

A Model of Deleveraging

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Abstract

What are the persistent effects of a decrease in firms' ability to borrow? I develop a tractable model of deleveraging that emphasizes (i) firms as suppliers of financial assets to consumers and (ii) the ability of firms and consumers to alleviate financial frictions by accumulating wealth. In the model, a permanent decrease in the ability of firms to borrow leads to: increased capital misallocation and decreased total factor productivity (TFP); an increased wedge between the average marginal product of capital and the interest rate; and increased riskiness of consumption. An endogenous decrease in the interest rate is shown to amplify these effects by discouraging wealth accumulation. In a calibration using U.S. firm-level data, I find these amplification effects are large. I study a reduction in firms' ability to borrow that is consistent with the recent post-crisis decline in long-term interest rates. In general equilibrium, the resulting TFP losses are more than twice as large as they would be if the interest rate were constant, as in a small open economy. These results underscore the general-equilibrium interactions between firms and consumers in the asset market.

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1 Introduction

A decline in firms' ability to borrow can lead to a reduction in firms' supply of financial assets. The resulting scarcity of financial assets can amplify the distortions caused by the decline in firms' ability to borrow. This scarcity can also adversely affect other agents, such as workers, who need financial assets.

To explore this idea formally, this paper builds a tractable dynamic model of the persistent effects of a permanent decrease in firms' ability to borrow. In the model, firms and workers can overcome financial frictions by accumulating liquid wealth. However, the scarcity of financial assets limits the extent to which they do so. A decline in firms' ability to borrow leads to greater scarcity of financial assets and a decrease in the interest rate. This discourages the accumulation of wealth and results in larger investment distortions for firms and increased riskiness of consumption for workers. These amplification and spillover effects underscore the importance of a general equilibrium framework for analyzing the economy's response to deterioration in the quality of the financial system.

In the model, workers use financial assets as self-insurance against income risk. The decline in firms' ability to borrow requires workers to hold fewer financial assets and thus workers are less able to smooth consumption.

Economists are very familiar with how shocks to firms affect consumers through general equilibrium in the labor market: if firms want to hire more labor, wages increase and workers benefit. This paper emphasizes how shocks to firms, such as a credit crunch, affect consumers through the financial market.

By understanding how firms and workers are affected by scarcity of financial assets and low interest rates, this model can shed light on how the economy responds to a shift from loose credit for firms to tight credit. Such a shift may accompany financial crises and explain the long-lasting reductions in output and leverage that typically follow financial crises.

The dynamic accumulation of liquid wealth is central to the model. In the model, there are two forms of liquid wealth: financial assets and physical capital. Workers invest only in financial assets, because they do not have access to a technology in which to invest physical capital. Firms, which are run by entrepreneurs, can invest capital productively, and hence they are natural suppliers of financial assets to the workers. However, entrepreneurs also need liquid wealth, because they face moral hazard and liquid wealth allows them to create

the collateral required to take advantage of investment opportunities.

An entrepreneur can always choose to accumulate more liquid wealth, by reducing consumption and investing in capital or supplying fewer financial assets. By accumulating more liquid wealth, the entrepreneur would be less constrained in the future. Therefore, a key question for the long-run properties of the economy is the extent to which entrepreneurs choose to accumulate sufficient liquid wealth to overcome their borrowing constraints. For an individual entrepreneur, the answer depends on the equilibrium interest rate. I show that if the interest rate is equal to the rate of time preference, an entrepreneur eventually accumulates enough liquid wealth to completely overcome the borrowing constraints. In contrast, if the interest rate is less than the rate of time preference, then at least some of the time, the entrepreneur will have a binding borrowing constraint.

I also show that, due to this endogenous wealth-accumulation channel, steady-state investment distortions are decreasing in the interest rate. One measure of investment distortions that I consider is the difference between total factor productivity (TFP) and the TFP that would obtain if there were no borrowing constraints. Another measure that I study is the wedge between the marginal product of capital and the interest rate; this wedge is the finance premium firms would be willing to pay for an additional unit of borrowing. Investment distortions are decreasing in the interest rate because taking advantage of investment opportunities requires holding liquid wealth, and holding liquid wealth is more costly when the interest rate is lower.

For the same reason, the riskiness of workers' and entrepreneurs' consumption is decreasing in the interest rate. When the interest rate falls, workers and entrepreneurs alike must bear more consumption risk in order to reduce the costs of holding liquid wealth.

Thus, the long-run properties of the economy depend crucially on the equilibrium interest rate and any analysis of a decrease in firms' ability to borrow should take into account the liquidity environment and how the interest rate changes. In a baseline scenario where workers are assumed to be hand-to-mouth consumers who do not save, the unique steady-state equilibrium interest rate is equal to the rate of time preference. Hence, a decrease in firms' ability borrow has no long-run effect on the wedge between the marginal product of capital and the interest rate. In contrast, when workers are allowed to accumulate financial assets, the steady-state interest rate is below the rate of time preference. As in Aiyagari (1994) and other Bewley-like models, this is due to the precautionary motive for savings: if

the interest rate were greater than or equal to the rate of time preference, then workers would seek to accumulate infinite wealth, which in turn would be inconsistent with equilibrium. Unlike in Aiygari (1994), however, the workers' precautionary demand for financial assets in my model opens the door to an important general-equilibrium interaction between the workers and the firms: a decrease in firms' ability to borrow leading to increased investment distortions due to a decreased interest rate.

The financial constraints on firms in this economy arise endogenously, due to the moral hazard that firms face. In particular, if an entrepreneur reneges on a promise to repay, creditors cannot seize all of the entrepreneur's income and assets. As a result, entrepreneurs can pledge to investors only a fraction of their income and undepreciated capital.

Importantly, there are no financial constraints on firms except for those that arise endogenously due to this moral hazard. Firms can issue state-contingent assets and assets of any maturity. Thus, this paper focuses on a single source of financial constraints – the moral hazard of reneging – and all of the results can be traced clearly to this moral hazard. Moreover, the model maintains its tractability even though firms can choose any maturity structure.

The financial constraints due to moral hazard limit the ability of firms both to take advantage of investment opportunities and to diversify their idiosyncratic risks. The accumulation of wealth can alleviate the bite of financial constraints along both of these dimensions.

In the model, a decline in firms' borrowing ability leads to a decrease in firms' supply of financial assets at a given interest rate. This occurs for two reasons. The first is a direct effect: conditional on the firms' capital choices, the maximum amount of financial assets that firms can supply decreases. The second is an indirect effect: conditional on the interest rate, a decrease in firms' ability to borrow implies firms need more liquid wealth to achieve a given level of investment, and because holding wealth is expensive, firms respond by choosing lower capital.

The general equilibrium amplification and spillover effects identified here will be stronger when the decrease in firms' ability to borrow affects a large economy (like the U.S.) or an economy that is less open to international financial markets.

In a numerical analysis using firm-level data for U.S. entrepreneurs, I study a reduction in firms' borrowing ability that is consistent with the recent post-crisis decline in real long-term interest rates. I calculate that this decrease in firms' ability to borrow results in significant

increases in investment distortions for firms and consumption riskiness for workers. General-equilibrium amplification plays a meaningful role. For instance, in general equilibrium, TFP losses – the gap between TFP and the TFP that would obtain if firms were unconstrained – increase by 29 percent. If the interest rate were held constant, as in a small open economy, TFP losses would increase only 13 percent.

A decrease in firms' ability to borrow is one potential explanation of the persistent reductions in output and leverage associated with financial crises. Reinhart and Reinhart (2010) find that financial crises are followed by large and long-lasting declines in the ratio of domestic bank credit to GDP. For example, four Asian emerging-market countries hit by the 1997 crisis required a full decade or more before the ratio of bank credit to GDP bottomed. Across the four countries, bank credit to GDP one decade after the crisis was, on average, 46 percentage points lower than its highest level around the crisis. Rajan and Zingales (2003) analyze measures of financial development such as the ratio of bank deposits to output and the ratio of equity issuance to capital formation. By such measures, countries were less financially developed in 1980 than in 1913, before the Great Depression, and only surpassed their 1913 levels of financial development in the 1990s. IMF (2010) finds that seven years after a financial crisis, output and total factor productivity are, respectively, about 10 and 4 percentage points below levels predicted from a pre-crisis trend.

Further results. The core of my analysis focuses on the long-run response of the economy to an adverse shock to the financial constraints faced by firms. While this seems to be one of the key aspects of financial crises, there are other changes in the economy that may accompany a financial crisis. For example, the economy appears to be subject to more volatility at both the macro and the micro level. My model provides a useful framework for understanding either how these changes could emerge endogenously, or how they could be amplified and propagated in the economy because of the aforementioned general-equilibrium interaction through asset markets.

Consider an increase in the volatility of idiosyncratic labor-income risk. Storesletten, Telmer and Yaron (2004) provide evidence that this volatility increases during recessions.¹ An increase in labor-income risk leads consumers to increase their savings, resulting in a decrease in the interest rate. Aggregate capital increases. These predictions are present

¹Krueger and Perri (2006) provide evidence that this volatility has been increasing at low frequency in recent decades.

in my model and in the model of Aiyagari (1994). However, my model features additional effects: because the lower interest rate leads entrepreneurs to supply more financial assets, there is a decrease in productive efficiency and an increase in the average wedge between the marginal product of capital and the interest rate. These effects are not present in standard macroeconomic models, including models like Aiyagari (1994) that are able to capture an increase in labor-income risk. The analysis for an increase in consumer risk aversion is qualitatively similar.

This shows how a shock to labor-income risk can have an amplified effect on consumption riskiness and can have spillover effects on firms. The model also suggests how increased labor-income risk could arise endogenously from a decrease in firms' ability to borrow. For example, suppose that a worker's idiosyncratic labor-income risk originates from the idiosyncratic risk that his employer faces.² An extension of the model along these lines could then explain the increase in workers' unemployment or labor-income risk as a direct consequence of the endogenous increase in the riskiness of firms.³

Related literature. This paper is related to the long literature on the macroeconomic effects of financial frictions that dates back to the work of Keynes and Fisher. Since then, economists have developed models that emphasize the importance of liquid wealth for overcoming borrowing constraints. Early work, such as Evans-Jovanovic (1989), featured a static model in which liquid wealth was exogenous and wealth accumulation played no role. Kiyotaki-Moore (1997) incorporated a dynamic framework, but one without wealth accumulation.

In contrast, more recent work by Holmstrom-Tirole (1998) and Albuquerque-Hopenhayn (2004) features a crucial role for wealth accumulation as a means of overcoming financial frictions. Holmstrom-Tirole (1998), like this paper, emphasize general-equilibrium limits to liquidity accumulation.

Wealth accumulation also plays a central role in recent contributions to the entrepreneurship literature in macroeconomics, such as Quadrini (2000), Angeletos-Calvet (2006), Cagetti-DiNardi (2006), Buera (2009), Banerjee-Moll (2010), Buera-Kaboski-Shin (2010)

²This could be due to search frictions in the labor market or firm-specific human-capital investments that tie the fate of a worker to the fate of his employer.

³However, if workers' labor-income is tied to the fate of their employer, and the employer issues state-contingent assets, as in this model, one would have to take into account the workers' ability to reduce their idiosyncratic risk by short-selling the equity-like securities. In practice, workers do not seem to pursue this strategy (Massa and Simonov 2006), although there appear to be large gains from doing so.

and Moll (2010). Indeed, for some of these papers, the empirical motivation is study how financial frictions shape the wealth distribution, rather than how the distribution of liquid wealth affects financial frictions.

This paper also builds on previous work on limited enforcement and endogenous solvency constraints, including Kehoe-Levine (1993), Alvarez-Jermann (2000), Lorenzoni-Walentin (2007), Lorenzoni (2008) and Rampini-Viswanathan (2010a,b). A common theme of this literature that also appears in this paper is that endogenous solvency constraints limit risk sharing and depress the interest rate. One paper in this literature, Cooley-Marimon-Quadrini (2004), studies the aggregate effects of limited enforcement; however, Cooley-Marimon-Quadrini (2004) features a constant interest rate and hence the endogenous changes in the interest rate that are crucial in my analysis are absent in their paper.

The two papers most closely related to this paper are Midrigan-Xu (2010) and Buera-Shin (2010a). Midrigan-Xu (2010)'s primary contribution is to thoughtfully parametrize the borrowing friction and other elements of their model using plant-level data about inputs and revenue for manufacturing firms in Korea and Colombia and aggregate data about leverage for manufacturing firms in these countries. As an exercise, Midrigan-Xu (2010) study a permanent decrease in Korean firms' borrowing ability sufficient to reduce a leverage ratio for Korea to that for Colombia.

One important difference between this paper and Midrigan-Xu (2010) is that Midrigan-Xu (2010) assume a small open economy and hence in their paper the interest rate is exogenous and unaffected by the decrease in firms' ability to borrow. In contrast, the central focus of this paper is the general equilibrium change in the interest rate and its amplification effects. Of course, assuming that Korea is a small open economy may be a suitable modeling choice for their question, whereas this paper contemplates a shock affecting a large economy like the U.S. where general equilibrium is important.

Buera-Shin (2010a) compare the transition dynamics associated with three reform scenarios: removal of exogenous idiosyncratic distortions; removal of exogenous distortions combined with an increase in firms' ability to borrow; and removal of idiosyncratic distortions combined with capital-account liberalization, which increases the interest rate from the general-equilibrium closed-economy rate to the world interest rate. In Buera-Shin (2010a), as in this paper, an increase in firms' ability to borrow is associated with a higher steady-state interest rate and higher measured TFP. However, because the central exercises of their

paper are meant to capture reform scenarios, the main numerical results reflect the variety of economic forces unleashed by the multi-faceted reforms.

One contribution of this paper to the existing entrepreneurship literature is to focus on the role of entrepreneurs as suppliers of financial assets to workers and the spillover effect on workers from a change in the interest rate when entrepreneurs' ability to borrow decreases. This paper derives several theoretical results that provide a clearer understanding of several issues, such as the role of general-equilibrium changes to the interest rate. Also, the financial constraints in this paper arise endogenously from a single friction, the ability of firms to partially renege on their promises. In contrast, many papers in the entrepreneurship literature, such as Angeletos-Calvet (2006) or Cagetti-DiNardi (2006), feature multiple and ad-hoc forms of market incompleteness, including a restriction to safe debt with one-period maturity.

Much of the literature on entrepreneurship is concerned with cross-country differences in financial development. This highlights an alternative interpretation of many of my results, which is to explain how differences in financial development result in differences across countries in the levels of investment distortions and the abilities of workers to smooth consumption.

Lorenzoni and Guerrieri (2010) also study the effects of a credit crunch. In particular, Lorenzoni and Guerrieri (2010) study the response of an economy of Bewley-Aiyagari consumers to a permanent, unexpected tightening of their borrowing constraint. Like this paper, Lorenzoni and Guerrieri (2010) emphasizes precautionary saving and the scarcity of liquid assets. One important difference between these papers is that Lorenzoni and Guerrieri (2010) focus on changes in the borrowing constraints of consumers, whereas this paper emphasizes shocks to the borrowing constraints of entrepreneurs. Although my paper also includes an analysis of shocks to Bewley-Aiyagari consumers, the focus in my paper is on the general-equilibrium interactions between firms and workers. In their model, in contrast, the production side is frictionless.

Jermann-Quadrini (2009), like this paper, emphasizes the macroeconomic effects of financial shocks. Earlier work, such as Bernanke-Gertler (1989) and Bernanke-Gertler-Gilchrist (1999), examined how financial frictions affect short-run fluctuations.

This paper is also related to Mendoza-Quadrini-Ríos-Rull (2009) and Angeletos-Panousi (2010), which attempt to explain current-account dynamics by investigating entrepreneurs'

precautionary saving.

The workers in this paper cannot insure their labor-income risk. This assumption is the focus of the income fluctuations problem in macroeconomics. This paper draws on key contributions to this literature by Bewley (1983), Clarida (1990), Aiyagari (1994) and Chamberlain-Wilson (2000). In the numerical analysis, I find a wedge between the inverse of the discount rate and the interest rate that is much larger than the wedge found in Aiyagari (1994). I show how this difference can be attributed to the inclusion of frictions for firms, which are absent in Aiyagari (1994).

2 The environment

There are two types of agents in the economy, entrepreneurs and workers.

There is only one good in the economy. It is produced by entrepreneurs. The entrepreneurs use the good and labor as inputs for production.

Entrepreneurs

In period t , an entrepreneur produces $F(k_{t-1}, l_t, s_t)$, where k_{t-1} is capital, l_t is labor, and s_t is the entrepreneur's idiosyncratic productivity.

Productivity $s_t \in S$ follows a Markov process with transition function Q . S is discrete. The probability that an entrepreneur with productivity s_t obtains a productivity s_{t+1} tomorrow is given by $Q(s_t, s_{t+1})$. I assume that Q is monotone in the usual sense.⁴

In period t , an entrepreneur has a history of idiosyncratic productivity shocks $s^t = \{s_0, s_1, \dots, s_t\}$ and invests $k(s^t)$ in the production technology. In period $t+1$, the entrepreneur learns productivity s_{t+1} . The entrepreneur then hires labor $l(s^{t+1})$. The total output produced in period $t+1$ is $F(k(s^t), l(s^{t+1}), s_{t+1})$.

