# Moral Hazard and Efficiency in General Equilibrium with Anonymous Trading*

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April 27, 2010

**Abstract**

A "folk theorem" originating, among others, in the work of Stiglitz maintains that competitive equilibria are always or "generically" inefficient (unless contracts directly specify consumption levels as in Prescott and Townsend, thus bypassing trading in anonymous markets). This paper critically reevaluates these claims in the context of a general equilibrium economy with moral hazard. We first formalize this folk theorem. Firms offer contracts to workers who choose an effort level that is private information and that affects worker productivity. To clarify the importance of trading in anonymous markets, we introduce a *monitoring partition* such that employment contracts can specify expenditures over subsets in the partition, but cannot regulate how this expenditure is subdivided among the commodities within a subset. We say that preferences are *nonseparable* (or more accurately, not weakly separable) when the marginal rate of substitution across commodities within a subset in the partition depends on the effort level, and that preferences are *weakly separable* when there exists no such subset. We prove that the equilibrium is always inefficient when a competitive equilibrium allocation involves less than full insurance and preferences are nonseparable. This result appears to support the conclusion of the above-mentioned folk theorem. Nevertheless, our main result highlights its limitations. Most common-used preference structures do not satisfy the nonseparability condition. We show that when preferences are weakly separable, competitive equilibria with moral hazard are constrained optimal, in the sense that a social planner who can monitor all consumption levels cannot improve over competitive allocations. Moreover, we establish $\varepsilon$-optimality when there are only small deviations from weak separability. These results suggest that considerable care is necessary in invoking the folk theorem about the inefficiency of competitive equilibria with private information.

**Keywords:** competitive equilibrium, double deviations, efficiency, general equilibrium theory, monitoring partition, moral hazard.

**JEL Classification:** D52, D61, D62, D82

*We thank Alberto Bisin, Glenn Ellison, Bengt Holmstrom, Robert Townsend, Muhamet Yilidiz and seminar participants at NYU and MIT for useful comments and suggestions.

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# 1 Introduction

A central question for economic theory is the efficiency of competitive markets. In economies with complete markets, this question is conclusively answered by the celebrated First and Second Welfare Theorems, which show that, under some regularity conditions, competitive equilibria are Pareto optimal and every Pareto optimal allocation can be decentralized as a competitive equilibrium. Nevertheless, the complete market benchmark does not cover many empirically-relevant economies where missing markets are ubiquitous. Arguably the most important reason for missing markets in practice is *private information*. Individual agents know more about their preferences, risks and actions than the market can observe. Despite a sizable literature on this topic, efficiency properties of economies with private information are not yet fully understood. In this paper, we investigate the efficiency of competitive equilibria in a subclass of economies with private information, those with *moral hazard*, where individuals take privately-observed actions affecting their endowments (and/or production).

One approach to the study of efficiency in moral hazard economies has been pioneered by Prescott and Townsend (1984a, 1984b). Prescott and Townsend propose the important idea of considering insurance contracts as commodities that should also be priced in equilibrium. Prescott and Townsend show that competitive equilibria with moral hazard are (constrained) Pareto optimal under two key assumptions: *exclusivity* and *full monitoring.* The first implies that individuals can sign exclusive contracts and is a good starting point for the study of employment contracts.[1] We focus on exclusive contracts throughout the paper. The second assumption, full monitoring, is more problematic. Under full monitoring, contracts specify complete consumption bundles for individuals in different states of nature. This essentially implies that firms or some other outside agency can fully monitor individual consumptions. This assumption is not only unrealistic but also goes against the spirit of "competitive markets." Competitive markets should allow *anonymous trading*, so that individuals are able to buy at least a subset of commodities in anonymous markets without a central agency keeping track of their exact transactions.

A systematic analysis of the structure and efficiency of competitive equilibria with anonymous trading is not available, but a series of papers by Stiglitz and coauthors, most notably, Greenwald and Stiglitz (1986), and also Arnott and Stiglitz (1986, 1990, 1991), claim that competitive equilibria under these circumstances are always or "generically" Pareto subopti-

[1] Exclusivity may be a less satisfactory assumption for insurance contracts, in particular, when informal insurance is also possible; see, e.g., Arnott and Stiglitz (1991) and Bisin and Guaitoli (2003). Real-world insurance and financial contracts often explicitly regulate what other contracts individuals can sign for the same risks, or whether they can pledge the revenues of the same business. Exclusivity is much more natural in the context of employment contracts we focus on in this paper.

mal. These claims are supported by local analysis of first-order conditions, though without a rigorous proof that this type of local analysis is valid or economically important.[2] Hence one may say that the inefficiency of competitive equilibria with anonymous trading has emerged as a *folk theorem.* This folk theorem is not only of theoretical interest but has been very influential in applied work. It is often invoked to argue that decentralized allocations in insurance, labor and credit markets are inefficient and necessitate government intervention (or to provide the intuition for specific models in which this is the case).

In this paper, we consider a general equilibrium environment where the structure and efficiency of competitive equilibria with anonymous trading can be studied. The economy consists of a large number of firms and risk-averse individuals. Individuals accept employment contracts from firms and choose an effort level, which determines the probability distribution over a vector of production. Individual effort is private information. Commodities in this economy are partitioned, such that expenditures over subsets in a given *monitoring partition* of commodities are observable (for example, how much an individual spends on vacation can be determined but not how this spending is distributed across different activities in the vacation resort). Employment contracts specify payments to workers and expenditure levels over the subsets in the monitoring partitions as a function of the realization of the state of nature. The Prescott-Townsend economy is a special case where each subset in the monitoring partition is a singleton.[3] After all uncertainty is resolved (the underlying states of the world are realized), individuals allocate the contractually-specified expenditures within the subsets in the partition at given market prices.

We establish the existence of a competitive equilibrium and an indirect maximization problem that characterizes equilibrium allocations (Theorem 1 and Proposition 1). We then formalize the above-mentioned folk theorem. We say that there is *no full insurance* at an equilibrium if the marginal rate of substitution of some good between any two states is not one. can we say that preferences are *nonseparable* (or more accurately, "not weakly separable"), if there is a subset in the monitoring partition such that the marginal rate of substitution between the goods in the subset change if the effort level is modified. Conversely, we say that preferences

[2]This is particularly concerning for three reasons. First and more importantly, this local analysis assumes that a range of Lagrange multipliers exist and are strictly positive, though there is no mathematical or economic reason for them to be so. One of our main results will establish the (constrained) optimality of competitive equilibria under certain conditions explained below, thus invalidating this line of analysis. Second, the local analysis makes use of differentiability assumptions and the first-order approach, which do not generally apply in these environments (see Grossman and Hart, 1983, Rogerson, 1985, Jewitt, 1988 on the first-order approach). Third, as we will show, it may well be that even if efficiency is "nongeneric," small deviations from this efficiency benchmark might still lead to allocations that are approximately ($\varepsilon$-) efficient and many inefficiencies identified via this method may not be first order.

[3]Another special case is one in which the partition consists of a number of singleton elements, which correspond to "monitored goods," and the remainder, which comprises "nonmonitored goods."

are *weakly separable* when there exists no such subset (this is significantly weaker than the standard separability assumptions often adopted in theoretical and applied work). Our first result (contained in Theorem 2) shows that when preferences are nonseparable and there is no full insurance at an equilibrium, then this equilibrium is constrained suboptimal (inefficient), in the sense that a social planner who is constrained by the same moral hazard problems (but who is allowed to monitor expenditures on all goods) can improve over the equilibrium allocation.[4] Note that this theorem is silent on whether a social planner who is also constrained by the *same* monitoring technology can implement such a Pareto improvement, and we show that this is not necessarily the case.[5]

While Theorem 2 appears to give some support to the folk theorem, the rest of our analysis sheds doubt on its general validity and applicability. Theorem 3 shows that competitive equilibria are constrained Pareto optimal when preferences are weakly separable (see also Theorem 6 for the case in which contracts with randomization are allowed). This is an important result for two reasons. First, most preferences used in applied work satisfy this weak separability condition.[6] Second, it establishes that the equilibrium is constrained optimal relative to a very strong notion in which the social planner has access to more instruments than the market (she can monitor and specify expenditures for all goods, whereas contracts can only do so for goods within a subset in the partition). This result suggests that, at least in most of environments considered in applied work, the inefficiencies emphasized by the folk theorem do not arise.

Finally, in Theorem 7 we show that when there are only small deviations from this benchmark environment with weak separability, competitive equilibria remain approximately efficient (or are $\varepsilon$-efficient for the right choice of $\varepsilon$). This result highlights another important conceptual point, that a large set of economically relevant environments may not feature meaningful inefficiencies even if there is a "generic" inefficiency result.

Overall, although our results do not imply that competitive equilibria are always efficient in private information economies, they delineate a range of benchmark situations in which equilibria have very strong optimality properties. They also show that when such benchmark situations are a good approximation to the actual environment, efficiency will hold approx-

[4] Prescott and Townsend (1984a,b) emphasized the importance of contracts that allow for randomization for efficiency in their analysis. Such randomization is important for existence in their environment, but is not central for the baseline efficiency results in our or their framework. To highlight this and to simplify the exposition, we start with an environment that does not allow for such randomization. We then establish the equivalent results when randomization is allowed (e.g., Theorem 5 generalizes Theorem 2, etc.).

[5] Nevertheless, the notion of efficiency we use for most of the analysis may be more relevant than this latter weaker notion, since in a production economy the social planner can often achieve the allocations under full monitoring by using taxes and subsidies. To simplify notation, we do not explicitly study such tax policies.

[6] See, for example, Atkeson and Lucas (1992), Golosov and Tsyvinski (2006), Golosov, Kocherlakota and Tsyvinski (2003).

imately. Suppose for example that the worker's preferences are defined at two levels: the worker has preferences over a number of needs (such as food, entertainment, procrastination, vacation, health care and so on) and each one of these needs is satisfied by consumption of various goods in the economy. Suppose also that there are only a few needs that interfere with the worker's effort choice such as vacation, procrastination and health care. In particular, a higher effort choice might make the worker enjoy vacations and procrastination less relative to other needs (since she spends most of her time working) and enjoy health care more relative to other needs (since the cost of effort decreases with level of health), but the worker's preferences for other needs may not depend on effort level. Suppose also that employers can monitor the consumption levels for the few needs that interfere with effort choice (but not necessarily the consumption of the particular goods that make up these needs). This amounts to assuming that firms can enforce how long a vacation the worker takes, how much time she spends in the office, and how good a health care she receives. Under these assumptions, our Theorem 3 applies and shows that the equilibrium will be constrained optimal. This scenario thus constitutes a counterexample to the conjectured suboptimality of competitive equilibria under private information and anonymous trading, and suggests considerable caution in appealing to the above-mentioned folk theorem. Moreover, in this example, the folk theorem would suggest taxing the goods that provide procrastination services (such as TVs), but our results indicate that this might be the wrong policy recommendation since firms already monitor the consumption of TVs (and other procrastination goods) by making the workers come to the office during work hours.

As the above discussion clarifies, our paper is related to a number of literatures. The relationship of our paper to Prescott and Townsend (1984a, 1984b) and to Greenwald and Stiglitz (1986) has already been discussed. Another set of closely related papers are by Geanakoplos and Polemarchakis (1986, 2008) and Citanna, Kajii and Villanacci (1998), which establish the generic inefficiency of competitive equilibria in economies with (exogenously-given) incomplete markets. Our work extends the Geanakoplos-Polemarchakis results to environments with *endogenously incomplete markets* (because of moral hazard). It also highlights that in such environments constrained optimality may result if the appropriate subsets of goods are monitored. Similar issues arise in other economies with price externalities due to endogenously incomplete markets. For example, Kehoe and Levine (1993) provide results similar to ours for an economy with participation constraints. Our paper is also related to Citanna and Villanacci (2000), which establishes generic inefficiency of equilibria for a moral hazard economy with exclusive contractual relationships in which the principal has all the bargaining power. Our work shows that their inefficiency result crucially depends on the assumption that the principal has the bargaining power. Given the separability assumptions they impose on preferences, our results imply that the equilibrium would be efficient in the polar opposite case in which the agent has all the "bargaining power," i.e. when insurance contracts are exclusive and the insurance market is competitive.

In addition to these works, the paper most closely related to our paper is Lisboa (2001), which establishes the Pareto optimality of competitive equilibria in the context of an economy with moral hazard and fully separable utility. A number of key differences between our work and Lisboa are worth emphasizing. First, Lisboa considers a special case of the model studied here, in which utility functions are fully separable across all goods and effort and there is no monitoring of consumption in any subset of goods. Second, Lisboa's analysis relies on the first-order approach applying everywhere, which is restrictive and not used in our analysis. Third, Lisboa's analysis contains neither our characterization results on inefficiency of comparative equilibria nor our result on approximate efficiency.

There is also a large literature on various different aspects of moral hazard in general equilibrium. Bennardo and Chiappori (2003) and Gottardi and Jerez (2007) discuss the problems that arise in general equilibrium economies with moral hazard because of potential non-transferability of utility. They show how Bertrand competition might lead to equilibria with positive profits. This is an issue that also arises in our model and we provide sufficient conditions (that are not very restrictive) for Bertrand competition to lead to zero profits. Bisin, Geanakoplos, Gottardi, Minelli and Polemarchakis (2007) consider an alternative approach to moral hazard in general equilibrium, where, in contrast to our setup, contracts are not necessarily individualized. This introduces natural externalities across the actions of different individuals signing the same type of contract.[7]

Finally, some of the same issues we emphasize in the context of general equilibrium also arise in the public finance and mechanism design literatures. See, for example, Hammond (1987), Allen (1985), Atkeson and Lucas (1992), Guesnerie (1998), Cole and Kocherlakota (2001), Werning (2001), Kocherlakota (2004), Golosov and Tsyvinski (2006), and Doepke and Townsend (2006). None of these studies derive results similar to our main theorems in this paper.

The rest of the paper is organized as follows. Section 2 presents the environment, defines a competitive equilibrium, and establishes the existence of equilibrium. Section 3 introduces the notion of constrained optimality and provides sufficient conditions under which the equilibrium is not constrained optimal. Section 4 introduces the notion of weak separability and presents

[7] Also related are recent papers considering adverse selection in general equilibrium, for example, Bisin and Gottardi (1999, 2006) and Jerez (2003).

our main result, which shows that the equilibrium is constrained optimal when the preferences are weakly separable. Section 5 introduces the environment with stochastic contracts and generalizes the efficiency results to this setting. Section 6 presents our second main result, which shows that the equilibrium is approximately constrained optimal when the preferences are approximately weakly separable. Section 7 concludes. Appendix A.1 discusses additional results omitted from the main text, and Appendix A.2 contains the proofs of all the results stated in the text.

# 2 Environment and Equilibrium

## 2.1 Preferences

We consider a static production economy with a finite set of goods denoted by $G$ and a finite set of (individual-specific) states of nature denoted by $S$. We use $g \in G$ and $s \in S$ to index goods and states, and use $|G|$ and $|S|$ to denote the cardinality of these sets. There is a continuum of individual workers, denoted by $\mathcal{N}$, with measure normalized to 1. To simplify the analysis and the exposition, we assume that all workers have identical utility and identical production technology.[8] In particular, each worker chooses an effort level $e \in E$, where $E = \left\{e_1, .., e_{|E|}\right\} \subset \mathbb{R}$ is a finite set. The effort choice of the worker induces a probability distribution over an endowment (production) vector $y \in \mathbb{R}_+^{|G|}$. We represent this probability distribution by the function $q$, whereby $q_s(e) \in [0,1]$ is the probability of state $s \in S$ for the worker in question when she exerts effort $e$ (naturally with $\sum_{s \in S} q_s(e) = 1$ for all $e \in E$). Each state $s \in S$ is, in turn, associated with a production vector $y_s \in \mathbb{R}_+^{|G|}$. For each $g$, there exists $s \in S$ such that $y_s^g \neq 0$, which ensures that each good $g$ is in positive supply in some states. We also assume throughout that the realization of states in $S$ (conditional on effort) is independent across individuals. Thus, with a law of large numbers type argument there is no aggregate uncertainty.[9]

We assume that each worker has VNM preferences over consumption of goods and effort choice represented by

$$U(x, e) = \sum_{s \in S} q_s(e)\, u(x_s, e),$$

where $x_s \equiv \left(x_s^1, ..., x_s^{|G|}\right) \in \mathbb{R}_+^{|G|}$ is the vector of consumption in state $s$ and $u(\cdot)$ denotes the state utility function. Throughout, we use the notation $x_s \equiv (x_s^g)|_{g \in G}$ to designate vectors,

[8] The results generalize to multiple types straightforwardly provided that worker type is observable and contractible.

[9] See, for example, Bewley (1986) or Malinvaud (1973). Nevertheless, some care is necessary in defining the right notion of integral in applying such a law of large numbers. The simplest approach, proposed by Uhlig (1996), is sufficient here.

and $x = (x_s^g)|_{s \in S, g \in G}$ to designate matrices. We refer to the pair $(x, e)$ as an allocation, and let $A \equiv \mathbb{R}_+^{|S| \times |G|} \times E$ denote the set of allocations. We make the following standard assumption on the utility function:

**Assumption A1 (Preferences)** The state utility function $u(\cdot)$ is twice continuously differentiable in $x_s$, strictly increasing in each $x_s^g$, and strictly concave in $x_s$.

Throughout, the effort choice of the worker is her private information, so there is a *moral hazard problem* and employment contracts cannot be conditioned on effort choices. The realized production vector is publicly observable and employment contracts can condition on these realizations. Motivated by the discussion in the introduction, we consider a partition $\mathcal{G} = \{G_1, .., G_{|\mathcal{G}|}\}$ of the set of goods (i.e., a collection of disjoint subsets of goods the union of which is equal to the set of all goods, $G$). The employment contracts can specify the worker's expenditure $w_s^m \in \mathbb{R}_+$ on the monitoring subset $G_m$, for each $m \in M \equiv \{1, .., |\mathcal{G}|\}$. The goods within each monitoring subset $G_m$ are traded in spot markets that operate after all production vectors are realized (and at market clearing prices as described below). We use $w = \{w_s^m\}_{s \in S, m \in M}$ to denote the matrix where each element denotes the individual's expenditure on each monitoring subset at a given state. We denote the vector of prices by $p \in \mathbb{R}_+^{|G|}$. We choose good 1 as the numeraire, i.e. $p^1 = 1$. For any subset of commodities $G' \subset G$, we denote by $p^{G'}$ the corresponding price sub-vector, and by $x^{G'}$ the corresponding consumption sub-matrix.

## 2.2 Firms and Employment Contracts

A large finite number of firms can sign employment contracts with the workers. We denote the set of firms by $J = \{1, 2, .., |J|\}$. Firms are owned by the workers and maximize the expected profits. Since, as we will see shortly, in equilibrium firms will make zero profits, we do not introduce additional notation to specify the allocation of their profits.

Throughout we impose *exclusivity* and assume that each worker can only contract with a single firm. A*n employment contract* between a firm and a worker gives the property rights over the worker's production, $y_s$, to the firm and specifies worker's expenditures, $w = \{w_s^m\}_{s \in S, m \in M}$, and prescribes consumption levels for goods, $x = (x_s^g)|_{s \in S, g \in G}$, and effort choice, $e \in E$. Note that at this point we are not allowing *stochastic contracts*, which would instead determine a probability distribution over outcomes. We return to stochastic contracts in Section 5.

We denote a contract by the tuple $c = (w, x, e)$, and we denote the set of contracts by $C = \mathbb{R}_+^{|S| \times |M|} \times A$ (where recall that $A \equiv \mathbb{R}_+^{|S| \times |G|} \times E$ denotes the set of allocations). Notice that $C$ is a subset of a finite dimensional space and its elements, denoted by $c \in C$, are simply vectors. An incentive compatible contract is $c = (w, x, e) \in C$ such that the effort choice and the level of consumption of goods are *incentive compatible* given prices $p$ and wage schedule $w$. More formally, this means that for a given market price vector $p > 0$,

$$\begin{aligned} (x, e) &\in \arg\max_{(\tilde{x},\tilde{e})\in A} U(\tilde{x}, \tilde{e}) \\ \text{subject to} \quad & \tilde{x}_s^{G_m} p_s^{G_m} \leq w_s^m \text{ for each } s \text{ and } m. \end{aligned} \tag{1}$$

We denote the set of incentive compatible contracts by $C^I(p)$.

