Monetary Policy and Corporate Default*

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Abstract

When a corporation issues debt with a fixed nominal coupon, the real value of future payments decreases with the price level. Forward-looking corporate default decisions therefore depend on monetary policy through its impact on expected inflation. We build a general equilibrium economy with deadweight bankruptcy costs that demonstrates how nominal rigidities in corporate debt create an important role for monetary policy even in the absence of standard nominal frictions such as staggered price setting in the output market. Under a passive interest rate target, the direct effects of a negative productivity shock combine with deflation to produce strong incentives for corporate default. A debt-deflationary spiral results when there are real costs of financial distress. Inflation targeting eliminates this amplification mechanism, but full inflation targeting requires permitting the nominal interest rate to depend explicitly on credit market conditions.

JEL Classification:

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1 Introduction

The financial crisis of 2007/2008 and subsequent global recession has had a severe impact on both the default rates and credit spreads of firms. According to Moody’s (Emery et al. (2009)), the global default rate on speculative grade debt reached 13% in 2009, close to the previous high of 15.4% set in 1933, as shown in Figure 1. Credit spreads similarly surged during the recent financial crisis, with the Baa-Aaa spread reaching a peak of just under 3.5% in 2008-2009. The last time the Baa-Aaa spread surpassed this level was also during the 1930's, as shown in Figure 2. Hence, from the perspective of credit conditions, the recent recession has been the worst since the Great Depression.

While the nominal interest rate declined with GDP during both the recent crisis and the Great Depression, the behaviour of inflation has been markedly different. During the period 2007-2009 inflation has declined, but any deflation has, so far, been negligible as seen in Panel A of Figure 5. By contrast, the 1930’s were marked by substantial deflation. The decline in real GDP during the recent crisis has also been on a much smaller scale than during the Great Depression as shown in Panel B of Figure 5. A vast literature studies the impact of monetary policy on inflation and output,¹ and a growing subset of this work develops links between monetary policy and the microfoundations of corporate default and credit spreads.²

In this paper we build on the observation that fixed-rate corporate obligations are typically denominated in nominal dollars, and are often long-lived. Hence a decrease in expected inflation, due for example to a monetary policy shock, increases the incentives of a corporation to default. Unlike the financial accelerator literature (discussed in footnote 1) the channel we consider does not depend on differences across agents in access to credit and investment opportunities, and our framework assumes a representative agent.

We instead consider a cross-section of heterogeneous firms whose output depends on systematic as well as idiosyncratic productivity shocks. Following Merton (1974), Fischer et al.

¹See for example Woodford (2003) and Gali (2008)
²See, for example, Curdia and Woodford (2010) who model credit spreads in a monetary economy where default on single-period household debt is exogenously specified. Christiano et al. (2009) develop a high-dimensional dynamic stochastic general equilibrium model where unexpected variations in inflation impact the distribution of wealth across agents due to a nominal rigidity in debt contracts. Bernanke et al. (1996) embed a financial accelerator model of output fluctuations into a New Keynesian monetary economy. Bernanke et al. (1996) discuss the financial accelerator literature, which is based on the idea that negative aggregate shocks tend to reduce borrowers’ access to credit, amplifying the initial shock (e.g., Bernanke and Gertler (1989), Kiyotaki and Moore (1997), Bernanke et al. (1999)).
(1989), and Leland (1994), firms optimally issue perpetual risky debt to take advantage of a tax benefit to debt. They choose to default when the present value of coupon payments to bond holders is greater than the present value of future dividends. The corporate finance literature emphasizes substantial costs of financial distress, in the range of 5-20% of firm value for firms ranging from investment grade to bankrupt.\(^3\) Our model correspondingly permits that in the event of default, there are bankruptcy costs and bond holders take over the firm. Bankruptcy costs consist of transfers that leave aggregate output and the consumption of our aggregate consumer unchanged, as well as deadweight losses which do affect consumption. The endogenous relationship between inflation and output in our model implies a Phillips curve. We depart from the New Keynesian approach by abstracting from price setting in product or labor markets, and hence our Phillips curve is not a consequence of standard nominal rigidities.\(^4\) Instead, the positive relation between output and inflation in our model derives solely from the sticky nature of nominal debt contracts combined with deadweight costs of financial distress.

The stickiness of debt in our model stems from the assumptions of nominal debt contracts, perpetual maturity, and the inability to refinance debt except in default. Together these assumptions produce strong persistence in leverage ratios. Fama and French (2002) find that firms’ debt ratios adjust slowly toward their targets. Leary and Roberts (2005) provide evidence that corporations rebalance their debt infrequently. Hence while our assumptions that generate stickiness in capital structure are strong, we believe that these features of the model capture an important nominal rigidity.

Monetary policy in our model takes the form of an interest rate rule. The policy instrument is the short-term nominal interest rate, which may depend on the aggregate state variables of the model. The aggregate state variables are the systematic productivity shock, a monetary policy shock, and the distribution of firms across idiosyncratic shocks and leverage. Since the distribution of firms is high dimensional, we approximate its impact on output

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\(^3\)These costs involve both direct expenses of bankruptcy and a variety of indirect effects that impair operating activities in the neighborhood of distress. The real costs of financial distress have been estimated in different settings by Warner (1977), Cutler and Summers (1988), Weiss (1990), Andrade and Kaplan (1998), Bris et al. (2006), Almeida and Philippon (2007), and van Binsbergen et al. (2010).

\(^4\)The New Keynesian paradigm departs in two important dimensions from the frictionless model, where real quantities such as output and consumption are determined independently of monetary policy. First, firms sell differentiated products for which they can set the price, consistent with the idea of imperfect competition in the goods market. Second, firms cannot reset their product price in every period, but instead price changes are staggered as in Calvo (1983). See Woodford (2003), Goodfriend (2004b), and Gali (2008).
using the aggregate default rate of the economy. We follow Gallmeyer et al. (2007) who determine the endogenous inflation process under an interest rate rule using the Euler equation, and show how inflation incorporates risk premia stemming from fluctuations in the aggregate state variables of the economy. In our setting assets correlated with the aggregate default rate earn a risk premium because of the impact of default on aggregate consumption.

Our quantitative analysis focuses on two principal monetary policy rules. Prior literature has examined tradeoffs in central bank policies aimed at stabilizing interest rates versus prices (e.g., Goodfriend (1987), Khan et al. (2003), Goodfriend and King (2009)). The first monetary policy rule we consider sets the short-term interest rate equal to a constant target plus noise, as in the ‘Friedman rule’ prescription of low and stable interest rates. Such a policy is sometimes described as ‘passive’ in the sense that the target policy rate does not respond to aggregate quantities or inflation. Under this rule, our endogenous inflation process is procyclical. As a consequence, in bad times when aggregate productivity is low, firms have at least two motivations to default on debt. First, low aggregate productivity implies low current and future output. Second, anticipated deflation in bad times implies that the expected real value of committed future coupons is increasing. Since these two motives for default are both countercyclical and thus reinforcing, corporate defaults cluster strongly together.

When deadweight bankruptcy costs are present an additional amplification effect occurs under the constant target interest rate policy. During a default wave the consumption and output trough is lower in the presence of deadweight bankruptcy costs than without such costs. The additional drop in consumption that occurs because of deadweight bankruptcy costs adds to the deflation that occurs during bad times, further increasing incentives for default. This cycle in which deflation contributes to default, which causes additional deflation captures important aspects of the debt deflation theory of Fisher (1933). Previous research develops aspects of the debt deflation theory using other transmission channels.5

The second monetary policy rule we consider is a constant inflation target. We show that with deadweight costs of default, implementation of such a rule requires that the short-term interest rate respond to credit market conditions, in our case the default rate. Inflation is naturally acyclical under this policy. Corporations still tend to default in bad times due to the direct effect of the aggregate productivity shock on cash flows, but variations in expected

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inflation do not amplify the waves of default. As a consequence, default waves are milder and the distribution of aggregate default rates is less volatile. Default rates have a similar unconditional expectation as under the interest rate target rule, but defaults tend to be more evenly spread through time and corporations have higher average leverage ratios.\textsuperscript{6}

Our paper is related to recent work studying default in the context of monetary policy. Curdia and Woodford (2010) examine the effects of modifying monetary policy rules to depend on interest rate differentials or credit conditions. Their model is driven by heterogeneous preferences between borrowers and lenders but default is exogenous. Our model has a representative agent, but default is endogenous. Goodfriend and McCallum (2007) study the impact of interest rate differentials on monetary policy in a model with a banking sector and money. While our framework has neither of these features, it differs from Goodfriend and McCallum (2007) because default is possible in equilibrium. The way we model default decisions builds on structural models of optimal capital structure and the pricing of corporate debt based on Leland (1994).\textsuperscript{7} Following Gomes and Schmid (2011), we extend this approach to general equilibrium by allowing for default to have deadweight costs. Gomes and Schmid (2011) also model real investment and endogenize the entry of new firms, whereas our framework does not incorporate investment or endogenous entry. Instead, we introduce monetary policy, which impacts output via its effect on endogenous leverage and default decisions.