I assume that $F(k, l, s)$ has the standard neoclassical properties except for constant returns to scale.⁵ In place of constant returns to scale, I assume that F has decreasing returns to scale in capital and labor, consistent with the notion of limited "span of control" for

⁴The monotonicity of Q means that $\sum_{s \in S} h(s)Q(s_-, s)$ is non-decreasing in s_- for any non-decreasing function $h : S \rightarrow \mathbb{R}$. Since S is discrete, it is immediate that the Feller property also holds.

⁵That is, $F_k > 0$, $F_l > 0$, $F_{kk} < 0$ and $F_{ll} < 0$, for all $k > 0$, $l > 0$ and all $s \in S$. In addition, the Inada conditions hold for capital and labor. The Inada conditions for capital are: $\lim_{k \rightarrow 0} F(k, l, s) = \infty$ for all $l > 0, s > 0$; and $\lim_{k \rightarrow \infty} F(k, l, s) = 0$ for all l and all s . Moreover, $F(k, l, 0) = 0$.

entrepreneurs (Lucas 1978). I make the technical assumption that F is strictly concave in capital and labor and that $F(\cdot, s)$ is increasing in s .

In period $t + 1$, the value of undepreciated capital is $(1 - \delta)k(s^t)$.

The labor market is competitive. The wage rate in period $t + 1$ is given by ω_{t+1} . Since the economy features only idiosyncratic shocks, the process for wages will be deterministic. Entrepreneurs' output net of labor costs is $F(k(s^t), l(s^{t+1}), s_{t+1}) + (1 - \delta)k(s^t) - \omega_{t+1}l(s^{t+1})$. In period t , the entrepreneur can correctly forecast next-period output net of labor costs $f(k(s^t), s_{t+1})$ for any capital choice $k(s^t)$ and productivity s_{t+1} . Static profit maximization for the entrepreneur requires

$$f(k(s^t), s_{t+1}) = \max_l F(k(s^t), l, s_{t+1}) + (1 - \delta)k(s^t) - \omega_{t+1}l$$

The assumption of strict concavity in capital and labor is a necessary and sufficient condition for the period- t expected marginal product of capital, $E[f_k(k(s^t), s_{t+1})|s_t]$, to be strictly decreasing in k .⁶

The entrepreneur can access financial markets by trading state-contingent promises that pay out conditional on the realization of productivity. At history s^t , the entrepreneur sells $d(s^{t+1})$ Arrow-Debreu securities that represent a promise to pay one if state s_{t+1} is realized. The unit price in period t is $p(s^{t+1})$.

Although the promises are state-contingent, markets are incomplete because entrepreneurs can renege on payment. In particular, if an entrepreneur reneges, the most that creditors can seize is a fraction $\theta \leq 1$ of the entrepreneur's output net of labor costs. When an entrepreneur reneges, the unmet portion of the entrepreneur's debt is erased. (When the entrepreneur is allowed to trade promises due in more than one period, all future promises by the entrepreneur are also erased). Given this, the entrepreneur will keep his promise to pay $d(s_{t+1})$ if and only if

$$d(s_{t+1}) \leq \theta f(k(s^t), s_{t+1}) \tag{1}$$

This constraint can limit both the ability to borrow and the ability to diversify risks.

In Appendix B, I show that it is without loss of generality to restrict attention to one-period promises in this environment. Any allocation that can be implemented with any maturity structure can also be implemented with one-period promises. This generality

⁶For a proof, please see the appendix.

contrasts with many papers, such as Angeletos-Calvet (2006) or Cagetti-DiNardi (2006), that exogenously impose a restriction to one-period debt.⁷

By allowing state-contingent promises, I am able to focus exclusively on a single financial friction, the possibility of reneging. In reality, entrepreneurs can make state-contingent promises in a variety of ways. Through default or renegotiation, putatively non-contingent debt becomes state-contingent. Entrepreneurs can also sell equity stakes in their businesses. Once an entrepreneur is no longer the sole owner of the business, the entrepreneur may be able to separate payments to herself and to other equity holders by paying herself a salary.

It should be noted that with $\theta = 1$, the entrepreneur's financial constraint can still bind and limit the financial decisions of the entrepreneurs. However, it will be shown later that with $\theta \geq 1$, an entrepreneur invests in capital as if there were no financial constraint.

The entrepreneur's budget constraint at state s^t is:

$$c(s^t) + k(s^t) - \sum_{s^{t+1}|s^t} p(s^{t+1})d(s^{t+1}) \leq f(k(s^{t-1}), s_t) - d(s^t) \quad (2)$$

The entrepreneur's choices must also satisfy a no-Ponzi condition.

An entrepreneur's liquid wealth in period $t + 1$ is defined as $w(s^{t+1}) = f(k(s^t), s_{t+1}) - d(s^{t+1})$.

Workers. Workers have a stochastic endowment of labor. In period t , a worker's labor endowment is z_t and her labor income is $\omega_t z_t$, where ω_t is the wage.

Labor productivity z_t follows a Markov process with transition function Q_Z . I assume that Q_Z is monotone and has the Feller property.

Consistent with Bewley (1983) and Aiyagari (1994), workers can only trade a safe asset; they cannot insure against labor endowment shocks using assets with a payoff linked to their labor process.⁸

Again, because there are no aggregate shocks, the path for the gross interest rate, $\{R_t\}_{t=0}^\infty$, will be deterministic.

The budget constraint of the workers is:

⁷Of course, in reality, maturity structure does matter, as highlighted by Broner-Lorenzoni-Schmukler (2008) and the recent financial crisis. One reason maturity structure matters is that, when the interest rate is stochastic, non-contingent bonds of different maturities facilitate risk sharing, as emphasized by Angeletos (2002) and Buera-Nicolini (2004). These concerns are absent here, however, because the environment in this paper features state-contingent debt and a deterministic path for the interest rate.

⁸This can be endogenized as in Allen (1985) and Cole-Kocherlakota (2001), which study environments where workers' income shocks are unobservable and they can privately store resources.

$$c(z^t) + \frac{1}{R_t}a(z^t) = a(z^{t-1}) + w_t z_t \quad (3)$$

Workers face a borrowing limit:

$$a(z^t) \geq \underline{a}$$

As in Aiyagari (1994), this could be a natural or ad-hoc borrowing limit.

Financial market. Each firm takes a set of state-contingent positions in the financial market.

Consider a firm with history s^t . The firm makes promises $d(s^{t+1})$ with payout contingent on the realization of s_{t+1} . At t , it is known that the expected amount of these payouts in $t + 1$ will be

$$E[d(s^{t+1})|s^t]$$

where the expectation is integrating over the realization of s_{t+1} .

Of course, in the economy, there are firms with different histories. Because the payouts are contingent on idiosyncratic shocks, by a law of large numbers, the total payout from the entrepreneurs is certain even before the idiosyncratic uncertainty is realized. At t , it is known that the total payouts by entrepreneurs in $t + 1$ will be

$$E[d(s^{t+1})]$$

where the expectation is integrating over all histories s^{t+1} .

The financial market could be organized as follows: firms sell claims that are contingent on the firms' idiosyncratic shocks and consumers buy a portfolio of claims; since the consumers will not want to be exposed to entrepreneurs' idiosyncratic risk, they will buy a diversified pool of claims that amount to a safe asset. Alternatively, one could imagine a financial intermediary pooling claims sold by firms and selling a safe asset backed by the claims to workers.

Workers with history z^t purchase assets requiring payout $a(z^t)$ next period. Hence, the financial assets of workers require a period $t + 1$ repayment equal to:

$$E[a(z^t)]$$

Hence, market clearing for financial assets requires:

$$E[d(s^{t+1})] = E[a(z^t)]$$

Because the entrepreneur's shocks are idiosyncratic, a no-arbitrage condition requires that $p(s^{t+1}) = \frac{1}{R_t}Q(s_t, s_{t+1})$. Thus, the only interesting asset price in the model is the interest rate.

Preferences.

The preferences of entrepreneurs and workers are given by

$$\sum_{t=0}^{\infty} \beta^t u(c_t) \tag{4}$$

with $u' > 0$ and $u'' < 0$, where c_t is contingent upon the history s^t for entrepreneurs and the history z^t for consumers.

The model emphasizes the creation and allocation of liquidity. Liquidity refers to instruments that enable an agent to transfer wealth to the times and occasions when it is needed. There are two forms of liquidity in the model: financial assets and physical capital. Workers can invest in financial assets, but they do not have access to a technology in which to productively invest physical capital. Entrepreneurs can invest in physical capital, making them natural suppliers of financial assets to the workers. Nonetheless, both entrepreneurs and workers need liquidity, to smooth consumption in the case of workers, and to take advantage of investment opportunities in the case of entrepreneurs.

3 Steady-state equilibrium characterization

In this environment, an equilibrium is a deterministic interest-rate and wage sequence $\{r_t, w_t\}_{t=0}^{\infty}$ and collections of plans for entrepreneurs $\{k(s^t), l(s^{t+1}), d(s^t)\}$ and for consumers $\{c(z^t), a(z^t)\}$ such that:

- (1) the plans $\{k(s^t), l(s^{t+1}), d(s^t)\}$ maximize the utility of each entrepreneur;
- (2) the plans $\{c(z^t), a(z^t)\}$ maximize the utility of households;

(3) the bond market clears:

$$E[d(s^{t+1})] = E[a(z^t)]$$

(4) the labor market clears:

$$E[l(s^t)] = E[z_t]$$

3.1 The entrepreneur's problem

In this section, I formulate the entrepreneur's problem recursively and analyze the first-order conditions. I postpone until Section 4 a detailed discussion of conditions under which the value function is well defined and how the transition functions are specified.

To fix notation, let ζ be the distribution of entrepreneurs over liquid wealth w and current productivity s_- . Denote by φ the distribution of consumers over assets a and current productivity z . Let $\mu = (\zeta, \varphi)$ be the distributions for both entrepreneurs and consumers. Write $R(\mu)$ for the market-clearing interest rate given distribution μ .

The entrepreneur's problem can be written recursively as:

$$V(w, s_-; \mu) = \max_{k, \{d_s\}} u\left(w + \frac{1}{R(\mu)} E[d_s | s_-] - k\right) + \beta E[V(f(k, s) - d_s, s; \mu') | s_-]$$

subject to

$$d_s \leq \theta f(k, s)$$

and the transition function for μ . Equivalently, the entrepreneur can choose w_s directly, allowing the problem to be written as:

$$V(w, s_-; \mu) = \max_{k, \{w_s\}} u\left(w + \frac{1}{R(\mu)} E[f(k, s) - w_s | s_-] - k\right) + \beta E[V(w_s, s; \mu') | s_-] \quad (5)$$

subject to

$$w_s \geq (1 - \theta) f(k, s) \quad (6)$$

and the transition function for μ .

Denote the Lagrange multiplier on the financial constraint (6) for state s by ϕ_s . We will say that the financial constraint for state s binds if $\phi_s > 0$.

The first-order condition for capital is:

$$u'(c)\left(\frac{1}{R(\mu)}E[f_k(k, s)] - 1\right) - (1 - \theta)E[\phi_s f_k(k, s)] = 0 \quad (7)$$

The first-order condition for capital immediately implies that if any of the financial constraints are binding, then capital k will be strictly less than the level of capital that maximizes expected output next period, discounted at the market interest rate, less investment. This latter capital level will be a useful benchmark throughout the paper:

$$k^u(R, s_-) \equiv \max_k \frac{1}{R}E[f(k, s)|s_-] - k$$

The first-order condition for wealth tomorrow in state s is:

$$-u'(c)\frac{1}{R} + \beta V_w(w_s, s; \mu') + \phi_s = 0 \quad (8)$$

and the envelope condition is:

$$V_w(w, s_-; \mu) = u'(c) \quad (9)$$

Consider any next-period idiosyncratic states s for which the financial constraint is not binding in the current period. Conditions (8) and (9) imply that next-period consumption will be equal across these states. Also, next-period consumption in these states will be lower than this-period consumption if $\beta R < 1$.

Now consider any next-period idiosyncratic states s for which the financial constraint is binding in the current period. Entrepreneurs would like to transfer wealth away from these next-period states, but the binding financial constraint prevents them from doing so. Next-period consumption in these states will be higher than in any states for which the constraint is not binding.

Define the rate of time preference ρ by $\beta = (1 + \rho)^{-1}$. A net interest rate lower than the rate of time preference provides an incentive for the entrepreneur to have a downward-tilted consumption path. For a given investment k , however, the downward-tilting may not be possible; the entrepreneurs' reneging constraint requires that she have a claim to at

least $(1 - \theta)$ share of tomorrow’s output net of labor costs and undepreciated capital. This trade-off between downward-tilting the consumption path and investing enough to maximize expected discounted profits is central to the entrepreneur’s problem.

3.2 The “bite” of entrepreneurs’ financial constraints

Throughout the rest of the paper, whenever I refer to the marginal product of capital, I mean the expected marginal increase in output net of labor costs, $E[f_k(k(s^t), s_{t+1})|s^t]$. This differs from the realized marginal product of capital, $f_k(k(s^t), s_{t+1})$. When there are no financial constraints, it is $E[f_k(k(s^t), s_{t+1})|s^t]$ that will be equal to the interest rate and hence also equal across entrepreneurs. Even without financial constraints, the realized marginal product of capital will still differ across entrepreneurs if productivity is stochastic.

This paper emphasizes a positive analysis of the equilibrium impact of entrepreneurs’ borrowing constraints. In particular, the first-order condition for capital makes it clear that in this setting, there may be differences between the interest rate and an entrepreneur’s marginal product of capital, and between the marginal product of capital of different entrepreneurs.

A wedge between the marginal product of capital and the interest rate is a measure of the finance premium firms would be willing to pay for an additional unit of borrowing. One interesting statistic is the mean wedge across entrepreneurs. If the mean wedge is positive, then there is a kind of conditional under-investment; at the equilibrium interest rate, firms would like to borrow and invest more. It should be noted, however, that despite this conditional under-investment, aggregate capital may be greater than if there were no frictions, since the financial frictions may depress the interest rate.

It is also interesting to study how the marginal product of capital differs across entrepreneurs. If the marginal product of capital differs across entrepreneurs, there is a loss of productive efficiency, in the sense that a social planner could achieve higher next-period output with same amount of aggregate capital by re-allocating current-period capital across entrepreneurs. One useful way to capture this loss of productive efficiency is through measured TFP. When technology is Cobb-Douglas, with $F(k, l, s) = sk^a l^b$, it is possible to compute TFP in this economy for any allocation of capital to entrepreneurs $k(w, s_-)$ and any distribution $\zeta(w, s_-)$ over wealth and productivity.

Lemma 1 *Suppose technology is Cobb-Douglas. Then aggregate output is given by*

$$Y = ZK^aL^b$$

where Z is measured TFP

$$Z = \left[\sum_{s_- \in S} E[s^{\frac{1}{1-b}} | s_-] \int \left(\frac{k(w, s_-)}{K} \right)^{\frac{a}{1-b}} \zeta(w, s_-) dw \right]^{1-b} \quad (10)$$

and K and L are aggregate capital and labor.

With Cobb-Douglas technology, measured TFP is a function of the shares of aggregate capital deployed by firms with given wealth and productivity. Maximizing TFP requires that the marginal product of capital, $E[f_k(k, s) | s_-]$, be equal across firms. Hence, if the wealth distribution is not degenerate, maximizing measured TFP requires that capital not depend on liquid wealth w .

Definition 2 *First-best TFP is defined as*

$$\overline{TFP} = \max_{x(w, s_-)} \left[\sum_{s_- \in S} \int E[s^{\frac{1}{1-b}} | s_-] x(w, s_-)^\alpha \zeta(w, s_-) dw \right]^{1-b}$$

subject to

$$\sum_{s_- \in S} \int x(w, s_-) \zeta(w, s_-) dw \leq 1.$$

Denote $\hat{q}(s_j) = \int \zeta(w, s_j) dw$. First-best TFP is given by

$$\overline{TFP} = \left[\sum_{s_- \in S} \hat{q}(s_-) E[s^{\frac{1}{1-b}} | s_-] \left(\frac{E[s^{\frac{1}{1-b}} | s_-]^{\frac{1}{1-\alpha}}}{\sum_{s_j \in S} E[s^{\frac{1}{1-b}} | s_j]^{\frac{1}{1-\alpha}} \hat{q}(s_j)} \right)^\alpha \right]^{1-b} \quad (11)$$

where $\alpha = \frac{a}{1-b}$.

Note that it is possible to have a wedge between each entrepreneurs' marginal product of capital and the interest rate and still have measured TFP equal to first-best TFP, so long as there is no dispersion in the marginal product of capital. Thus, this paper differentiates between the average wedge between the marginal product of capital and the interest rate,

which can be considered a measure of conditional under-investment, and the dispersion of this wedge across entrepreneurs, which can be considered an indicator of productive inefficiency. This approach to studying TFP is similar to the analysis of TFP in Midrigan-Xu (2010) and Moll (2010).

3.3 Steady state definition

For a given interest rate and wage, the solution to the entrepreneur's problem (5) is associated with optimal policies for capital and next-period state-contingent wealth, $\{k^*(w, s_-), \{w_s^*(w, s_-)\}_{s \in S}\}$. Under certain conditions discussed below, the entrepreneur's problem is well-defined and the optimal policies are continuous and single-valued. The single-valued optimal policies, together with the transition function Q , define a new transition function P_E . Given a distribution $\zeta(w, s_-)$ over entrepreneurs' wealth and productivity today, the transition function P_E determines the distribution tomorrow.

