S}} q_t(s^t, s) d_t(s^t, s) = [1 - \tau_t(s^t)] Y_t(s^t) + d_{t-1}(s^t). \tag{3}$$

The consumer's problem consists of maximizing his life-time utility, $\mathbb{E}_0 \mathcal{U}_0$, subject to the series of budget constraints above.

Definition 1. An allocation $\{C_t(.), L_t(.)\}_{t=0}^{\infty}$ and a policy $\{\tau_t(.)\}_{t=0}^{\infty}$ are implementable with state-contingent debt if there are prices $\{q_t(.)\}_{t=0}^{\infty}$ and contingent debt promises $\{d_t(.)\}_{t=0}^{\infty}$ such that: (**i**) Given $\{\tau_t(.), q_t(.)\}_{t=0}^{\infty}$, $\{C_t(.), L_t(.), d_t(.)\}_{t=0}^{\infty}$ maximize $\mathbb{E}_0 \mathcal{U}_0$ subject to (3). (**ii**) Given $\{C_t(.), L_t(.), q_t(.)\}_{t=0}^{\infty}$, $\{\tau_t(.), d_t(.)\}_{t=0}^{\infty}$ satisfy (2).

The equilibrium price of an Arrow security is equal to the intertemporal rate of substitution in consumption: $q_t(s^{t+1}) = \beta[\mu(s^{t+1})\ \partial U_{t+1}(s^{t+1})/\partial C]/[\mu(s^t)\ \partial U_t(s^t)/\partial C]$, $s^{t+1} \in \sigma^{t+1}(s^t)$. Moreover, the marginal rate of substitution between consumption and leisure must equal the after-tax wage rate:

$$\partial U_t(s^t)/\partial X = [1 - \tau_t(s^t)] \cdot \partial F_t(L_t(s^t), s_t)/\partial L \cdot \partial U_t(s^t)/\partial C. \tag{4}$$

Using equilibrium prices and integrating (2) over all $s^n \in \sigma^n(s^t), n \geq t$, we derive the *intertemporal* budget at (t, s^t):

$$d_{t-1}(s^t) = PV_t(s^t) \equiv \sum_{n=t}^{\infty} \sum_{s^n \in \sigma^n(s^t)} \frac{\beta^n \mu(s^n) \partial U_n(s^n)/\partial C}{\mu(s^t) \partial U_t(s^t)/\partial C} [\tau_t(s^t) Y_t(s^t) - G_t(s^t)]. \quad (5)$$

$PV_t(s^t)$ is the present value of surpluses at (t, s^t), which must always cover the debt obligation of the government.

Under complete Arrow-Debreu markets, the government can *freely* transfer fiscal funds across all different periods and realizations of uncertainty. As a result, the intertemporal budgets for all $t \geq 1$ are redundant:

Proposition 1. An allocation and a policy are implementable with state-contingent debt if and only if: (**i**) They satisfy (1) and (4) at all dates and events; and (**ii**) they satisfy the intertemporal budget at date 0, namely:

$$b_{-1} = \sum_{t=0}^{\infty} \sum_{s^t \in \mathbb{S}^t} \beta^t \mu(s^t) \frac{\partial U_t(s^t)/\partial C}{\partial U_0/\partial C} [\tau_t(s^t) F_t(L_t(s^t), s_t) - G_t(s^t)]. \quad (6)$$

C. Incomplete Markets and Non-Contingent Debt

In this section I introduce the *incomplete-markets* economy, defined by the collection $(\mathbf{E}, N) = (\{U_t, F_t, G_t\}_{t=0}^{\infty}, \beta, \mu, \mathbb{S}, N)$. The government issues only non-contingent zero-coupon bonds. These are real securities that promise to pay one unit of consumption at maturity, independent of the realization of uncertainty. Maturity is indexed by $j \in \{1, 2, ..., N\}$. $b_{t,j}(s^t)$ denotes the stock of bonds issued at date t, event s^t, and promising to pay one unit of consumption at date $t + j$. Their issue price is denoted by $p_{t,j}(s^t)$.

The maturity structure is $B_t = (b_{t,j})_{1 \leq j \leq N} \in \mathbb{R}^N$, and $P_t = (p_{t,j})_{1 \leq j \leq N}$ corresponds to the term structure of interest rates. Finally, $p_{t,0} \equiv 1$.

Without loss of generality, I assume that the government restructures the public debt every period: She first redeems all outstanding debt, and then issues fresh debt at all maturities. The withdrawal of outstanding debt must be financed by the primary surplus, plus the revenue from the sale of new debt. Therefore, the fiscal budget at (t, s^t) is:

$$\sum_{j=0}^{N-1} p_{t,j}(s^t) b_{t-1,j+1}(s^{t-1}) = \left[\tau_t(s^t) Y_t(s^t) - G_t(s_t)\right] + \sum_{j=1}^{N} p_{t,j}(s^t) b_{t,j}(s^t). \tag{7}$$

The left-hand side represents the *effective* debt burden in period t. Although the stock of bonds $b_{t-1,j}$ is non-contingent at $t-1$, its market value at t depends endogenously on s^t, through prices $p_{t,j}$, provided $N \geq 2$.

The consumer's budget at (t, s^t) is symmetrically given by

$$C_t(s^t) + \sum_{j=1}^{N} p_{t,j}(s^t) b_{t,j}(s^t) = [1 - \tau_t(s^t)] Y_t(s^t) + \sum_{j=0}^{N-1} p_{t,j}(s^t) b_{t-1,j+1}(s^{t-1}). \tag{8}$$

The consumer's problem consists of maximizing his life-time utility, $\mathbb{E}_0 \mathcal{U}_0$, subject to the series of budget constraints above.

Definition 2. An allocation $\{C_t(.), L_t(.)\}_{t=0}^{\infty}$ and a policy $\{\tau_t(.)\}_{t=0}^{\infty}$ are implementable with non-contingent debt if there exist interest rates $\{(p_{t,j}(.))_{1 \leq j \leq N}\}_{t=0}^{\infty}$ and non-contingent debt issues $\{(b_{t,j}(.))_{1 \leq j \leq N}\}_{t=0}^{\infty}$ such that: **(i)** given $\{\tau_t(.), (p_{t,j}(.))\}_{t=0}^{\infty}$, $\{C_t(.), L_t(.), (b_{t,j}(.))\}_{t=0}^{\infty}$ maximize $\mathbb{E}_0 \mathcal{U}_0$ subject to (8) and **(ii)** given $\{C_t(.), L_t(.), (p_{t,j}(.))\}_{t=0}^{\infty}$, $\{\tau_t(.), (b_{t,j}(.))\}_{t=0}^{\infty}$ satisfy (7).

The equilibrium price of a non-contingent bond equals the expected marginal rate of substitution in consumption between issuance and maturity: $p_{t,j}(s^t) = \sum_{s^{t+j} \in \sigma^{t+j}(s^t)} \beta^j$ $\left[\mu(s^{t+j}) \partial U_{t+1}(s^{t+j})/\partial C\right] / \left[\mu(s^t) \partial U_t(s^t)/\partial C\right]$. Moreover, condition (4) holds again. Using equilibrium prices and integrating (7) over all $s^n \in \sigma^n(s^t), n \geq t$, we derive the *intertemporal* budget at (t, s^t) as

$$\sum_{j=0}^{N-1} p_{t,j}(s^t) b_{t-1,j+1}(s^{t-1}) = PV_t(s^t). \tag{9}$$

Thus, the present value of surpluses must cover the market value of outstanding debt. (The derivation of (9) is included in the proof of Proposition 2.)

