Question 1. A basic RBC machine. 30 points

Consider an economy described by the optimization problem:

\[
\max V(K_t) = \sum_{i=0}^{\infty} \beta^i \log C_{t+i}
\]

subject to:

\[
C_{t+i} + K_{t+i+1} = Y_{t+i} = K_{t+i}^\alpha
\]

(1) (5 points) Write down the Bellman equation (the recursive relation followed by the value function)

(2) (15 points) Guess that the value function is of the form \(V(K) = a + b \log(K)\), with parameters \(a\) and \(b\) to be determined.

Solve for the optimal consumption rule, that is optimal \(C\) as a function of \(K\), for given \(a\) and \(b\).

Solve for the parameter \(b\) (the parameter \(a\) is more complicated and not needed) and so for the optimal consumption rule in terms of the underlying parameters of the model, \(\alpha\) and \(\beta\).

(Let be me nice here, and give you the end result. The optimal consumption rule is given by \(C = (1 - \alpha \beta)Y\).

(3) (5 points) Explain in words the dependence of the consumption rule on \(\alpha\) and \(\beta\)
(4) (5 points) The optimal consumption rule appears totally myopic: consumption depends only on current income. Yet, one would have thought agents were forward looking. Can you explain what is going on?

1 Question 2. Shocks, output and welfare. 30 points

No math, just intuition.

Take a standard monopolistic competition macro model, and assume prices are predetermined each period.

(1) (10 points) Suppose that, after prices have been set for this period, consumers become more optimistic about future output. What will happen to consumption? to output? to welfare this period?

(2) (10 points) Should the monetary authority contract the nominal money supply so as to keep output at its normal level? Why, or why not?

(3) (10 points) Following on your conclusions to (2), what do you think about adopting the following monetary rule in this economy:

In case of unexpected positive shocks to demand, do not contract money and let output be higher.

In case of unexpected negative shocks to demand, expand money so that output remains at its normal level.

2 Question 3. Inflation and unemployment. 30 points

Many price staggering models yield an equation of the type:

\[ p_t = .5p_{t-1} + .5Ep_{t+1} - au_t \]

where \( p \) is the logarithm of the price level, and \( u \) is the unemployment gap, the deviation of unemployment from its natural level.

(1) (5 points) In what sense if any does this equation exhibit price stickiness?
(2) (10 points) Using the fact that inflation is defined by $\pi_t \equiv p_t - p_{t-1}$, rewrite this equation in terms of inflation, expected future inflation, and the unemployment gap.

In what sense if any does this equation exhibit inflation stickiness? Contrast your answer with your answer to (1), and explain.

(3) (15 points) How well does the inflation equation you derived in (2) fit the facts?