To be more precise, consider the measurable spaces (W, \mathcal{W}) and (S, \mathcal{S}) . Define $W = [0, \hat{w}]$ and \mathcal{W} as the collection of all Borel sets that are subsets of W . The definition of \hat{w} will be more precise below. Let $(X, \mathcal{X}) = (W \times S, \mathcal{W} \times \mathcal{S})$ be the product space. Recall that Q is the transition function on (S, \mathcal{S}) . We can define the transition function P_E as

$$P_E((w, s_-), A \times B) = \sum_{s \in B} Q(s_-, s) \mathbb{I}\{w_s^*(w, s_-) \in A\} \quad (12)$$

for all $w \in W, s_- \in S, A \in \mathcal{W}, B \in \mathcal{S}$.

For any probability measure ζ on $(W \times S, \mathcal{W} \times \mathcal{S})$, the operator mapping this probability measure into next period's probability measure is given by:

$$(T^*\zeta)(A) = \int P_E(x, A) \zeta(dx) \quad (13)$$

for all $A \in \mathcal{W} \times \mathcal{S}$. An invariant distribution under P_E is a fixed point of (13).

One can analogously define a transition function P_C from the transition function for consumer productivity, Q_Z , and the optimal policy of consumers, $a^*(a, z)$, for given R and w .

Now a steady-state equilibrium can be defined.

Definition 3 *A steady-state equilibrium consists of: (i) an equilibrium with $R_t = R$ and*

$\omega_t = \omega$ for all t ; (ii) a distribution ζ over the wealth and productivity of entrepreneurs that is invariant with respect to P_E ; (iii) a distribution φ over wealth and productivity of consumers that is invariant with respect to P_C .

4 Steady state comparisons

This section studies the steady-state properties of the economy, in partial and general equilibrium. The long-run effects of a decrease in firms' ability to borrow are studied by comparing an initial steady-state with the steady-state that arises when firms' moral hazard problem worsens.

In the remainder of this section, I focus on two special cases: (i) constant productivity, and (ii) productivity that is i.i.d. across time. For the case of constant productivity, the entrepreneurs' steady-state marginal product of capital can be calculated in closed form for given R and ω (Proposition 4). Thus, I can study how the wedge between the marginal product of capital and the interest rate varies with the ability to borrow θ and the interest rate R . Moreover, the general equilibrium steady-state can be partially characterized, allowing insight into the extent to which entrepreneurs escape financial frictions by accumulating liquid wealth and how this depends on workers' demand for financial assets.

A key result is Proposition 7, which shows that, without consumer demand for financial assets, the general-equilibrium steady-state interest rate equals the rate of time preference, whereas with consumer demand for financial assets, the equilibrium interest rate must be less than the rate of time preference. It has long been recognized that an interest rate less than entrepreneurs' rate of time preference is important if financial frictions are to matter in the long run. Thus, some papers, such as Lorenzoni and Walentin (2007), assume a difference between the discount factors of entrepreneurs and consumers in order to achieve an interest rate less than entrepreneurs' rate of time preference. In contrast, this paper assumes a common discount factor for entrepreneurs and consumers and arrives endogenously at such an interest rate. The key is consumers' demand for liquidity, which lowers the interest rate.

Having an endogenous wedge between the steady-state interest rate and the entrepreneurs' rate of time preference allows consideration of how shocks – such as a decrease in firms' ability to borrow – affect the wedge. In contrast, in papers where the wedge is exogenous and comes from an ad-hoc assumption of entrepreneurs being less patient than other

households, shocks such as a decrease in firms' ability to borrow will not affect this wedge.

For the case of i.i.d. productivity, I will show that, in steady state, the average marginal product of capital is greater than the interest rate and the variance in the marginal product of capital is positive if and only if the interest rate is below the rate of time preference and moral hazard is sufficiently severe ($\theta < 1$).

4.1 Constant productivity

Productivity is constant if the productivity transition function Q is the identity matrix. If Q has more than one element, the economy features a non-degenerate distribution over entrepreneurial productivity, with the feature that each entrepreneurs' productivity is constant over time.

When productivity is constant, entrepreneurs' long-run behavior can be characterized in closed form for any utility function and any production function that satisfy the basic technical assumptions described above. Also, though this proposition and some of the results that follow are partial-equilibrium results that take the interest rate as given, they can also be given a general-equilibrium interpretation, by assuming consumers are "CARA-normal."⁹

Proposition 4 *Suppose that productivity is constant over time. In steady state:*

(i) *the marginal product of capital equals the harmonic mean of R and $\frac{1}{\beta}$, where the weights are θ and $(1 - \theta)$, respectively. That is,*

$$R \leq f_k(k, s) = \frac{1}{\theta \frac{1}{R} + (1 - \theta)\beta} \leq \frac{1}{\beta} \quad (14)$$

where the inequalities are strict and the financial constraint is binding if and only if $R\beta < 1$.

(ii) *the supply of financial assets is decreasing in the interest rate.*

According to this proposition, if the interest rate is less than the rate of time preference, then in steady-state, the financial constraint will bind for every entrepreneur.

This result builds on the observation that in steady state with constant-productivity entrepreneurs, each entrepreneur's wealth and consumption must be constant over time.

⁹If consumers have constant absolute risk aversion (CARA) preferences and experience Gaussian shocks, consumers' steady-state demand for assets is elastic at an interest rate R that depends on consumers' risk aversion and the variance of the shocks, as shown in Angeletos-Calvet (2006). With positive variance of the shocks, this interest rate satisfies $R\beta < 1$.

This intuitive observation is proved in Lemma 6 below. With this observation, Proposition 4 then follows directly from the first order conditions (7) and (8).

In particular, the first-order condition for wealth (8) implies that

$$\phi = u'(c) \frac{1}{R} - \beta u'(c_s) \quad (15)$$

Thus, in steady state, $\phi > 0$ and the financial constraint is binding if and only if $\beta R < 1$.

Now, suppose an entrepreneur increases capital at the margin, holding debt constant. The first-order condition for capital implies

$$-u'(c) + \beta u'(c_s) f_k(k, s) + \phi \theta f_k(k, s) = 0 \quad (16)$$

Increasing capital at the margin results in a decrease in consumption today, with cost $u'(c)$, and an increase in consumption tomorrow, with benefit $\beta u'(c_s) f_k(k, s)$. Increasing capital also serves to loosen the financial constraint, with benefit $\phi \theta f_k(k, s)$. Combining (15) and (16) with the observation that consumption is constant over time in steady state, we obtain (14).

This result can also be understood by thinking about the dynamic accumulation of wealth. As will be shown in the proof for Lemma 6, if the interest rate is less than the rate of time preference, entrepreneurs with capital below [above] the level of capital defined in (14) will have higher [lower] next-period wealth and capital.

The expression for the marginal product of capital in (14) can be thought of as a user cost of capital, as in Jorgenson (1963) and Rampini and Viswanathan (2010b). Because output net of labor costs and undepreciated capital are not fully pledgeable to investors, entrepreneurs need to hold liquid wealth in order to take advantage of their investment opportunity each period, and if the interest rate is less than the rate of time preference, holding liquid wealth is expensive.

The closed-form solution for steady-state marginal product of capital allows a clear understanding of how investment distortions depend on the equilibrium interest rate, as described in the following corollary.

Corollary 5 *Suppose productivity is constant. In steady state,*

(i) There is a wedge between the marginal product of capital and the interest rate if and only if $\theta < 1$ and $\beta R < 1$.

(ii) If $\theta < 1$, the ratio of the marginal product of capital to the interest rate is decreasing in R .

(iii) If $\beta R < 1$, the ratio of marginal product of capital to the interest rate is decreasing in θ .

(iv) There is no dispersion in the marginal product of capital.

Part (i) of the corollary shows that in the case of constant productivity, for there to be a positive average finance premium in the long-run, it is required both that moral hazard is sufficiently severe ($\theta < 1$) and that the interest rate is below the rate of time preference. Part (ii) and (iii) of the corollary can be interpreted in terms of the user cost of capital. A lower interest rate implies that it is more expensive to hold liquid wealth; thus, entrepreneurs are willing to bear greater investment distortions to economize on the costs of holding liquid wealth when the interest rate is lower. Likewise, greater moral hazard, or a lower value for θ , implies that more liquid wealth is needed to undertake a given level of investment; if holding liquid wealth is expensive, greater moral hazard increases the user cost of capital.

With constant productivity, there is no dispersion in the marginal product of capital. Hence, if the production technology is Cobb-Douglas, there are no endogenous losses in measured TFP. The absence of capital misallocation and the irrelevance of the interest rate for steady-state TFP is driven by two special assumptions: (i) unchanging productivity; and (ii) the equality across entrepreneurs of the moral hazard parameter or, equivalently, the ability to pledge future income and assets. Dropping either of these assumptions will give rise to capital misallocation and steady-state TFP losses if the interest rate is less than the rate of time preference, as shown in Proposition 8 (for the case of heterogeneity in entrepreneurs' ability to pledge future income and assets) and Corollary 12 (for the case of productivity shocks). Moreover, with heterogeneity in θ , Proposition 8 shows that TFP is increasing in the interest rate, if the interest rate is less than the rate of time preference. In the calibration, which features productivity shocks, TFP is also increasing in the interest rate.

In the case of constant productivity, it is possible to fully characterize the invariant distributions for a given interest rate and show how the uniqueness or multiplicity of the invariant distribution for a given interest rate depend on whether the interest rate is equal to the rate of time preference. This is a helpful step in characterizing the supply of financial assets and moving toward the general equilibrium results below.

Lemma 6 *Suppose entrepreneurs' productivity is constant.*

(i) *For $R < \frac{1}{\beta}$, there is a unique invariant distribution $\zeta(w, s_-)$. In the invariant distribution, the wealth of entrepreneurs with productivity s is $w = (1-\theta)f([f_k(\cdot, s)]^{-1}(\frac{1}{\theta\frac{1}{R}+(1-\theta)\beta}), s)$.*

(ii) *For $R = \frac{1}{\beta}$, a distribution over wealth and productivity is an invariant distribution if and only if the measure of entrepreneurs with wealth greater than or equal to $(1-\theta)f(k^u(R), s)$ is equal to one.*

(iii) *In any invariant distribution, each entrepreneur's wealth is constant over time.*

As shown above, if the interest rate is less than the rate of time preference, all entrepreneurs will be constrained and have positive capital and hence the net supply of financial assets by entrepreneurs will be positive. This leads to the next proposition, which illustrates the importance of considering workers' demand for financial assets when studying how a decrease in firms' ability to borrow affects workers and firms. I contrast general equilibrium in the model with the equilibrium that would obtain if workers were hand-to-mouth consumers. Define workers as hand-to-mouth consumers if each period they simply consume their labor income, because they are not allowed to or do not wish to save.

Proposition 7 (i) *Suppose workers are hand-to-mouth consumers. In general-equilibrium steady state, $\beta R = 1$ and there is no wedge between the marginal product of capital and the interest rate.*

(ii) *Suppose consumers can save. Then $\beta R < 1$ and there is a positive wedge between marginal product of capital and the interest rate in general-equilibrium steady-state.*

The intuition for this result is straightforward. With hand-to-mouth workers, financial market equilibrium requires that net supply of financial assets by firms be zero. But we have already seen that $\beta R < 1$ implies that all firms are constrained and hence the net supply of financial assets is positive. Moreover, there cannot be a general-equilibrium steady-state with $\beta R > 1$, since firms would have perpetually increasing wealth. In contrast, with $\beta R = 1$, there are multiple invariant distributions consistent with zero net supply of financial assets by entrepreneurs. In particular, any distribution of wealth satisfying the lower-bound on wealth in part (ii) of the previous lemma and such that aggregate entrepreneurial wealth equals aggregate unconstrained entrepreneurial output is an invariant distribution with zero net supply of financial assets. Hence, in general-equilibrium steady state, $\beta R = 1$ and there is no wedge between the marginal product of capital and the interest rate.

In contrast, when workers can save, $\beta R = 1$ can no longer be an equilibrium; a well-known result about the income fluctuations problem is that, when $\beta R = 1$, workers' financial assets will converge to infinity almost surely (e.g., Aiyagari 1994). Hence, general-equilibrium steady state will feature $\beta R < 1$ and, by the previous corollary, a wedge between the marginal product of capital and the interest rate.

Importantly, to shift the general equilibrium from $\beta R = 1$ to $\beta R < 1$, it is not necessary that workers' demand for financial assets be such that savers' assets converge to infinity when $\beta R = 1$. All that is required is a sufficiently large worker demand for financial assets when the interest rate equals the rate of time preference; the threshold is given by $\theta \sum_{s_- \in S} f(k^u(\frac{1}{\beta}, s_-), s_-) \hat{q}(s_-)$ where \hat{q} is the distribution over entrepreneurial productivity.

An implication of this proposition is that, with hand-to-mouth workers, a decrease in the ability of firms to borrow has no long-run effect on the interest rate or on the wedge between the marginal product of capital and the interest rate. Allowing workers to save opens the door to the predictions of the model that a decrease in firms' ability to borrow leads to increased distortions in firms' investment and a decreased interest rate.

Nonetheless, at this point, we cannot answer theoretically the question of how a decrease in firms' ability to borrow affects firms and workers in general equilibrium. Although we have characterized firms' supply of financial assets in closed form for any production technology and utility function satisfying the basic technical assumptions, much less can be said about the workers' problem at this level of generality. This is because of the well-known theoretical difficulties of the income fluctuations problem. Indeed, workers' steady-state demand for financial assets may not be everywhere increasing in the interest rate, especially since the equilibrium wage is decreasing in the interest rate. Hence, a further exploration of general equilibrium response of the economy to a decrease in firms' ability to borrow will be postponed until the calibrated example.¹⁰

¹⁰However, if we make assumptions not about the primitive parameters of the workers' problem but about the steady-state demand for financial assets (an equilibrium object), then more can be said about a close variant to this economy. Consider this alternate economy: the entrepreneurs' technology does not depend on labor input and the workers' wage is ϖ irrespective of the interest rate. The later would be the case if workers were themselves self-employed with productivity ϖ or if firms with a technology ϖL , where L is the labor input, are added to the model. If workers' steady-state demand for financial assets, $\int a^*(a, y; \varpi, R) \varphi(a, y) da dy$, is increasing in the interest rate, then: (i) the wedge between entrepreneurs' marginal product of capital and the interest rate is decreasing in θ ; and (ii) the interest rate and workers' holdings of financial assets are increasing in θ .

Note that if the workers have constant relative risk-aversion (CRRA) utility and their borrowing constraint is a multiple of the wage, as with the natural borrowing constraint, then workers' steady-state demand for

4.1.1 Heterogeneity in the ability to pledge future cash flows

Throughout the paper, I abstract from heterogeneity in entrepreneurs' ability to pledge future cash flows. In particular, I assume that the moral hazard parameter θ is equal for all entrepreneurs. However, it is worth considering what happens when there is cross-sectional heterogeneity in the ability to pledge future cash flows. One reason that θ may vary across entrepreneurs is differences in asset tangibility, a topic which has recently received considerable attention in macroeconomics and finance (McGrattan and Prescott 2010, Rampini and Viswanathan 2010b).

Hence, in this section, I allow the moral hazard parameter θ to differ across entrepreneurs. Denote the set of possible values for θ by $\Theta \subset [0, 1]$, where Θ is a discrete set. Define $g(\theta, s) : \Theta \times S \rightarrow [0, 1]$ as the joint distribution function for the pledgeability parameter θ and productivity s .

Given the possibility of heterogeneity in entrepreneurs' ability to pledge future income and assets, one might want to be flexible in how ability to pledge is related to productivity. Hence, I allow any joint distribution function $g(\theta, s)$.

When the distribution of θ is non-degenerate (i.e., there is positive cross-sectional variance in θ), there will be capital misallocation if the interest rate is less than the rate of time preference, even though entrepreneurs' productivity is constant. This is because, all else equal, firms with a higher ability to pledge will invest more than firms with a lower ability to pledge. A decrease in the interest rate biases the composition of output toward entrepreneurs with a high ability to pledge future income and assets. Moreover, with Cobb-Douglas technology, measured TFP is increasing in the interest rate. This is formalized in the following proposition.

Proposition 8 *Suppose there is positive cross-sectional variance in the pledgeability parameter θ . In steady state, for any distribution function $g(\theta, s)$ over pledgeability and productivity:*

- (i) *There is dispersion in the marginal product of capital if and only if $\beta R < 1$.*
- (ii) *If technology is Cobb-Douglas, TFP is increasing in the interest rate for $\beta R < 1$.*

The compositional effect underlying this proposition suggests potential ways to evaluate this theory using panel data on firms' financing and investment decisions.

financial assets $\int a^*(a, y; \omega) \varphi(a, y) da$ is homogenous of degree one in ω . Hence if $\int a^*(a, y; \varpi, R) \varphi(a, y) da$ is increasing in the interest rate for any ϖ , it is increasing in the interest rate for all ϖ .

The proposition also shows that the model is sufficiently tractable not only to admit a closed-form solution for steady-state TFP, but also to permit comparative statics with respect to the interest rate, even though the production side here is composed of a distribution of entrepreneurs with financial constraints and with two dimensions of heterogeneity.

In the calibration, I abstract from heterogeneity in firms' ability to pledge or asset tangibility. In future work, it would be interesting to compare the share of TFP losses due to capital misallocation accounted for by: (i) productivity shocks; and (ii) heterogeneity in firms' ability to pledge.

