Predatory Trading and Credit Freeze

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Abstract

This paper studies how predatory trading affects the ability of banks and large trading institutions to raise capital in times of temporary financial distress in an environment in which traders are asymmetrically informed about each others' balance sheets. Predatory trading is a strategy in which a trader can profit by trading against another trader’s position, driving an otherwise solvent but distressed trader into insolvency. The predator, however, must be sufficiently informed of the distressed trader’s balance sheet in order to exploit this position. I find that when a distressed trader is more informed than other traders about his own balances, searching for extra capital from lenders can become a signal of financial need, thereby opening the door for predatory trading and possible insolvency. Thus, a trader who would otherwise seek to recapitalize is reluctant to search for extra capital in the presence of potential predators. Predatory trading may therefore make it exceedingly difficult for banks and financial institutions to raise credit in times of temporary financial distress.

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1 Introduction

The inter-bank lending market and the discount window of the Fed are two facilities which allow banks to borrow short-term in order to meet temporary liquidity needs. However, these opportunities are not always availed by traders. Financial institutions often appear reluctant to borrow, even at times when liquidity is most needed. In this paper I study how strategic interactions among banks may deter financial institutions from raising money in times of temporary financial distress.

Financial markets are often modeled as interactions between small traders in perfectly competitive markets taking prices as given. However, in reality these markets are not devoid of large players with market impact. For this reason, a recent literature has begun to emphasize strategic behavior among large financial institutions. “Predatory trading” is one such strategic interaction. Brunnermeier and Pedersen (2005) define predatory trading as “trading that induces and/or exploits the need of other investors to reduce their positions.” That is, predatory trading is a strategy in which a trader can profit by trading against another trader’s position, driving an otherwise solvent but distressed trader into insolvency. The forced liquidation of the distressed trader leads to price swings from which the predator can then profit. Brunnermeier and Pedersen (2005) provide a framework to study this type of interaction, and show how predatory trading can in fact induce a distressed trader’s need to liquidate.

In this paper I explore how predatory trading may affect the incentives of banks to seek loans in times of financial distress. In general, a distressed bank or trader may wish to raise money in order to temporarily bridge financial short-falls. However, in an environment in which banks have private information about their own finances, searching for extra capital from outside lenders may become a signal of financial weakness. Traders can then exploit this information, and predatorily trade against funds that they infer to be sufficiently weak. Therefore, the mere act of searching for loans may expose a distressed firm to predatory trading and possible insolvency.

The key assumption behind this result is that there exists asymmetric information among traders—that is, ex ante, traders have private information about their own balance sheet that is not available to other traders. Within an asymmetric information environment, actions undertaken by banks to relieve financial distress may convey information about its underlying financial state. Hence, in deciding whether to search for a loan, a distressed bank faces a trade-off between the financial cushion provided by a loan and the information this act reveals. In equilibrium, I find that some distressed funds who would otherwise seek to recapitalize may be reluctant to search for extra capital in the presence of potential predators. Predatory trading may therefore deter banks and financial institutions from raising funds in times when they need it the most.

Anecdotal Evidence. The findings of this paper support certain anecdotal evidence about strategic trading and the reluctance of financial institutions to find loans in times of distress. One of the most often-cited examples of predatory trading is the alleged front-running against the infamous hedge fund Long-Term Capital Management (LTCM) in the fall of 1998. After realizing losses in a number of markets, it is reported that LTCM began searching for capital from
a number of Wall Street banks, most notably Goldman Sachs & Co. LTCM alleges that with
this information Goldman then traded heavily against LTCM’s positions in credit-default swaps,
front-running LTCM’s eventual unwinding. *Business Week* writes,\(^1\)

..if lenders know that a hedge fund needs to sell something quickly, they will sell the same asset—driving the price down even faster. Goldman Sachs & co. and other counterparties to LTCM did exactly that in 1998. Goldman admits it was a seller but says it acted honorably and had no confidential information.

Similarly, in *When Genius Failed: The rise and fall of Long-Term Capital Management*, Lowenstein writes,\(^2\)

As it scavenged for capital, Long-Term had been forced to reveal bits and pieces and even the general outline of its portfolio... Meriwether bitterly complained to the Fed’s Peter Fisher that Goldman, among others, was “front-running”, meaning trading against it on the basis of inside knowledge. Goldman, indeed, was an extremely active trader in mid-September, and rumors that Goldman was selling Long-Term’s positions in swaps and junk bonds were all over Wall Street.

Furthermore, an interesting study by Cai (2007), uses a unique data set of audit transactions to examine the trading behavior of market makers in the Treasury bond futures market during LTCM’s collapse. Cai finds strong evidence of predatory front running behavior by market makers, based on their informational advantages.\(^3\)

The findings of this paper also provide a possible explanation as to why financial firms may not obtain loans in times of financial shortfall. During the 2008-2009 financial crisis, sources report that Lehman Brothers was reluctant to publicly raise liquidity, a month or so before its collapse. *The Wall Street Journal* writes,\(^4\)

As the credit crunch deepened, the Fed had set up a new lending facility for investment banks. Although the central bank doesn’t reveal who borrows from it, the market generally figures it out, and there’s a stigma associated with it. Lehman didn’t do so over the summer, because it didn’t want to be seen as needing Fed money, says one person familiar with the matter.

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2 According to the author, *When Genius Failed: The Rise and Fall of Long-Term Capital Management* was based on interviews with former employees and partners of the firm, as well as interviews conducted at the major Wall street investment banks.

3 Although identities are concealed in the transactions dataset, Cai finds one large clearing firm (coded “PI7”) with large customer orders during the crisis period which closely match various features of LTCM’s trades executed through Bear Stearns, including trade size, pattern and timing. More importantly, Cai finds that market makers traded on their own accounts in the same direction just one or two minutes before before PI7 customer orders were executed.

The *WSJ* further reports that Lehman eventually tried to secretly raise funds from the European Central Bank:

In the weeks before it collapsed, Lehman Brothers Holdings Inc. went to great lengths to conceal how fast it was careening toward the financial precipice. The ailing securities firm quietly tapped the European Central Bank as a financial lifeline.

Eventually, any funds Lehman could acquire were apparently not enough, and the investment bank declared bankruptcy on September 15, 2008. The *Associated Press* writes,\(^5\)

If the mortgage meltdown is like a financial hurricane, then think of Lehman Brothers as a casualty that waited too long to cry for help.

**Related Literature.** Brunnermeier and Pedersen (2005) provides the basic framework for predatory trading used in this paper. Brunnermeier and Pedersen (2005) show that if a distressed firm is forced to liquidate a large position, other traders have the incentive to trade in the same direction, in order to profit from large price swings. Furthermore, they show that predatory trading can even induce the distressed trader’s need to liquidate. In their analysis, the predator is perfectly informed of the distressed trader’s balance sheet, whereas in this paper I relax this assumption and allow traders to have private information about their own finances. This is motivated by the observation that banks often know more about their own balance sheet (and portfolio) than other institutions.

This paper more generally emphasizes the importance of considering non-competitive markets in which large strategic traders do not take prices as given. Strategic trading based on private information about security fundamentals is studied by Glosten and Milgrom (1985) and Kyle (1985), while speculative trading by investors with no knowledge of fundamental values, but who do possess superior knowledge of the trading environment is studied by Madrigal (1996) and Vayanos (2001). Allen and Gale (1992), on the other hand, study stock price manipulation in which an investor buys and sells shares, incurring profits by convincing others that he is informed. Finally, Carlin, Lobo, and Viswanathan (2007) offer a complementary theory of predatory trading; they show how predation is a manifestation of a breakdown in cooperation between market participants.

This paper furthermore examines how lending problems may arise from the strategic interactions among banks. In this way, this paper is related to Acharya, Gromb, and Yorulmazer (2009), who study market power in the interbank lending market. They show that during crises episodes, the profits a surplus bank may gain from buying fire-sale assets and increasing market share may lead to a lower willingness to supply interbank loans. Similarly, this paper is related to the literature on the role of the central bank during episodes of aggregate liquidity shortages or interbank-lending market breakdown, see for example Allen and Gale (1998), Holmstrom and Tirole (1998), Diamond and Rajan (2005), and Gorton and Huang (2006). Finally, Bolton and Scharfstein (1990) show

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that an optimal lending contract may leave a firm unable to fully counter predation risk. They consider product market predation, not financial market predation. Finally, while all of these papers emphasize the provision of liquidity by banks and central banks, i.e. the suppliers of funds, they do not consider the signal value of searching for liquidity by distressed financial firms and how that endogenously affects the demand for funds.