The outline of our paper is as follows. In Section 2, we develop a structural equilibrium model of heterogeneous firms that make optimal capital structure decisions trading off the tax benefits of debt against costs of financial distress. The monetary authority sets policy according to an interest rate rule, and firms incorporate this rule in forming their capital structure and default decisions. Deadweight bankruptcy costs are incurred when firms default. In Section 3, we calibrate the model and demonstrate the effects of monetary policy on aggregate default and output. Section 4 concludes.

\textsuperscript{6}Although we do not include such an example in our calibrations, it is clear to see that more ‘active’ policies that seek to produce countercyclical inflation can potentially remove the cyclicality of default waves altogether. This can occur, for example, if the cash flow effect of aggregate productivity shocks is fully offset by countercyclical inflation, which reduces the incentives of corporations to default in bad times.

\textsuperscript{7}See for example, Goldstein et al. (2001), Hackbarth et al. (2006), Carlson and Lazrak (2010), Bhamra et al. (2010a,b,c) and Chen (2010) who introduce exogenous consumption within structural models. Gomes and Schmid (2010) introduce investment with an exogenous pricing kernel.
2 The Model

We consider a consumption-based, representative household economy where a cross-section of heterogeneous firms generate output according to exogenously specified aggregate and idiosyncratic productivity shocks. Firms use debt and equity financing, choosing the nominal debt coupon and default policy optimally according to the trade-off between tax benefits to debt and potential bankruptcy costs, as in Leland (1994). The default of an individual firm generates losses to bondholders. Bankruptcy costs may include transfers as well as deadweight economic losses that reduce aggregate output. The monetary authority chooses an interest rate rule, which via no arbitrage, pins down the path of inflation, as in Gallmeyer et al. (2007). We do not model sticky prices, but our model nonetheless contains a nominal rigidity since coupons are paid in nominal dollars and refinancing is infrequent. Consequently, changes in inflation have real consequences for default, output, and asset prices, implying an important role for monetary policy.

2.1 Representative Household, Real and Nominal Pricing Kernels

The infinitely-lived representative household has power utility over consumption, modeled in discrete time with the time discount rate $\beta$ and relative risk aversion $\gamma$. The real pricing kernel is thus

$$M_{t+1} = \beta \left( \frac{C_{t+1}}{C_t} \right)^{-\gamma},$$

where $C_t$ is consumption at date $t$. The log pricing kernel satisfies

$$m_{t+1} = \ln M_{t+1} = \beta - \gamma \Delta c_{t+1},$$  \hspace{1cm} (1)

where $\Delta c_{t+1} = c_{t+1} - c_t = \ln C_{t+1} - \ln C_t$.

Denote the nominal price index by $P$. The log inflation rate is $\pi_t$, where $\pi_t = \ln P_t - \ln P_{t-1}$. The nominal pricing kernel is

$$M_{t+1}^* = M_t e^{-\pi_t},$$

and the log nominal pricing kernel satisfies

$$m_{t+1}^* = m_t - \pi_t.$$  \hspace{1cm} (2)

Throughout the paper we denote nominal variables by superscripting with an asterisk. Variables without asterisks are real. Consumption and inflation are to be determined endogenously.
in equilibrium.

2.2 Output, Coupons and Dividends for Individual Firms

Firms are endowed with a project of size $k = 1$ which is constant over time. Firm $i$ produces real output $Y_{i,t}$ according to the production function

$$Y_{i,t} = e^{x_t + z_{i,t}} k,$$

where $x_t$ and $z_{i,t}$ denote real aggregate and firm specific productivity shocks. The shocks follow AR(1) processes:

$$x_t = \rho_x x_{t-1} + \sigma_x \varepsilon_x^t,$$

$$z_{i,t} = \rho_z z_{i,t-1} + \sigma_z \varepsilon_{i,t}^z,$$

where $\varepsilon_x^t, \varepsilon_{i,t}^z$ are standard normal i.i.d. innovations. Firm-specific shocks are uncorrelated across firms.

Firms can issue nominal debt in the form of a consol bond that pays a fixed nominal coupon $b_i^*$ as long as the firm does not default. At $t = 0$, the real and nominal coupon are equal, $b_i^* = b_{i,0}$. The real coupon at date $t$ is

$$b_{i,t} = \frac{b_i^*}{P_t}.$$  

Following (6), since the nominal coupon is fixed the real coupon changes with inflation. To write $b_{i,t}$ in terms of stationary state variables, note that $\ln b_{i,t} - \ln b_{i,t-1} = -\pi_t$. The real coupon therefore obeys

$$b_{i,t} = b_{i,t-1} e^{-\pi_t}.$$  

In addition to the Gaussian shocks $\varepsilon_x^t, \varepsilon_{i,t}^z$, we also allow idiosyncratic technological obsolescence. For simplicity, we assume obsolescence implies the immediate death of the firm, and the loss of all future cash flows. This assumption provides additional flexibility in matching average credit spreads and default rates while maintaining reasonable leverage ratios, but will not impact the dynamics of default and their impact on output, which is the focus of our study. The probability of firm death per unit time is denoted $p_d$. After death a firm is immediately replaced by a new firm with a new draw of $z_{i,t}$. 

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Following these assumptions, after-tax nominal earnings of a firm less the coupon payment to debtholders are

\[ e_{i,t}^* = \varphi_{i,t}(1 - \eta)(P_t Y_{i,t} - b_{i,t}^*), \]  

(8)

where \( \varphi_{i,t} \) is an indicator variable for whether the firm is still alive at date \( t \) and \( \eta \) is the corporate tax rate on profits. We can rewrite (8) in real terms as

\[ e_{i,t} = \varphi_{i,t}(1 - \eta)(Y_{i,t} - b_{i,t}). \]  

(9)

All positive earnings are immediately distributed as dividends to shareholders. Negative earnings require firms to raise equity from shareholders, which is costly at rate \( \lambda \). Nominal dividends are therefore given by

\[ d_{i,t}^* = (1 + \lambda 1\{e_{i,t}^* < 0\})e_{i,t}^*. \]  

(10)

2.3 Default, Leverage and Credit Spreads

Default occurs if at any time of negative earnings, equity holders decide not to provide the new capital necessary to make payments to bondholders. This is the standard assumption of the structural approach to endogenous default introduced by Leland (1994). Equity holders decide when to default by maximizing the firm’s equity value. Nominal equity value thus solves

\[ S_{i,t}^* = \max\{0, d_{i,t}^* + \mathbb{E}_t[M_{t+1}^* S_{i,t+1}^*]\}, \]  

(11)

and equivalently real equity value \( S_i = S_{i,t}^*/P \) solves

\[ S_{i,t} = \max \left\{ 0, \frac{d_{i,t}^*}{P_t} + \mathbb{E}_t[M_{t+1} S_{i,t+1}] \right\}. \]  

(12)

An important aspect of the default decision is that equity holders are forward-looking in deciding whether to continue operations. In standard formulations of monetary economies (e.g., Gali (2008)), firms set prices myopically and hence monetary policy impacts the real economy only under nominal frictions such as sticky prices. The dependence of the real coupon on inflation combined with the forward-looking nature of the default decision implies that inflation has real consequences despite the absence of sticky prices. In our model it is capital structure which is sticky and acts as a nominal rigidity.