4.2 Productivity independent across time

In the previous section, it was shown that when workers can save, the general-equilibrium steady state features a wedge between the marginal product of capital and the interest rate. Moreover, with heterogeneity in firms' ability to pledge future cash flows, there is capital misallocation in steady state and total factor productivity is less than first best. This section introduces productivity shocks for entrepreneurs. With productivity shocks for entrepreneurs, the economy will feature capital misallocation in steady state even if all entrepreneurs have the same ability to pledge future cash flows. Incorporating productivity shocks for entrepreneurs allows the model to make richer predictions about how a decrease in firms' ability to borrow affects capital misallocation, risk management and TFP.

Preliminaries

The property that productivity is independent across time is equivalent to $Q(s_-, s) = Q(s'_-, s)$ for all $s_-, s'_- \in S$. When productivity is independent across time, I will restrict W to $W = [0, \hat{w}]$. For $\theta < 1$, I take $\hat{w} = \bar{w} \equiv (1 - \theta)f(k^u(R, \cdot), \bar{s})$. For $\theta = 1$, any finite $\hat{w} > 0$ can be used. The restrictions on W will be discussed throughout the text and the proofs.

Proposition 9 *Suppose productivity is independent across time. Then:*

- (i) *The optimal policies $k^*(w)$ and $w_s^*(w)$ are continuous, single-valued functions;*
- (ii) *V is strictly increasing and strictly concave in w and continuously differentiable at any $w \in \text{int}(W)$.*

This proposition also applies to the case of constant productivity; to see this, for any productivity s , take $S = s$ and $Q = 1$.

For the next proposition, I will need a technical assumption. When productivity is independent across time, there is a possibility of zero productivity if

$$0 \in S \text{ and } Q(., 0) > 0$$

Proposition 10 *Suppose productivity is independent across time. Then, given R , there is a distribution over wealth that is invariant under P_E . Suppose further that there is a possibility of zero productivity. Then P_E has a unique invariant distribution ζ^* .*

If $\beta R < 1$, uniqueness of the invariant distribution and weak convergence apply for any upper-bound on W that is greater than or equal to $(1 - \theta)f(k^u(R, .), \bar{s})$. If $\beta R = 1$ and if the domain of W is enlarged by taking $\hat{w} > \bar{w}$, then a distribution composed of a convex combination of ζ^* and any distribution over $[\bar{w}, \hat{w}]$ will also be an invariant distribution, and hence there will not be a unique invariant distribution. This is similar to the case of constant productivity, which also featured a unique invariant distribution when the interest is less than the rate of time preference, but featured a multiplicity of invariant distributions when the interest rate is equal to the rate of time preference. This multiplicity will not affect the results below, in the sense that the results hold for any upper-bound on wealth greater than or equal to \bar{w} .

Steady-state analysis

The following partial-equilibrium result highlights that an interest rate below the rate of time preference leads entrepreneurs to have less liquid wealth than they would need to avoid financial constraints.

Proposition 11 *In steady state, the measure of firms with at least one binding financial constraint is strictly positive if and only if $\beta R < 1$.*

Suppose, by contradiction, that $\beta R = 1$ and the measure of firms with a binding financial constraint is positive under some distribution ζ . Then, by the first-order condition for wealth (6), for any next-period states for which the financial constraint is binding, next-period consumption will be greater than current consumption. For any next-period states for which the financial constraint is not binding, next-period consumption will be the same as current consumption. Hence, next-period aggregate consumption would be greater than current-period consumption. Since the consumption policy is a function of wealth and

productivity, this must mean that current distribution ζ is not the invariant distribution ζ^* . The intuition for the $\beta R < 1$ case is similar.

Studying wealth dynamics for a given interest rate is useful in understanding how steady-state properties vary with the interest rate. If $\beta R = 1$, any entrepreneur with $w < \bar{w}$ will have next-period wealth w_s , with $w \leq w_s \leq \bar{w}$; if $\theta < 1$, the first inequality is strict for \bar{s} . Entrepreneurs will eventually accumulate exactly enough wealth to overcome their financial constraints; for any initial distribution of wealth over $[0, \bar{w}]$, the steady-state wealth distribution is degenerate: $\Pr(w = \bar{w}) = 1$. Once an entrepreneur has wealth $(1 - \theta)f(k^u(R, \cdot), \bar{s})$, the entrepreneur will thereafter have constant wealth and consumption and invest as if she were unconstrained.

In contrast, if $\beta R < 1$, an entrepreneur with wealth $\bar{w} = (1 - \theta)f(k^u(R, \cdot), \bar{s})$ will have strictly lower next-period wealth.¹¹ This de-cumulation of wealth fostered by an interest rate less than the rate of time preference leads to binding financial constraints.

The next proposition shows how moral hazard and an interest rate below the rate of time preference can together result in certain distortions in steady state.

Proposition 12 *Suppose that S has more than one element and consider the steady state. If $\beta R < 1$ and $\theta < 1$, then and only then the following hold:*

- (i) *the average marginal product of capital is greater than the interest rate;*
- (ii) *the variance in the marginal product of capital is positive;*
- (iii) *entrepreneurs' consumption is risky;*

Corollary 13 *When technology is Cobb-Douglas, aggregate TFP is less than first-best, $TFP < \overline{TFP}$, if and only if $\beta R < 1$ and $\theta < 1$.*

This proposition allows an analysis of a specific worsening in entrepreneurs' moral hazard, holding the interest rate constant. That is, the steady-state behavior of entrepreneurs for $\theta = 1$ and $\theta < 1$ can be compared for a given interest rate. Although this is a very coarse change, it provides some insight nonetheless. If the interest rate equals the rate of time

¹¹To see this, suppose $\beta R < 1$ and consider an entrepreneur with wealth $w = (1 - \theta)f(k^u(R, \cdot), \bar{s})$. Suppose that the entrepreneur's financial constraint for state s is not binding. Then wealth in state s will be strictly less than w , since the first-order condition for wealth implies $V_w(w_s) = \frac{1}{\beta R}V_w(w)$, and V is concave. Suppose instead that the entrepreneur's financial constraint for state s is binding. Then capital k must be less than unconstrained capital, $k < k^u(R, \cdot)$. Since the constraint is binding, wealth in state s must be given by $(1 - \theta)f(k(R, \cdot), s) < (1 - \theta)f(k^u(R, \cdot), \bar{s}) = w$.

preference, then a worsening of moral hazard has no long-run effect on investment distortions. That is, if $\beta R = 1$, there will be no finance premium, no dispersion in the marginal product of capital, and no endogenous TFP losses, regardless of whether θ is less than one or equal to one.

In contrast, if the interest rate is less than the rate of time preference, a decrease in θ from $\theta = 1$ to $\theta < 1$ will result in a positive average finance premium, positive variance of marginal product, and endogenous TFP losses.

Likewise, we can consider how the long-run properties of the economy depend on the equilibrium interest rate. With $\theta < 1$, there are investment distortions if and only if $R < \frac{1}{\beta}$.

In the numerical analysis below, we will see that the direction of these comparative statics (“lower theta, higher steady-state investment distortions,” “lower interest rate, higher steady-state investment distortions”) hold when the space for θ and the interest rate are not partitioned as coarsely as here.¹²

An analysis of the well-studied problem of the workers shows that it is not possible to have a steady-state equilibrium with $\beta R = 1$, because at this interest rate, the workers’ asset demand is not finite, as emphasized by Aiyagari (1994), Chamberlain and Wilson (2000), and others.

Lemma 14 (Chamberlain-Wilson (2000), Sargent-Ljungqvist (2000)) *Suppose that $\beta R = 1$ and that workers’ productivity is independent across time. Then $\lim_{t \rightarrow \infty} a(z^t) = \infty$ almost surely.*

The reason why entrepreneurs’ asset position is finite when $\beta R = 1$, whereas consumers’ asset position converges to infinity, is due to entrepreneurs’ access to state-contingent promises. In contrast, consumers trade only a safe asset.

Thus, it is not possible to have a steady state equilibrium with $\theta < 1$ and without binding financial constraints.

The individual entrepreneur

This section studies further the problem of the individual entrepreneur in partial equilibrium. The results here provide some further intuition for understanding steady-state properties like endogenous TFP losses.

¹²Note that when capital misallocation is a result of heterogeneity in entrepreneurs’ ability to pledge future cash flow, we can show analytically that steady-state TFP is increasing in the interest rate (Proposition 8).

In the model, entrepreneurs invest less than or equal to $k^u(R, s)$, the amount that would be invested by an entrepreneur who faced no moral hazard. The next proposition shows that, for wealth levels such that capital is less than $k^u(R, s)$, capital is increasing in liquid wealth.

Lemma 15 *If any of the financial constraints bind, $k^*(w, s_-)$ is strictly increasing in w . Optimal consumption $c^*(w, s_-)$ and state-contingent next-period wealth $\{w_s^*(w, s_-)\}_{s \in S}$ are strictly increasing in w . c^* increases with w at a rate strictly less than one-for-one.*

The next lemma further highlights the link between liquid wealth and whether financial constraints bind. The lemma requires defining $w_{l,s_-} = -\max_k -k + \frac{\theta}{1+r}E[f(k, s)]$. When productivity is independent across time, this value does not depend on s_- . This lemma is similar to a result in Rampini-Viswanathan (2010a).

Lemma 16 *For every s_- , there is a threshold $\underline{w}_{s_-} > w_{l,s_-}$ such that, for $w < \underline{w}_{s_-}$, the financial constraints are binding tomorrow in all states s .*

5 Calibration and Numerical Results

In this section, U.S. data on entrepreneurs' revenues, capital and labor inputs, and financing are used to calibrate the model. I use micro data on U.S. entrepreneurs to find reasonable parameters for the production technology and productivity process of U.S. entrepreneurs. The workers' income process is calibrated using standard parameters from Heaton and Lucas (1996). I then choose the moral-hazard parameter, θ , and the discount factor β to match an aggregate leverage measure for U.S. entrepreneurs and a target interest rate.

Using these parameters, I study the steady-state properties of the economy, such as TFP losses from misallocation. The central calibration exercise is to study the long-run effects of an increase in moral hazard, which corresponds to a decrease in θ . I do so by comparing the steady-states for the calibrated value θ_1 and a decreased value $\theta_2 < \theta_1$. I choose the size of the decrease in θ to match the decline in long-run real interest rates associated with the recent financial crisis.

The purpose of the exercise is to study further the properties of the model and the mechanisms discussed above, and to give a sense of the potential importance of the amplification and spillover effects previously identified.

The entrepreneurial sector in the United States is large. According to data from the 2007 Survey of Consumer Finances (SCF), 15 million U.S. households have a family member with an "active management role" in a privately-held business which the family owns in whole or in part. Based on the SCF data, these entrepreneurial firms had total sales in 2006 equal to more than 13 trillion dollars; total sales of all businesses in the US in 2006 were 28 trillion dollars. The total number of employees of these entrepreneurial firms was 103 million; according to the Current Population Survey (CPS), the number of employed U.S. civilians in 2006 was 144 million.¹³ Using SCF data, the reported value of entrepreneurs' stakes in their businesses was 9.7 trillion dollars; for comparison, according to Federal Reserve Flow of Funds data, the value of corporate equities held by U.S. households and non-profits was 9.6 trillion dollars.

In order to obtain parameters for the productivity process and production technology, I use data on U.S. entrepreneurs from the Kauffman Firm Survey (KFS), a longitudinal dataset of entrepreneurial firms. Virtually all macroeconomic studies of U.S. entrepreneurs rely on the Survey of Consumer Finances, the Panel Study of Income Dynamics, the Consumer Expenditure Survey or the National Longitudinal Survey of Youth. The estimation approach I use could not be implemented with these data sets, because they lack information on the capital and labor inputs used by the firms. Moreover, among these data sets, the SCF has the best coverage of entrepreneurs, because it oversamples the wealthy, but the Survey of Consumer Finances (SCF) has only a very limited panel component, comprising the 1983 and 1989 surveys. Thus, any attempt to obtain parameters for the productivity process and production technology from the Survey of Consumer Finances (SCF) has to rely either on a two-period panel or on identification from cross-sectional observations. In contrast, I can make use of the longitudinal aspect of the KFS. The data sources are described in more detail in Appendix C.

For entrepreneurs, I assume a Cobb-Douglas production technology:

$$F(k, l, s) = sk^a l^b$$

Here $a + b$ is a measure of the returns to scale for entrepreneurs' production technology, which can be interpreted as entrepreneurs' "span of control" (Lucas 1978). Equally of

¹³This estimate of the number of employees of entrepreneurial firms is biased downward because the number of employees for a given firm is top-coded at 5,000.

interest is the elasticity with respect to capital of the entrepreneurs' output net of labor cost function f . This elasticity is given by $\alpha = \frac{a}{1-b}$.

I estimate a and b using a procedure similar to Olley-Pakes (1996); the details are given in Appendix C. This procedure also produces estimates of idiosyncratic firm-level productivity. Using the estimated productivities, I perform the following regression

$$\log s_{it} = \rho \log s_{it-1} + \varepsilon_{it} \tag{17}$$

to obtain the process for productivity.

This approach to finding parameters for the entrepreneurs' production technology and productivity process is similar to the one used by Moll (2010) to study the impact of financial frictions for Chilean and Colombian manufacturing firms.

Table 1 provides the results of this procedure. The elasticity with respect to labor, 0.841, is higher than the typical value of two-thirds corresponding to the historical aggregate U.S. labor share of total income. Correspondingly, the elasticity with respect to capital is lower than the typical value of one-third. This is likely related to entrepreneurial nature of these firms; for example, there are virtually no mining firms or utilities in the KFS sample, and these industries tend to be very capital intensive. Many development-focused papers on entrepreneurship, including Moll (2010), focus exclusively on manufacturing industries and, not surprisingly, the elasticities with respect to capital that they estimate are quite high.

Table 1. Parameters of the entrepreneurs' productivity process and technology

Parameter	Description	Value
a	Elasticity of output with respect to capital	0.130
b	Elasticity of output with respect to labor	0.841
σ_ε	Standard deviation of productivity innovations	0.110
ρ	Autocorrelation of productivity	0.538

Source: Estimated using micro-data for U.S. entrepreneurs. See text or Appendix C.

The span of control value, $a + b$, is 0.97, close to constant returns to scale. The elasticity with respect to capital of the output net of labor costs function f is 0.82. This is slightly less than the elasticity calibrated for U.S. entrepreneurs by Cagetti-DiNardi (2006), 0.88, but much higher than the elasticity calibrated by Buera-Shin (2010a), 0.56. In Buera-Shin

(2010a), identification comes mostly from matching the employment share of the top tenth of establishments and the earnings share of the top twentieth of the population.

For the workers' labor income process, I use standard values estimated by Heaton and Lucas (1996), who estimate households' income processes from the PSID. Heaton and Lucas (1996) assume that labor productivity is log-normally distributed and find that the average autocorrelation is 0.529 and that the average standard deviation of innovations to log productivity is 0.251.¹⁴ For the workers' borrowing limit, I take the natural borrowing limit, which is zero, given the assumption of log-normal productivity.

I assume that the utility function u takes the form

$$u(c) = \begin{cases} \frac{c^{1-\gamma}}{1-\gamma} & \text{if } \gamma \neq 1 \\ \log(c) & \text{if } \gamma = 1 \end{cases}$$

where γ is the coefficient of relative risk aversion. In the primary specification, I assume the coefficient of relative risk aversion is equal to one, a conservative assumption that also facilitates computation.

This leaves two parameters to be calibrated, β and θ . To pin down these parameters, I seek to match: the aggregate ratio of debt and equity to sales for entrepreneurial firms; and a target interest rate. The aggregate ratio of debt and equity to sales is 0.3547. Data for the aggregate value of debt and sales of U.S. entrepreneurial firms comes from the Internal Revenue Service Statistics of Income. I use the SCF to estimate the aggregate value of equity in U.S. entrepreneurial firms owned by people without an active management role in the firms.¹⁵ This notion of equity is consistent with the model's definition of financial assets as claims on entrepreneurial firms held by outside investors. I include equity, rather than only debt, because the model features state-contingent claims and I seek to match the total value of investors' claims on entrepreneurial firms.

I choose a target real yearly interest rate of 2.5 percentage points. Because $E[d]$ in my model refers to the face-value of financial claims on the entrepreneurial firm plus interest payment, I seek to match $E[d] = 0.3547(1 + r) = 0.3634$. I am able to match the leverage ratio and target interest rate almost exactly, as shown in Table 2.

¹⁴Average here refers to the average across households.

¹⁵Equity in the entrepreneurial firms I study can, in general, only be held by people; the equity cannot be held by corporations, for example. See Appendix C for details.

Table 2. Calibration

	Target	Model
Leverage ratio, $\frac{E[debt+equity]}{E[sales]}$	0.363	0.364
Real interest rate	0.025	0.025

The calibrated discount rate is $\beta = 0.94$. This corresponds to a difference between the inverse discount rate and the interest rate, $\frac{1}{\beta} - R$, equal to about four percentage points. This is less than the difference found by Buera and Shin (2010a) for US data with a model where the only friction is the lack of state-contingent debt, but it is far more than the differences found in Aiyagari (1994), which featured no firm-side frictions and a constant returns to scale production technology. The difference relative to Aiyagari (1994) will be revisited at the end of the next section.