This paper is organized as follows. Section 2 describes the model. In Section 3, I define the equilibrium of the economy and analyze the optimal decision for each type of trader. Section 4 examines the benchmark case in which there is no predator. Section 5 characterizes the equilibria in the full model with predatory trading. Section 6 concludes.

2 The Model

There are 3 periods: \( t \in \{1, 2, 3\} \) and two tradeable assets: a riskless bond and a risky asset. The risk-free rate is normalized to 0. The risky asset has an aggregate supply \( Q \) and a final payoff \( z \) at time \( t = 3 \), where \( z \) is a random variable with an expected value of \( \mathbb{E}[z] = \bar{z} > 0 \). The price of the risky asset at any time \( t \) is denoted \( s_t \).

There are two strategic traders, the distressed trader and the (potential) predator, which are denoted by \( i \in \{d, p\} \). Both traders are risk neutral and seek to maximize their expected profit at time \( t = 3 \), which I denote as \( \tilde{w}_i \). Each strategic trader is large, and hence, his trading impacts the equilibrium price. Traders can be thought of as hedge funds or the proprietary trading desks of large investment banks. Let \( x_{i,t} \) denote trader \( i \)'s holding of the stock at time \( t \). Each strategic trader has a given initial endowment, \( x_{i,1} \), of the risky asset and is restricted to hold \( x_{i,t} \in [-\bar{x}, \bar{x}] \). For simplicity I assume that each trader’s initial endowment is equal to its maximum long position, that is, \( x_{p,1} = x_{d,1} = \bar{x} \).

In addition to the two large strategic traders, the market is populated by long-term investors. The long-term traders are price-takers and have at each point in time an aggregate demand curve given by

\[
Y(s_t) = \frac{1}{\lambda} (\bar{z} - s_t) .
\]  

(1)

This demand schedule has two important attributes. First, it is downward sloping: in order for long-term traders to hold more of the risky asset, they must be compensated in terms of lower prices. Second, the long-term traders’ demand depends only on the current price \( s_t \), that is, they do not attempt to profit from future price swings.

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6This position limit can be interpreted more broadly as a risk limit or a capital constraint.

7This could be due to risk aversion or due to institutional frictions that make the risky asset less attractive for long-term traders. For instance, long-term traders may be reluctant to buy complicated derivatives such as asset-backed securities.

8Long-term investors may be interpreted as pension funds and individual investors. Under this interpretation, long-term investors may not have sufficient information, skills, or time to predict future price changes.
The market clearing price solves \( Q = Y(s_t) + x_{p,t} + x_{d,t} \). Market clearing implies that the equilibrium stock price is given by \( s_t = \bar{z} - \lambda [Q - (x_{p,t} + x_{d,t})] \). Due to the constraint on asset holdings, strategic traders cannot take unlimited positions. Assuming the case of limited capital, i.e. \( 2\bar{x} < Q \), the equilibrium price is always lower than the fundamental value: \( s_t < \bar{z}, \forall t \). Therefore, strategic traders can expect positive profits from holding the asset until time \( t = 3 \).

In addition to the risky asset, each strategic trader is endowed with a non-tradeable investment. At time \( t = 3 \), this investment, if not liquidated, yields a payoff of \( u \), where \( u \) is a random variable with an expected value of \( E_i u = \bar{u} > 0 \). This investment is non-tradeable in the following sense: it cannot be sold by the trader at any point in time before the investment has materialized in the last stage. I let \( v_{i,t} \) represent the paper value at time \( t \) of this investment. For example, if the trader is an investment bank, \( v_{i,t} \) may be thought of as the value of investments made by the lending side of the bank which, perhaps due to agency reasons, cannot be securitized.

The paper value of the distressed’s investment is subject to liquidity shocks, such that \( v_{d,t} \) is not necessarily equal to \( \bar{u} \) at every point in time. In particular, \( v_{d,t} \) at any point in time takes one of three values: \( v_{d,t} \in V_d \equiv \{v_l, v_m, v_h\} \), where without loss of generality \( v_l < v_m < v_h \). The realizations of \( v_{d,t} \) are however independent of \( u \), so that the trader’s expected final payoff from his non-tradeable investment is always given by \( \bar{u} \). On the other hand, the predator’s valuation of non-tradeable assets is constant over time, and equal to its expected payoff: \( v_{p,t} = \bar{u}, \forall t \).

At any time \( t \), a trader’s “mark-to-market” wealth is given by \( w_{i,t} = x_{i,t} s_t + v_{i,t} \). If the trader survives to period 3, its expected payoff from holding its portfolio is \( E_i [w_{i,3}] = x_{i,3} \bar{z} + \bar{u} \). Let \( \bar{w} \) denote the maximum expected wealth of a trader’s portfolio, that is, \( \bar{w} \equiv \bar{z} \bar{z} + \bar{u} \). However, if at any time before the last period a trader’s wealth drops below some threshold level \( \bar{w} \), then the trader must liquidate all assets at fire sale prices. This assumption of forced liquidation could be due to margin constraints, risk management, or other considerations in connection with low wealth. Let \( L < \bar{w} \) be the fire-sale value of the entire portfolio if the trader is forced to liquidate before the last stage, and let \( \Delta \equiv \bar{w} - L \) denote the difference between the expected payoff from the portfolio and its fire sale value. One may think of \( \Delta \) as the penalty the trader incurs for liquidating prematurely.

**Timing and Information.** Before stage 1, Nature draws initial value \( v_{d,1} \in V_d = \{v_l, v_m, v_h\} \) of the distressed’s non-tradeable holdings according to the following distribution:

\[
v_{d,1} = \begin{cases} 
  v_l & \text{with probability } q_l \\
  v_m & \text{with probability } q_m \\
  v_h & \text{with probability } q_h 
\end{cases}
\]

where \( q_l + q_m + q_h = 1 \). For simplicity, I let \( q_l = q_m = q_h = 1/3 \). One may think of this as the initial “type” of the distressed trader. That is, the distressed is initially a low type if \( v_{d,1} = v_l \), a medium type if \( v_{d,1} = v_m \), and a high type if \( v_{d,1} = v_h \).
Stage 1. In stage 1, the distressed trader learns his initial type $v_{d,1}$ (or valuation of his non-tradeable investment), but the distressed’s type is not observed by the predator. This can be interpreted as investors conducting a valuation of the financial firm, but this value is not released publicly. Once it observes $v_{d,1}$, the distressed trader then has the option to search for additional resources from an outside lender. I let $a_d \in A_d \equiv \{S, NS\}$ denote the action taken by the distressed, where $S$ denotes the decision to “search” for a loan, and $NS$ denotes the decision to “not search”. For this reason, I refer to stage 1 as the “loan-seeking stage”.

If the distressed decides to search, he receives a loan which may increase his wealth. Before deciding to search, however, the distressed does not know the value of this loan. In particular, I assume that the loan is stochastic, and with some probability may bring the distressed trader into a higher valuation state for stage 2. For example, if the distressed initially is a medium type ($v_{d,1} = v_m$) and decides to search for a loan, with probability $\pi_{mh}$ he receives a loan which makes him a high-type firm for stage 2 ($v_{d,2} = v_h$). On the other hand, with probability $\pi_{mm}$ the distressed does not receive a loan and stays a medium type for stage 2. More formally, if the distressed searches, his stage 2 type $v_{d,2}$ is determined by the following transition matrix

$$
\pi(v_{d,2}|v_{d,1}) = \begin{bmatrix}
\pi_{ll} & \pi_{lm} & \pi_{lh} \\
0 & \pi_{mm} & \pi_{mh} \\
0 & 0 & 1
\end{bmatrix}
$$

(2)

where $\pi_{ll} + \pi_{lm} + \pi_{lh} = 1$ and $\pi_{mm} + \pi_{mh} = 1$. Like the bond, the loan has zero interest but does have a fixed cost of $c > 0$ which is incurred in the last period. Finally, if the distressed decides to not search for a loan, his type remains constant; that is, $v_{d,2} = v_{d,1}$.