The maximization problem in (1) can be conceptually divided into two parts. Given an effort level $\tilde{e}$, the consumption choice at each state $s$ is uniquely determined as the solution to:

$$\begin{aligned} & x_s \in \arg\max_{\tilde{x}_s\in\mathbb{R}_+^{|G|}} u(\tilde{x}_s, \tilde{e}) \\ \text{subject to} \quad & \tilde{x}_s^{G_m} p_s^{G_m} \leq w_s^m \text{ for each } m. \end{aligned}$$

We denote the consumption choice at state $s$ by the function $\mathbf{x}_s(w_s, p, \tilde{e})$, and the consumption choice at all states by the function $\mathbf{x}(w, p, \tilde{e})$. By Berge's Maximum Theorem and the strict concavity of $u(\cdot)$ in $x$, the function $\mathbf{x}(\cdot)$ is continuous in its arguments. The incentive compatible effort choice is then a solution to the following problem:

$$e \in \arg\max_{\tilde{e}\in E} U(\mathbf{x}(w, p, \tilde{e}), \tilde{e}). \tag{2}$$

Since $E$ is finite, this problem always has a solution. The solution is represented by a correspondence $\mathbf{e}(w, p)$. We also define the *indirect utility function*

$$V(c, p) = \max_{\tilde{e}\in E} U(\mathbf{x}(w, p, \tilde{e}), \tilde{e}) \tag{3}$$

for each $p > 0$ and $c \in C^I(p)$. By Berge's Maximum Theorem, the correspondence $\mathbf{e}(w, p)$ is upper hemicontinuous, and the indirect utility function is continuous in its arguments (here recall that $c \in C^I(p)$ is simply a vector).

### 2.3 Worker's Contract Choice

Each individual worker $\nu$ faces a menu of incentive compatible contracts, one from each firm, $\{c(\nu, \tilde{j})\}_{\tilde{j}\in J}$, and she chooses the contract that maximizes her utility. The worker can also reject all contract offers. In this case, she generates an alternative production vector $\tilde{y}_s \leq y_s$ in each $s$ (since she will be less productive without the firm). Since, as we will see, in equilibrium the worker will never reject all contract offers, there is no loss of generality in assuming that $\tilde{y}_s = y_s$, and we adopt this assumption to simplify the notation. So when the worker rejects all contract offers, she solves

$$\max_{(\tilde{x},e)\in A} U\left(\tilde{x},\tilde{e}\right), \tag{4}$$
$$\text{subject to} \qquad \tilde{x}_s p \leq y_s p \text{ for each } s.$$

Let $(x,e)$ be a solution to the preceding problem, and define the contract $c\left(\nu,0 \mid p\right) \equiv \left(\left(w^m \equiv x^{G_m} p^{G_m}\right)_{m\in M}, x, e\right)$ as *the outside option* of the worker. Rejecting every contract is equivalent for the worker to accepting the contract $c\left(\nu,0 \mid p\right)$.

Hence, a *strategy* for the worker $\nu$ is a function $\mathbf{J}_\nu{:}C^I\left(p\right)^{|J|} \mapsto J\cup\{0\}$ that specifies the index of the firm she chooses or 0 if she chooses her outside option:

$$\mathbf{J}_\nu\left[\left\{c\left(\nu,\tilde{j}\right)\right\}_{\tilde{j}\in J}\right] \in \arg\max_{\tilde{j}\in J\cup\{0\}} V(c\left(\nu,\tilde{j}\right),p). \tag{5}$$

## 2.4 Firm's Problem

We assume that firm $j$ offers a continuum of incentive compatible contracts, one for each worker, taking the price vector $p$, the contracts offered by other firms $\left(c\left(\nu,\tilde{j}\right)\right)_{\nu\in\mathcal{N},\ \tilde{j}\ \in J-\{j\}}$, and the worker strategies $\left(\mathbf{J}_\nu\right)_{\nu\in\mathcal{N}}$ as given.[10] Formally, the contract offer of firm $j$ is a Lebesgue measurable function $c\left(\cdot,j\right) : [0,1] \to C^I\left(p\right)$. Let $\mathcal{L}\left(\mathcal{C}^I\left(p\right)^{[0,1]}\right)$ denote the set of Lebesgue measurable functions from $[0,1]$ to $C^I\left(p\right)$, and let $\pi\left(c,p\right)$ denote the profit of the firm from an accepted contract $c=(w,x,e)$, given by:

$$\pi\left(\left(w,x,e\right),p\right) = \sum_{s\in S} q_s(e)(y_s p - \sum_{m\in M} w_s^m).$$

Note that the firm's profit from a contract that is not accepted is equal to 0. Hence, firm $j$ solves the following problem:

$$\max_{c(\cdot,j)\in\mathcal{L}\left(\mathcal{C}^I(p)^{[0,1]}\right)} \int_{\left\{\nu\subset\mathcal{N}\ \mid\ \mathbf{J}_\nu\left[\left\{c\left(\nu,\tilde{j}\right)\right\}_{\tilde{j}\in J}\right]=j\right\}} \pi\left(c\left(\nu,j\right),p\right) d\nu. \tag{6}$$

## 2.5 Definition of Equilibrium

Let us refer to the economy described in the previous section with $\mathcal{E}$. In this section, we define a competitive equilibrium for economy $\mathcal{E}$ and show that such an equilibrium exists.

**Definition 1.** *A* ***competitive equilibrium*** *in economy $\mathcal{E}$ is a collection of contract offers $\left[c\left(\nu,j\right)\right]_{j\in J,\nu\in\mathcal{N}}$ by the firms, a collection of strategies for the workers $\left(\mathbf{J}_\nu\right)_{\nu\in\mathcal{N}}$, a price vector $p$, and accepted contracts, $\left[w\left(\nu\right),x\left(\nu\right),e\left(\nu\right)\right]_{\nu\in\mathcal{N}}$, such that*

[10] To justify the assumption that firms take the price of goods as given, we could put an exogenous limit on the measure of contracts a firm can sign (a capacity constraint). If the capacity constraint is sufficiently small and the number of firms is sufficiently large, each firm will be too "small" to influence equilibrium prices while there will be sufficiently many firms to provide each worker with employment contracts. This extension is straightforwad, but we present the model without capacity constraints to simplify the exposition.

1. *Workers' contract choice is optimal, i.e., for each* $\nu \in \mathcal{N}$*, the strategy* $\mathbf{J}_\nu$ *satisfies* (5)*.*

2. *Firms maximize expected profits, i.e., for each* $j \in J$*, the contract offer* $c(\cdot, j)$ *solves problem* (6)*.*

3. *Goods markets clear, i.e., for each* $g \in G$*,*

$$\int_{\mathcal{N}} \sum_{s \in S} q_s(e(\nu))(y_s^g - x_s^g(\nu))d\nu \geq 0, \text{ with equality if } p^g > 0. \tag{7}$$

## 2.6 Firm Competition and the Indirect Problem

To facilitate the characterization of equilibrium, we also impose the following assumptions.

**Assumption A2 (Probability function)** The probability function $q_s$ is strictly positive, that is, $q_s(e) > 0$ for each $s \in S$ and $e \in E$.

**Assumption A3 (Local Transferability)** There exists a monitoring subset $G_1$ and functions $u^{G_1}(\cdot)$ and $u^{G \backslash G_1}(\cdot)$ such that

$$u(x_s, e) = u^{G_1}\left(x_s^{G_1}\right) + u^{G \backslash G_1}\left(x_s^{G \backslash G_1}, e\right). \tag{8}$$

In addition, $u^{G_1}(\cdot)$ satisfies:

$$\lim_{\left\|x_s^{G_1}\right\| \to 0} \left\| \frac{\partial u^{G_1}\left(x_s^{G_1}\right)}{\partial x_s^{G_1}} \right\| = \infty. \tag{9}$$

Assumption A2 is standard. Assumption A3 is less standard, but relatively weak. The first part of the assumption, for example, will hold if there is one good which is completely separable from effort choice, and consumption of which can be monitored by the firm. The second part of Assumption A3 is a relatively weak form of the standard Inada condition (in particular, it implies a minimum consumption requirement on the consumption vector, $x_s^{G_1}$, as opposed to separate requirements on the consumption of each good, $x_s^g$). This requirement, along with Assumption A2, ensures that the worker's expenditure on the goods in $G_1$ is strictly positive at each state $s$, i.e., $w_s^1 > 0$. The first part of Assumption A3 ensures that the firm can slightly increase (or decrease) the worker's expenditure on the goods in $G_1$ while keeping the incentive compatible effort level the same. Consequently, the assumption allows for at least a small amount of utility transfer between the worker and the firm, while respecting the incentive compatibility constraints (see Lemma 2 in Appendix A.2 for a formalization). Without a condition that allows for this type of utility transfer, Bertrand competition may not drive profits to zero (see Bennardo and Chiappori, 2003). Since this problem is already well understood and is orthogonal to our main concerns, Assumption A4 enables us to focus on the questions of interest for us.

The following proposition provides an equivalent characterization for the equilibrium. In particular, a collection of accepted contracts is part of an equilibrium if and only if all but a measure zero of them maximize the utility of the worker subject to the incentive compatibility constraint and a non-negative profit constraint for the firm. As with all the other proofs in this paper, the proof of this Proposition is in Appendix A.2.

**Proposition 1.** ***(The Indirect Problem)*** *Consider an economy $\mathcal{E}$ that satisfies Assumptions A1-A3. Then, the prices and accepted contracts, $\left(p, c(\nu) = [w(\nu), x(\nu), e(\nu)]_{\nu \in \mathcal{N}}\right)$, are part of an equilibrium if and only if:*

1. *The contract $c(\nu)$ is a solution to*

$$\max_{\tilde{c} \in C^I(p)} \quad V(\tilde{c}, p) \tag{10}$$

$$\text{subject to} \quad \pi(\tilde{c}, p) \geq 0, \tag{11}$$

   *for all but a measure zero of $\nu \in \mathcal{N}$.*

2. *The goods markets clear [cf. Eq. (7)].*

*Moreover, at the solution to problem (10), the constraint (11) binds, that is, each firm makes zero profits in equilibrium.*

## 2.7 Existence of Equilibrium

The assumptions we made so far do not guarantee the existence of an equilibrium. If the solution set to problem (10), denoted by $S(p)$, is not upper hemicontinuous in the price vector $p$, then the equilibrium may not exist. In particular, the correspondence $S(p)$ could fail to be upper hemicontinuous if the constraint set, $C^I(p)$, is discontinuous. In this setting, the worker's incentive compatible effort choice can be discontinuous in the price vector $p$. Nevertheless, because the elements of $C^I(p)$ are contracts, $c = (w, x, e)$ (and not simply the effort choice, $e$), the correspondence $C^I(p)$, which lies in the larger contract space $C$, could be continuous even when effort choice is discontinuous in $p$. The following assumption is sufficient to establish the continuity of the constraint set, $C^I(p)$ (see Lemma 3 in Appendix A.2).

**Assumption A4 (Effort Targeting)** For each $e \in E$, there exists a vector of utility transfers $t \in \mathbb{R}^{|S|}$ such that

$$\sum_{s \in S} q_s(e)\, t_s > \sum_{s \in S} q_s(\hat{e})\, t_s \text{ for each } \hat{e} \in E \setminus \{e\}. \tag{12}$$

This assumption is natural and quite weak. It loosely corresponds to the requirement that firms should be able to induce (target) any level of effort if they wish to do so. It is also not difficult to satisfy. For example, it holds when $|E| \leq |S|$ and the probability vectors, $(q_s(e))_{s \in S}$ for each $e \in E$, are linearly independent. In the commonly studied case of two effort levels, this assumption holds when there are at least two states and the efforts do not lead to identical success probabilities. Assumption A4 ensures the continuity of $C^I(p)$, which in turn implies that the solution correspondence to problem (10), $S(p)$, is upper hemicontinuous (by a version of Berge's Maximum Theorem). The existence of equilibrium then follows from standard arguments.

**Theorem 1.** ***(Existence of Equilibrium)*** *Consider economy $\mathcal{E}$ that satisfies Assumptions A1-A4. There exists a competitive equilibrium for the economy $\mathcal{E}$.*

**Remark 1.** ***(The Role of Assumptions A3 and A4)*** An alternative to imposing Assumptions A3 and A4 would be to assume continuous effort choice, e.g., $E = [0, 1]$, and to make sufficiently strong assumptions to ensure that the effort choice changes continuously in the price vector $p$. The following set of sufficient conditions are typically adopted in general equilibrium analyses of moral hazard economies with continuous effort (see, for example, Arnott and Stiglitz, 1991, 1993, or Lisboa, 2001): (1) The state utility function is fully separable between consumption and effort choice, i.e., $u(x_s, e) = v(x_s) - c(e)$ for some function $v(\cdot)$ and a cost function $c(\cdot)$. (2) There are only two individual states, i.e., $S = \{h, l\}$ (corresponding to high output and low output). (3) The probability function $q_h(e)$ is strictly increasing and strictly concave in $e$, and the cost function $c(e)$ is strictly increasing and convex in $e$. Under these conditions, it can be shown that the first-order approach is valid, which in turn implies that the worker's effort choice is continuous in $p$. However, these conditions are too restrictive for our purposes since they rule out nonseparable state utility functions (defined in Definition 3 below), which play a crucial role in our main efficiency results in Sections 3 and 4. Our Assumptions A3 and A4 are considerably weaker than the standard assumptions, and as such, they enable us to establish the existence of equilibrium for nonseparable as well as separable state utility functions. The analytical difficulties that emerge in the setting with nonseparable state utility functions have been emphasized by Arnott and Stiglitz (1988b, 1993). In this light, Theorem 1 could also be viewed as a methodological contribution to general equilibrium analyses of moral hazard economies.

It is also worth noting that allowing for stochastic contracts as in Prescott and Townsend (1984a) does not bypass the need to impose Assumptions A3 and A4 (see Section 5). Intuitively, allowing for stochastic contracts convexifies the incentive compatibility set, $C^I(p)$, which simplifies the analysis in some dimensions. However, convexifying a discontinuous correspondence does not necessarily make it continuous. Hence, the essential difficulty for establishing the existence of equilibrium remains in the setting with stochastic contracts. This difficulty does not arise in the Prescott and Townsend (1984a) economy, because contracts directly prescribe consumption levels for each good and thus endogenous prices do not affect the set of incentive compatibility contracts.

## 3 Generic Inefficiency of Equilibrium

This section formalizes the folk theorem for imperfect information economies, by providing sufficient conditions under which the equilibrium is inefficient. We start by describing the notion of efficiency used in our analysis.

An allocation in our setting, $(x, e) \in A$, is *effort-incentive compatible* if the effort choice is optimal given the level of consumption prescribed, that is, if

$$U(x, e) \geq U(x, \tilde{e}) \text{ for each } \tilde{e} \in E.$$

We denote the set of effort-incentive compatible allocations with $A^I$. An economy-wide allocation $([x(\nu), e(\nu)]_{\nu \in \mathcal{N}}$ is *effort-incentive compatible* and *feasible* if $(x(\nu), e(\nu)) \in A^I$ for each $\nu \in \mathcal{N}$, and the resource constraints hold, that is,

$$\int_{\mathcal{N}} \sum_{s \in S} q_s(e(\nu)) \left(y_s^g - x_s^g(\nu)\right) d\nu \geq 0, \text{ for each } g. \tag{13}$$

**Definition 2.** *An economy-wide allocation $[x(\nu), e(\nu)]_{\nu \in \mathcal{N}}$ is* ***constrained (Pareto) optimal*** *if it is effort-incentive compatible and feasible, and there does not exist another effort-incentive compatible and feasible economy-wide allocation $[\hat{x}(\nu), \hat{e}(\nu)]_{\nu \in \mathcal{N}}$ such that $U(\hat{x}(\nu), \hat{e}(\nu)) \geq U(x(\nu), e(\nu))$ for all $\nu \in \mathcal{N}$, with strict inequality for a positive measure of $\nu \in \mathcal{N}$.*

*Consider an equilibrium of the economy $\mathcal{E}$ with price vector and accepted contracts, $\left(p, [w(\nu), x(\nu), e(\nu)]_{\nu \in \mathcal{N}}\right)$. We say that the equilibrium is constrained optimal if the economy-wide allocation $[x(\nu), e(\nu)]_{\nu \in \mathcal{N}}$ is constrained optimal.*

**Remark 2.** ***(Full Monitoring by the Social Planner)*** Our notion of efficiency provides the social planner with the same informational constraints as the firms but with better contracting (monitoring) technology. In particular, the planner cannot observe the effort choice of the worker, but can specify the consumption of all goods in the employment contract. This notion of optimality is arguably strong. Nevertheless, it is natural for us for two reasons. First, this notion helps us to develop our main point more succinctly. For example, our main result

(Theorem 3) delineates a range of benchmark situations in which the equilibrium is efficient in this strong sense. Second, the social planner can approximate a constrained optimal outcome according to our definition using more limited instruments, i.e., a tax and transfer system. For example, the government can reduce the consumption of a particular good by levying a linear tax on that good (though a tax and transfer system is not equivalent to full monitoring: Appendix A.1 considers a weaker notion of optimality and provides an example economy that is constrained suboptimal according to the strong optimality notion, but not according to this weaker optimality notion).

The main result in this section establishes sufficient conditions under which the equilibrium is constrained suboptimal. These conditions are related to the notions of nonseparability and no full insurance, which we define next.[11]

**Definition 3.** *Consider an allocation $(x, e)$. The state utility function is **nonseparable** (not weakly separable) at $(x, e)$ if there exists a state $s$, a monitoring subset $G_m$, and two goods $g_1, g_2 \in G_m$ such that the marginal rate of substitution between $g_1$ and $g_2$ at state $s$ changes when effort level is modified, that is*

$$\frac{\partial u(x_s, e) / \partial x_s^{g_1}}{\partial u(x_s, e) / \partial x_s^{g_2}} \neq \frac{\partial u(x_s, \hat{e}) / \partial x_s^{g_1}}{\partial u(x_s, \hat{e}) / \partial x_s^{g_2}} \text{ for any } \hat{e} \in E \setminus \{e\}.$$

*There is **no full insurance** at $(x, e)$, if there exists a good $g \in G$ and two states $s_1, s_2 \in S$ such that the marginal rate of substitution for good $g$ between states $s_1, s_2$ is not equal to 1, that is*

$$\frac{\partial u(x_{s_1}, e) / \partial x_{s_1}^{g}}{\partial u(x_{s_2}, e) / \partial x_{s_2}^{g}} \neq 1.$$

The next theorem is our main inefficiency result. It shows that the equilibrium is constrained suboptimal whenever the state utility function is nonseparable and there is no full insurance at the equilibrium allocation for a positive measure of workers.

**Theorem 2.** ***(Nonseparability and Inefficiency)*** *Consider an economy $\mathcal{E}$ that satisfies Assumptions A1-A4. Let $\left[p, (w(\nu), x(\nu), e(\nu))_{\nu \in \mathcal{N}}\right]$ denote the prices and accepted contracts in an equilibrium. Suppose that there is a positive measure set $\mathcal{N}^* \subset \mathcal{N}$ such that for each $\nu \in \mathcal{N}^*$, the state utility function is nonseparable and there is no full insurance at the equilibrium allocation $(x(\nu), e(\nu))$. Then, the equilibrium is **not constrained optimal**.*

The intuition for the result is closely related to *double deviations* by the worker, that is, deviations in which a worker switches to a different effort level and reoptimizes her consumption

[11] As Definition 4 below makes it clear, "nonseparable" preferences are the converse of "weakly separable" preferences. We use the terminology "nonseparable" since its easier to use than "not weakly separable" or "weakly nonseparable".

of nonmonitored goods for the new effort level. When the preferences are nonseparable and there is no full insurance at the equilibrium allocation, double deviations bind in the incentive compatibility constraints. That is, they prevent firms from providing more insurance to the workers. A social planner who can also prescribe the consumption of nonmonitored goods is not constrained by double deviations, and can therefore provide better insurance without violating the incentive compatibility constraints.

