Idiosyncratic Sentiments and Coordination Failures

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Motivation

- how rational investors can have **differing degrees of optimism** regarding the prospects of economy

- even if they share the same information regarding all economic fundamentals
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- how rational investors can have differing degrees of optimism regarding the prospects of economy
- even if they share the same information regarding all economic fundamentals
- key insight: idiosyncratic extrinsic uncertainty
This Paper

- model 1: simple real investment game
- model 2: a financial market
This Paper

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▶ model 2: a financial market

▶ no exogenous heterogeneity: identical preferences, identical constraints, identical information about fundamentals

▶ independence ⇒ unique equilibrium, identical choices
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- model 1: simple real investment game
- model 2: a financial market

- no exogenous heterogeneity: identical preferences, identical constraints, identical information about fundamentals

- independence $\Rightarrow$ unique equilibrium, identical choices

- complementarity $\Rightarrow$ endogenous heterogeneity, despite strong incentive to coordinate
This Paper

- modeling instrument: **private sunspots**
- payoff-irrelevant (like public sunspots), but imperfect (Aumann)
- examples: “how bright is the sun?”, “what did the leader say?”
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- payoff-irrelevant (like public sunspots), but imperfect (Aumann)
- examples: “how bright is the sun?”, “what did the leader say?”

- devices that permit the construction of equilibria with **self-fulfilling heterogeneity in beliefs**
Novel Positive and Normative Properties

- capture strategic uncertainty
- rationalize idiosyncratic investor sentiment
- source of heterogeneity in investment/portfolio choices
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- smoother fluctuations
Novel Positive and Normative Properties

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- rationalize idiosyncratic investor sentiment
- source of heterogeneity in investment/portfolio choices
- sustain richer aggregate outcomes
- smoother fluctuations
- higher welfare
- render apparent coordination failures evidence of efficiency
Model 1: Real Investment Game
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- continuum of investors, each choosing \( k = 0 \) or \( k = 1 \)
- return to investment increasing in \( K \):

\[
A(K) \equiv \begin{cases} 
1 & \text{if } K \geq \hat{\kappa} \\
0 & \text{if } K < \hat{\kappa}
\end{cases}
\]

for some \( \hat{\kappa} \in (0, 1) \)
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for some $\hat{\kappa} \in (0, 1)$

- best response:

$$k_i = BR(K) \equiv \begin{cases} 
1 & \text{if } K \geq \hat{\kappa} \\
0 & \text{if } K < \hat{\kappa}
\end{cases}$$

- no or only public sunspots $\implies$ two equilibrium outcomes, $K = 0$ or $K = 1$
Private Sunspots

- nature draws an unobserved payoff-irrelevant random variable $s$, with support $\mathbb{S} \subseteq \mathbb{R}$ and c.d.f. $F : \mathbb{S} \rightarrow [0, 1]$
- each investor observes a private signal $m$ regarding $s$
- conditional on $s$, $m$ is i.i.d. across investors, with support $\mathbb{M} \subseteq \mathbb{R}$ and c.d.f. $\Psi : \mathbb{M} \times \mathbb{S} \rightarrow [0, 1]$
- $(\mathbb{S}, F, \mathbb{M}, \Psi)$ defines the “sunspot structure”
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**Definition**

An equilibrium with private sunspots consists of a sunspot structure $(\mathbb{S}, F, \mathbb{M}, \Psi)$ and a strategy $k : \mathbb{M} \rightarrow \{0, 1\}$ such that

$$k(m) \in \arg \max_{k \in \{0, 1\}} \int_{\mathbb{S}} U(k, K(s))dP(s|m) \quad \forall m \in \mathbb{M},$$

with $K(s) = \int_{\mathbb{M}} k(m)d\Psi(m|s) \quad \forall s \in \mathbb{S}$, and with $P(s|m)$ being the c.d.f. of the posterior about $s$ conditional on $m$ (as implied by Bayes’ rule).
Gaussian Private Sunspots

- \( s \sim N(\mu_s, \sigma_s^2) \)
- \( m_i = s + \varepsilon_i, \varepsilon_i \sim N(0, \sigma_\varepsilon^2) \)