Bondholders receive the nominal coupon \( b_{i,t}^* \) as long as firm \( i \) does not default. In the case of default, bondholders receive a fraction \( 1 - \phi \) of the nominal after-tax value of the unlevered
firm. We denote the after-tax nominal and real values of the unlevered firm by

\[ A^*_i,t = (1 - \eta)P_i,t + \mathbb{E}_t[M_{t+1}^*A^*_{i,t+1}], \]

\[ A_{i,t} = (1 - \eta)Y_{i,t} + \mathbb{E}_t[M_{t+1}A_{i,t+1}]. \]

The real value accruing to bondholders in default is then \((1 - \phi)A_{i,t}\). The real firm value lost in default

\[ L_{i,t} = \phi A_{i,t}, \]

may be composed of transfers redistributed to the representative agent as well as deadweight losses to the economy. To capture both types of losses, define the transfer component of bankruptcy costs by \(T_{i,t} = \phi_T A_{i,t}\) and the deadweight losses by \(D_{i,t} = \phi_D A_{i,t}\). Total losses are the sum of the two components

\[ L_{i,t} = T_{i,t} + D_{i,t}, \]

which implies that \(\phi_T + \phi_D = \phi\). We assume that after the payment of bankruptcy costs, the firm is reorganized with a new draw of the idiosyncratic shock \(z_{i,t+1}\). Given the value accruing to bondholders in default, the real market value of debt is

\[ B_{i,t} = (b_{i,t} + \mathbb{E}_t[M_{t+1}B_{i,t+1}]1_{\{S_{i,t} > 0\}} + (1 - \phi)A_{i,t}1_{\{S_{i,t} = 0\}}. \]

We now outline how credit spreads are impacted by inflation. Credit spreads are defined as

\[ cs_{i,t} \equiv \frac{b^*_i}{B^*_i,t} - \frac{b^*_{i,t}}{B^*_{i,t}} = \frac{b_{i,t}}{B_{i,t}} - \frac{b_{i,t}}{B^*_{i,t}}, \]

where \(B^*_{i,t}\) is the nominal value of a default-free bond with the same nominal coupon \(b^*_i\), i.e.

\[ B^*_{i,t} = b^*_i,t + \mathbb{E}_t[M_{t+1}B^*_{i,t+1}], \]

and so

\[ B^*_{i,t} = b_{i,t} + \mathbb{E}_t[M_{t+1}B^*_{i,t+1}]. \]

Inflation impacts both the real coupon, \(b_{i,t}\), and the real pricing kernel via aggregate output. To see how inflation impacts real output, note that inflation impacts leverage, which impacts default probabilities. Deadweight bankruptcy costs ensure that default impacts output. Consequently inflation affects credit spreads through both cash flow risk and how those risks are priced using the real pricing kernel.
The optimal coupon is chosen at date 0 to maximize firm value $V_i,0$:

$$V_i,0 = \max_{b_i^*} \{ S_i,0 + B_i,0 \}. \quad (21)$$

The default and leverage decisions fully specify firm actions in our model.

### 2.4 Aggregation, the Monetary Authority and Equilibrium

In this section we describe how aggregate consumption is related to individual firm output and show how the monetary authority’s choice of interest rate rule determines inflation. We then summarize the equilibrium conditions and describe how we solve for equilibrium quantities. Finally, we use equilibrium quantities to determine the real pricing kernel and thus via no-arbitrage derive endogenous inflation explicitly for a given nominal interest rate rule.

The representative household holds all debt and equity claims. We assume throughout the paper that taxes, equity issuance costs, and the transfer component of bankruptcy costs are redistributed to the representative household and therefore have no aggregate effect. In contrast, the deadweight loss component of bankruptcy costs reduces aggregate consumption and thereby directly impacts the real pricing kernel. Additionally, interest rates and inflation may also be impacted by the presence of deadweight losses through monetary policy, which induces additional effects through corporate capital structure and default decisions.

Let $\mu_t$ denote the time-varying distribution of firms over the firm level state space $(z_{i,t}, b_{i,t})$. Real aggregate output is determined by

$$Y_t = \int Y_{i,t} \, d\mu_t - D_t. \quad (22)$$

where $D$ denotes aggregate deadweight bankruptcy costs

$$D_t = \int 1_{\{S_{i,t} = 0\}} D_{i,t} \, d\mu_t. \quad (23)$$

Market clearing implies that consumption equals output, i.e.

$$C_t = Y_t. \quad (24)$$

We now describe how monetary policy drives inflation, and the connection with optimal leverage and default. We consider monetary policy in the form of an interest rate rule that in general depends on the aggregate state variables of the economy. The aggregate state variables include the systematic productivity shock $x_t$ and the time varying distribution of
firms \( \mu_t \). We additionally permit as an aggregate state variable a stationary monetary policy shock, which we denote \( s_t \). Throughout the paper we assume that the monetary policy shock follows the dynamics

\[
    s_t = \rho_s s_{t-1} + \sigma_s \varepsilon_s^t,
\]

where \( \varepsilon_s^t \) is standard normal i.i.d. and independent of all other shocks. The aggregate state variables of the economy are then described by the vector \( \chi_t = (x_t, \mu_t, s_t) \). The interest rate rule of the monetary authority is a function of the aggregate state variables:

\[
    i_t = i(\chi_t). \tag{26}
\]

We make the simplifying assumption that the policy rule of the monetary authority is fully credible. Hence in rational expectations equilibrium investors price assets assuming that the policy \( i(\chi) \) is fixed in perpetuity. Consideration of imperfect credibility is important to many interpretations of monetary phenomena but is beyond the scope of our paper. We assume that there exists a complete set of financial markets, including a one-period nominal riskless bond. By no-arbitrage, the nominal interest rate must satisfy

\[
    i_t = -\ln(\mathbb{E}_t[e^{\pi_{t+1} + \pi_{t+1}}]). \tag{27}
\]

Consequently, inflation \( \pi_t = \pi(\chi_t) \) is determined by the aggregate state variables of the economy.

We now summarize the conditions that define equilibrium. Consumption is related to firm level output and deadweight costs by the market clearing condition (24). Deadweight costs are determined by firm-level optimal default and leverage decisions. The optimal (nominal) coupon is chosen at date 0 according to (21). The boundary condition for default can be expressed as the level of the firm-specific shock at which equity value reaches zero given the values of the aggregate and firm-level state variables:

\[
    z_{i,d}(\chi_t, b_{i,t}) = \min\{z_i : S_i(\chi_t, z_{i,t}, b_{i,t}) = 0\}, \tag{28}
\]

where \( S_{i,t} = S(\chi_t, z_{i,t}, b_{i,t}) \) is the state-dependent equity value of the firm. Default at the aggregate level impacts consumption, which itself plays a role in determining the inflation path via (27).

The equilibrium conditions are all intertwined, leading to a fixed point problem. Endogenous consumption, firm level default and coupon decisions, and endogenous inflation will
in general be functions of the aggregate state vector \( \chi_t = (x_t, \mu_t, s_t) \). The distribution of firms’ idiosyncratic shocks and real coupon levels, \( \mu_t \), is infinite dimensional. Krusell and Smith (1998), Khan and Thomas (2008) and Gomes and Schmid (2011) approximate the high-dimensional distribution of firms using different lower-dimensional linear systems. We follow a similar approach and approximate log output \( y_t = \ln Y_t \) as a linear function

\[
y_t = \nu_0 + \nu_x x_t + \nu_s s_t + \nu_f f_t,
\]

(29)

where \( f_t \) is the log default rate, which is linearly approximated by

\[
f_{t+1} = \xi_0 + \xi_x x_t + \xi_s s_t + \xi_f f_t.
\]

(30)

When default leads to deadweight losses to the economy, an increase in the default rate will lower aggregate output, and so \( \nu_f < 0 \). Also, since inflation impacts real coupon levels and hence default probabilities, the monetary policy shock will impact aggregate output, and so \( \nu_s \neq 0 \). We must also assume a specific functional form for the interest rate rule used by the monetary authority. In line with (29), we assume the log short-term nominal interest rate to be linear in inflation, output, and the log default rate \( f_t \):

\[
i_t = \tau_0 + \tau_\pi \pi_t + \tau_y y_t + \tau_f f_t + s_t,
\]

(31)

The coefficients \( \tau_0, \tau_\pi, \tau_y, \tau_f \) thus act as policy parameters. By permitting \( \tau_f \neq 0 \), we allow monetary policy to depend explicitly on credit market conditions, which departs from the standard assumption that the policy rate depends on inflation and output only.