It is also worth noting that Theorem 2 is a generic inefficiency result, since nonseparable utility functions are open and dense (or weakly separable utility functions, as defined in Definition 4, are nowhere dense) in the set of all continuous utility function (with the sup norm). Put differently, if $u$ is weakly separable and $\tilde{u}$ is nonseparable, then $\varepsilon u + (1-\varepsilon)\tilde{u}$ is nonseparable for any $\varepsilon \in (0,1)$.

The next example illustrates the intuition of Theorem 2.

**Example 1.** Suppose that there are two states, $S = \{h, l\}$, corresponding to high output and low output, and two goods, $G = \{1, 2\}$. The monitoring partition is given by $\mathcal{G} = \{G\}$, which implies that the firm can only specify wages and otherwise cannot monitor worker's consumption. For simplicity, consider a partial equilibrium setting in which the relative price of the goods is fixed and is equal to 1, i.e., suppose $p^1 = p^2 = 1$.[12] Suppose there are two effort levels, i.e., $E = \{0, 1\}$, which respectively correspond to shirking and working. Assume the firm makes zero profits in the low output state and positive profits in the high output state, that is, $\pi_l \equiv y_l p = 0$, and $\pi_h \equiv y_h p > 0$. Assume $q_h(e=1) = 1/2$ and $q_h(e=0) = 0$. Assume also that good 2 is relatively more complementary to leisure than good 1. More specifically, the worker's state utility function is given by:

$$u(x_s, e) = \ln\left(x_s^1 + \left(\frac{1}{2} + (1-e)\right)x_s^2\right). \tag{14}$$

Note that the worker enjoys good 2 relatively more (in comparison to good 1) when she does not work, and relatively less when she works. Consider a social planner that chooses the allocation $\left(\left(\hat{x}_h^1, \hat{x}_h^2, \hat{x}_l^1, \hat{x}_l^2\right), \hat{e}\right)$, subject to effort-incentive compatibility and resource constraints. It can be seen that the social planner implements $\hat{e} = 1$ and provides the worker with the consumption of only good 1, i.e., she chooses $\hat{x}_h^2 = \hat{x}_l^2 = 0$. Moreover, given that the worker does not consume good 2, the state utility function in (14) implies that there is no incentive problem. Thus, the

[12] The partial equilibrium setting is a special case of our model in which there is a linear production technology (that operates without moral hazard considerations) which can convert the two goods to each other. To simplify the notation, we present our main results in an environment without this type of production technology. Appendix B, which is available on request, shows that all of our results continue to hold when we introduce a general (potentially nonlinear) production technology that is not subject to moral hazard. Also, again to simplify the exposition, in this example we use functional forms that do not satisfy Assumptions A2 and A3.

social planner provides the worker with full insurance. That is, the worker's consumption of the first good is given by:

$$\hat{x}_h^1 = \hat{x}_l^1 = \frac{\pi_h}{2}, \tag{15}$$

and the worker's utility is given by

$$U(\hat{x}, \hat{e}) = \frac{1}{2}\ln\left(\frac{\pi_h}{2}\right) + \frac{1}{2}\ln\left(\frac{\pi_h}{2}\right) = \ln\left(\frac{\pi_h}{2}\right). \tag{16}$$

Next consider a firm that offers a wage contract $(w_h, w_l)$. It can be seen that the social planner's full insurance solution is not incentive compatible. That is, given the wages just enough to consume the bundle (15), the worker would instead not work and consume a different bundle. Since $\ln\left(\frac{\pi_h}{2}\right) < \ln\left(\frac{3}{2}\frac{\pi_h}{2}\right)$, the worker can increase her utility with a double deviation in which she changes her effort choice and reoptimizes her consumption for the new effort decision. The firm will instead offer the wage contract $(w_h, w_l)$ that is the solution to the following equations:

$$\frac{\tilde{w}_h + \tilde{w}_l}{2} = \frac{\pi_h}{2} \qquad \text{(Budget constraint)},$$

$$\frac{1}{2}\ln(\tilde{w}_h) + \frac{1}{2}\ln(\tilde{w}_l) \geq \ln\left(\frac{3}{2}\tilde{w}_h\right) \qquad \text{(Incentive compatibility)}.$$

The equilibrium wages and consumption are given by:

$$w_h = \frac{9}{13}\pi_h,\ x_h^1 = w_h, x_h^2 = 0, \text{and } w_l = \frac{4}{13}\pi_h, x_l^1 = w_l, x_l^2 = 0. \tag{17}$$

The equilibrium utility is given by:

$$U(x, e) = \frac{1}{2}\ln\left(\frac{9}{13}\pi_h\right) + \frac{1}{2}\ln\left(\frac{4}{13}\pi_h\right) = \ln\left(\frac{6}{13}\pi_h\right). \tag{18}$$

Comparing Eqs. (17)-(18) with (15)-(16), note that the firm is only partially insuring the worker, and the contract offered by the firm is strictly worse for the worker than the allocation offered by the social planner.

Theorem 2 establishes the inefficiency of equilibrium under conditions on the equilibrium allocation. A natural question is whether there are any economies for which these conditions hold. To address this question, Theorem 8 in Appendix A.1 characterizes a class of economies in which any equilibrium is constrained optimal. The result essentially provides conditions on the preferences and the technology such that Theorem 2 applies at any equilibrium. To ensure that the equilibrium allocation of a worker always features less than full insurance, we assume that there exists a shirking effort level which is always preferred by the worker under full insurance, and which yields the firm (almost) zero profits. To ensure that every equilibrium allocation satisfies the nonseparability property, we assume that there exists a monitoring subset $G_m$ and two goods $g_1, g_2 \in G_m$ such that the marginal rate of substitution between $g_1$ and $g_2$ always change monotonically in the effort level.

Theorems 2 and 8 provide some support for the folk theorem for the inefficiency of the equilibrium. Recall, however, that these theorems rely on a strong notion of optimality which essentially provides the social planner with a better monitoring technology than the firms (cf. Remark 2). Theorems 2 and 8 are silent on whether a social planner who is also constrained by the same monitoring technology can implement such a Pareto improvement. To address this issue, Appendix A.1 introduces a weaker notion of optimality which constrains the social planner with the same monitoring technology as the firms. This appendix also provides an example economy with nonseparable utility, which is constrained suboptimal as implied by Theorem 8, but is weakly constrained optimal. The example shows that the inefficiency of equilibrium established in Theorems 2 and 8 in part stems from the strong notion of optimality which gives the social planner a technological advantage in monitoring. This suggests that care must be taken in invoking these theorems.

# 4 Efficiency of Equilibrium under Weak Separability

We next provide our main result, which is the converse of Theorem 2. In particular, the result shows that the equilibrium is constrained optimal when worker preferences are weakly separable. The next definition formalizes the notion of weak separability.

**Definition 4.** *The state utility function $u(\cdot)$ is* ***weakly separable*** *if, for any monitoring subset $G_m$ and two goods $g_1, g_2 \in G_m$, the marginal rate of substitution between $g_1$ and $g_2$ is independent of effort level. That is,*

$$\frac{\partial u(x_s, e) / \partial x_s^{g_1}}{\partial u(x_s, e) / \partial x_s^{g_2}} = \frac{\partial u(x_s, \hat{e}) / \partial x_s^{g_1}}{\partial u(x_s, \hat{e}) / \partial x_s^{g_2}} \text{ for each } g_1, g_2 \in G_m,\ x_s \in \mathbb{R}_+^{|G|} \text{ and } e, \hat{e} \in E. \quad (19)$$

Note that the state utility function is weakly separable if and only if it is not nonseparable at any allocation $(x, e) \in A$. Our next result shows that weak separability is sufficient for the competitive equilibrium to be constrained Pareto optimal.

**Theorem 3.** ***(Efficiency under Weak Separability)*** *Consider an economy $\mathcal{E}$ that satisfies Assumptions A1-A4. Assume also that the state utility function $u(\cdot)$ is weakly separable. Then, any equilibrium of the economy $\mathcal{E}$ is* ***constrained optimal.***

The intuition of this result is that, under weak separability, the social planner chooses for the worker the consumption bundle which the worker would have chosen by herself in the anonymous trading market. Hence, there is no benefit to additional monitoring, and competition among firms leads to the allocations that the social planner would have chosen. A complementary intuition is that double deviations in which the worker changes effort level and reoptimizes her consumption bundle accordingly are not valuable. This further implies that the relative price changes caused by other contracts in the economy does not change the insurance-incentive trade-off for a worker, rendering pecuniary externalities ineffective. We demonstrate this theorem with a simple example in which preferences are fully separable.

**Example 2.** Consider the same setup as in Example 1 with two differences. First, to emphasize that price externalities do not create inefficiencies in this setting, we consider more general relative prices than in Example 1, that is, we consider the price vector $\left(p^1 = 1, p^2 \in \mathbb{R}_+\right)$ (we continue to assume that the prices are fixed). Second, we assume that the worker's state utility function is given by

$$u\left(x_s, e\right) = u^{sep}\left(v\left(x_s^1, x_s^2\right), e\right),$$

where $u^{sep}\left(v, e\right)$ is a scalar valued function that is strictly increasing and strictly concave in $v$ and decreasing in $e$, and $v\left(x_s^1, x_s^2\right)$ is a scalar valued function that is strictly increasing and strictly concave in both arguments (a particular instance is $u^{sep}\left(v, e\right) = \ln\left(v\right) - e$ and $v\left(x_s^1, x_s^2\right) = \left(\left(x_s^1\right)^{\frac{\varepsilon-1}{\varepsilon}} + \left(x_s^2\right)^{\frac{\varepsilon-1}{\varepsilon}}\right)^{\frac{\varepsilon}{\varepsilon-1}}$ for some $\varepsilon > 0$). Here, the inner function $v\left(x_s^1, x_s^2\right)$ can be thought of as the consumption level of a particular "need," which is satisfied by consuming goods 1 and 2. The outer function $u^{sep}\left(v, e\right)$ gives the worker's utility corresponding the level of the need, $v$, and the effort choice, $e$. Note that the state utility function, $u\left(\cdot\right)$, is weakly separable, since

$$\frac{\partial u\left(x_s, e\right) / \partial x_s^1}{\partial u\left(x_s, e\right) / \partial x_s^2} = \frac{\partial v\left(x_s^1, x_s^2\right) / \partial x_s^1}{\partial v\left(x_s^1, x_s^2\right) / \partial x_s^2},$$

which is independent of the effort choice.

Consider first a planner who determines an effort-incentive compatible allocation $\left(\left(x_h^1, x_l^1, x_h^2, x_l^2\right), e\right)$ for each worker. Suppose, for simplicity, that the planner offers every worker the same employment allocation (the result in Theorem 3 is more general and does not rely on this assumption). The planner maximizes the representative worker's utility subject to effort-incentive compatibility and resource constraints, that is:

$$\begin{aligned}
&\max_{(\tilde{x},\tilde{e})\in A} q_h\left(\tilde{e}\right) u^{sep}\left(v\left(\tilde{x}_h^1, \tilde{x}_h^2\right), \tilde{e}\right) + \left(1 - q_h\left(\tilde{e}\right)\right) u^{sep}\left(v\left(\tilde{x}_l^1, \tilde{x}_l^2\right), \tilde{e}\right) \\
\textit{subject to}\quad &\sum_{s\in S} q_s\left(\tilde{e}\right)\left(\tilde{x}_s^1 + p^2 \tilde{x}_s^2\right) = \sum_{s\in S} q_s\left(\tilde{e}\right)\pi_s, \\
&U\left(\tilde{x}, \tilde{e}\right) \geq U\left(\tilde{x}, \hat{e}\right) \text{ for all } \hat{e} \in E.
\end{aligned}$$

To solve this problem, the planner computes the value from implementing any $\tilde{e} \in E$. The planner then implements the effort level that yields the highest utility to the worker. Similar to the analysis in Grossman and Hart (1983), the social planner's problem is simplified in view of the weak separability of the state utility function. In particular, the planner's problem is equivalent to first deciding how much of the need to provide in each state, $\tilde{v}_s$, and then deciding the optimal consumption bundle that provides this level of the need. Hence, the optimal utility from implementing $\tilde{e} \in E$ is given by

$$\begin{aligned} & \max_{\tilde{x} \in \mathbb{R}_+^{|S| \times |G|}, \tilde{v}_h \in \mathbb{R}, \tilde{v}_l \in \mathbb{R}} q_h(\tilde{e})\, u^{sep}(\tilde{v}_h, \tilde{e}) + (1 - q_h(\tilde{e}))\, u^{sep}(\tilde{v}_l, \tilde{e}) \qquad (20) \\ \text{subject to} \quad & \sum_{s \in S} q_s(\tilde{e}) \left(\tilde{x}_s^1 + p^2 \tilde{x}_s^2\right) = \sum_{s \in S} q_s(\tilde{e})\, \pi_s \\ & \sum_{s \in S} q_s(\tilde{e})\, u^{sep}(\tilde{v}_h, \tilde{e}) \geq \sum_{s \in S} q_s(\hat{e})\, u^{sep}(\tilde{v}_h, \hat{e}) \text{ for all } \hat{e} \in E, \end{aligned}$$

and two additional constraints $v\left(\tilde{x}_h^1, \tilde{x}_h^2\right) = \tilde{v}_h$ and $v\left(\tilde{x}_l^1, \tilde{x}_l^2\right) = \tilde{v}_l$.

Note that, given $\tilde{v}_s$, the planner would like to minimize the cost of providing this level of the need. That is, for each $s \in \{h, l\}$, the vector $x_s$ is the solution to:

$$\begin{aligned} & w_{pl}(\tilde{v}_s) = \min_{\tilde{x}_s \in \mathbb{R}_+^{|G|}} \tilde{x}_s^1 + p^2 \tilde{x}_s^2 \\ \text{subject to} \quad & v\left(\tilde{x}_s^1, \tilde{x}_s^2\right) = \tilde{v}_s. \end{aligned}$$

This is a strictly convex minimization problem that provides a one-to-one relationship between $w_{pl}(\cdot)$ and the optimum choice of the consumption vector $x_s$. By duality, the optimum vector $x_s$ also maximizes $v\left(x_s^1, x_s^2\right)$ subject to the expenditure being not greater than $w_{pl}(\tilde{v}_s)$. Using these observations, problem (20) can be rewritten as

$$\begin{aligned} & \max_{\substack{w_{pl}(\tilde{v}_h) \in \mathbb{R}_+, \\ w_{pl}(\tilde{v}_l) \in \mathbb{R}_+}} \sum_{s \in S} q_s(\tilde{e}) \left( \max_{\left\{\tilde{x}_s \in \mathbb{R}_+^{|G|} \,\mid\, \tilde{x}_s^1 + p^2 \tilde{x}_s^2 \leq w_{pl}(\tilde{v}_s)\right\}} u^{sep}\left(v\left(\tilde{x}_s^1, \tilde{x}_s^2\right), \tilde{e}\right) \right) \qquad (21) \\ \text{subject to} \quad & \sum_{s \in S} q_s(\tilde{e})\, w_{pl}(\tilde{v}_s) = \sum_{s} q_s(\tilde{e})\, \pi_s \\ & \textstyle\sum_{s \in S} q_s(\tilde{e}) \left( \max_{\left\{\tilde{x}_s \in \mathbb{R}_+^{|G|} \,\mid\, \tilde{x}_s^1 + p^2 \tilde{x}_s^2 \leq w_{pl}(\tilde{v}_s)\right\}} u^{sep}\left(v\left(\tilde{x}_s^1, \tilde{x}_s^2\right), \tilde{e}\right) \right) \geq \\ & \textstyle\sum_{s \in S} q_s(\hat{e}) \left( \max_{\left\{\tilde{x}_s \in \mathbb{R}_+^{|G|} \,\mid\, \tilde{x}_s^1 + p^2 \tilde{x}_s^2 \leq w_{pl}(\tilde{v}_s)\right\}} u^{sep}\left(v\left(\tilde{x}_s^1, \tilde{x}_s^2\right), \hat{e}\right) \right) \text{ for each } \hat{e} \in E. \end{aligned}$$

Next consider the problem of a firm offering a wage contract $\{w_g, w_b\}$. To implement effort level $\tilde{e} \in E$, the firm will solve

$$\max_{w_h \in \mathbb{R}_+, w_l \in \mathbb{R}_+} \quad \sum_{s \in S} q_s(\tilde{e}) \left( \max_{\left\{\tilde{x}_s \in \mathbb{R}_+^{|G|} \;|\; \tilde{x}_s^1 + p^2 \tilde{x}_s^2 \le w_s\right\}} u^{sep}\left(v\left(\tilde{x}_s^1, \tilde{x}_s^2\right), \tilde{e}\right) \right) \tag{22}$$

$$\text{subject to} \quad \sum_{s \in S} q_s(\tilde{e}) w_s = \sum_s q_s(\tilde{e}) \pi_s$$

$$\textstyle\sum_{s \in S} q_s(\tilde{e}) \left( \max_{\left\{\tilde{x}_s \in \mathbb{R}_+^{|G|} \;|\; \tilde{x}_s^1 + p^2 \tilde{x}_s^2 \le w_s\right\}} u^{sep}\left(v\left(\tilde{x}_s^1, \tilde{x}_s^2\right), \tilde{e}\right) \right) \ge$$

$$\textstyle\sum_{s \in S} q_s(\hat{e}) \left( \max_{\left\{\tilde{x}_s \in \mathbb{R}_+^{|G|} \;|\; \tilde{x}_s^1 + p^2 \tilde{x}_s^2 \le w_s\right\}} u^{sep}\left(v\left(\tilde{x}_s^1, \tilde{x}_s^2\right), \hat{e}\right) \right) \text{ for each } \hat{e} \in E.$$

A comparison of problems (21) and (22) shows that they are equivalent. Consequently, the social planner and the firm will choose to implement the same effort level, $e$, and the worker will receive the same utility in each case. Hence, in this example, monitoring is not valuable, and a firm that can only offer a wage contract provides incentives as well as a social planner who can monitor worker's consumption.

The analysis in this example also suggests a general class of preferences that satisfies the weak separability condition. Suppose that the state utility function can be written as

$$u(x_s, e) = u^{sep}\left(v^1\left(x_s^{G_1}\right), v^2\left(x_s^{G_2}\right), .., u^{|M|}\left(x_s^{G_{|\mathcal{G}|}}\right), e\right), \tag{23}$$

where $u^{sep}\left(v^1, v^2, .., v^{|M|}, e\right)$ is a scalar valued function which is strictly increasing and strictly concave in its first $|M|$ arguments, and for each $m$, $v^m : \mathbb{R}_+^{|G_m|} \mapsto \mathbb{R}$ is a scalar valued function which is increasing and concave in its arguments. This functional form is intuitive, as it implies that the worker receives utility over a number of higher order needs (such as food, entertainment, procrastination, vacation, health care, education etc.) each of which is provided by the consumption of a distinct set of goods. Clearly, the state utility function in (23) is weakly separable. Thus, Theorem 3 implies that equilibria of economies in which preferences can be represented as in (8) are constrained optimal. This analysis the suggests that, if firms can monitor the consumption of broad aggregates that interfere with effort choice, then the equilibrium will be efficient even though firms are unable to monitor workers' exact consumption choices.

## 5 Equilibrium and Efficiency with Stochastic Contracts

This section extends our setup to allow for contracts with random outcomes, briefly "stochastic contracts." Allowing for randomization can be beneficial in this economy because of the nonconvexity of problem (2) in Section 2 (e.g., Prescott and Townsend, 1984a, 1984b, Arnott and Stiglitz, 1988a). Our analysis in this section shows that allowing for stochastic contracts does not change the efficiency properties of this economy. In particular, analogues of our main results, Theorems 2 and 3, apply for the economy with stochastic contracts.