I earlier established (Proposition 1) that the intertemporal budgets for all $t \geq 1$ were redundant when markets were complete. Thanks to state-contingent debt, the government could *freely* transfer funds across different *states* just as well as across different *dates*. Now that markets are incomplete, however, the asset structure restricts the ways the government may transfer funds across different states. As a result, the intertemporal budgets at $t \geq 1$ are no longer redundant.

To understand the nature of these restrictions, consider $N = 1$ as an example. This is the case of risk-free debt as in Marcet, Sargent, and Seppala [2000]. (9) then reduces to $PV_t(., s) = PV_t(., s')$ for all $s, s' \in \mathbb{S}$. That is, for a policy to be implementable with risk-free debt, the present value of surpluses must not vary across states. The example highlights two facts: First, the lack of state-contingent debt restricts the stochastic variation in present value surpluses. Second, the case of risk-free debt is extreme in the sense that it eliminates *any* cross-state variation in present-value surpluses.

To formalize the restrictions that the lack of state-contingent debt implies, I introduce the following two objects, $\forall t \geq 1$: The $S \times N$ matrix

$$Q_t(s^{t-1}) \equiv \begin{bmatrix} 1 & p_{t,1}(s^{t-1},1) & \dots & p_{t,N-1}(s^{t-1},1) \\ 1 & p_{t,1}(s^{t-1},2) & \dots & p_{t,N-1}(s^{t-1},2) \\ \vdots & \vdots & \ddots & \vdots \\ 1 & p_{t,1}(s^{t-1},S) & \dots & p_{t,N-1}(s^{t-1},S) \end{bmatrix},$$

and the $S \times 1$ vector

$$V_t(s^{t-1}) \equiv \begin{bmatrix} PV_t(s^{t-1},1) \\ \vdots \\ PV_t(s^{t-1},S) \end{bmatrix}.$$

$Q_t(s^{t-1})$ corresponds to the span of the term structure of real returns at date t, given history s^{t-1}. $V_t(s^{t-1})$ gives the contemporaneous variation in present value surpluses. I then arrange the intertemporal budgets (9) in matrix form as

$$Q_t(s^{t-1}) \cdot B_{t-1}(s^{t-1}) = V_t(s^{t-1}). \tag{10}$$

Because the choice of $B_{t-1}(s^{t-1})$ is otherwise free, (10) simply requires that $V_t(s^{t-1})$ is spanned by $Q_t(s^{t-1})$. That is, the stochastic variation in the term structure of interest rates must support the contemporaneous variation in present value surpluses. I thus conclude:

Proposition 2. An allocation and a policy are implementable with non-contingent debt if and only if: (**i**) They satisfy (1) and (4) at all dates and events; (**ii**) they satisfy

(6), the intertemporal budget at date 0; and (**iii**) equilibrium interest rates and surpluses satisfy

$$V_t(s^{t-1}) \in Span\left[Q_t(s^{t-1})\right], \quad \forall t \geq 1, s^{t-1} \in \mathbb{S}^{t-1}. \tag{11}$$

D. Interest Rates, Taxes, and Fiscal Shocks

In general equilibrium, allocations and interest rates depend on the rate of taxation and the state of the economy, which includes the level of government spending. An increase in the tax rate induces substitution toward leisure without implicating a wealth effect. Labor supply, output, and consumption thus decrease, causing the marginal utility of consumption to increase. By implication, interest rates increase with contemporaneous taxes and decrease with taxes at the date of maturity. On the other hand, a positive innovation in fiscal expenditure means a negative wealth effect. Consumption and leisure fall, output increases, and marginal utility raises. An adverse fiscal shock thus raises contemporaneous interest rates. These intuitions are formalized below:

Proposition 3. There are mappings χ_t, λ_t, and ω_t ($t \geq 0$) such that $C_t(s^t) = \chi_t(\tau_t(s^t), s_t)$, $L_t(s^t) = \lambda_t(\tau_t(s^t), s_t)$, and $\partial U_t(s^t)/\partial C = \omega_t(\tau_t(s^t), s_t)$. $\partial \chi_t/\partial \tau < 0$, $\partial \lambda_t/\partial \tau < 0$, and $\partial \omega_t/\partial \tau \neq 0$ for generic (U, F). If $\partial^2 U_t/(\partial C \partial X) \geq 0$, $\partial \omega_t/\partial \tau > 0$ and therefore $\partial p_{t,j}(s^t)/\partial \tau_t(s^t) < 0 < \partial p_{t,j}(s^t)/\partial \tau_{t+j}(s^{t+j})$.

Proposition 4. Provided $\partial^2 U_t/(\partial C \partial X) \geq 0$, a positive innovation in fiscal expenditure depresses consumption, expands employment and output, and raises interest rates across all maturities.

Remark. I will use the dependence of interest rates on tax rates to prove that non-contingent debt can replicate state-contingent debt. I will later use the correlation of interest rates with fiscal expenditure to characterize the optimal maturity structure.

IV. The Maturity Structure of Non-Contingent Debt

Define $\mathcal{SCD}$ and $\mathcal{NCD}$ as the sets of policies that are implementable with state-contingent debt and non-contingent debt, respectively. Following Propositions 1 and 2, the only difference between $\mathcal{SCD}$ and $\mathcal{NCD}$ is the series of spanning restrictions (11). But, *when do these restrictions really matter?*

A. Arrow-Debreu Allocations with Non-Contingent Debt

V_t is a vector in $\mathbb{R}^S$, while Q_t is collection of N column vectors in $\mathbb{R}^S$. Whenever $N < S$, $Span[Q_t]$ is necessarily a *proper* subspace of $\mathbb{R}^S$. It follows that:

Lemma 1. (**a**) $\mathcal{NCD} \subsetneqq \mathcal{SCD}\ \forall N, S$. (**b**) $N < S \Rightarrow Closure[\mathcal{NCD}] \subsetneqq \mathcal{SCD}$.

(**c**) $N_1 < N_2 \leq S \Rightarrow Closure\left[\mathcal{NCD}|_{N=N_1}\right] \subsetneqq \mathcal{NCD}|_{N=N_2}$.