Using (29) and the market clearing condition (24) it follows that the log real pricing kernel is given by (2) where

\[
\Delta c_{t+1} = \Delta y_{t+1} = \nu_x \Delta x_t + \nu_s \Delta s_t + \nu_f \Delta f_t.
\]

(32)

Consequently, the real pricing kernel is impacted not only by default, but also by monetary policy shocks.

The inflation process \( \pi \) must satisfy (27). If we restrict date-\( t \) inflation to be a linear function solely of \( x_t, f_t, \) and \( s_t \), then inflation is given uniquely by

\[
\pi_t = \kappa_0 + \kappa_x x_t + \kappa_f f_t + \kappa_s s_t,
\]

(33)
where

\[ \kappa_0 = \frac{1}{2} \Sigma \frac{1 - \tau_y}{\xi_0 - \nu_y}, \]

\[ \kappa_x = \frac{-\kappa_x}{\xi_x - \nu_x}, \]

\[ \kappa_f = \frac{\nu_f(1 - \xi_f) + \xi_f \tau_f + \nu_f \tau_x}{\xi_f - \tau_x}, \]

\[ \kappa_s = \frac{1 - \kappa_s}{\xi_s - \nu_s}. \]

(34) (35) (36) (37)

A proof is given in the Appendix.\(^8\)

The relation between inflation and output implies that a Phillips curve similar to the New Keynesian Phillips curve will be present in our model, despite the absence of sticky output prices. It is natural in our setting to define the output gap in period \( t \) as the difference in log output assuming no default less the log of actual output with default: \( \ln Y_t - \ln \left( \int Y_{i,t} \, di \right) = \ln \left( 1 - \frac{D_t}{Y_t} \right) \). Using this definition we can estimate the slope of the Phillips curve in our model by regressing \( \pi_t - \beta E_t[\pi_{t+1}] \) against \( \ln Y_t - \ln \left( \int Y_{i,t} \, di \right) \), using simulated data.\(^9\)

### 2.5 Monetary Policy Rules

We focus on two special cases of the monetary policy rule (31). First, we consider a fixed interest rate target, in which the Fed has a target nominal interest rate independent of inflation and output, subject to the random policy shocks \( s_t \). This leads to the rule \( i_t = \tau_0 + \ln s_t \).

The interest rate target policy includes the “Friedman rule” specification of a zero nominal target interest rate when \( \tau_0 = 0 \). The idea of a fixed nominal interest rate target also relates to Goodfriend (1987) and Goodfriend and King (2009), who study interest rate smoothing. The interest rate targeting policy is “passive” in the sense of Hetzel (2007), since the Fed does not attempt to actively adjust its policy instrument to output or inflation shocks. The process for inflation corresponding to interest rate targeting is given in the Appendix.

Intuition for the interest rate target can be gained in the special case where deadweight

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\(^8\) The Appendix also shows how to derive inflation as a function of \( y_t, f_t \), and \( s_t \).

\(^9\) The output gap will always be negative according to our definition, and will only be significant when there is a wave of defaults, as can been seen from the approximation \( \ln Y_t - \ln \left( \int Y_{i,t} \, di \right) \approx -\frac{D_t}{Y_t} \).
bankruptcy costs are zero. In this case the coefficients for inflation simplify to

\[ \kappa_0 = \ln \beta + \frac{1}{2} \Sigma + \tau_0 \]  
(38)

\[ \kappa_x = \frac{\gamma (1 - \rho_x)}{\rho_x} \]  
(39)

\[ \kappa_s = \frac{1}{\rho_s}. \]  
(40)

Following a positive shock to aggregate productivity, inflation rises. Intuitively, since productivity is mean reverting a high level of productivity implies low real interest rates. In order to keep nominal rates fixed, inflation must then be high to compensate.

The second monetary policy rule we consider is inflation targeting. A large literature investigates inflation targeting and related policies that focus on strongly anchoring inflation expectations. See, for example, Goodfriend (2004a), Goodfriend and King (2005, 2009), and Hetzel (2007). Some of these studies emphasize the benefits of anchoring inflation expectations under imperfect credibility, and the Volcker Disinflation provides one example of regaining credibility for inflation control. Our focus is different, since we do not consider the effects of imperfect credibility. Nonetheless inflation targeting has interesting implications for default rates and output in our setting. We model the inflation targeting policy as a constant target inflation rate independent of output and inflation, impacted only by the monetary policy shock:

\[ \pi_t = \kappa_0 + \kappa_s s_t. \]  
(41)

The main theoretical result of this section is that in the presence of deadweight bankruptcy costs inflation targeting cannot be implemented by a standard nominal interest rate rule of the form

\[ i_t = \tau_0 + \tau_\pi \pi_t + \tau_y y_t + s_t, \]  
(42)

which only depends on current inflation, current log output and the current monetary policy shock. To implement inflation targeting the nominal interest rate rule must, in addition, depend on the aggregate default rate, which is the state variable related to credit market conditions.

**Proposition 1**  The inflation targeting rule (41) can be implemented by a nominal interest rate rule of the form

\[ i_t = \tau_0 + \tau_\pi \pi_t + \tau_y y_t + \tau_f f_t + s_t, \]  
(43)
where

\[
\tau_\pi = -\frac{\gamma \nu_f \nu_s \xi_x}{\kappa_s \nu_x} + \frac{\gamma}{\kappa_s} [\nu_f \xi_s + \nu_s (\rho_s - \rho_x)] + \rho_s - \frac{1}{\kappa_s}, \tag{44}
\]

\[
\tau_y = -\gamma (1 - \rho_x) + \frac{\gamma \xi_x \nu_f}{\nu_x}, \tag{45}
\]

\[
\tau_f = -\gamma \nu_f \frac{1 - \xi_f}{\xi_f} - \frac{\tau_y \nu_f}{\xi_f}. \tag{46}
\]

and \( \Sigma = (\gamma \nu_x)^2 \sigma_x^2 + (\gamma \nu_s + \kappa_s)^2 \sigma_s^2 \).

In the absence of deadweight bankruptcy costs the coefficients simplify to

\[
\tau_\pi = \rho_s - \frac{1}{\kappa_s}, \tag{47}
\]

\[
\tau_y = -\gamma (1 - \rho_x), \tag{48}
\]

\[
\tau_f = 0. \tag{49}
\]

When the effect of the monetary policy shock on the interest rate target is sufficiently large, i.e. \( \kappa_s > 1/\rho_s \), then the nominal interest rate increases when inflation is high.

3 Model Calibration and Implications

We calibrate the economy under both interest rate and inflation targeting to compare the impacts of these policies on default rates, credit spreads and output.

3.1 Calibration

Our calibration is summarized in Table 1. We set the annualized time discount factor equal to around 0.96 (0.99 in quarterly units), which is within the range commonly chosen in the literature (for example, 0.99 in Abel (1999), 0.93 in Abel (1990), 0.89 in Campbell and Cochrane (1999) and 0.998 in Bansal and Yaron (2004)). We choose a coefficient of relative risk aversion of 10. Risk aversion is usually chosen to be in the range 3-10 (see, for example Mehra and Prescott (1985) who argue that relative risk aversion is less than or equal to 10 and Bansal and Yaron (2004) who set relative risk aversion equal to 10). Project size is set to one, without loss of generality. The corporate tax rate is 10% per annum (close to the mean tax rate on equity income of 12% estimated in Graham (2000)). Equity issuance costs are equal to 5 %, slightly lower than 8.3 %, as estimated by Hennessy and Whited (2007) for the sample of Compustat firms, and close to the estimate of 5.14% in Altinkilic and Hansen.
We set total bankruptcy costs equal to 20% (see, for example Andrade and Kaplan (1998) who report default costs of about 10–25% of asset value and Hennessy and Whited (2007) who estimate bankruptcy costs to be 10%). Deadweight bankruptcy costs are equal to 0.5%, which is a fraction of asset value, and is equivalent to a loss of about 25% of firm output for one year.

Our estimates for the volatility and persistence of aggregate shocks are in line for those based on the Solow residual in Cooley and Prescott (1995). Standard practice is to apply the HP filter to real GDP data from the Bureau of Economic Analysis, and to concentrate on the detrended data. We choose the persistence and volatility of shocks to $x$ (0.9, 0.75) accordingly. Monetary policy shocks are persistent with small volatility as in Gallmeyer et al. (2008). In all cases, we assume exogenous stochastic obsolescence at the rate of 10 basis points per quarter.