The calibrated moral hazard parameter is $\theta_1 = 0.62$. This corresponds to firms' being able to pledge to investors about two-thirds of their next-period output net of labor costs and undepreciated capital.

Although no information about the size distribution of firms is used to calibrate the model, the calibrated model captures the concentration of production in the largest firms. For example, in US data, the share of total employment accounted for by the top decile (measured by employment) of establishments is 67 percent, according to Buera and Shin (2010a); in the calibration, it is 74 percent. We can also compare the size distribution in the model to the size distribution of entrepreneurial firms in the SCF. In the 2007 SCF, the top decile (measured by employment) of entrepreneurial firms accounted for 71 percent of total entrepreneurial employment.

Moreover, because of a homotheticity property of the entrepreneurs' and workers' problems, the average firm productivity here is a normalization; this is made precise in the following lemma. Thus, we can set average firm productivity to match the empirical capital-labor ratio.

Lemma 17 *Suppose there is a steady-state equilibrium with interest rate R and wage w and distributions over entrepreneurs $\zeta(w, s_-)$ and over workers $\varphi(a, z)$. Suppose technology increases by a factor of x . Then there is a steady-state equilibrium with interest rate R and wage $x^{\frac{1}{1-a}}$ and distributions over entrepreneurs $\zeta'(w, s_-)$ and workers $\varphi'(a, z)$, where*

$$\zeta'(w, s_-) = \zeta(x^{-\frac{1}{1-a}}w, x^{-1}s_-)$$

and

$$\varphi'(a, z) = \varphi(x^{-\frac{1}{1-a}}a, z)$$

The cross-sectional distribution of the marginal product of capital is unchanged. TFP losses as a share of first-best TFP – the difference between first-best TFP and actual TFP, divided by first-best TFP – are unchanged.

Similarly, the ratio of firms to workers is also a normalization. Thus, we can set this ratio in the calibration to match average employment per firm.

5.1 Main numerical exercise: Decrease in firms' ability to borrow

This section compares the steady-state properties of the benchmark economy to the steady-state properties of an economy where firms' ability to borrow is lower than in the benchmark economy. That is, I compare the steady-states of two economies that have different levels of moral hazard but otherwise have identical parameters. The goal is to understand the persistent effects of a decrease in firms' ability to borrow.

In the benchmark economy, the pledgeability parameter is $\theta_1 = 0.62$. I choose the new value of θ such that the endogenous decrease in the steady-state interest rate is consistent with the recent decline in long-term real interest rates associated with the financial crisis. This is simply for a point of reference, to anchor the change in θ in a way that can be readily understood.¹⁶

The new value of θ is $\theta_2 = 0.49$, reflecting a 13 percentage point decrease in firm's ability to borrow. This lower value for θ corresponds to greater moral hazard and lower ability to commit to repayment.

Table 3 compares several steady-state properties of the two economies. The first column lists the steady-state properties of the economy at the calibrated value for θ . The second column (" $\theta = \theta_2$: Partial Equilibrium") lists the steady-state properties if θ decreases from θ_1 to θ_2 , but the interest rate remains unchanged. This is the outcome that would obtain in a small open economy with a world interest rate such that, before the decrease in firms' ability to borrow, the economy had zero current account surplus. This scenario is an analytical tool

¹⁶For the real long-term interest rate, I use the rate on 20 year US Treasury Inflation Protected Securities (TIPS). On May 31, 2007, before the major summertime subprime downgrades, this interest rate was 2.53 percent. On October 4, 2010, as the Troubled Asset Relief Program (TARP) expired, this rate was 1.49 percent. The ten year real interest rate showed a greater decline, from 2.54 to 0.49. Which maturity one should use for this exercise is not obvious, as the paper's model does not include term premia.

for thinking about the general equilibrium effects of a decrease in firms' ability to borrow. The third column lists the new steady-state properties, after the decrease in θ from θ_1 to θ_2 .

The first row shows losses in measured total factor productivity (TFP). If investment were unconstrained, the economy would obtain first-best TFP, as defined in (11). At the calibrated value for θ , measured TFP is 2.45 percentage points less than first-best TFP. The second row shows the average wedge between the marginal product of capital and the interest rate. This is a measure of average finance premium firms would be willing to pay for an additional unit of borrowing. At the calibrated value for θ , the average wedge is 1.62 percentage points. The third row shows the standard deviation of the marginal product of capital. Like measured TFP losses, this is a common measure of misallocation. At the calibrated value for θ , the standard deviation of this wedge is 2.55 percentage points.

Now consider a decrease in firms' ability to borrow. If the interest rate were held constant, TFP losses would increase by 0.33 percentage points (column " $\theta = \theta_2$: Partial Equilibrium"). However, in order to obtain steady-state equilibrium, the interest rate must fall, because of the decreased supply of financial assets. Hence, the actual increase in TFP losses is 0.72 percentage points (column " $\theta = \theta_2$: General Equilibrium"). Thus, with general equilibrium, TFP losses increase by 29 percent, whereas if the interest rate is unchanged, TFP losses increase only 13 percent.

The average wedge between the marginal product of capital and the interest rate increases from 1.62 percentage points to 2.74 percentage points, an increase of about 69 percent. Likewise, the standard deviation of the marginal product of capital increases from 2.55 percentage points to 3.51 percentage points, an increase of about 38 percent. Again, these increases are much larger than the increases that would obtain if the interest rate were held constant.

Table 3. Decrease in firms' ability to borrow: Steady-state distortions

	$\theta=\theta_1$	$\theta=\theta_2$		Increase
		Partial equilibrium	General equilibrium	
TFP losses	2.45	2.78	3.17	0.72
E[marginal product of capital]-r	1.62	2.24	2.74	1.12
Sd[marginal product of capital]	2.55	3.18	3.51	0.96
Workers' consumption risk, $(1 - Ex)$	2.47	.	2.82	0.35

All values in percentage points. See text for details. The final column gives the difference between the steady-state value for $\theta = \theta_2$ and the steady-state value for $\theta = \theta_1$

The final row in Table 3 illustrates how firms' supply of financial assets affects workers' ability to smooth consumption. To measure workers' ability to smooth consumption, I examine the share of average consumption that workers would be willing to give up to have constant consumption over time. This measure is a standard tool in public finance and has been used by Lucas (1987, 2003) in a celebrated study of the welfare costs of business cycles. For a given worker, the thought experiment is: what share $(1 - x)$ of the worker's average consumption \bar{c} provided in perpetuity would generate the same level of utility as the worker's stochastic consumption stream? To measure the workers' ability to smooth consumption, I take an average of $(1 - x)$ across all workers. Due to a homotheticity property of the workers' problem, this measure x does not depend on the wage. The homotheticity comes from the assumption of constant relative risk aversion and the linearity of the natural borrowing limit in the wage.

In the initial steady state, workers would be willing to give up 2.47 percent of their average consumption to have smooth consumption. At the new steady state, because of the decreased availability of financial assets, workers' consumption is riskier. The amount of average consumption that workers would be willing to give up to have smooth consumption increases to 2.82 percent. This increase of 35 basis points is far larger than the share of consumption that a representative consumer would be willing to give up to eliminate business cycles, which Lucas (2003) calculated to be just 5 basis points for the same coefficient of relative risk aversion used here.

The spillover effect of increased riskiness of workers' consumption is also meaningful compared with the spillover effect through the labor market, which is a decrease in the

wage. Due to the decrease in firms' ability to borrow, the steady-state wage declines by less than one percent.

The change in investment distortions and workers' consumption smoothing can be better understood by examining the change in certain quantities and prices.

Table 4. Decrease in firms' ability to borrow: Steady-state quantities and prices

		$\theta=\theta_1$	$\theta=\theta_2$		Increase
			Partial equilibrium	General equilibrium	
Quantities					
E[d]	Financial assets	0.36	0.14	0.27	-0.09
E[w]	Firms' liquid wealth	0.88	1.02	0.96	0.08
E[k]	Capital	1.15	1.07	1.14	-0.01
Price					
r	Interest rate	2.50	.	1.44	-1.06

As shown in Table 4, if the interest rate were held constant, firms' equilibrium supply of financial assets would decrease from 0.36 to 0.14, as a share of initial steady-state output.¹⁷ However, this decrease in firms' supply of financial assets is not consistent with an equilibrium. To obtain equilibrium, the interest rate falls from 2.50 percent to 1.44 percent. Relative to the small-open-economy counterfactual, firms' supply of financial assets increases, from 0.14 to 0.27. This still represents a reduction in the amount of financial assets supplied relative to the initial steady state, but it is a smaller reduction than would have occurred if the interest rate were unchanged. Of course, the equilibrium counterpart of a reduction in the financial assets supplied by firms is a reduction in the assets held by workers.

The decrease in firms' borrowing ability results in an increase in the steady-state liquid wealth of firms, as firms require more liquid wealth to pursue investment opportunities. However, the increase in liquid wealth is less than it would have been if the interest rate were unchanged. This, again, reflects the endogenous decrease in the interest rate.

While the change in the interest rate does amplify the distortions due to a decrease in firms' borrowing ability, it also dampens the decrease in capital.

¹⁷All quantities are measured as a share of steady-state average sales conditional on $\theta = \theta_0$.

The general-equilibrium amplification of investment distortions can be further understood by considering how distortions vary with the interest rate. In this exercise, for a given interest rate, the steady-state investment distortions are computed. For each interest rate, the wage is chosen so as to clear the labor market. The results are given in Figure 1. The steady-state distortions are decreasing in the interest rate.

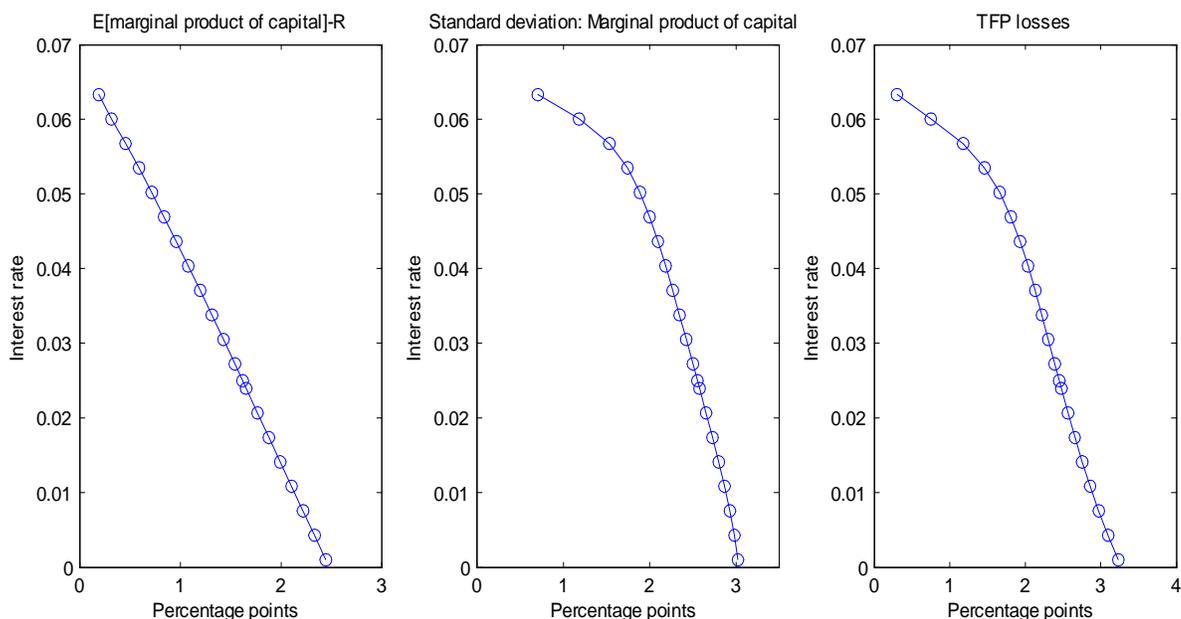


Figure 1. Steady-state distortions as a function of the interest rate, for $\theta = \theta_1$

Note: The first panel shows the expected wedge between the marginal product of capital and the interest rate, where the expectation is taken across entrepreneurs. The second panel shows the standard deviation, across entrepreneurs, of the marginal product of capital. The final panel shows TFP losses, relative to first-best TFP. See text for details.

When firms' ability to borrow decreases, conditional on the interest rate, steady-state investment distortions increase, as shown in Figure 2.

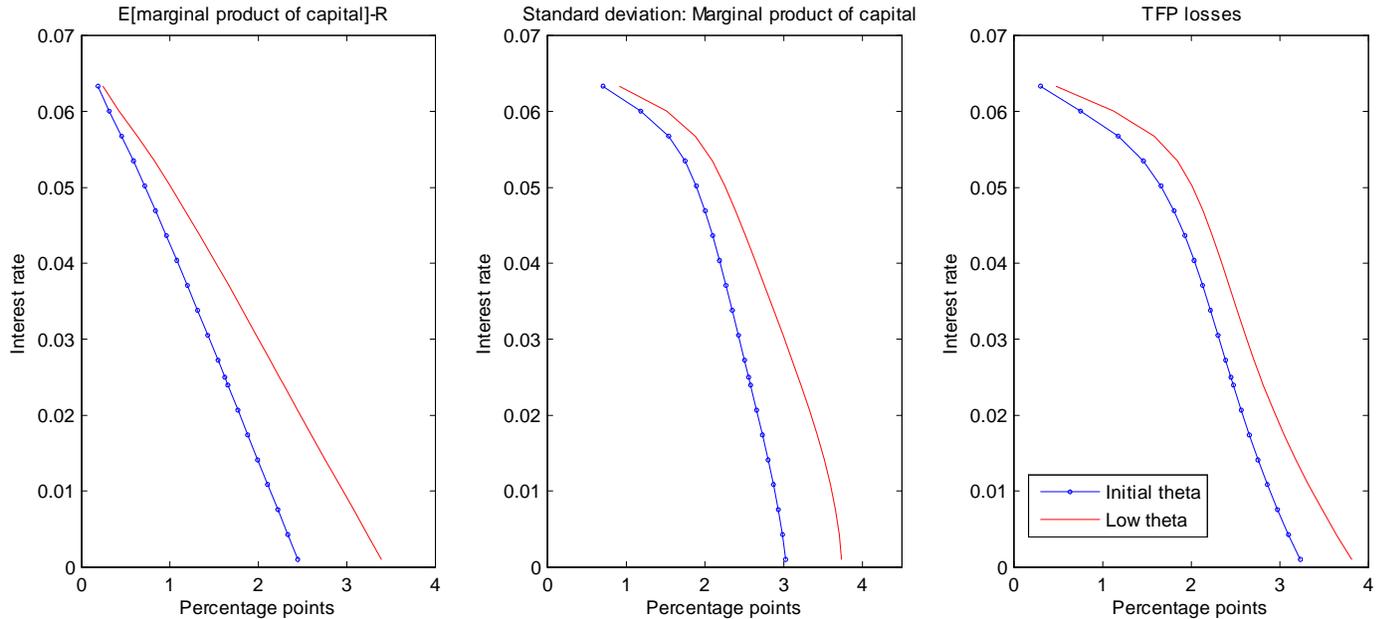


Figure 2. Steady-state distortions as a function of the interest rate, for $\theta \in \{\theta_1, \theta_2\}$

Note: See Figure 1 or text for details.

Increase in consumers' risk aversion

This model is also useful for thinking about an increase in workers' risk aversion or the riskiness of workers' labor income. One might think that workers' risk aversion or the riskiness of workers' income increase as the result of financial crises. The effect of either change is to increase workers' demand for financial assets, leading to a decrease in the interest rate and an increase in investment distortions for firms.

I consider an increase in consumers' risk aversion from $\gamma_1 = 1$ to $\gamma_2 = 1.5$, holding constant the moral hazard parameter at the calibrated value. As shown in Table 5, there are increases in capital misallocation and the average finance premium firms would be willing to pay for an additional unit of borrowing. These are driven by an endogenous decrease in the interest rate from 2.50 to 1.52. These increases in investment distortions are much smaller than those caused by a decrease in firms' ability to borrow, in part because the increases here are driven purely by the decrease in the interest rate.

Table 5. Increase in consumer risk aversion: Steady-state distortions

	$\gamma_1=1$	$\gamma_2=1.5$	Difference
TFP losses	2.45	2.72	0.28
E[marginal product of capital]-r	1.61	1.96	0.35
Sd[marginal product of capital]	2.55	2.78	0.23

There is also an increase in steady-state capital. An increase in steady-state capital is the same qualitative prediction that one would obtain from a standard macroeconomic model, such as Aiyagari (1994), that one would use to study an increase in workers' risk aversion or an increase in the riskiness of workers' labor income. However, standard macroeconomic models would not predict the increase in investment distortions for firms.

The decrease in the interest rate also limits workers' ability to smooth consumption. If the interest rate were held constant, then with the increased risk aversion, workers, on average, would be willing to give up 2.84 percent of their mean consumption in order to have a flat consumption path. However, with the decrease in the interest rate, this increases to 3.19 percent.

Low moral hazard and Aiyagari (1994)

The calibrated wedge between the inverse of the discount rate, $\frac{1}{\beta}$, and the interest rate is 4.33 percentage points. In contrast, Aiyagari (1994) finds a much smaller wedge, and indeed the smallness of the wedge he finds is a central point of his paper. Aiyagari (1994) uses a variety of parameterizations; for the parameterization of the worker's problem that is similar to mine, Aiyagari (1994) finds a wedge of 0.41 percentage points.