Proposition

For any \((\mu_s, \sigma_s^2, \sigma_\varepsilon^2)\), there exists an equilibrium in which the following are true:

- An investor invests when \( m > m^* \) and not when \( m < m^* \), for some \( m^* \in \mathbb{R} \).
- The aggregate level of investment is stochastic, with full support on \((0, 1)\).
- The cross-sectional distribution of expectations regarding the aggregate level of investment, \( E[K|m] \), has full support on \((0, 1)\).
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Gaussian Private Sunspots

Proof. Given the proposed strategy,

\[ K(s) = \Pr(m \geq m^* | s) = \Phi\left(\frac{s - m^*}{\sigma_{\epsilon}}\right) \]

\[ K(s) \geq \hat{\kappa} \text{ iff } s \geq s^*, \text{ where } s^* = m^* + \sigma_{\epsilon} \Phi^{-1}(\hat{\kappa}) \]

Since the posterior about \( s \) conditional on \( m \) is Normal,

\[ \mathbb{E}[A(K(s))|m] = \Pr(s \geq s^*|m) - c = \Phi(\ldots) - c \]

Proposed strategy is an equilibrium iff \( m^* \) satisfies \( \mathbb{E}[A|m^*] = 0 \).
Equivalently,

\[ m^* = \mu_s - \sigma_s \left\{ \frac{\sigma_s^2 + \sigma_{\epsilon}^2}{\sigma_s \sigma_{\epsilon}} \Phi^{-1}(\hat{\kappa}) + \sqrt{1 + \left(\frac{\sigma_s}{\sigma_{\epsilon}}\right)^2 \Phi^{-1}(c)} \right\} \]

QED
Extension: Dynamics and Learning

- $s_t = \rho s_{t-1} + \nu_t$
- $m_{it} = s_t + \varepsilon_{it}$
- sufficient statistic $\hat{m}_{it}$
Extension: Dynamics and Learning

▶ \( s_t = \rho s_{t-1} + \nu_t \)
▶ \( m_{it} = s_t + \varepsilon_{it} \)
▶ sufficient statistic \( \hat{m}_{it} \)
▶ stationary equil where an agent invests at \( t \) iff \( \hat{m}_t > \hat{m}^* \)

\[
K_t(s_t) = \Phi \left( \frac{s_t - \hat{m}^*}{\hat{\sigma}} \right)
\]

▶ up to a monotone transformation, \( K_t \) follows a smooth AR(1) process
Extension: Dynamics and Learning

- $s$ constant over time, but learning through new signals
- Non-stationary equil where an agent invests at $t$ iff $\hat{m}_t > \hat{m}_t^*$

$$K_t(s) = \Phi \left( \frac{s - \hat{m}_t^*}{\hat{\sigma}_t} \right)$$

- More and more coordination over time:

$$\lim_{t \to \infty} K_t(s) \in \{0, 1\}$$
Model 2: Financial Market
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▶ re-interpret $k$ as investment in an asset traded in a financial market
▶ dividend of the asset $A(K)$
▶ price of the asset $p$
▶ payoff of an investor

$$\pi = \Pi(k, K, p) \equiv [A(K) - p] k$$

▶ exogenous supply of the asset:

$$Q = Q(p, u)$$

where $u$ is an unobserved supply shock
sunspot structure \((S, F, M, \Psi)\) as before
but now equilibrium price partially reveals \(s\)
Aumann meets Grossman-Stiglitz!
Private Susnspots: Correlated Eq meets REE

**Definition**

A REE with private sunspots consists of a sunspot structure \((S, F, M, \Psi)\), a price function \(P : S \times \mathbb{R} \rightarrow \mathbb{R}\), an individual demand function \(k : M \times \mathbb{R} \rightarrow [k, \bar{k}]\), and a belief \(\mu : S \times \mathbb{R} \times M \times \mathbb{R} \rightarrow [0, 1]\), such that:

- \(\mu\) consistent with Bayes rule, given \(P\)
- given \(\mu\) and \(P\), the demand function satisfies individual rationality:

\[
 k(m, p) \in \arg \max_{k \in \{0, 1\}} \int_{S \times U} \Pi(k, K(s, P(s, u), P(s, u)))d\mu(s, u|m, p) \forall (m, p)
\]

where \(K(s, p) \equiv \int_{M} k(m, p)d\Psi(m|s) \forall s \in S\).
- given the demand function, the price function satisfies market-clearing:

\[
 K(s, P(s, u)) = Q(s, u) \forall (s, u).
\]
Gaussian example

- Normality: $u \sim N(0, \sigma_u^2)$, $s \sim N(\mu_s, \sigma_s^2)$, $m_i = s + \varepsilon_i$, $\varepsilon_i \sim N(0, \sigma_{\varepsilon}^2)$

- Functional forms:

$$A(K) = \begin{cases} 
1 & \text{if } K \geq 1/2 \\
0 & \text{otherwise} 
\end{cases} \quad \text{and} \quad Q(p, u) = \Phi(u + \lambda \Phi^{-1}(p))$$
Gaussian example

Proposition

For any \((\sigma_u, \lambda)\), there exists a REE with private sunspots in which:

- The equilibrium price is \(p = P(s, u)\), where \(P\) is a continuously increasing function of \(s\) and a continuously decreasing function of \(u\).
- An investor’s equilibrium demand is
  
  \[
  k(m, p) = \begin{cases} 
  1 & \text{if } m \geq m^*(p) \\
  0 & \text{otherwise}
  \end{cases}
  \]

  where \(m^*(p)\) is a continuous decreasing function of \(p\).
- The aggregate demand for the asset, \(K(s, p)\), is continuously increasing in \(s\) and continuously decreasing in \(p\).
Private Sunspots and Efficiency
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- In model 1, equilibrium with $K = 1$ is first-best efficient
- but not in general: investment booms could be excessive (congestion, bubbles, adverse price effects)
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- **key point to take:** too high $K$ in best sunspot-less equilibrium
variant of model 1:

\[ A(K) = \begin{cases} 
1 - c - hK & \text{if } K \geq \hat{\kappa} \\
-c - hK & \text{if } K < \hat{\kappa}
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Proposition

Suppose \( 0 < 1 - c - h < h \).

There exist only two sunspot-less equilibria: \( K = 1 \) and \( K = 0 \).

The equilibrium in which \( K = 1 \) achieves higher welfare than the equilibrium in which \( K = 0 \), as well as than any equilibrium with public sunspots.

The first-best level of aggregate investment is \( K^* \in [\hat{\kappa}, 1) \).
Private Sunspots and Efficiency

- public sunspots can not improve welfare
- low investment \((K = 0)\) evidence of coordination failure, symptom of inefficiency
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- neither of the above true once we allow for private sunspots
Proposition

Suppose $0 < 1 - c - h < h(1 - h)$, allow for private sunspots, and consider the set of equilibria that maximize welfare. There exists a unique pair $(q^*, p^*)$, with $K^* < q^* < 1$ and $0 < p^* < 1$, such that all these equilibria are characterized by the following properties:

1. $K(s) = q^*$ with probability $p^*$ and $K(s) = 0$ with probability $1 - p^*$; that is, the economy fluctuates between “normal times”, events during which aggregate investment is positive, and “crashes”, events during which investment collapses to zero.

2. $q^*$ and $p^*$ decrease with $c$ or $h$; that is, the probability of a crash increases, and the level of investment in normal times decreases, as fundamentals get worse.
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Best Equilibrium with Private Sunspots

Figure: Comparative statics of best private-sunspot equilibrium.
Conclusion
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- private sunspots in macro/finance applications
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- idiosyncratic sentiment, endogenous heterogeneity
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- private sunspots in macro/finance applications
- intriguing positive and normative properties
- idiosyncratic sentiment, endogenous heterogeneity
- richer aggregate outcomes, smoother fluctuations
- apparent coordination failures become evidence of efficiency
- policies that fight such coordination failures may reduce efficiency