### 3.2 Equilibrium Laws of Motion and Annual Moments

Table 2 describes the equilibrium laws of motion for interest rates, inflation, output, and aggregate default rates under interest rate targeting and inflation targeting. The constant in the interest rate target policy is set to $\tau_0 = 0.0063$, in order to obtain average inflation of approximately zero, which aids comparison across the different policies. Otherwise the other coefficients are all zero: $\tau_\pi = \tau_y = \tau_f = 0$. The policy thus approximates the Friedman rule objective of low and stable interest rates. The interest rate target policy is ‘passive’ in the sense that the monetary authority does not adjust its policy instrument in response to output and inflation shocks, or the threat of rising aggregate default. The policy thus provides a coarse representation of monetary policy during the Great Depression, during which strong deflationary pressures existed (see Figure 3, with sustained inflation rates of -10%), yet interest rates were brought down slowly, not reaching a level of 1% until 1934. In our calibration the monetary authority does not seek to aggressively fight deflation, and as a consequence deflation is strong after negative shocks to aggregate productivity ($\kappa_x \approx 1.1 > 0$). The average inflation rate is approximately zero $\kappa_0 \approx 0$, and increases in the aggregate default rate cause deflation ($\kappa_f \approx -0.52 < 0$) due to the impact on aggregate consumption. Examining output dynamics, we observe that output increases almost one-for-one with shocks to aggregate productivity as expected ($\nu_x \approx 1$), while shocks to the default rate decrease output due to deadweight
bankruptcy costs \((\nu_f \approx -0.25)\). The default rate under interest rate targeting is persistent \((\xi_f \approx 0.83)\) and increases after a negative shock to aggregate productivity \((\xi_x < 0)\).

The equilibrium laws of motion show important differences under inflation targeting. This policy aims to achieve low and stable inflation \((\pi_0 = \pi_x = \pi_f = 0)\). The policy thus approximates the prescription of Fisher (1911, 1923) that monetary policy should focus on maintaining low and stable inflation.\(^{10}\) Recent discussions of implicit and explicit inflation targeting include Bernanke (2003), Goodfriend (2004a), McCallum (2007), and Hetzel (2007).\(^{11}\) Inflation targeting implies that nominal interest rates should fluctuate with drivers of the real rate of interest. The interest rate coefficient on output \(\tau_y\) is negative due to mean reversion in aggregate productivity, since high current output implies low output growth and a low real interest rate. The interest rate loading on aggregate default \(\tau_f\) is positive since a high current default rate implies low current consumption and therefore high expected consumption growth and a high real interest rate. The output law of motion is driven by both aggregate productivity \((\nu_x \approx 1)\) and default \((\nu_f \approx -0.35)\). The default rate dynamics are persistent, but less so than under interest rate targeting \((\xi_f \approx 0.76)\).

Table 3 shows moments of the data under interest rate and inflation targeting policies. Each moment is calculated by simulating 1000 samples each consisting of 500 firms, for 100 years. The full set of moments is calculated for each simulation and the table reports the average, 10th, and 90th percentiles of each statistic across the 1000 independent samples. The table shows that the consumption growth volatility is comparable across both policies, with an average of about 1.6% annually, and small variations across samples. The average default rates are also similar under both policies. The average default rates across all simulations is approximately 0.8% annually. The 10th percentile of the average default rate is about 0.4% annually and the 90th percentile of the average default rate reaches 1.5% annually in a 100 year sample. Variability in average default rates even across 100 year samples indicates the presence of considerable persistence and large outliers. Importantly, the default rate volatility is larger under the policy of passive interest rate targeting, as is the maximum default rate.

\(^{10}\)See Khan et al. (2003) for discussion and analysis of the different objectives of price, output, and interest rate stabilization motivated by the prescriptions of Fisher, Keynes, and Friedman.

\(^{11}\)One commonly cited benefit of explicit inflation targeting is to improve central bank credibility and thus aid in the management of long-run inflation expectations. Goodfriend and King (2005) provide an analysis of central bank credibility focusing on the Volcker disinflation, and Goodfriend and King (2009) discuss the inflation drift of the 1970’s in terms of oscillating central bank objectives between output stabilization and inflation fighting.
This occurs because default waves are stronger under passive interest rate targeting than under inflation targeting. Hence while average default rates are similar, defaults are more correlated and countercyclical, and default rates are correspondingly more volatile, under the interest rate targeting policy.

Average credit spreads are also noticeably larger under interest rate targeting, even though the average default rates are very similar. The result can be explained by the fact that defaults tend to be concentrated in bad times moreso under interest rate targeting than under inflation targeting. Bondholders thus demand higher compensation for default risk under the interest rate target policy. The average leverage level is much higher under inflation targeting than under interest rate targeting. Firms perceive debt as less attractive under interest rate targeting because defaults and deadweight losses tend to occur in bad times, and also because inflation volatility is much larger. By contrast, the volatility of nominal riskless rates is much smaller under the interest rate target, as expected.

Finally, both policies result in positive Phillips curve coefficients, and the slope is larger under the interest rate target policy. In our setting the output gap is naturally defined as the difference between actual output and output assuming no default. Hence the output gap is directly related to aggregate default. The larger slope for the interest rate target policy is due to the fact that defaults are more strongly concentrated in deflationary, low-output times, whereas defaults are more dispersed under the inflation target policy.

As a consequence of concentrated waves of default and high default rate volatility, the unconditional distribution of default rates has a thicker right tail and is more positively skewed under the interest rate target policy than under inflation targeting. The unconditional distributions of default rates under both policies is shown in Figure 4, based on simulating 5000 firms for 4000 years. The skewness coefficient of the unconditional distribution of default rates is approximately 10 for the interest rate target policy, but is only 4 for the inflation targeting policy. Hence, the interest rate target policy by permitting procyclical inflation produces stronger clustering in default rates than inflation targeting.

### 3.3 Deadweight Bankruptcy Costs and Amplification

Under interest rate targeting, the presence of deadweight bankruptcy costs in our model generates additional amplification effects in output, deflation, and default rates. Consider
an initial shock to aggregate productivity $x_t$. As discussed in the previous subsection, under interest rate targeting this shock will be accompanied by deflation, and the real output and deflation channels will jointly contribute to an increase in the default rate. In the absence of deadweight bankruptcy costs there would be no impact on aggregate output, and the shock would immediately begin to dissipate.

Now consider the additional effects of deadweight bankruptcy costs in default. Due to the initial wave of default caused by the shock to $x_t$, we will have the direct effect of an immediate shock to output associated with the initial default wave. We focus on the additional amplification effects that follow. Specifically, the drop in output due to default will cause a drop in expected inflation, and potentially deflation, since $\kappa_f < 0$. The additional anticipated deflation further increases incentives for default, which leads to additional declines in output, and so on. This cycle captures well some important aspects of the debt deflation theory of Fisher (1933). We note that prior studies have also developed aspects of the debt deflation theory (e.g., Bernanke (1983); Bernanke and Gertler (1989, 1990); Kiyotaki and Moore (1997); Bernanke et al. (1999)). These typically work through the financial accelerator channel described by Bernanke et al. (1996), in which an initial shock impacts the balance sheets of borrowers in an economy with heterogeneous agents, reducing the ability to finance investment. The channel we focus on is different, since it operates in a representative agent economy and is a purely monetary phenomenon having to do with how an initial shock is propagated via the impact of the price channel on default rates and output.

To observe the amplification effect of deadweight bankruptcy costs, we show in Figure 5 Panel A the impulse response functions of default, inflation, and output, following an initial shock to aggregate productivity $x_t$. The dashed line shows the impulse response functions when there are no deadweight bankruptcy costs, and the solid line shows the functions in the presence of deadweight bankruptcy costs. We first note that, following the initial shock to output, the subsequent spikes in default rates and deflation are much larger in the presence of deadweight bankruptcy costs. These effects are persistent, resulting in a sustained lower level of output in the deadweight bankruptcy costs scenario following the initial shock.

Panel B of Figure 5 shows that these amplification effects of deadweight bankruptcy costs are eliminated under the inflation targeting policy. Although the initial shock does produce an initial drop in output, this is not exacerbated by deflation and the initial spike in default rates
is not as large. Moreover, while presence of deadweight bankruptcy costs does imply lower output, the drop in output does not produce additional deflation, and hence the propagation of the initial productivity shock does not continue through the inflation channel. Hence, defaults in this scenario are driven by the initial productivity shock and the exogenous persistence in the productivity shock, but are not amplified by deflation and the cycle of reduced output generating further deflation. The monetary authority by holding inflation constant eliminates the debt deflation cycle under this policy.