Given a finite dimensional space $Z$, we let $\mathcal{B}(Z)$ denote the Borel $\sigma$-algebra of $Z$ and $\mathcal{P}(Z)$ denote the set of probability measures over $(Z, \mathcal{B}(Z))$. We endow $\mathcal{P}(Z)$ with the weak* topology. A *stochastic allocation* in our setting is a probability measure $\eta$ over the allocation space $\left(A = \mathbb{R}_+^{|S|\times|G|} \times E, \mathcal{B}(A)\right)$. The worker's utility from a stochastic allocation $\eta$ is denoted by $U^R(\eta)$, and it is equal to $\int_{(x,e)\in A} U(x,e)\, d\eta$. Similarly, a *stochastic contract* is a probability measure, $\mu \in \mathcal{P}(C)$, over the contract space $\left(C = \mathbb{R}_+^{|S|\times|M|} \times A, \mathcal{B}(C)\right)$. Given a stochastic contract $\mu$, we let $\mu|_{(x,e)} \in \mathcal{P}(A)$ denote the marginal measure over the allocation space $A$.

Depending on when the contract uncertainty is resolved, a stochastic contract $\mu$ can be interpreted as having either ex-ante or ex-post randomization. With ex-post randomization, the contract uncertainty is resolved after the effort decision is made, that is, the worker chooses an effort level and then learns which contract she will receive. With ex-ante randomization, the contract uncertainty is resolved before the effort decision is made, i.e., the worker learns her contract before she chooses an effort level (see, Arnott and Stiglitz, 1988a, for further discussion and the role of each type of randomization). For analytical and notational convenience, we analyze the case with only ex-ante randomization. All of the results in this section generalize to the case with both ex-ante and ex-post randomization.[13]

Given the assumption of only ex-ante randomization, a stochastic contract $\mu$ is *incentive compatible* if and only if its support lies in the set of deterministic incentive compatible contracts, that is, $supp(\mu) \subset C^I(p)$. Equivalently, an incentive compatible stochastic contract is a probability measure $\mu \in \mathcal{P}\left(C^I(p)\right)$. The expected utility of the worker and the expected profits of the firm are respectively given by $\int_{C^I(p)} V(c,p)\, d\mu$ and $\int_{C^I(p)} \pi(c,p)\, d\mu$. As in the deterministic case, each worker $\nu$ faces a menu of incentive compatible contracts $\left\{\mu_{(\nu,j)}\right\}_{j\in J}$, and chooses the contract that maximizes her utility. Each firm $j$ offers a continuum of contracts to maximize its expected profit, taking the workers' strategies and other firms' contract offers as given. We denote the economy with stochastic contracts with $\mathcal{E}^R$, and we define the equilibrium for this economy as follows.

**Definition 5.** *A **competitive equilibrium** in economy $\mathcal{E}^R$ is a collection of incentive compatible contract offers $\left\{\mu_{(\nu,j)}\right\}_{j\in J,\nu\in\mathcal{N}}$ by the firms, a collection of strategies for workers, $\left[\mathbf{J}_\nu^R\right]_{\nu\in\mathcal{N}}$, a price vector $p$, allocations $\left[\mu_\nu\right]_{\nu\in\mathcal{N}}$, such that: workers' contract choice is optimal, firms max-*

[13] It is well known that ex-post randomization in this setting could be useful to provide the worker with additional incentives (see Arnott and Stiglitz, 1988a). However, as noted by Bennardo and Chiappori (2003), this additional incentive provision does not interfere with the existence or efficiency properties of equilibrium.

*imize expected profits, and goods markets clear, that is, for each g:*

$$\int_{\mathcal{N}} \int_{(w,x,e) \in C^I(p)} \sum_{s \in S} q_s(e)(y_s^g - x_s^g) d\mu_\nu d\nu \geq 0, \text{ with equality for } p^g > 0. \tag{24}$$

The following result is the counterpart of Proposition 1 and Theorem 1 with stochastic contracts.

**Theorem 4.** ***(Existence of Equilibrium with Stochastic Contracts)*** *Consider an economy $\mathcal{E}^R$ that satisfies Assumptions A1-A4. There exists a competitive equilibrium for the economy $\mathcal{E}^R$. The prices and accepted contracts, $(p, [\mu_\nu]_{\nu \in \mathcal{N}})$, are part of an equilibrium if and only if:*

1. *For all but a measure zero of workers $\nu \in \mathcal{N}$, the contract $\mu_\nu$ is a solution to*

$$\max_{\tilde{\mu} \in \mathcal{P}(C^I(p))} \quad \int_{C^I(p)} V(c, p)\, d\tilde{\mu} \tag{25}$$

$$\text{subject to} \quad \int_{C^I(p)} \pi(c, p)\, d\tilde{\mu} \geq 0. \tag{26}$$

2. *The goods markets clear [cf. Eq. (24)].*

*Moreover, at the solution to Problem (25), the profit constraint (26) binds, that is, every firm makes zero profits in equilibrium.*

Analysis of the efficiency of the equilibrium closely parallels the case with deterministic contracts. First, we provide the analogous definition of constrained optimality with stochastic contracts.

Given the assumption of only ex-ante randomization, a stochastic allocation $\eta$ is effort-incentive compatible if and only if the support of $\eta$ lies in the set of deterministic effort-incentive compatible allocations, that is, $\eta \in \mathcal{P}(A^I)$. An economy-wide stochastic allocation $[\eta_\nu]_{\nu \in \mathcal{N}}$ is effort-incentive compatible and feasible if and only if $\eta_\nu \in \mathcal{P}(A^I)$ for each $\nu$ and the resource constraints hold:

$$\int_{\mathcal{N}} \int_{(x,e) \in A^I} \sum_{s \in S} q_s(e)(y_s^g - x_s^g) d\eta_\nu d\nu \geq 0 \text{ for each } g. \tag{27}$$

**Definition 6.** *An economy-wide stochastic allocation $[\eta_\nu]_{\nu \in \mathcal{N}}$ is* ***constrained optimal*** *if it is effort-incentive compatible and feasible, and there does not exist another effort-incentive compatible and feasible economy-wide allocation $[\hat{\eta}_\nu]_{\nu \in \mathcal{N}}$ such that $U^R(\hat{\eta}_\nu) \geq U^R(\eta_\nu)$ for all $\nu \in \mathcal{N}$ with strict inequality for a positive measure of $\nu$.*

*Consider an equilibrium of the economy $\mathcal{E}^R$ with the prices and accepted contracts, $(p, [\mu_\nu]_{\nu \in \mathcal{N}})$. We say that the equilibrium is constrained optimal if the economy-wide allocation $\left[\mu_\nu|_{(x,e)}\right]_{\nu \in \mathcal{N}}$ is constrained optimal.*

Our next result is the analogue of Theorem 2 for stochastic contracts, and establishes sufficient conditions under which the equilibrium is not constrained optimal. To state the result, we generalize the notions of nonseparability and no full insurance to stochastic allocations.

**Definition 7.** *Consider a stochastic allocation $\eta$ and a compact set $A^* \subset A$ such that $\eta(A^*) > 0$. The state utility function $u(\cdot)$ is **nonseparable** at $(\eta, A^*)$, if there exists a state $s$, a monitoring subset $G_m$, and two goods $g_1, g_2 \in G_m$ such that the marginal rate of substitution between $g_1$ and $g_2$ at state $s$ changes when the effort level is modified, that is,*

$$\frac{\partial u(x_s, e) / \partial x_s^{g_1}}{\partial u(x_s, e) / \partial x_s^{g_2}} \neq \frac{\partial u(x_s, \hat{e}) / \partial x_s^{g_1}}{\partial u(x_s, \hat{e}) / \partial x_s^{g_2}} \text{ for each } (x, e) \in A^* \text{ and } \hat{e} \in E \setminus \{e\}.$$

*There is **no full insurance** at $(\eta, A^*)$ if there exists a good $g \in G$ and two states $s_1, s_2 \in S$ such that*

$$\frac{\partial u(x_{s_1}, e) / \partial x_{s_1}^g}{\partial u(x_{s_2}, e) / \partial x_{s_2}^g} \neq 1 \text{ for each } (x, e) \subset A^*.$$

The following is our main inefficiency result for the economy with stochastic contracts.

**Theorem 5.** ***(Inefficiency under Stochastic Contracts)*** *Consider an economy $\mathcal{E}^R$ that satisfies Assumptions A1-A4. Let $\left(p, [\mu_\nu]_{\nu \in \mathcal{N}}\right)$ denote the prices and accepted contracts in an equilibrium. Suppose that there is a positive measure set $\mathcal{N}^* \subset \mathcal{N}$ such that for each $\nu \in \mathcal{N}^*$, there exists a compact set $A^* \subset A$ with $\mu_\nu|_{(x,e)}(A^*) > 0$ such that the state utility function is nonseparable and there is no full insurance at $\left(\mu_\nu|_{(x,e)}, A^*\right)$. Then, the equilibrium is **not constrained optimal**.*

We next provide the analogue of Theorem 3 for stochastic contracts. The following result shows that, when the utility function is weakly separable, any equilibrium with stochastic contracts is constrained optimal.

**Theorem 6.** ***(Efficiency under Stochastic Contracts)*** *Consider economy $\mathcal{E}^R$ that satisfies Assumptions A1-A4. Assume also that the state utility function $u(\cdot)$ is weakly separable. Then, any equilibrium of the economy $\mathcal{E}^R$ is **constrained optimal**.*

# 6 Approximate Efficiency of Equilibrium

As noted in Section 3, weak separability is a nongeneric property (in the sense that any weakly separable utility function will become nonseparable after a small perturbation). However, there is also a sense in which such genericity results may not be relevant for understanding the economic importance of certain types of inefficiencies. In particular, such genericity results do not preclude the possibility that in most economically relevant situations equilibria might be "approximately" efficient. In the present context, weakly separable utility functions might be a fairly good approximation to most economic situations (including almost all cases considered in applied work). Thus, what might be relevant is whether there are significant inefficiencies when there are only small deviations from such weak separability. In this section, we investigate this question. We introduce a notion of approximate efficiency ($\varepsilon$-constrained optimality) and show that when the economy is "close to" an alternative economy with weakly separable utility functions, its equilibrium will be approximately constrained optimal.

**Definition 8.** *For each $\varepsilon \geq 0$, an economy-wide allocation $[\eta_\nu]_{\nu\in\mathcal{N}}$ is **$\varepsilon$-constrained optimal** if it is effort-incentive compatible and feasible, and there does not exist another effort-incentive compatible and feasible economy-wide allocation $[\hat{\eta}_\nu]_{\nu\in\mathcal{N}}$ such that $U^R(\hat{\eta}_\nu) \geq U^R(\eta_\nu) + \varepsilon$ for all $\nu \in \mathcal{N}$, with strict inequality for a positive measure of $\nu \in \mathcal{N}$.*

*Consider an equilibrium of the economy $\mathcal{E}^R$ with the prices and accepted contracts, $\left(p, [\mu_\nu]_{\nu\in\mathcal{N}}\right)$. We say that the equilibrium is $\varepsilon$-constrained optimal if the economy-wide allocation $\left[\mu_\nu|_{(x,e)}\right]_{\nu\in\mathcal{N}}$ is $\varepsilon$-constrained optimal.*

The notion of $\varepsilon$-constrained optimality is a generalization of the notion of constrained optimality (cf. Definition 2) since the two notions are equivalent for $\varepsilon = 0$. We next introduce the notion of a perturbation of the state utility function. Recall that, by Assumption A3, the state utility function has the representation

$$u(x_s, e) = u^{G_1}\left(x_s^{G_1}\right) + u^{G\setminus G_1}\left(x_s^{G\setminus G_1}, e\right),$$

for some functions $u^{G_1}(\cdot)$ and $u^{G\setminus G_1}(\cdot)$. For simplicity, we keep the transferable component, $u^{G_1}(\cdot)$, unchanged and we focus on perturbations of $u^{G\setminus G_1}(\cdot)$ (this is without loss of any generality). Let $u^{G_1}(\cdot)$ denote a continuously differentiable, strictly increasing, strictly concave and bounded function, and consider the set of state utility functions:

$$\mathcal{U} = \left\{ \left( \begin{array}{c} u^{G_1}, \\ u^{G\setminus G_1} : \mathbb{R}_+^{G\setminus G_1} \times E \to \mathbb{R} \end{array} \right) \middle| \; u \text{ satisfies Assumption A1 and is bounded.} \right\}.$$

Note that any state utility function $u \in \mathcal{U}$ satisfies Assumptions A1 and A3. We endow the set $\mathcal{U}$ with the sup norm, which makes $\mathcal{U}$ into a normed vector space, and thus a metric space.[14]

For a state utility function $\overline{u} \in \mathcal{U}$, we let $B(\overline{u}, \delta) = \{u \in \mathcal{U} \mid \|u - \overline{u}\| \leq \delta\}$ denote the $\delta$ neighborhood of $\overline{u}$. When $\delta > 0$, the functions in $B(\overline{u}, \delta)$ can be thought of as small

[14] The assumption that $u$ is bounded is without loss of generality, because each worker's production of each good $g$ is bounded above by $\max_{s\in S} y_s^g$, which implies that the utility function can be restricted to a bounded consumption set. More specifically, for any unbounded state utility function, $\tilde{u}$, there exists a bounded state utility function, $u$, which agrees with $\tilde{u}$ on the relevant consumption set and which leads to the same equilibrium set.

perturbations of the function $\overline{u}$. If $\overline{u}$ is weakly separable, then the state utility functions in $B(\overline{u}, \delta)$ are close to being weakly separable. Our next result shows that equilibria of economies with utility functions in $B(\overline{u}, \delta)$ are approximately efficient ($\varepsilon$-constrained optimal).

**Theorem 7.** ***(Approximate Efficiency)*** *Let $\left[\mathcal{E}^R(u)\right]_{u\in\mathcal{U}}$ denote the class of economies that satisfy Assumptions A1-A4 and that differ only in the state utility function, $u \in \mathcal{U}$. Consider a weakly separable state utility function $\overline{u} \in \mathcal{U}$. For any $\varepsilon > 0$, there exists $\delta > 0$ such that, if $u \in B(\overline{u}, \delta)$, then any equilibrium of the economy $\mathcal{E}^R(u)$ is $\varepsilon$-constrained optimal.*

We provide a sketch proof of this result, which is completed in Appendix A.2. We first present a social planner's problem for this economy, which is useful to characterize the efficiency properties of the equilibrium. Consider the problem of maximizing an (equal) weighted utility of the workers subject to effort-incentive compatibility and resource constraints:

$$\begin{aligned} \max_{\left[\tilde{\eta}_\nu \in \mathcal{P}(A^I)\right]_{\nu\in\mathcal{N}}} \quad & \int_{\mathcal{N}} \int_{(x,e)\in A^I} U(x,e)\, d\tilde{\eta}_\nu d\nu \\ \text{subject to} \quad & \int_{\mathcal{N}} \int_{(x,e)\in A^I} \sum_{s\in S} q_s(e)\,(y_s^g - x_s^g) d\tilde{\eta}_\nu d\nu \geq 0. \end{aligned} \tag{28}$$

Note that problem (28) is a linear optimization problem and the constraint set, $\mathcal{P}\left(A^I\right)$, is convex. Hence, problem (28) has the same optimal value as the following simpler problem:

$$\begin{aligned} \max_{\tilde{\eta} \in \mathcal{P}(A^I)} \quad & \int_{(x,e)\in A^I} U(x,e)\, d\tilde{\eta} \\ \text{subject to} \quad & \int_{(x,e)\in A^I} \sum_{s\in S} q_s(e)\,(y_s^g - x_s^g) d\tilde{\eta} \geq 0, \end{aligned} \tag{29}$$

where $\tilde{\eta} \in \mathcal{P}\left(A^I\right)$ is the average measure defined by:

$$\tilde{\eta}\left(\tilde{A}\right) = \int_{\mathcal{N}} \eta_\nu \left(\tilde{A}\right) d\nu \text{ for each } \tilde{A} \in \mathcal{B}\left(A^I\right). \tag{30}$$

In other words, the social planner can be thought of as choosing a single stochastic allocation to maximize a worker's expected utility subject to resource constraints. Let $U^R_{planner}(u)$ denote the optimal value of problem (29) for the economy with the state utility function $u \in \mathcal{U}$.

The next lemma characterizes approximate constraint optimality of the equilibrium by comparing the optimal value of the social planner's problem (29) with the optimal value of the equilibrium problem (25). To state the result, let $U^R_{eq}(p,u)$ denote the value of problem (25) when the price vector is given by $p$ and the state utility function is $u \in \mathcal{U}$.

**Lemma 1.** *Consider an equilibrium of the economy $\mathcal{E}^R$ with the price vector $p$. For any $\varepsilon \geq 0$, the equilibrium is $\varepsilon$-constrained optimal if and only if $U^R_{planner}(u) \in \left[U^R_{eq}(p,u), U^R_{eq}(p,u) + \varepsilon\right]$. In particular, the equilibrium is constrained optimal if and only if $U^R_{planner}(u) = U^R_{eq}(p,u)$.*

Given this lemma, Theorem 7 intuitively follows from Theorem 6 and the continuity properties of problems (25) and (29). To see this, first note that Theorem 6 implies that any equilibrium of the economy $\mathcal{E}(\overline{u})$ is constrained optimal. By Lemma 1, this implies

$$U^R_{planner}(\overline{u}) = U^R_{eq}(p, \overline{u}), \tag{31}$$

for any equilibrium price vector $p$ of the economy $\mathcal{E}(\overline{u})$. Next, the analysis in Appendix A.2 establishes that a version of Berge's Maximum Theorem applies to problem (29), and thus $U^R_{planner}(u)$ is a continuous function of $u$. Similarly, $U^S_{eq}(p, u)$ is a continuous function of $(p, u)$. Moreover, it can be seen that the equilibrium price correspondence,

$$P(u) = \left\{ p \in \mathbb{R}^{|S|}_+ \mid p \text{ is an equilibrium price vector of } \mathcal{E}^R(u) \right\}, \tag{32}$$

is upper hemicontinuous in $u$. These observations, along with Eq. (31), imply that $U^R_{planner}(u)$ and $U^R_{eq}(p, u)$ are close to each other for a state utility function, $u$, that lies in a neighborhood of $\overline{u}$. Appendix A.2 formalizes and completes this argument, establishing a proof of Theorem 7.

Theorem 7 also highlights why the notion of generic inefficiency is not always economically useful. Take the set of economies corresponding to utility functions in the set $B(\overline{u}, \delta)$ for some $\delta > 0$. With the same argument as above, weakly separable utility functions are nowhere dense within the set $B(\overline{u}, \delta)$. But our result shows that all of the corresponding economies have equilibria that are approximately efficient.

**Remark 3.** ***(The Role of Stochastic Contracts in Approximate Efficiency)*** Note that Theorem 7 concerns the economy $\mathcal{E}^R$ with stochastic contracts, and we do not have an analogous approximate efficiency result for an economy $\mathcal{E}$ with deterministic contracts. We conjecture that the result generalizes to economy $\mathcal{E}$, but this conjecture is not straightforward to prove.

The proof of Theorem 7 does not generalize to economy $\mathcal{E}$, mainly because there appears to be no analogue of Lemma 1 in that setting. In particular, due to the nonconvexity of the set of incentive compatible contracts, $C^I(p)$, the Pareto frontier of economy $\mathcal{E}$ is not necessarily characterized as the solution to a weighted social planner's problem. One can construct example economies in which the equilibrium is constrained optimal; but neither problem (28) nor problem (29) (nor any other weighted social planner's problem) yields the worker the same utility as the equilibrium. Since our proof of Theorem 7 exploits the continuity properties of the social planner's problem, this proof does not generalize to economy $\mathcal{E}$. Stochastic contracts are useful in this context because they convexity the set of incentive compatible contracts, which in turn enables us to characterize constrained optimality using a social planner's problem (as formalized by Lemma 1).

## 7 Conclusion

This paper investigated the efficiency of competitive equilibria in environments with private information. Prescott and Townsend (1984a, 1984b) establish the constrained optimality of competitive equilibrium in such environments when (insurance, employment or credit) contracts can fully specify consumption bundles. Though important, these results are not applicable to situations in which individuals are allowed to trade in anonymous markets. We view such anonymous trading to be an essential feature of competitive equilibria. Less is known about the structure and efficiency of competitive equilibria in the presence of such anonymous trading.