The key difference between the models in this paper and Aiyagari (1994) is that the later does not include frictions on the production side that limit firms' ability to create financial assets. In Aiyagari (1994), each period, all claims on physical capital are liquid assets held by consumers. For any $\theta < 1$, this is ruled out in my model.

To better understand how the firm-side friction in my model contributes to the low calibrated interest rate, consider a counterfactual in which $\theta = 1$. With $\theta = 1$, the firm is never constrained in its choice of investment, as the first-order condition for capital (7) shows. Thus, $\theta = 1$ corresponds to a "low" level of moral hazard. (A zero level of moral hazard would have $\theta = \infty$, whereas a higher level of moral hazard would have $\theta < 1$.) Calculating this counterfactual also serves a check on the numerical code, because the investment distortions are analytically known to be zero.

With $\theta = 1$, the wedge between the inverse of the discount factor and the interest rate is 0.71 percentage points. This is much closer to Aiyagari (1994)'s calibrated value. There are two reasons why the wedge is smaller for $\theta = 1$ than for $\theta = \theta_1 < 1$. First, for any choice of capital, a higher θ allows a greater amount of financial assets to be created. Second, a higher θ results in a larger capital stock.

With $\theta = 1$, workers would be willing to forgo only 0.79 percent of their mean consumption in order to have a smooth consumption path, rather than the 2.47 percent they would be willing to forgo when $\theta = \theta_0$. As predicted, all three measures of investment distortions (TFP losses, the average wedge between the marginal product of capital and the interest rate, and the standard deviation of the marginal product of capital) are zero.

This underlines the importance of considering frictions on the firm side when studying the capacity of workers to self-insure by accumulating wealth.

6 Conclusion

This paper has studied how endogenous changes to the scarcity of liquidity affect the economy's response to a decrease in firms' ability to borrow. In my framework, distortions due to a decrease in firms' ability to borrow are amplified by a reduced availability of liquidity.

The framework features a tractable model of the dynamic financing and investment choices of firms. It also features an important role for workers' demand for financial assets. Workers' need for financial assets creates an interesting channel through which firms' decreased supply of liquidity can affect workers. Workers' need for financial assets also affects firms' response to tighter borrowing constraints, by making it attractive for firms to be suppliers of liquidity. Indeed, in the baseline economy with hand-to-mouth workers and constant entrepreneurial productivity, a decrease in firms' ability to borrow has no effect on long-run distortions. This implies that any analysis of deleveraging should take into account the liquidity environment and how deleveraging changes the equilibrium scarcity of liquidity.

A calibration using firm-level data for U.S. entrepreneurs provides evidence that the amplification and spillover effects identified in the paper are important. A decrease in firms' ability to borrow consistent with the recent decline in the long-term real interest rate results in an increase in TFP losses from capital misallocation of about 29 percent. If the interest

rate were constant, as in a small open economy, the increase would be only 13 percent.

In current research, I am exploring the transition dynamics that arise after a decrease in firms' ability to borrow. Understanding the transition dynamics will make possible a welfare analysis, in contrast to the positive analysis in this paper. Also, once the transition dynamics are better understood, we can ask whether temporary shocks to firms' ability to borrow can generate the persistent effects that we associate with financial crises.

I am also exploring the business-cycle properties of the model and the ability of scarcity of assets to amplify macroeconomic volatility at business-cycle frequency. In the present model, an increase in foreigners' demand for financial assets, for example, leads to a decrease in the interest rate and hence an increase in investment distortions. In a model with aggregate productivity shocks, it would be interesting to understand how the endogeneity of the interest rate and the wealth accumulation channel might amplify aggregate productivity shocks, or whether an increase in demand for financial assets from foreigners makes the economy more vulnerable to aggregate shocks.

This paper also raises interesting questions about policy responses to deterioration in the financial system.

First, for governments with credible regalian powers of taxation, there may be scope to reduce distortions through fiscal policy, by creating financial assets backed by future taxation. Ricardian equivalence does not hold here. My model could be used to analyze the optimal issuance of government debt, given that government debt can alleviate the shortage of financial assets. Aiyagari and McGrattan (1998) perform such an analysis in a Bewley economy; in my model, such an analysis would also include the liquidity benefits to firms of additional government debt. Since the introduction of frictions on the firm side can lead to a much larger wedge between the rate of time preference and the interest rate than in a Bewley economy (see Section 5), the quantitative results of this analysis may be quite different when borrowing frictions for firms are taken into account.

Second, if the government can provide insurance, for example, to workers, this may decrease their demand for liquidity and hence reduce distortions for firms. Of course, if the government cannot provide more insurance than the agents can achieve through trading in safe debt – for example, due to the hidden storage and hidden information problems in Cole Kocherlakota (2001) – then this policy implication has little bite. However, the lack of insurance might reflect, for example, a standard trade off between insurance and incentives

to avoid moral hazard. Floden and Linde (2001) perform such an analysis in a Bewley economy. In my model, in making the trade off, one should take into account the effects on entrepreneurs of the decrease in workers' demand for liquidity that occurs if insurance is improved.

Finally, this paper points out costs of a government-imposed decrease in firms' borrowing ability; these costs come not only from increasing firms' investment distortions, but also in the form of a decrease in the liquidity that workers can use to hedge their idiosyncratic shocks.

Appendix A Proofs and additional lemmas

Lemma 18 *The expected marginal product of capital, $E[f_k(k, s)|s_-]$, is strictly decreasing in k if and only if F is strictly concave in capital and labor.*

Proof. Define $l(\omega, k)$ by:

$$F_L(k, l(\omega, k), s) = \omega$$

By the envelope theorem,

$$\frac{\partial f}{\partial k} = F_k(k, l(\omega, k))$$

Hence

$$\begin{aligned} \frac{\partial^2 f}{\partial k^2} &= F_{kk}(k, l(\omega, k)) + F_{kl}(k, l(\omega, k)) \frac{\partial l(\omega, k)}{\partial k} \\ &= F_{kk} - F_{kl} \frac{F_{lk}}{F_{ll}} \end{aligned}$$

Thus,

$$\frac{\partial^2 f}{\partial k^2} < 0 \text{ if and only if } F_{kk}F_{ll} > F_{kl}^2.$$

■

Proof of Lemma 1

Suppose that the production function is Cobb-Douglas

$$F(k, l, s) = sk^a l^b$$

where the elasticities of output to capital and labor are given by a and b , respectively. Then output net of labor costs is given by

$$f(k, s) = s^{\frac{1}{1-b}} k^{\frac{a}{1-b}} \left(\frac{b}{\omega}\right)^{\frac{b}{1-b}} (1-b)$$

where $\alpha = \frac{a}{1-b}$. Define an entrepreneur's liquid wealth at date t

$$w(s^t) = f(k(s^{t-1}), s_t) - d(s^t)$$

and consider the distribution over wealth w_t and productivity s_t given by $\zeta_t(w, s)$.

Aggregate output in period $t + 1$ will be given by:

$$\begin{aligned} Y_{t+1} &= \int \sum_{s_- \in S} \sum_{s \in S} s k(w, s_-)^a l(w, s_-, s)^b Q(s_-, s) \zeta_t(w, s_-) dw \\ &= \int \sum_{s_- \in S} \left(\frac{b}{\omega}\right)^{\frac{b}{1-b}} E[s^{\frac{1}{1-b}} | s_-] k(w, s_-)^\alpha \zeta_t(w, s_-) dw \end{aligned} \quad (18)$$

$$= A_{t+1} K_t^a L_{t+1}^b \quad (19)$$

where K_t is aggregate capital used in period $t + 1$, L_{t+1} is aggregate labor hired in $t + 1$, and A_{t+1} is measured total factor productivity (TFP).

$$K_t = \int \sum_{s_- \in S} k(w, s_-) \zeta_t(w, s_-) dw \quad (20)$$

$$L_{t+1} = \int \sum_{s_- \in S} \sum_{s \in S} s^{\frac{1}{1-b}} k(w, s_-)^\alpha \left(\frac{b}{\omega}\right)^{\frac{1}{1-b}} Q(s_-, s) \zeta_t(w, s_-) dw \quad (21)$$

$$A_{t+1} = \left[\int \sum_{s_- \in S} E[s^{\frac{1}{1-b}} | s_-] \left(\frac{k(w, s_-)}{K}\right)^\alpha \zeta_t(w, s_-) dw \right]^{1-b} \quad (22)$$

To obtain (19), divide (18) by L_{t+1}^b , using the expression in (21) for L_{t+1} . Then divide by K_t^a .

Proof of Proposition 4

This proof relies on the result that if each entrepreneur's productivity is constant over time, then in steady-state, each entrepreneur's wealth and consumption is constant over

time. This intuitive technical result is proved in Lemma 6.

Part (i). When productivity is constant, the first-order conditions for capital and next-period wealth:

$$u'(c)\left[\frac{1}{R}f_k(k, s) - 1\right] = \phi(1 - \theta)f_k(k, s)$$

and

$$\phi = -\beta V_w(w, s) + u'(c)\frac{1}{R}$$

In steady-state, each entrepreneur's wealth is constant over time, as shown in Lemma 6. Substituting $V_w(w, s) = u'(c_s) = u'(c)$ yields the expression for $f_k(k, s)$ in the proposition and shows that the financial constraint is binding if and only if $\beta R < 1$.

Part (ii). In steady state, an entrepreneur with productivity s invests $k = [f_k(\cdot, s)]^{-1}\left(\frac{1}{\theta\frac{1}{R} + (1-\theta)\beta}\right)$. If $\beta R < 1$, an entrepreneur with productivity s borrows $d = (1 - \theta)f(k, s)$. If $\beta R = 1$, an entrepreneur with productivity s borrows $d \leq (1 - \theta)f(k, s)$. The inequality is due to the multiplicity of invariant distributions when $\beta R = 1$, as discussed in Lemma 6. Thus, the supply of financial assets is increasing in the interest rate.

Proof of Lemma 6

This proof relies on the concavity of the value function, which is demonstrated in Proposition 8.

Re-arranging the first-order conditions, we can obtain:

$$\frac{1}{R} - \frac{1}{f_k(k)} = (1 - \theta)\left(\frac{1}{R} - \beta \frac{V'(w_s)}{V'(w)}\right) \quad (23)$$

where w is current-period wealth and w_s is next-period wealth.

(i) Suppose $R < \frac{1}{\beta}$. The following is a steady-state: an entrepreneur with productivity s has wealth $(1 - \theta)f\left([f_k(\cdot, s)]^{-1}\left(\frac{1}{\theta\frac{1}{R} + (1-\theta)\beta}\right), s\right)$, invests capital $[f_k(\cdot, s)]^{-1}\left(\frac{1}{\theta\frac{1}{R} + (1-\theta)\beta}\right)$, and has next-period wealth equal to current wealth. This satisfies the first-order conditions and the complementary slackness condition; in particular, the constraint is binding. We can prove by contradiction that there is no other steady state. Consider a candidate invariant distribution

with positive mass on wealth levels not equal to $(1-\theta)f([f_k(\cdot, s)]^{-1}(\frac{1}{\theta\frac{1}{R}+(1-\theta)\beta}), s)$. Condition (23) implies that all entrepreneurs with wealth less than $(1-\theta)f([f_k(\cdot, s)]^{-1}(\frac{1}{\theta\frac{1}{R}+(1-\theta)\beta}), s)$ will have next-period wealth greater than current wealth. Likewise, all entrepreneurs with wealth greater than $(1-\theta)f([f_k(\cdot, s)]^{-1}(\frac{1}{\theta\frac{1}{R}+(1-\theta)\beta}), s)$ will have next-period wealth less than current wealth. Hence the candidate distribution is not an invariant distribution.

(ii) Suppose $R = \frac{1}{\beta}$. Any distribution over wealth and productivity is a steady state if the measure of entrepreneurs with wealth greater than or equal to $(1-\theta)f(k^u(R), s)$ is equal to one. To see this, note that with wealth greater than or equal to $(1-\theta)f(k^u(R), s)$, next-period wealth equal to current wealth and capital equal to $k^u(R)$ satisfies the first-order conditions and the complementary slackness condition; in particular, the financial constraint is not binding. To see that there cannot be an invariant distribution with a positive measure on wealth less than $(1-\theta)f(k^u(R), s)$, note that an entrepreneur with less than $(1-\theta)f(k^u(R), s)$ will have capital less than $k^u(R)$ and hence, from condition (7), next-period wealth that exceeds current wealth.

(iii) In the invariant distributions identified in parts (i) and (ii), next-period wealth equals current wealth.

Proof of Proposition 7

(i) Assume that workers are hand-to-mouth workers.

Suppose there were a steady-state with $\beta R < 1$. Then, based on the partial-equilibrium analysis of Proposition 4, the financial constraint of each entrepreneur would bind and capital would equal $k = [f_k(\cdot, s)]^{-1}(\frac{1}{\theta\frac{1}{R}+(1-\theta)\beta})$. Hence each entrepreneur would supply a positive amount of financial assets and entrepreneurs' aggregate supply of financial assets would be strictly positive. Since workers demand for savings is zero, this cannot be a steady-state equilibrium.

If $\beta R > 1$, entrepreneurs' wealth would be increasing over time. This is inconsistent with an invariant distribution for wealth and with market clearing, as it would result in an aggregate supply of financial assets that converges to negative infinity.

If $\beta R = 1$, there is a multiplicity of general-equilibrium steady states. Consider the condition:

$$\sum_{s \in S} \int f(k^u(R, s), s) \zeta(w, s) dw = \sum_{s \in S} \int w \zeta(w, s) dw \quad (24)$$

where $\zeta(w, s)$ is a distribution over wealth and productivity. This condition says that the total liquid wealth of the entrepreneurs equals the total output net of labor costs when investment is unconstrained. With hand-to-mouth workers, this condition is equivalent to market-clearing in the financial market. Consider also the condition:

$$Pr_{\zeta}[w \geq (1 - \theta)f(k^u(R), s)] = 1 \quad (25)$$

where the probability distribution over wealth and productivity is given by ζ . This condition says that the wealth of each entrepreneur is sufficiently high such that the choice of investment is unconstrained. For $R = \frac{1}{\beta}$, this condition is required for a steady state, as stated in Lemma 6.

Any distribution $\zeta(w, s)$ that satisfies (24) and (25) is a general equilibrium steady state. Moreover, a comparison of the two conditions shows that there is an infinity of distributions ζ that satisfy both.

(ii) Assume that workers can save.

There cannot be a general equilibrium steady state with $\beta R = 1$, because with $\beta R = 1$, workers' financial assets converge to infinity almost surely, as shown in Lemma 13. In contrast, for any invariant distribution ζ^* for $\beta R \leq 1$, firms' capital is finite and hence firms' supply of financial assets must be finite.

There is at least one general equilibrium steady state with $\beta R < 1$. To see this, first define the workers' steady state asset demand as a function of the interest rate:

$$A(R) = \int \int a^*(a, y; R) \varphi^*(a, y; R) da dy$$

Note that the policy function a^* and the distribution φ^* depend on the interest rate and the wage, which is a continuous function of the interest rate. A standard result is that, for a given interest rate, there is a unique invariant distribution $\varphi^*(a, y; R)$. From Bewley (1984) and Clarida (1990), we have the following results:

$$\lim_{R \uparrow \frac{1}{\beta}} A(R) = \infty$$

and

$$\lim_{R \downarrow 0} A(R) = \underline{a}$$

From Aiyagari (1994), and by application of a standard theorem in Stokey-Lucas-Prescott (1989), we know that $A(R)$ is continuous in R . We also know that firms' supply of financial assets

$$D(R) = \theta \sum_{s \in S} \int f([f_k(\cdot, s)]^{-1}(\frac{1}{\theta \frac{1}{R} + (1-\theta)\beta}, s) \zeta^*(w, s; R) dw$$

is strictly positive, finite and continuous in R for any $R \in (0, \frac{1}{\beta})$. Moreover,

$$\lim_{R \downarrow 0} D(R) = \infty$$

Hence there is at least one general-equilibrium steady state and any general-equilibrium steady state features $\beta R < 1$.

Proof of Proposition 8

With constant productivity, the marginal product of capital of a firm with productivity s and moral hazard θ is given by (14). Thus, it is immediate that part (i) of the proposition holds.

For part (ii), we can combine the assumption of Cobb-Douglas technology with (14) to solve for capital $k(R, \theta, s)$ as a function of s and θ , and substitute into (10) to find a closed-form expression for TFP.

In particular, TFP equals X^{1-b} , where X is given by

$$X = \frac{1}{K^\alpha} \sum_s s^{\frac{1}{1-b}} \sum_\theta k(R, \theta, s)^\alpha g(\theta, s) d\theta \quad (26)$$

where aggregate capital K is given by

$$K = \sum_s \sum_\theta k(R, \theta, s) g(\theta, s) d\theta \quad (27)$$

and $g(\theta, s)$ is the distribution function over θ and s .

Firm capital $k(R, \theta, s)$ is given by:

$$k = (\alpha s^{\frac{1}{1-b}})^{\frac{1}{1-\alpha}} \left(\frac{1}{R} \theta + \beta(1 - \theta) \right)^{\frac{1}{1-\alpha}} \quad (28)$$

Substitute (28) into (27) and (26) and take the derivative of (26) with respect to R . Suppose that $\beta R < 1$. Let $S = \{s_1, s_2, \dots, s_{\#S}\}$. Let $\Theta = \{\theta_1, \theta_2, \dots, \theta_{\#\Theta}\}$.