4 Conclusion

Monetary policy impacts corporate default through its influence on inflation and inflation expectations. Passive monetary policy – as some would argue occurred during the Great Depression – generates procyclical inflation. Adverse real shocks thus generate strong deflationary pressures, compounding the incentives of corporations to default and thereby generating a potentially strong amplification mechanism. Inflation targeting or other policies that seek to eliminate procyclical inflation can dampen this amplification mechanism, reducing default rates and credit spreads and stabilizing output.

Recent policy discussions and literature have placed considerable emphasis on incorporating credit market conditions into policy rules (e.g., Taylor (2008), Curdia and Woodford (2010)). Our framework naturally suggests that credit conditions should be a part of monetary policy rules, whether through default rates as in our model, or through alternative indicators such as credit spreads.

We see many directions for further research. First, it would be useful to incorporate nominal rigidities in prices or wages along with the sticky debt channel we have focused on to allow comparison with typical New Keynesian models. Second, our model assumes perfect credibility and commitment of the monetary authority to a fixed policy over time, and weakening this assumption may shed additional light on corporate default decisions. Credibility and commitment are clearly important policy considerations in light of recent discussions over moral hazard in corporate decision-making. Finally, the model we have considered assumes fixed-rate perpetual debt, but other tradeoffs may naturally appear when corporations issue shorter-maturity, floating-rate obligations.
References


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The figure shows the percentage annual global default rate for speculative grade debt from 1920 until 2009, using data from Moody’s (see Exhibit 5 in Emery et al. (2009)).
The figure shows the spread (in annualized percentage units) between Baa and Aaa Moody’s rated debt from 1919 until 2010.
Figure 3: Nominal Interest Rates, Inflation and GDP, 1929-1934 v 2006-2009

The upper panels show the nominal risk-free rate in units of percent per annum (solid line), together with real GDP (with real GDP in 1929 normalized to 1) and CPI inflation (in percent per annum) during the period 1929-1934. The lower panels show the same quantities (with real GDP in 2006 normalized to 1). Data on the nominal risk-free rate and CPI inflation is from Robert Shiller’s website and real GDP data is from the Bureau of Economic Analysis.
This figure shows the histogram of annual default rates in percent based on simulated data of the model. The left panel displays the outcome of interest rate targeting policy and the right panel of inflation targeting policy. The distribution of default rates is more volatile and more positively skewed under interest rate targeting than under inflation targeting.
This figure shows the impulse responses of annual default rates, inflation, and output when the economy is subjected to a negative productivity shock. The solid line depicts results under our base calibration of deadweight losses in default of 0.5% of unlevered firm value. The dashed line sets deadweight losses to zero. Panel A displays results for interest rate targeting and Panel B for inflation targeting.
Table 1: Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time discount rate, $\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>Risk aversion, $\gamma$</td>
<td>10</td>
</tr>
<tr>
<td>Project size, $k$</td>
<td>1</td>
</tr>
<tr>
<td>Corporate tax rate, $\eta$</td>
<td>0.1</td>
</tr>
<tr>
<td>Equity issuance costs, $\lambda$</td>
<td>0.05</td>
</tr>
<tr>
<td>Total bankruptcy costs, $\phi$</td>
<td>0.20</td>
</tr>
<tr>
<td>Deadweight bankruptcy costs, $\phi_D$</td>
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</tr>
<tr>
<td>Idiosyncratic shock volatility, $\sigma_z$</td>
<td>0.15</td>
</tr>
<tr>
<td>Idiosyncratic shock persistence, $\rho_z$</td>
<td>0.95</td>
</tr>
<tr>
<td>Aggregate shock volatility, $\sigma_x$</td>
<td>0.007</td>
</tr>
<tr>
<td>Aggregate shock persistence, $\rho_x$</td>
<td>0.90</td>
</tr>
<tr>
<td>Monetary policy shock volatility, $\sigma_s$</td>
<td>0.005</td>
</tr>
<tr>
<td>Monetary policy shock persistence, $\rho_s$</td>
<td>0.90</td>
</tr>
</tbody>
</table>

This table summarizes our calibration at quarterly frequency.
Table 2: Equilibrium Law of Motions

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Interest Rate Targeting</th>
<th>Inflation Targeting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Interest rate constant</td>
<td>$\tau_0$</td>
<td>0.0063</td>
</tr>
<tr>
<td>Interest rate sensitivity w.r.t. inflation</td>
<td>$\tau_{\pi}$</td>
<td>0</td>
</tr>
<tr>
<td>Interest rate sensitivity w.r.t. output</td>
<td>$\tau_y$</td>
<td>0</td>
</tr>
<tr>
<td>Interest rate sensitivity w.r.t. $f$</td>
<td>$\tau_f$</td>
<td>0</td>
</tr>
<tr>
<td>Inflation rate constant</td>
<td>$\kappa_0$</td>
<td>0.0003</td>
</tr>
<tr>
<td>Inflation sensitivity w.r.t. $x$</td>
<td>$\kappa_x$</td>
<td>1.1013</td>
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<tr>
<td>Inflation sensitivity w.r.t. $s$</td>
<td>$\kappa_s$</td>
<td>1.0998</td>
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<tr>
<td>Inflation sensitivity w.r.t. $f$</td>
<td>$\kappa_f$</td>
<td>-0.5215</td>
</tr>
<tr>
<td>Output rule constant</td>
<td>$\nu_0$</td>
<td>0.1149</td>
</tr>
<tr>
<td>Output sensitivity w.r.t. $x$</td>
<td>$\nu_x$</td>
<td>1.0011</td>
</tr>
<tr>
<td>Output sensitivity w.r.t. $s$</td>
<td>$\nu_s$</td>
<td>0.0027</td>
</tr>
<tr>
<td>Output sensitivity w.r.t. $f$</td>
<td>$\nu_f$</td>
<td>-0.2528</td>
</tr>
<tr>
<td>Default rate rule constant</td>
<td>$\xi_0$</td>
<td>0.0001</td>
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<tr>
<td>Default rate sensitivity w.r.t. $x$</td>
<td>$\xi_x$</td>
<td>-0.0034</td>
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<tr>
<td>Default rate sensitivity w.r.t. $s$</td>
<td>$\xi_s$</td>
<td>-0.0043</td>
</tr>
<tr>
<td>Default rate sensitivity w.r.t. lagged $f$</td>
<td>$\xi_f$</td>
<td>0.8278</td>
</tr>
</tbody>
</table>

This table describes the equilibrium laws of motion for interest rates, inflation, output and default rates under interest rate targeting and inflation targeting. In the table, $f$ denotes the aggregate default rate, $x$ the aggregate productivity shock and $s$ the monetary policy shock.
We simulate 1000 economies of 5000 firms for 100 years under the interest rate targeting policy and inflation targeting policy. For each policy, we report the mean, the 10th percentile, and the 90th percentile for each moment listed.

