

Problem 1. [Sudoku as a game of cooperation]

In the popular game of *Sudoku*, the number placement puzzle, the objective is to fill a 9×9 grid so that each column, each row, and each of the nine 3×3 boxes (also called blocks or regions) contains the digits from 1 to 9 only one time each (see figure). The puzzle setter provides a partially completed grid and the puzzle solver is asked to fill in the missing numbers. Consider now a multi-agent extension of the Sudoku game, in which each of the 81 boxes is controlled by one agent. The agent's action space is choosing a number from 1 to 9 to put in her box, i.e., $\mathcal{A}_i = \{1, \dots, 9\}$, where \mathcal{A}_i denotes the action space of agent i . Let α denote the vector of choices of the agents. Then, agent i 's utility is given by:

$$u_i(\alpha) = -(n_i^R(\alpha) + n_i^C(\alpha) + n_i^B(\alpha)),$$

where $n_i^R(\alpha)$, $n_i^C(\alpha)$, $n_i^B(\alpha)$ is the number of repetitions of numbers in i 's row, column and block respectively. Show that the multi agent Sudoku game is a potential game and write down explicitly the associated potential function.

5	3		7					
6			1	9	5			
	9	8					6	
8				6				3
4			8		3			1
7				2				6
	6					2	8	
			4	1	9			5
				8			7	9

Figure 1: Sudoku

Problem 2. Consider the network formation game discussed in Lecture 12.

- (i) Find conditions for which the ring network, in which the agents form a circle (each one has two neighbors), is a pairwise stable equilibrium of the game.
- (ii) Find conditions for which a network that consists of two disjoint rings in a pairwise stable equilibrium of the game.

Problem 3. [The Stag Hunt Game - A Game of Social Cooperation]

The stag hunt is a game which describes a conflict between safety and social cooperation. Other names for it or its variants include "assurance game", "coordination game", and "trust dilemma". Inspired by the philosopher Jean-Jacques Rousseau, the game involves two individuals that go out on a hunt. Each can individually choose to hunt a stag or hunt a hare. If an individual hunts a stag, he must have the cooperation of his partner in order to succeed. An individual can get a hare by himself, but a hare is worth less than a stag. The game is succinctly described by the payoff matrix below:

	stag	hare
stag	a, a	$0, b$
hare	$b, 0$	$b/2, b/2$

In particular, if they both cooperate and hunt a stag, they succeed and get a . Alternatively, one goes for hare, succeeds and get a lower payoff b , whereas the other that went for stag gets 0, since stag hunting needs cooperation. Finally, if both go for hare, then they both obtain $b/2$. The main assumption is that $a > b > 0$.

- (i) Compute all Nash Equilibria of the stag hare game, both in pure and mixed strategies.
- (ii) Show that the pure strategy Nash Equilibria are evolutionary stable. How about the mixed strategy equilibrium?

- (iii) Consider the continuous time replicator dynamics for the stag hare game. Write down their expression and show that the pure strategy Nash equilibria are asymptotically stable.

Problem 4. [The Chain Store Paradox]

Consider a monopolist with a store in each one of n towns. There is a separate entrant considering entry into each one of the towns. Entry is sequential, and this is a perfect information game. In particular, at $n = 1$, the first entrant decides whether to enter, and then the monopolist decides whether to fight or not; at $n = 2$, the second entrant decides after observing past actions etc. The monopolist makes a profit equal to 2 if there is no entry into the relevant market, a profit equal to 1 if there is entry and no fighting, and -1 if there is fighting. The entrant gets 0 if it does not enter, 1 if it enters and there is no fighting, and -1 if there is fighting. The monopolist's total payoff is the discounted sum of the profits from the n towns, where the discount factor is $0 < \delta < 1$.

- (i) Show that there exists a unique subgame perfect equilibrium of the game.
- (ii) Consider now the case when $n = \infty$ (infinite horizon game). Show that for $\delta > 2/3$, there exists a subgame perfect equilibrium, in which no entrant ever chooses to enter. Describe in detail the strategy profile that supports this subgame perfect equilibrium. Finally, draw a parallel between this equilibrium and the cooperation equilibrium in the infinitely repeated prisoner's dilemma.