Part (**a**) states that there are always Arrow-Debreu allocations that *cannot* be implemented with non-contingent debt. Part (**b**) gives more bite by establishing that $\mathcal{NCD}$ cannot be dense in $\mathcal{SCD}$ if $N < S$: When $N < S$, there are clusters of Arrow-Debreu allocations that are remote from any allocation that can be implemented with state-contingent debt. Finally, part (**c**) means that $\mathcal{NCD}$ increases with N, in a 'dense' sense, as long as $N < S$.[5]

On the other hand, when $N \geq S$, the converse to part (**b**) holds. That is, $\mathcal{NCD}$ is dense in $\mathcal{SCD}$ if $N \geq S$:

Lemma 2. *$N \geq S \Rightarrow Closure[\mathcal{NCD}] = \mathcal{SCD}$.*

The formal proof is presented in Appendix A, but it is important to outline the main argument here: I pick an arbitrary policy $\tau = \{\tau_t(.)\}_{t=0}^{\infty} \in \mathcal{SCD}$, calculate V_t and Q_t at all nodes $t \geq 1, s^{t-1}$, and check whether $V_t \in Span[Q_t]$ at all (t, s^{t-1}). If $V_t \notin Span[Q_t]$ at some (t, s^{t-1}), then and only then the policy fails to be in $\mathcal{NCD}$. In that case, I perturb the allocation so as to force $rank[Q_t] = S$, and thus $V_t \in Span[Q_t]$, at all nodes. I can do this because (from Proposition 3) tax rates at $t+j$ give me control over interest rates at t. It is important, however, that I keep interest rates before t fixed while I perturb interest rates at t. This makes sure that, while I force $rank[Q_t] = S$ at a date t, I do not introduce a pathology before t. Finally, I make sure that the change in tax rates is arbitrarily small in every node, and that it does not break the period-0 intertemporal budget. Therefore, I am able to construct a policy in $\mathcal{NCD}$ that is arbitrarily close to the any policy in $\mathcal{SCD}$.

Combining Lemmas 1 and 2, I conclude:[6]

[5]In general, introducing an asset in an incomplete-markets economy might decrease social welfare. In the present context, however, (**c**) implies that $\sup_{\tau \in \mathcal{NCD}} \mathbb{E}_0 \mathcal{U}_0$ is (generically) increasing in N.

[6]Greg Mankiw first pointed out that my result was reminiscent of results in the finance literature regarding the spanning of contingent claims by long-lived securities. Although I was unaware of this, my formal argument is related to Kreps [1979] and Duffie and Huang [1985].

Theorem 1. Let $S \geq 1$ be the number of states and $N \geq 1$ the number of maturities. When $N \geq S$, then and only then any policy/allocation that is implementable with Arrow-Debreu markets can either be implemented with non-contingent debt or be approximated arbitrarily well.

Corollary. When $N = S$, a generic Arrow-Debreu allocation is implemented with non-contingent debt by setting

$$B_t(s^t) = Q_{t+1}(s^t)^{-1} V_{t+1}(s^t), \; \forall t \geq 0. \tag{12}$$

Remark. In the model, S is identified with the number of different values the state variable can take. However, the number of different sources of uncertainty is what matters quantitatively. For example, if there are only shocks in fiscal expenditure, one long maturity ($N = 2$) is enough. (See Section V.)

B. The Optimal Maturity Structure

The Ramsey plan equates the marginal welfare loss of taxation across *all dates and all events.* Because preferences and the resource constraint at any given t depend only on s_t and not on s^{t-1}, optimal tax rates and optimal allocations (under complete markets) are independent of history. By implication:

Lemma 3. Along the Ramsey plan, $Q_t(s^{t-1}) = \overline{Q}_t$ and $V_t(s^{t-1}) = \overline{V}_t$, $\forall s^{t-1}, t \geq 1$. If the economy is stationary and uncertainty follows a Markov process, then $\overline{Q}_t = \overline{Q}$ and $\overline{V}_t = \overline{V}$, $\forall t \geq 1$.

Theorem 1 implies that the Ramsey allocations can at least be approximated with non-contingent debt. Furthermore, it can be proved that if the state follows a generic Markov process, then $\overline{Q}$ has full rank and thus the Ramsey plan is *itself* implemented with non-contingent debt. (See ch.1 in Angeletos [2001a].)

Definition 3. The optimal maturity structure is the one that implements the Ramsey plan with non-contingent debt when $N = S$.

In an economy without capital, the history-independence of the Ramsey allocations implies that the optimal maturity structure is invariant, not only over all past shocks, but also over the contemporaneous shock:

Theorem 2. The optimal maturity structure at t is independent of s^t. If the economy is stationary and uncertainty follows a Markov process, then the optimal maturity structure is independent of t as well.

C. The Optimal Maturity Structure in an Economy with Capital

My model is an economy without capital, like Lucas and Stokey [1983]. It is important, however, to extend the results to an RBC economy as in Chari, Christiano, and Kehoe [1994]:

Theorem 3. In an a stationary economy with capital, the optimal maturity structure is a function of the contemporaneous optimal investment *alone* and is otherwise independent of s^t and t.

Therefore, the optimal maturity structure is invariant with contemporaneous and past disturbances only once it is conditioned on contemporaneous investment. Moreover, the level of public debt and its maturity structure inherit the cyclical and persistence properties of investment.

Simulating the optimal maturity structure for an RBC economy could provide useful insights about the cyclical properties of optimal debt management. The recipe for this kind of quantitative exercise is now complete: First, simulate the Ramsey allocations under complete markets, along the lines of Chari, Christiano, and Kehoe [1994] and Chari and Kehoe [1999]. Next, calculate interest rates and present-value surpluses along the Ramsey plan. Finally, use $B_t = [Q_{t+1}]^{-1} V_{t+1}$ to uncover the optimal maturity structure.

V. Optimal Policy with Fiscal Shocks

In this section, I characterize the optimal maturity structure when the only (or main) source of uncertainty is innovation in fiscal expenditure. I start with a stylized economy which only has two states.

A. A Stylized Example: War and Peace

The state space is $\mathbb{S} = \{w, n\}$ $\forall t \geq 1$. w represents 'war' and n represents 'normal times' or 'peace'. w and n are serially uncorrelated with probabilities μ_w and μ_n. A 'war' is simply a period of high fiscal expenditure: $G(w) > G(n)$. $\overline{R}(s), \overline{PV}(s)$, and $1/\overline{p}(s)$

denote, respectively, the primary surplus, the present-value of surpluses, and the risk-free rate under the Ramsey allocation, $\forall t \geq 1, s \in \{w, n\}$. R_0, PV_0, and $1/p_0$ correspond to $t = 0$. Finally, for any variable x, $\mathbb{E}x \equiv \mu_n x(n) + \mu_w x(w)$ and $\Delta x \equiv |x(n) - x(w)|$.

Proposition 4 established that, *for a fixed tax rate*, a war increases output, depresses private consumption, and raises interest rates. Because the optimal adjustment in the tax rate is small, these results (typically) extend to the Ramsey allocation as well. I normalize $\partial U/\partial C = 1$ during peace and $\partial U/\partial C = 1/\delta > 1$ during war. To simplify, I set $\partial U/\partial C = 1$ in period 0. It follows that $p_0 = \overline{p}(n) = \beta\left(\mu_n + \mu_w/\delta\right)$ and $\overline{p}(w) = \delta\overline{p}(n)$. $(1-\delta) = \Delta\overline{p}/p_0$ measures the increase in interest rates during a war. A war worsens the fiscal budget in two ways: It reduces the contemporaneous surplus and raises the interest rate at which future surpluses are discounted. Both effects imply $\overline{PV}(w) < \overline{PV}(n)$. Moreover,[7] $\Delta\overline{PV} = \Delta\overline{R} + (\Delta\overline{p}/p_0)(b_{-1} - R_0)$. That is, the variation in the present value of surpluses equals the variation in the primary surplus plus the variation in the cost of refinancing the historical debt.