Table 3: Annual Moments

<table>
<thead>
<tr>
<th>Moment</th>
<th>Interest Rate Targeting</th>
<th>Inflation Targeting</th>
</tr>
</thead>
<tbody>
<tr>
<td>Consumption growth volatility</td>
<td>1.58</td>
<td>1.56</td>
</tr>
<tr>
<td>10-% percentile</td>
<td>1.44</td>
<td>1.43</td>
</tr>
<tr>
<td>90-% percentile</td>
<td>1.73</td>
<td>1.70</td>
</tr>
<tr>
<td>Average default rate (%)</td>
<td>0.77</td>
<td>0.79</td>
</tr>
<tr>
<td>10-% percentile</td>
<td>0.39</td>
<td>0.40</td>
</tr>
<tr>
<td>90-% percentile</td>
<td>1.51</td>
<td>1.46</td>
</tr>
<tr>
<td>Default rate volatility (%)</td>
<td>1.49</td>
<td>0.97</td>
</tr>
<tr>
<td>10-% percentile</td>
<td>0.09</td>
<td>0.09</td>
</tr>
<tr>
<td>90-% percentile</td>
<td>4.94</td>
<td>2.59</td>
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<tr>
<td>Maximum default rate (%)</td>
<td>12.17</td>
<td>6.03</td>
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<tr>
<td>10-% percentile</td>
<td>0.62</td>
<td>0.62</td>
</tr>
<tr>
<td>90-% percentile</td>
<td>43.29</td>
<td>15.25</td>
</tr>
<tr>
<td>Average credit spread (b.p.)</td>
<td>86.83</td>
<td>77.19</td>
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<tr>
<td>10-% percentile</td>
<td>60.34</td>
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</tr>
<tr>
<td>90-% percentile</td>
<td>121.27</td>
<td>103.69</td>
</tr>
<tr>
<td>Credit spread volatility (b.p.)</td>
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<tr>
<td>10-% percentile</td>
<td>4.12</td>
<td>5.04</td>
</tr>
<tr>
<td>90-% percentile</td>
<td>26.73</td>
<td>20.27</td>
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<tr>
<td>Average market leverage (%)</td>
<td>28.90</td>
<td>51.22</td>
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<tr>
<td>10-% percentile</td>
<td>12.60</td>
<td>33.87</td>
</tr>
<tr>
<td>90-% percentile</td>
<td>46.64</td>
<td>65.42</td>
</tr>
<tr>
<td>Average nominal risk-free rate (%)</td>
<td>2.54</td>
<td>2.85</td>
</tr>
<tr>
<td>10-% percentile</td>
<td>1.24</td>
<td>0.72</td>
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<tr>
<td>90-% percentile</td>
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<td>4.98</td>
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<td>2.21</td>
<td>3.99</td>
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<tr>
<td>10-% percentile</td>
<td>1.95</td>
<td>3.50</td>
</tr>
<tr>
<td>90-% percentile</td>
<td>2.49</td>
<td>4.50</td>
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<td>4.60</td>
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<tr>
<td>10-% percentile</td>
<td>3.92</td>
<td>1.94</td>
</tr>
<tr>
<td>90-% percentile</td>
<td>5.34</td>
<td>2.50</td>
</tr>
</tbody>
</table>
Appendix

Derivation of the real risk-free rate and market price of risk

The real risk-free rate is given by
\[ e^{-r_t} = E_t[e^{m_{t+1}}]. \]
If \( m_{t+1} \) is linear in the aggregate state variables and the underlying shocks are Gaussian, then
\[ r_t = -E_t[m_{t+1}] - \frac{1}{2} Var_t[m_{t+1}]. \]
From (1) and (32), it follows that the real log pricing kernel is
\[ m_{t+1} = -\delta - \gamma [\nu_x(x_{t+1} - x_t) + \nu_s(s_{t+1} - s_t) + \nu_f(f_{t+1} - f_t)]. \tag{A1} \]
Hence
\[ E_t[m_{t+1}] = -\delta - \gamma [\nu_x(\rho_x - 1)x_t + \nu_s(\rho_s - 1) + \nu_f(f_{t+1} - f_t)], \]
and
\[ m_{t+1} - E_t[m_{t+1}] = -\gamma [\nu_x(x_{t+1} - \rho_x x_t) + \nu_s(s_{t+1} - \rho_s s_t)] = -\gamma (\nu_x \sigma_x \xi_{t+1}^x + \nu_s \sigma_s \xi_{t+1}^s), \]
which implies that the date-\( t \) market price of risk is given by
\[ Var_t[m_{t+1}] = \gamma^2[(\nu_x \sigma_x)^2 + (\nu_s \sigma_s)^2], \]
and the real risk free rate is given by
\[ r_t = \delta - \gamma [\nu_x(1 - \rho_x) x_t + \nu_s(1 - \rho_s) s_t - \nu_f(f_{t+1} - f_t)] - \frac{1}{2}\gamma^2[(\nu_x \sigma_x)^2 + (\nu_s \sigma_s)^2]. \]

Derivation of (38) – (40)

From (33) and (A1), it follows that the nominal log pricing kernel is
\[ m_{t+1}^* = -\delta - \gamma [\nu_x(x_{t+1} - x_t) + \nu_s(s_{t+1} - s_t) + \nu_f(f_{t+1} - f_t)] - [\kappa_0 + \kappa_x x_{t+1} + \kappa_f f_{t+1} + \kappa_s s_{t+1}]
= -(\delta + \kappa_0) - (\gamma \nu_f + \kappa_f) \xi_0 - (\gamma \nu_x + \kappa_x) x_{t+1} - (\gamma \nu_s + \kappa_s) s_{t+1}
+ (\gamma \nu_x - (\gamma \nu_f + \kappa_f) \xi_x) x_t + (\gamma \nu_s - (\gamma \nu_f + \kappa_f) \xi_s) s_t + (\gamma \nu_f - (\gamma \nu_f + \kappa_f) \xi_f) f_t. \]
Since \( m_{t+1}^* \) is linear in the aggregate state variables and the underlying shocks are Gaussian, we have
\[ i_t = -E_t[m_{t+1}^*] - \frac{1}{2} Var_t[m_{t+1}^*]. \tag{A2} \]
Now

\[ \mathbb{E}_t [m_{t+1}^s] = m_0 + m_x x_t + m_s s_t + m_f f_t, \]

where

\[
\begin{align*}
m_0 &= -\delta - \kappa_0 - (\gamma \nu_f + \kappa_f) \xi_0 \\
m_x &= \gamma \nu_x - (\gamma \nu_f + \kappa_f) \xi_x - (\gamma \nu_x + \kappa_x) \rho_x \\
m_s &= \gamma \nu_s - (\gamma \nu_f + \kappa_f) \xi_s - (\gamma \nu_s + \kappa_s) \rho_s \\
m_f &= \gamma \nu_f - (\gamma \nu_f + \kappa_f) \xi_f,
\end{align*}
\]

and the date-\( t \) nominal market price of risk is given by

\[
\text{Var}_t (m_{t+1}^s) = (\gamma \nu_x + \kappa_x)^2 \sigma_x^2 + (\gamma \nu_s + \kappa_s)^2 \sigma_s^2 = \Sigma.
\]

Hence

\[ i_t = -(m_0 + \frac{1}{2} \Sigma) - m_x x_t - m_s s_t - m_f f_t. \]

The monetary authority follows an interest rate rule of the form (31). Hence

\[
\begin{align*}
i_t &= \tau_0 + \tau_x \nu_t + \tau_y y_t + \tau_f f_t + s_t \\
&= \tau_0 + \tau_x (\kappa_0 + \kappa_x x_t + \kappa_s s_t + \kappa_f \ln f_t) + \tau_y (\nu_0 + \nu_x x_t + \nu_s s_t + \nu_f f_t) + s_t \\
&= \tau_0 + \tau_x \nu_0 + \tau_y \nu_0 + (\tau_x \kappa_0 + \tau_y \nu_0) x_t + (\tau_x \kappa_0 + \tau_y \nu_0 + 1) s_t + (\tau_x \kappa_f + \tau_y \nu_f + \tau_f) f_t,
\end{align*}
\]

and so

\[
-(m_0 + \frac{1}{2} \Sigma) - m_x x_t - m_s s_t - m_f f_t = \tau_0 + \tau_x \nu_0 + \tau_y \nu_0 + (\tau_x \kappa_0 + \tau_y \nu_0) x_t \\
+ (\tau_x \kappa_0 + \tau_y \nu_0 + 1) s_t + (\tau_x \kappa_f + \tau_y \nu_f + \tau_f) f_t.
\]

Equating coefficients gives

\[
\begin{align*}
-(-\delta - \kappa_0 - (\gamma \nu_f + \kappa_f) \xi_0 + \frac{1}{2} \Sigma) &= \tau_0 + \tau_x \kappa_0 + \tau_y \nu_0 & (A3) \\
-(\gamma \nu_x - (\gamma \nu_f + \kappa_f) \xi_x - (\gamma \nu_x + \kappa_x) \rho_x) &= \tau_x \kappa_0 + \tau_y \nu_x & (A4) \\
-(\gamma \nu_s - (\gamma \nu_f + \kappa_f) \xi_s - (\gamma \nu_s + \kappa_s) \rho_s) &= \tau_x \kappa_0 + \tau_y \nu_s + 1 & (A5) \\
-(\gamma \nu_f - (\gamma \nu_f + \kappa_f) \xi_f) &= \tau_x \kappa_f + \tau_y \nu_f + \tau_f & (A6)
\end{align*}
\]