A "folk theorem" originating in the work of Stiglitz and coauthors maintains that competitive equilibria are always or "generically" inefficient in such environments. This folk theorem has widespread applicability in both applied models and in policy discussions, though it has not been formally investigated. This paper critically reevaluates this folk theorem in the context of a general equilibrium economy with moral hazard. In our economy, firms offer contracts to workers who choose an effort level that is private information and affects the probability distribution of endowment and production vectors. We establish the existence of a competitive equilibrium and characterize some of its properties.

To investigate the efficiency properties of competitive equilibrium, we introduce a *monitoring partition* such that employment contracts can specify expenditures over subsets in the partition but cannot regulate how this expenditure is subdivided among the commodities within a subset. We say that preferences are *nonseparable* (or not weakly separable) when the marginal rate of substitution across commodities within a subset in the partition depends on the effort level. We prove that the equilibrium is always inefficient when a competitive equilibrium allocation involves less than full insurance and preferences are nonseparable. While this result is consistent with the folk theorem on the inefficiency of competitive equilibrium, our main result shows why such inefficiency does not always arise and can be mitigated by partial monitoring. We show that when there is *weak separability* in preferences, a condition satisfied by preference is used in most applied theory work, competitive equilibria with moral hazard are constrained optimal, in the sense that a social planner who can regulate and monitor all consumption levels cannot improve over these competitive allocations. We also show that equilibria in economies that have utility functions that are approximately weakly separable will be approximately efficient. These results imply that the strong suboptimality claims of the folk theorem for competitive equilibria in private information economies may be somewhat exaggerated. At the very least, considerable care is necessary in concluding that competitive equilibria are inefficient and government intervention is necessary without knowing the details of preference and information structure.

Our results also emphasize that the efficiency properties of competitive equilibria depend on the monitoring partition, which raises the question of how the monitoring partition is determined in practice. In related work, we develop a framework for the analysis of competitive equilibria in which firms pay a cost to choose which subsets of commodities and actions to monitor. Among other things, this framework shows that endogenous monitoring will create another force towards efficiency. In particular, additional welfare loss in equilibrium compared to the constrained efficient allocation is bounded above by the cost of monitoring a particular partition (which depends on the worker's preferences). This further implies that, when the cost of monitoring this partition is sufficiently small, the competitive equilibrium is approximately constrained optimal, despite the costs of monitoring that it incurs (relative to the social planner who does not incur them). This result reinforces our point that considerable care is necessary in invoking the folk theorem about the inefficiency of competitive equilibria with private information.

# A Appendices

## A.1 Omitted Results.

The appendix presents results omitted from the main text.

**Class of economies in which any equilibrium is inefficient.** Theorem 2 in the main text established sufficient conditions under which the equilibrium allocation is constrained suboptimal. The next result identifies a class of economies in which any equilibrium satisfies the conditions of Theorem 2, and thus, is inefficient. To ensure that the equilibrium always features less than full insurance, we assume that there exists a shirking effort level which is always preferred by the worker under full insurance, and which yields the firm (almost) zero profits. To ensure that every equilibrium has the nonseparability property, we assume that there exists two goods within the same monitoring partition such that the marginal rate of substitution between the goods change monotonically in the effort level. The result then follows from Theorem 2 (proof omitted).

**Theorem 8.** ***(Sufficient Conditions For Inefficiency)*** *Consider a class of economies* $[\mathcal{E}(\theta)]_{\theta\in(0,1)}$ *which differ only in the parameter* $\theta$ *defined below. Suppose that*

1. *There exists an effort level,* $e_{shirk} \in E$*, such that* $u(x, e_{shirk}) > u(x, e)$ *for all* $e \in E \setminus \{e_{shirk}\}$ *and* $x \in \mathbb{R}_{++}^{|S| \times |G|}$*. There also exists a state* $s_{low} \in S$ *such that* $y_{s_{low}} = 0$ *and* $q_{s_{low}}(e_{shirk}) = 1 - \theta$*. Moreover,* $U(\mathbf{0}, e_{shirk}) < \max_{e \in E} U(y, e)$*, i.e., the allocation* $(x = \mathbf{0}, e = e_{shirk})$ *is less desirable than the allocation that offers no insurance (and lets the worker choose the effort level).*

2. *There exists a monitoring subset* $G_m$ *and two goods* $g_1, g_2 \in G_m$ *such that* $\frac{\partial u(x,e)/\partial x^{g_1}}{\partial u(x,e)/\partial x^{g_2}}$ *is strictly increasing in* $e \in E \subset \mathbb{R}$ *for any* $x \in \mathbb{R}_{++}^{|S| \times |G|}$*.*

*There exists* $\bar{\theta} \in (0, 1)$ *such that, for each* $\theta \leq \bar{\theta}$*, Assumptions A1-A4 hold,* $\mathcal{E}(\theta)$ *has at least one competitive equilibrium, and any equilibrium of the economy* $\mathcal{E}(\theta)$ *is not constrained optimal.*

**Alternative notions of efficiency.** We next consider a weaker notion of optimality than studied in the main text. This notion of optimality constrains the social planner with the same monitoring technology as the firms.

**Definition 9.** *A price and contract allocation pair,* $\left(p, [c(\nu) = (w(\nu), x(\nu), e(\nu)]_{\nu \in \mathcal{N}}\right)$*, is* ***market feasible*** *if the contracts are incentive compatible given the price vector, i.e.,* $c(\nu) \in C^I(p)$ *for each* $\nu \in \mathcal{N}$*, and the resource constraints in* (13) *hold. The pair is weakly constrained optimal if it is market feasible and there does not exist another market feasible pair,* $\left(\hat{p}, [\hat{c}(\nu) = (\hat{w}(\nu), \hat{x}(\nu), \hat{e}(\nu)]_{\nu \in \mathcal{N}}\right)$*, such that* $U(\hat{x}(\nu), \hat{e}(\nu)) \geq U(x(\nu), e(\nu))$ *for all* $\nu \in \mathcal{N}$ *with strict inequality for a positive measure of* $\nu \in \mathcal{N}$*.*

We next provide an example with nonseparable state utility function in which the equilibrium is not constrained optimal, as implied by Theorem 2, but which is weakly constrained optimal. The example shows that the inefficiency of equilibrium established in Theorems 2 and 8 in part stems from the strong notion of optimality which gives the social planner a technological advantage in monitoring.

**Example 3** (Nonseparable Preferences, Weak Optimality)**.** Consider the same setup as in Example 1 but assume, additionally, that workers are heterogeneous in their preferences. In particular, there are two types of workers, denoted by $T_1$ and $T_2$, of equal measure, 1/2. The state utility functions of type $T_1$ and $T_2$ workers are respectively given by:

$$\begin{aligned} u(x_s, e\,; T_1) &= \ln\left(x_s^1 + \left(\frac{1}{2} + (1-e)\right)x_s^2\right) \text{ and} \\ u(x_s, e\,; T_2) &= \ln\left(\left(\frac{1}{2} + (1-e)\right)x_s^1 + x_s^2\right). \end{aligned} \tag{A.1}$$

Let $\left(p^1 = 1, p^2 = 1, (c[T_i] = w[T_i], x[T_i], e[T_i])_{i\in\{1,2\}}\right)$ denote an equilibrium price and contract allocation pair in which workers of the same type accept the same contract. The equilibrium allocation for type $T_1$ workers is characterized exactly as in Example 1 [cf. Eq. (17)], while the equilibrium allocation for type $T_2$ workers is the mirror image allocation in which the consumption of goods 1 and 2 are reversed. We claim that this equilibrium is weakly constrained optimal.

First consider the Pareto improving allocation in Example 1 (the allocation chosen by a social planner that can fully monitor consumption), given by $\hat{e}[T_i] = 1$ for each $i \in \{1,2\}$, and the consumption allocations:

$$\textit{For type } T_1 \quad : \quad \hat{x}^1_h[T_1] = \hat{x}^1_l[T_1] = \frac{\pi_h}{2} \textit{ and } \hat{x}^2_h[T_1] = \hat{x}^2_l[T_1] = 0, \tag{A.2}$$

$$\textit{For type } T_2 \quad : \quad \hat{x}^1_h[T_2] = \hat{x}^1_l[T_2] = 0 \textit{ and } \hat{x}^2_h[T_2] = \hat{x}^2_l[T_2] = \frac{\pi_h}{2}. \tag{A.3}$$

We claim that the corresponding contract allocation, $\{\hat{c}[T_i] = (\hat{w}[T_i] = \hat{x}[T_i]\hat{p}, \hat{x}[T_i], \hat{e}[T_i])\}_{i\in\{1,2\}}$, is not market feasible for any relative price level $\hat{p}^2$ (recall that $\hat{p}^1$ is normalized to 1). Given the state utility function in $(A.1)$ (and $\hat{e}[T_1] = 1$), the allocation in $(A.2)$ is incentive compatible for the type $T_1$ workers only if $\hat{p}^2 \leq \frac{1}{2}$. However, similarly, the allocation in $(A.3)$ is incentive compatible for the type $T_2$ workers only if $\hat{p}^2 \geq 2$. Hence, there is no relative price level that can make the contract allocation $\{\hat{c}[T_1], \hat{c}[T_2]\}$ market feasible, verifying the claim.

It remains to show that there is no other market feasible price and contract allocation pair which is a Pareto improvement over the equilibrium. Suppose, to reach a contradiction, that there is such a price and allocation pair, denoted by $\left(\hat{p}^1 = 1, \hat{p}^2, (\hat{w}[T_i] = \hat{x}[T_i]\hat{p}, \hat{x}[T_i], \hat{e}[T_i])_{i\in\{1,2\}}\right)$. It can be seen that $\hat{e}[T_i] = 1$ for each $i \in \{1,2\}$.[15] In view of the conversion technology, the resource constraints can be reduced to a single constraint:

$$\sum_{i\in\{1,2\}} \frac{1}{2} \sum_{s\in\{l,h\}} \frac{1}{2} \sum_{g\in\{1,2\}} x^g_s[T_i] \leq \frac{\pi_h}{2}. \tag{A.4}$$

First consider the case $\hat{p}^2 \in \left(\frac{1}{2}, 2\right)$, so that type $T_1$ workers always choose to consume good 1 and type $T_2$ workers always choose to consume good 2, i.e., $x^g_s[T_i] = 0$ for $i \neq g$. Then, the incentive compatibility constraints can be written as:

$$\begin{aligned}
\frac{1}{2}\ln\left(\hat{x}^1_h[T_1]\right) + \frac{1}{2}\ln\left(\hat{x}^1_l[T_1]\right) &\geq \ln\left(\frac{3}{2}\frac{\hat{x}^1_l[T_1]}{\hat{p}_2}\right) \\
\frac{1}{2}\ln\left(\hat{x}^2_h[T_2]\right) + \frac{1}{2}\ln\left(\hat{x}^2_l[T_2]\right) &\geq \ln\left(\frac{3}{2}\hat{x}^2_l[T_2]\,\hat{p}_2\right).
\end{aligned}$$

[15] More specifically, for the case in which only one type works, it can be seen that it is not possible to attain a Pareto improvement without violating the resource constraints.

These constraints can be simplified to

$$\frac{\hat{x}_h^1[T_1]}{\hat{x}_l^1[T_1]} \geq \frac{9}{4}\frac{1}{(\hat{p}_2)^2} \text{ and } \frac{\hat{x}_h^2[T_2]}{\hat{x}_l^2[T_2]} \geq \frac{9}{4}(\hat{p}_2)^2 . \tag{A.5}$$

Note also that this allocation is a Pareto improvement over the equilibrium allocation (cf. Eqs. (18) – (17)) if only if:

$$\frac{1}{2}\ln\left(\hat{x}_h^i[T_i]\right) + \frac{1}{2}\ln\left(\hat{x}_l^i[T_i]\right) \geq \ln\left(\frac{6}{13}\pi_h\right) \text{ for each } i \in \{1,2\}, \tag{A.6}$$

with strict inequality for some $i \in \{1,2\}$. It can be seen that conditions $(A.5)$ and $(A.6)$ establish a lower bound on the amount of resources that need to be spent on each type workers, that is:

$$\begin{aligned} \hat{x}_h^1[T_1] + \hat{x}_l^1[T_1] &\geq \left(\frac{3}{2\hat{p}_2} + \frac{2\hat{p}_2}{3}\right)\frac{6}{13}\pi_h \text{ and} \\ \hat{x}_h^2[T_2] + \hat{x}_l^2[T_2] &\geq \left(\frac{3\hat{p}_2}{2} + \frac{2}{3\hat{p}_2}\right)\frac{6}{13}\pi_h, \end{aligned}$$

with strict inequality for some $i \in \{1,2\}$. Combining the last two inequalities with the resource constraint $(A.4)$ (along with the fact that $x_s^g[T_i] = 0$ for $i \neq g$), we have:

$$\frac{3}{2\hat{p}_2} + \frac{2\hat{p}_2}{3} + \frac{3\hat{p}_2}{2} + \frac{2}{3\hat{p}_2} < \frac{13}{3}. \tag{A.7}$$

On the other hand, using the arithmetic-mean geometric-mean inequality, we have:

$$\begin{aligned} \frac{3}{2\hat{p}_2} + \frac{3\hat{p}_2}{2} &\geq 2\sqrt{\frac{3}{2\hat{p}_2}\frac{3\hat{p}_2}{2}} = 3 \text{ and} \\ \frac{2\hat{p}_2}{3} + \frac{2}{3\hat{p}_2} &\geq 2\sqrt{\frac{2\hat{p}_2}{3}\frac{2}{3\hat{p}_2}} = \frac{4}{3}. \end{aligned}$$

Combining these inequalities with Eq. $(A.7)$ yields a contradiction. A similar contradiction can be obtained for the case in which $\hat{p}_2 \notin \left[\frac{1}{2}, 2\right]$, proving that the equilibrium is weakly constrained optimal.

The key intuition behind this result is captured by the incentive compatibility constraints, $(A.5)$. Note that a high level of the relative price $\hat{p}_2$ relaxes the incentive compatibility constraints for the type $T_1$ workers (who have a greater temptation to shirk when they consume good 2), but it also simultaneously tightens the incentive compatibility constraints for the type $T_2$ workers (who have a greater temptation to shirk when they consume good 1). Hence, the Pareto improving allocation constructed in Example 1 cannot be decentralized with any price vector, and the equilibrium is weakly constrained optimal despite the fact that it is not constrained optimal.

## A.2 Proofs

This appendix presents proofs of the results in the main text.

### A.2.1 Proofs for Section 2

We first show that Assumptions A2 and A3 imply the following lemma, which we use in the subsequent analysis.

**Lemma 2.** ***(Local Transferability)*** *Suppose Assumptions A1-A3 hold and consider an incentive compatible contract,* $c = (w, x, e) \in C^I(p)$*. Then, for each* $\varepsilon > 0$*, there exists contracts* $c_+, c_- \in C^I(p)$ *such that*

$$V(c_-, p) > V(c) - \varepsilon, \quad \pi(c_-, p) > \pi(c, p), \tag{A.8}$$

*and*

$$V(c_+, p) > V(c, p), \quad \pi(c_+, p) > \pi(c, p) - \varepsilon. \tag{A.9}$$

**Proof of Lemma 2.** We first define a transfer function that will be useful in this and some of the subsequent proofs. For each non-zero consumption vector $x_s^{G_1}$, price vector $p$, and transfer level $t_s \in \mathbb{R}$, we define $\mathbf{w}_s^1\left(x_s^{G_1}, p, t_s\right) \in \mathbb{R}$ as the optimum value and $\tilde{\mathbf{x}}_s^{G_1}\left(x_s^{G_1}, p, t_s\right)$ as the solution of the following strictly convex optimization problem:

$$\begin{aligned} \mathbf{w}_s^1\left(x_s^{G_1}, p, t_s\right) &= \min_{\tilde{x}_s^{G_1} \geq 0} \tilde{x}_s^{G_1} p^{G_1} \\ \text{subject to} \quad & u^{G_1}(\tilde{x}_s^{G_1}) = u^{G_1}\left(x_s^{G_1}\right) + t_s. \end{aligned} \tag{A.10}$$

That is, $\mathbf{w}_s^1\left(x_s^{G_1}, p, t_s\right)$ describes the minimum wage level necessary to increase the state $s$ utility obtained from the goods in $G_1$ by $t_s$, given that the current consumption is $x_s^{G_1}$ and the current price vector is $p$. Note that, since $x_s^{G_1}$ is non-zero, there exists a sufficiently small interval $\left(-\overline{\delta}, \overline{\delta}\right)$ such that $\mathbf{w}_s^1\left(x_s^{G_1}, p, t_s\right)$ is well defined for each $t_s \in \left(-\overline{\delta}, \overline{\delta}\right)$. Note also that, over this interval, $\mathbf{w}_s^1\left(x_s^{G_1}, p, t_s\right)$ is continuous and strictly increasing in $t_s$. Given the transfer vector $t = (t_s)_{s \in \mathcal{S}}$, we also define $\mathbf{w}^1\left(x^{G_1}, p, t\right) = \left(\mathbf{w}_s^1\left(x_s^{G_1}, p, t_s\right)\right)_{s \in S}$ and $\tilde{\mathbf{x}}^{G_1}\left(x^{G_1}, p, t\right) = \left(\tilde{\mathbf{x}}_s^{G_1}\left(x_s^{G_1}, p, t_s\right)\right)_{s \in S}$ as the corresponding transfer functions over all states.

We next prove Lemma 2. Consider an incentive compatible contract $c = (w, x, e) \in C^I(p)$ and a positive scalar $\varepsilon > 0$. We first construct a contract $c_+$ that satisfies Eq. $(A.9)$. Note that we have $\mathbf{w}_s^1\left(x_s^{G_1}, p, 0\right) = w_s^1$ since $c \in C^I(p)$. Then, from the continuity of $\mathbf{w}_s^1(\cdot)$, there exists $\delta > 0$ such that

$$\mathbf{w}_s^1\left(x_s^{G_1}, p, \delta\right) \in \left(w_s^1, w_s^1 + \varepsilon\right) \text{ for each } s. \tag{A.11}$$

By definition of $\mathbf{w}_s^1(\cdot)$ (and incentive compatibility of $c$) we also have

$$u^{G_1}\left(\tilde{\mathbf{x}}_s^{G_1}\left(x_s^{G_1}, p, \delta\right)\right) = u^{G_1}\left(x_s^{G_1}\right) + \delta, \text{ for each } s \in S. \tag{A.12}$$

Define the transfer vector $t(\delta) = (t_s \equiv \delta \text{ for each } s)$. Since $c$ is incentive compatible and $u(\cdot)$ satisfies condition (8), Eq. $(A.12)$ implies that $e$ remains incentive compatible after the utility transfer $t(\delta)$. It follows that $c_+ = \begin{pmatrix} \left(\mathbf{w}^1\left(x^{G_1}, p, t(\delta)\right), w^{M\setminus\{1\}}\right), \\ \left(\tilde{\mathbf{x}}^{G_1}\left(x^{G_1}, p, t(\delta)\right), x^{G\setminus G_1}\right), e \end{pmatrix}$ is incentive compatible. By Eq. $(A.11)$, this contract costs the firm at most $\varepsilon$ more than the contract $c$. Thus, $c_+$ satisfies Eq. $(A.9)$. Next recall that Assumptions A2 and A3 ensure $w_s^1 > 0$ for each $s$. Hence, a similar argument establishes the existence of a contract $c_-$ that satisfies Eq. $(A.8)$. This completes the proof of the lemma.