In Appendix B, I show how to implement the optimal policy with continuous trading of a one-period and a two-period bond. I also present a numerical example and discuss comparative statics of the optimal maturity structure. Below, I show how to implement the optimal policy with an alternative scheme that involves small transactions: *Selling a perpetuity once in period* 0 *and investing each period in a reserve fund.*

[7] It follows from $b_{-1} = PV_0$ and from $PV_0 = R_0 + b^*$, $PV(n) = R(n) + b^*$, and $PV(w) = R(w) + \delta b^*$, where $b^* \equiv \beta[\mu_n PV(n) + \mu_w PV(w)/\delta]$.

First, consider period 0. The government sells a perpetuity which promises to pay a fixed coupon κ in every $t \geq 1$. The price of this perpetuity is $\pi = \sum_{t=1}^{\infty} \beta^t (\mu_n \kappa + \mu_w \kappa/\delta) = p_0 \kappa/(1-\beta)$. The government also invests an amount z in a short-term asset, which I call the 'reserve fund.' The period-0 budget implies $z + b_{-1} = \pi + R_0$. Next, consider period 1. (Along the Ramsey plan, periods $t > 1$ are isomorphic to period 1.) The short-term rate between 0 and 1 is $1/p_0$. The reserve fund thus opens with balance z/p_0. The government pays κ for the coupon of the perpetuity. The market value of the remaining perpetuity is $\sum_{t=2}^{\infty} \beta^{t-1} (\mu_n \kappa + \mu_w \kappa/\delta) = \pi$ during peace and $\sum_{t=2}^{\infty} \beta^{t-1} \delta (\mu_n \kappa + \mu_w \kappa/\delta) = \delta\pi$ during war. Therefore, the intertemporal budget is $\kappa + \pi - z/p_0 = \overline{PV}(n)$ in the event of peace, and $\kappa + \delta\pi - z/p_0 = \overline{PV}(w)$ in the event of war. It follows that $(\Delta \overline{p}/p_0)\pi = \Delta \overline{PV}$. That is, the right π makes sure that the realized variation in the market value of the perpetuity matches the desired variation in the present value of surpluses. Finally, combining with $\Delta \overline{PV} = \Delta \overline{R} + (\Delta \overline{p}/p_0)(b_{-1} - R_0)$ and $p_0 z + b_{-1} = \pi + R_0$, I conclude:

$$z = \frac{\Delta \overline{R}}{\Delta \overline{p}/p_0} \quad \text{and} \quad \pi = [b_{-1} - R_0] + \frac{\Delta \overline{R}}{\Delta \overline{p}/p_0}.$$

The optimal investment in the reserve funds makes sure that the increase in real returns during war is just enough to compensate for the shortfall in the primary surplus. The optimal perpetuity, on the other hand, is equal to this investment plus the historical level of debt.

I now can describe the optimal policy *in action*: At date 0, the government transforms all outstanding public debt to a perpetuity and sells an additional perpetuity to invest in the reserve fund. As long as peace prevails, the government rolls over a constant balance in the reserve fund. Whenever a war occurs, the government raises neither taxes nor debt. She simply withdraws the expenses of the war from the reserve fund. The reserve fund thus closes with a lower balance. A war, however, brings higher real returns, and the increase in return compensates for the drop in balance. This way, the government manages to insure herself against fiscal shocks and to implement a roughly invariant tax rate.

Remark 1. The positive correlation between real returns and fiscal expenditure drives the result that holding long-term debt and a short-term asset hedges against fiscal shocks. In this aspect, I am standing on fairly sound foundations: The prediction that real returns, output, and work hours increase with government spending is common ground in macro theory. (See, e.g., Blanchard [1981], Turnovsky and Fisher [1992], Aiyagari et al. [1992], Baxter and King [1993], and Barro [1995].) Most importantly, there is substantial empirical evidence that real returns, together with output and work hours, increase with a positive innovation in fiscal expenditure. (See, e.g., Barro [1987], Plosser [1982, 1987], Plosser and Rouwenhorst [1994], and Evans [1987].)

Remark 2. If $\overline{R}(n) = \overline{R}(w)$, $z = 0$ and $\pi = b_{-1} - R_0$. If the primary surplus is invariant, there is no need to hold a reserve fund. However, it is still optimal to transform the initial debt to a perpetuity, so as to insulate the budget from the risk of refinancing public debt at variable interest rates.

Remark 3. Prescott et al. [1983], Plosser [1987], Fama and French [1989], King and Watson [1996], and Seppala [2000] provide evidence that real returns increase with negative innovations in output. If this is the case, long-term debt hedges the budget against output fluctuations as well. Seppala [2000] reports standard deviations for the real returns of British indexed bonds in the range of $4\% - 5\%$. If the British government kept a long-term debt position in the range of $50\% - 100\%$ of GDP, and if a recession raised the real return by about one standard deviation, the treasury department would enjoy a capital gain in the range of $2\% - 5\%$ of GDP during a recession. These numbers are of course only suggestive and serious quantitative analysis is still required.

B. The Number of Maturities, States, and Shocks

In the stylized economy I presented above, there were only two states ($S = 2$). Two maturities ($N = 2$) were thus enough to implement the optimal policy. But, what if S is much larger, say $S = 100$? If we take Theorem 1 *too* literally, $N = 100$ is necessary for *perfect* insurance. However, $N = 2$ (one short and one long maturity) can still provide substantial hedging. For a plausible numerical example that I present in Appendix B, as much as 97% of the optimal variation in present-value surpluses can be supported with $N = 2$ when $S = 100$. The reason for this result is that when there is only one macroeconomic shock in the economy, present value surpluses and interest rates are monotonic transformations of that shock and are nearly collinear with each other.

This insight suggests that what really matters is not S per se, but rather the types of different macroeconomic shocks.[8] If shocks in fiscal expenditure are the only (or main)

[8] In continuous time, a natural conjecture would be the following: If there are $F \geq 1$ aggregate random factors, $N = F + 1$ implements the Ramsey policy with non-contingent debt.

source of aggregate uncertainty, holding long-term debt and investing in a short-term asset is enough to implement the optimal policy. But, suppose we introduce uncertainty in productivity and tastes (interest rates) as well. How well can the government do with just two debt instruments? Holding a perpetuity and a short-term asset will continue to hedge the budget against both fiscal and output shocks, as well as against the risk of refinancing the debt at variable interest rates. On the other hand, this hedging will be incomplete, and optimal taxes and debt will necessarily exhibit a unit-root component.

Remark. The complete-markets paradigm predicts that optimal tax rates should be roughly invariant, while the random-walk paradigm suggests that they should exhibit unit-root persistence. One might infer that testing for a unit root in tax rates is enough to distinguish the two paradigms. However, *any* degree of incomplete insurance generates a unit-root component in taxes. The latter is just the manifestation of intertemporal tax smoothing. To confront the two paradigms, one needs to compare the variance of this unit-root component with the variance of the innovation in the annuity value of fiscal spending. The complete-markets paradigm predicts that the ratio of the two is close to zero. The random-walk paradigm predicts that this ratio is close to one. Whether actual policy is closer to zero or one is still an open question.