Solving the above equation system gives the unique solution (38) – (40).
We now derive inflation given the nominal interest rate rule
\[ i_t = \tau_0 + \tau_y y_t + \tau_f f_t + \tau_s s_t. \]  
(A7)

Suppose that
\[ \pi_t = \kappa_0 + \kappa_y y_t + \kappa_f f_t + \kappa_s s_t. \]

Therefore
\[ \pi_t = \kappa_0 + \kappa_y (\nu_0 + \nu_x x_t + \nu_s s_t + \nu_f f_t) + \kappa_f f_t + \kappa_s s_t \]
\[ = \kappa_0 + \kappa_y \nu_0 + \kappa_y \nu_x x_t + (\kappa_y \nu_f + \kappa_f) f_t + (\kappa_y \nu_s + \kappa_s) s_t. \]

Hence
\[ m_{t+1} = -\delta - \gamma [\nu_x (x_{t+1} - x_t) + \nu_s (s_{t+1} - s_t) + \nu_f (f_{t+1} - f_t)] \]
\[ -[\kappa_0 + \kappa_y \nu_0 + \kappa_y \nu_x x_{t+1} + (\kappa_y \nu_f + \kappa_f) f_{t+1} + (\kappa_y \nu_s + \kappa_s) s_{t+1}] \]
\[ = -[\delta + \kappa_0 + \kappa_y \nu_0] - (\gamma + \kappa_y) \nu_x x_{t+1} - (\gamma \nu_s + \kappa_y \nu_s + \kappa_s) s_{t+1} - [\gamma \nu_f + (\kappa_y \nu_f + \kappa_f)] f_{t+1} \]
\[ + \gamma \nu_x x_t + \gamma \nu_s s_t + \gamma \nu_f f_t \]
\[ = -[\delta + \kappa_0 + \kappa_y \nu_0 + ((\gamma + \kappa_y) \nu_f + \kappa_f) \xi_0] - (\gamma + \kappa_y) \nu_x x_{t+1} - ((\gamma + \kappa_y) \nu_s + \kappa_s) s_{t+1} \]
\[ + [\gamma \nu_x - ((\gamma + \kappa_y) \nu_f + \kappa_f) \xi_x] x_t + [\gamma \nu_s - ((\gamma + \kappa_y) \nu_f + \kappa_f) \xi_s] s_t \]
\[ + [\gamma \nu_f - ((\gamma + \kappa_y) \nu_f + \kappa_f) \xi_f] f_t. \]

Using the above expression is it straightforward to compute \( E_t[m_{t+1}^*] \) and \( Var_t[m_{t+1}^*] \), and we substitute these expressions, together with (A7) and (4) into (A2). Equating coefficients gives
\[ \tau_x (\kappa_0 + \kappa_y \nu_0) + \nu_0 \tau_y + \tau_0 = -\frac{\Sigma}{2} + \xi_0 (\gamma \nu_f + \kappa_f + \kappa_y \nu_f) + \delta + \kappa_0 + \kappa_y \nu_0 \]
\[ (\kappa_y \tau_x + \tau_0) \nu_x = \xi_x ((\gamma + \kappa_y) \nu_f + \kappa_f) + \rho_x \nu_x (\gamma + \kappa_y) - \gamma \nu_x \]
\[ \kappa_s \tau_x + \kappa_y \nu_s \tau_x + \nu_x \tau_y + 1 = \xi_s (\gamma \nu_f + \kappa_f + \kappa_y \nu_f) + \rho_s (\gamma \nu_s + \kappa_s + \kappa_y \nu_s) - \gamma \nu_s \]
\[ \kappa_f \tau_x + \kappa_y \nu_f T_x + \nu_f \tau_y + \tau_f = \xi_f (\gamma \nu_f + \kappa_f + \kappa_y \nu_f) - \gamma \nu_f \]

The above equation system has a unique solution for \( \kappa_0, \kappa_y, \kappa_f \) and \( \kappa_s \).

**Proof of Proposition 1**

If inflation is given by (41), then setting \( \kappa_x = \kappa_f = 0 \) in (A3) – (A6) and solving gives the unique solution (44) – (46). If the interest rate rule is independent of \( f \), i.e. \( \tau_f = 0 \), then
we obtain two distinct expressions for $\tau_y$. Thus, by reductio ad absurdum $\tau_f \neq 0$. Hence, to implement inflation targeting, the interest rate rule must depend on $f$.

**Inflation under interest rate targeting**

By setting $\pi = \tau_y = \tau_f = 0$ in (38) – (40), we see that inflation under interest rate targeting is given by

$$\pi_t = \kappa_0 + \kappa_x x_t + \kappa_f f_t + \kappa_s s_t,$$

where

$$\begin{align*}
\kappa_0 &= \frac{1}{2} \Sigma - (\delta + \kappa_f \xi_0 + \gamma \nu_f \xi_0 - \tau_0), \\
\kappa_x &= -\kappa_f \xi_0 + \gamma (1 - \rho_s) \nu_s - \nu_f \xi_x, \\
\kappa_f &= \frac{\nu_f (1 - \xi_f)}{\xi_f}, \\
\kappa_s &= \frac{1 - \kappa_f \xi_s + \gamma (1 - \rho_s) \nu_s - \nu_f \xi_s}{\rho_s}.
\end{align*}$$

**Interest rate rule under perfect inflation rate targeting, when $\kappa_s = 0$**

If inflation targeting as defined by (41) is perfect, then $\kappa_s = 0$ and inflation is constant, i.e. $\pi_t = \kappa_0$. Setting $\kappa_x = \kappa_f = \kappa_s = 0$ in (A3) – (A6) gives

$$\begin{align*}
(-\delta - \kappa_0 - (\gamma \nu_f) \xi_0 + \frac{1}{2} \Sigma) &= \tau_0 + \tau_x \kappa_0 + \tau_y \nu_0, \\
-(\gamma \nu_x - (\gamma \nu_f) \xi_x - (\gamma \nu_x) \rho_x) &= \tau_x \nu_x, \\
-(\gamma \nu_s - (\gamma \nu_f) \xi_s - (\gamma \nu_s) \rho_s) &= \tau_y \nu_s + 1, \\
-(\gamma \nu_f - (\gamma \nu_f) \xi_f) &= \tau_x \nu_f + \tau_f.
\end{align*}$$

Solving the second and third equations in the above system gives two distinct expressions for $\tau_y$. These expressions are equal if and only if

$$\nu_s = \frac{\nu_s}{\gamma \nu_x + \nu_s (\rho_s - \rho_x)}.$$

If the above equation is not satisfied, it follows from reductio ad absurdum that the monetary policy shock cannot impact any variables in the approximately solved economy. Hence,

$$\begin{align*}
y_t &= \nu_0 + \nu_s x_t + \nu_f f_t, \\
f_{t+1} &= \xi_0 + \xi_x x_t + \xi_f f_t, \\
i_t &= \tau_0 + \tau_y y_t + \tau_f f_t,
\end{align*}$$

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and

\[-(\delta - \kappa_0 - (\gamma \nu_f) \xi_0 + \frac{1}{2} \Sigma) = \tau_0 + \tau_\pi \kappa_0 + \tau_y \nu_0 \]
\[-(\gamma \nu_x - (\gamma \nu_f) \xi_x - (\gamma \nu_x) \rho_x) = \tau_y \nu_x \]
\[-(\gamma \nu_f - (\gamma \nu_f) \xi_f) = \tau_y \nu_f + \tau_f.\]

Solving the above equation system gives the unique solution

\[
\tau_0 = -\delta - \kappa_0 - \gamma \nu_f \xi_0 - \frac{1}{2} \Sigma, \\
\tau_y = -\gamma (1 - \rho_x) + \gamma \frac{\nu_f}{\nu_x} \xi_x, \\
\tau_f = -\gamma \frac{1 - \xi_f}{\xi_f} \nu_f - \tau_y \frac{\nu_f}{\xi_f},
\]

where \( \Sigma = (\gamma \nu_x)^2 \sigma_x^2 \). The above rule is unique within the class of rules which are log-linear and depend only on current values of \( y \) and \( f \).

Note also that (A2) implies that the nominal interest rate is given by

\[
i_t = -E_t [m_{t+1}] + \kappa_0 - \frac{1}{2} \text{Var}_t [m_{t+1}] = \kappa_0 + r_t.
\]