After the distressed decides whether or not to search for a loan, the value $v_{d,2}$ is realized. This value is again observed by the distressed but not by the predator.

Stage 2. Although the predator does not observe the distressed’s type $v_{d,2}$ directly, the predator does however observe whether the distressed decided to search or not. Specifically, the predator observes $a_d$. After observing $a_d$, the predator then decides whether or not to predatorily trade against the distressed. I let $a_p \in A_p \equiv \{P, NP\}$ denote the action taken by the potential predator, where $P$ denotes the decision to “predate”, and $NP$ denotes the action to “not predate”. For this reason, I refer to this stage as the “predatory phase”.

If the predator decides to predatorily trade, then the strategic traders engage in a “predation war”. The results of this predation war are derived from Brunnermeier and Pedersen (2005). The mechanics of this predation war are not the main focus of this paper. For this reason, in this section I only present the important (reduced-form) results that are pertinent to understanding the model. A more detail description of the predation war is given in the Appendix.

If the predator decides to predate, then the strategic traders engage in a “predation war” in which both traders sell the risky asset as fast as possible. This predation war continues until one of

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9The predator’s type $v_p$ is constant and common knowledge throughout the game.
the traders is forced to leave the market. The trader who is forced to leave the market is the trader whose wealth falls below the minimum wealth threshold $w$ first—that is, the trader with the lower amount of wealth will be forced to leave the market. The predator therefore wins the predation war and the distressed loses if and only if $v_p > v_{d,2}$. In this case, the predator buys back up to its optimal position $\bar{x}$, receives strictly positive profits $m > 0$ and moves on to stage 3, while the distressed trader is forced into liquidation and receives final payoff $\tilde{w}_d = L$. On the other hand, if the predator loses and the distressed wins the predation war, the predator must liquidate its assets at fire sale prices and receives final payoff $\tilde{w}_p = L$, while the distressed buys back up to its optimal position $\bar{x}$ and continues on to stage 3.$^{10}$

If the predator decides not to predate, then there is no predation war. Both traders move on to the next period with no change to their current or expected wealth.

Stage 3. In stage 3, conditional on making it to this stage (either not engaging or winning the predation war in stage 2), the predator receives the final realized wealth from his portfolio, $\tilde{w}_p = x_{p,3}z + u$.

In addition, the distressed trader, conditional on making to this stage (either not engaging or winning the predation war in stage 2), is subject to an exogenous income shock. This income shock has two outcomes, either it results in stage 3 wealth below the threshold $w$, forcing the distressed trader to liquidate, or it results in stage 3 wealth above the threshold.

The distressed in period 3, has valuation $v_{d,3}$ equal to its valuation in the previous period, $v_{d,2}$. The probability of hitting the lower bound on wealth after the income shock depends on the distressed’s current type. In particular, the probability that the trader’s wealth after the income shock is above the threshold is increasing in $v_{d,3}$. If the distressed trader is low type, then his wealth after the income shock is above the threshold $\tilde{w}$ with probability $p_l$. If the trader is medium type, then his wealth after the income shock is above the threshold with probability $p_m$. Finally, if the trader is high type, then his wealth after the income shock is above the threshold with probability $p_h$. I assume $p_l < p_m < p_h$, so that the high type has the lowest probability of hitting the lower bound on wealth, and the low type has the highest probability of hitting the lower bound.

If the distressed hits the lower bound on wealth after the income shock, he is forced to liquidate all assets at fire-sale prices and receives final payoff $\tilde{w}_d = L$. If instead the distressed’s wealth is above the threshold after the income shock, then he receives the final payoffs from holding the portfolio, $\tilde{w}_d = x_{d,3}z + u$. If the distressed took out a loan in stage 1, he now pays the fixed cost $c$ to the outside lender.

Remarks and Assumptions. The extensive form of this game is presented in Figure 1 for stages 1 and 2. I omit the last stage for simplicity and because no player moves in the last stage. Therefore, what is omitted in this figure is the stochastic income shock in stage 3 and the realization of payoffs

$^{10}$Note that the distressed does not make profits from winning the predation war. This may be interpreted as the distressed isn’t trying, or does not have the skills, to profit from the exit of the predator. Thus, in the event that the distressed wins a predation war, it receives zero gains: $m_d = 0$. 

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for each player. From this figure, one can see that Nature (denoted by “N”) moves first, choosing the distressed’s non-tradable investment value $v_l, v_m, v_h$ with probabilities $q_l, q_m, q_h$ respectively. After observing its own type, the distressed decides whether to search (S) or not search (NS) for a loan. If the distressed searches, Nature then draws another type based on the transition matrix given in (2). Finally, the predator, although he does not observe the distressed’s type, does get to observe the distressed’s action. Given this information set, the predator then decides whether to predate (P) or not (NP).

This figure therefore illustrates the two main decisions nodes in the game: first, the decision of the distressed to search for a loan, and second, the decision of the predator to predate.

The key decision for the distressed trader occurs in stage 1, the loan seeking stage. In this stage, after observing his initial value (or type), the distressed trader decides whether or not to search for a loan. In making this decision, there are two future risks that the distressed trader faces: predatory trading risk in stage 2 and exogenous income risk in stage 3. Searching for a loan is the distressed trader’s only way of potentially protecting itself against these risks. In stage 3, the lower the trader’s valuation, $v_d, 3$, the greater the probability of hitting the lower bound on wealth. For this reason a loan would be desirable. However, the main caveat of searching for a loan is its possible signal value—that is, the potential predator sees whether the distressed searched for a loan, and hence infers some information from this action. Therefore, in deciding whether to search for a
loan, a distressed bank faces a trade-off between the financial cushion provided by a loan and the information it conveys.

The key decision for the predator occurs in stage 2. While the predator does not observe the distressed’s type, he does observe whether or not the distressed searched for a loan. This is motivated by the following. Financial firms must contact outside lenders, counterparties, or central banks when seeking loans. Although any loan amount received may not be observed by the market, the act of seeking liquidity is likely to become public. Thus, potential predators may be able to infer information from this action. In the next section, I show how the predator forms beliefs about the distressed’s type optimally via Bayes rule.

Finally, note that is optimal for each trader to always hold $\tilde{x}$ of the risky asset, unless engaged in a predation war. This corner solution is due to the long-term investor’s demand curve and to the fact that traders have limited capital, so that the equilibrium price is always lower than the fundamental value.

**Assumptions.** I make the following assumptions on parameter values.

**Assumption.** $v_m < v_p < v_h$

That is, if the predator predates in stage 2, he succeeds if $v_{d,2} = v_l$ or $v_m$, but fails if $v_{d,2} = v_h$. Note that even after receiving a loan, the distressed firm may not be a high type. Thus, this assumption implies that a loan will not always bring the firm into the range where it is not subject to predation risk. Bolton and Scharfstein (1990) show that an optimal financial contract may leave an agent cash constrained even if the agent is subject to predation risk.\footnote{They consider product market predation, not financial market predation. Furthermore, they do not consider the signal value of searching for liquidity when information is asymmetric.}

**Assumption.** $p_h = 1$ and $p_l = 0$

This simply states that in stage 3, the high type never hits the lower bound on wealth while the low type always hits the lower bound on wealth. This will imply that the distressed’s type space has dominance regions. That is, for the two extreme types—the low type and the high type—searching and not searching, respectively, are strictly dominant. The dominant strategies will be proven and shown in the following section.

**Assumption.** $0 < c < \Delta$ and $0 < m < \Delta$

This assumption states that the gain incurred from successfully predating is less than the loss due to a forced liquidation, and that the fixed cost from obtaining a loan is less than the loss due to a forced liquidation.

Finally, I assume the following condition for the transition probabilities.