VI. Concluding Remarks

If the maturity structure of non-contingent debt is carefully chosen, the equilibrium variation in the market value of public debt can hedge the budget against random variation in fiscal conditions. Within a standard neoclassical general-equilibrium economy, almost every Arrow-Debreu allocation can be implemented with non-contingent debt of different maturities. The optimal maturity structure is that which provides perfect insurance and permits the government to sustain an invariant rate of taxation.

Critical for my argument that the maturity structure can provide *perfect* insurance is that the government knows *exactly* how equilibrium interest rates will fluctuate with relevant macroeconomic shocks. It is precisely this knowledge that permits the government to *exactly* implement the desirable variation in the market value of public debt by carefully choosing the maturity structure.

There are at least two reasons why the government may not enjoy this kind of knowledge: Noise in asset prices and uncertainty about the model of the economy. If real returns are subject to shocks that are orthogonal to the variation in present-value surpluses, the optimal maturity structure will face a trade off between hedging against fiscal shocks and infusing noise in the debt burden.

It is worth formalizing the above intuitions within a model that permits the government to design the maturity structure of public debt, but introduces noise in interest rates. The presence of uninsurable risks will generate a martingale in tax rates as in Marcet, Sargent, and Seppala [2000]. However, because interest rates will continue to increase with positive innovations in fiscal expenditure or negative innovations in output, the nature of the optimal maturity structure will survive.

To recap, this paper proved that, within the neoclassical framework, the maturity structure of non-contingent debt can replicate state-contingent debt and permit the government to sustain an invariant tax rate. Of course, I do not suggest by this result that we should abandon research on policy design under *incomplete* markets. To the contrary. Governments are unavoidably subject to uninsurable shocks and work such as Marcet, Sargent, and Seppala [2000] and Marcet and Scott [2001] is very important for understanding both the positive and the normative aspects of fiscal policy. However, I will reiterate the main policy conclusion of this paper:

> Holding long-term debt and investing in a short-term asset can hedge the budget against both random variation in fiscal expenditure and aggregate income, as well as against the risk of refinancing the outstanding debt at variable interest rates.

This is probably more important for economies that face large fiscal and interest-rate risks, but also could be an input in the recent debate about reducing and restructuring the public debt of the United States.

Appendix A: Proofs

Proof of Proposition 1. This result is standard. See, e.g., Lucas and Stokey [1983] or Chari and Kehoe [1999].

Proof of Proposition 2. Let $R_t \equiv \tau_t Y_t - G_t$, $P_t \equiv (p_{t,1}, ..., p_{t,N})$, $B_t \equiv (b_{t,1}, ..., b_{t,N})$, and rewrite the budget (7) as

$$R_t(s^t) = \left[\begin{smallmatrix}1\\P_t(s^t)\end{smallmatrix}\right]' \left[\begin{smallmatrix}\mathbf{I}\\\mathbf{0}'\end{smallmatrix}\right] B_{t-1}(s^{t-1}) - P_t(s^t)' B_t(s^t).$$

A prime ($'$) denotes vector transpose, $\mathbf{I}$ is the $N \times N$ identity matrix, and $\mathbf{0}$ is a column vector of N zeros. Define $q_{t,n}(s^{t+n}|s^t) \equiv \beta^n[\mu(s^{t+n})\partial U_{t+n}(s^{t+n})/\partial C]/[\mu(s^t)\partial U_t(s^t)/\partial C]$. Observe that $q_{t,1}(s^{t+1}|s^t) \equiv q_t(s^t, s_{t+1})$, $q_{t,n+1}(s^{t+n+1}|s^t) = q_{t,n}(s^{t+n}|s^t)q_{t+n}(s^{t+n}, s_{t+n+1})$. Step 1 below proves that the budgets imply (11). Step 2 proves the converse.

Step 1. I fix (t, s^t). I take the budget at $(t+n, s^{t+n})$, multiply both sides with $q_{t,n}(s^t, s^{t+n})q_{t,n+1}(s^{t+n+1}|s^t) = q_{t,n}(s^{t+n}|s^t)q_{t+n}(s^{t+n}, s_{t+n+1})$, and sum over all $s^{t+n} \in \sigma^{t+n}(s^t), n \geq 0$, to obtain:

$$\begin{aligned} PV_t(s^t) &\equiv \sum_{n=0}^{\infty}\sum_{s^{t+n}} q_{t,n}(s^{t+n}|s^t)R_{t+n}(s^{t+n}) = \\ &= \sum_{n=0}^{\infty}\sum_{s^{t+n}} q_{t,n}(s^{t+n}|s^t)\left\{\left[\begin{smallmatrix}1\\P_{t+n}(s^{t+n})\end{smallmatrix}\right]'\left[\begin{smallmatrix}\mathbf{I}\\\mathbf{0}'\end{smallmatrix}\right]B_{t+n-1}(s^{t+n}) - P_{t+n}(s^{t+n})'B_{t+n}(s^{t+n})\right\} = \\ &= \left[\begin{smallmatrix}1\\P_t(s^t)\end{smallmatrix}\right]'\left[\begin{smallmatrix}\mathbf{I}\\\mathbf{0}'\end{smallmatrix}\right]B_{t-1}(s^{t-1}) + \sum_{n=0}^{\infty}\sum_{s^{t+n}} q_{t,n}(s^{t+n}|s^t)\times \\ &\quad\times\left\{-P_t(s^{t+n})' + \sum_{s^{t+1}} q_{t+n}(s_{t+n}, s_{t+n+1})\left[\begin{smallmatrix}1\\P_{t+n+1}(s^{t+n}, s_{t+n+1})\end{smallmatrix}\right]'\left[\begin{smallmatrix}\mathbf{I}\\\mathbf{0}'\end{smallmatrix}\right]\right\}B_{t+n}(s^{t+n}). \end{aligned}$$

Because $p_{t+n,j}(s^{t+n}) = \sum_{s_{t+1}}[q_{t+n}(s_{t+n}, s_{t+n+1})p_{t+n+1,j-1}(s^{t+n}, s_{t+n+1})], \forall j$, the term in the brackets vanishes, implying $PV_t(s^t) = \sum_{j=0}^{N-1} p_{t,j}(s^t)b_{t-1,j}(s^{t-1})$. Evaluating this at $t = 0$ gives (6). Arranging over all $s_t \in S$ at $t \geq 1, s^{t-1}$, gives $V_t(s^{t-1}) = Q_t(s^{t-1})B_{t-1}(s^{t-1})$, which implies (11).

Step 2. (11) implies that, $\forall s^{t-1} \in \mathbb{S}^{t-1}, t \geq 1$, there is *some* $B_{t-1}(s^{t-1}) \in \mathbb{R}^N$ such that $V_t(s^{t-1}) = Q_t(s^{t-1})B_{t-1}(s^{t-1})$, and therefore $\sum_{j=0}^{N-1} p_{t,j}(s^t)b_{t-1,j+1}(s^{t-1}) = PV_t(s^t)$, $\forall s^t \in \sigma^t(s^{t-1})$. By definition, $PV_t(s^t) = R_t(s^t) + \sum_{s\in\mathbb{S}} q_t(s^t, s)PV_{t+1}(s^t, s)$. Using $PV_{t+1}(s^t, s) = \sum_{j=0}^{N-1} p_{t+1,j}(s^t, s)b_{t,j+1}(s^t, s)$ and $\sum_s[q_t(s_t, s)p_{t+1,j-1}(s^t, s)] = p_{t,j}(s^t)$, I obtain $\sum_{j=0}^{N-1} p_{t,j}(s^t)b_{t-1,j+1}(s^{t-1}) = R_t(s^t) + \sum_{j=1}^{N} p_{t,j}(s^t)b_{t,j}(s^t)$.