**Proof of Proposition 1.** First consider the only if part of the proposition. Let $\left(p, [w(\nu), x(\nu), e(\nu)]_{\nu \in \mathcal{N}}\right)$ denote an equilibrium price and contract allocation pair. The second claim holds holds by definition of equilibrium. To prove the first claim, first note that firms can always guarantee themselves 0 profits (by offering contracts that will not be accepted). This implies $\pi(c(\nu), p) \geq 0$, that is, the contract $c(\nu)$ is in the constraint set of problem (10) for all $\nu$ (except potentially a measure zero set). We claim that $c(\nu)$ solves problem (10) for all but a measure zero set of workers. Suppose, to reach a contradiction, that there exists a positive measure set $\mathcal{N}^* \subset \mathcal{N}$ and contracts $(\hat{c}(\nu))_{\nu \in \mathcal{N}^*}$ such that $c(\nu)$ and $\hat{c}(\nu)$ are both in the constraint set of problem (10), but $V(\hat{c}(\nu), p) > V(c(\nu), p)$. Since $\mathcal{N}^* \subset \mathcal{N}$ is of positive measure and the set of firms is finite, there exists a firm $j$ such that the set $\mathcal{N}^j = \{\nu \in \mathcal{N}^* \mid J(\nu) = j\}$ is of positive measure. Consider a contract $c(\nu)$, with $\nu \in N^j$. Applying Lemma 2 to this contract for $\varepsilon = V(\hat{c}(\nu), p) - V(c, p) > 0$, there exists another incentive compatible contract $\hat{c}_-(\nu)$ such that

$$V(\hat{c}_-(\nu), p) > V(\hat{c}(\nu), p) \text{ and } \pi(\hat{c}_-(\nu), p) > \pi(\hat{c}(\nu), p) \geq 0. \tag{A.13}$$

Then, we claim that another firm $j' \neq j$ can strictly increase its profits by changing its contract offers to the workers in $\mathcal{N}^j$ to $(\hat{c}_-(\nu))_{\nu \in N^j}$. Note that, after this change, all the workers $\nu \in \mathcal{N}^j$ switch to firm $j'$. Since each contract $\hat{c}_-(\nu)$ makes the firm positive profits (cf. $(A.13)$), the expected profits of firm $j'$ strictly increase after this deviation, proving our claim. This contradicts the fact that firm $j'$ maximizes profits in equilibrium, completing the proof for the only if part of the proposition.

We next prove the claim in the proposition that the constraint $\pi((w, x, e), p) \geq 0$ binds for any solution $(w, x, e)$ to problem (10). Suppose, to reach a contradiction, that the contract

$c = (w, x, e)$ is a solution to problem (10) and satisfies $\pi(c, p) > 0$. Then, by Lemma 2, there exists another incentive compatible contract $c_+$ such that $\pi(c_+, p) \geq 0$ and $V(c_+, p) > V(c, p)$. This contradicts the fact that $c$ is a solution to problem (10), showing that the profit constraint binds.

Next consider the if part of the proposition. Let $p$ and $[w(\nu), x(\nu), e(\nu)]_{\nu \in \mathcal{N}}$ be a price and allocation system that satisfies the two claims of the proposition. We conjecture a symmetric equilibrium in which every firm offers every worker the same contract, i.e., $(c(\nu, j) = [w(\nu), x(\nu), e(\nu)])_{\nu \in \mathcal{N}, j \in J}$. Given these offers, any worker strategy is optimal. Moreover, the goods market clearing condition is satisfied by assumption. Hence, we are only left with by proving that firms' contract offers are optimal. Suppose, to reach a contradiction, that there exists a firm $j'$ that can make strictly positive profits by offering a collection of incentive compatible contracts $[\hat{c}(\nu, j')]_{\nu \in \mathcal{N}}$. Then, there exists $\mathcal{N}^* \subset \mathcal{N}$ with positive measure such that for each $\nu \in \mathcal{N}^*$,

$$J(\nu) = j' \text{ after the deviation by firm } j', \text{ and}$$
$$\pi\left(\hat{c}\left(\nu, j'\right), p\right) > \pi(c(\nu), p) = 0. \tag{A.14}$$

Since the worker $\nu \in \mathcal{N}^*$ prefers the contract offered by $j'$ over the contract $c(\nu)$, we have

$$V\left(\hat{c}\left(\nu, j'\right), p\right) \geq V(c(\nu), p). \tag{A.15}$$

By Lemma 2 and equations $(A.14)$ and $(A.15)$, there exists an incentive compatible contract $\hat{c}_+(\nu, j')$ such that

$$\pi\left(\hat{c}_+\left(\nu, j'\right), p\right) > 0, \text{ and } V\left(\hat{c}_+\left(\nu, j'\right), p\right) > V(c(\nu), p).$$

It follows that $c(\nu)$ is not a solution to problem (10) for $\nu \in \mathcal{N}^*$, which is a contradiction. This shows that the price vector and contract allocations constructed above constitute an equilibrium, completing the proof for the if part of the proposition. ■

We next show that Assumptions A3 and A4 imply the following lemma, which we need to establish the existence of equilibrium.

**Lemma 3.** ***(Continuity of Incentive Compatible Contracts)*** *Suppose Assumptions A1-A4 hold Then, the correspondence $C^I(p)$ is lower hemicontinuous in $p$. That is, for any incentive compatible contract $c \in C^I(p)$ and any sequence $p[n] \to p$, there exists a sequence of contracts $c[n] \in C^I(p[n])$ such that $c[n] \to c$.*

Since $C^I(p)$ is also upper hemicontinuous, Lemma 3 establishes the continuity of $C^I(p)$ in $p$.

**Proof of Lemma 3.** Consider the price vector $p$, an incentive compatible contract $c \equiv (w, x, e)$, and a sequence $\{p[n]\}_{n=1}^{\infty} \to p$. We claim that, for any $\varepsilon > 0$, there exists an index $n$ and a contract $c[n] \equiv (x[n], e, w[n])$ such that

$$c[n] \in C^I(p[n]) \text{ and } w[n] \in B(w, \varepsilon). \tag{A.16}$$

Given this claim, a sequence $\{c[n]\}_{n=0}^{\infty}$ can be constructed such that $c[n] \in C^I(p[n])$ and $c[n] \to c$. It thus follows that $C^I(p)$ is lower hemicontinuous.

To prove the claim in $(A.16)$, first note that Assumption A4 implies that there exists a transfer vector $t \in \mathbb{R}^{|S|}$ such that Eq. (12) holds for the effort level $e$ and the vector $t$. Given this vector $t$, note that Eq. (12) also holds for the transfer vector $\eta t$, where $\eta > 0$ is an arbitrary positive scalar.

Next consider the function $\mathbf{w}^1(\cdot)$ defined in $(A.10)$. Since $\mathbf{w}^1(\cdot)$ is continuous in $p$ and $t$, and since $\mathbf{w}^1\left(x^{G_1}, 0, p\right) = w^1 > 0$ (by Assumptions A2 and A3), there exists $\delta > 0$ such that $\mathbf{w}^1\left(x^{G_1}, \tilde{p}, \tilde{t}\right) \in B\left(w^1, \varepsilon\right) \cap \mathbb{R}_{++}$ for each $\tilde{p}$ and $\tilde{t}$ that satisfies $\left\|\tilde{t}\right\| \leq \delta$ and $||\tilde{p} - p|| \leq \delta$. Let $\eta$ be sufficiently small so that $||\eta t|| \leq \delta$ and define

$$\bar{\varepsilon} = \sum_{s \in S} \eta t_s q_s(e) - \max_{\hat{e} \in E \setminus \{e\}} \sum_{s \in S} \eta t_s q_s(\hat{e}), \tag{A.17}$$

which is strictly positive in view of Eq. (12). Since the indirect utility function $V(c, p)$ and the function $U(\mathbf{x}(w, p, e), e)$ are continuous in $p$, there exists a sufficiently large $n$ such that $||p[n] - p|| \leq \delta$ and

$$\begin{aligned} V(c, p[n]) &< V(c, p) + \bar{\varepsilon}/2, \text{ and} \\ U(\mathbf{x}(w, p[n], e), e) &> U(\mathbf{x}(w, p, e), e) - \bar{\varepsilon}/2. \end{aligned}$$

Since $V(c, p) = U(\mathbf{x}(w, p, e), e)$, these two inequalities jointly imply

$$U(\mathbf{x}(w, p[n], \hat{e}), \hat{e}) < U(\mathbf{x}(w, p[n], e), e) + \bar{\varepsilon} \text{ for all } \hat{e} \in E \setminus \{e\}. \tag{A.18}$$

Given the constructed $\eta t$ and $p[n]$, we define the contract $c[n] = \begin{pmatrix} x[n] = \left(\tilde{\mathbf{x}}^{G_1}\left(x^{G_1}, p[n], \eta t\right), x^{G \setminus G_1}\right) \\ e \\ w[n] = \left(\mathbf{w}^1\left(x^{G_1}, p[n], \eta t\right), w^{M \setminus \{1\}}\right) \end{pmatrix}$ and we claim that $c[n]$ satisfies $(A.16)$. By construction of $x[n]$ and $w[n]$ (and $\delta$), we have that $w[n] \in B\left(w^1, \varepsilon\right)$. Moreover, $x[n]$ is incentive compatible given the wages $w[n]$ and the effort level $e$. Hence, we only need to show that the effort level $e$ is incentive compatible. Suppose, to reach a contradiction, that there exists $\hat{e} \neq e$ such that

$$U(\mathbf{x}(w[n], p[n], \hat{e}), \hat{e}) \geq U(\mathbf{x}(w[n], p[n], e), e). \tag{A.19}$$

Note that the construction of $w[n]$ differs from $w$ only for the wages for the monitoring subset $G_1$. Note also that, from condition (8), the consumption choice for goods within $G_1$ does not affect the consumption choice for the goods within other monitoring subsets. This implies $\mathbf{x}^{G\backslash G_1}(w[n], p[n], \tilde{e}) = \mathbf{x}^{G\backslash G_1}(w, p[n], \tilde{e})$ for any effort level $\tilde{e}$. These observations, along with the definition of the function $\mathbf{w}^1(\cdot)$ (cf. Eq. $(A.10)$), further imply that:

$$U(\mathbf{x}(w[n], p[n], \tilde{e}), \tilde{e}) = \sum_{s \in S} \eta t_s q_s(\tilde{e}) + U(\mathbf{x}(w, p[n], \tilde{e}), \tilde{e}).$$

Considering this equality for $\tilde{e} \in \{e, \hat{e}\}$, and plugging in the inequality $(A.19)$ implies:

$$\sum_{s \in S} \eta t_s q_s(\hat{e}) + U(\mathbf{x}(w, p[n], \hat{e}), \hat{e}) \geq \sum_{s \in S} \eta t_s q_s(e) + U(\mathbf{x}(w, p[n], e), e).$$

Combining this inequality with the inequality in $(A.18)$, we have

$$\sum_{s \in S} \eta t_s q_s(\hat{e}) > \sum_{s \in S} \eta t_s q_s(e) - \overline{\varepsilon}.$$

This inequality yields a contradiction to the definition of $\overline{\varepsilon}$ in $(A.17)$, proving that the effort level $e$ is incentive compatible. This shows that $c[n]$ satisfies the claim in $(A.16)$ and completes the proof of the lemma. ■

**Proof of Theorem 1.** The key step in establishing the existence of the equilibrium is to show that the solution correspondence for the problem (10), denoted by $S(p)$, is upper hemicontinuous in $p$. We establish this using the continuity of $C^I(p)$ (cf. Lemma 3) along with the local transferability condition (cf. Lemma 2).

Let $\{(w_n, x_n, e_n), p_n\}_{n=1}^{\infty}$ denote a sequence such that $(w_n, x_n, e_n) \in S(p_n)$ for each $n$ and

$$\lim_{n \to \infty} (w_n, x_n, e_n) = (w, x, e) \text{ and } \lim_{n \to \infty} p_n = p.$$

Note that $(w, x, e)$ satisfies the constraints of problem (10) for the price vector $p$. We claim that $(w, x, e) \in S(p)$. Suppose, to reach a contradiction, that there exists $(\hat{w}, \hat{x}, \hat{e})$ that satisfies the constraints of problem (10) and that yields the worker $V((\hat{w}, \hat{x}, \hat{e}), p) > V((w, x, e), p)$. By Lemma 2, there exists another incentive compatible contract $(\overline{w}, \overline{x}, \overline{e})$ which satisfies the non-zero profit constraint strictly,

$$\pi((\overline{w}, \overline{x}, \overline{e}), p) > 0, \tag{A.20}$$

and which yields a strictly greater utility for the worker,

$$V((\overline{w}, \overline{x}, \overline{e}), p) > V((w, x, e), p). \tag{A.21}$$

Since $(\overline{w}, \overline{x}, \overline{e}) \in C^I(p)$, by Lemma 3, there exists $(\overline{w}_n, \overline{x}_n, \overline{e}_n) \to (\overline{w}, \overline{x}, \overline{e})$ such that $(\overline{w}_n, \overline{x}_n, \overline{e}_n) \in C^I(p_n)$ for each $n$. Since $\pi(\cdot)$ and $V(\cdot)$ are continuous functions, Eqs. $(A.20)$ and $(A.21)$ imply that there exists a sufficiently large $n$ such that

$$\pi\left(\left(\overline{w}_n, \overline{x}_n, \overline{e}_n\right), p_n\right) > 0 \text{ and } V\left(\left(\overline{w}_n, \overline{x}_n, \overline{e}_n\right), p_n\right) > V\left(\left(w_n, x_n, e_n\right), p_n\right).$$

But since $(\overline{w}_n, \overline{x}_n, \overline{e}_n) \in C^I(p_n)$, these inequalities contradict the fact that $(w_n, x_n, e_n)$ is a solution to problem (10) given the price vector $p_n$. It follows that $(w, x, e) \in S(p)$, which implies that $S(p)$ is upper hemicontinuous.

The rest of the proof follows standard arguments. Consider the excess demand correspondence $D : \{1\} \times \mathbb{R}_+^{|G|-1} \rightrightarrows \mathbb{R}^{|G|}$ (recall that the price of good 1 is normalized to 1), given by:

$$D(p) = \left\{ \begin{array}{c} \int_{\mathcal{N}} \sum_{s \in S} (\tilde{x}_s(\nu) - y_s) q_s(\tilde{e}(\nu)) d\nu \in \mathbb{R}^{|G|} \\ \mid \ (\tilde{w}(\nu), \tilde{x}(\nu), \tilde{e}(\nu)) \in S(p) \ \text{ for each } \nu \in \mathcal{N} \end{array} \right\}. \tag{A.22}$$

Since there is a continuum of workers, the demand correspondence $D(p)$ is convex valued. Since $S(p)$ is upper hemicontinuous, the correspondence $D(p)$ is also upper hemicontinuous. We have, $\lim_{p^g \to 0} D^g(p) > 0$, since the supply of good $g \neq 1$ is bounded from above (since $y_s < \infty$ for each $s$) while the demand for good $g$ tends to infinity as $p^g \to 0$. Similarly, $\lim_{p^g \to \infty} D^g(p) < 0$ since the supply of good $g$ is bounded away from zero, while the demand for good $g$ tends to zero as $p^g \to \infty$. Therefore, Kakutani's Fixed Point Theorem applies to this economy, which implies that there exists a price vector $p$ such that $0 \in D(p)$. By definition of $D(p)$, there exists $(w(\nu), x(\nu), e(\nu))_{\nu \in \mathcal{N}}$ such that $(w(\nu), x(\nu), e(\nu)) \in S(p)$ for each $\nu \in \mathcal{N}$ and the goods markets clear. By Proposition 1, the price vector $p$ and the contract allocations $(w(\nu), x(\nu), e(\nu))_{\nu \in \mathcal{N}}$ correspond to an equilibrium, completing the proof of the theorem. ■

### A.2.2 Proofs for Section 3

**Proof of Theorem 2.** Consider a worker $\nu \in \mathcal{N}^*$ and denote her allocation by $c \equiv (w, x, e)$. Since $(x, e)$ does not feature full insurance, there exists $s_1, s_2 \in S$ and $g \in G$ such that the MRS for good $g$ between states $s_1$ and $s_2$ is not equal to 1. We will reallocate the consumption of good $g$ across states so that the worker receives better insurance while her equilibrium effort choice $e$ remains effort-incentive compatible. Formally, we claim that there exists $\hat{x}^g \in \mathbb{R}_+^{|S|}$ such that

$$\sum_{s \in S} q_s(e)\, \hat{x}_s^g = \sum_{s \in S} q_s(e)\, x_s^g, \tag{A.23}$$

$$e \in \arg\max_{e \in E} U\left(x^{G \setminus \{g\}}, \hat{x}^g, e\right), \tag{A.24}$$

$$\text{and } U\left(x^{G \setminus \{g\}}, \hat{x}^g, e\right) > U(x, e). \tag{A.25}$$

Once we prove this claim, it follows that there is an effort-incentive compatible and Pareto improving deviation for each worker $\nu \in \mathcal{N}^*$, which implies that the equilibrium allocation is not constrained optimal.

To prove the claim, we first show that there exists a deviation $\hat{x}_s^g$ that satisfies $(A.23)$ and $(A.25)$. Recall that $\frac{\partial u(x_{s_1},e)/\partial x_{s_1}^g}{\partial u(x_{s_2},e)/\partial x_{s_2}^g} \neq 1$. Suppose, without loss of generality, that

$$\frac{\partial u\left(x_{s_1},e\right)}{\partial x_{s_1}^g} > \frac{\partial u\left(x_{s_2},e\right)}{\partial x_{s_2}^g}. \tag{A.26}$$

For any $\varepsilon > 0$, consider the deviation vector $v[\varepsilon] \in \mathbb{R}^{|S|}$ defined by $v_s[\varepsilon] = 0$ for any $s \notin \{s_1, s_2\}$ and

$$v_{s_1}[\varepsilon] = \frac{\varepsilon K}{q_{s_1}(e)} \text{ and } v_{s_2}[\varepsilon] = -\frac{\varepsilon K}{q_{s_2}(e)}, \tag{A.27}$$

where the constant $K = \frac{\min\left(q_{s_1}(e),q_{s_2}(e)\right)}{2} > 0$ ensures that $\|v[\varepsilon]\| \leq \varepsilon$ for any $\varepsilon > 0$. Note that, by construction, the vector $\hat{x}^g = x^g + v[\varepsilon]$ satisfies the resource constraints in $(A.23)$. Note also that

$$\begin{aligned} U\left(x^{G\setminus\{g\}}, x^g + v[\varepsilon], e\right) &= \sum_{s\in S} q_s(e)\, u\left(x_s^{G\setminus\{g\}}, x_s^g + v_s[\varepsilon], e\right) \\ &= \left( \begin{array}{c} \sum_{s\in S} q_s(e)\, u(x_s, e) + \\ q_{s_1}(e) \frac{\partial u(x_{s_1},e)}{\partial x_{s_1}^g} \frac{\varepsilon K}{q_{s_1}(e)} - q_{s_2}(e) \frac{\partial u(x_{s_2},e)}{\partial x_{s_2}^g} \frac{\varepsilon K}{q_{s_2}(e)} \\ +o(\varepsilon) \end{array} \right) \\ &= U(x,e) + \varepsilon K \left( \frac{\partial u(x_{s_1},e)}{\partial x_{s_1}^g} - \frac{\partial u(x_{s_2},e)}{\partial x_{s_2}^g} \right) + o(\varepsilon) \end{aligned} \tag{A.28}$$

where the second line considers a first order Taylor expansion for the functions $u(x_{s_1}, e)$ and $u(x_{s_2}, e)$, and the notation $o(\varepsilon)$ captures the residual which satisfies $\lim_{\varepsilon\to 0} \frac{o(\varepsilon)}{\varepsilon} = 0$. From Eqs. $(A.28)$ and the inequality in $(A.26)$, it follows that there exists $\bar{\varepsilon} > 0$ such that $U\left(x^{G\setminus\{g\}}, x^g + v[\varepsilon], e\right) > U(x,e)$ for each $\varepsilon \in (0, \bar{\varepsilon})$. This proves our claim that there exists a deviation $\hat{x}^g$ that satisfies $(A.23)$ and $(A.25)$.