**Assumption.** $0 < \pi_{lh} < \pi_{mh}$

This assumption states that, conditional on searching for a loan, the probability of the medium type becoming a high type is greater than the probability of the low type becoming a high type.
3 Equilibrium Definition

Both strategic traders are risk-neutral and expected payoff maximizers. There are two stages in this game where the traders make choices, and the choices each trader makes may affect their final payoff \( \tilde{w}_i \). In this section I define the equilibrium in this game and characterize the decision rules for each agent.

Note that in this game, each agent—the distressed and the predator—may face the risk of liquidating prematurely and receiving final payoff \( \tilde{w}_i = L \). For the predator, this could be the outcome of the predation war. For the distressed, this could either be the outcome of the predation war, or the outcome of the exogenous income shock in stage 3. In terms of final outcomes of the game, I say that a particular trader “survives” if he is never forced to liquidate. That is, “survival” refers to the event that the trader makes it through the entire game without liquidating and receives the final value of holding its portfolio, \( \tilde{w}_i = x_{i,3} z + u \).

In stage 1, the distressed trader, after observing its initial type, \( v_{d,1} \), decides whether or not to search for a loan. To make this decision the distressed trader forms beliefs about his survival probability that depend not only on his chosen action and its initial type, but also on the strategy of the predator. Given an initial type \( v_{d,1} \), let \( \alpha (a_d, v_{d,1} | r_p) \) denote the distressed’s belief it survives if it chooses action \( a_d \), conditional on the predator following strategy \( r_p \). Thus, given initial type \( v_{d,1} \), the distressed’s expected payoff conditional on not searching is given by

\[
E_{d,1} [\tilde{w}_d | NS, v_{d,1}, r_p] = \tilde{w} \alpha (NS, v_{d,1} | r_p) + (\tilde{w} - \Delta) (1 - \alpha (NS, v_{d,1} | r_p)),
\]

since he gets expected payoff \( \tilde{w} \) if he survives, and liquidation value \( L = \tilde{w} - \Delta \) otherwise. On the other hand, given initial type \( v_{d,1} \), the distressed’s expected payoff conditional on searching is given by

\[
E_{d,1} [\tilde{w}_d | S, v_{d,1}, r_p] = (\tilde{w} - c) \alpha (S, v_{d,1} | r_p) + (\tilde{w} - \Delta) (1 - \alpha (S, v_{d,1} | r_p))
\]

since he gets expected payoff \( \tilde{w} \) if he survives, minus the fixed cost of the loan, and liquidation value \( L = \tilde{w} - \Delta \) otherwise.

Likewise, in stage 2, the predator, after observing the action of the distressed, \( a_d \), decides whether or not to predate. To make this decision the predator forms beliefs about his survival probability that depend not only on his chosen action, but also on the observed action of the distressed and the distressed’s strategy. Given an observed action \( a_d \), let \( \beta (a_p, a_d | r_d) \) denote the predator’s belief he survives if he chooses action \( a_p \), conditional on the predator following strategy \( r_d \). If the distressed chooses to predate, then the survival probability is merely the posterior probability that \( v_{d,2} < v_p \), i.e. \( \beta (P, a_d | r_d) = \Pr [v_{d,2} < v_p | a_d, r_p] \). On the other hand, if the predator does not predate, then \( \beta (NP, a_d | r_d) = 1 \), for any \( a_d, r_d \). Therefore, given observed action \( a_d \), the predator’s expected payoff conditional on predatorily trading is given by

\[
E_{p,2} [\tilde{w}_p | P, a_d, r_d] = (\tilde{w} + m) \beta (P, a_d | r_d) + (\tilde{w} - \Delta) (1 - \beta (P, a_d | r_d))
\]
since he gets expected payoff $\bar{w}$ if it survives, plus profits $m$, and liquidation value $L = \bar{w} - \Delta$ otherwise. On the other hand, given observed action $a_d$, the predator’s expected payoff conditional on not predatorily trading is given by

$$E_{p,2} [\bar{w}_p | NP, a_d, r_d] = \bar{w}$$

A strategy of the distressed trader is merely a mapping from the distressed’s type space to an action, that is $r_d : V_d \rightarrow A_d$. A strategy for the predator is merely a mapping from its information set to an action, that is, $r_p : A_d \rightarrow A_p$. The equilibrium of this game is then defined as follows.

**Definition 1.** An equilibrium is a strategy for the distressed $r_d : V_d \rightarrow A_d$, a strategy for the predator $r_p : A_d \rightarrow A_p$, and a belief system $\alpha : A_d \times V_d \rightarrow [0, 1]$ and $\beta : A_p \times A_d \rightarrow [0, 1]$

(i) For each $v_{d,1} \in V_d$, the distressed of initial type $v_d$ searches for a loan if and only if his expected payoff from doing so is greater than his expected payoff from not searching

$$r_d (v_{d,1}) = S \text{ if and only if } E_{d,1} [\bar{w}_d | S, v_{d,1}, r_p] > E_{d,1} [\bar{w}_d | NS, v_{d,1}, r_p],$$

conditional on the predator following strategy $r_p$.

(ii) For each $a_d \in A_d$, the predator who observes $a_d$ predates if and only if his expected payoff from doing so is greater than his expected payoff from not predating

$$r_p (a_d) = P \text{ if and only if } E_{p,2} [\bar{w}_p | P, a_d, r_d] > E_{p,2} [\bar{w}_p | NP, a_d, r_d]$$

conditional on the distressed following strategy $r_d$.

(iii) The survival belief of the distressed, $\alpha$, is based on the predator following strategy $r_p$, and the survival belief of the predator $\beta$ is formed using Bayes rule and based on the distressed following strategy $r_d$.

Conditions (i) and (ii) of Definition 1 require that the strategies of the distressed and the predator are sequentially rational given their beliefs. Condition (iii) states that the belief system must be consistent given the strategy profile of the players. Thus, the equilibrium definition is that of a standard perfect-Bayesian equilibrium, in which the distressed is the sender and the predator is the receiver. Finally, I prove shortly that in this game there are no out-of-equilibrium beliefs.

**Decision rule for the distressed trader.** I first consider the decision for the distressed trader in stage 1. The expected payoffs for the distressed from searching and from not searching are given in (4) and (3), respectively. Combining these with the distressed’s decision rule stated in (7), it follows that optimal action for the distressed trader may be expressed as follows.

**Lemma 1.** Given initial valuation $v_{d,1}$ and conditional on the predator following strategy $r_p$, the distressed trader searches for a loan if and only if

$$\frac{\alpha (NS, v_{d,1} | r_p)}{\alpha (S, v_{d,1} | r_p)} < \frac{\Delta - c}{\Delta}$$

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where \( \Delta - \xi \in (0, 1) \).

The above Lemma gives a simple cut-off rule, in terms of the distressed’s beliefs, for when it is optimal for the distressed to search for a loan. To see this, note that the left-hand side of equation (9) is merely the ratio of the probability of survival from not searching to the probability of survival from searching. Lemma 1 states that the distressed trader will search if and only if this ratio is sufficiently low. The cut-off for this ratio, i.e. the right-hand side of equation (9), is a constant which is decreasing in the fixed cost of obtaining a loan, \( c \), but increasing in the liquidation penalty \( \Delta \). Therefore, the distressed finds it optimal to search if the probability of surviving from not searching is low relative to that of searching. That is, searching is optimal if it greatly increases the distressed’s chances of survival. However, a higher fixed cost of the loan or a lower liquidation penalty makes it less likely that the distressed will find it optimal to search.

Lemma 1 gives a simple decision rule for the distressed trader, given his initial type. Using this decision rule, it is now clear that for the two extreme types—the low type \( v_d \) and the high type \( v_h \)—searching and not searching, respectively, are strictly dominant. This is stated in the following lemma.

**Lemma 2.** For any strategy of the predator, the low type always finds it optimal to search. Likewise, for any strategy of the predator, the high type always finds it optimal to not search.