Proof of Proposition 3. To simplify notation, drop the dependence on t. $C = \chi(\tau, s)$ and $L = \lambda(\tau, s)$ are defined by $U_X(C, 1-L, s) = (1-\tau)F_L(L, s)U_C(C, 1-L, s)$ and $C + G(s) = F(L, s)$. Applying the implicit function theorem, $\partial\chi/\partial\tau = (\partial F/\partial L)^2/D$ and $\partial\lambda/\partial\tau = (\partial F/\partial L)/D$, where D is the Jacobian determinant. Concavity of U and F (with at least one being strictly concave) imply $D < 0$. Therefore, $\partial\chi/\partial\tau < 0$ and $\partial\lambda/\partial\tau < 0$. Next, define $\upsilon(\tau, s) \equiv U(\chi(\tau, s), 1-\lambda(\tau, s), s)$ and $\omega(\tau, s) \equiv U_C(\chi(\tau, s), 1-\lambda(\tau, s), s)$. $\partial\upsilon/\partial\tau = (1/D)F_L(U_C F_L - U_X) = (1/D)F_L^2 U_C \tau$. Thus, $\tau = 0 \Rightarrow \partial\upsilon/\partial\tau = 0$ and $\tau > 0 \Rightarrow \partial\upsilon/\partial\tau < 0$. $\partial\omega/\partial\tau = (1/D)F_L[U_{CC}F_L - U_{CX}]$. Thus, $\partial\omega/\partial\tau \neq 0$ for generic (U, F) and $\partial\omega/\partial\tau > 0 \Leftrightarrow U_{CX} > U_{CC}F_L$ (< 0). Finally, $p_j(s^t) = \sum_{\varsigma\in\sigma^{t+j}(s^t)}[\beta^j\mu(\varsigma)\omega(\tau_{t+j}(\varsigma), \varsigma)]/[\mu(s^t)\omega(\tau_t(s^t), s^t)] \equiv \phi_j(\tau_t(s^t), \tau_{t+j}(\sigma^{t+j}(s^t)), s^t)$. (If μ is a Markov process, $\phi_j(., s^t)$ can be redefined as $\phi_j(., s_t)$.) The derivatives of ϕ_j then follow from those of ω.

Proof of Proposition 4. Assume G is sufficient statistic for the state and U, F are independent of G. Redefine $C = \chi(\tau, G)$ and $L = \lambda(\tau, G)$ by $U_X = (1-\tau)F_L U_C$ and $C + G = F(L)$, and $\omega(\tau, G) \equiv \partial U(\chi(\tau, G), 1-\lambda(\tau, G))/\partial C$. Letting again $D < 0$ be the Jacobian determinant, $\partial\lambda/\partial G = -(1/D)(U_C U_{CX} - U_X U_{CC})/(U_{CC})^2$ and $\partial\chi/\partial G = -(1/D)[U_C U_{XX} - U_X U_{CX} + (1-\tau)U_C^2 F_{LL}]/(U_{CC})^2 \leq -(1/D)(U_C U_{XX} - U_X U_{CX})/(U_{CC})^2$. Thus, $U_{CX} \geq 0$ is sufficient for $\partial\chi/\partial G > 0$ and $\partial\lambda/\partial G > 0$, and then $\partial\omega/\partial G > 0$.

Proof of Lemma 1. Part (**a**) and is obvious from Propositions 1 and 2. The proof of parts (**b**) and (**c**) uses a perturbation argument similar to that of Lemma 2 (see below): When $N < S$, $rank[Q_t] \leq N < S$, implying that $Span[Q_t]$ is never as large as $\mathbb{R}^S$, where V_t lives. If $\tau \in \mathcal{SCD}$ implies $V_t \notin Span[Q_t]$ at some t, any $\tau' \in \mathcal{SCD}$ close enough to τ will fail to induce $V_t \in Span[Q_t]$. Similarly, $Span[Q_t|_{N=N_1}]$ is never as large as $Span[Q_t|_{N=N_2}]$, provided $N_1 < N_2 \leq S$. If a particular $\tau \in \mathcal{NCD}|_{N=N_2}$ implies $V_t \notin Span[Q_t|_{N=N_1}]$, any τ' close enough to τ will fail $V_t \in Span[Q_t|_{N=N_1}]$.

Proof of Lemma 2. Assume $N \geq S \geq 2$ ($S = 1$ is the trivial deterministic case). $V_t \in \mathbb{R}^S$, $Q_t = Q_t(s^{t-1})$ is an $S \times N$ matrix, $rank[Q_t] \leq \min\{S, N\} = S$, and $rank[Q_t] = S$ is sufficient for $V_t \in Span[Q_t]$. Take any $\tau = \{\tau_t(.)\}_{t=0}^{\infty} \in \mathcal{SCD}$. If $V_t \in Span[Q_t]$ for *all* t, s^{t-1}, then $\tau \in \mathcal{NCD}$, and I am done. Consider τ such that $V_t \notin Span[Q_t]$ for *some* t, s^{t-1}. I start ascending the date-event tree till I hit the first pathological node. I then follow Steps 1 and 2 below:

Step 1. From Proposition 3, $\partial U_{t+j}(s^{t+j})/\partial C = \omega_{t+j}(\tau_{t+j}(s^{t+j}), s_{t+j}), \partial\omega_{t+j}/\partial\tau_{t+j} \neq 0$, and $p_{t,j}(s^t) = \beta^j \mathbb{E}_t(\partial U_{t+j}/\partial C)/(\partial U_t/\partial C) = \phi_{t,j}(\tau_t(s^t), \tau_{t+j}(\sigma^{t+j}(s^t)), s^t), \partial\phi_{t,j}/\partial\tau_{t+j} \neq 0$. Perturbing $\partial U_{t+j}(s^{t+j})/\partial C$ is thus equivalent to perturbing $\tau_{t+j}(s^{t+j})$ and $\tau_{t+j}(\sigma^{t+j}(s^t)) \equiv \{\tau_{t+j}(s^{t+j}) \mid s^{t+j} \in \sigma^{t+j}(s^t)\}$ is a set of controls for $p_{t,j}(s^t)$. Moreover, $\forall s, s' \in \mathbb{S}$ and $j, j' \geq 1$, $p_{t,j'}(s^{t-1}, s')$ is invariant with $\tau_{t+j}(\sigma^{t+j}(s^{t-1}, s))$, if and only if $s' \neq s$ or $j' \neq j$. Thus, for given $j \in \{0, ..., N-1\}, s \in \mathbb{S}$, perturbing $\tau_{t+j}(\sigma^{t+j}(s^{t-1}, s))$ affects only element $(s, j+1)$ and no other element of matrix $Q_t(s^{t-1})$. By perturbing $\tau_{t+j}(\sigma^{t+j}(s^{t-1}, s))$ for *all* $j \in \{0, ..., N-1\}, s \in \mathbb{S}$, I can definitely obtain $rank[Q_t(s^{t-1})] = S$.