We next consider a sequence of vectors $\{v[\varepsilon[n]]\}_{n=1}^\infty$, where $\{\varepsilon[n]\}_n$ is a sequence of scalars such that $\varepsilon[n] \in (0, \bar{\varepsilon})$ for each $n$ and $\varepsilon[n] \to 0$. We claim that there exists $n$ such that $\hat{x}^g = x^g + v[\varepsilon[n]]$ also satisfies the effort-incentive compatibility constraint in $(A.24)$. Suppose, to reach a contradiction, that there is no such $n$. Then, for each vector $v[\varepsilon[n]]$, there exists $e[n] \in E\setminus\{e\}$ such that

$$U\left(x^{G\setminus\{g\}}, x^g + v[\varepsilon[n]], e[n]\right) > U(x,e). \tag{A.29}$$

Since the space $E\setminus\{e\}$ is finite (and thus compact), the sequence $\{e[n]\}_{n=1}^\infty$ has a convergent subsequence. Let $\hat{e} \in E \setminus \{e\}$ be a limit point of this sequence. Since $\lim_{n\to\infty} v[\varepsilon[n]] = 0$, Eq.

$(A.29)$ implies

$$U(x,\hat{e}) \geq U(x,e). \tag{A.30}$$

Note that, since preferences are nonseparable at $(x,e)$, there exists $s,m$ and $g_1,g_2 \in G_m$ such that

$$\frac{\partial u(x_s,\hat{e})/\partial x_s^{g_1}}{\partial u(x_s,\hat{e})/\partial x_s^{g_2}} \neq \frac{\partial u(x_s,e)/\partial x_s^{g_1}}{\partial u(x_s,e)/\partial x_s^{g_2}} = \frac{p^{g_1}}{p^{g_2}},$$

where the last equality follows since $c \in C^I(p)$. The last equation further implies that $\mathbf{x}_s^{\{g_1,g_2\}}(w_s,p,\hat{e}) \neq x_s^{\{g_1,g_2\}}$. This further implies

$$U(\mathbf{x}(w,p,\hat{e}),\hat{e}) > U(x,\hat{e}) \geq U(x,e),$$

where the last inequality uses $(A.30)$. This yields a contradiction to of the fact that the contract $(w,x,e)$ is incentive compatible. It follows that there exists a vector $v[\varepsilon[n]]$ such that $\hat{x}^g = x^g + v[\varepsilon[n]]$ satisfies equations $(A.23)-(A.25)$, which completes the proof of the theorem. ■

### A.2.3 Proofs for Section 4

The proof of Theorem 3 requires a preliminary step. It is intuitively clear that, if preferences satisfy the separability condition (19), then the indifference curves between any two goods in the same monitoring partition $m$ should be independent of the effort choice $e$. The next lemma formalizes this observation.

**Lemma 4.** *Suppose the separability condition* (19) *in Definition* 4 *holds for the monitoring subset $G_m$. Consider a vector $(x_s,e) \in \mathbb{R}_+^{|G|} \times E$, and let $\hat{x}_s^{G_m}$ denote a reallocation of the goods in $G_m$ such that the level of the utility is kept constant, i.e., suppose:*

$$u\left(\hat{x}_s^{G_m}, x_s^{G \setminus G_m}, e\right) = u(x_s,e).$$

*Then, this reallocation keeps the level of utility constant also for any other effort level, i.e.:*

$$u\left(\hat{x}_s^{G_m}, x_s^{G \setminus G_m}, \tilde{e}\right) = u(x_s,\tilde{e}) \text{ for any } \tilde{e} \in E. \tag{A.31}$$

**Proof of Lemma 4.** We first claim that, under the separability condition (19), the partial derivative of the utility admits the following representation:

$$\frac{\partial u(\tilde{x}_s,\tilde{e})}{\partial \tilde{x}_s^{G_m}} = h(\tilde{x}_s,\tilde{e})\,F(\tilde{x}_s), \tag{A.32}$$

where $F: \mathbb{R}_+^{|G|} \to \mathbb{R}^{|G_m|}$ is a vector valued function and $h: \mathbb{R}^{|G|} \times E \to \mathbb{R}_{++}$ is a strictly positive scalar valued function. To prove this claim, fix some good $g \in G_m$ and define the scalar valued function

$$h(\tilde{x}_s, \tilde{e}) = \frac{\partial u(\tilde{x}_s, \tilde{e})}{\partial \tilde{x}_s^g},$$

which is strictly positive. Applying condition (19) for all pairs $(\tilde{g} \in G_m, g)$, we have that the ratio,

$$\frac{\partial u(\tilde{x}_s, \tilde{e}) / \partial \tilde{x}_s^{G_m}}{\partial u(\tilde{x}_s, \tilde{e}) / \partial \tilde{x}_s^g} = \frac{\partial u(\tilde{x}_s, \tilde{e}) / \partial \tilde{x}_s^{G_m}}{h(\tilde{x}_s, \tilde{e})},$$

is independent of $\tilde{e}$. Hence, it can be denoted by a vector valued function $F(\tilde{x}_s)$. This completes the proof of the claim in $(A.32)$.

Next, to prove the lemma, note that there exists a continuously differentiable function $\hat{\mathbf{x}}_s^{G_m} : [0,1] \to \mathbb{R}_+^{G_m}$ which satisfies $\hat{\mathbf{x}}_s^{G_m}(0) = x_s^{G_m}$, $\hat{\mathbf{x}}_s^{G_m}(1) = \hat{x}_s^{G_m}$, and

$$u\left(\hat{\mathbf{x}}_s^{G_m}(t), x_s^{G \backslash G_m}, e\right) = u(x_s, e) \text{ for each } t \in [0,1]. \tag{A.33}$$

Note that $\hat{\mathbf{x}}_s^{G_m}(t)$ represents a curve that lies inside the indifference surface (which is the higher dimensional analogue of an indifference curve). Totally differentiating Eq. $(A.33)$ with respect to $t$, we have

$$\frac{\partial u\left(\hat{\mathbf{x}}_s^{G_m}(t), x_s^{G \backslash G_m}, e\right)'}{\partial x_s^{G_m}} \frac{d\hat{\mathbf{x}}_s^{G_m}(t)}{dt} = 0 \text{ for each } t \in [0,1].$$

Plugging in the representation in $(A.32)$, the previous equality implies:

$$h\left(\hat{\mathbf{x}}_s^{G_m}(t), x_s^{G \backslash G_m}, e\right) F\left(\hat{\mathbf{x}}_s^{G_m}(t), x_s^{G \backslash G_m}\right)' \frac{d\hat{\mathbf{x}}_s^{G_m}(t)}{dt} = 0 \text{ for each } t \in [0,1]. \tag{A.34}$$

Since $h(\cdot)$ is a strictly positive scalar valued function, the previous equality implies that $F\left(\hat{\mathbf{x}}_s^{G_m}(t), x_s^{G \backslash G_m}\right)' \frac{d\hat{\mathbf{x}}_s^{G_m}(t)}{dt} = 0$. This further implies that

$$h\left(\hat{\mathbf{x}}_s^{G_m}(t), x_s^{G \backslash G_m}, \tilde{e}\right) F\left(\hat{\mathbf{x}}_s^{G_m}(t), x_s^{G \backslash G_m}\right)' \frac{d\hat{\mathbf{x}}_s^{G_m}(t)}{dt} = 0 \text{ for each } t \in [0,1],$$

which is the same as Eq. $(A.34)$, except for the fact that the effort level, $e$, is replaced by an arbitrary $\tilde{e} \in E$. Using the representation in $(A.32)$ one more time, the previous equality implies

$$\frac{\partial u\left(\hat{\mathbf{x}}_s^{G_m}(t), x_s^{G \backslash G_m}, \tilde{e}\right)'}{\partial x_s^{G_m}} \frac{d\hat{\mathbf{x}}_s^{G_m}(t)}{dt} = 0 \text{ for each } t \in [0,1].$$

Integrating this equation and using the fundamental theorem of calculus, we establish Eq. $(A.31)$, completing the proof of the lemma.

We next use this lemma to provide a proof of Theorem 3. ■

**Proof of Theorem 3.** By the equilibrium conditions, $[x(\nu), e(\nu)]_{\nu \in \mathcal{N}}$ is incentive feasible. Assume, to reach a contradiction, that $[x(\nu), e(\nu)]_{\nu \in \mathcal{N}}$ is constrained suboptimal. Then, there exists an incentive feasible allocation $[\overline{x}(\nu), \hat{e}(\nu)]_{\nu \in \mathcal{N}}$ such that

$$U(\overline{x}(\nu), \hat{e}(\nu)) \geq U(x(\nu), e(\nu)) \tag{A.35}$$

with strict inequality for a positive measure of $\nu \in \mathcal{N}$.

Consider $\nu \in \mathcal{N}$ such that the inequality $(A.35)$ holds strictly, and drop the $\nu$'s from the notation for convenience. We will show that the allocation $(\overline{x}, \hat{e})$ violates the profit constraint of the indirect problem (10), that is, we claim

$$\sum_{s \in S} q_s(\hat{e})(y_s - \overline{x}_s)p < 0. \tag{A.36}$$

And similarly, we claim that the same inequality holds weakly whenever the inequality in $(A.35)$ holds weakly. Once we establish the claim in $(A.36)$, the result follows from the standard proof of the first welfare theorem. In particular, integrating Eq. $(A.36)$ over all $\nu \in \mathcal{N}$ yields a contradiction to the fact that $[\overline{x}(\nu), \hat{e}(\nu)]_{\nu \in \mathcal{N}}$ satisfies the resource constraints, which proves that the equilibrium is constrained optimal.

To prove the claim in $(A.36)$, we will construct a contract $\hat{c} = (\hat{w}, \hat{x}, \hat{e})$ which satisfies the following three properties:

(P1) $\hat{c}$ is incentive compatible.

(P2) $\hat{c}$ yields the worker the same utility than the allocation $(\overline{x}, \hat{e})$.

(P3) $\hat{c}$ costs the firm less than the allocation $(\overline{x}, \hat{e})$, i.e.,

$$\sum_{s \in S} q_s(\hat{e}) \hat{x}_s p \leq \sum_{s \in S} q_s(\hat{e}) \overline{x}_s p. \tag{A.37}$$

Once we construct a contract $\hat{c}$ with these properties, we have the following implications. By (P1), the contract $\hat{c}$ satisfies all the constraints of problem (10) except for the profit constraint (11). By (P2), the contract $\hat{c}$ yields the worker greater utility than the equilibrium contract. Since the equilibrium contract solves problem (10), the contract $\hat{c}$ must violate the remaining constraint of problem (10), that is, it violates the profit constraint. By (P3), the contract $\hat{c}$ costs the firm less than the allocation $(\overline{x}, \hat{e})$, which in turn implies that $(\overline{x}, \hat{e})$ also violates the profit constraint. This shows the claim in Eq. $(A.36)$, completing the proof of the theorem.

Hence, we are left with constructing a contract $\hat{c} = (\hat{w}, \hat{x}, \hat{e})$ that satisfies the properties (P1)-(P3). We will construct the wage and allocation pair, $(\hat{w}, \hat{x})$ as the limit of a sequence $\{\hat{w}[n], \hat{x}[n]\}_{n=0}^{\infty}$, where the sequence will be constructed recursively. In particular, let $(\hat{w}[0], \hat{x}[0]) \equiv (\hat{w} \equiv \overline{x}p, \overline{x})$ denote the initial vector. Suppose

$\{(\hat{w}[0], \hat{x}[0]), .., (\hat{w}[n-1], \hat{x}[n-1])\}$ is constructed for $n \geq 1$, and consider the construction of $(\hat{w}[n], \hat{x}[n])$. Let $m \in \{1, .., |M|\}$ denote the modulo $|M|$ value of $n$. For each $s$, define $\hat{x}[n]_s = \left(\hat{\mathbf{x}}_s^{G_m}, \hat{x}[n-1]_s^{G \backslash G_m}\right)$ and $\hat{w}[n]_s = \left(\hat{\mathbf{w}}_s^m, \hat{w}[n-1]_s^{M \backslash \{m\}}\right)$, where $\hat{\mathbf{w}}_s^m$ is the minimum value and $\hat{\mathbf{x}}_s^{G_m}$ is the unique solution to the following strictly convex optimization problem:

$$\min_{\tilde{x}_s^{G_m} \geq 0} \quad \tilde{x}_s^{G_m} p^{G_m} \tag{A.38}$$

$$\text{subject to} \qquad u\left(\tilde{x}_s^{G_m}, \hat{x}[n-1]_s^{G \backslash G_m}, \hat{e}\right) = u\left(\hat{x}[n-1]_s, \hat{e}\right). \tag{A.39}$$

That is, at each step, the vector, $(\hat{w}[n]_s, \hat{x}[n]_s)$, is constructed by reallocating the goods within one partition, $G_m$, in a way to minimize the costs while providing the worker with the same utility as before. The partitions are subject to this operation one at a time and in an order. Once we operate over all partitions, we start the process over (which is formally captured above by taking the modulo $|M|$ value of $n$). We claim that the sequence $\{\hat{w}[n], \hat{x}[n]\}$ converges to a vector $(\hat{w}, \hat{x})$, and that the contract $\hat{c} = (\hat{w}, \hat{x}, \hat{e})$ satisfies (P1)-(P3).

We first show that the sequence $\{\hat{w}[n], \hat{x}[n]\}$ converges. Note that the allocations and the wage for each partition, $\left(\hat{w}[n]_s^m, \hat{x}[n]_s^{G_m}\right)$, is updated once every $|M|$ turns. Moreover, problem $(A.38)$ shows that the wage corresponding to each partition, $\hat{w}[n]_s^m$, weakly decreases each time an updating occurs (and it is constant if an updating does not occur). In particular, $\{\hat{w}[n]_s^m\}_{n=0}^{\infty}$ is a decreasing sequence. Moreover, it is bounded below by 0. This means that, for each $m$ and $s$, $\hat{w}[n]_s^m$ has a unique limit point which we denote by $\hat{w}_s^m$. This further implies that $\hat{w}[n] \to \hat{w}$. Next, note that, the solution $\hat{x}[n]_s^{G_m}$ to problem $(A.38)$ satisfies the first order conditions:

$$p^g \quad \geq \quad \lambda[n]_s^m \frac{\partial u(\hat{x}[n]_s, e_0)}{\partial \hat{x}_s^g} \text{ for each } s, m, \text{ and } g \in G_m, \tag{A.40}$$
$$\text{with equality if } \hat{x}_s^{G_m} > 0,$$

where $\lambda[n]_s^m$ is a strictly positive Lagrange multiplier for each $s$ and $m$. Hence, given the wages $\hat{w}[n]$, the solutions $\hat{x}[n]$ are uniquely characterized by the conditions in $(A.40)$ along with the equations

$$\hat{w}[n]_s^m = \hat{x}[n]_s^{G_m} p^{G_m} \text{ for each } s \text{ and } m, \tag{A.41}$$

which hold since the minimum of the problem $(A.38)$ is attained. Since the equations in $(A.40) - (A.41)$ are continuous in $\hat{w}[n]$, and since $\hat{w}[n] \to \hat{w}$, the solutions $\hat{x}[n]$ also converge to a vector $\hat{x}$. The limiting vector, $\hat{x}$, is the solution to Eqs. $(A.40) - (A.41)$ corresponding to the limiting wages $\hat{w}$. This establishes that the sequence we have constructed converges to a vector, $(\hat{w}, \hat{x})$.

We next show that the contract $\hat{c} = (\hat{w}, \hat{x}, \hat{e})$ satisfies (P1), that is, $\hat{c}$ is incentive compatible. We break this argument into two steps. First, we claim that the allocation $(\hat{x}, \hat{e})$ is effort-incentive compatible, that is,

$$\sum_{s \in S} q_s(\hat{e})\, u(\hat{x}_s, \hat{e}) \geq \sum_{s \in S} q_s(e)\, u(\hat{x}_s, \tilde{e}) \text{ for each } \tilde{e} \in E. \tag{A.42}$$

We will then establish that the contract $\hat{c}$ satisfies the stronger incentive compatibility condition (4). The proof of the claim in $(A.42)$ relies on Lemma 4. In particular, note that Eq. $(A.39)$ implies that the utility is preserved at each step of the above construction, that is:

$$u\left(\hat{x}[n]_s, \hat{e}\right) = u\left(\hat{x}[n-1]_s, \hat{e}\right) \text{ for each } s.$$

Moreover, note that, at each step, the above operation reallocates the goods within exactly one monitoring subset $G_m$. That is, the allocations $\hat{x}[n]_s$ and $\hat{x}[n-1]_s$ are the same except (potentially) for the allocations $\hat{x}[n]_s^{G_m}$. Hence, Lemma 4 applies and shows that

$$u\left(\hat{x}[n]_s, \tilde{e}\right) = u\left(\hat{x}[n-1]_s, \tilde{e}\right) \text{ for each } s \text{ and } \tilde{e} \in E.$$

This further implies that the limiting utility is equal to the initial utility for any effort level $\tilde{e}$, that is:

$$u(\hat{x}_s, \tilde{e}) = u(\overline{x}_s, \tilde{e}) \text{ for each } s \text{ and } \tilde{e} \in E. \tag{A.43}$$

Since the allocation $(\overline{x}, \hat{e})$ is effort-incentive compatible, Eq. $(A.43)$ implies that the allocation $(\hat{x}, \hat{e})$ is also effort-incentive compatible, proving the claim in $(A.42)$.

We next prove that the contract $\hat{c}$ is incentive compatible. Given wages $\hat{w}$ and some effort choice $\tilde{e} \in E$, the worker's consumption choice satisfies the following first order conditions:

$$\frac{\partial u(\tilde{x}_s, \tilde{e})}{\partial \tilde{x}_s^g} \leq \gamma_s^m p^g \text{ for each } s, m,\ g \in G^m, \tag{A.44}$$
$$\text{with equality if } \tilde{x}_s^{G_m} > 0,$$

where $\gamma_s^m$ is a positive Lagrange multiplier for each $s$ and $m$. In particular, the workers' consumption choice is uniquely determined by the conditions in $(A.44)$ along with the budget constraints:

$$\hat{w}_s^m = \tilde{x}_s^{G_m} p^{G_m} \text{ for each } s \text{ and } m. \tag{A.45}$$

Note that the first order conditions in $(A.44)$ are identical to the first order conditions in $(A.40)$, and Eq. $(A.41)$ is identical to Eq. $(A.45)$. Recall also that $\hat{x}$ is the unique solution to Eqs. $(A.40) - (A.41)$. Hence, the worker's consumption choice is independent of her effort choice $\tilde{e}$, and is equal to $\hat{x}$. This implies that the effort-incentive compatibility condition $(A.42)$ implies the incentive compatibility of the contract $\hat{c}$, establishing (P1).

We next show that $\hat{c}$ satisfies (P2) and (P3). Note that, by construction, $\hat{c}$ yields the same utility to the worker as the allocation $(\overline{x}, \hat{e})$, establishing (P2). Recall also that $\{\hat{w}[n]_s^m\}_{n=0}^{\infty}$ is a decreasing sequence, which implies that $\hat{w} = \hat{x}p \leq \hat{w}[0] = \overline{x}p$. Hence, contract $\hat{c}$ costs the firm less than the allocation $(\overline{x}, \hat{e})$ and Eq. $(A.37)$ holds, establishing (P3). This completes the proof of Theorem 3. ■

### A.2.4 Proofs for Section 5

**Proof of Theorem 4.** We first claim that the pair, $\left(p, [\mu_\nu]_{\nu\in\mathcal{N}}\right)$, is part of an equilibrium if and only if conditions 1-2 of the theorem are satisfied. Since the space, $C^I(p)$, satisfies the transferability condition in 2, the space, $\mathcal{P}\left(C^I(p)\right)$, also satisfies the analogous transferability condition. Given this result, the argument in the proof of Proposition 10 applies unchanged with stochastic contracts, proving the claim.

We next show that an equilibrium exists. First, it can be seen that the constraint set of problem (25) is compact. Since the objective function is linear (and thus continuous) in $\tilde{\mu}$, a solution to problem (25) always exists. Next, we claim that the solution set to problem (25), denoted by $S^R(p) \subset \mathcal{P}\left(C^I(p)\right)$, is upper hemicontinuous in $p$ (recall that $\mathcal{P}(C)$ is endowed with weak* topology). By Lemma 3, $C^I(p)$ is a continuous correspondence in $p$. It follows that $\mathcal{P}\left(C^I(p)\right) : \mathbb{R}^{|S|} \rightrightarrows \mathcal{P}(C)$ is also a continuous correspondence in $p$. Then, a similar argument to Theorem 1 establishes that $S^R(p)$ is upper hemicontinuous. Recall also that $S^R(p)$ is convex valued since problem (10) is linear.