The proof of Lemma 2 is simple. Consider first the low-type’s decision. For any strategy of the predator, if the low type decides not to search, his probability of survival is zero, since no matter what happens in stage 2, the exogenous income shock in stage 3 will force the firm to liquidate \( (p_l = 0) \). On the other hand, for any strategy of the predator if the low type decides to search for a loan, his probability of survival is strictly positive. Condition (9) is hence satisfied for all \( r_p \). Therefore, no matter the strategy of the predator, the low type always finds it optimal to search.

Second, consider the decision of the high type in stage 1. For any strategy of the predator, the high type’s probability of survival, whether it searches or not, is always equal to 1. This is due to the fact that neither the predator in stage 2 nor the income shock in stage 3 can force the high type to liquidate. Therefore, according to condition (9), the high type always finds it optimal to not search.

Lemma 2 clarifies the type of equilibria that may exist in this game. The property that the low type always searches and the high type never searches, i.e. that there are dominance regions in the type space, implies that any possible equilibrium in this game must be a separating (or semi-separating) equilibrium. Any action observed by the predator is consistent with an equilibrium path, and hence no off-the-equilibrium path beliefs need be specified.

Finally, note that Lemma 2 also contributes to understanding the signalling nature in this game. Because the predator does not observe the type of the distressed, it can only infer information from the distressed’s action. From Lemma 2, we see that regardless of the predator’s strategy, it is strictly dominant for the low type to search, and it is strictly dominant for the high type to not
search. Therefore, when the medium type makes its decision whether or not to search, part of its trade-off is whether to be pooled with the low types or to be pooled with the high types, and in this way convey information to the predator.

**Decision rule of the predator.** I now consider the decision for the predator in stage 2. The expected payoffs for the predator from trading and from not trading are given in (5) and (6), respectively. Combining these with the predator’s decision rule stated in (8), it follows that optimal action for the predator may be expressed as follows.

**Lemma 3.** Given observed action $a_d$ and conditional on the distressed following strategy $r_d$, the predator predatorily trades if and only if

$$
\frac{1 - \beta(P, a_d | r_d)}{\beta(P, a_d | r_d)} < \frac{m}{\Delta}
$$

where $\frac{m}{\Delta} \in (0, 1)$.

Much like Lemma 1, Lemma 3 gives a simple cut-off rule, in terms of the predator’s beliefs, for when it is optimal for the predator to predatorily trade. To see this, note that the left-hand side of equation (9) is merely the ratio of the probability of failing to the probability of succeeding, if the trader decides to predatorily trade. Lemma 3 states that the predator will predate if and only if this ratio is sufficiently low. The cut-off for this ratio, i.e. the right-hand side of equation (9), is a constant which is increasing in the gain from winning a predation war, $m$, but decreasing in the liquidation penalty $\Delta$. Therefore, the predator finds it optimal to predate if the probability of surviving conditional on predatorily trading is high enough. However, a lower gain from predatorily trading or a higher liquidation penalty makes it less likely that the predator will find it optimal to predate.

The decision rules stated in Lemmas 1 and 3 greatly simplify the equilibrium analysis. In what follows, in Section 4 I first look at the equilibrium when there is no predator, and then in Section 5 I turn to the equilibrium of the full game with predation.

### 4 Equilibrium without Predator

In this section I analyze the equilibrium in a benchmark case in which there is no predator. That is, I consider a setting identical to that described in Section 2, but without stage 2, the predatory stage. Within this predator-less setting, I need only to consider the optimal strategy of the distressed.

In this environment, Lemma 2 continues to hold; that is, it is optimal for the low types to search and for the high types to not search. Thus, in terms of the distressed’s strategy, one needs only to find what is optimal for the medium type. Although there is no predation risk, the medium type still faces income shock risk. Hence if the medium type decides not to search, his probability of survival is given by $\alpha (NS, v_m) = p_m$. On the other hand, if the medium decides to search for a
loan, there is some probability he becomes a high type; for this reason the probability of surviving is given by \( \alpha (S, v_m) = \pi_{mm} p_m + \pi_{mh} \). Combining these probabilities with the decision rule in (9), I find that the medium-type searches if and only if

\[
\frac{p_m}{\pi_{mm} p_m + \pi_{mh}} < \frac{\Delta - c}{\Delta}
\]

Therefore, depending on parameter values, the medium type could find either choice optimal in the benchmark with no predator.

For the remainder of this paper, I focus on the case where the medium type searches for a loan in the absence of predators. In other words, I impose the following condition.

**Condition 1.** \( \frac{p_m}{\pi_{mm} p_m + \pi_{mh}} < \frac{\Delta - c}{\Delta} \)

The above condition is imposed for the following reason. The focus of this paper is to study the effect of predatory trading on the incentives of financial firms to seek liquidity in times of distress. For this reason, it seems reasonable to consider an environment in which there is a clear incentive for a bank to seek out a loan in the absence of predators. Condition 1 imposes that in the absence of predation risk, searching for a loan greatly increases the distressed’s chances of avoiding the lower wealth bound, making it preferable for him search. Another way to read this condition is that the liquidation penalty is sufficiently high relative to the cost of a loan that the medium type prefers to search. In any case there is a clear incentive for the medium type to seek out a loan when there are no predators.¹²

Using this condition, the following proposition characterizes the optimal strategy for the distressed in the benchmark with no predatory trading.

**Proposition 1.** When there are no predators, under condition 1, the distressed follows a strategy in which the low and medium types search for a loan and the high type does not search.

## 5 Equilibrium with Predator

I now study the equilibrium (or equilibria) of the full game with predatory trading, as laid out in Section 2. As this is a signalling game, there can in principle be multiple equilibria. In order to characterize the set of all possible equilibria, I consider the entire set of possible strategies of one of the traders. Here I choose to focus on the set of strategies of the predator. For each of the predator’s strategies, I determine under what conditions the strategy may be compatible with an equilibrium. By systematically considering each of the predator’s strategies, this procedure allows me to characterize the set of all possible equilibria.

A strategy of the predator is merely a mapping from its information set to an action, \( r_p : A_d \rightarrow A_p \). Since \( A_d \) and \( A_p \) each contain only two elements, there are only four possible strategies for the predator. I label these strategies as \( \{ r^1_p, r^2_p, r^3_p, r^4_p \} \) and describe each as follows:

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¹² Another way to justify this condition is to imagine there were a continuum of types. Then there would exist a type, strictly greater than the low type that would search.
Let $r^1_p$ be strategy in which the predator never predates

$$r^1_p(a_d) = \begin{cases} NP & \text{if } a_d = S \\ NP & \text{if } a_d = NS \end{cases}$$

Let $r^2_p$ be strategy in which the predator always predates

$$r^2_p(a_d) = \begin{cases} P & \text{if } a_d = S \\ P & \text{if } a_d = NS \end{cases}$$

Let $r^3_p$ be strategy in which the predator predates if and only if he observes that the distressed did not search.

$$r^3_p(a_d) = \begin{cases} NP & \text{if } a_d = S \\ P & \text{if } a_d = NS \end{cases}$$

Finally, let $r^4_p$ be strategy in which the predator predates if and only if he observes that the distressed did search.

$$r^4_p(a_d) = \begin{cases} P & \text{if } a_d = S \\ NP & \text{if } a_d = NS \end{cases}$$

In this section, I consider each strategy $r_p \in R_p \equiv \{r^1_p, r^2_p, r^3_p, r^4_p\}$ separately. For a given proposed strategy $r_p$ of the predator, I find the best response of the distressed trader. That is, I find the survival probabilities of the distressed based on the belief that the predator is following strategy $r_p$, and given these survival beliefs I find the optimal strategy of the distressed. Allow me to denote this strategy as $r^*_d = BR_d(r_p)$, where $BR_d$ signifies that is is the best response of the distressed. Next, given the distressed’s best response strategy $r^*_d$, I then find the best response of the predator. That is, I find the survival probabilities of the predator based on the belief that the distressed is following strategy $r^*_d$, and then find the optimal strategy of the predator based on these survival beliefs. Allow me to denote this strategy as $r^*_p = BR_p(r^*_d)$, where $BR_p$ signifies that is is the best response of the predator.