After some algebra, one finds that

$$\frac{dX}{dR} > 0$$

if and only if

$$\begin{aligned} & \sum_{i=1}^{\#S} \sum_{j=1}^{\#\Theta} \sum_{m=1}^{\#S} \sum_{n=1}^{\#\Theta} [z_i y_j^\alpha g(\theta_j, z_i) z_m y_n^\alpha \theta_n g(\theta_n, z_m)] \\ & > \sum_{i=1}^{\#S} \sum_{j=1}^{\#\Theta} \sum_{m=1}^{\#S} \sum_{n=1}^{\#\Theta} [z_i z_m y_j^{2\alpha-1} \theta_j g(\theta_j, z_i) y_n g(\theta_n, z_m)] \end{aligned} \quad (29)$$

where, to economize on notation,

$$\begin{aligned} z_i &= s_i^{\frac{1}{1-b}} \\ y_j &= \left(\frac{1}{R} \theta_j + \beta(1 - \theta_j) \right)^{\frac{1}{1-\alpha}} \end{aligned}$$

There are $(\#S)^2(\#\Theta)^2$ terms on the left-hand side and an equal number on the right-hand side of (29). Of these, $(\#S)^2(\#\Theta)$ terms on each side feature $j = n$, and all of these terms cancel. In particular, if $j = n$, the left-hand term (i,j,m,n) equals

$$\theta_j z_i z_m y_j^{2\alpha} g(\theta_j, z_i) g(\theta_j, z_m)$$

which is equal to the right-hand term (i,j,m,n) .

This leaves $(\#S)^2((\#\Theta)^2 - (\#\Theta))$ terms with $j \neq n$. Pair terms (i,j,m,n) and (m,n,i,j) . I will show that the sum of each left-hand-side pair is strictly greater than the sum of each

right-hand-side pair. That is, I will show that

$$\begin{aligned} & z_i z_m \theta_n y_j^\alpha y_n^\alpha g(\theta_j, z_i) g(\theta_n, z_m) + z_m z_i \theta_j y_n^\alpha y_j^\alpha g(\theta_n, z_m) g(\theta_j, z_i) \\ > & z_i z_m \theta_j y_j^{2\alpha-1} y_n g(\theta_j, z_i) g(\theta_n, z_m) + z_m z_i \theta_n y_n^{2\alpha-1} y_j g(\theta_n, z_m) g(\theta_j, z_i) \end{aligned}$$

With some algebra, this inequality reduces to

$$\theta_n \left(\frac{(\frac{1}{R}\theta_n + \beta(1 - \theta_n)) - (\frac{1}{R}\theta_j + \beta(1 - \theta_j))}{(\frac{1}{R}\theta_n + \beta(1 - \theta_n))} \right) > \theta_j \left(\frac{(\frac{1}{R}\theta_n + \beta(1 - \theta_n)) - (\frac{1}{R}\theta_j + \beta(1 - \theta_j))}{(\frac{1}{R}\theta_j + \beta(1 - \theta_j))} \right) \quad (30)$$

Suppose, without loss of generality, that $\theta_n > \theta_j$, remembering that we are only considering terms with $j \neq n$. The assumption $\theta_n > \theta_j$ implies that (30) holds.

Proof of Proposition 9

Because the utility function may be unbounded, I restrict the domain of V to be $W \times S$, where $W = [0, \hat{w}]$. If \hat{w} is chosen to be sufficiently large, this restriction of the domain will not affect the analysis.¹⁸ In order to prove the proposition, I will verify the conditions required in Exercise 9.7 of Stokey-Lucas-Prescott (1989), hereafter SLP. To parallel SLP, the notation is:

$$V(w, s_-) = \max_{\{k, \{d_s\}\} \in \Gamma(w, s_-)} G(w, k, \{d_s\}, s_-) + \beta E[V(\psi(w, k, \{d_s\}, s), s)]$$

First, some important preliminaries.

- . W is a convex Borel set in \mathbb{R} , with its Borel subsets W .
- . S is a countable set and \mathcal{S} is the σ -algebra containing all subsets of S .
- . Define $\Gamma : W \times S_- \rightarrow K \times D^{\#S}$ by

$$\Gamma(w, s_-) = \{k, \{d_s\} : d_s \leq \theta f(k, s) \text{ and } w + \frac{1}{1+r} E[d_s] - k \geq \varepsilon \text{ and } f(k, s) - d_s \leq \hat{w}\}$$

The second and third conditions in the definition of Γ are not part of the economic problem; they are technical conditions useful for the proof. Because they never bind, it is

¹⁸For $\theta < 1$, we can use any $\hat{w} \geq \bar{w} \equiv (1 - \theta)f(k^u(R, \cdot), \bar{s})$. For $\theta = 1$, any $\hat{w} > 0$ will suffice.

acceptable to neglect them in the main text.

To obtain the proposition, I need to verify that the following conditions hold:

Condition 1: The constraint set Γ is non-empty, compact-valued and continuous. To see that it is non-empty valued, note that given s_-

$$k_{\min} = \max_k -k + \frac{1}{R}\theta E[f(k, s)|s_-]$$

and

$$d_s = \theta f(k_{\min}, s)$$

are always feasible. Γ is compact valued because it is closed and bounded.

Condition 2: The payoff function G is bounded, continuous and strictly increasing in w . Let A be the graph of Γ . Define $G : A \rightarrow \mathbb{R}$ by

$$G(w, k, \{d_s\}, s_-) = u(w + \frac{1}{1+r}E[d_s|s_-] - k)$$

G is bounded and continuous, and $\beta \in (0, 1)$. G is bounded below by $u(\varepsilon)$. G is bounded above by

$$\max_{s_-} u(\bar{w} + \frac{\theta}{R}E[f(k_{\min}(s_-), s)|s_-] - k_{\min}(s_-))$$

For each $(k, \{d_s\}, s_-) \in R_+ \times R^{\#S} \times S$, we have $u(w + \frac{1}{1+r}E[d_s|s_-] - k)$ strictly increasing in w , by the assumption that $u(c)$ is strictly increasing.

Condition 3: The constraint set is increasing in w . For each $s_- \in S$, $\Gamma(\cdot, s_-) : W \rightarrow K \times \mathbb{R}^{\#S}$ is increasing in the sense that $w \leq w'$ implies $\Gamma(w, s) \subseteq \Gamma(w', s)$. The first and last sets of restrictions in the definition of Γ do not involve w ; a higher w makes it easier to satisfy the second set of restrictions.

Condition 4: Concavity restriction on the payoff function. For each $s_- \in S$, $G : A_{s_-} \rightarrow \mathbb{R}$ satisfies the following concavity restriction:

$$\begin{aligned} & u(\varphi w + (1 - \varphi)w' + \frac{1}{R}E[\varphi d_s + (1 - \varphi)d'_s|s_-] - (\varphi k + (1 - \varphi)k')) \\ & \geq \varphi u(w + \frac{1}{R}E[d_s|s_-] - k) + (1 - \varphi)u(w' + \frac{1}{1+r}E[d'_s|s_-] - k') \end{aligned}$$

for all $\varphi \in (0, 1)$ and $(w, k, \{d_s\}), (w', k', \{d'_s\}) \in A_{s_-}$. This is due to the concavity of u .

Condition 5: No "increasing returns" in Γ . For all $s_- \in S$ and all $w, w' \in W$, we have $\{k, \{d_s\}\} \in \Gamma(w, s_-)$ and $\{k', \{d'_s\}\} \in \Gamma(w', s_-)$ implies $\varphi\{k, \{d_s\}\} + (1 - \varphi)\{k', \{d'_s\}\} \in \Gamma(\varphi w + \varphi w', s_-)$ for all $\varphi \in (0, 1)$. The first and last set of constraints satisfy this condition because they are linear. The second set of constraints satisfies this condition by weak concavity of f .

Condition 6: Differentiability of the payoff function. For given $s_- \in S$,

$$G(w, k, \{d_s\}, s_-) = u\left(w + \frac{1}{1+r}E[d_s|s_-] - k\right)$$

is continuously differentiable in $(w, k, \{d_s\})$ on the interior of A_{s_-} . This is immediate from the assumption that u is twice differentiable.

Condition 7: Restrictions on the mapping to next-period state variables. Define $\psi(w, k, \{d_s\}, s) = f(k, s) - d_s$.

ψ is non-decreasing in w .

For each s , $\psi(\cdot, s)$ is weakly concave, because f is concave in k .

$\psi(\cdot, s)$ does not depend on w .

Proof of Proposition 10

This proof is based on theorem 12.12 of SLP (1989). I need to verify that the transition function P_E defined by (12) is monotone, has the Feller property and satisfies a mixing condition.

Feller property. P has the Feller property if $\int h(w_s, s)P((w, s_-), (w_s, s))$ is bounded and continuous in w for any function h that is bounded and continuous in w . SLP theorem 9.14 implies that P has the Feller property if $g^w(w, s_-)$ is continuous in w , as verified in Proposition 8.

Monotonicity. The monotonicity condition requires that $\int h(w_s, s)P((w, s_-), (w_s, s))$ be increasing in w if h is bounded, continuous, and increasing in w . The monotonicity of w_s with respect to w shown in Lemma 14, together with SLP exercise 12.11, imply monotonicity.

Mixing condition. I must show that there exists $w^* \in [0, \bar{w}]$, $\xi > 0$, and $N \geq 1$ such that $P^N(0, [w^*, \bar{w}]) \geq \xi$ and $P^N(\bar{w}, [0, w^*]) \geq \xi$. Take $w^* = f(k_{\min}, \min_s(s \in S : s \neq 0))$. For arbitrary w , consider the optimal policy for w_0 , next-period wealth if realized productivity

is zero. If ϕ_0 is not binding, then $V'(w_0) = u'(c_0) = \frac{1}{\beta R} u'(c) = \frac{1}{\beta R} V'(w)$. If ϕ_0 is binding, then $w_0 = 0$. Thus, $P^N(\bar{w}, [0, w^*]) \geq \xi \equiv Q(0)^N$ for N equal to the smallest integer larger than $\frac{\log[\frac{u'(c^*(w^*))}{u'(c^*(\bar{w}))}]}{\log(\frac{1}{\beta R})}$.

Proof of Proposition 11

First, I show that $\beta R = 1$ implies the measure of constrained firms is zero.

Suppose $\beta R = 1$. Suppose that the distribution today ζ_t is an invariant distribution: $\zeta_t = T^* \zeta_t$. Suppose further, by contradiction, that the mass of entrepreneurs with at least one financial constraint is binding has positive measure. For these entrepreneurs, consumption tomorrow will be strictly higher than today, as indicated by the first-order condition (8). For all other entrepreneurs, consumption tomorrow will be the same as consumption today. Hence, aggregate consumption tomorrow is higher than aggregate consumption today:

$$\sum_{s_-} \int c^*(w, s_-) \zeta_t(dw, s_-) < \sum_{s_-} \int c^*(w, s_-) \zeta_{t+1}(dw, s_-)$$

Thus, $\zeta_t \neq \zeta_{t+1}$ and hence ζ_t is not an invariant distribution.

Second, I show that $\beta R < 1$ implies the measure of constrained firms is positive.

This proof with very similar to the previous analysis, with appropriate modifications. Suppose $\beta R < 1$. Suppose that the distribution today ζ_t is an invariant distribution: $\zeta_t = T^* \zeta_t$. Suppose further, by contradiction, that the mass of entrepreneurs with at least one binding financial constraint has zero measure. Hence, using the first order condition (8), aggregate consumption tomorrow is lower than it is today:

$$\sum_{s_-} \int c^*(w, s_-) \zeta_t(w, s_-) dw > \sum_{s_-} \int c^*(w, s_-) \zeta_{t+1}(w, s_-) dw$$

Thus, $\zeta_t \neq \zeta_{t+1}$ and hence ζ_t is not an invariant distribution.

From Proposition 9, we know that an invariant distribution exists. Combining these two negative results with the existence of an invariant distribution delivers Proposition 10.

Proof of Proposition 12

Part (i). In steady state, there is a positive measure of firms with at least one binding financial constraint if and only if $\beta R < 1$, according to Proposition 10. The first-order

condition for capital (7) implies that firms have a marginal product of capital greater than the interest rate if and only if the firm has binding financial constraints and $\theta < 1$. Part (i) holds whether or not S has more than one element.

Parts (ii). Suppose $\beta R < 1$. Any firm with a binding constraint and $\theta < 1$ will have next-period wealth that depends on realized productivity. Hence, the wealth distribution will not be degenerate. The measure of firms with at least one binding financial constraint is either: less than one; or one. If it is less than one, then immediately we have that there is a positive measure of firms with marginal product of capital equal to the interest rate, and a positive measure of firms with marginal product of capital greater than the interest rate. If the measure of constrained firms is one, then the variance of marginal product of capital is positive, since capital is increasing in wealth if a constraint is binding, according to Lemma 14, and the wealth distribution is non-degenerate.

If $\beta R = 1$, then in steady-state, the financial constraints are not binding. The first-order condition for capital therefore implies that the marginal product of capital equals the interest rate. If $\theta = 1$, then the marginal product of capital equals the interest rate, regardless of whether financial constraints bind.

Part (iii). Suppose $\beta R < 1$. Any firm with a binding constraint and $\theta < 1$ will have next-period wealth, and hence next-period consumption, that depends on realized productivity. If $\beta R = 1$, in steady state, next-period consumption is the same as today's, regardless of realized productivity.

Proof of Lemma 15

Denote the optimal policy for consumption by:

$$c^*(w, s_-; \mu) = w + \frac{1}{1 + r(\mu)} E[f(k^*, s) - w_s^*] - k^*$$

That c^* is increasing in w is immediate from the envelope condition, $V'(w, s_-) = u'(c^*)$, and the concavity of V and u .

Consider a given (w, s_-) . Consider any states of the world for which the financial constraint is not binding. The first-order condition for wealth in state s when the financial

constraint in state s is not binding is

$$u'(c) \frac{1}{\beta R} = V_w(w_s, s; \mu')$$

Since c is increasing in w , this first-order condition implies that w_s must be increasing in w for any s for which the financial constraint is not binding.

In the states for which the financial constraint is binding, $w_s = (1 - \theta)f(k, s)$. Hence if we can show that k is increasing in w whenever the financial constraint is binding for at least one state of the world, we will have shown that $\{w_s\}$ is increasing in w .

Combining the first-order conditions for w_s and k , we obtain the following, which holds almost everywhere:

$$\frac{1}{R(\mu)} E[f_k(k, s)] - 1 - \frac{1}{R(\mu)} \sum_{s \in S_r} (1 - \theta) q_s f_k(k, s) = -(1 - \theta) \beta \sum_{s \in S_r} q_s \frac{V_w((1 - \theta)f(k, s), s; \mu')}{u'(c)} f_k(k, s)$$

The left-hand-side is decreasing in k , due to the concavity of f in k . The right hand side is increasing in k , due to the concavity of f in k and V in w . Hence, because c is increasing in w , we must have that k is increasing in w whenever any of the financial constraints bind (that is, whenever S_r is not the empty set).

Finally, to see that c^* is increasing less than one-for-one with w , consider w and w' , with $w' > w$, with optimal policies $(k, \{w_s\})$ and $(k', \{w'_s\})$. Suppose that $c' \geq c + (w' - w)$, or, equivalently, $\frac{1}{1+r(\mu)} E[f(k, s) - w_s] - k \leq \frac{1}{1+r(\mu)} E[f(k', s) - w'_s] - k'$. Then $(k', \{w'_s\})$ would have been a better choice than $(k, \{w_s\})$ when wealth equals w ; it is feasible, produces higher consumption than $(k, \{w_s\})$, and, since w_s^* is increasing in w , it must offer higher utility tomorrow.

Proof of Lemma 16

For any (w, s_-) , it must be case that $w_s \geq (1 - \theta)f(k_{\min}, s) > 0$ for all s . The first inequality follows from the fact that the firm will never choose a k below k_{\min} and from the financial constraint for s ; the second follows from the assumptions on f . Denote the maximum possible marginal utility of consumption tomorrow in state s by $x_s = u'(c^*((1 - \theta)f(k_{\min}, s), s)) = V_w((1 - \theta)f(k_{\min}, s), s) < \infty$. This is the maximum possible marginal

utility tomorrow in state s because $V(\cdot, s)$ is concave.

Note that for any (w, s_-) , c is bounded above by $w - w_{l, s_-}$.

Consider some s_- . For every s , we can pick an $\varepsilon_{s, s_-} > 0$ such that, if $w \leq w_{l, s_-} + \varepsilon_{s, s_-}$, we have

$$\frac{1}{\beta R} u'(c) > \frac{1}{\beta R} u'(\varepsilon_{s, s_-}) > x_s \geq V_w(w_s, s)$$

Suppose that constraint s is not binding tomorrow. Then $V_w(w_s, s)$ must equal $\frac{1}{\beta R} u'(c)$, as shown by (7). This is a contradiction.

Since the set of s is discrete, we can define

$$\underline{w_{s_-}} = \min_s (\varepsilon_{s, s_-}) > 0$$

Appendix B: Multi-period promises

The following proposition makes precise the previous assertion that it is without loss of generality to restrict entrepreneurs to trading one-period promises.