The rest of the proof follows standard arguments. For a given price $p$, define the excess demand correspondence $D : \mathbb{R}_+^{|S|} \rightrightarrows \mathbb{R}^{|S|}$ by

$$D(p) = \left\{ \begin{array}{c} \int_{\mathcal{N}} \int_{(w(\nu),x(\nu),e(\nu))} \sum_{s\in S} (x_s(\nu) - y_s) q_s(e(\nu)) d\tilde{\mu}_\nu d\nu \\ \mid \tilde{\mu}_\nu \in S^R(p) \text{ for each } \nu \in \mathcal{N} \end{array} . \right\} \tag{A.46}$$

Note that, for any collection of allocations, $\left[\tilde{\mu}_\nu \in S^R(p)\right]_\nu$, the excess demand in $(A.47)$ is equivalent to

$$\int_{\mathcal{N}} \int_{(w,x,e)} \sum_{s\in S} (x_s - y_s) q_s(e(\nu)) d\tilde{\mu},$$

where $\tilde{\mu} \in \mathcal{P}(C)$ is the average measure defined by

$$\tilde{\mu}\left(\tilde{C}\right) = \int \tilde{\mu}_\nu \left(\tilde{C}\right) d\nu \text{ for each } \tilde{C} \in \mathcal{B}(C).$$

Note that $\tilde{\mu} \in S^R(p)$, since $\tilde{\mu}_\nu \in S^R(p)$ for each $\nu$ and $S^R(p)$ is convex valued. Hence, problem $(A.46)$ can equivalently be written as:

$$D(p) = \left\{ \int_{(w,x,e)} \sum_{s\in S} (x_s - y_s) q_s(e) d\tilde{\mu} \mid \tilde{\mu} \in S^R(p) \right\}. \tag{A.47}$$

Since $S^R(p)$ is upper hemicontinuous in $p$, $D(p)$ is also upper hemicontinuous in $p$. Then, the same arguments in the proof of Theorem 1 show that an equilibrium exists, completing the proof of Theorem 4. ■

**Proof of Theorem 5.** As in the proof of Theorem 2, we will show that there is an incentive compatible allocation, $\hat{\eta}$, that satisfies the resource constraints and improves the utility of the worker over the equilibrium allocation, $\eta \equiv \mu_\nu|_{(x,e)}$.

Let $A^{**} = A^* \cap supp(\eta)$, and note that $A^{**}$ is compact (since $A^*$ is compact and $supp(\eta)$ is closed) and that $\eta(A^{**}) = \eta(A^*) > 0$. Let $(x,e) \in A^{**}$ and note that Definition 7 implies that preferences are nonseparable at the deterministic allocation $(x,e)$. Then, consider the deviation constructed in the proof of Theorem 2. In particular, for each $\varepsilon > 0$, let $v[\varepsilon \mid (x,e)] \in \mathbb{R}^{|S|}$ denote the vector constructed in $(A.27)$. Note that $\|v[\varepsilon \mid (x,e)]\| \leq \varepsilon$, and that $v[\varepsilon \mid (x,e)]$ is continuous in $(x,e)$. Recall also that, by the proof of Theorem 2, there exists $\overline{\varepsilon}$ such that

$$U\left(x^{G\setminus\{g\}}, x^g + v[\varepsilon \mid (x,e)], e\right) > U(x,e) \tag{A.48}$$

for each $\varepsilon < \overline{\varepsilon}$ and each $(x,e) \in A^{**}$. For each $\varepsilon \in (0,\overline{\varepsilon})$, define the function $\zeta(\cdot,\varepsilon) : A \to A$ with:

$$\zeta((x,e),\varepsilon) = \begin{cases} (x,e) \text{ if } (x,e) \in A \setminus A^{**}, \\ \left(x^{G\setminus\{g\}}, x^g + v[\varepsilon \mid (x,e)], e\right) \text{ if } (x,e) \in A^{**}. \end{cases}$$

Note that $\zeta(\cdot,\varepsilon)$ is a measurable function from $(A,\mathcal{B}(A))$ to $(A,\mathcal{B}(A))$, because the perturbation function $v[\varepsilon \mid (x,e)]$ is continuous in $(x,e)$. Hence, given the probability measure $\eta$, the function $\zeta(\cdot,\varepsilon)$ induces another probability measure over $(A,\mathcal{B}(A))$ (the push-forward measure), defined by:

$$\eta[\varepsilon]\left(\tilde{A}\right) = \eta\left(\zeta^{-1}\left(\tilde{A},\varepsilon\right)\right) \text{ for each } \tilde{A} \in \mathcal{B}(A).$$

Note that $\eta[\varepsilon]$ is a stochastic allocation. Moreover, by equation $(A.48)$ and by the definition of $\eta[\varepsilon]$, we have that $U^R(\eta[\varepsilon]) > U^R(\eta)$ and that $\eta[\varepsilon]$ satisfies the resource constraints (27).

We next consider the sequence of stochastic allocations $\{\eta[\varepsilon[n]]\}_{n=1}^\infty$, where $\{\varepsilon[n]\}_{n=1}^\infty$ is a sequence of scalars such that $\varepsilon[n] \in (0,\overline{\varepsilon})$ for each $n$ and $\varepsilon[n] \to 0$. We claim that there exists $n$ such that $\eta[\varepsilon[n]]$ is also incentive compatible. Suppose, to reach a contradiction, that for each $n$, $supp(\eta[\varepsilon[n]])$ is not a subset of the set of deterministic incentive compatible allocations, $A^I$. By the construction of $\eta[\varepsilon[n]]$, this implies that there exists a vector $(x[n], e[n]) \in A^{**}$ such that the perturbed vector $\left(x[n]^{G\setminus\{g\}}, x[n]^g + v[\varepsilon[n] \mid (x[n], e[n])], e[n]\right)$ is not an element of $A^I$. That is, there exists $\hat{e}[n] \in E \setminus \{e[n]\}$ such that

$$U\left(x[n]^{G\setminus\{g\}}, x[n]^g + v[\varepsilon[n] \mid (x[n], e[n])], \hat{e}[n]\right) > U(x[n], e[n]). \tag{A.49}$$

Since the space $A^{**} \times E$ is compact, the sequence of vectors, $\{(x[n], e[n]), \hat{e}[n]\}_{n=1}^{\infty}$, has a convergent subsequence. Let $(\overline{x}, \overline{e}, \hat{e}) \in A^{**} \times E$ be a limit point of this sequence and note that $\hat{e} \neq e$ (since $\hat{e}[n] \neq e[n]$ for each $n$). Note also that Eq. $(A.49)$ implies

$$U(x, \hat{e}) \geq U(x, e).$$

Since preferences are nonseparable at the deterministic allocation $(x, e) \in A^{**}$, by the proof of Theorem 2, we have

$$U(\mathbf{x}(w \equiv xp, p, \hat{e}), \hat{e}) > U(x, \hat{e}) \geq U(x, e).$$

This yields a contradiction to the fact that the contract, $c = (w \equiv xp, x, e)$ is incentive compatible (the contract $c$ is incentive compatible since the allocation, $(x, e) \in A^{**}$, is in the support of the equilibrium stochastic allocation, $\eta = \mu_{\nu}|_{(x,e)}$). This completes the proof of Theorem 5.

**Proof of Theorem 6.** Suppose to obtain a contradiction that $(\eta_{\nu})|_{\nu \in \mathcal{N}}$ is not constrained optimal. Then, there exists an effort-incentive compatible and feasible stochastic allocation $(\overline{\eta}_{\nu})|_{\nu \in \mathcal{N}}$ such that

$$U^R(\overline{\eta}_{\nu}) \geq U^R(\eta_{\nu}) \tag{A.50}$$

with strict inequality for a positive measure of $\nu \in \mathcal{N}$.

Consider an allocation $\overline{\eta}_{\nu} \in \mathcal{N}$ for which the inequality in $(A.50)$ is satisfied strictly, and drop the $\nu$'s from the notation for convenience. We will show that the allocation $\overline{\eta}$ violates the profit constraint of the indirect problem (25), that is, we claim

$$\int_{(w,x,e) \in \mathcal{C}} \left( \sum_{s \in S} q_s(e)(y_s - x_s)p \right) d\overline{\eta} < 0. \tag{A.51}$$

And similarly, we claim that the same inequality holds weakly whenever the inequality in $(A.51)$ holds weakly. Once we establish the claim in $(A.51)$, the result follows from the standard proof of the first welfare theorem. In particular, integrating Eq. $(A.51)$ over all $\nu \in \mathcal{N}$ yields a contradiction to the fact that $(\overline{\eta}_{\nu})_{\nu \in \mathcal{N}}$ satisfies the resource constraint (27), which proves that the equilibrium is constrained optimal.

Consider $(\overline{x}, \hat{e}) \in supp(\overline{\eta})$ and note that $(\overline{x}, \hat{e}) \subset A^I$ since $\overline{\eta}$ is incentive compatible. By the proof of Theorem 3, there exists a contract $\hat{c} = (\hat{w}, \hat{x}, \hat{e})$ that satisfies properties (P1)-(P3), where recall that (P1) $\iff$ $\hat{c} \in C^I(p)$, (P2) $\iff$ $U(\hat{x}, \hat{e}) = U(\overline{x}, \hat{e})$, and

$$\text{(P3)} \iff \sum_{s \in S} q_s(\hat{e}) \hat{x}_s p \leq \sum_{s \in S} q_s(\hat{e}) \overline{x}_s p. \tag{A.52}$$

From the construction in Theorem 3, it can also be seen that the contract $\hat{c}[\overline{x}, \hat{e}] \equiv (\hat{w}[\overline{x}, \hat{e}], \hat{x}[\overline{x}, \hat{e}], \hat{e})$ is a continuous function of $(\overline{x}, \hat{e})$, for each $(\overline{x}, \hat{e}) \in supp(\overline{\eta})$. Extend $\hat{c}[\cdot]$ to a measurable function over all of $A$ by defining $\hat{c}[\tilde{x},\tilde{e}] = (\tilde{w} \equiv \tilde{x}p, \tilde{x}, \tilde{e})$ for each $(\tilde{x},\tilde{e}) \notin supp(\overline{\eta})$. Note that $\hat{c}[\cdot]$ is a measurable mapping from $(A, \mathcal{B}(A))$ to $(C, \mathcal{B}(C))$. Hence, given the probability measure $\overline{\eta}$ over $(A, \mathcal{B}(A))$, the mapping $\hat{c}[\cdot]$ induces a probability measure over $(C, \mathcal{B}(C))$ (push-forward measure), defined by:

$$\hat{\mu}\left(\tilde{C}\right) = \overline{\eta}\left(\hat{c}^{-1}\left(\tilde{C}\right)\right) \text{ for each } \tilde{C} \in \mathcal{B}\left(\tilde{C}\right).$$

Note that the support of $\hat{\mu}$ is in $C^I(p)$, since property (P1) implies that $\hat{c}[\overline{x},\hat{e}] \in C^I(p)$ for each $(\overline{x},\hat{e}) \in supp(\hat{\eta})$. Hence, the stochastic contract $\hat{\mu}$ is incentive compatible. Moreover, property (P2) and the fact that $(A.50)$ is satisfied strictly implies that $\hat{\mu}$ yields the worker strictly greater utility than the equilibrium allocation $\mu$. Since the equilibrium allocation $\mu$ solves problem $(A.50)$, it follows that $\hat{\mu}$ violates the remaining constraint of $(A.50)$, that is:

$$\int_{(x,e)} \left(\sum_{s\in S} q_s(e)(y_s - x_s)p\right) d\hat{\mu}|_{(x,e)} < 0.$$

By property (P3) and the definition of $\hat{\mu}$, we also have that the stochastic allocation $\hat{\mu}|_{(x,e)}$ costs the firm more than the allocation $\overline{\eta}$. This implies the inequality $(A.51)$, completing the proof of Theorem 6. ■

### A.2.5 Proofs for Section 6

**Proof of Lemma 1.** We first claim that $U^R_{planner}(u) \geq U^R_{eq}(p,u)$ holds for all $u \in \mathcal{U}$ and equilibrium price vector $p \in P(u)$. Note that the solution to problem (28) is always weakly greater than the solution to problem (25) (because the equilibrium allocation $\left[\mu_\nu|_{(x,e)}\right]_{\nu\in\mathcal{N}}$ is always in the constraint set of problem (28)). Since problems (28) and (29) are equivalent, it follows that $U^R_{planner}(u) \geq U^R_{eq}(p,u)$, proving the claim.

We next prove the lemma. First consider the only if part, that is, suppose the equilibrium allocation, $\left[\mu_{\nu|(x,e)}\right]_\nu$, is $\varepsilon$-constrained optimal. Suppose, to reach a contradiction, that $U^R_{planner}(u) \notin \left[U^R_{eq}(p,u), U^R_{eq}(p,u)+\varepsilon\right]$. Since $U^R_{planner}(u) \geq U^R_{eq}(p,u)$, we have $U^R_{planner}(u) > U^R_{eq}(p,u)+\varepsilon$. Let $\hat{\eta}$ denote the solution to problem (29) and consider the allocation $[\hat{\eta}_\nu]_{\nu\in\mathcal{N}}$, defined by $\hat{\eta}_\nu = \hat{\eta}$ for each $\nu$. The allocation $[\hat{\eta}_\nu]_{\nu\in\mathcal{N}}$ is effort-incentive compatible and feasible, and it improves every worker's utility by more than $\varepsilon$. This contradicts the fact that the equilibrium allocation $\left[\mu_\nu|_{(x,e)}\right]_{\nu\in\mathcal{N}}$ is constrained optimal, proving the only if part of the lemma.

Next consider the if part, that is, suppose $U^R_{planner}(u) \in \left[U^R_{eq}(p,u), U^R_{eq}(p,u)+\varepsilon\right]$. Suppose, to reach a contradiction, that the equilibrium is not constrained optimal. Then, there exists an allocation, $[\hat{\eta}_\nu]_{\nu\in\mathcal{N}}$, which is effort-incentive compatible and feasible, and which improves the utility of all workers by $\varepsilon$ (and strictly so for a positive measure workers). Consider the average allocation $\hat{\eta}$ defined in Eq. (30), which satisfies the resource constraints, and which is incentive compatible since $\mathcal{P}\left(A^I\right)$ is a convex set. Hence, $\hat{\eta}$ is in the constraint set of problem (29). Moreover, since $[\hat{\eta}_\nu]_\nu$ is a Pareto improvement, the average allocation $\hat{\eta}$ improves the utility of all workers by strictly more than $\varepsilon$, that is, $U^R(\hat{\eta}) > U^R_{eq}(p,u) + \varepsilon$. It follows that $U^R_{planner}(u) > U^R_{eq}(p,u) + \varepsilon$, which yields a contradiction, completing the proof of the lemma. ■

**Proof of Theorem 7.** We first claim that $U^R_{planner}(u)$ is continuous in $u \in \mathcal{U}$ (where recall $\mathcal{U}$ is a metric space with the sup norm). To show this, first consider the set of deterministic incentive compatible allocations, $A^I(u)$:

$$A^I(u) = \left\{(x,e) \in A \mid \sum_s q_s(e)\, u(x,e) \geq \sum_s q_s(\tilde{e})\, u(x,\tilde{e}) \text{ for each } \tilde{e} \in E\right\}.$$

Note that $A^I(u)$ is an upper hemicontinuous correspondence of $u$. Next, since each $u \in \mathcal{U}$ satisfies Assumption A3 for the same function $u^{G_1}(\cdot)$, an argument similar to the proof of Lemma 3 shows that $A^I(u)$ is a lower hemicontinuous correspondence of $u$. It follows that $A^I(u)$ is a continuous correspondence of $u$. This further implies that $\mathcal{P}\left(A^I(u)\right)$ is also a continuous correspondence of $u$ (when viewed as a correspondence from $\mathcal{U}$ to $\mathcal{P}(A)$). Then, an argument similar to the proof of Proposition 1 shows that the solution to problem (29) is upper hemicontinuous, and the optimal value is continuous (i.e., a version of the Maximum Theorem applies to problem (29)). This shows that $U^R_{planner}(u)$ is continuous in $u$.

Consider next the correspondence $\overline{U}^R_{eq} : \mathcal{U} \rightrightarrows \mathbb{R}$ defined by

$$\overline{U}^R_{eq}(u) = \left\{U^R_{eq}(p,u) \mid p \in P(u)\right\}, \tag{A.53}$$

where recall that $P(u)$ is the equilibrium price correspondence defined in (32). We claim that the correspondence $\overline{U}^R_{eq}(u)$ is upper hemicontinuous in $u$. To see this, let $S^R(p,u) \subset \mathcal{P}\left(A^I\right)$ denote the solution to problem (25), and recall that $U^R_{eq}(p,u)$ denotes the optimal value of the same problem. A similar argument to the previous paragraph establishes that $U^R_{eq}(p,u)$ is a continuous function and $S(p,u)$ is a continuous correspondence of $(p,u)$. By the same argument as in the proof of Theorem 4, the excess demand correspondence is given by

$$D(p,u) = \left\{\int_{(w,x,e)} \sum_{s \in S} (x_s - y_s) q_s(e) d\tilde{\mu} \mid \tilde{\mu} \in S^R(p,u)\right\}.$$

Since $S^R(p,u)$ is upper hemicontinuous, $D(p,u)$ is also upper hemicontinuous in $(p,u)$. Moreover, note that the equilibrium price correspondence satisfies:

$$P(u) = \left\{p \in \mathbb{R}^{|G|}_+ \mid 0 \in D(p,u)\right\}.$$

Since $D(p,u)$ is upper hemicontinuous in $(p,u)$, it has a closed graph, which implies that $P(u)$ is an upper hemicontinuous correspondence of $u$. Since $P(u)$ is upper hemicontinuous and $U_{eq}^{R}(p,u)$ is continuous, Eq. $(A.53)$ implies that $\overline{U}_{eq}^{R}(u)$ is upper hemicontinuous, proving the claim.

We have thus established that $U_{planner}^{R}(u)$ is a continuous function and $\overline{U}_{eq}^{R}(u)$ is a continuous correspondence of $u \in \mathcal{U}$. Next consider the values of these expressions for $u = \overline{u}$. Recall that any equilibrium of the economy $\mathcal{E}(\overline{u})$ is constrained optimal by Theorem 6. Then, by Lemma 1, it follows that $\overline{U}_{eq}^{R}(\overline{u})$ is a singleton and it is equal to $U_{planner}^{R}(\overline{u})$, that is,

$$\overline{U}_{eq}^{R}(\overline{u}) = \left\{U_{planner}^{R}(\overline{u})\right\}. \tag{A.54}$$

Fix some $\varepsilon > 0$. From the continuity of $U_{planner}^{R}(\cdot)$, there exists $\delta_{planner} > 0$ such that, if $||u - \overline{u}|| < \delta_{planner}$, then

$$\left|U_{planner}^{R}(u) - U_{planner}^{R}(\overline{u})\right| \leq \varepsilon/2. \tag{A.55}$$

Similarly, from the upper hemicontinuity of $\overline{U}_{eq}^{R}(\cdot)$ and Eq. $(A.54)$, there exists $\delta_{eq}$ such that if $||u - \overline{u}|| < \delta_{planner}$, then

$$\left|\tilde{U}_{eq} - U_{planner}^{R}(\overline{u})\right| \leq \varepsilon/2 \text{ for any } \tilde{U}_{eq} \in \overline{U}_{eq}^{R}(u). \tag{A.56}$$

Let $\delta = \min(\delta_{planner}, \delta_{eq})$ and note that Eqs. $(A.55)$ and $(A.56)$ imply $\left|U_{planner}^{R}(u) - \tilde{U}_{eq}\right| \leq \varepsilon$ for any $u \in B(\overline{u}, \delta)$ and any $\tilde{U}_{eq} \in \overline{U}_{eq}^{R}(u)$. By Lemma 1, it follows that any equilibrium of any economy $\mathcal{E}^{R}(u)$, with $u \in B(\overline{u}, \delta)$, is $\varepsilon$-constrained optimal. This completes the proof of Theorem 7. ■

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