I then characterize under what conditions the predator’s best response strategy coincides with the proposed strategy. If $r^*_p = r_p$, then there exists a fixed point in the traders’ best responses: $r^*_d = BR(r^*_p)$ and $r^*_p = BR(r^*_d)$. In this case, the strategy profile $\{r^*_p, r^*_d\}$ and corresponding survival beliefs therefore constitute an equilibrium. On the other hand, if under no conditions the fixed point exists, I then conclude that an equilibrium in which the predator follows the proposed strategy does not exist.

I now consider each strategy.

**The predator follows strategy $r^1_p$.** Suppose the predator follows a strategy in which he never predates.

**Distressed trader’s best response.** If the predator follows a strategy in which he never predates, in terms of the distressed trader’s decision making process it is as if the predator did not exist. In
other words, the distressed never faces any predation risk. The distressed trader will thus follow the same strategy outline above in Section 4 for the benchmark case in which there is no predator. Proposition 1 implies that the distressed’s best response to $r_p^1$ is a strategy in which the low and medium types search, while the high type does not search.

**Predatory trader’s best response.** The predator forms its survival beliefs based on the presumption that the distressed is following a strategy in which the low and medium types search, while the high type does not search. If the predator observes that the distressed did not search, the predator infers that he must be facing a high type. Hence, the predator knows that if he predates, his probability of surviving is zero. On the other hand, if he does not predatorily trade he receives a final payoff of $\bar{w}$. Therefore, if the predator observes that the distressed did not search, it is optimal for the predator to not predate.

On the other hand, if the predator observes that the distressed searched for a loan, then the probability of the predator surviving a predation war is given by

$$\beta (P, S|r_d^p) = \frac{(\pi_{ll} + \pi_{lm})ql + \pi_{mm}qm}{ql + qm}$$

This is simply the probability that the distressed’s wealth after seeking a loan is less than the wealth of the predator. Combining this with the predator’s decision rule in (10), the predator finds it optimal to predate if and only if

$$\kappa_1 < \frac{m}{\Delta}$$

where

$$\kappa_1 \equiv \frac{(1 - \pi_{ll} - \pi_{lm}) + (1 - \pi_{mm})}{\pi_{ll} + \pi_{lm} + \pi_{mm}}$$

(11)

where I have used the fact that $q_l = q_m$. Therefore the proposed equilibrium in which the predator never predates exists if and only if the above condition is not satisfied, that is, when $\frac{m}{\Delta} < \kappa_1$.

**The predator follows strategy $r_p^2$.** Suppose the predator follows a strategy in which he always predates.

**Distressed trader’s best response.** From Lemma 2, we know that the low type chooses to search and the high type chooses to not search. Now consider the optimal choice of the medium type. Given that the predator is following a strategy in which he always predates, if the medium type chooses to not search, then he will be engaged in a predation war which he will surely lose, since $v_{d,2} = v_m < v_p$. Hence, the medium type’s probability of survival from not searching, $\alpha (NS, v_m|r_p^2)$, is equal to zero. On the other hand, if the medium type chooses to search, then he still faces a predation war. In this case, however, there is some positive probability that the distressed receives a loan that makes it a high type and hence wins the predation war. Therefore, the medium type’s probability of survival from searching for a loan is given by $\alpha (S, v_m|r_p^2) = \pi_{mb}$, which is strictly greater than zero. According to the distressed’s decision rule (9), it is optimal for the medium type to search. The distressed’s best response to $r_p^2$ is therefore a strategy in which the low and medium types search, while the high type does not search.
Predatory trader’s best response. The predator forms its survival beliefs based on the presumption that the distressed is following a strategy in which the low and medium types search, while the high type does not search. If the predator observes that the distressed did not search, the predator infers that he must be facing a high type. Hence, the predator knows that if he predates, his probability of surviving is zero. On the other hand, if he does not predatorily trade he receives a final payoff of $\bar{w}$. Therefore, if the predator observes that the distressed did not search, it is optimal for the predator to not predate. Thus, given the strategy of the distressed, under no conditions does the best response of the predator coincide with $r_p^2$. Therefore, no equilibrium exists in which the predator follows a strategy in which he always predates.

The predator follows strategy $r_p^3$. Suppose the predator follows a strategy in which he predates if and only if the distressed does not search.

Distressed trader’s best response. The low type searches and the high type does not. The medium type forms his beliefs based on the strategy of the predator. Given that the predator is following a strategy in which it predates if and only if the distressed does not search, then he will be engaged in a predation war which he will surely lose, since $v_{d;2} = v_m < v_p$. Hence, the medium type’s probability of survival from not searching, $\alpha (NS, v_m|r_p^3)$, is equal to zero. On the other hand, given the predator’s strategy, if the medium type chooses to search, then he will not face any predation risk and the only risk he faces is the exogenous income shock in stage 3. In this case, his probability of survival is given by $\alpha (S, v_m|r_p^3) = \pi_{mm}p_m + \pi_{mh}$, which is strictly greater than zero. According to the distressed’s decision rule (9), it is optimal for the medium type to search. The distressed’s best response to $r_p^3$ is therefore a strategy in which the low and medium types search, while the high type does not search.

Predatory trader’s best response. The predator forms its survival beliefs based on the presumption that the distressed is following a strategy in which the low and medium types search, while the high type does not search. If the predator observes that the distressed did not search, the predator infers that he must be facing a high type. Hence, the predator knows that if he predates, his probability of surviving is zero. On the other hand, if he does not predatorily trade he receives a final payoff of $\bar{w}$. Therefore, if the predator observes that the distressed did not search, it is optimal for the predator to not predate. Thus, given the strategy of the distressed, under no conditions does the best response of the predator coincide with $r_p^3$. Therefore, no equilibrium exists in which the predator follows a strategy in which he predates if and only if the distressed does not search.

The predator follows strategy $r_p^4$. Finally suppose the predator follows a strategy in which he predates if and only if the distressed searches.

Distressed trader’s best response. The low type searches and the high type does not. The medium type forms his beliefs based on the strategy of the predator. Given that the predator is following a strategy in which he predates if and only if the distressed searches, if the medium type
chooses to not search, then he will not face any predation risk and the only risk he faces is the exogenous income shock in stage 3. Hence, the medium type’s probability of survival from not searching is given by $\alpha (NS, v_m|r_p,4) = p_m$. On the other hand, given the predator’s strategy, if the medium type chooses to search, then he will be engaged in a predation war with the predator in stage 2, which the distressed will lose if he is still a medium type, but will win if he receives a loan that makes him a high type. Therefore, the medium type’s probability of survival from searching for a loan is given by $\alpha (S, v_m|r_p,4) = \pi_{mh}$. According to the distressed’s decision rule (9), the medium type searches if and only if

$$\frac{p_m}{\pi_{mh}} < \frac{\Delta - c}{\Delta}$$

Therefore, depending on parameter values, the medium type could find either choice optimal. If $\frac{p_m}{\pi_{mh}} < \frac{\Delta - c}{\Delta}$, then conditional on the predator’s strategy it is optimal for the medium type to search. In this case, the distressed’s best response to $r_p^4$ is a strategy in which the low and medium types search, while the high type does not search. On the other hand, if $\frac{p_m}{\pi_{mh}} > \frac{\Delta - c}{\Delta}$, then conditional on the predator’s strategy, it is optimal for the medium type to not search. In this case, the distressed’s best response to $r_p^4$ is a strategy in which the low type searches for a loan, and the medium and high types do not.

**Predatory trader’s best response.** To characterize the best response of the predator, I consider separately the two cases: first, the case in which $\frac{p_m}{\pi_{mh}} < \frac{\Delta - c}{\Delta}$ and second the case in which $\frac{p_m}{\pi_{mh}} > \frac{\Delta - c}{\Delta}$.

First, suppose that $\frac{p_m}{\pi_{mh}} < \frac{\Delta - c}{\Delta}$. In this case, the predator forms his survival beliefs based on the presumption that the distressed is following a strategy in which the low and medium types search, while the high type does not search. Note that this strategy of the distressed is identical to the distressed’s best response to $r_p^1$. Using the findings of that discussion, one may infer that if the predator observes that the distressed does not search, it is optimal for the predator to not predate. On the other hand, if the predator observes that the distressed does search for a loan, then the predator finds it optimal to predate if and only if $\kappa_1 < \frac{\Delta}{\Delta}$, where $\kappa_1$ is given in (11). In this case, the predator’s best response coincides with $r_p^4$. Therefore the proposed equilibrium in which the predator predatorily trades if and only if the distressed searches exists whenever $\kappa_1 < \frac{\Delta}{\Delta}$ and $\frac{p_m}{\pi_{mh}} < \frac{\Delta - c}{\Delta}$.