Step 2. $\partial U_{t+j}/\partial C$ enters, not only in $p_{t,j} = \beta^j \mathbb{E}_t(\partial U_{t+j}/\partial C)/(\partial U_t/\partial C)$, but also $p_{t-k,k+j} = \beta^j \mathbb{E}_{t-k}(\partial U_{t+j}/\partial C)/(\partial U_{t-k}/\partial C)$ for *all* $k \in \{1, ..., t\}$. Therefore, perturbing $\tau_{t+j}(\sigma^{t+j}(s^{t-1}, s))$ generally affects not only $Q_t(s^{t-1})$, but also $Q_k(s^{k-1})$, $\forall k \in \{t - N +$

$1 \leq k \leq t-1\}$, $k \geq 1$, $s^{k-1} = \sigma^{k-1}(s^{t-1})$. This feedback from τ_{t+j} to Q_k, however, works only through $\mathbb{E}_{t-k}(\partial U_{t+j}/\partial C) = \mathbb{E}_{t-k}[\mathbb{E}_{t-1}(\partial U_{t+j}/\partial C)]$. And $Q_k(s^{k-1})$ is unaffected if $s^{k-1} \neq \sigma^{k-1}(s^{t-1})$ or $k \leq t-N$. To make sure that no earlier node is affected, it is thus sufficient to keep $\mathbb{E}[(\partial U_{t+j}/\partial C)|s^{t-1}]$ remains invariant, $\forall j \in \{1, ..., N-1\}$. From Proposition 3, $\mathbb{E}[(\partial U_{t+j}/\partial C)|s^{t-1}]$ is a single non-linear constraint on $\{\tau_{t+j}(\sigma^{t+j}(s^{t-1}, s)) : 1 \leq s \leq S\}$. Moreover, letting $\theta(s) \equiv \omega_t(\tau_t(s^{t-1}, s), (s^{t-1}, s))\mu(s^{t-1}, s)/\mu(s^{t-1})$, we can write $\beta^{-j}\mathbb{E}[(\partial U_{t+j}/\partial C)|s^{t-1}] = \sum_{s \in \mathbb{S}} \theta(s) \cdot p_{t,j}(s^{t-1}, s)$. The latter is a simple linear constraint over the S elements of column $j+1$ of matrix $Q_t(s^{t-1})$. Therefore, I can freely perturb $S-1$ elements of column $j+1$ of $Q_t(s^{t-1})$ so as to obtain full rank for $Q_t(s^{t-1})$ and at the same time ensure no effect on earlier nodes.

I proceed forward in the event tree, repeating Steps 1 and 2 at all pathological nodes. Finally:

Step 3. $\mathcal{SCD}$ and $\mathcal{NCD}$ are subsets of the space of sequences $x = \{x_t\}_{t=0}^{\infty}$, $x_t \in \mathbb{R}^{S^t}$. I endow this space with the norm $||x||^* \equiv \sup_{t \geq 0}\{S^{-t/2}||x_t||\}$, $||x_t|| \equiv \sqrt{x_t' x_t}$. Let $\tau = \{\tau_t(.)\}_{t=0}^{\infty} \in \mathcal{SCD}$ denote the original policy and $\widehat{\tau} = \{\widehat{\tau}_t(.)\}_{t=0}^{\infty}$ the perturbed one; and consider any $\varepsilon, \varepsilon_0 > 0$. $\tau_t(s^t)$ enters $Q_{t-k}(s^{t-k})$, $s^{t-k} = \sigma^{t-k}(s^t)$, for all $k \in \{1, ..., N-1\}$. Therefore, $\tau_t(s^t)$ is perturbed at most $N-1$ times. I limit each such perturbation to be $< \varepsilon/(N-1)$. Thus, $|\tau_t(s^t) - \widehat{\tau}_t(s^t)| < \varepsilon \ \forall \geq 1, s^t$. Next, I need to adjust τ_0 to make sure that $\widehat{\tau}$ satisfies (6), the period-0 intertemporal budget. (6) is continuous in $\tau_t(s^t)$ $\forall s^t, t \geq 1$. The first partial derivative of (6) with respect to τ_0 is non-zero for almost every $\tau \in \mathcal{SCD}$ (τ_0 is not on the top of the Laffer curve). Therefore, the adjustment in τ_0 is both feasible and small: $\exists\varepsilon > 0$ such $\widehat{\tau} \in \mathcal{SCD}$ and $|\tau_0 - \widehat{\tau}_0| < \varepsilon_0$. Therefore, $||\tau_t - \hat{\tau}_t|| = \{\sum_{s^t}[\tau_t(s^t) - \hat{\tau}_t(s^t)]^2\}^{1/2} < S^{t/2}\varepsilon$ for all all $t \geq 1$ and $|\tau_0 - \widehat{\tau}_0| < \varepsilon_0$, implying $||\tau - \hat{\tau}||^* \leq \max\{\varepsilon_0, \varepsilon\}$.

To recap: I just constructed a $\widehat{\tau} \in \mathcal{SCD}$ that is arbitrarily close to $\tau \in \mathcal{SCD}$ and obtains full rank at all $t \geq 1$.

Proof of Theorem 1. It follows from Lemmas 1 and 2.

Proof of Lemma 3. That the Ramsey allocations and tax rates are independent of s^{t-1} is established, e.g., in Lucas and Stokey [1983] and Chari and Kehoe [1999]. It follows that $PV_t, p_{t,j}$ and thus V_t, Q_t, are independent of $s^{t-1}: V_t(s^{t-1}) = \overline{V}_t, Q_t(s^{t-1}) = \overline{Q}_t$. If $(U_t, F_t, G_t) = (U, F, G)$ and $\mu(s^{t-1}, s, s') = \mu(s^{t-1})\mu_{s,s'}$, the Ramsey allocations, and therefore $\overline{V}_t, \overline{Q}_t$, are independent of t as well.

Proof of Theorem 2. Assume $N = S = rank[\overline{Q}_t], \forall t \geq 1$. From (12) and Lemma 3, $B_t(s^t) = \overline{B}_t \equiv \overline{Q}_{t+1}^{-1}\overline{V}_{t+1}, \forall t \geq 0$.