Consider an environment parameterized by $n \geq 1$ in which entrepreneurs at history s^t can trade state-contingent promises that pay out at history s^{t+i} , for $i = \{1, \dots, n\}$. Denote by $x(s^t, s^{t+i})$ the amount of payment that the entrepreneur at history s^t promises to pay at history s^{t+i} . The entrepreneur's budget constraint is

$$c(s^t) + k(s^t) \leq f(k(s^{t-1}), s_t) - x(s^{t-1}, s^t) + \sum_{i=1}^n \sum_{s^{t+i}|s^t} \frac{1}{R^i} E_t[x(s^t, s^{t+i}) - x(s^{t-1}, s^{t+i})] \quad (31)$$

where $x(s^{t-1}, s^{t+n}) = 0$ since at $t - 1$, the entrepreneur cannot trade promises to pay in period $t + n$.

If the entrepreneur reneges at s^t , this eliminates both his debt that is immediately due $x(s^{t-1}, s^t)$ and his promises to repay in future periods, $\{x(s^{t-1}, s^{t+i})\}_{i=1}^{n-1}$. As above, when the entrepreneur reneges, his creditors can seize only a fraction θ of his current period output,

$f(k, s)$. Hence, given the entrepreneur's budget constraint, the entrepreneur will not renege at s^t if and only if

$$\sum_{i=0}^{n-1} \sum_{s^{t+i}|s^t} \frac{1}{R^i} x(s^{t-1}, s^{t+i}) \leq (1 - \theta) f(k(s^t), s_t) \quad (32)$$

The entrepreneur now chooses a collection $\{k(s^t), c(s^t), \{x(s^t, s^{t+i})\}_{i=1}^n\}$ to maximize utility (4) subject to a no-Ponzi condition and a collection of budget constraints (31) and financial constraints (32) for each s^t .

Lemma 19 *Consider a solution to the entrepreneur's problem with n securities given by $\{k(s^t), c(s^t), \{x(s^t, s^{t+i})\}_{i=1}^n\}$. There exists a policy of one-period positions $d(s^{t+1})$ promised at s^t to pay out at s^{t+1}*

$$d(s^{t+1}) = \sum_{i=1}^n \frac{1}{R^{i-1}} E_{t+1}[x(s^{t+i}, s^t) | s^{t+1}]$$

that, together with the same policies for capital and consumption $\{k(s^t), c(s^t)\}$, satisfy the budget (2) and financial (1) constraints of the problem with one-period promises and achieve the same utility.

Proof. This comes from comparing the expression for $d(s^{t+1})$ in the lemma to condition (32). ■

Appendix C Numerical approach

The parameters of the Cobb-Douglas production technology and the productivity process were estimated using firm-level data for U.S. entrepreneurs from the Kauffman Firm Survey. The estimator is a modified Olley-Pakes (1996) estimator. This estimator addresses the simultaneity bias by semi-parametrically controlling for unobserved productivity.

Firms from finance and insurance (NAICS code 52) and real estate rental and leasing (NAICS code 53) were excluded from the analysis. Also, attention was restricted to sole proprietorships and Subchapter S corporations, for which aggregate leverage ratios could be obtained.

The results of the estimation are given in Table 1. Bootstrapped standard errors will be included pending data-disclosure approval.

The aggregate leverage ratio is a measure of aggregate debt and equity to aggregate sales. Data on debt and sales is obtained from the Internal Revenue Service (IRS) Statistics of Income (SOI), which provides data for sole proprietorships and S corporations disaggregated by NAICS code. I use the average ratio of debt to sales for 1998-2003, the period for which data is available.

By equity, this paper means equity held by investors, rather than the entrepreneur. For sole proprietorships, all the equity is held by the entrepreneur. For S corporations, there can be up to 100 equity holders. Data on equity in S corporations held by non-entrepreneurs is obtained from the Survey of Consumer Finances (SCF). I use the standard definition of non-entrepreneur, namely, people owning a stake in S corporations but reporting that they have no active management role in the firm. The SCF has weights that allow aggregate calculations; see Curtin, Juster, and Morgan (1989) and Cagetti-DiNardi (2006) for details. Corresponding to the IRS data availability, I used data from the 1998, 2001 and 2004 SCF.

Computing the stationary equilibrium. The central exercise is to calculate the steady-state equilibrium for given fundamental parameters. To do so, the following procedure is used:

Step 1. Guess a value for the interest rate R .

Step 2. Solve the firm's problem, conditional on R and a guess for the wage ω . Since the model features state contingent financing, the firm's problem requires each firm to choose capital and a schedule of next-period repayments. Solving for a schedule of repayments rather than an uncontingent repayment increases the number of choice variables, but it

does not increase the number of state variables required. Because the value function is concave in wealth and because the set of productivities S is discrete, the value function can be approximated by a collection of one-dimensional Chebyshev polynomials, which are computationally efficient.

At each value-function iteration, the new value function is calculated based on the previous guess and the firm's optimization. This new value function is calculated for a grid of points, where spacing of the grid points is (roughly) exponential, as opposed to the traditional Chebyshev spacing. This spacing of grid points allows a high density of points at the wealth levels of interest while avoiding unintentional restrictions on the state space for wealth. The firm's choice of capital and next-period wealth are not restricted to lie on a grid. The new guess for the value function is then determined by Chebyshev interpolation.

Step 3. Simulate 12,000 firms for T periods, where T is taken to be 500 periods, to find the steady-state distribution over wealth and productivity.

Step 4. Solve for wage rate that would clear the labor market. This is straightforward, since the firm's problem is homothetic in the following sense: a change in the wage from ω to $z\omega$ will result in an increase in steady-state capital from k to $z^{-\frac{b}{1-b}\frac{1}{1-\alpha}}k$. This property comes from the assumption of constant relative risk aversion and simplifies the numerical task considerably.

Step 5. Solve the worker's problem and determine workers' demand for financial assets.

Step 6. Check financial market clearing. If workers' demand is greater [less] than firms' supply of financial assets, choose a new guess for R that is lower [higher] than the previous guess.

Step 7. Repeat steps 2-6 until the financial market clears.

Like the firm's problem, the worker's problem has a useful homotheticity property. In particular, asset demand at a given interest rate is linear in the wage. This is due to the assumption of constant relative risk aversion; this assumption is made in the calibration exercise. The homotheticity also depends on the borrowing constraint being zero or linear in the wage, a property satisfied by the natural borrowing constraint. The homotheticity of the workers' problem means that, once the workers' asset demand is calculated for a given interest rate, it can be calculated immediately for any wage required for the various counter-factual exercises.

How to deal with non-entrepreneurial firms has been a persistent problem in this litera-

ture. One approach has been to include non-entrepreneurial firms as a constant-returns-to-scale sector that does not face any financial constraints (Quadrini 2000, Cagetti and DiNardi 2006). One problem with this approach is that even large, public firms face financial frictions, if not exactly of the type or severity faced by entrepreneurs. A second approach has been to ignore broad swaths of the economy, even in general equilibrium exercises. For example, in a general-equilibrium calibration meant to quantify TFP losses from financial frictions, Moll (2010) combines aggregate current-account and output data with estimates of productivity-process and production-function parameters for manufacturing plants, ignoring non-manufacturing firms; Moll’s calibration also features hand-to-mouth consumers (or, equivalently for steady-state analysis of workers’ demand for financial assets, workers without labor-productivity and unemployment risk and with an ad-hoc zero borrowing limit) and thus misses the important role of the savings of non-entrepreneurs.

In this part of the appendix, I incorporate non-entrepreneurial firms as a Lucas tree. This creates a gap between the equilibrium supply of financial assets by entrepreneurs and the equilibrium demand for financial assets by workers. In the calibration, I require that the value of the Lucas tree equal an estimate of the historical value of U.S. corporate equities and bonds held by households as a share of GDP, or 0.58. It should be noted that this value likely overstates the importance of non-entrepreneurial firms, since some entrepreneurial firms are incorporated, using Subchapter S, Chapter C or other corporate forms. (For example, Koch Industries, Inc., is a privately-held conglomerate run by Chairman and CEO Charles G. Koch, who has a major ownership stake in the firm. Koch Industries, Inc. has about 70,000 employees according to its website and is estimated by Forbes to have annual revenue of 100 billion dollars.)

With the non-entrepreneurial Lucas tree, the calibrated value for the discount factor β is 0.96, and the calibrated gap between the interest rate and the rate of time preference is thus smaller. Likewise, the calibrated value for firms’ ability to borrow is $\theta_{1a} = 0.88$, which is greater than in the analysis without the non-entrepreneurial Lucas tree. As a result, the calibrated steady-state TFP losses from entrepreneurial firms are significantly smaller, as shown in Table 6 below. Here again the steady-state distortions are studied before and after a decrease in firms’ ability to borrow. Qualitatively, we see that a decrease in firms’ ability to borrow still leads to a decrease in the interest rate and an amplification of the increase in investment distortions. However, with a corporate Lucas tree, the general-equilibrium

amplification effect is smaller; the decrease in TFP in general equilibrium is only 14 percent larger than it would be if the interest rate were constant, as in a small open economy.

Table 6. Decrease in firms' ability to borrow: Steady-state distortions

	$\theta=\theta_{1a}$	$\theta=\theta_2$		Increase
		PE	GE	
TFP losses	0.50	2.19	2.42	1.91
E[marginal product of capital]-r	0.17	0.83	1.14	0.96
Sd[marginal product of capital]	0.66	1.83	2.06	1.40
Workers' consumption risk, $(1 - Ex)$	1.40	.	1.74	0.33

Notes: Please see Table 3 or main text.

A full analysis should take into account differences in the productivity process, production function and borrowing capacity of different sectors of the economy, and of the types of financial frictions present in each. With this approach, one could study the TFP losses due to heterogeneity in firms' ability to borrow, which are decreasing in the interest rate in the case of constant-productivity entrepreneurs. This would enable a comparison of the two sources of TFP losses in the model – differences in firms' ability to borrow, and productivity shocks that endogenously result in some firms having high productivity and low wealth – and their relative contribution to steady-state distortions and general-equilibrium amplification.

References

- [1] Aiyagari, S. Rao, 1994. "Uninsured Idiosyncratic Risk and Aggregate Saving," *Quarterly Journal of Economics*, Vol. 109, No. 3, pp. 659-684.
- [2] Albuquerque, Rui and Hugo A. Hopenhayn, 2004. "Optimal Lending Contracts and Firm Dynamics." *Review of Economic Studies*, Vol. 71, pp. 285–315.
- [3] Alvarez, Fernando and Urban J. Jermann, 2000. "Efficiency, Equilibrium, and Asset Pricing with Risk of Default," *Econometrica*, Vol. 68, No. 4, pp. 1468-0262.
- [4] Angeletos, George-Marios and Laurent-Emmanuel Calvet, 2006, "Idiosyncratic Production Risk, Growth, and the Business Cycle," *Journal of Monetary Economics*, Vol. 53, No. 6, pp. 1095-1115.
- [5] Angeletos, George-Marios and Vasia Panousi, 2010. "Financial Integration, Entrepreneurial Risk and Global Dynamics," mimeo.
- [6] Banerjee, Abhijit and Benjamin Moll, 2010. "Why does misallocation persist?," *American Economic Journal: Macroeconomics*, forthcoming.
- [7] Bernanke, Ben, Mark Gertler and Simon Gilchrist, 1999. "The financial accelerator in a quantitative business cycle framework," in John B. Taylor and Michael Woodford, eds., *Handbook of Macroeconomics*, Vol. 1, Ch. 21, pp. 1341-1393.
- [8] Bernanke, Ben and Mark Gertler, 1989. "Agency Costs, Net Worth, and Business Fluctuations," *The American Economic Review*, Vol. 79, No. 1, pp. 14-31.
- [9] Buera, Francisco and Yongseok Shin, 2010a. "Financial Frictions and the Persistence of History: A Quantitative Exploration," mimeo.
- [10] Buera, Francisco and Yongseok Shin, 2010b. "Productivity Growth and Capital Flows: The Dynamics of Reforms," mimeo.
- [11] Cagetti, Marco and Mariacristina De Nardi, 2006. "Entrepreneurship, Frictions, and Wealth," *Journal of Political Economy*, 2006, vol. 114, no. 5, pp. 835-870.
- [12] Clarida, Richard H., 1990, "International Lending and Borrowing in a Stochastic, Stationary Equilibrium," *International Economic Review*, vol. 31, no. 3, pp. 543-558.

- [13] Cooley, Thomas, Ramon Marimon, and Vincenzo Quadrini, 2004. "Aggregate Consequences of Limited Contract Enforceability," *The Journal of Political Economy*, vol. 112, no. 4, pp. 817-847.
- [14] Curtin, Richard T., F. Thomas Juster, and James N. Morgan, 1989. "Survey Estimates of Wealth: An Assessment of Quality," in Lipsey, Robert E. and Helen Stone Tice, eds., *The Measurement of Saving, Investment and Wealth*, University of Chicago Press.
- [15] Evans, David S. & Jovanovic, Boyan, 1989. "An estimated model of entrepreneurial choice under liquidity constraints," *Journal of Political Economy*, Vol. 97, No. 4, pp. 808–27.
- [16] Farhi, Emmanuel and Jean Tirole, 2010. "Bubbly Liquidity," mimeo.
- [17] Heaton, John and Deborah J. Lucas, 1996. "Evaluating the Effects of Incomplete Markets on Risk Sharing and Asset Pricing," *Journal of Political Economy*, Vol. 104, No. 3, pp. 443-487.
- [18] Holmström, Bengt and Jean Tirole, 1998. "Private and Public Supply of Liquidity," *The Journal of Political Economy*, Vol. 106, No. 1, pp. 1-40.
- [19] Jorgenson, Dale W., 1963. "Capital Theory and Investment Behavior," *The American Economic Review, Papers and Proceedings*, Vol. 53, No. 2, pp. 247-259.
- [20] Kehoe, Timothy J. and David K. Levine, 1993. "Debt-Constrained Asset Markets," *The Review of Economic Studies*, Vol. 60, No. 4, pp. 865-888.
- [21] Krueger, Dirk and Fabrizio Perri, 2006. "Does Income Inequality Lead to Consumption Inequality? Evidence and Theory," *Review of Economic Studies*, Vol. 73, No. 1, pp. 163–193.
- [22] Lorenzoni, Guido, 2008. "Inefficient Credit Booms," *Review of Economic Studies*, Vol. 75, No. 3, pp. 809-833.
- [23] Lorenzoni, Guido and Veronica Guerrieri, 2010. "Credit Crises, Precautionary Savings and the Liquidity Trap," mimeo.

- [24] Lorenzoni, Guido and Karl Walentin, 2007. "Financial Frictions, Investment and Tobin's q ," mimeo.
- [25] Lucas, Robert E., 1978. "On the Size Distribution of Business Firms," *The Bell Journal of Economics*, Vol. 9, No. 2, pp. 508-523.
- [26] Lucas, Robert E., 1987. *Models of Business Cycles*. New York: Blackwell.
- [27] Lucas, Robert E., 2003. "Macroeconomic Priorities," mimeo., University of Chicago.
- [28] Massa, Massimo and Andrei Simonov, 2006. "Hedging, Familiarity and Portfolio Choice," *The Review of Financial Studies*, Vol. 19, No. 2, pp. 633-685.
- [29] McGrattan, Ellen R. and Edward C. Prescott, 2010. "Unmeasured Investment and the Puzzling U.S. Boom in the 1990s," Federal Reserve Bank of Minneapolis, Research Department Staff Report 369.
- [30] Mendoza, Enrique G., Vincenzo Quadrini, and José-Víctor Ríos-Rull, 2009. "Financial Integration, Financial Development, and Global Imbalances," *Journal of Political Economy*, Vol. 117, No. 3, pp. 371-416.
- [31] Midrigan, Virgiliu and Daniel Yi Xu, 2010. "Finance and Misallocation: Evidence from Plant-Level Data," NBER Working Paper 15647.
- [32] Moll, Benjamin, 2010. "Productivity Losses from Financial Frictions: Can Self-Financing Undo Capital Misallocation?," mimeo.
- [33] Olley, G. Steven and Ariel Pakes, 1996. "The Dynamics of Productivity in the Telecommunications Equipment Industry," *Econometrica*, Vol. 64, No. 6, pp. 1263-1297.
- [34] Quadrini, Vincenzo, 2000. "Entrepreneurship, Saving and Social Mobility," *Review of Economic Dynamics*, Vol. 3, No. 1, pp. 1-40.
- [35] Rajan, Raghuram G. and Luigi Zingales, 2003. "The great reversals: the politics of financial development in the twentieth century," *Journal of Financial Economics*, Vol. 69, No. 1, Tuck Symposium on Corporate Governance, pp. 5-50.
- [36] Rampini, Adriano A. and S. Viswanathan, 2010a. "Collateral, Risk Management, and the Distribution of Debt Capacity," *Journal of Finance*, Vol. 65, pp. 2293-2322.

- [37] Rampini, Adriano A. and S. Viswanathan, 2010b. "Collateral and capital structure," mimeo., Duke University.
- [38] Reinhart, Carmen M. and Vincent R. Reinhart, "After the fall," NBER Working Paper 16334.
- [39] Storesletten, Kjetil, Chris I. Telmer and Amir Yaron, 2004. "Cyclical Dynamics in Idiosyncratic Labor Market Risk," *The Journal of Political Economy*, Vol. 112, No. 3, pp. 695-717.