Second, suppose that $\frac{p_m}{\pi_{mh}} > \frac{\Delta - c}{\Delta}$. In this case, the predator forms his survival beliefs based on the presumption that the distressed is following a strategy in which the low type searches for a loan, but the medium and high types do not. If the predator observes that the distressed does not search, he infers that it must be facing either a medium or high type. In this case, the probability of the predator surviving a predation war is given by $\beta (P, NS|r_p^d) = \frac{q_m}{q_m + q_h}$. Combining this with the predator’s decision rule in (10), I find that it is optimal for the predator to not predate. On the other hand, if the predator observes that the distressed does search for a loan, then the probability of the predator surviving a predation war is given by $\beta (P, S|r_p^d) = \pi_{il} + \pi_{lm}$. Combining this with
the predator’s decision rule in (10), the predator finds it optimal to predate if and only if

\[ \kappa_2 < \frac{m}{\Delta} \]

where\(^{13}\)

\[ \kappa_2 \equiv \frac{\pi_{lh}}{\pi_l + \pi_{lm}} \] (12)

Therefore the proposed equilibrium in which the predator predatorily trades if and only if the distressed searches exists whenever \( \kappa_2 < \frac{\mu}{\Delta} \) and \( \frac{p_m}{\pi_{mh}} > \frac{\Delta - c}{\Delta} \).

5.1 Equilibrium Characterization

Given the above analysis, I first state the following non-existence result.

**Lemma 4.** No equilibrium exists in which the predator follows a strategy in which he always predates. Moreover, no equilibrium exists in which the predator follows a strategy in which he predates if and only if he observes that the distressed did not search.

This lemma is useful in that it implies that in any equilibrium, the predator is either playing a strategy in which he never predated, or one in which he predated if and only if he observes that the distressed searched for a loan. In the former case, the equilibrium of the game will be similar to that in the benchmark with no predator. In the latter case, the fact that the predator predates if and only if he observes that the distressed searched for a loan, implies that the presence of predators creates an incentive for the distressed to refrain from searching.

I now characterize the set of all possible equilibria in this game in Propositions 2 and 3. In Proposition 2, I first consider the case in which the ratio \( \frac{p_m}{\pi_{mh}} \) is low, that is, when the medium type’s probability of surviving the exogenous income shock is low relative to the transition probability of becoming a high type after searching for a loan.

**Proposition 2.** Suppose \( \frac{p_m}{\pi_{mh}} < \frac{\Delta - c}{\Delta} \).

(i) if \( \kappa_1 < \frac{\mu}{\Delta} \), then there exists a unique equilibrium such that the distressed trader follows a strategy in which the low and medium types search for a loan while the high type does not search, and the predator follows a strategy in which he predates if and only if the distressed searches.

(ii) if \( \frac{\mu}{\Delta} < \kappa_1 \), then there exists a unique equilibrium such that the distressed trader follows a strategy in which the low and medium types search for a loan while the high type does not search, and the predator follows a strategy in which he never predatorily trades.

Thus, when the ratio \( \frac{p_m}{\pi_{mh}} \) is sufficiently low, the distressed trader always behaves as he does in the benchmark with no predator. That is, in any equilibrium the distressed follows a strategy in which the low and medium types search, and the high type does not search. Note that when \( \frac{\mu}{\Delta} < \kappa_1 \), the equilibrium is almost identical to the benchmark without predation, except that in this case it is optimal for the predator to never predatorily trade.

\(^{13}\)Note that, given the assumptions on the parameter values, \( \kappa_2 < \kappa_1 \).
On the other hand, when $\kappa_1 < \frac{m}{\Delta}$, the predator predatory trades if and only if he observes that the distressed searched. When the predator observes that the distressed searched, he knows that the distressed must be initially a low or medium type. Using Bayes rule, the predator can then form posterior beliefs over the probability of winning in a predation war. Under these beliefs, it is optimal for the predator to predate. The key insight here is that by searching for a loan, the distressed is signalling that he is either a low or medium type. Hence by taking measures to increase its financial viability, the distressed is in effect signalling its financial weakness.

Furthermore, note that the predator’s strategy provides an incentive for the distressed to refrain from searching. However, the increase in survival probability the medium type gains from searching is high enough to compensate for the increased predation risk. Thus, the medium type still finds it optimal to search.

I now consider the case in which the ratio $p_m/\pi_{mh}$ is high, that is, when the medium type’s probability of surviving the exogenous income shock is high relative to the transition probability of becoming a high type after searching for a loan.

**Proposition 3.** Suppose $\frac{p_m}{\pi_{mh}} > \frac{\Delta - c}{\Delta}$.

(i) if $k_2 < k_1 < \frac{m}{\Delta}$, then there exists a unique equilibrium such that the distressed trader follows a strategy in which the low type searches for a loan while the medium and high types do not search, and the predator follows a strategy in which he predates if and only if the distressed searches.

(ii) if $k_2 < \frac{m}{\Delta} < k_1$, there exist two (pure-strategy) equilibria:\(^{14}\)

(a) the distressed trader follows a strategy in which the low type searches for a loan while the medium and high types do not search, and the predator follows a strategy in which he predates if and only if the distressed searches.

(b) the distressed trader follows a strategy in which the low and medium types search for a loan while the high type does not search, and the predator follows a strategy in which he never predatory trades.

(iii) if $\frac{m}{\Delta} < k_2 < k_1$, then there exists a unique equilibrium such that the distressed trader follows a strategy in which the low and medium types search for a loan while the high type does not search, and the predator follows a strategy in which he never predatory trades.

Therefore, when the ratio $p_m/\pi_{mh}$ is sufficiently high, under certain parameter values there exists an equilibrium in which the distressed trader follows a strategy such that the low type searches while the medium and high types do not search, and the predator follows a strategy in which it predates if and only if the distressed searches. This equilibrium is interesting because the medium type finds it optimal to not search, and hence plays a different action than he would in the benchmark without predation.

\(^{14}\)Of course, there also exists a third equilibria in mixed strategies. In this equilibrium, the distressed strategy will be one in which the low type searches, the high type does not search, and the medium type randomizes between searching and not searching, while the predator’s strategy is one in which it randomizes between predatory trading and not predatory trading.
In this equilibrium, the predator finds it optimal to predate if he observes that the distressed searched. Of course, this is because the predator knows that only the low types search, and hence the predator has a high chance of winning in a predation war. Thus, even though the predator cannot directly observe types, searching for a loan is a strong signal that the distressed has a very weak financial status. For this reason the predator finds it optimal to predate.

The predator’s equilibrium strategy provides a strong incentive for the distressed to refrain from searching. Consider the decision of the medium type distressed trader. If the medium type chooses to not search, then he will not face any predation risk; the only risk he faces is in the exogenous income shock in stage 3. On the other hand, if the medium type chooses to search for a loan, he then engages in a predation war in stage 2, which he can win only if he receives a large enough loan to become a high type. Therefore, when the probability of surviving the income shock as a medium type is high relative to the transition probability of becoming a high type after searching for a loan, that is, when the ratio \( p_m / \pi_{mh} \) is sufficiently high, then the medium type finds it optimal to not search. In other words, the medium type prefers to pool himself with high types by not searching and consequently facing greater income risk, over pooling himself with low types but consequently facing predation risk. One can think of this as a financially weak firm that tries to ride out a temporary financial shortfall on its own, without signalling any weakness to predators by seeking outside liquidity.