Proof of Theorem 3. k_t denotes the capital stock at t. The state at t is (s_t, k_t). Allocations and prices are conditional on (s^t, k_t). Redefine $B_t(s^t, k_t) \equiv [b_{t,j}(s^t, k_t)]_{1\leq j\leq N}$, $V_t(s^{t-1}, k_t) \equiv [PV(s^{t-1}, s_t, k_t)]_{1\leq s_t\leq S}$, and $Q_t(s^{t-1}, k_t) \equiv [p_{t,j-1}(s^{t-1}, s_t, k_t)]_{1\leq s_t\leq S}^{1\leq j\leq N}$, and rewrite (10) as $Q_t(s^{t-1}, k_t) \cdot B_{t-1}(s^{t-1}, k_{t-1}) = V_t(s^{t-1}, k_t)$. Zhu [1992] and Chari, Christiano, and Kehoe [1994] prove that the Ramsey allocations and policies at t are independent of s^{t-1}. Let $\overline{k}(s, k)$, $\overline{p}_j(s, k)$, and $\overline{PV}(s, k)$ denote, respectively, the rate of investment, the price at maturity j, and the present value of surpluses at state (s, k) under the Ramsey plan. Define $\overline{V}(k) \equiv \left[\overline{PV}(s, k)\right]_{1\leq s\leq S}$, $\overline{Q}(k) \equiv \left[1\ \overline{p}_1(s, k)\ ...\ \overline{p}_{N-1}(s, k)\right]_{1\leq s\leq S}$, and assume $N = S = rank[\overline{Q}(k)]$, $\forall k$. Then, $B_{t-1}(s^{t-1}, k_{t-1}) = \left[\overline{Q}(k_t)\right]^{-1} \overline{V}(k_t)$, $k_t = \overline{k}(s_{t-1}, k_{t-1})$.

Appendix B: A Numerical Example

Consider the wars-and-peace economy I introduced in Section V. $\mathbb{S} = \{w, n\}$ and $G(w) > G(n) \geq 0$. To derive closed-form results, I assume linear technology and isoelastic preferences: $Y = L$ and $U(C, X) = (C^{\alpha} X^{1-\alpha})^{1-\gamma}/(1-\gamma)$, $\gamma > 0$, $0 < \alpha < 1$. $\overline{B} = (\overline{b}_{short}, \overline{b}_{long})$ denotes the optimal maturity structure. The government trades a one-period (short) and a two-period (long) bond. $\overline{B} = \overline{Q}^{-1}\overline{V}$ reduces to $\overline{b}_{short} = \mathbb{E}\overline{PV} - \overline{b}_{long}\mathbb{E}\overline{p}_1$ and $\overline{b}_{long} = \Delta\overline{PV}/\Delta\overline{p}_1$. $\overline{PV}(w) < \overline{PV}(n)$ and $\overline{p}(w) < \overline{p}(n)$ imply $\overline{b}_{long} > 0$. Besides, following the results in Section V, $\overline{b}_{long} \approx \pi \approx b_{-1} + \Delta\overline{R}/\Delta\overline{p}$ and $\overline{b}_{short} \approx -z \approx \Delta\overline{R}/\Delta\overline{p}$, where π is the perpetuity and z is the reserve fund.

Baseline example. Preferences are $\beta = .97, \gamma = 3$, and $\alpha = 2/3$. (The fraction of labor time devoted to production is 2/3 and the coefficient of relative risk aversion is $\rho \equiv \alpha\gamma = 2$.) Probabilities are $\mu_w = \mu_n = .5$. $\mathbb{E}G$ is calibrated to 25% of $\mathbb{E}Y$, a 'war' means $\Delta G = 5\%$ of $\mathbb{E}Y$, and initial debt is 40% of $\mathbb{E}Y$. For this parametrization, $\overline{B} \equiv (\overline{b}_{short}, \overline{b}_{long}) = (-31\%, +72\%) \times \mathbb{E}Y$. That is, the government transforms the initial debt to long-term debt and issues another 30% of GDP in long-term debt, which she invests in a short-term asset. I next consider some comparative statics.

Initial debt. Any change in the historical debt is reflected almost exclusively in the long maturity. $b_{-1} - R_0 = 0 \Rightarrow \overline{B} = (-32\%, +33\%)$; $b_{-1} - R_0 = 40\% \Rightarrow \overline{B} = (-31\%, +72\%)$, and $b_{-1} - R_0 = 50\% \Rightarrow \overline{B} = (-31\%, +82\%)$.

Magnitude of shocks. $\Delta\overline{R}$ and $\Delta\overline{p}$ tend to increase almost linearly with ΔG. As a result, ΔG has almost no effect on $\overline{B}$. $\Delta G = 10\% \Rightarrow \overline{B} = (-30\%, +71\%)$, while $\Delta G = 5\% \Rightarrow \overline{B} = (-31\%, +72\%)$.

Risk Aversion. Given ΔG, $\Delta\overline{p}$ tends to increase with $\rho \equiv \alpha\gamma$, the coefficient of relative risk aversion, while $\Delta\overline{R}$ is almost invariant with ρ. As a result, $z \approx -\overline{b}_{short}$ tends to decrease with ρ. $\rho = 1 \Rightarrow \overline{B} = (-60\%, +102\%)$; $\rho = 2 \Rightarrow \overline{B} = (-31\%, +72\%)$; and $\rho = 4 \Rightarrow \overline{B} = (-16\%, +56\%)$.

Likelihood of war. The unconditional probabilities per se do not appear to matter. $\mu_w = .1, \mu_n = .9 \Rightarrow \overline{B} = (-29\%, +71\%)$, while $\mu_w = \mu_n = .5 \Rightarrow \overline{B} = (-31\%, +72\%)$.

The Persistence of Shocks. In an economy without capital, more persistence in exogenous shocks translates to more persistence in optimal consumption and less variation in interest rates. By implication, the investment in the short-term asset increase with persistence. If I increase the probability of staying in the same state from .50 (baseline example) to .75, I get $\overline{b}_1 = -160\%$ and $\overline{b}_2 = +206\%$. As persistence tends to unit root, interest-rate variation shrinks to zero, and $-\overline{b}_1, \overline{b}_2$ explode to infinity. *However*, this disturbing result is mostly an artefact an economy without capital. In a standard RBC economy, a permanent increase in fiscal expenditure triggers an increase in the steady-state level of capital. Consumption growth and interest rates initially increase and then slowly decrease as the economy converges to the new steady state. (See, e.g., Turnovsky and Fisher [1992], Aiyagari et al. [1992], Baxter and King [1993].) Therefore, the variation of interest rates stays away from zero even when persistence tends to unit root. The maturity structure can then hedge the budget against even perfectly persistent shocks.

The number of states. $N = 2$ and $S = 100$. I calibrate $G(s)$ to vary linearly between 20% and 30% of GDP. The rest of the parameter values are as in the baseline case. I simulate the Ramsey plan and calculate $\overline{V}$ and $\overline{Q}$. $\overline{V}$ is 100×1 and $\overline{Q}$ is 100×2. The first column of $\overline{Q}$ is a column of units. The second column corresponds to the variation in the short-term rate. I regress $\overline{V}$ on $\overline{Q}$ and compute the coefficient of determination, R^2. The latter measures the portion of variability in $\overline{V}$ that can be sustained with $N = 2$. I get $R^2 = 97\%$. Besides, the coefficients of this regression, $(\widehat{b}_{short}, \widehat{b}_{long}) = [\overline{Q}'\overline{Q}]^{-1}\overline{Q}'\overline{V}$, can be interpreted as an 'approximate' optimal maturity structure. Not surprisingly, $\widehat{b}_{short} < 0 < \widehat{b}_{long}$.

Remark. Buera and Nicolini [2001] explore in some more detail the quantitative properties of the optimal maturity structure. A problem with their simulations, however, is that they impose high persistence in fiscal shocks. In an economy without capital, this makes the variation in interest rates artificially small and the spread in the maturity structure artificially large. It is thus important to extend the quantitative analysis to a standard RBC economy, with endogenous capital.

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