This equilibrium clearly illustrates how predator trading may adversely affect the incentives of banks to seek loans in times of financial distress. In the benchmark without predation, the medium-type distressed trader searches for a loan in order to protect itself against exogenous income risk. However, when there are predators who cannot directly observe the wealth of traders, actions undertaken by these traders to relieve financial distress may convey information about their underlying financial state. For this reason, predators have the incentive to predate when they see a large trader searching for a loan. Thus, in deciding whether or not to search for a loan, the distressed firms face a trade-off between the financial cushion provided by a loan and the information this act reveals. There are equilibria in which medium-type distressed funds who would otherwise seek to recapitalize may be reluctant to search for loans in the presence of predators.

Finally, this analysis may have further implications for regulation and policy. Policies that may break the separating equilibria found in this analysis, i.e. lead to pooling equilibria, may weaken the adverse signal value of searching for funds. For example, an interesting policy to consider would be one in which the government forces all firms to obtain a loan. If all traders--low types, medium types, and high types--are forced to take a loan, then the predator cannot use this information to infer underlying financial states. However, under this policy one would need to then consider the cost of saving funds or banks that may in fact be insolvent.

Another policy to consider is one which increases the probability of successful search--that is, a policy that increases \( \pi_{mh} \). This policy would then shift the equilibrium from the one in Proposition 3 in which the medium type does not search for a loan, to an equilibrium in Proposition 2 in which the medium type does find it optimal to search. In this case, the distressed trader would act exactly
as it would in the benchmark without predation. This would then be an interesting new perspective on central bank policy during crises episodes—as opposed to the lender of last resort rationale, the central bank may be important as an institution that improves and facilitates inter-bank lending.

6 Conclusion

This paper analyzes how predatory trading may affect the incentives of banks to seek loans in times of financial distress. I find that when a distressed trader is more informed than other traders about its own balances, searching for extra capital from lenders can become a signal of financial need, thereby opening the door for predatory trading and possible insolvency. I find equilibria in which some distressed traders who would like borrow short-term in order to meet temporary liquidity needs, may be reluctant to do so in the presence of potential predators. Predatory trading may therefore deter banks and financial institutions from raising funds in times when they need it the most.

A Appendix: The Predation War

In this appendix I provide a more detailed analysis of the predation war discussed in Section 2. In stage 2, if the predator decides to predatorily trade, then the strategic traders engage in a “predation war”. The results of this predation war are derived from Brunnermeier and Pedersen (2005).

Suppose that time is continuous within this stage and denoted by \( \tau \in [0, \tilde{\tau}] \). That is, traders may now trade continuously in the asset. Let \( x_i(\tau) \) denote the position of trader \( i \) in the asset at time \( \tau \), and let \( s(\tau) \) denote the price of that asset. At the beginning of stage 2, each trader has an initial position, \( x_i(0) = \tilde{x} \), of the risky asset. Within stage 2 the trader can now continuously trade the asset by choosing his trading intensity, \( a_i(\tau) \). Hence, at time \( \tau \) the trader’s position in the risky asset is given by

\[
x_i(\tau) = x_i(0) + \int_0^\tau a_i(u)du
\]

As mentioned previously, each strategic trader is restricted to hold \( x_i(\tau) \in [-\tilde{x}, \tilde{x}] \). Finally, I consider the case of limited capital, such that \( 2\tilde{x} < Q \). In addition to the two large strategic traders, the market is populated by long-term investors, whose aggregate demand curve is given in (1).

Furthermore, it is assumed that traders cannot sell infinitely fast. Strategic traders can as a whole can trade at most \( A \in R \) shares per time unit at the current price. That is, at any moment \( \tau \), the aggregate amount of trading must satisfy \( a_d(\tau) + a_p(\tau) \leq A \). This implies that if both strategic traders are trading, the greatest intensity at which each trader may trade is \( A/2 \).

15Brunnermeier and Pedersen (2005) assume that strategic traders can as a whole trade at most \( A \in R \) shares per time unit at the current price. Rather than simply assuming that orders beyond \( A \) cannot be executed, they assume
Trader $i$’s within-stage objective is to maximize his expected wealth at the end of the stage. His earnings from investing in the risky asset is given by the final value of his stock holdings, $x_i (\bar{\tau}) z$, minus the cost of buying shares. That is, a strategic trader’s objective is to choose a trading process so as to maximize

$$\max E \left[ x_i (\bar{\tau}) z + \int_0^\tau a_i(\tau) s(\tau) \, d\tau \right]$$

Due to limited capital of the strategic traders, $s(\tau) < \bar{z}$ at any time, and hence, any optimal trading strategy satisfies $x_i (\bar{\tau}) = \bar{x}$ for the surviving trader. That is, any surviving trader ends up with the maximum capital in the arbitrage position. The qualitative results presented in this appendix will then depend on the following: (i) strategic traders have limited capital, that is $2\bar{x} < Q$, otherwise $s(\tau) = \bar{z}$ and (ii) markets are illiquid in the sense that large trades move prices ($\lambda > 0$) and traders cannot trade arbitrarily fast ($A < \infty$).

Let $\tau_d$ and $\tau_p$ denote the amount of time it takes for the distressed trader and the predator to hit their lower bounds on wealth, respectively, if both were trading simultaneously at their highest intensity. That is,

$$\tau_d \equiv \frac{w_{d,2}(0) - w}{A/2} \quad \text{and} \quad \tau_p \equiv \frac{w_{p,2}(0) - w}{A/2},$$

where $w_{d,2} = \bar{x}s(0) + v_{d,2}$ and $w_{p,2} = \bar{x}s(0) + v_p$. In equilibrium, both traders sell as fast as possible until one of the traders is forced to leave the market. Specifically, both traders trade at constant speed $-A/2$ from from time $0$ to time $\tau^*$, where $\tau^* \equiv \min \{\tau_d, \tau_p\}$. Therefore, the pivotal time $\tau^*$ is determined by the wealth of the trader who is closest to the threshold; in other words, the trader who begins the period with lower wealth (i.e. the lower $v$) is the trader who is forced to leave the market. I assume that $w$ is high enough such that at least one trader hits the lower bound.

More precisely, the trader which is forced to leave the market trades according to the following process

$$a_i(\tau) = \begin{cases} -A/2 & \text{for} \quad \tau \in [0, \tau^*] \\ 0 & \text{for} \quad \tau \geq \tau^* \end{cases}$$

While the surviving trader trades according to the following process

$$a_i(\tau) = \begin{cases} -A/2 & \text{for} \quad \tau \in [0, \tau^*] \\ A & \text{for} \quad \tau \in [\tau^*, \tau^* + \frac{\bar{x} - x(\tau^*)}{A}] \\ 0 & \text{for} \quad \tau \geq \tau^* + \frac{\bar{x} - x(\tau^*)}{A} \end{cases}$$

Thus, both traders trade as fast as they can at constant speed $-A/2$ for $\tau^*$ periods, at which point one trader is forced to leave the market. This liquidation strategy is known by both strategic traders. At time $\tau^*$, the surviving trader then buys at a constant rate back up to the original arbitrage position $\bar{x}$; this takes $\frac{\bar{x} - x(\tau^*)}{A}$ periods. From then on, the surviving trader remains in this position.
Finally, the equilibrium price follows the following trajectory

\[
s(\tau) = \begin{cases} 
  s(0) - \lambda A\tau & \text{for } \tau \in [0, \tau^*] \\
  s(0) - \lambda 2\bar{x} + \lambda A(\tau - \tau^*) & \text{for } \tau \in [\tau^*, \tau^* + \frac{\bar{x} - x(\tau^*)}{A}] \\
  \bar{z} - \lambda (\bar{x} - Q) & \text{for } \tau \geq \tau^* + \frac{\bar{x} - x(\tau^*)}{A}
\end{cases}
\]

The simultaneous selling by both strategic traders leads to price “overshooting.” This implies that the surviving trader may yield a gain from winning the predation war. This gain is given by

\[
m = \int_0^{\tau^*} a_i(\tau) s(\tau) d\tau.
\]

This is because the surviving trader sells his assets for an average price that is higher than the price at which he buys them back after the other trader has left the market. Therefore, the predator has an incentive to predate in order to profit from the price swings that occur in the wake of the liquidation. Furthermore, the overshooting price due to simultaneous selling makes liquidation excessively costly for the trader who is ultimately forced to leave the market.

References


